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Technical Report 771

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ON BOARD SATELLITE PROCESSING STUDY — FINAL REPORT

By: D. L. Reed

May 1973

Prepared under NASA Contract NAS5-21645
for Goddard Space Flight Center
National Aeronautics & Space Administration
Greenbelt, Maryland 20771

Operations Research, Inc. A LEASCO Company

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OPERATIONS RESEARCH, Inc.

SILVER SPRING, MARYLAND

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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont)

LIST OF FIGURES

LIST OF FIGURES (Cont)

LIST OF FIGURES (Cont)

I. INTRODUCTION

The utility of satellite-based, world-wide data collection and location systems is becoming more and more apparent to various users such as oceanographers, meteorologists, ecologists, etc. Consequently, the quantity of data that must be handled by the satellites of these systems and their ground data links will increase rapidly in the coming years. To prevent premature saturation, the data collection systems, and in particular, the satellite borne equipment, must be carefully configured to provide maximum capacity without degrading performance. This report presents analyses that describe this capacity for one particular type of data collection and location system— namely, one based upon random time and frequency access to the satellite.

An important aspect of the analyses is that they are parametric in nature. Specific requirements in terms of the amount and characteristics of data as well as the need for location estimates in the systems being considered are imposed only in general terms akin to those employed to develop current data collection and location systems. Because of this, major results are in terms of the anticipated parameters to be used when specific system requirements become available.

To describe these results, the report is divided into five major sections. The first of these, Section 2, is a general description of the concept of a random access system and a discussion of the considerations which make this type of system attractive relative to time and/or frequency ordered systems. With this as a basis for further analysis, Section 3 investigates the quantitative characteristics of random access systems in order to establish criteria whereby different concepts can be compared.

In Section 4 the model for comparing performance of different systems is developed. Basically, this model combines satellite kinematics and bit error characteristics of digital data communication with the statistical characteristics of a random access system. The results derived from this model provide the basis for the satellite processor concepts presented in Section 5.

The performance of random access data collection and location systems is characterized in Section 4 by the quantity of data which can be handled by the system without excessive interference between the asynchronous transmissions of platforms. This performance, in terms of average number of bits of data per second acquired by the satellite is quantitatively determined by:

- The statistics of time and frequency random access that determine different levels of saturation of the time-bandwidth product of a system in terms of the anticipated bit error rate, assuming no degradation is encountered either on-board the satellite or in subsequent ground data processing;
- The satellite on-board processing equipment to assure statistically that platform transmissions can be acquired, as a function of complexity in terms of parallel detection and data channels;
- The characteristics of platform transmissions in terms of data rate, data quantity per transmission (or transmission duration), and the interval between transmissions of the platforms.

The major results derived from these analyses indicate the following:

- The singularly most important parameter in maximizing the capacity of random access systems is platform data rate (directly related to, platform effective radiated power). The higher this is, the greater the capacity of the random access system, and the less complex the on-board processing equipment need be.
- The probability of interference between platform transmissions is directly proportional to the average rate at which transmissions occur and the quantity of data contained within a single transmission.
- Of the two phase modulation schemes analyzed, namely NRZ and split phase (Manchester coding), NRZ provides nearly twice the system capacity at equal levels of interference because of its narrower power spectrum.

- The performance of random access systems is improved with lower satellite altitude because of the reduced number of platforms simultaneously in view and the wider separation of transmissions in frequency due to the larger doppler shifts.

- The probability of a particular platform transmission being interferred with is directly proportional to the ratio of system data rate (average bit rate through the satellite processor) to system bandwidth, with the proportionality factor being between two and ten depending upon modulation scheme.

- For satellite altitudes corresponding to two and three hour periods, the probability of bit error during any given transmission is on the order of one in ten to one in twenty for system data rates measured in hundreds of bits per second. These error rates are independent of the sophistication of satellite on-board processing equipment.

Based upon the results of Section 4, Section 5 presents concepts of satellite on-board processors from the viewpoint of minimizing the quantity of bits which are stored and re-transmitted to the ground station(s). Three generic types of processors are presented.

Noise elimination processor — By eliminating those regions of the time-frequency space at the satellite that do not contain platform transmissions, the storage and retransmission load can be significantly reduced in comparison to satellite equipment which stores and retransmits the entire time-bandwidth product. However, whether analog or digital, the compression achievable is insignificant compared to the potential compression available as measured by system data rate compared to system bandwidth.

Demodulating Processor — By detecting and demodulating platform transmissions when they are received at the satellite, the quantity of data stored and re-transmitted from the satellite can be made comparable to the actual quantity of data transmitted by the platforms. Furthermore, depending upon actual platform transmission characteristics and format, further compression can be accomplished by logical manipulation (preliminary data reduction), prior to storage, on-board the satellite.

Because mutual interference is necessarily severe for high capacity random access systems, some combination of error coding and/or after-the-fact correlation of redundant transmissions to recognize and correct errors is necessary. By which technique this improvement in effective bit error rate is achieved, however, is not at all apparent. Both error coding as well as acquisition of redundant transmissions in themselves serve to increase interference and are therefore, to some extent, self defeating. Furthermore, both directly increase the complexity of on-board processing, particularly in the case wherein the error correction process is performed on-board the satellite.

II. RANDOM ACCESS SYSTEMS

2.1 GENERAL DESCRIPTION

The fundamental characteristic of random access systems being considered herein is the completely asynchronous nature of both the time of arrival and frequency of arrival of transmissions received at the satellite from the data collection platforms. These platforms operate independently by transmitting bursts of data at reasonably regular intervals without command or interrogation from the satellite or any other source—i.e. the timing and periodicity for the burst transmissions result from timing mechanisms internal to the platform itself. A graphical presentation of the elements of a random access system based upon this transmit-only type of platform is shown in Figure 2.1.

Four distinct elements are required for the functioning of the system. First there are the data collection platforms themselves. These acquire data to be transmitted to the satellite, format and encode these data along with a identification code specifying the platform, and lastly, transmit these data periodically.

The second element in the data collection process is the satellite. Contained within the satellite is equipment performing several possible functions depending upon system design-these functional elements make up what is referred to throughout this report as the satellite on-board processor. Regarding these functions, three will be present in all processor concepts. These are the receive, store and re-transmit functions. The receive function is the RF front end of the processor wherein the bandwidth within which platform signals arrive is isolated and amplified for further processing.

The store function must also be present in all processor concepts. The satellites envisioned for the data collection and location systems are

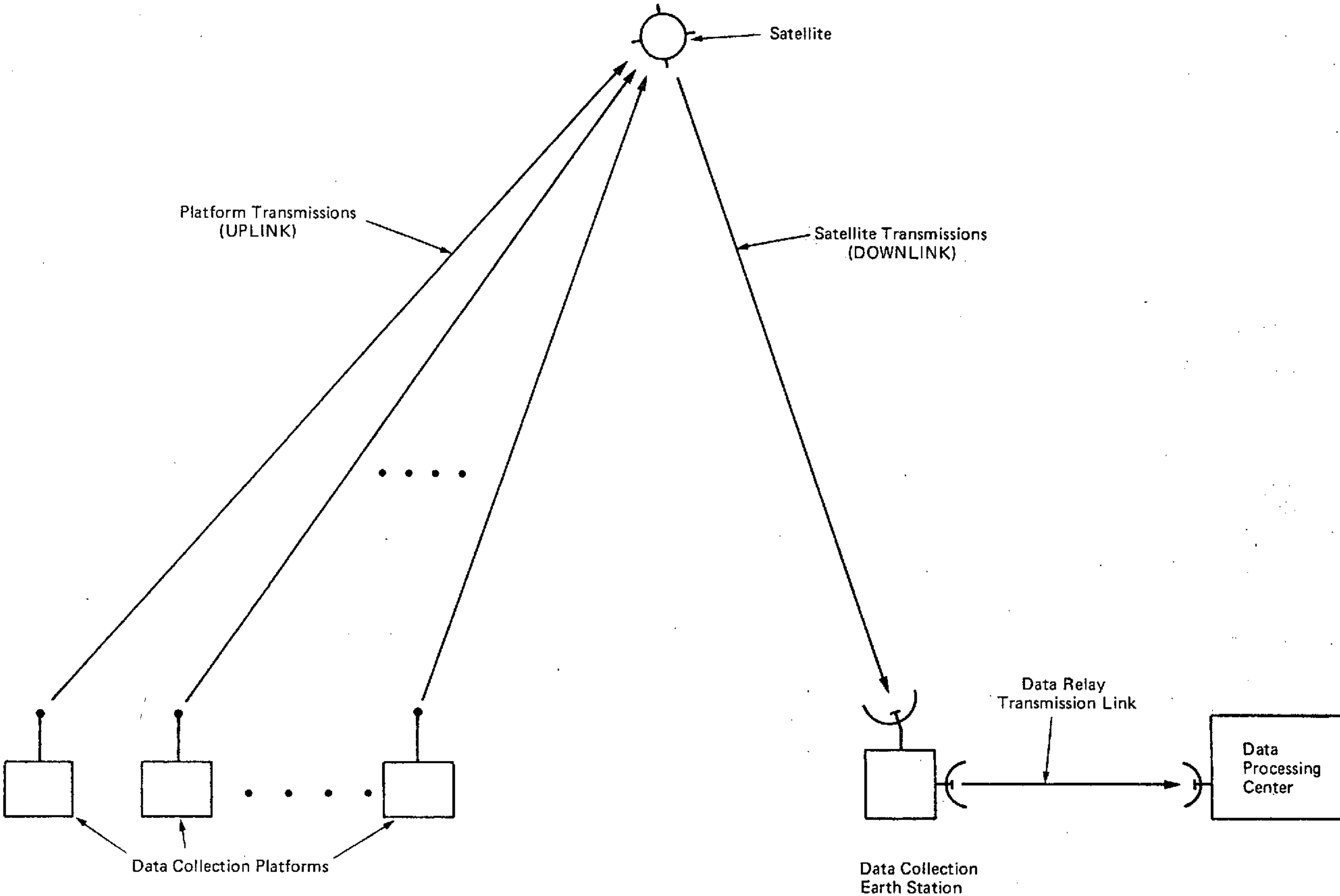

FIGURE 2.1. BASIC ELEMENTS OF A SATELLITE DATA COLLECTION AND LOCATION SYSTEM

low altitude to enable location of platforms by means of kinematic measurements between the satellite and platforms. Because of this, a realtime data relay between the platforms and a data collection earth station is not possible over most of the satellite's orbital path particularly for the remotely located platforms. Therefore, the information contained within the received bandwidth of platform transmissions must be preserved on the satellite at least until re-transmission to a ground station is possible.

The third function common to all systems is the re-transmit function. This comprises the command and sequencing functions whereby platform data stored on-board the satellite is transferred to the ground station during those periods wherein satisfactory communication links are established.

The two remaining elements of the random access systems are the data collection earth station and the data processing center. As mentioned previously, the data collection earth station performs the function of receiving the transmission from the satellite of the stored platform data. To increase the frequency with which these transmissions can take place, the stations are normally located in geographic areas remote from the terminal point for the data-namely the data processing center.

Regarding the data processing center, two types of operations are performed on the data received from the earth station. One operation is translation of the data as received to a format useable by the users of the system. The extent of this operation will be largely determined by the processor on-board the satellite as will be seen in subsequent discussions. The second operation performed at the data processing center is data dissemination.

2.2 ADVANTAGES OF ON-BOARD PROCESSING

From the general description provided, the performance of a random access data collection system can be shown to depend heavily upon the nature and extent of signal processing performed by the satellite equipment—i.e. the on-board processor. In this regard, performance in this context refers to the relative ability of different random access systems to handle large quantities of data with minimum effective error rate as seen by the user. The dependence of this performance on the on-board processor can be seen by evaluating the three basic communication links within the system.

2.2.1 Platform Transmission (Uplink)

Because of the transmit-only nature of the data collection platforms, the ability of the uplink from platform to satellite in transferring error free data is strictly a function of platform transmission characteristics only. In particular, the potential quantity of data will be determined by two factors

- the bandwidth allocated to the uplink or more particularly, the bandwidth of platform transmissions received at the satellite

- the statistics of the platform transmissions with regard to mutual interference caused by asynchronous transmission in time and frequency.

The bandwidth of the transmissions received at the satellite will be determined by two factors. First, the bandwidth can be made arbitrarily large to increase the capacity for data transmission by assigning different transmission frequencies to each platform. Secondly, the motion of the satellite relative to the platforms will cause an increase in received bandwidth at the satellite because of doppler shifts.

Regarding increased bandwidth achieved by assigning different frequencies (or frequency bands) to different platforms, this can always be implemented regardless of the version of random access system being considered. From a relative performance viewpoint then, achieving increased performance in this manner should not provide an inherent advantage to one system or another. For this reason, the nominal transmission frequency of all platforms are presumed to be the same except for deviations which might occur due to drifting of platform equipment parameters.

Satellite motion however, will have a distinct effect on the uplink bandwidth as seen by the satellite. Because the doppler shift experienced by different platforms is determined by their location relative to the satellite, separation in frequency between simultaneous platform transmissions will be accomplished because of the doppler shift will be a monotonic function of satellite altitude. Therefore, the data capacity of the platform-to-satellite uplink from a frequency occupancy viewpoint will be determined by satellite altitude.

For a given satellite altitude then, the capacity of the uplink will be limited by the mutual interference experienced between platform transmissions. This capacity is entirely independent of any signal processing performed on-board the satellite. In essence then, the upper limit of performance of random access systems is fixed as soon as the transmissions characteristics of the platforms is established—i.e., how frequently individual platforms transmit their data burst, how long the bursts are, the number of platforms in view at any given point in time, and, the portion of the received frequency spectrum at the satellite that is taken up by each transmission.

2.2.2 Satellite Transmission Downlink

With the limiting quantity of data from platforms fixed and independent of any signal processing functions on-board the satellite, the satellite-to-earth station communication link must be sufficiently wide to transmit all information stored on-board the satellite that contains the platform data. The first, and by far the most important, function performed by an on-board processor is to decrease the burdens placed on this link. This can be seen as follows:

A straightforward mechanism to store platform data on-board the satellite when not in view of an earth station is to store, analog or digital, the entire time-bandwidth product received. However, because the systems are not ordered, this will mean storage and re-transmission of time-bandwidth regions wherein no platform transmissions are present—i.e., noise. The ability of an onboard processor to eliminate these regions of noise prior to storage and transmission will reduce the burden on the satellite down link and any other transmission link up to the point of detection of platform data. Subsequent analyses will show this compression function of the on-board processor is inherently significant for random access systems.

2.2.3 Data Relay Transmission Link

The remaining communication link is that one between the data collection earth station and the data processing center. This link is singled out because it is usually more limited in terms of data transfer capability than the satellite downlink. Therefore, the importance of further reduction of the data requiring transfer to the ground is emphasized.

The second inherent advantage of on-board processing is to achieve this further reduction or data compression. This is achieved by a combination of one or more possible functions.

- Demodulation of platform data prior to storage inherently reduces the quantity of data because overhead data bits such as bit and frame synch, for example, can be eliminated
- The repetitive nature of platform transmissions will result in redundant data being present which might be eliminated by the processor if in no other way than storing platform identification codes only once
- Preliminary data reduction might be performed, for example, by partial or complete solution of a location algorithm or to a lesser extent such reduction as may be afforded by storage of differential data only.

Each additional degree of data compression provided by the on-board processor will be at the expense of increased complexity and decreased reliability. One of the goals of the analyses performed subsequently is to minimize these effects.

III. COMPARISON OF RANDOM ACCESS SYSTEMS

One system concept for collection data from remotely located platforms is based upon random access to the satellite in both time and frequency. Specifically, the platforms are configured to transmit bursts of data on a nominally periodic basis governed solely by timing equipment contained within the platform—i.e., the platforms contain no receivers for the purpose of interrogation or order among platform transmissions. Assuming a communication link has been established with the satellite, receipt of any given transmission such that data can be extracted on-board the satellite then requires both of the following:

- No other transmission can be occurring at the same time such that its received frequency at the satellite is close enough to cause "unacceptable" interference to the given transmission
- At least one data channel is available within the satellite processor to accept the given signal and extract the data contained therein.

The non-existence of either one of these conditions, when a transmission appears at the satellite, constitutes interference. In the first case, the data of the transmission will experience "high" bit error rates. In the second case, the transmission will be totally lost. Determination of the probability of either one or both of these types of interference will, then, characterize the performance of various random access systems including the techniques employed by the on-board processing equipment to minimize interference.

The probability of interference provides the basis for establishing comparison criteria for different system concepts. However, before a suitable criteria can be developed, a specification of system requirements is necessary.

3.1 SYSTEM SPECIFICATIONS

In general, the requirements for a data collection and location system can be stated in terms of five separate parameters.

3.1.1 Number of Platforms

A prime requirement for a data collection and location system will be the number of users or platforms which can access the system. In this regard, there are two different ways of establishing such a number depending upon two different limitations that will be imposed by the random access system.

One limitation imposed by the system will be a direct result of the interference experienced between platform transmissions. For this limitation to apply in establishing a number of platforms, it is only the number of platforms, within the view of the satellite at any point in time that is of concern. Stated differently, this limitation is really a limitation on the density of platforms (e.g., number per square mile) as seen by the satellite.

The other limitation on number of platforms is a result of the data storage capacity on-board the satellite and the frequency with which the data can be transferred to a ground station. This limitation reflects a constraint on the time averaged number of platforms in view coupled with the amount of data to be collected from individual platforms and the degree of data compression performed by the on-board processor.

In the context of this report, the former of these limitations will be assumed for all systems. Namely, interference between transmissions will limit the number of users/platforms to be serviced by the system as opposed to storage limitations on-board the satellite.

3.1.2 Data Quantity

Another requirement of data collection systems is the quantity of data to be transferred from the platforms during a single overpass of the satellite. This may be achieved in a number of ways all of which are governed by a requirement or lack thereof to locate the platforms.

Platform location will require a number of successive transmissions from each platform during an overpass. For a given quantity of data to be transferred from a platform during an overpass then, the options of sending different data during each transmission versus sending all the data during each transmission is available. There are advantages and disadvantages to both options.

If all data is sent during each transmission, the transmission duration will will be long compared to sending different data on each transmission. The longer transmissions, however, tend to increase the likelihood of interference if a fixed number of transmissions are necessary for the location requirement

In contrast, the relatively short transmissions tend to minimize interference but not in direct proportion to transmission duration because of the overhead bits (platform I.D., frame synch, etc.) required with each transmission. Furthermore, if each transmission contains different information, then there is no opportunity during ground processing to recognize and correct bit errors (caused by either interference or noise) through correlation.

In summary, while the total quantity of data acquired from a platform during an overpass is a system requirement, the manner in which this data is transferred to the satellite will depend on a requirement to locate the platform and in turn, on the level of interference between platforms.

3.1.3 Transmission Interval

The time interval between platform transmissions is also a direct consequence of the system requirement to locate platforms and the location algorithm employed. In this regard, "location" in the context of this report is used in its most general sense to comprise the requirement for estimating two coordinates of platform position, two coordinates of platform velocity, and the requirement to minimize (by correction techniques) the errors caused by instabilities in the platform's equipment. Therefore, a minimum of five independent measurements of the parameter(s) used in the location algorithm is assumed to be necessary during each satellite overpass.

The location algorithm will determine the number of platform transmissions that are required per overpass to obtain the five or more measurements which are assumed to be of the relative Kinematics between the satellite and platform If a single measurement is made during each platform transmission, such as the received frequency, then at least five separated transmissions must be received during the overpass. However, if both received frequency and time rate of change of received frequency can be measured during a single transmission, then three transmissions are sufficient. Similarly, if range difference is measured between successive transmissions, two transmissions can satisfy location requirements.

In theory, the single, double, or triple measurement systems can satisfy the same location requirement if the measurements can be made to the necessary precision. The significant impact for random access systems is then the specification of the time interval between transmissions from the platforms in order to acquire a given minimum number of transmissions. This time is assumed to be only a function of satellite altitude.

3.1.4 Platform Transmitter Power

Many data collection platforms are limited by a restriction of a maximum transmitter power. This limitation will be seen to fundamentally impact random access systems and in particular, their susceptibility to mutual interference.

If all system parameters are held constant and platform transmitter power is increased, then transmission duration can be proportionately shortened by increasing platform data rate. While it will be shown that the combined probability of temporal and spectral overlap of transmissions does not appreciably change by utilizing increased platform power in this way, the shorter transmissions tend to simplify signal processing by decreasing the number of parallel data channels necessary to limit an overall level of interference. Therefore, maximum platform power can be shown to directly influence overall interference level.

3.1.5 Bit Error Rate

The remaining major system requirement is the maximum bit error probability to be tolerated. However, because of the combined effects of mutual interference and multiple transmissions, bit error rate must be interpreted as the effective error rate as presented to a user after ground processing is completed instead of that occuring during individual platform transmissions.

By their very nature, random access systems must operate within an environment wherein the probability of mutual interference is relatively high compared to most acceptable levels of bit error probability. Therefore achieving low effective bit error rates requires some form of error detection and/or correction. For this report, this correction/detection is analyzed only from the viewpoint of recognizing conditions of high levels of interference and the ability to correct errors by means of comparing successive transmissions of the same data during a single overpass.

3.2 PERFORMANCE CRITERIA

From an overall viewpoint, a satellite based data collection system is merely a means of relaying data from geographically distributed platforms to a central location for distribution to the users. A natural measure for comparing various systems is then the average rate at which this data can be acquired and transferred subject to specifications of effective bit error rate, platform location capability, and platform transmitter power limit. For the purposes of this study, two factors establish this average rate.

One factor which will limit the average rate of data transfer is the level of interference existing prior to reception of platform signals at the satellite. In particular, regardless of the sophistication present in the satellite's signal processor, the two dimensional space of time and frequency is limited in the amount of data that can be transferred. In ordered systems, this point of saturation can be fixed deterministically. In random access systems, this point of saturation can be specified only in terms of the statistics of interference.

The second factor limiting the average system data rate is the capacity of the satellite processor to preserve the data contained in time and frequency. This can be accomplished by various means directly attributable to the amount of signal processing performed on the satellite compared to the amount performed at a ground processing center. Furthermore, the type of signal processing performed on-board the satellite will determine its susceptibility to saturation and therefore loss of data.

Combination of these two factors are used to compare various systems on the basis of their average data rate. In particular for each on-board processor configuration, the probability of acquiring and transferring platform data at a given rate with effective bit error probability less than a specified amount is estimated.

3.3 ON-BOARD PROCESSORS

As indicated above, the fundamental difference between on-board processors is the degree to which the two dimensional signal space of time and frequency is compressed. At one extreme, the entire time-bandwidth product of the satellite's receiver can be stored and retransmitted to a ground processing center wherein the information content is extracted. The other extreme is wherein only the desired information from the platform signals is stored on-board the satellite for retransmission to a ground station. In this case, the ground processing is limited in function to that of a data distribution center with no data processing performed. Comparing different on-board processors, then, is a basically a trade between complexity of equipment versus the compression of the received time-bandwidth product prior to retransmission to the ground.

These two basically different types of compression may be viewed as follows. One type of compression is concerned with the partial or total elimination of those parts of the time-frequency space wherein no information is present, i.e., no platform transmissions are present. The other type of compression is accomplished by logical operations on platform data subsequent to detection and storage on-board the satellite. The first type of compression is therefore, concerned with the extent of signal detection and demodulation performed. The second type can be viewed as eliminating superfluous data associated with the data transmission process such as redundant data, repetitively stored platform ID., and perhaps information in excess of that required to perform platform location.

IV. INTERFERENCE LIMITS

An upper bound on the quantity of data that can be transferred by a random access system is the point of statistical saturation of the time-frequency space available. This point can be defined by the probability of <u>not</u> being able to detect the information content of a platform's transmission regardless of the sophistication available—in essence, the point wherein the "noise" created by interferring signals becomes intolerable. This limit of internally generated noise can only be degraded by the performance limitations of physically realizeable on-board processors.

Analysis of this interference limit is presented in two ways. First, a qualitative "zero-one" model of interference is derived which indicates the basic interaction between system parameters. Secondly, a more quantitative model of interference is described which includes a more accurate description of system performance. The fundamental difference between these two is the manner in which spectral overlap of two simultaneous signals is characterized. In the "zero-one" model, transmissions are presumed to occupy finite portions of the received frequency spectrum and if there is any overlap of the spectra of the two signals, both signals are presumed to be lost. In the more precise model, the power spectra of two modulation schemes are used and the power from interfering signals is treated as an increase in thermal noise from which an increase in effective bit error rate is estimated.

