General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

STANFORD UNIVERSITY

CENTER FOR SYSTEMS RESEARCH

Final Report On

INVESTIGATION OF HALO SATELLITE ORBIT CONTROL

Principal Investigator

Professor John V. Breakwell

Guidance and Control Laboratory

(NASA-CR-132858) INVESTIGATION OF HALO SATELLITE ORBIT CONTROL Final Report (Stanford Univ.) 8 p HC $3.00 CSCL 22C

N74-17537

Unclas

G3/30 29639

Final Report On

INVESTIGATION OF HALO SATELLITE ORBIT CONTROL

Principal Investigator

Professor John V. Breakwell
Department Aeronautics and Astronautics
Guidance & Control Laboratory
STANFORD UNIVERSITY
Stanford, California 94305

Supported at Stanford University by

NASA GODDARD SPACE FLIGHT CENTER
Grant NGR 05-020-507

Dr. Robert Farquhar
Technical Monitor

October 1973

Final Report On

INVESTIGATION OF HALO SATELLITE ORBIT CONTROL

It is desired to maintain the communication station in a "Halo" orbit near the translunar libration point L_2 of the Earth-Moon system. Such a Halo orbit is to be "visible" at all times from Earth, and (except for perturbations due to solar gravity and mean Earth-Moon orbital eccentricity) is to be a possible periodic orbit for an unpowered vehicle. A one-parameter family of such periodic orbits, which are all unstable, has previously been computed analytically by a truncated Linstedt method [Ref. 1], and is shown in Fig. 1.

The accuracy of the truncated analytic description of a typical "nominal" Halo orbit is limited, leading to an acceleration error averaging about 10^{-6} g's, this being therefore the "cost" of a very tight control to the nominal path.

Since a looser control would be quite satisfactory, the station-keeping problem is posed in the following way:

$$\text{given} \quad \dot{x} = \underbrace{F(t)}_{\text{PERIODIC}}x + \underbrace{\begin{pmatrix} 0 \\ \vec{e}(t) \end{pmatrix}}_{\text{PERIODIC}} + \begin{pmatrix} 0 \\ \vec{u} \end{pmatrix},$$

where

$$\underset{6\times 1}{x} = \begin{pmatrix} \vec{\delta r} \\ \dot{\vec{\delta r}} \end{pmatrix},$$

$\vec{\delta r}$ is the deviation from the nominal path, $\vec{e}(t)$ is the known acceleration error along the nominal path, and $\vec{u}$ is the control acceleration; find $\vec{u}(t)$ to minimize

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \left\{ |\vec{u}(t)|^2 + k|\vec{\delta r}(t)|^2 \right\} dt ,$$

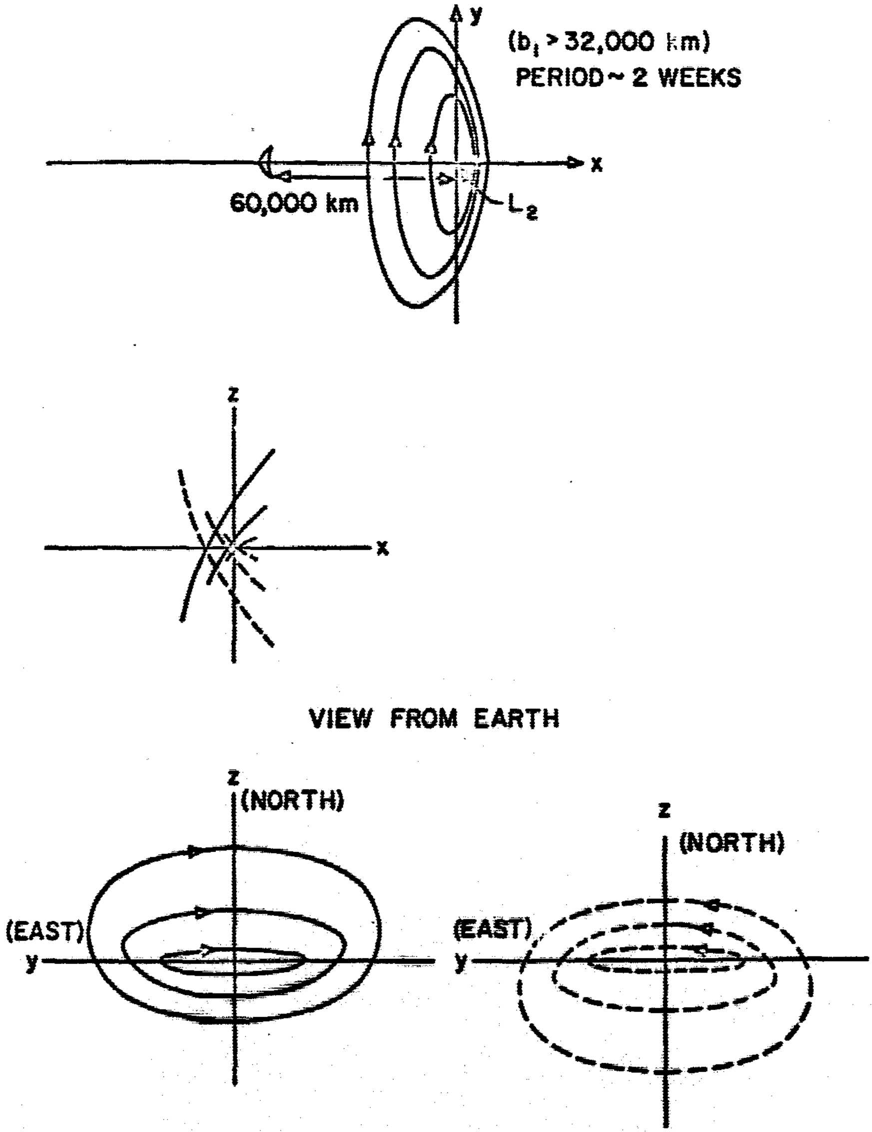

FIG. 1 HALO ORBITS

where the positive scalar k is a measure of the tightness of the desired control. The optimal three-axis control $\vec{u}$ takes the form