While the two interference models differ in characterization of spectral overlap, both models are based upon Poisson distributions in time for the occurrence of platform transmissions. For this reason, the following section describes this Poisson time model. The two sections subsequent to this then derive the "zero-one" and quantitative models of system interference.

4.1 TIME DISTRIBUTION OF TRANSMISSIONS

Basic assumptions in describing the distribution in time of platform transmissions are that (1) - start times are statistically independent of one another and (2) - the probability of a transmission beginning during some interval of time is proportional to the duration of the time interval. Analytically, if λ is the average rate over time at which transmissions are initiated, then the probability of n transmission beginning during a time interval $(\Delta t + dt)$ may be written as*

$$p(n, \Delta t + dt) = p(n, \Delta t)(1 - \lambda dt) + p(n-1, \Delta t)\, \lambda\, dt$$

where

$p(n, \Delta t + dt)$	=	probability of n transmissions starting during the time interval $\Delta t + dt$
$\lambda\, dt$	=	probability of a transmission starting during the time interval dt
$p(n, \Delta t)(1 - \lambda\, dt)$	=	probability of n transmissions starting during the time interval $\Delta t + dt$ given that n transmissions started during the interval Δt
$p(n-1, \Delta t)\, \lambda dt$	=	probability of n transmissions starting during the time interval $\Delta t + dt$ given that n-1 transmissions had started during the interval Δt

If this differential equation is solved with the initial condition that $p(0,0) = 1$, then the result is the Poisson distribution. Note, for this to be a valid approximation, the number of platforms must be relatively large when λ is assumed constant. This will be shown to be true for workable values of system parameters.

This Poisson distribution determines the probability of time overlap between two or more platform transmissions—or of more concern, the probability that time overlap will not occur. In particular, if the number of platforms in view of the satellite is N and they transmit asynchronously with periodicity T, then the average rate at which transmissions start is (N/T). Given that a transmission starts at some specific point in time, the probability of another platform transmission starting during a time interval τ equal to the duration of the platform's transmission is

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*"Notes on Operations Research 1959", MIT Operations Research Center, Technology Press, pps 14,15.

$$P_t = 1-p(O,\tau)$$

= probability of at least one or more other transmissions starting during the duration τ of the known or desired transmission that started at a specific point in time.

where from the Poisson distribution

$$P(O,\tau) = e^{-\lambda\tau} = e^{-N\tau/T}$$

If there were no frequency separation between platform transmissions arriving at the satellite, this expression for time overlap would escribe the probability of a particular platform transmission experiencing interference.

4.2 INTERFERENCE

While the above describes the probability of time overlap of platform transmissions, this probability does not constitute interference between transmissions unless spectral overlap also occurs. The probability of interference must be derived by computing the simultaneous occurrence of both time and spectral overlap. The first step in computing the probability of spectral overlap is to determine the statistical distribution of frequencies received at the satellite.

4.2.1 Distribution of Received Frequency

To derive the distribution of frequencies received at the satellite, the relative kinematics between the satellite and platforms must be derived in order to determine the doppler shifts relative to transmitted frequencies. Secondly, an assumption must be made regarding the geographic distribution of platforms relative to the satellite in order to determine the distribution of doppler shifts. Thirdly, if the transmitted frequencies from platforms are subject to drifting, then the statistics of this drifting must also be introduced.

To determine the doppler shifts between a platform and satellite, the position of the platform on the earth's surface relative to the satellite subpoint must be known. The orbital parameters of the satellite then serve to determine the satellite's position and velocity vectors relative to the platform from which the radial velocitv is determined. If the platform transmits at a frequency of f_t , the frequency received on-board the satellite will be shifted from f_t by the amount.

$$f_d = f_r - f_t = -V_R (f_t/c)$$

where

f_r = received frequency

V_R = radial velocity between satellite and platform

C = signal propagation velocity

The distribution of the doppler shifts is determined from the distribution of platforms on the earth's surface—i.e., the distribution translates monotonically into a distribution of V_R. Throughout this report, platforms in view of the satellite are assumed to be uniformly distributed over the earth's surface in a statistical sense.

The last assumption required to determine the distribution of received frequencies is that describing the distribution of transmitted frequencies. Throughout this report, the transmission frequencies from platforms are assumed to be normally distributed with a standard deviation of 2000 Hz and a mean frequency of 401 megahertz.

Numerical results derived from these assumptions and relationships are shown in Figures 4.1 and 4.2. Figure 4.1 is a presentation of the doppler shifts that occur due to the relative motion between the satellite and platforms. As noted, these data correspond to a satellite whose orbital period is two hours and whose inclination is 103° (for sun-synchronism). The satellite is assumed to be at zero degrees latitude and longitude traveling in a south-south westerly direction. The two circles indicate the limits of visibility of platforms depending upon the lowest angle (0° or 5° shown) above the platform's horizon that the satellite can still be seen. The nearly hyperbolic contours show the locations of platforms on the surface of the earth that give rise to equal doppler shifts if all platforms transmit at 401 megahertz. The notation "PLATFORM LOCATION" should be ignored at this point of discussion. It will be referred to subsequently.

Figure 4.2 presents the density and cumulative distribution functions for the conditions noted in Figure 4.1 and for the assumptions mentioned above. There are two aspects of these data that are of particular interest.

From the density function of received frequency, it is apparent that the most likely frequencies to be received at the satellite are about ±6000 hertz about the mean received frequency of 401 megahertz. This means that spectral overlap of two simultaneously received transmissions is most likely to occur if the frequency of one is ±6000 hertz about the mean - stated differently, this overlap is most likely to occur if one of the platforms is located near either the 6000 or -6000 hertz contour of Figure 4.1. The reason for this can be seen by noting the larger area of the visibility circle (and therefore larger number of platforms statistically) enclosed by the 5000 to 7000 hertz contours compared to, for example, the area enclosed by the ± 1000 hertz contours.

DOPPLER CONTOURS

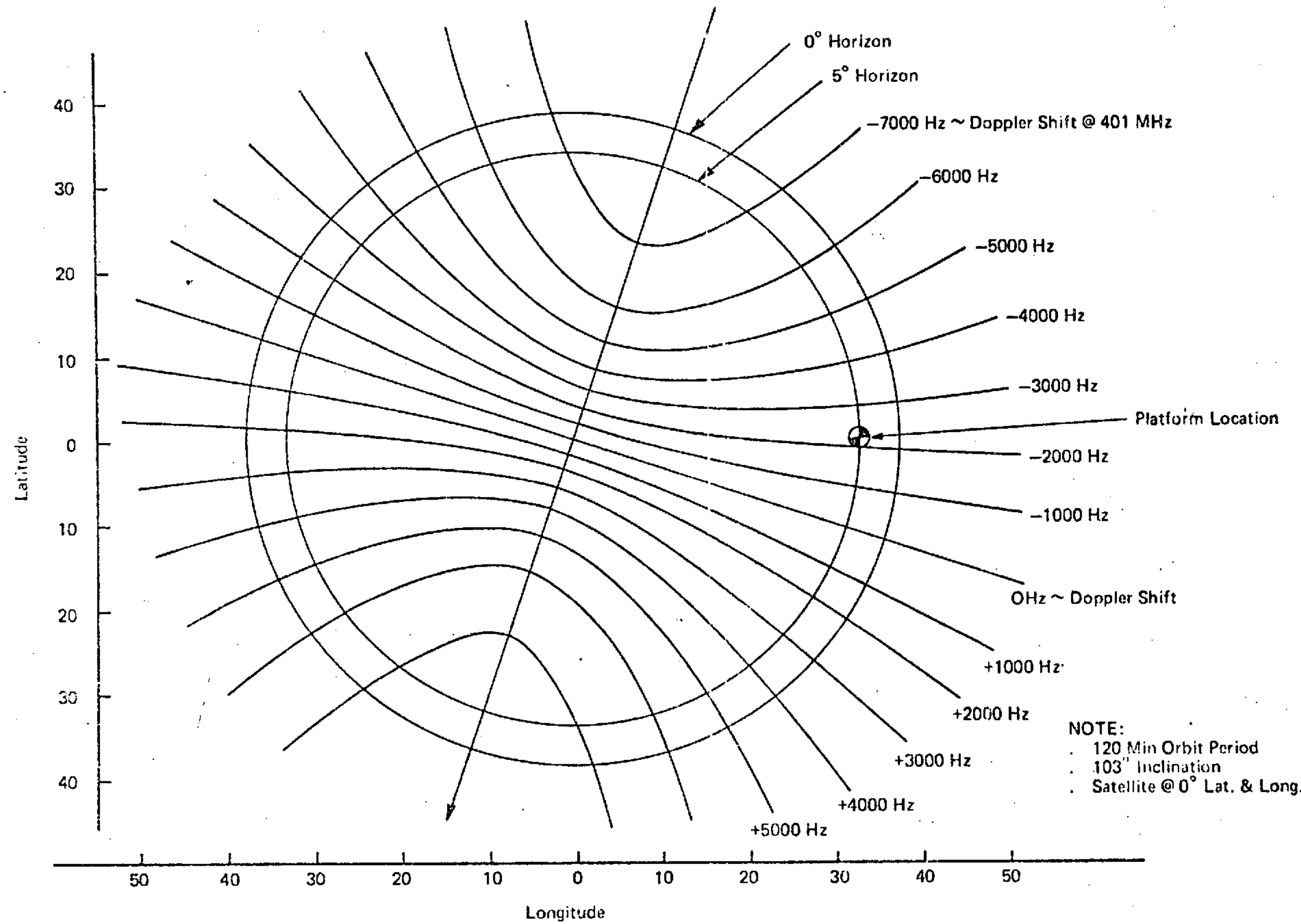

FIGURE 4.1 DOPPLER CONTOURS

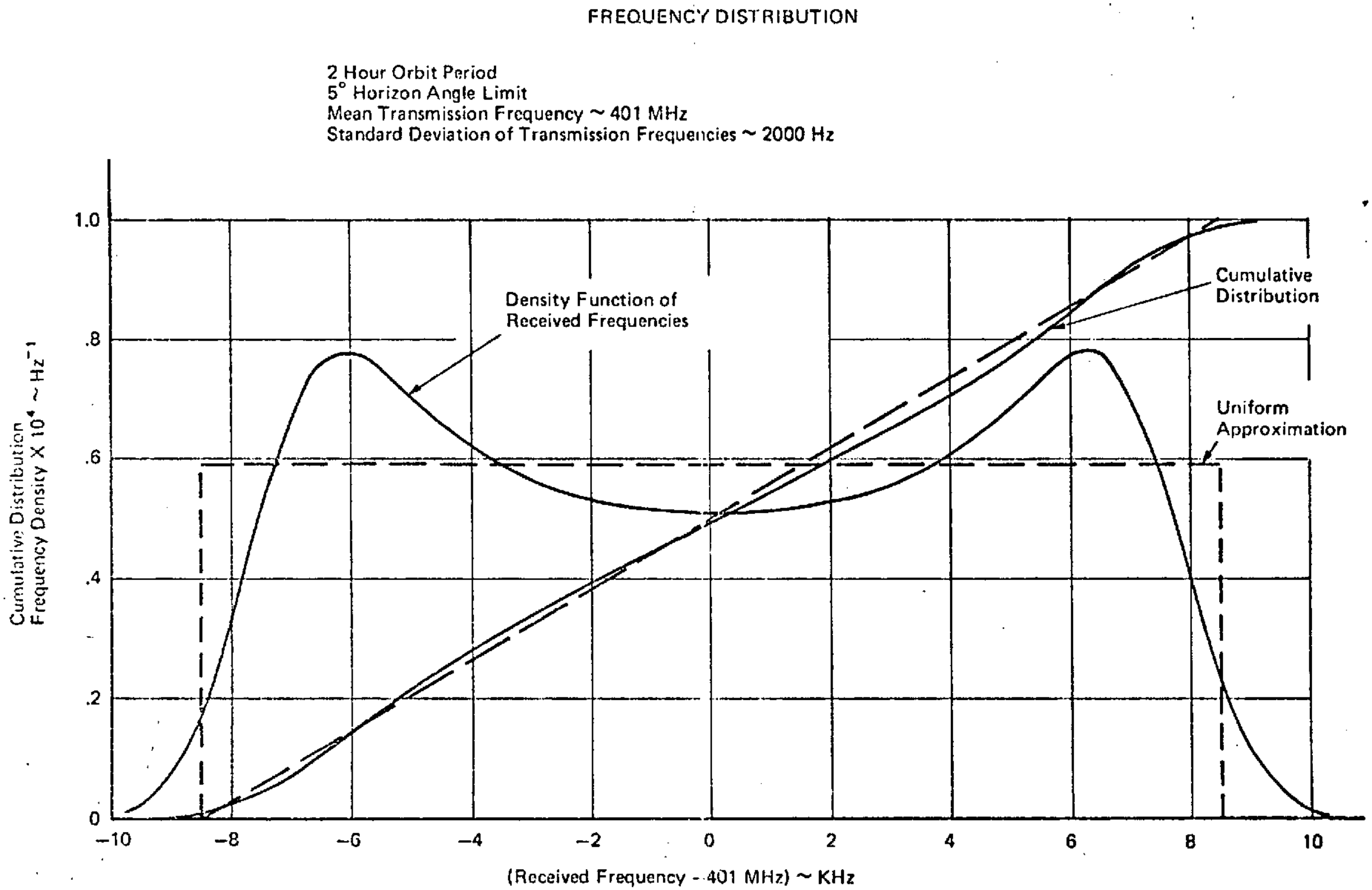

FIGURE 4.2 FREQUENCY DISTRIBUTION

The second point of particular interest from Figure 4.2 is the reasonable approximation that can be made by assuming received frequencies are uniformly distributed. The dashed lines of Figure 4.2 indicate this approximation. With this approximation, the total band of received frequencies is about 17,000 hertz from which the probability density is about $.58 \times 10^{-4}$ per hertz. These numbers are referred to below in the "zero-one" interference model.

4.2.2 "Zero-One" Interference Probability

With the statistical distribution of received frequencies, the probability of spectral overlap given two simultaneous signals can be determined. An analytical approximation to interference probability including both time and spectral overlap is the "zero-one" interference model. Its derivation is based upon two assumptions.

- The statistical distribution of received frequencies can be approximated by a uniform distribution as mentioned above and graphically shown in Figure 4.2.
- Interference occurs between two simultaneous signals if their received frequencies are separated by less than a specified frequency interval—f_I

The uniform distribution of received frequencies permits computation of the probability of interference given that two signals are simultaneously present. In particular, the probability of one signal being closer in frequency to a known or specified signal than f_I hertz is just $2f_I/F_T$ where I/F_T is the density function value for the uniform distribution—for the distribution of Figure 4.2, $I/F_T \sim 1/17000$ or about $.58 \times 10^{-4}$ per hertz. This is a valid approximation for the case where f_I is small relative to F_T which is as will be seen true for the random access systems being considered.

With this probability of spectral overlap, the "zero-one" probability of interference can be derived. This is accomplished in a manner entirely analogous to the derivation of the Poisson distribution for time overlap alone. In this case, the statistical event of interest is the start of a transmission whose frequency is within some specified frequency interval. The average rate at which this event will occur in time may be written now as the product of the time rate (N/T) and the probability of the frequency lying within the given band. In particular, the average rate λ in this case becomes:

$$\lambda = \frac{N}{T} \quad \frac{2f_I}{F_T}$$

Following the same procedure and with the same assumptions outlined in Section 4.1, the probability of at least one or more other transmissions starting during the duration τ of a known or specified transmission and being within F_I in frequency from the known transmission may be written as

$$1 - e^{-\left(\frac{N\tau}{T}\right)\left(\frac{2f_I}{F_T}\right)}$$

or for those conditions wherein P_I, the zero - one probability of interference, is small, this expression may be approximated by

$$P_I \sim \frac{2N\tau}{T} \frac{f_I}{F_T} \text{ for } \left(P_I^2 \ll 1\right) .$$

The unspecified parameter in the above expression is the frequency separation, f_I, required between two simultaneous signals in order to preclude interference. Its value will depend upon two things.

- The symbol or bit rate of the platform transmissions
- The modulation technique employed.

If NRZ signaling is employed, then the majority of signal power (and therefore potentially interfering power) is contained within a frequency band equal to twice the bit rate centered at the carrier. Therefore, for NRZ signaling, if two transmission carriers are separated by less than twice the bit rate, the "zero-one" interference assumption is presumed to apply in that the data contained within the two signals is lost (or is retrieved with high bit error probability). For NRZ signaling then, f_I is assumed equal to 2R where R is defined as the data or bit rate of the transmissions. Similarly, if split phase (Manchester coding) signaling is employed, most power is contained with a frequency band of four times the data rate so that f_I is 4R for split phase signaling.

In terms of data rate R, the above expression for the probability of interference can be written as:

for NRZ signaling $$P_I \sim \frac{4\,N\tau}{T} \frac{R}{F_T}$$

for split phase signaling $$P_I \sim \frac{8\,N\tau}{T} \frac{R}{F_T} .$$

However, the product of transmission duration (τ) and data rate (R) is just the number (quantity) of bits contained within a transmission. The first conclusion from these expressions then is that the level of interference within random access systems depends only upon the quantity of data in a platform transmission and not upon individual effects of transmission duration and data rate.

This conclusion also leads to the second conclusion that the level of interference is directly proportional to system capacity—i.e., (N/T) is the average number of transmissions from the platforms reaching the satellite that contains τR bits of data. Therefore, ($N\tau R/T$) is the average number of bits per second reaching the satellite from all platforms within view. Also, if interference is considered to be a condition wherein data is retrieved with high error rates, then the overall bit error rate for the system (on a per transmission basis without error correction) is directly proportional to the amount of data handled by the system. Stated differently, the accurate retrieval of "large" amounts of data with random access systems implies the necessity for some sort of error detection and correction.

Regarding the difference between NRZ and split phase signaling, the above expression indicates a factor of two advantage in system capacity when NRZ signaling is employed. This is probably not a quantitatively reliable conclusion because of the assumptions made in defining "zero-one" interference. In particular, if interfering signal power is equated to an effective increase in noise power within a demodulating band, then noise density increase (which will determine bit error rate for coherent detection) will be less for split phase compared to NRZ for equal amounts of interfering power. The comparison of modulation schemes should be based upon the more quantitative analysis of interference provided subsequently in Section 4.2.3.

Another interpretation of the P_I relationship is important. This regards the product of the time between platform transmissions (T) and the width of the frequency band of received transmissions under the assumption of uniform distribution (F_T). If the assumption is made that the requirement to locate a platform is equivalent to requiring receipt of n platform transmissions during a satellite overpass, then the value of T is only a function of n and the minimum viewtime between a satellite and platform during an overpass. However, this minimum viewtime is only a function of satellite altitude (for fixed horizon angle limit). Similarly, F_T is also only a function of satellite altitude. Therefore, interference probability depends upon satellite altitude and the product ($N\tau R$).

4.2.3 System Interference

While the "zero-one" interference model provides understanding of the interaction between system parameters and how they effect overall capacity, it does not provide a quantitative description of system performance. This is particularly true regarding the effective bit error rate to be expected as a function of system capacity. To acquire this information, a model must be developed to specify bit error rate during demodulation of a transmission as a function of two conditions.

- The frequency separation(s) between a signal being demodulated and other signal(s) simultaneously present

- The amplitude(s) of the interfering signal(s) relative to the amplitude of the signal being demodulated.

This model is derived in the discussions below and in detail in the appendix. Basically, the power within the demodulation bandwidth of the desired signal due to interfering signals and their distributed power spectra is treated as an increase in background or thermal noise density. Knowing the amplitude of the desired signal then enables computation of bit error rate in an interference environment. The interference environment is then determined from the statistics of signal(s) amplitudes and frequencies received at the satellite.

Gaussian Noise Model. The basis for establishing the effects of system interference is the assumption that the power of interfering transmissions can be approximated to be an increase in thermal noise density. This power is determined by integrating the power spectrum for the modulation technique over a band equal to the width of the main lobe (twice the data rate for NRZ and four times the data rate for split phase) that is centered a specified distance in frequency from the signal of interest. The increase in thermal noise density is then this integrated power divided by the band.

Two modulation techniques have been evaluated in terms of this noise model. One is NRZ with a power spectrum of $(\sin x/x)^2$ and the other is split phase or Manchester coding wherein the power spectrum is $\sin^4 (x/2)/(x/2)^2$. Performing the integration indicated above as a function of the separation between a desired and interfering signal leads to the data shown in Figure 4.3. As indicated, the ordinate of this figure is the ratio between the integrated power within the main lobe band of the desired signal as caused by an interfering signal separated in frequency in units of data rate to the power of the signal of interest. Note, in the case of NRZ, the width of the main lobe is twice the data rate while in split phase this band is four times the data rate. This is basically the cause for the higher values of the power ratio for split phase modulation.

Utilization of these data to eventually estimate bit error rate in the presence of multiple interfering signals is accomplished as follows:

Defining

(E_b/N_o) = energy per bit/thermal noise density

β = width of main lobe/data rate = B/R

B = width of main lobe

(P_I/P_o) = ratio of integrated power of interfering signal to desired signal over main lobe band of desired signal—i.e., the ordinate of Figure 4.3

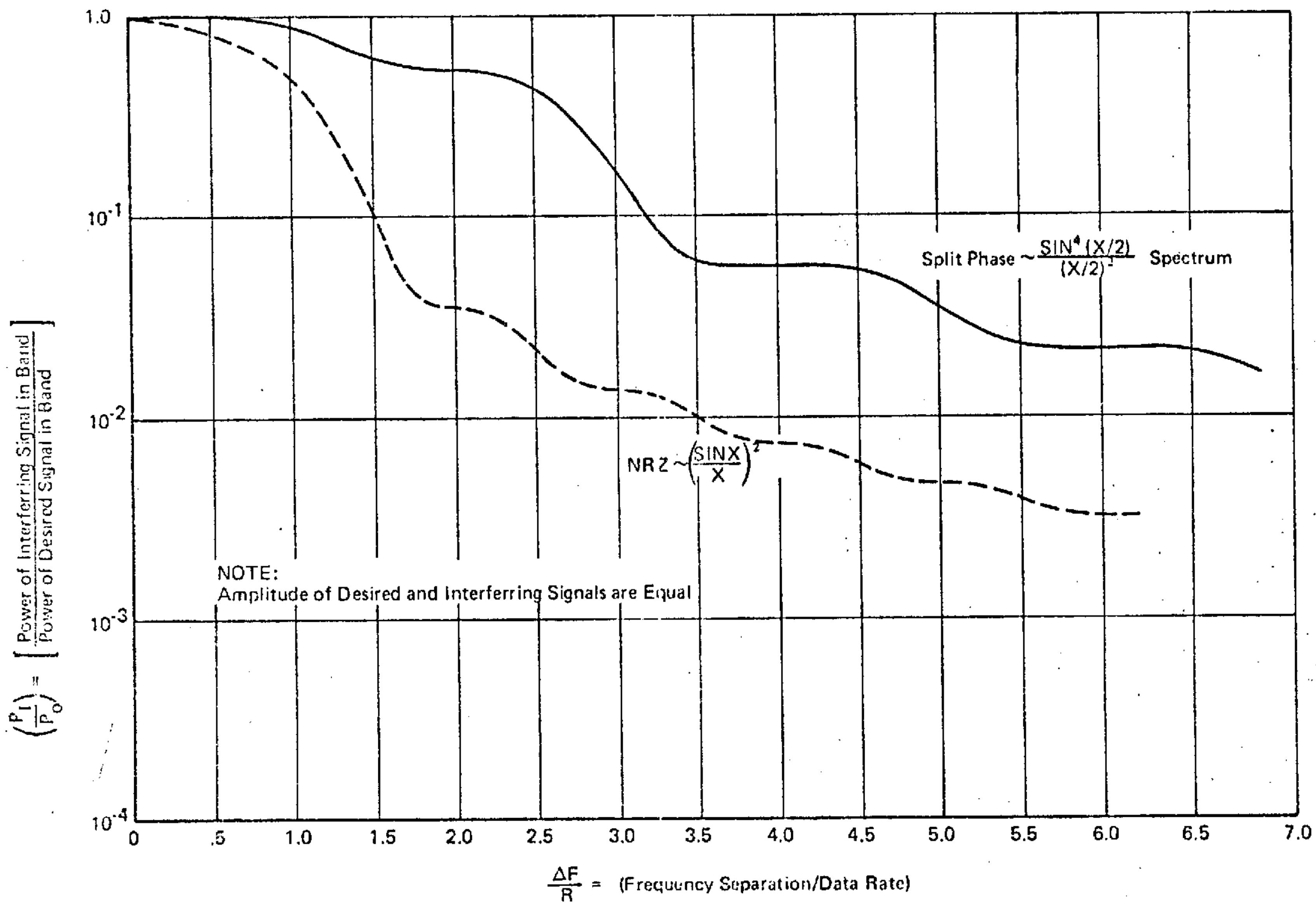

FIGURE 4.3 INTERFERRING POWER RATIO

(S_I/S_o) = ratio between main lobe power of interfering singal to main lobe power of desired signal.

With these definitions, an effective energy per bit per noise density is derived.

$$E_B/\eta \text{ Effective} = \frac{E_B}{\eta_o + \Sigma P_I/B} = \frac{1}{\eta_o/E_B + 1/\beta\, \Sigma}$$

$$= \left\{ \frac{E_B/\eta_o}{1 + \frac{E_B/\eta_o}{\beta} \Sigma(P_I/P_o)(S_I/S_o)} \right\}$$

where the summation is over all the interfering signals present—including consideration of their individual separations in frequency from the desired signal as well as their relative power as determined from different free space loss and/or antenna patterns.

With this relationship an estimate of bit error rate in the presence of interference can be obtained under an assumption of a given signal format—i.e., PSK, DPSK, etc. As an example if PSK signaling is assumed, then the variation of bit error rate with interference level $(\Sigma(P_I/P_o)(S_I/S_o))$ is that shown in Figure 4.4 for NRZ modulation and in Figure 4.5 for split phase modulation. As noted, the abcissa of these curves is the energy per bit to noise density with no interference present.

For purposes of evaluation, this model of interference can be compared with a more exact representation of the bit error rate for a PSK signal in the presence of one other interfering PSK signal. This representation is based upon unpublished notes of Dr. A. Ghais at NASA/GSFC. This comparison of the above interference model with this representation is shown in Figure 4.6 for NRZ modulation wherein the interfering signal power level is 10 dB above the desired signal and the bit error rate without interference is about 10^{-6}.

Based upon Figure 4.6, the interference model to be employed in the ensuing analyses appears to provide conservative estimates of bit error rates. On the other hand, where there are multiple interfering signals present, it is anticipated that the interference model will be reasonably accurate.

Distribution of Interference. With this noise model, the distribution of interference level within a system will give the distribution of energy per bit per noise density and therefore, the distribution of bit error rate. In this regard,

PROBABILITY OF BIT ERROR

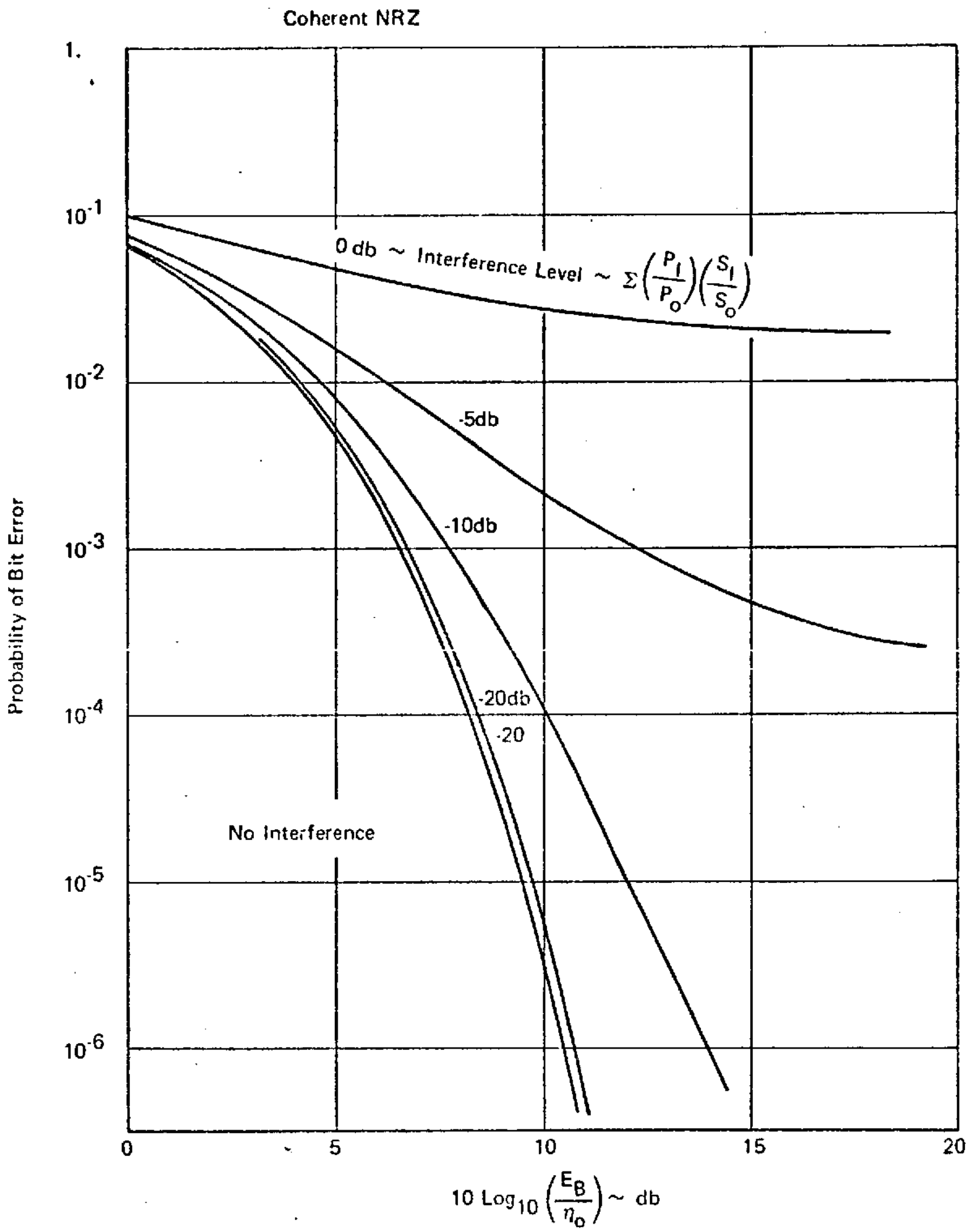

FIGURE 4.4 PROBABILITY OF BIT ERROR

PROBABILITY OF BIT ERROR

Coherent Split Phase

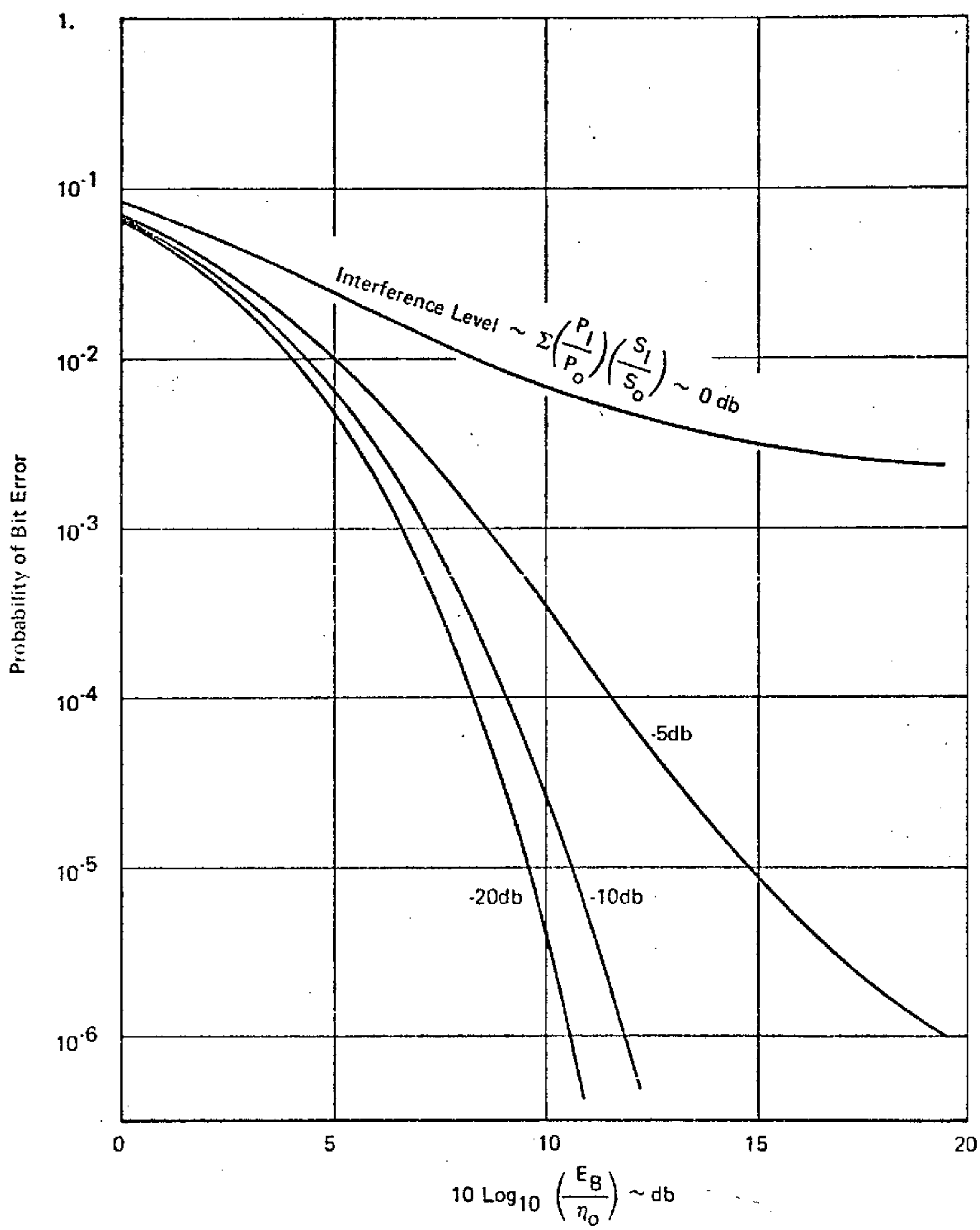

FIGURE 4.5 PROBABILITY OF BIT ERROR

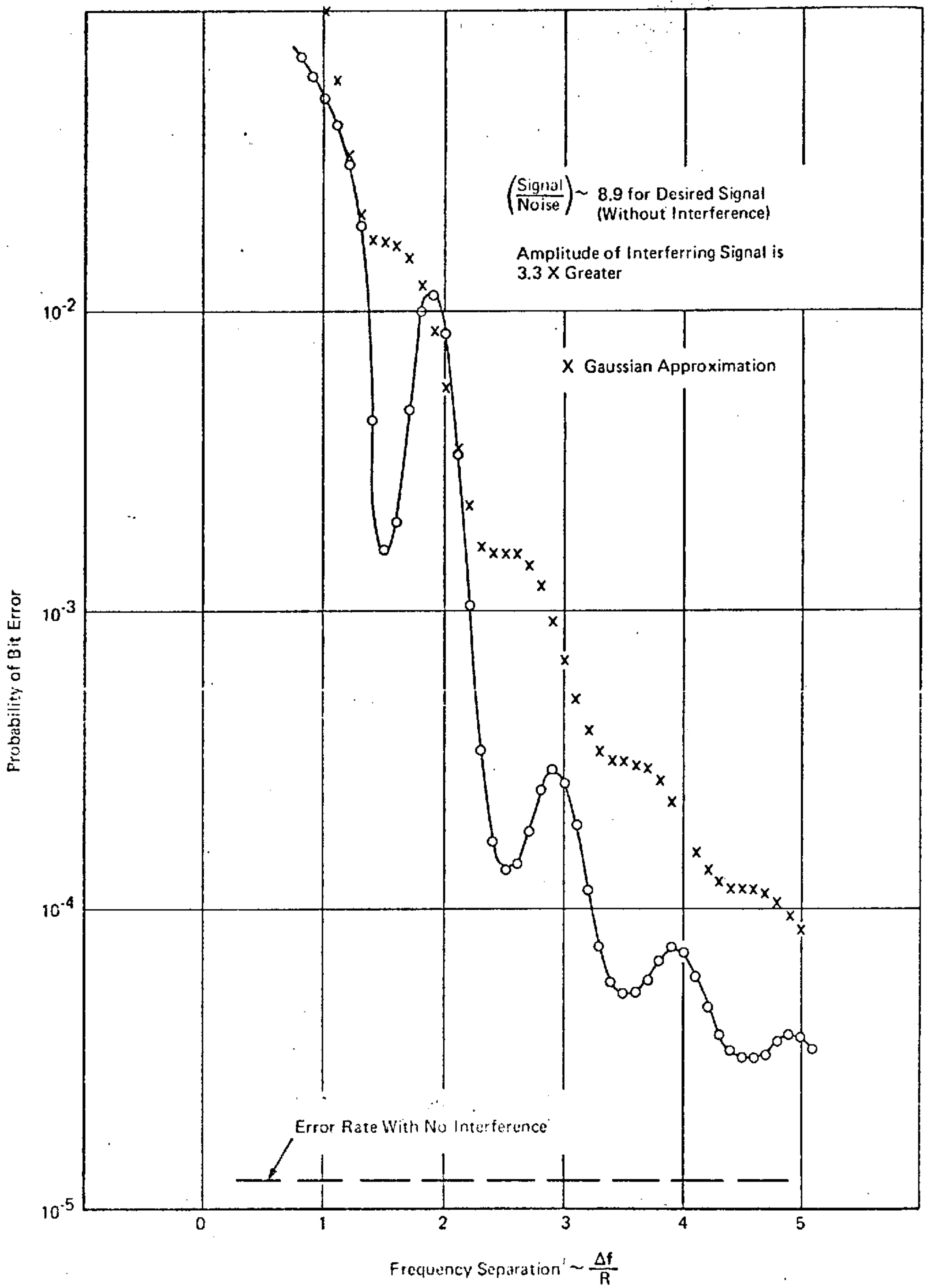

FIGURE 4.6 COMPARISON OF BIT ERROR PROBABILITIES

the term interference level refers to the sum $\Sigma (P_I/P_o)(S_I/S_o)$ appearing in the denominator of the above expression for E_B/η. The Appendix provides a detailed derivation of the means by which the interference levels are obtained. Numerical results are provided in Figures 4.7 to 4.10.

Figure 4.7 shows the variation of the probability density function of interference level for NRZ modulation and data rates of 100, 200, and 400 bits/second. As noted, these data assume free space loss variation of signal amplitudes reaching the satellite from different platform locations within the visibility circle. Also, the satellite orbit is a two hour sun-synchronous orbit. These data correspond to a platform located at the noted location in Figure 4.1 for the condition where there is only one interfering signal present. Note also, the ordinate of Figure 4.7 is not the density function itself but rather, the density function multiplied by the interference level itself. This product is shown instead of interference level for two reasons.

- The order of magnitude of the product is relatively constant which facilitates plotting
- By plotting this product versus the logarithm of the interference level, the area under a given portion of the curve is directly proportional to the probability of the interference lying within the interference points defining the area "under the curve."

For example, the probability of the interference level lying between -40 and -45 dB for the conditions of Figure 4.7 is approximately .259 if the data rate is 100 bit/second.

If instead of free space loss variation the signal amplitudes at the satellite are equal, then the density function of interference level becomes that shown in Figure 4.8. A close comparison of these data with that of Figure 4.7 will show that there is an expected decrease in interference probabilities at any given level of interference.

Another aspect of Figure 4.7 and particularly Figure 4.8 is the occurrance of spikes in the density functions at particular interference levels. The reason for these spikes can be seen by referring to the interference power ratio versus frequency separation given in Figure 4.3. At frequency separations that are integer multiples of the data rate, there are relatively large bands wherein the interference level changes by small amounts. Because the distribution of interference is derived from the distribution of received frequencies (and this distribution is relatively constant as discussed in Section 4.2.2), the values of interference at these points tend to occur with high probability. For

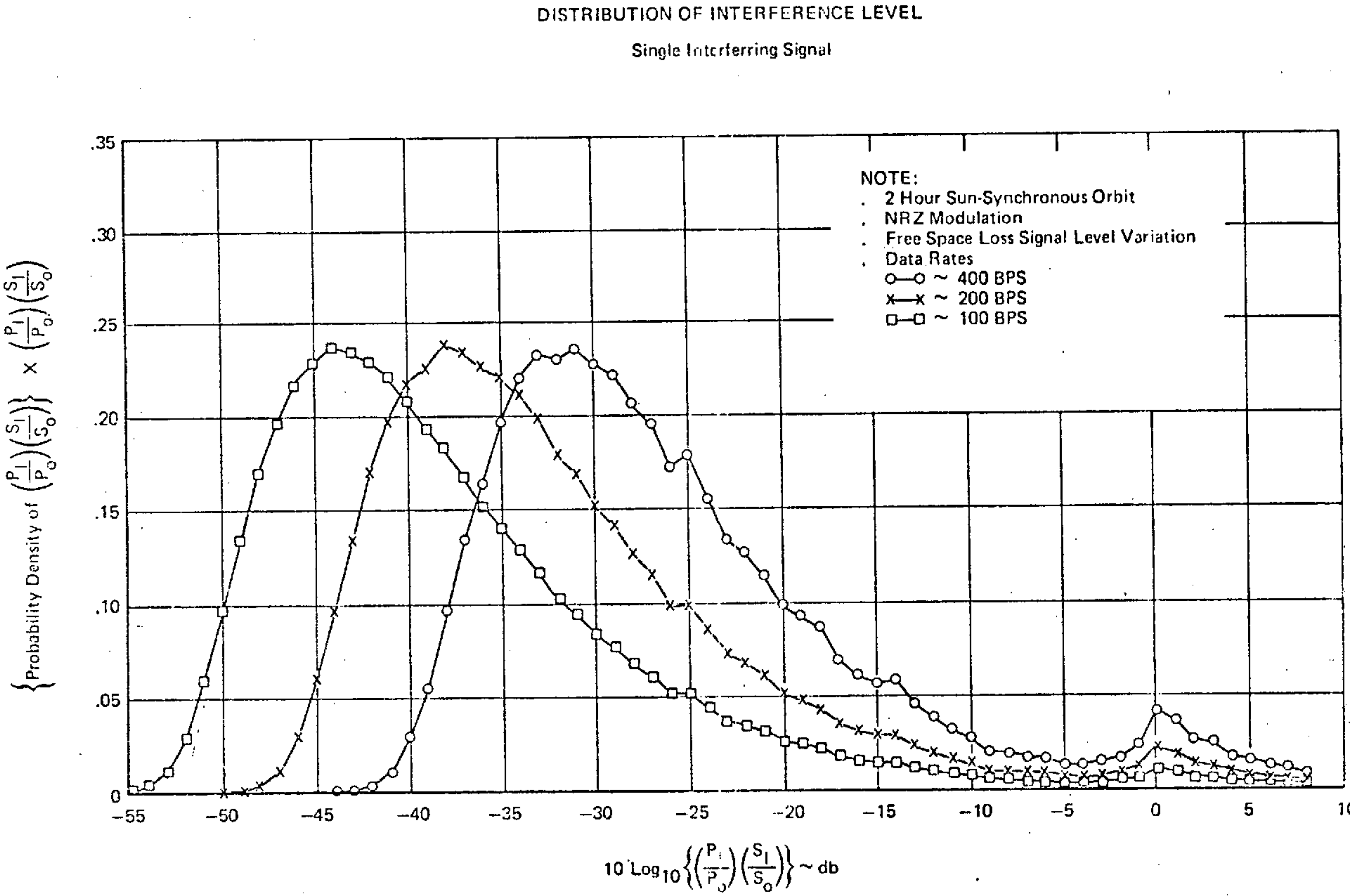

FIGURE 4.7 DISTRIBUTION OF INTERFERENCE LEVEL

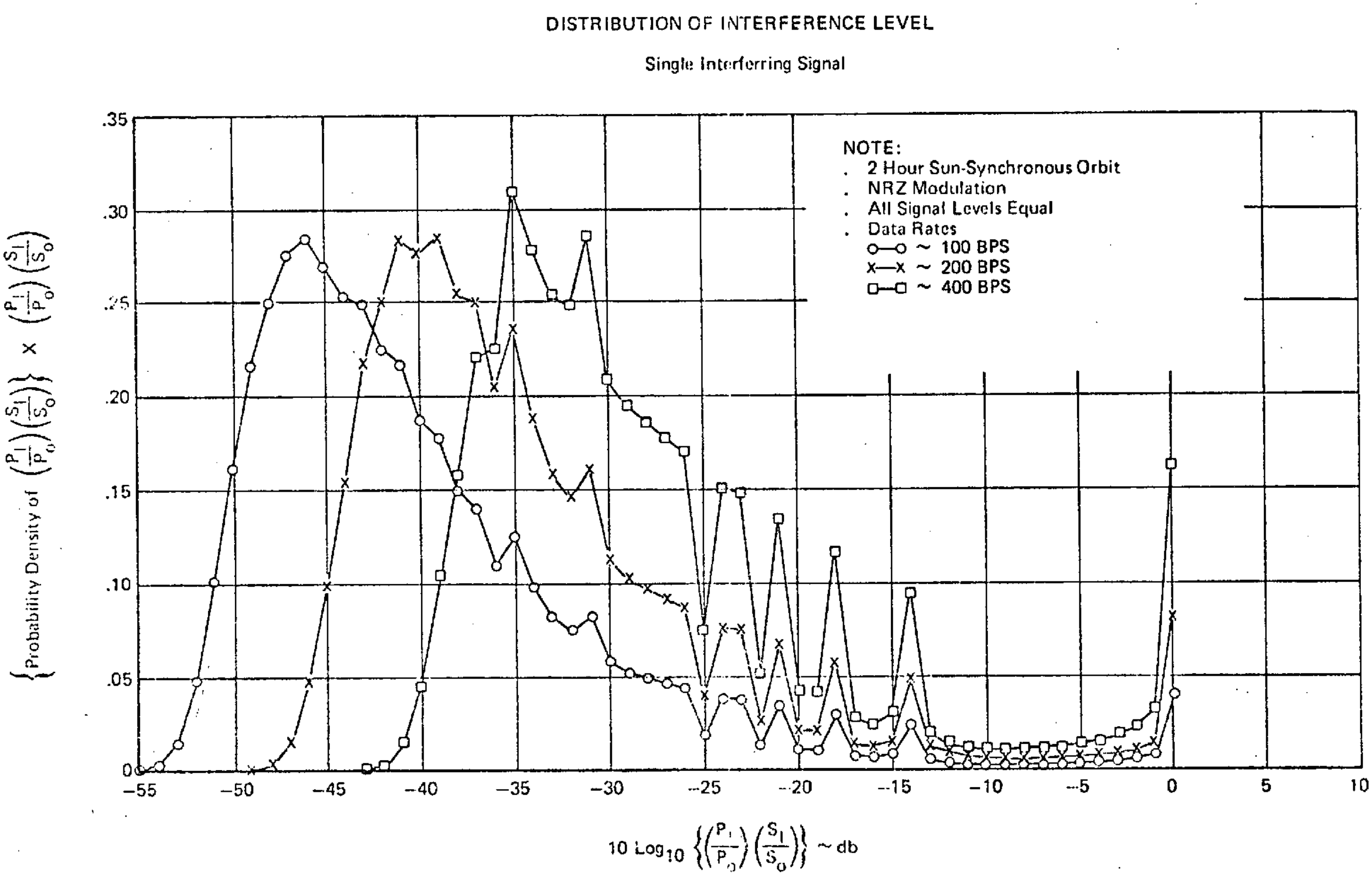

FIGURE 4.8 DISTRIBUTION OF INTERFERENCE LEVEL

example, the first point on Figure 4.1 for NRZ signaling wherein the interference level remains relatively constant while the frequency separation between two signals changes is where the frequency separation is twice the data rate. The interference level at this point is approximately 3.5×10^{-2} (or -14.6 dB) which corresponds to the first spike seen in Figure 4.8 below the 0 dB spike.