$$\vec{u} = -C(t)x - \vec{b}(t) ,$$

where $\overset{3\times 6}{C(t)}$, $\overset{3\times 1}{\vec{b}(t)}$ are periodic quantities, dependent on k, and computable with the aid of "spectral factorization" [2]. The resulting rms $|\vec{\delta r}|$ and $|\vec{u}|$ are plotted in Fig. 2, for single-axis as well as 3-axis control, together with an "error settling time" (= time for initial $|\vec{\delta r}_o|$ to decrease by a factor 10) which, like the rms $|\vec{\delta r}|$, increases as k decreases. The squares in Fig. 2 indicate that the average control can be reduced to less than 10^{-7}g's with an error settling time of less than three months.

More important, the resulting limiting motion in the 3-axis control case provides an improved nominal, correctable as in Ref. 1 for the non-commensurable perturbations, which permit an average control of only 10^{-8}g's with much tighter control, corresponding to settling times of the order of 1 day.

Lastly, the station-keeping cost will be slightly increased by imperfect knowledge of the state x. If it is assumed that a continuous control $\vec{u}$ is accompanied by a small random acceleration error of known statistics, and that observation errors have known statistics, the control law must make use of a Kalman estimator

$$\dot{\hat{x}} = F(t)\hat{x} + \begin{bmatrix} 0 \\ \vec{e}(t)+\vec{u} \end{bmatrix} + \overset{6\times m}{K(t)}\left[\overset{m\times 1}{z} - H(t)\hat{x}\right] ,$$

z being the usual linearized measurement vector, and, if H(t) is periodic, K(t) approaches a computable periodic gain matrix as t increases. The increase in the rms $|\vec{u}|$ is then computable.

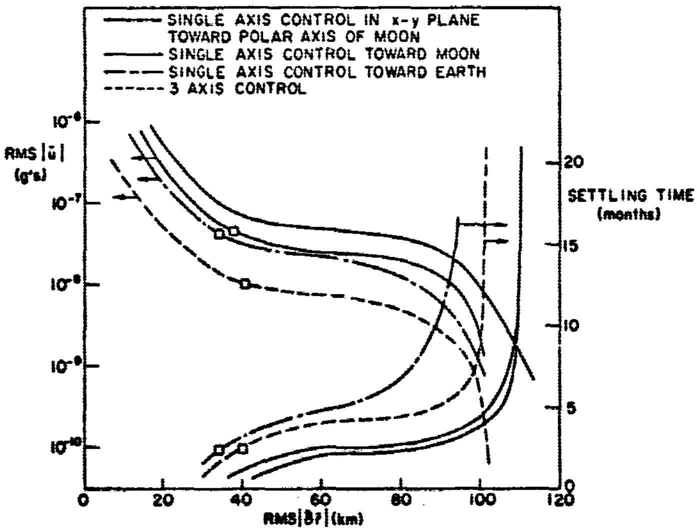

FIG. 2 CONTROL COST VS. SETTLING TIME.

In the case of range measurements from Earth, the information matrix $H^T(t)H(t)$, averaged over a day, is essentially constant, with information perpendicular to the Earth-Moon direction provided by simultaneous measurement from stations with different latitude and longitude. In the case of range-rate measurements, the Earth's rotation provides significant [3] but generally different NS and EW position information. The inclination, however, of the Moon's orbit to the equator spoils the periodicity of the information relative to the Halo orbit, which has a period of about two weeks not exactly commensurable with the Moon's orbital motion. A roughly comparable, but strictly periodic, information pattern is obtained if the inclination of the Moon's orbit is taken as zero. Assuming a random control execution error with rms value 10^{-9} g's and a correlation time of one day, as well as typical range and range-rate measurement errors, the increase in the rms $|\vec{u}|$ was not significant, even when reliance was placed entirely on range-rate measurement.

A comprehensive account of this work was written by M. Ratner as a Ph.D. dissertation [4].

REFERENCES

1. Farquhar, R.W., and A.A. Kamel, "Quasi-Periodic Orbits About the Translunar Libration Point," Celestial Mechanics, Vol. 7, No. 4 Jun 1973.

2. Nishimura, T., "Spectral Factorization in Periodically Time-Varying Systems and Applications to Navigation Problems," Jour. of Spacecraft, Vol. 9, No. 7, Jul 1972.

3. Curkendall, D.W., and S.R. McReynolds, "A Simplified Approach for Determining the Information Content of Radio Tracking Data," Jour. of Spacecraft, May 1969.

4. Ratner, M.J., "Station-Keeping For A Translunar Communication Station," Ph.D. Dissertation, Dept. Aeronautics and Astronautics, Guidance & Control Lab., Stanford University, Stanford, Calif., under NASA GSFC Grant NGR 05-020-507; Oct 1973, SUDAAR No. 466.