Another presentation of the data in Figures 4.7 and 4.8 is given in Figures 4.9 and 4.10 where the cumulative probability is shown as a function of interference level. Figure 4.9 is derived from the density function of Figure 4.7 for the case of free space loss variation in signal amplitudes. Figure 4.10 corresponds to equal amplitude signals whose density function is given in Figure 4.8.

Taken individually these cumulative distributions indicate the rapid increase in interference level as the data rate of the transmissions increase. At fixed values of cumulative probability less than about .9, doubling the data rate from 100 to 200 bps or from 200 to 400 bps results in a 6 dB or factor of four increase in the corresponding level of interference. However, for cumulative probabilities above .9, there is more than four fold increase. For example, from Figure 4,10 at probability .95, increasing the data rate from 200 bps to 400 bps results in the corresponding interference level increasing from about -16 dB to -4 dB or more than an order of magnitude increase.

Another aspect of Figures 4.9 and 4.10 is the value of cumulative probability at interference levels wherein the bit error rates might be considered acceptable. This can be determined by referring to the bit error rate data presented in Figure 4.4. If the energy per bit per noise density is assumed to be 10 dB with no interference present, Figure 4.4 indicates the degradation in bit error probability from the minimum value of $.3 \times 10^{-5}$ to 10^{-4} at an interference level of -10 dB and about 2×10^{-3} at -5 dB. From Figure 4.10 and for example a data rate of 200 bps, the probability of interference level being less than -10 dB is about .965 and for -5 dB about .972. Therefore, the bit error rate will be less than 10^{-4} with probability .965 and less than 2×10^{-3} with probability .972.

Comparing the data of Figures 4.9 and 4.10 indicates the effects of equal and free-space loss signal amplitude variations. At low interference levels, a slight reduction in interference level, 2 to 3 dB, is achieved at equal probability levels with equal amplitude signals. At high interference levels, the reduction is more pronounced. However, the location of the platform being interfered with should be emphasized. This platform, for free space loss signal variations, experiences the greatest free space loss relative to nearly all other platforms in view of the satellite. Therefore, the amplitudes of all interfering signals are higher.

DISTRIBUTION OF INTERFERENCE LEVEL

Single Interferring Signal

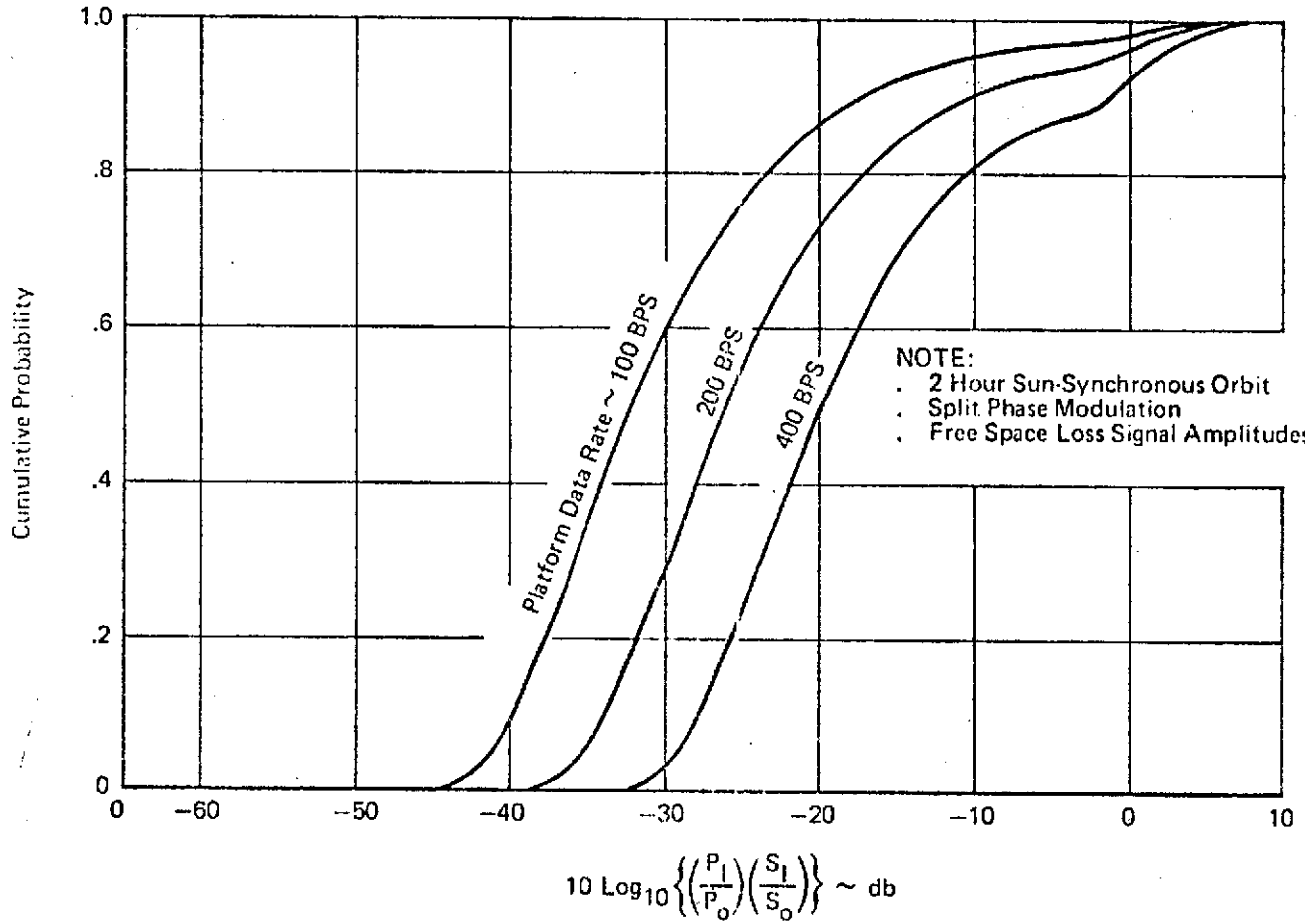

FIGURE 4.9 DISTRIBUTION OF INTERFERENCE LEVEL

DISTRIBUTION OF INTERFERENCE LEVEL

Single Interferring Signal

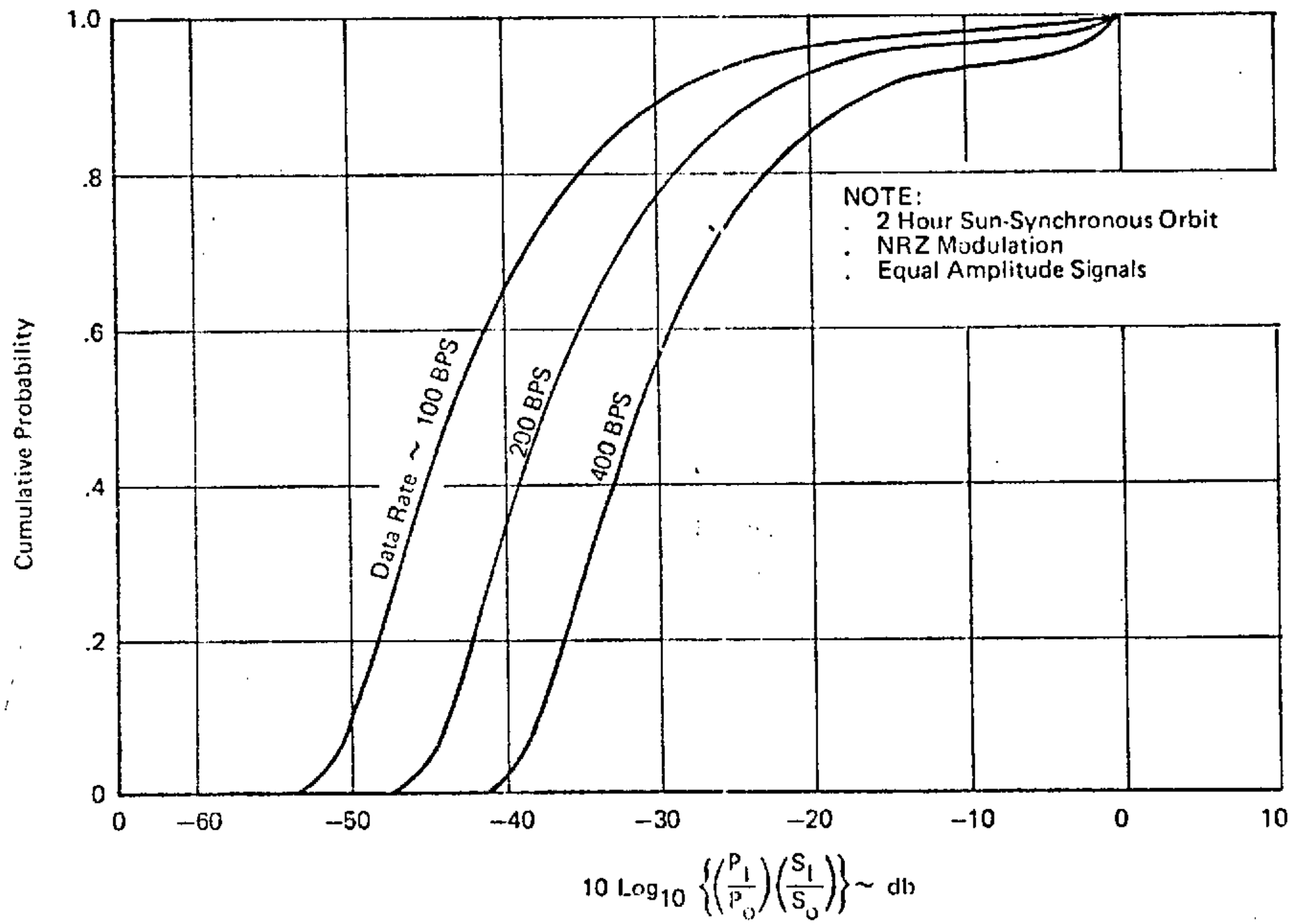

FIGURE 4.10 DISTRIBUTION OF INTERFERENCE LEVEL
SINGLE INTERFERRING SIGNAL

To evaluate the effects of modulation techniques, the cumulative distribution of single-interfering-signal interference levels for split phase modulation is shown in Figures 4.11 and 4.12—Figure 4.12 being equal amplitude signals and 4.11 being the free space loss case. The wider spectrum of the split phase modulation is seen to give considerably higher interference levels compared to the NRZ data of Figures 4.9 and 4.10. For example, with a data rate of 200 bits per second, there is probability .965 that the interference level will be less than -10 dB for NRZ modulation and equal amplitude signals. For split phase modulation with the same conditions, this probability is seen to be .93 from Figure 4.12. However, at equal bit error probabilities the reduction is not as severe.

For a 10 dB energy per bit per noise density without interference, an interference level of -10 dB will give about 10^{-4} bit error probability for NRZ modulation—Figure 4.4. At these same conditions for split phase modulation, the interference level is approximately -8 dB at the 10^{-4} bit error probability. From Figure 4.12, the cumulative probability of -8 dB interference level is seen to be about .935 which is slightly higher than the .93 probability of -10 dB or less.

The previous discussions have described the statistics of interference level for the case where a single interfering signal is present. For random access however, the presence of one or more or no interfering signals is also a statistical quantity. Therefore, two additional steps are required to characterize interference level statistics

- The single interferor data typified by the data presented in Figures 4.7 and 4.8 must be convolved to determine the density functions of interference level with multiple signals present
- The previously derived Poisson model of the time of occurrance statistics must be convolved with the interference statistics.

The effects of multiple interfering signals are indicated in Figures 4.13 and 4.14.

In Figure 4.13, the density function of interference level is shown for the case of NRZ modulation, free space loss signals and as many as 10 simultaneously interfering signals. As would be expected, the major effect of multiple signals is the elimination of the lower values of interference level and the enhancement of the higher values. Figure 4.14 indicates for two interfering signals present the density functions of interference for NRZ and split phase modulation at data rate of 400 bps. The greater susceptibility of split phase signals to significant interference is seen to be enhanced with the presence of multiple interferers compared to NRZ signals.

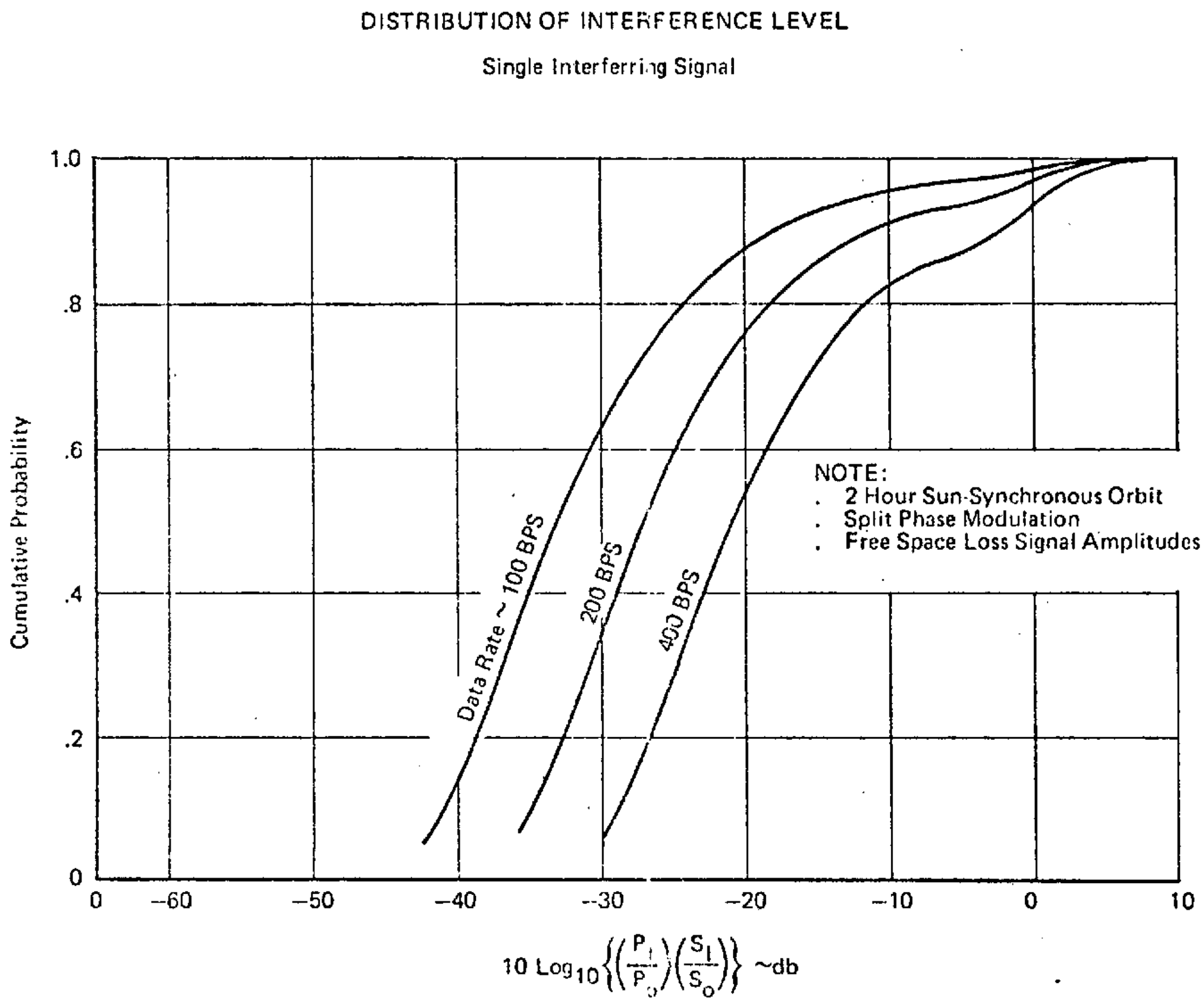

FIGURE 4.11 DISTRIBUTION OF INTERFERENCE LEVEL
SINGLE INTERFERRING SIGNAL

DISTRIBUTION OF INTERFERENCE LEVEL

Single Interferring Signal

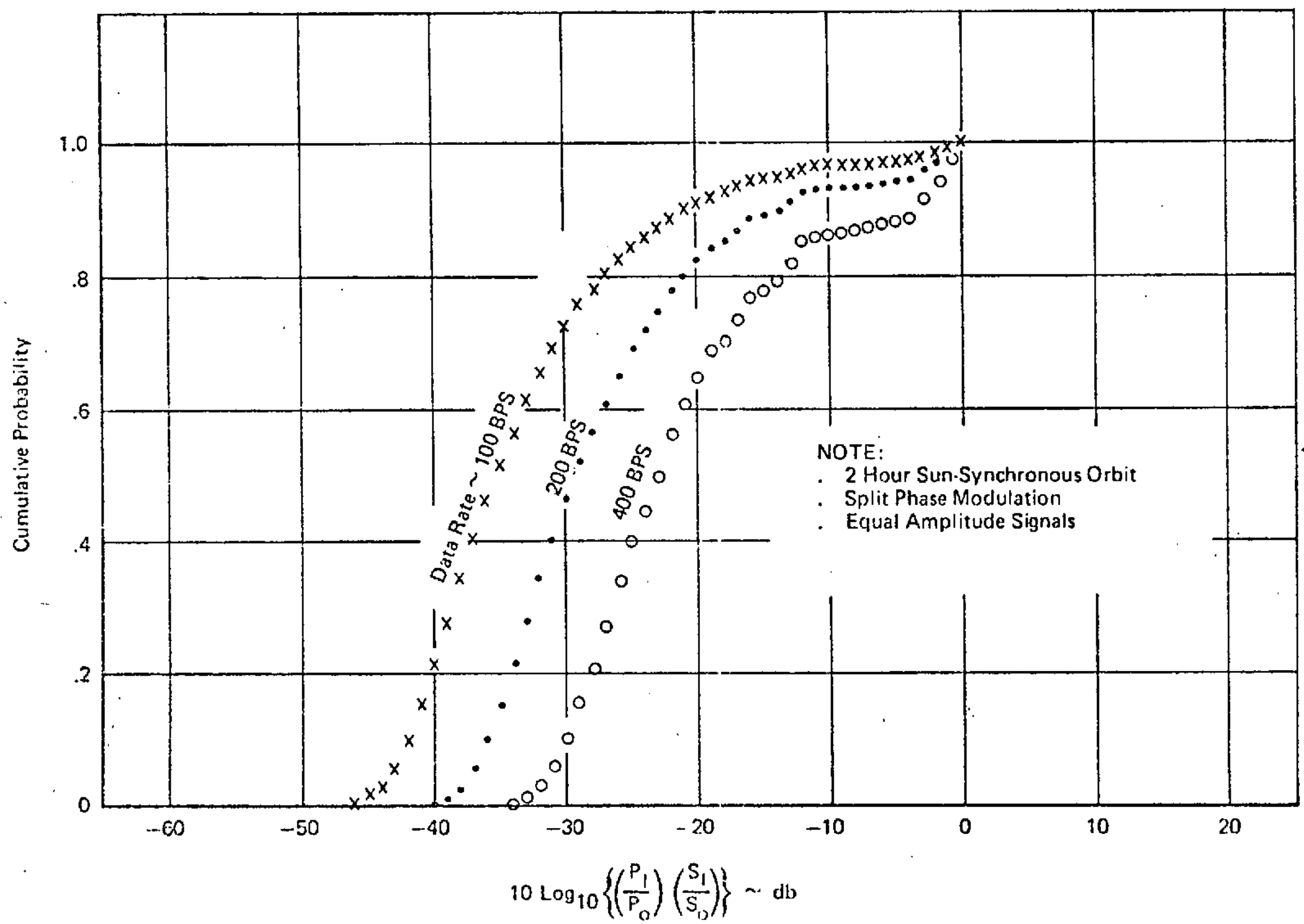

FIGURE 4.12 DISTRIBUTION OF INTERFERENCE LEVEL
SINGLE INTERFERRING SIGNAL

DISTRIBUTION OF INTERFERENCE LEVEL

Multiple Interferring Signals

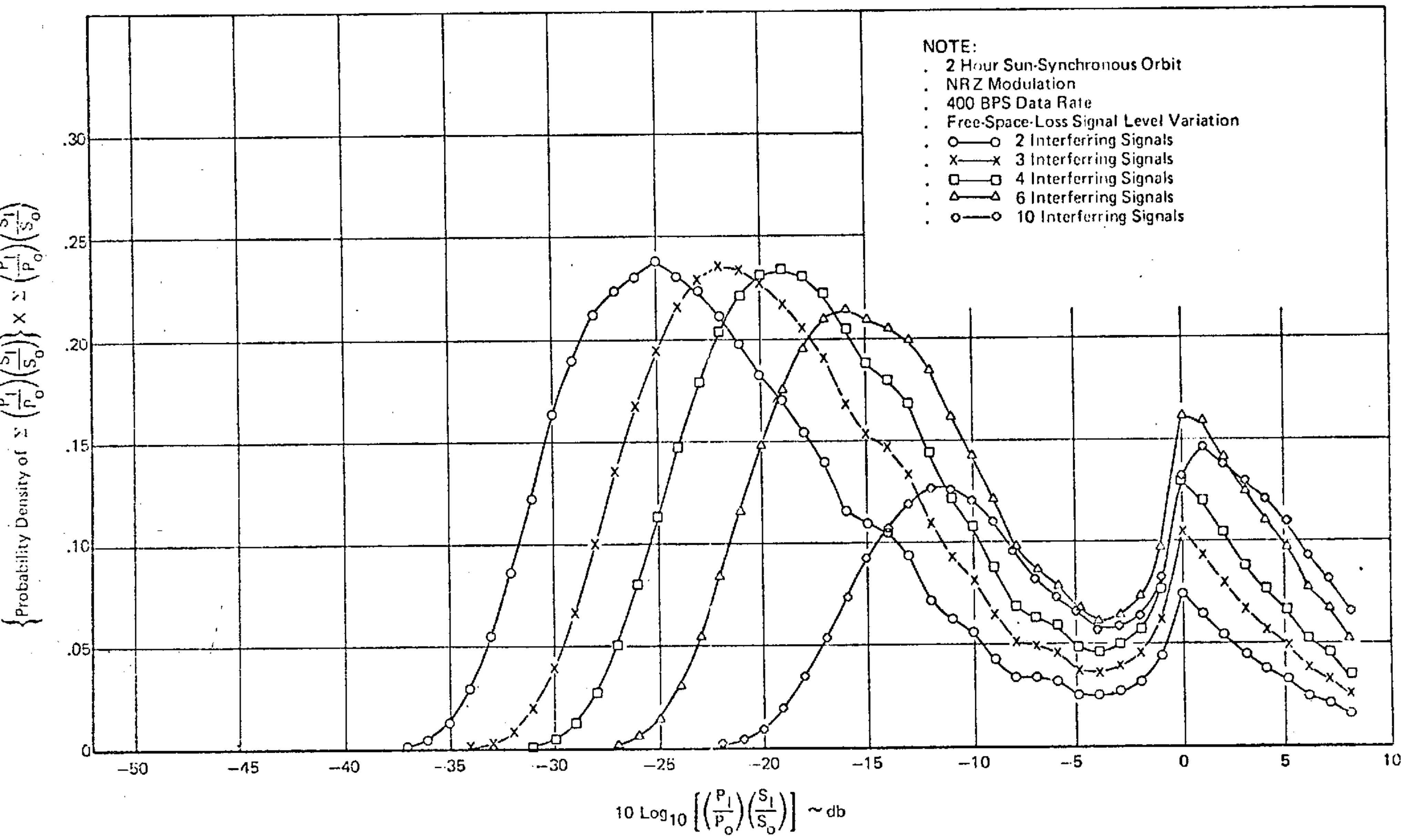

FIGURE 4.13. DISTRIBUTION OF INTERFERENCE LEVEL

(Multiple Interferring Signals)

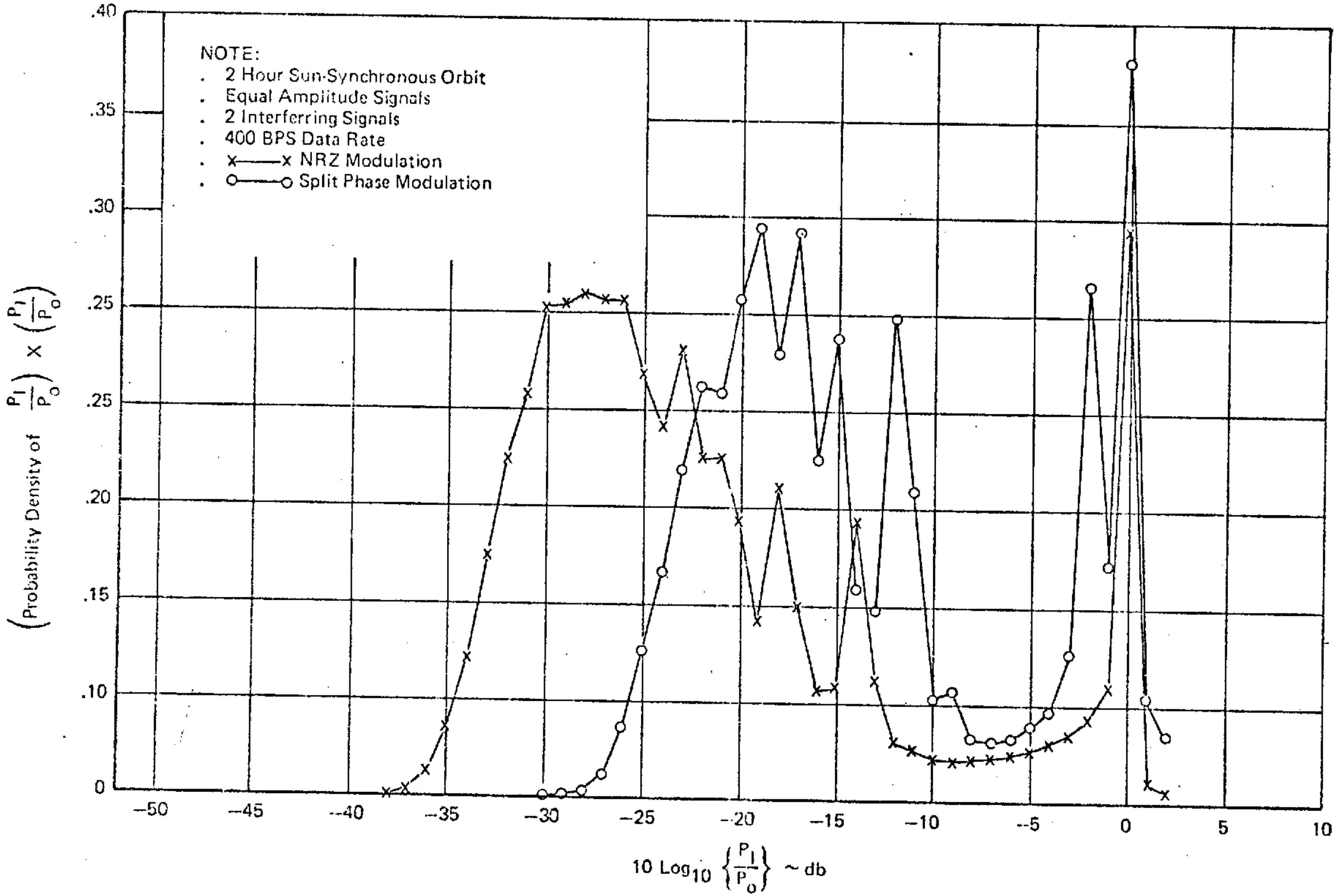

FIGURE 4.14 DISTRIBUTION OF INTERFERENCE LEVEL

By statistically combining data similar to that of Figure 4.13 with the Poisson statistics of platform transmissions, the statistics of both time and spectral overlap of platform transmissions can be obtained. These data represent the interference statistics of a random access system from the viewpoint of the limitations imposed by the finite time-frequency space. In otherwords, these data characterize systems wherein no performance degradation is caused by a signal processor on-board the satellite. These data are indicated in Figures 4.15 and 4.16.

In Figure 4.15, the probability of bit error being less than 10^{-4} is shown as a function of system data rate for NRZ and split phase modulation under the assumption that signals arriving at the satellite are of equal amplitudes. As noted, the bit error rate would be about 3×10^{-6} without interfering signals. For NRZ modulation, this error rate corresponds to energy per bit per noise density ratio of 10 dB as can be seen in Figure 4.4. Also, at 10^{-4} error rate, the interference level is about -10 dB. Therefore, the ordinate of Figure 4.15 is entirely equivalent to the probability of interference level being less than -10 dB. From Figure 4.5 for split phase signaling, the interference level for 10^{-4} error rate can be seen to be between -7 and -8 dB—again for the condition of 10 dB energy per bit per noise density. Similar data is presented in Figure 4.16 for the case where the amplitudes of received signals differ by free space losses.

The most important aspect of Figures 4.15 and 4.16 is the absence of platform data rate (R) as an independent parameter. The only platform parameter of concern is the total number of bits transmitted during each transmission (τR) and not the individual parameters of data rate or transmission duration.

This conclusion was also reached in the derivation of "zero-one" probability of interference presented in Section 4.2.2. For comparison purposes, the "zero-one" probability relationships are used to draw the noted curves of Figure 4.15 and 4.16. These can be seen to be conservative estimates particularly in the case of equal amplitude signals. In this regard, correspondence between the "zero-one" type interference and the more exact model can be accomplished by adjusting the constants for interference probability. For equal amplitude signals, this adjustment leads to

$$\text{for NRZ } P_I \sim 2.4 \frac{N\tau R}{TF_T}$$

$$\text{for split phase } P_I \sim 4.4 \frac{N\tau R}{TF_T}$$

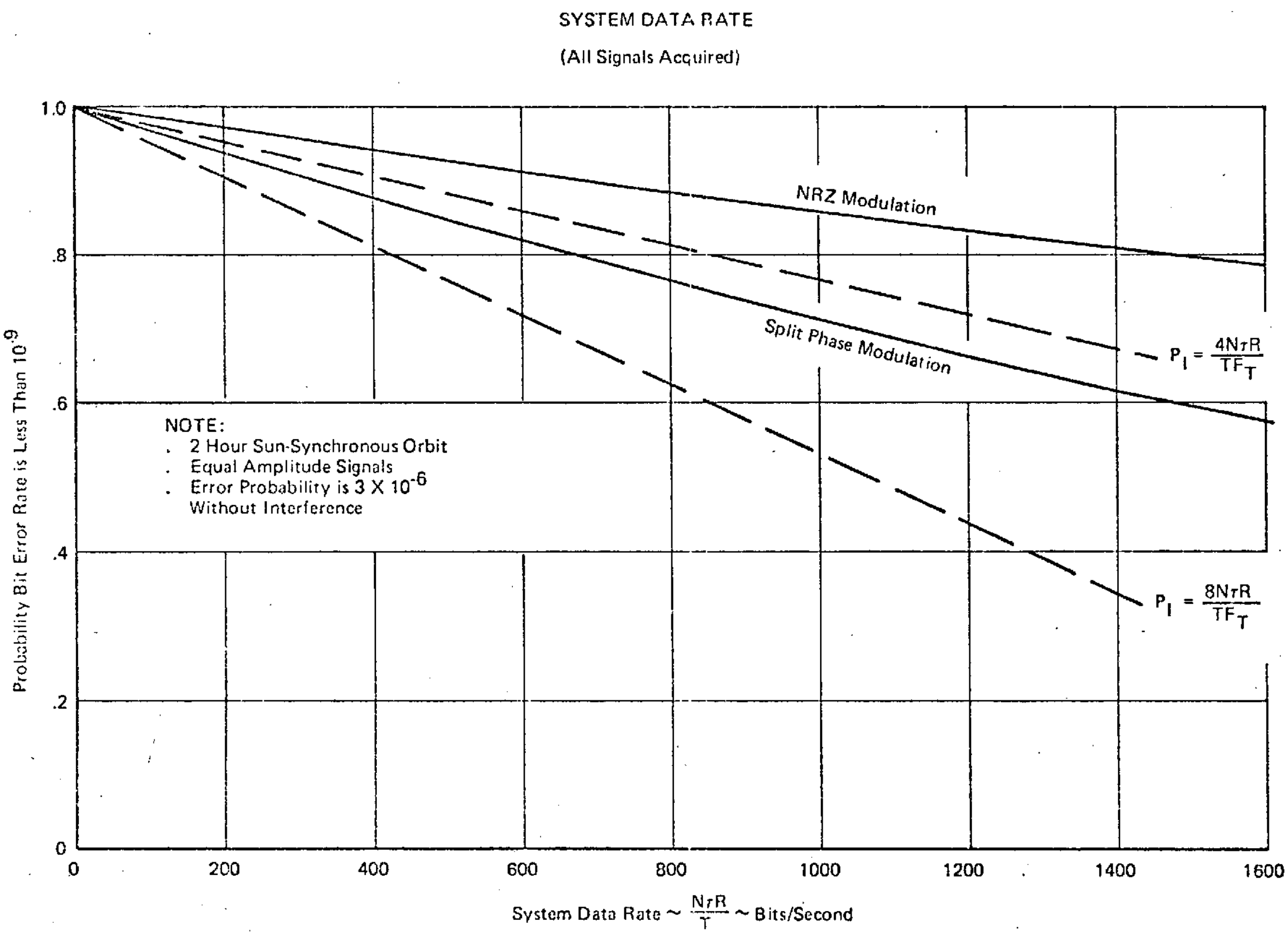

FIGURE 4.15 SYSTEM DATA RATE

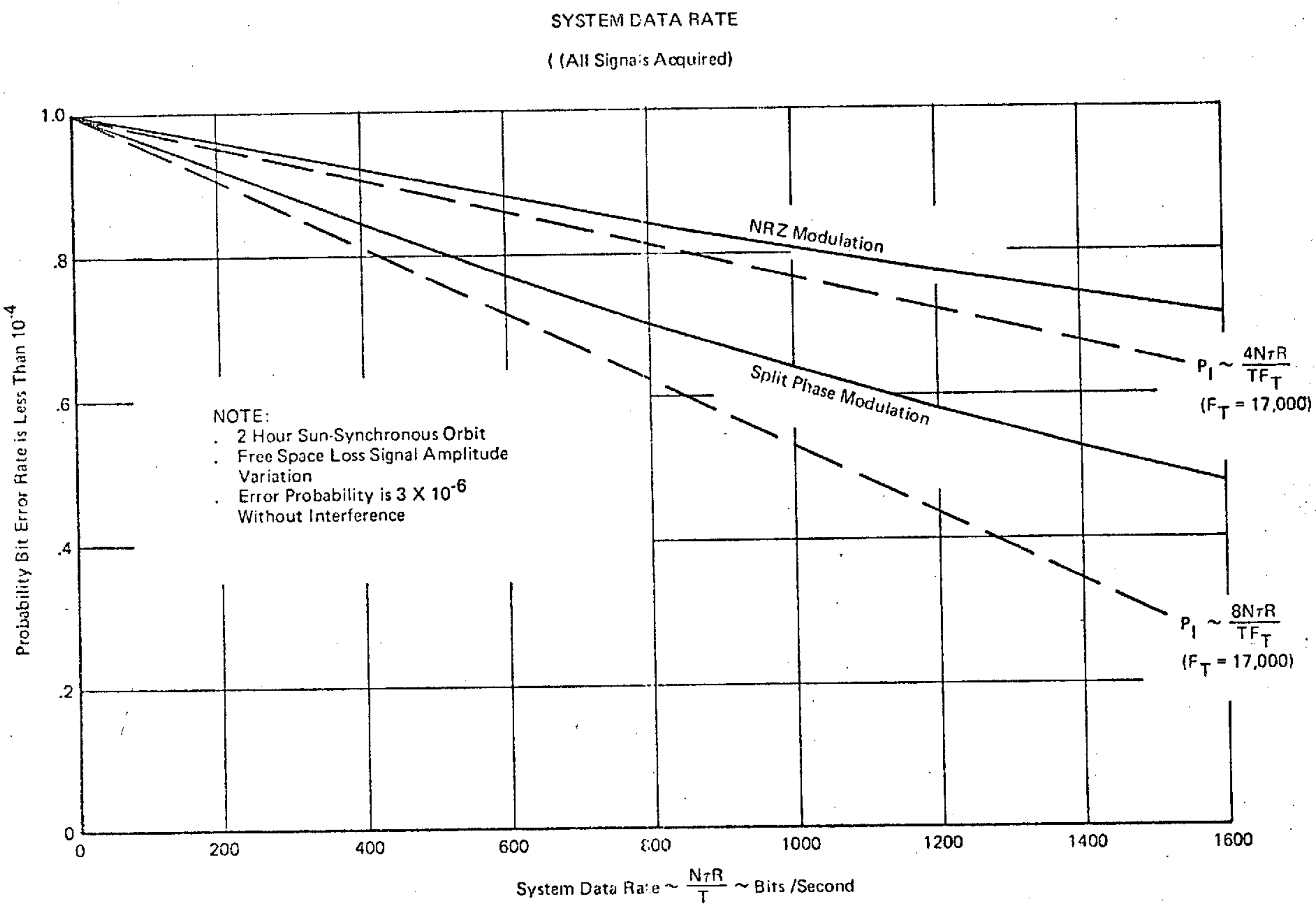

FIGURE 4.16 SYSTEM DATA RATE

where F_T is 17,000 Hertz for the two hour orbit used to compute the data of Figures 4.15 and 4.16. For the case where received signal amplitudes vary by free space losses, the "zero-one" interference constants can be adjusted to be

$$\text{for NRZ} \quad P_I \sim 3.2 \, \frac{N\tau R}{TF_T}$$

$$\text{for split phase} \quad P_I \sim 5.4 \, \frac{N\tau R}{TF_T} \, .$$

This adjustment of the "zero-one" interference constants can be carried one step further. In particular, if data similar to that of Figures 4.15 and 4.16 is computed for different levels of bit error probability, then the "zero-one" constants can be empirically determined as a function of bit error probability. For example, Figure 4.17 is a graph of the probability that bit error rate is less than a given value with this value as the parameter of the curves. As noted, these data correspond to NRZ signaling with equal amplitude signals. If this is done for split phase and for free space loss signal amplitudes, then the data of Figure 4.18 result.

In Figure 4.18, the constant of proportionality between "zero-one" interference and system data rate is shown as a function of bit error probability with modulation and signal amplitude distribution as parameters. Using these constants and limiting their usage to relatively high probabilities, the probability that bit error rate is less than a given value can be computed in close correspondence to the more exact interference model by means of

$$\text{probability bit error is less than BER} = 1 - P_I \sim \frac{KN\tau R}{TF_T} \quad (P_I^2 \ll 1)$$

where K is the proportionality constant shown in Figure 4.18 as a function of bit error rate (BER).

On-Board Processor Interference. The data previously presented for the performance of random access systems assumed that there was no degradation caused by the signal processor on-board the satellite. This meant, for example, that regardless of the number of transmissions simultaneously present, there is always, a data channel available in the processor—i.e., the on-board processor was assumed never to saturate. For asynchronous arrival of transmissions, such a non-saturating processor is not physically realizable because of the large number of parallel data channels required. Therefore, comparison of different random access systems requires consideration of the effects of the limitations of physically realizable processors.

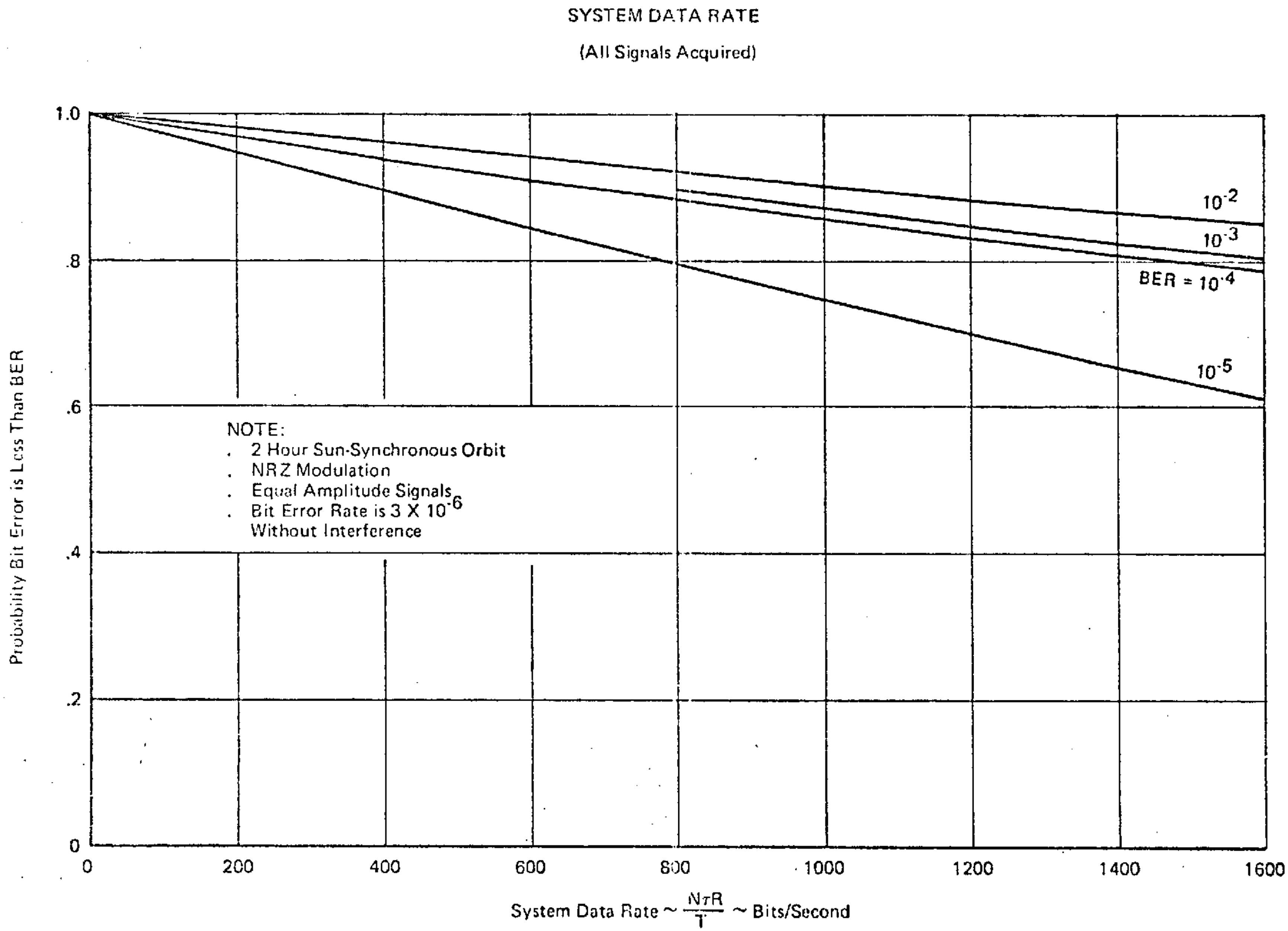

FIGURE 4.17 SYSTEM DATA RATE

"ZERO-ONE" INTERFERENCE CONSTANT

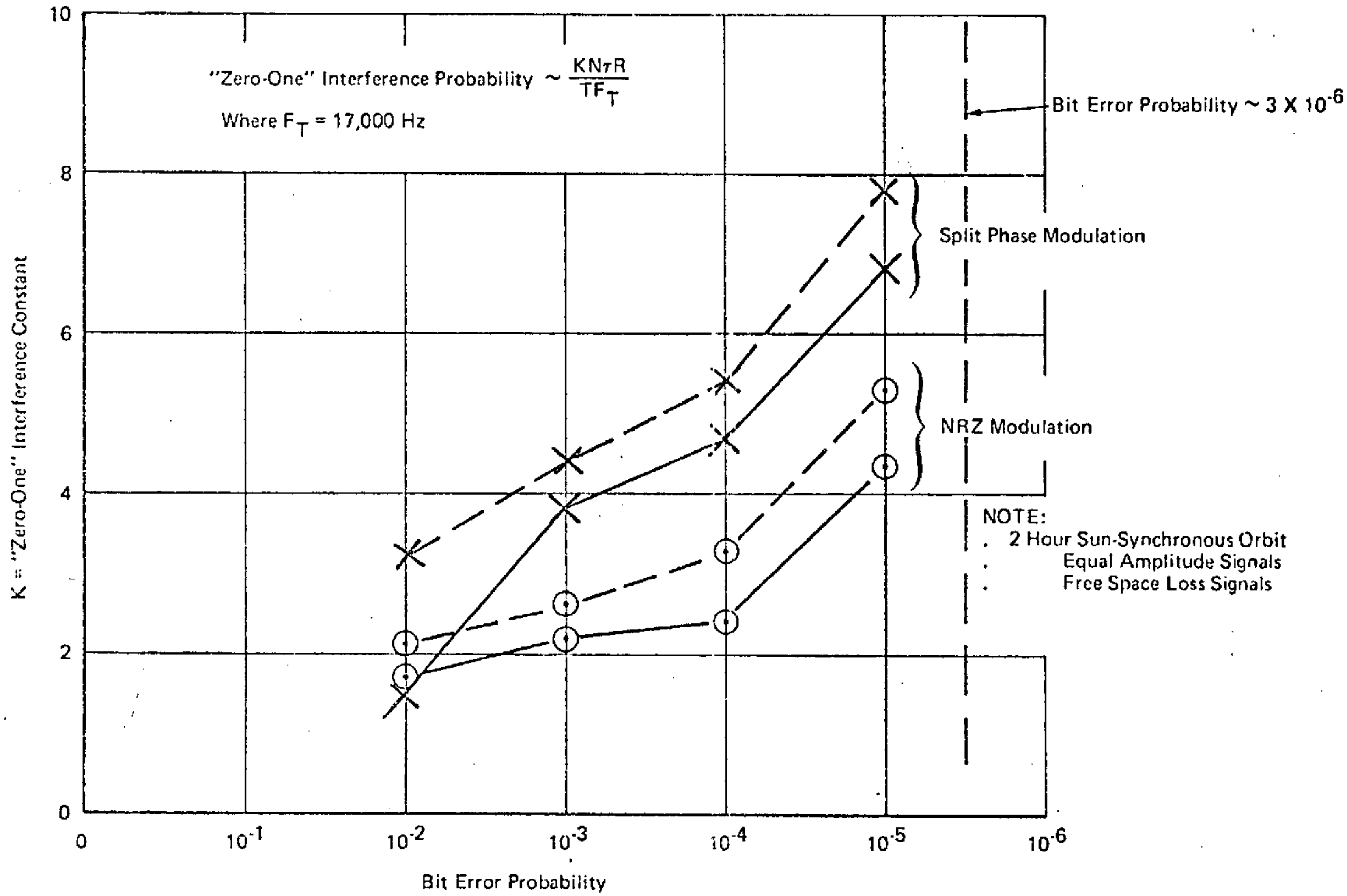

FIGURE 4.18 "ZERO-ONE" INTERFERENCE CONSTANT

In order to model the performance of on-board processors, three basic functions must be accommodated:

- Detection—the arrival of a platform transmission in both time and frequency must be recognized by the processor
- Demodulation—once detected, a platform transmission must be assigned to a data channel wherein the data contained within the message is extracted
- Storage—upon demodulation, the data derived from each message must be stored for subsequent re-transmission to the ground.

The detection function is separated from the demodulation function for the random access systems considered herein because of the data rates, platform power and received bandwidths. For example, the signal power required for a 100 Hertz data rate where demodulation at 10 dB signal/noise within a 200 Hertz band is performed is much less than the noise power with a 17 kilohertz receiver band. Therefore, a signal detection process whether by comb filters, sweeping filters, fast fourier transform etc. must take place prior to demodulation.

Regardless of the technique employed to perform the detection function, the statistical model describing its performance is assumed to be essentially the same. The total bandwidth of the receiver is effectively divided into a given number of cells within which the presence or absence of platform transmissions is sensed on a periodic basis. Aside from the probabilities of detection and false detection, the first type of interference experienced by a physically realizeable processor is the limited resolution of the detector(s). If two signals are simultaneously present, recognition of the fact that there are two signals instead of one will depend upon how widely separated in frequency the signals are. In this regard, however, the presence of two signals that are closer than some fixed amount may not be important because this condition would mean both time and spectral overlap causing excessive interference. To preclude assignment of a data channel to both of the obviously interfering signals, the detection function/logic can be established to ignore the presence of an interfering signal whose frequency is closer than a fixed interval to a previously detected signal.

This logic of ignoring signals closer than some frequency interval to a previously detected signal can be modeled by means of guard bands. In particular, a signal is presumed to be ignored and therefore lost during the detection process if its frequency is closer than a specified interval (guard band) to a previously detected signal. This concept of a guard band is closely related then to the frequency interval for interference defined in the "zero-one" interference model previously described.

Signal Detection. The detection in time and frequency of arriving signals is characterized by the number of signals present at a point in time and their distribution with regard to frequency separation. The probability of a given number (n) of signals being present at a specific point in time is determined from the Poisson distribution by computing the probability of n transmissions being initiated during the duration τ of a transmission. Therefore, a fundamental parameter is the Poisson parameter ($N\tau/T$)—i.e., the average arrival rate N/T multiplied by the time interval τ.

The frequency distribution of the n simultaneous signals is characterized by the distribution of n random transmissions in m discrete frequency cells. In this regard, the distribution of received frequencies for a 2 hour sun-synchronous orbit is shown in Figure 4.2 for the case of uniformly distributed platforms in the satellite's visibility circle and for transmission frequencies that are normally distributed about the mean frequency with standard deviation of 2,000 Hertz. However, to facilitate modeling of the detection process, the frequency distribution of Figure 4.2 is approximated by a uniform distribution over the significant part of the band—for the satellite of Figure 4.2 this band is about 17 kilohertz. With this assumption, the probability of a transmission lying within a particular frequency cell (Δf) is equal to Δf divided by the band (17 khz) or more simply, the reciprocal of the number of equal width frequency cells.

The statistics of the detection function modeled in this fashion are the distribution of n randomly thrown balls falling into m equally sized boxes—Maxwell-Boltzman occupancy statistics.* These statistics serve to determine two characteristics of an on-board detector and in turn the logic necessary to ignore obviously interfering signals.

Regarding the logic to ignore obviously interfering signals, a transmission from a platform is assumed to be ignored if at the moment in time its transmission starts, there is another signal within its frequency cell. The probability of this not occurring is the probability that none of the n signals present fall within the cell of the specified transmission which is $(1 - 1/m)^n$—where m is the number of discrete frequency cells.

The other part of the detector statistics are concerned with the number of cells occupied by the n signals present at the start of the specified transmission. These statistics are used to determine whether or not a data extractor will be available to demodulate the specified transmission. For example, if there are K data extractors in the processor, then the probability of one of these being available is the same as the probability that any K-1 of the discrete frequency cells are occupied by the n simultaneously present signals.

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* Feller, "An Introduction to Probability Theory and its Applications"

With this model of detection statistics and the associated logic for assigning data extractors, Figures 4.19 through 4.22 can be drawn. These figures present the probability of the specified signal being assigned to a data extractor as a function of the Poisson parameter $N\tau/T$ with the number of data extractors as a parameter for the case where the detection band is divided into equal size frequency cells. Figure 4.19 corresponds to 4 frequency cells. Figure 4.20 is for 10 cells and Figures 4.21 and 4.22 are for 20 and 40 cells respectively.

In order to properly evaluate these data, some interpretation should be given to the Poisson parameter $N\tau/T$. This is with regard to the ability of the detection mechanism to not only detect signals that are present but also its false detection rate. If the detection of actual signals is essentially unity for the detector and the effective rate of false detections is essentially zero, then (N/T) is exactly the time averaged rate at which signals arrive at the satellite. However, this should be decreased by probability of detection and increased by false alarm rate of the detector.

The consequence of detection probability being significantly less than one is obvious in that detection probability is a multiplicative factor in determining the probability of signal/data acquisition. However, an effective (N/T) that is significantly greater than the arrival rate of real signals as caused by false alarm rate of the detection mechanism will rapidly degrade system performance. Attainment of sufficiently low rates of false detection may be difficult to obtain as can be seen by the following example.

If the detection mechanism is a comb filter comprising m separate filters (frequency cells), then presumedly the width of each filter (and therefore the number required to cover the signal band) is established to achieve high probability of detection. Also minimum signal level and detection threshold could be set for each filter to insure a low probability of false detection. However, if there are m such filters, then the rate of false detection is the product of m and the false detection probability for an individual filter. For example, if there are 100 filters and the probability of false detection in each filter is .01, then presuming high probability of detecting a real signal, the rate of false detections will equal the rate of arrival of real signals.

The use of Figures 4.19 through 4.22 when the rate of false detection is significant requires more than an adjustment to (N/T) as indicated above. Additionally, the ability of the system to recognize a false detection must also be specified in terms of the time required to do so. In particular, if τ_{FA} is the time required to recognize that a false signal has been detected, then the effective value of $N\tau/T$ to be used is the following

$$(N\tau/T)_{\text{Effective}} = \left(\frac{N\tau}{T}\right) + \tau_{FA} \times \text{(False Detection Rate)}.$$

PROBABILITY OF SIGNAL ACQUISITION

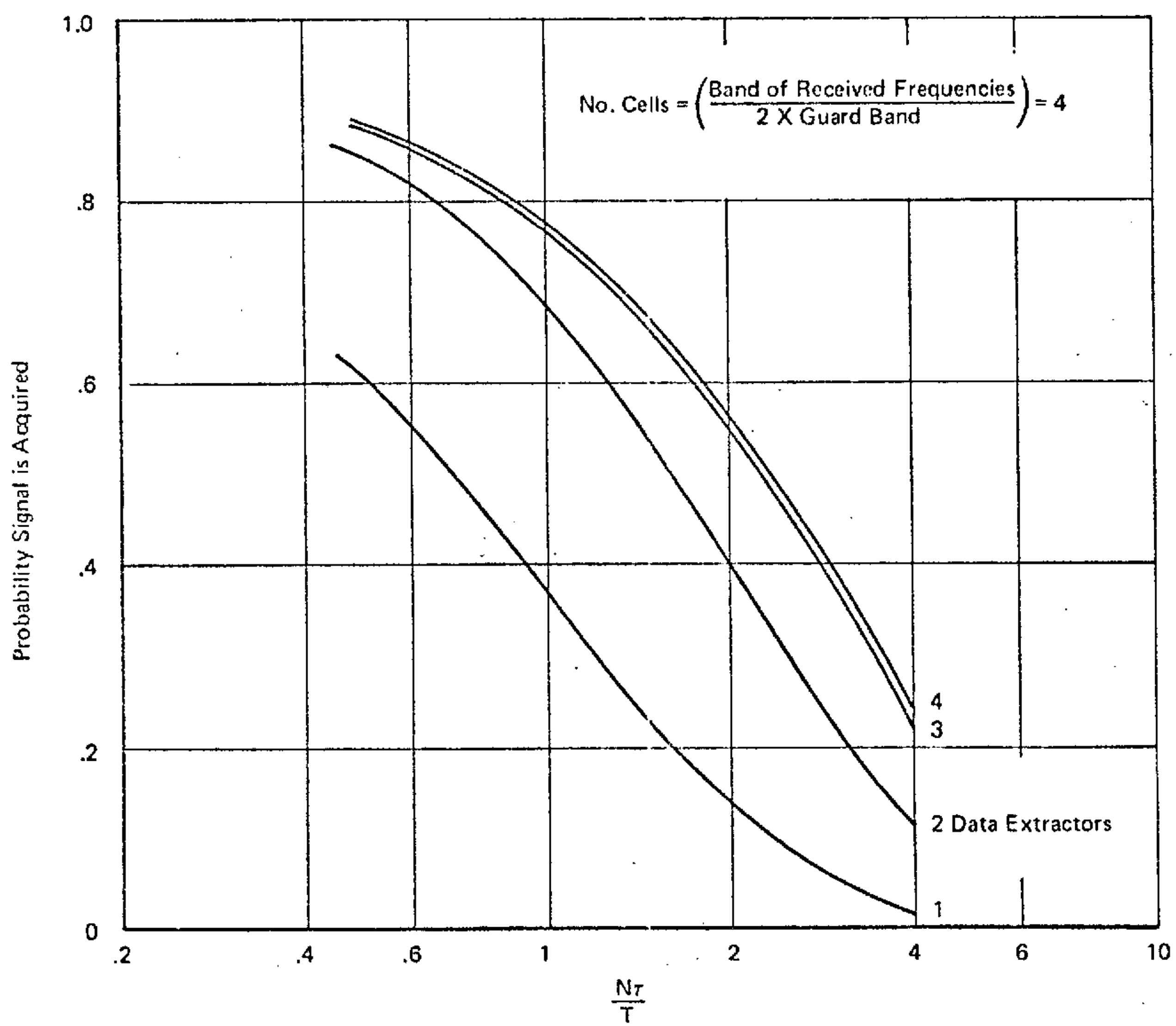

FIGURE 4.19 PROBABILITY OF SIGNAL ACQUISTION

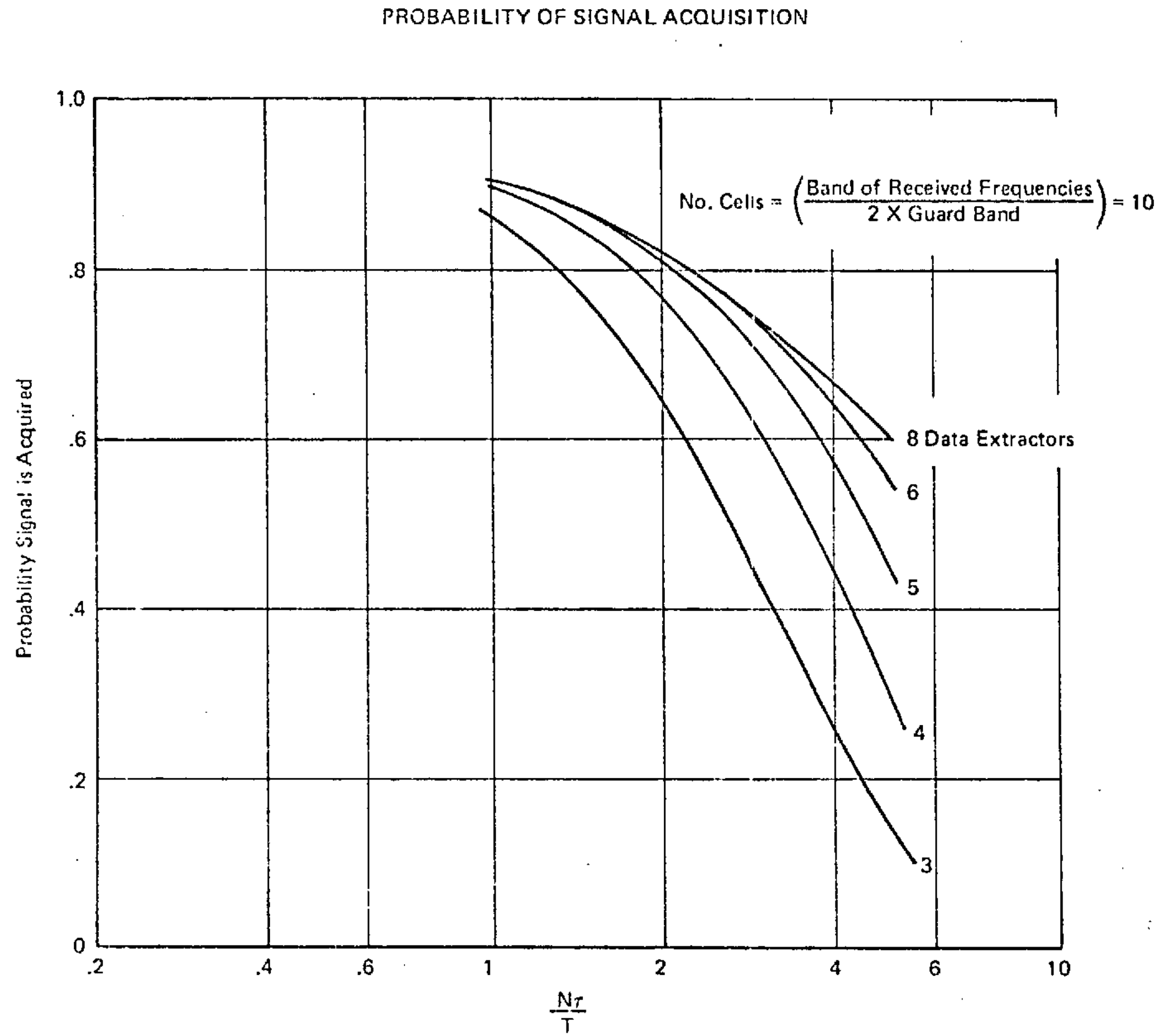

FIGURE 4.20 PROBABILITY OF SIGNAL ACQUISITION

PROBABILITY OF SIGNAL ACQUISITION

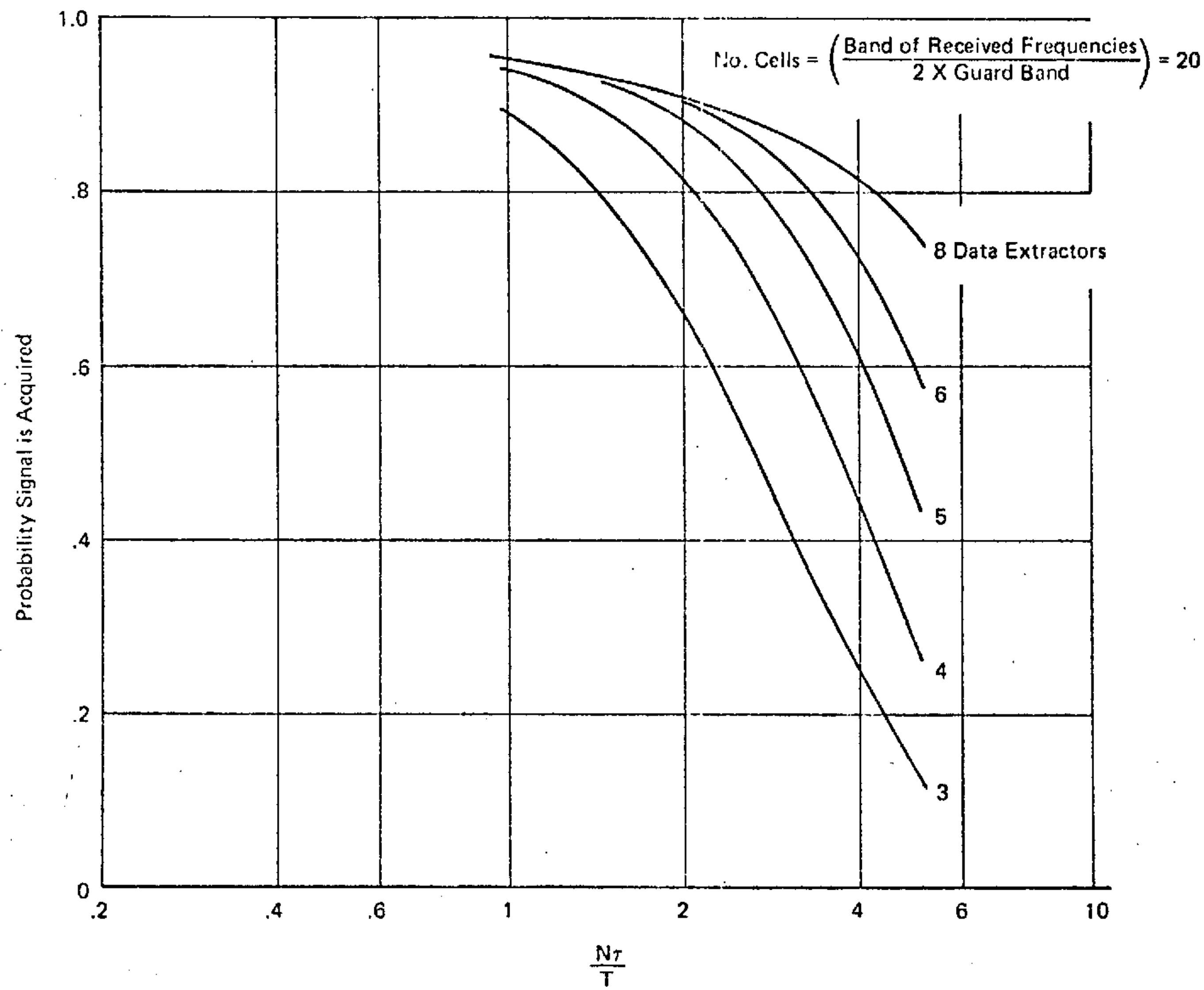

FIGURE 4.21 PROBABILITY OF SIGNAL ACQUISITION

PROBABILITY OF SIGNAL ACQUISITION

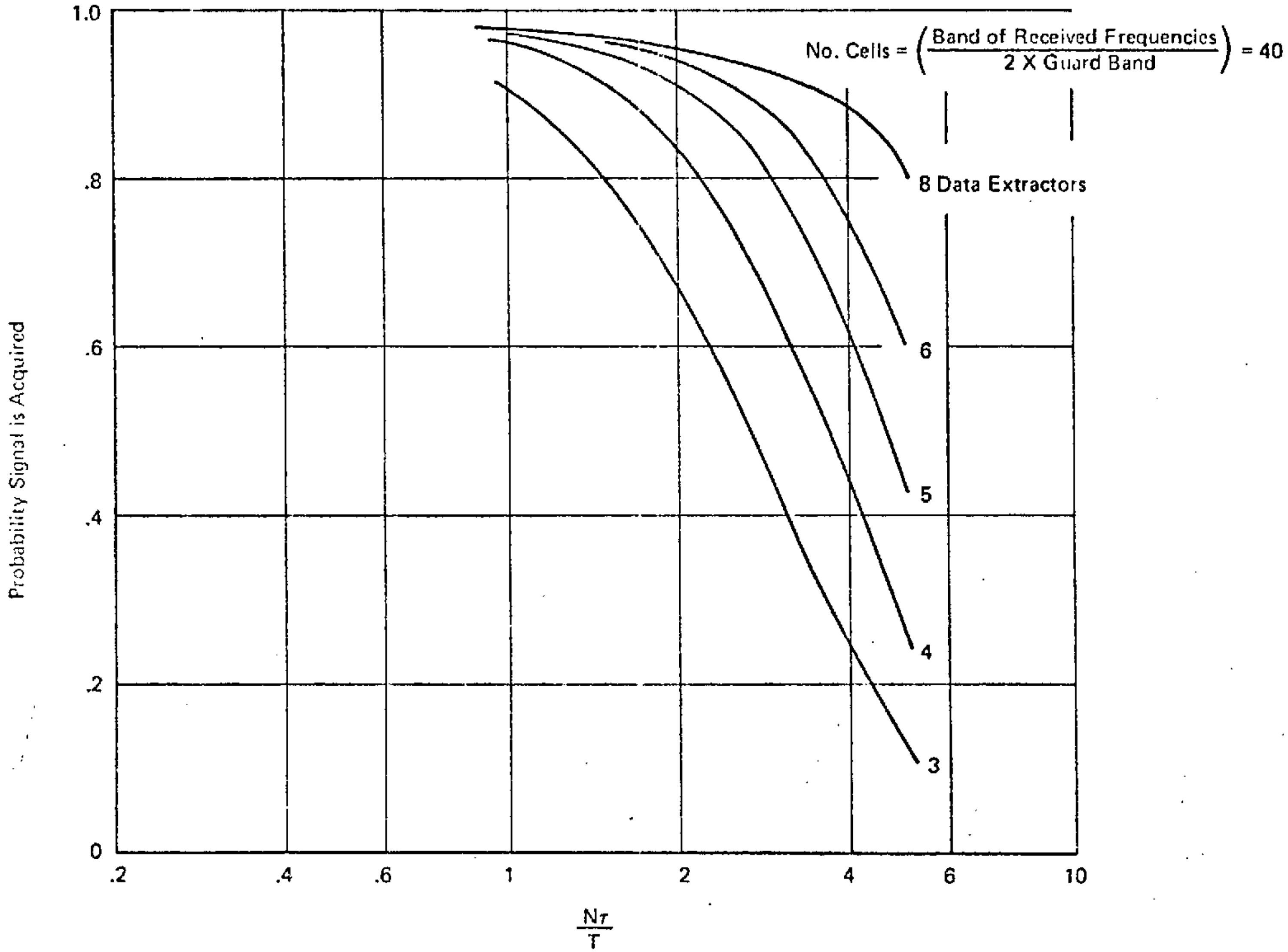

FIGURE 4.22 PROBABILITY OF SIGNAL ACQUISITION

Distribution of Bit Error. Having determined the probability of the specified signal being assigned to a data extractor, the remaining computation describing the processor is concerned with the probability of bit error during demodulation. This analysis utilizes the spectral interference model previously described and combines these data statistically as a function of ($N\tau/T$) and the number of data extractors to arrive at an average bit error probability throughout the transmission period of the specified platform. This analysis can be developed in the following manner.

For a given value of ($N\tau/T$), the statistics (Poisson) of the number of simultaneous interfering transmissions present at the beginning of the transmission of the specified platform can be determined. Similarly, the statistics of the number of interfering transmissions present at the end of the specified transmission can be determined. However, the interference distributions associated with those on at the beginning and those on at the end are significantly different. This difference arises from the logic utilized to assign detected transmissions to the data extractors.

The statistics of the number of transmissions on at the beginning of the specified transmission corresponds to those transmissions which begin during the interval τ prior to the start of the specified transmission. However, the logic of ignoring signals simultaneously present in a given frequency cell precludes the interference caused by signals closer in frequency than the width of the assumed guard band or frequency call. Therefore, given that the specified transmission has been assigned to a data extractor, (i.e., there is no other transmission closer than the width of a frequency cell to the received frequency of the specified transmission) then the interference level existing at the moment of assignment is determined by deleting (statistically) interference of other simultaneous transmissions closer than a cell width to the specified transmission. The effects of this deletion are shown in Figure 4.23 for NRZ modulation, a 400 bps data rate, and a guard band of 800 Hertz.

This is not true, however, for these transmissions on at the end of the specified transmission. These transmissions must begin at some time between the beginning and end of the specified transmission. Therefore, as the logic has been discussed above, all interfering signals regardless of frequency relative to the frequency of the specified transmission must be included.

In order to clarify these discussions further, an analytical description of performance computations is appropriate. This begins by determining the probability that the average interference will be equal to a specific value during demodulation of the specified transmission. In computing the quantity, the number of interfering transmissions on at the beginning of demodulation is given as well as the fact that a data extractor has been assigned to the specified transmission.

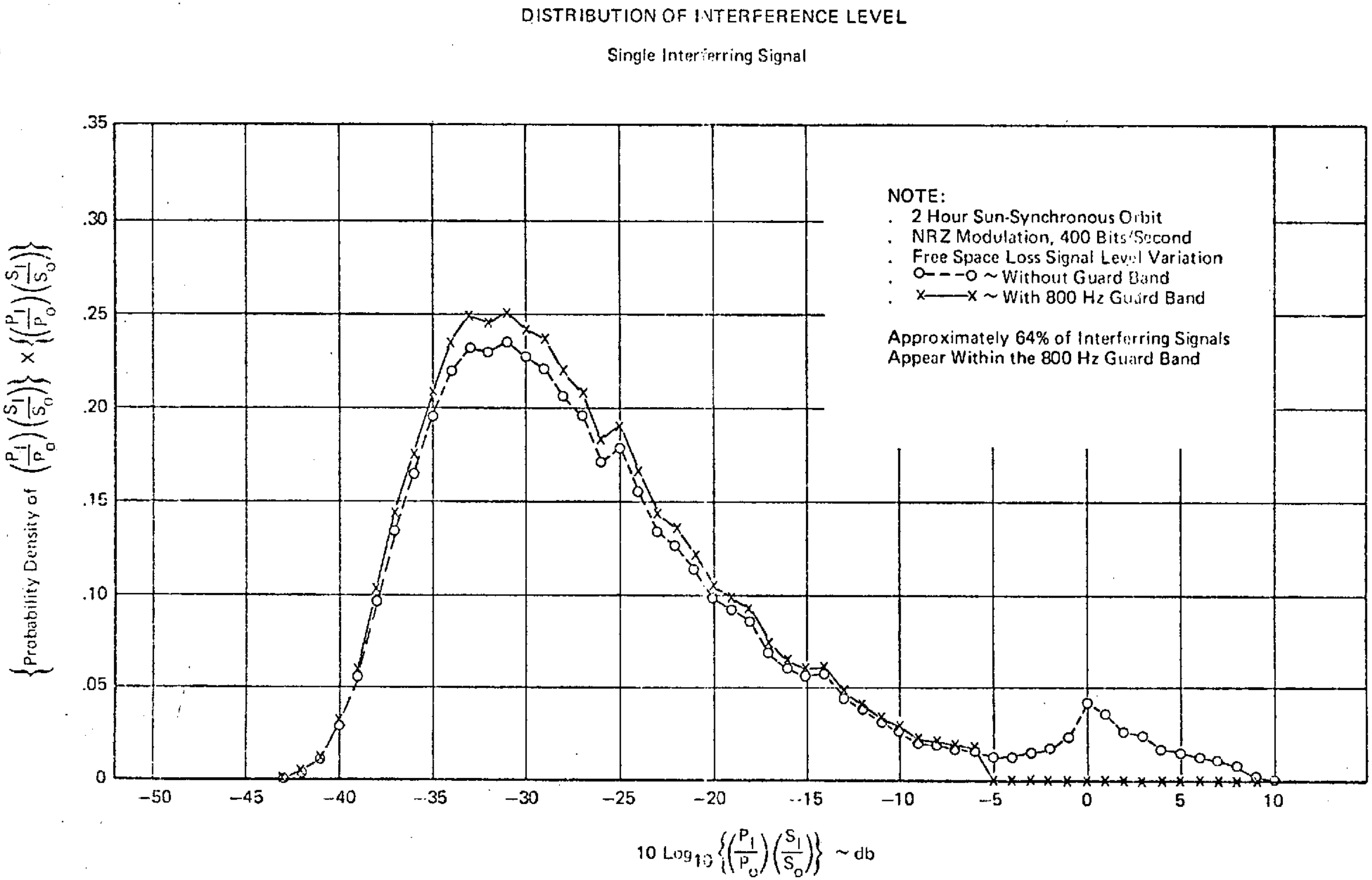

FIGURE 4.23 DISTRIBUTION OF INTERFERENCE LEVEL
SINGLE INTERFERRING SIGNAL

$$\mathrm{pdf}(I_{AV}|n,m,\text{ assignment}) = \tfrac{1}{2}\{\mathrm{pdf}(I|n,m) + \sum_{i=0}^{\infty} [\mathrm{pdf}(I|i)p(i|N\tau/T)]\}$$

where,

$\mathrm{pdf}(I_{AV}\|n,m,\text{ assignment})$	=	probability density function of the average level of interference existing during the demodulation of the specified transmission given there are n interfering transmissions on at the beginning of the specified transmission, given there are m frequency cells, and given a data extractor is assigned to the specified transmission.
$\mathrm{pdf}(I\|n,m)$	=	probability density function of the interference level existing given there are n interfering signals present and given that none of these interfering signals are closer than the width of a frequency cell, of which there are m, to the frequency of the specified transmission.
$\mathrm{pdf}(I\|i)$	=	probability density function of the interference level existing with i interfering transmissions with all possible interference frequencies included.
$p(i\|N\tau/T)$	=	probability i interfering transmissions are present at the end of the specified transmission given the Poisson parameter $N\tau/T$.

Note, the probability density functions of the level of interference given a specified number of transmissions are present is computed recursively from the density function of interference when only one transmission is present. Analytically, this is accomplished by

$$\mathrm{pdf}(I|n) = \int_{nI_{min}}^{I-I_{min}} \mathrm{pdf}(I-\xi|n-1)\mathrm{pdf}(\xi|i)d\xi$$

where I_{min} is the minimum interference level when there is a single transmission present.

The remaining computations provide the desired end result—namely, the probability the specified transmission is acquired and the average interference level (or bit error probability) is less than a certain value. To compute this, it is first necessary to add to the interference density function, derived above, the probability of assignment to a data extractor. In particular

$$\mathrm{pdf}(I_{AV}, \text{assignment}\,|\,m,k) = \sum_{n=0}^{\infty} p(\text{assignment}\,|\,n,m,k)\,p(n\,|\,N\tau/T)\,\mathrm{pdf}(I_{AV}\,|\,n,m, \text{assignment})$$

where:

$\mathrm{pdf}(I, \text{assignment}\,|\,m)$	=	probability the specified transmission is assigned to a data extractor and that the average interference level during demodulation will equal I_{AV} given m frequency cells and K data extractors are within the processor.
$p(\text{assignment}\,|\,n,m,k)$	=	probability a data extractor will be assigned (i.e., is available) to the specified transmission given n interfering transmissions are present at the initiation of the specified transmission, given m frequency cells, and given K data extractors.
$p(n\,|\,N\tau/T)$	=	probability n interfering transmissions are present at the beginning of the specified transmission given the Poisson parameter $N\tau/T$.

The final computation is then the determination of the cumulative distribution function. That is,

$$P(\text{assignment}, I_{AV} \le x\,|\,m,k) = \int_{I_{min}}^{x} \mathrm{pdf}(I_{AV}, \text{assignment}\,|\,m,k)\,dI_{AV}.$$

4.3 SYSTEM PERFORMANCE

Based upon the above derivations, the performance of on-board processors within random access systems can now be described as a function of system parameters and in particular system capability as discussed in Section 3. The measure of performance for the data to be presented is as follows. Given a platform located at the position noted in Figure 4.2 and, given that it initiates a transmission at a specific point in time, performance is characterized by the probability that this given transmission is acquired by the on-board processor and that during the demodulation of the data the average probability of bit error is less than 10^{-4}. In this regard, the energy per bit per noise density is assumed to be 10 dB per Hertz when there is no interference present.

The selection of 10^{-4} as the bit error rate for this performance parameter is based upon the data shown in Figure 4.24. This figure presents the combined probability of signal acquisition and the cumulative probability of the bit error rate as a function of bit error rate. The parameter of this figure is the Poisson parameter describing the arrival time statistics of transmissions—i.e., the average arrival rate of transmission (N/T) multiplied by the duration of the platform transmissions. The other system parameters are fixed as noted.

The important aspect of the data of Figure 4.24 is the generally flat (constant) nature of the curves as bit error probability changes from 10^{-2} to nearly 10^{-6}. This describes a general characteristic of random time and frequency access systems. In particular, transmissions tend to be severely interferred with or relatively interference free. For example, with a Poisson parameter value (Nτ/T of 2, signals will be acquired with bit error rates less than 10^{-2} with probability of .71 and yet, this probability decreases only to .68 if the bit error rate is specified to be less than 10^{-4}. Because of this, bit error rate is arbitrarily maintained at 10^{-4} in the subsequent presentations of performance.

Figures 4.25 through 4.28 present system performance as a function of system capacity (average data rate seen by the satellite from all platforms) for a two hour sun-synchronous orbit and for NRZ modulation. For these data, the guard band is set at twice the platform data rate and the difference between the four figures is the number of data extractors assumed and the distribution of signal amplitudes received at the satellite. A perusal of these figures will reveal the following.

For all conditions, the higher the platform data rate the higher the average system data rate (system capacity). However, the relative improvement in system capacity decreases with increasing data rate. For example, in Figure 4.25 at probability of .8, increasing platform data rate from 100 bits per second to 200 bits per second improves system capacity from 200 to 360 bits per second—an increase by a factor of 1.8 by doubling the data rate. However, doubling the platform data rate from 200 to 400 bits per second only increases system capacity from 360 to 540 bits per second or a factor of 1.5.

Another aspect of these figures that is common to all is the rate of system degradation as the system data rate increases. At the higher platform bit rates, the probability of signal acquisition and low error rate decreases more slowly with system data rate than at the low platform data rates. A localized concentration of platforms (N) within view of the satellite can be accommodated more easily at the high rates. For example, from Figure 4.26, a system designed for 100 bit per second platforms and nominally operating at a system data rate of 200 bits per second would result in a .8 probability of acquisition and 10^{-4} error rate. However, if there was a region of higher density of platforms such that the system data rate increased from 200 to 300 bits per second, then

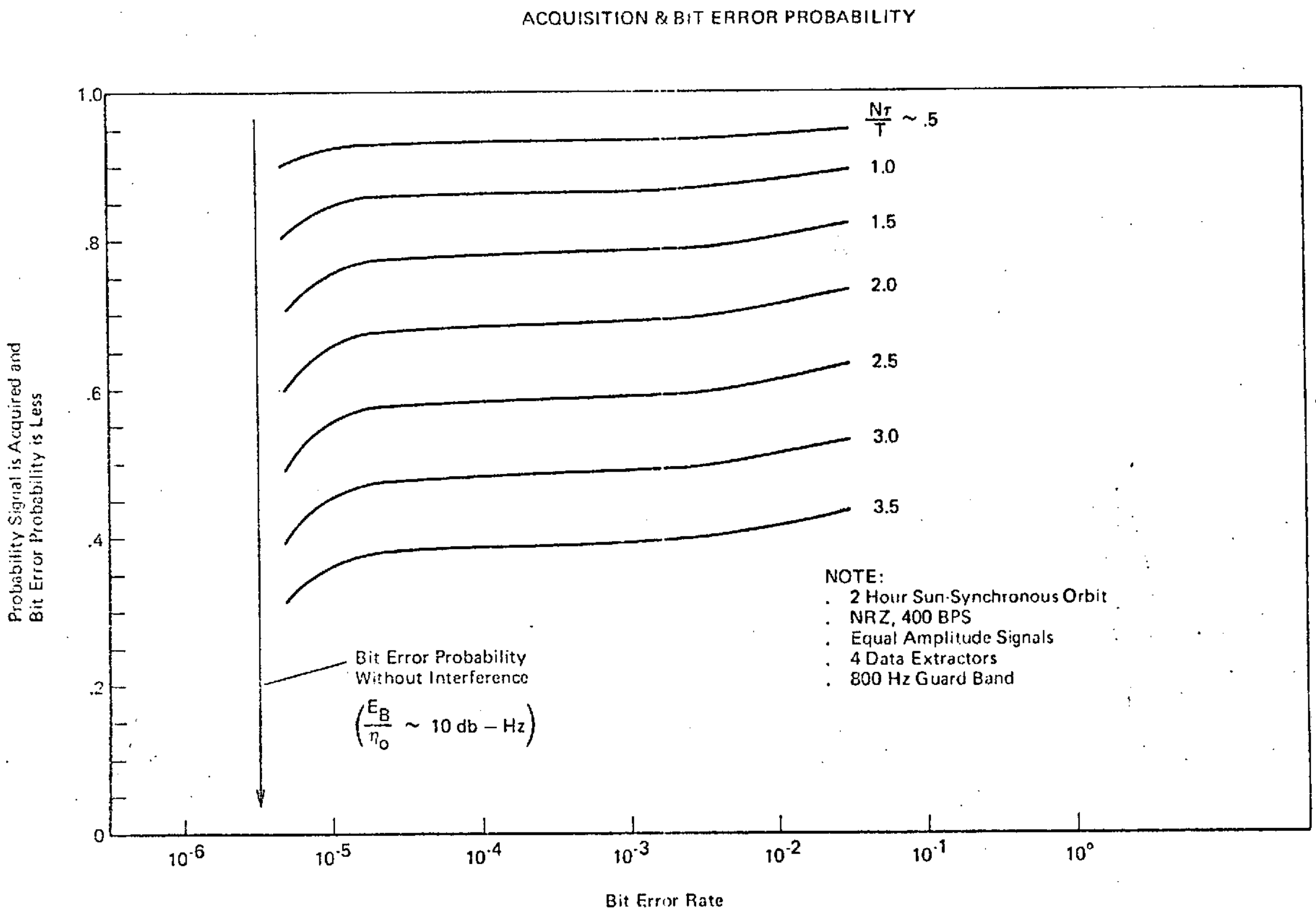

FIGURE 4.24 ACQUISITION AND BIT ERROR PROBABILITY

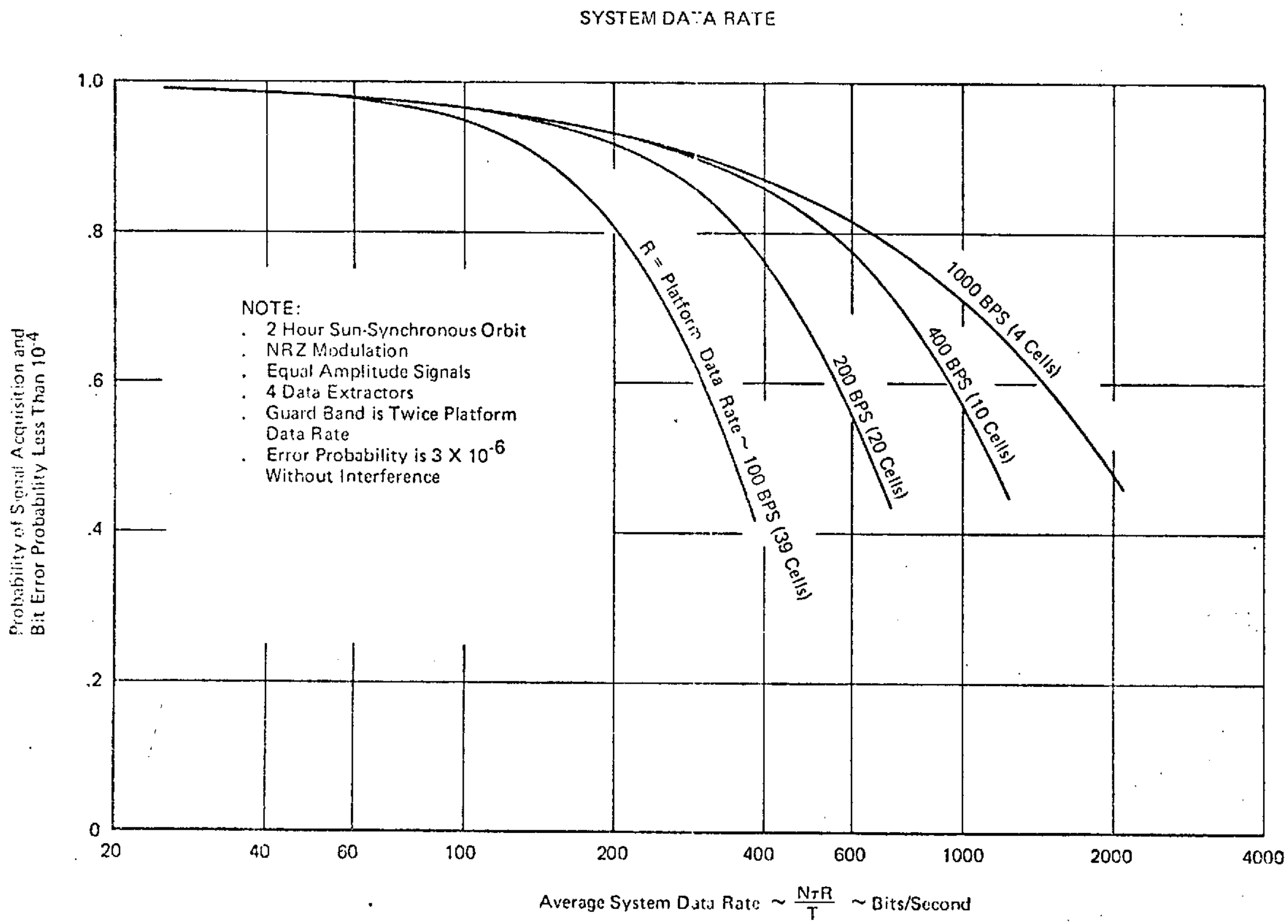

FIGURE 4.25 SYSTEM DATA RATE

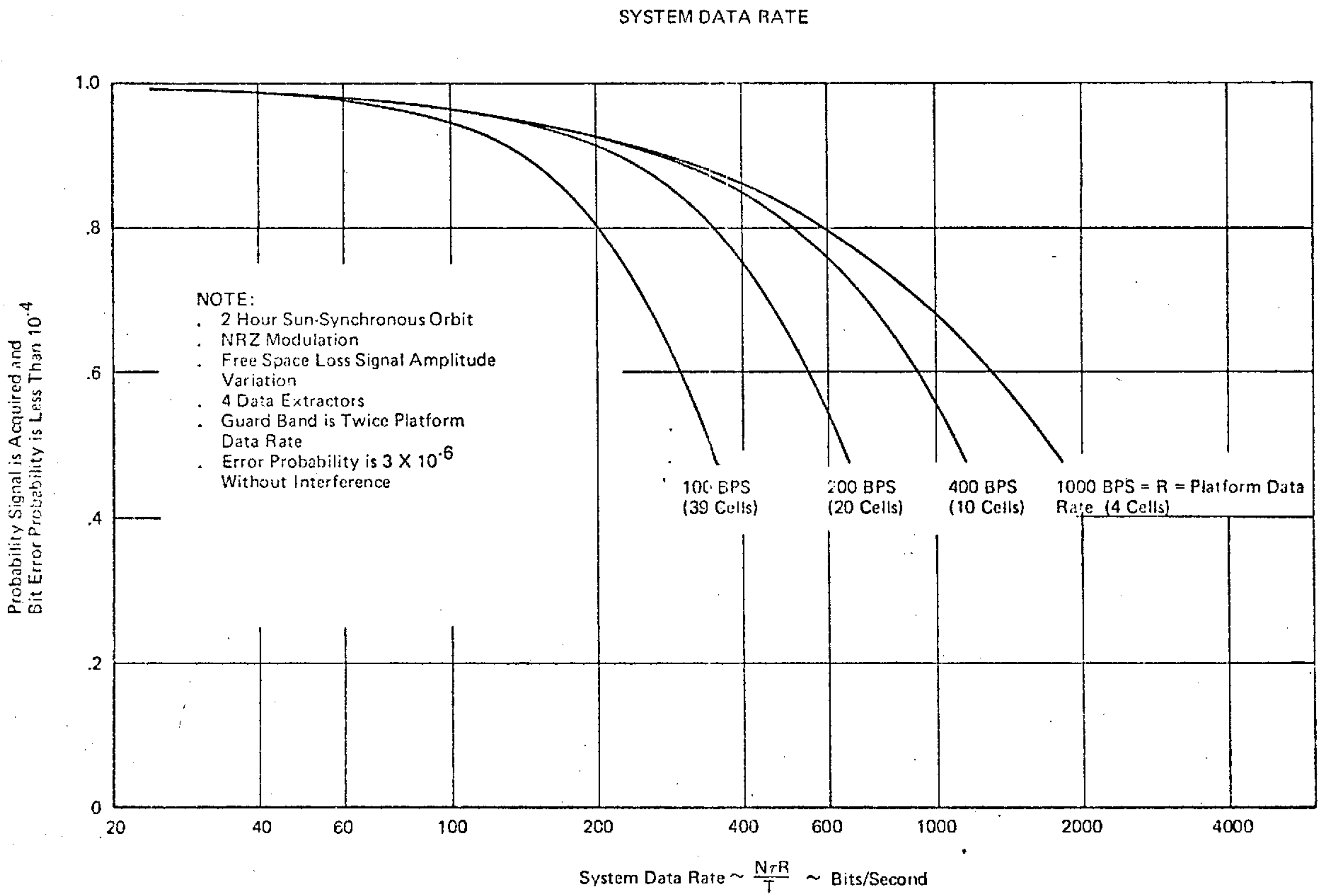

FIGURE 4.26 SYSTEM DATA RATE

this probability decreases to .6. Under the same conditions with a platform data rate of 400 bits per second, the degradation in performance as system data rate increases from 200 to 300 bits per second is only from .91 to .84.

This rapid fall-off phenomena and worse performance at the low platform data rates is primarily due to unavailability or saturation of the fixed number of data extractors available. This can be seen by referring to Figures 4.19 through 4.22. The abcissa of these figures is the Poisson parameter ($N\tau/T$) which when multiplied by platform data rate given system data rate—i.e., the abcissa of Figures 4.25 through 4.28. The improved performance at higher platform data rates can be traced directly to a reduction in length of platform transmissions. This enables a given number of data extractors to handle a large number of transmissions with the same probability of acquiring the signals. This more than compensates for the increased probabilities of interference at the higher platform data rates when multiple signals are present.

By comparing the data of Figure 4.25 with 4.26 and Figure 4.27 with 4.28, the effects of distribution of received signal amplitude can be assessed. This comparison shows little difference in performance at low platform data rates and a small margin for equal amplitude signals at the higher platform data rates. The reason for this is the relative importance of interference between signals due to simultaneous spectral overlap and the loss of signals because of the unavailability of data extractors. At low platform data rates, the unavailability of extractors is the dominant factor because of either the larger number of transmissions or their longer duration. At high platform data rates, simultaneous spectral overlap of signals becomes more important and therefore the difference between equal amplitude signals and signal amplitudes reflecting free space loss becomes a more significant factor.

By comparing Figure 4.25 with 4.27 and 4.26 with 4.28, the advantages of increasing the number of data extractors can be determined. As in the comparisons above, these advantages are highly dependent upon platform data rate. At all platform data rates, system capacity is essentially proportional to the number of data extractors at equal probabilities of signal acquisition and less than 10^{-4} bit error rate. However, another factor imposes a limit to the increases in system data rate as platform data rate increases. This factor is the number of frequency cells available within the total spectrum of received signals. Because of the logic function of precluding assignment of more than one data extractor to a given frequency cell, improvement in system capacity by increasing the number of data extractors is only possible until the number of data extractors is equal to the number of frequency cells. This point is apparently then the maximum performance point for random access systems.

Extrapolation of the conclusion suggests that peak performance corresponds to a system wherein platform data rate equals the frequency spectrum of received signals may not be valid. This point of operation violates several of the basic assumptions used to develop both the interference model and the statistical model describing the availability of data extractors. However, this point of operation also corresponds to greatest platform power.

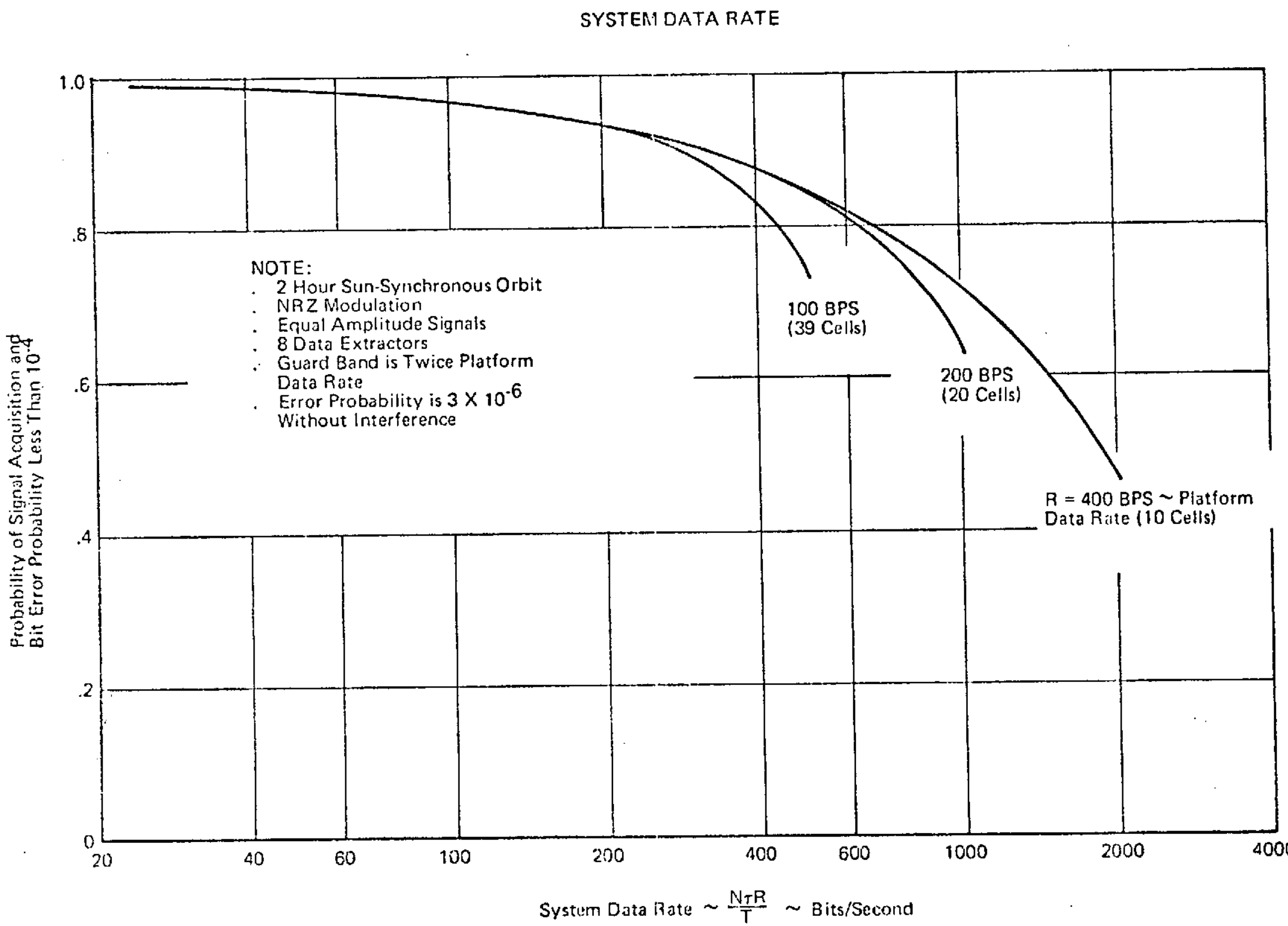

FIGURE 4.27 SYSTEM DATA RATE

SYSTEM DATA RATE

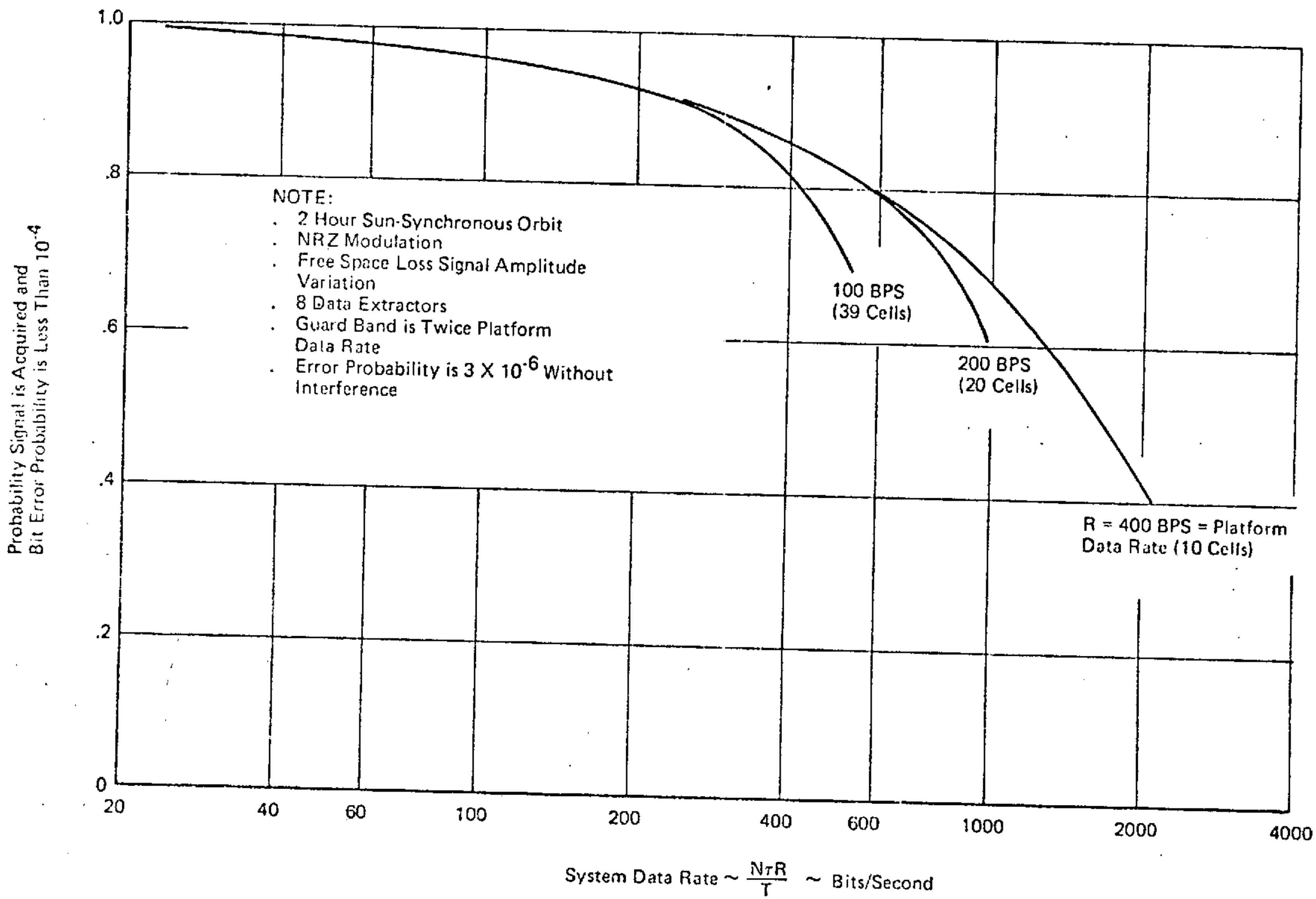

FIGURE 4.28 SYSTEM DATA RATE

The above discussions have been concerned with performance wherein the modulation utilized is NRZ. Figure 4.29 presents similar performance data when split phase or Manchester coding is employed. The conditions of these data correspond in all respects to the NRZ data presented in Figure 4.27. Comparison of these two figures indicates the loss in system performance caused by the wider spectrum of the split phase signaling. For example, at probability .8 of signal acquisition and error rate less than 10^{-4} with NRZ signaling (from Figure 4.27), the system data rate is approximately 660 bits per second at a platform data rate of 400 bits per second. For split phase signaling, the system data rate decreases to about 440 bits per second from Figure 4.29. This is approximately a 30% decrease in capacity. Perusal of these two figures also indicates that this 30% reduction for split phase systems compared to NRZ systems is virtually constant as the probability of acquisition and less than 10^{-4} bit error rate is changed.

For all figures from 4.25 to 4.29, the guard band used to prevent assignment of more than one data extractor to a given frequency cell is assumed to be twice the data rate of the platforms. Figure 4.30 indicates the change in system performance when this guard band is decreased. In particular, the performance of the split phase modulation system of Figure 4.29 is presented in Figure 4.30 for platform data rate of 400 Hertz when the guard band is set at 150 Hertz instead of 800 Hertz (twice the data rate). Comparison of the two Figures shows that the smaller guard band of 150 Hertz results in better system performance—i.e., higher probability at given system data rate. However, in contrast to this result, reference to Figure 4.31 indicates a decrease in system performance for an NRZ modulation system at a platform data rate of 200 bits per second.

These variations in system performance as guard band is altered suggests there may be guard band sizes for each modulation technique and platform data rate that maximizes system performance. While this may be true, other system considerations may be more important. For example, eliminating guard bands entirely will mean demodulation and storage of a greater number of platform transmissions during which interference levels and therefore high bit error rates existed.

A final consideration for random access system performance is the effect which satellite altitude will have on system capacity. An indication of this is shown in Figure 4.32 where the performance of a split phase system is presented under the assumption of a three hour satellite orbit instead of the two hour orbit assumed for the previous data. This figure is comparable to Figure 4.29 in that all assumptions are the same except for orbit altitude.

A comparison of Figure 4.32 and 4.29 shows there is a reduction in system data rate throughout for the higher altitude orbit. The reasons for this

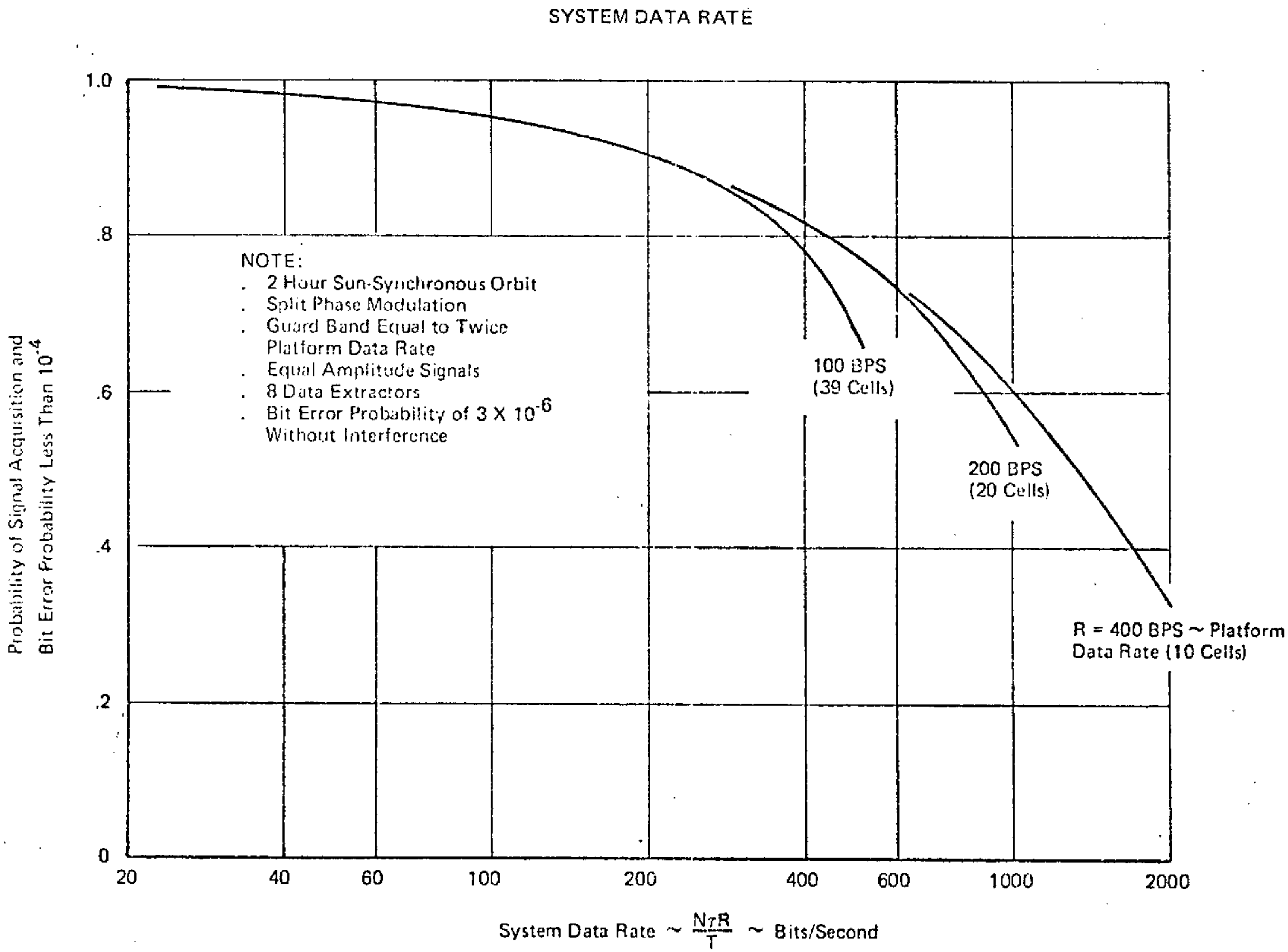

FIGURE 4.29 SYSTEM DATA RATE

SYSTEM DATA RATE

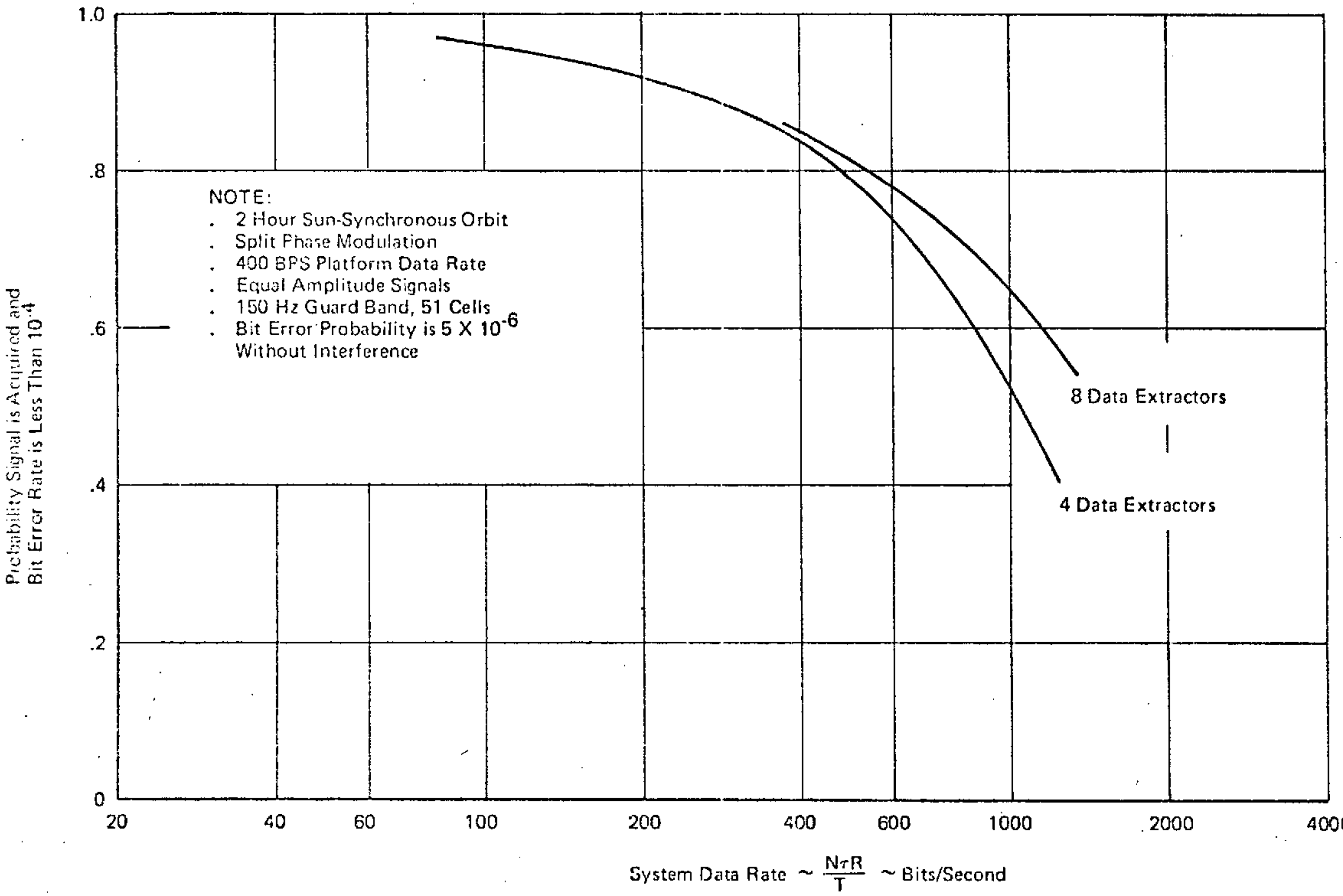

FIGURE 4.30 SYSTEM DATA RATE

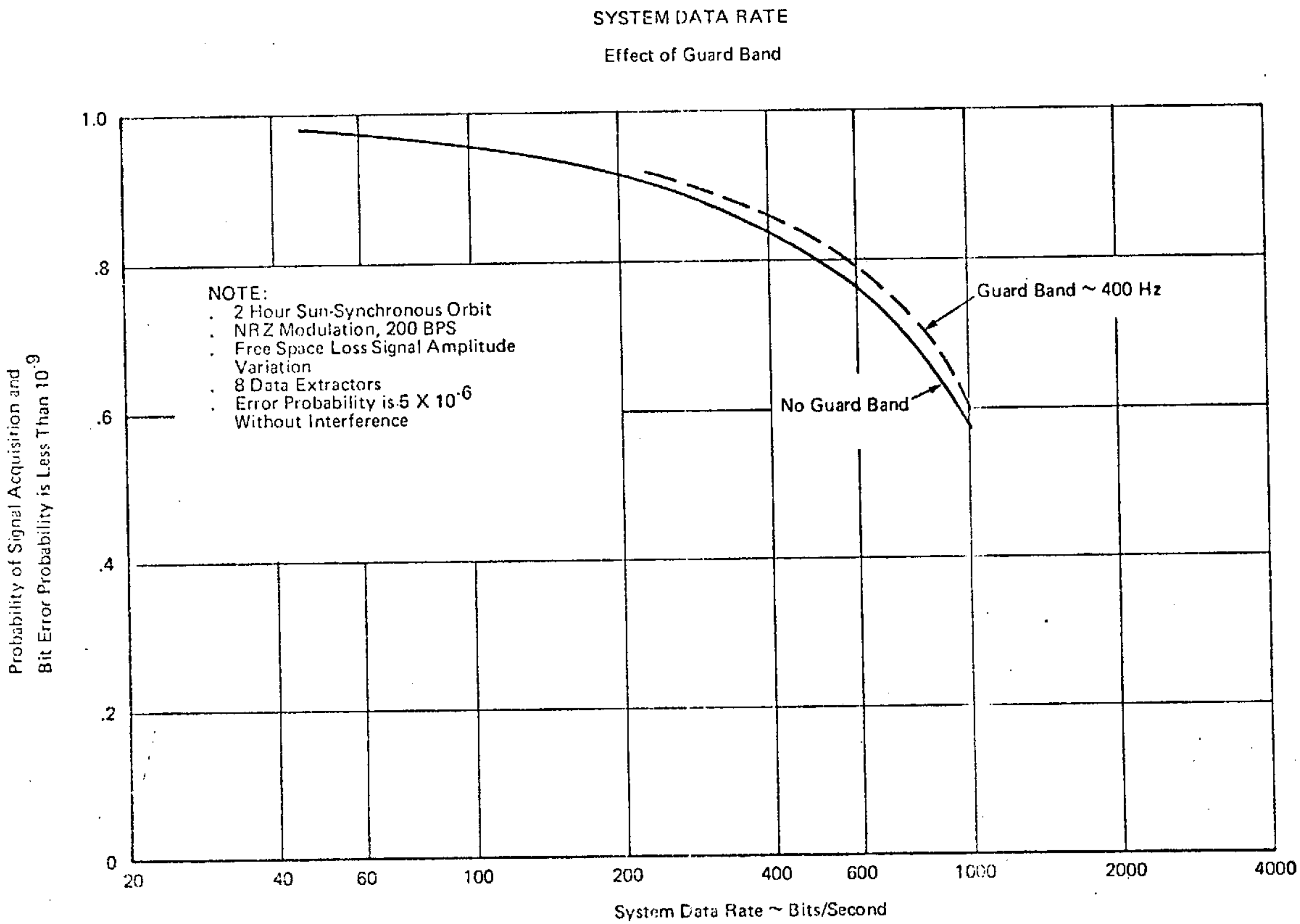

FIGURE 4.31. SYSTEM DATA RATE (Effect of Guard Band)

SYSTEM DATA RATE

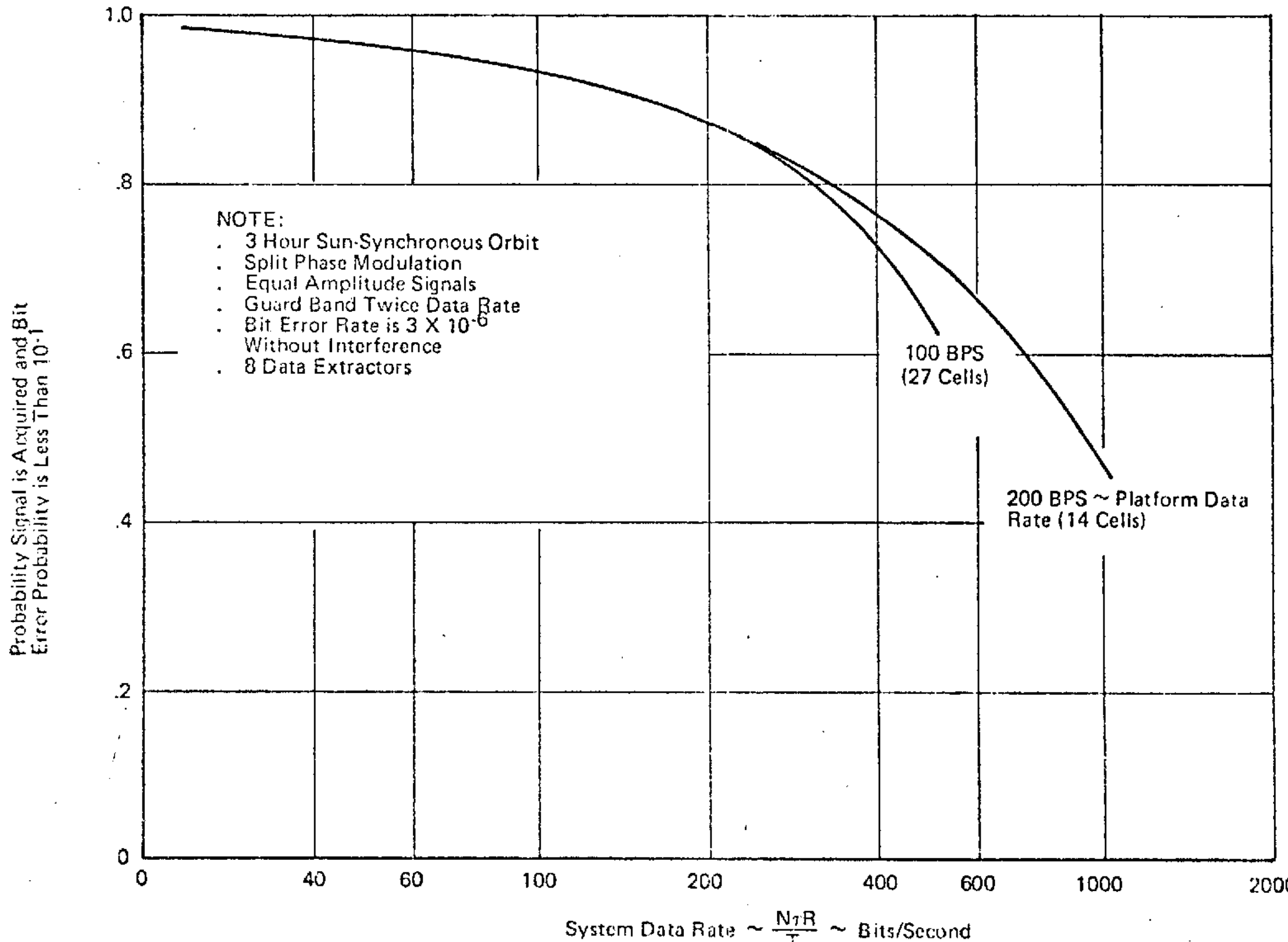

FIGURE 4.32 SYSTEM DATA RATE

are that (1) the size of the viewing circle and therefore the number of platforms in view (N) changes with orbital altitude and (2) the minimum viewtime between platforms and the satellite to acquire a given number of transmission is larger for the higher orbit—i.e., the time between transmissions (T) can be increased thereby permitting a larger N with fixed (N/T) ratio. Therefore, before a valid comparison can be made between systems at different orbital altitudes, the change in the (N/T) ratio with altitude must be determined under equal ground rules. This may be accomplished as follows.

If the platforms are uniformly distributed, over the entire earth then the number in view of a satellite at any point in time is a function of the altitude of the satellite and the minimum angle above the horizon of the satellite to permit communication. These data are presented in Figure 4.33. The ordinate is the ratio of number of platforms in view (N) to the total number over the entire earth (N_{TOT}). This ratio is given as a function of satellite altitude with the horizon angle limit noted parametrically. The orbital period is also indicated. For the following discussions, the 5^{o} horizon curve will be used to define the number of platforms in view.

The second type of data required is the minimum viewtime between the satellite and platform during an overpass. In this regard, it is necessary to establish whether or not successive overpasses of a satellite are necessary. This requirement comes about from the need to not only locate a platform but to estimate its mean velocity between overpasses. For present purposes this requirement is assumed to be present. The corresponding minimum viewtimes for the two and three hour sun-synchronous orbits are presented in Figures 4.34 and 4.35.

In Figure 4.34, the subtrack of a two hour sun-synchronous satellite are shown in latitude and longitude relative to a platform located at zero degrees latitude and longitude. Also, the limits of communication visibility are indicated parametrically as a function of the limiting horizon angle. The two subtracks indicated correspond to the geometry of successive overpasses wherein the minimum viewtime overpass is experienced—namely, this condition exists when the second successive overpass passes directly over the platform. As noted, the time ticks on the sub-tracks indicate the motion of the satellite sub-point in two minute intervals. From these data, the inset graph of Figure 4.34 can be derived. This graph presents the minimum viewtime as a function of the limiting horizon angle. For example, for the condition of a 5^{o} horizon angle, the minimum viewtime is approximately eleven minutes.

Figure 4.35 is entirely analogous to Figure 4.34 except the orbital altitude is three hours instead of two. Under these conditions, the minimum viewtime with a 5^{o} horizon angle limit can be seen to be about 28 minutes.

NUMBER OF PLATFORMS IN VIEW

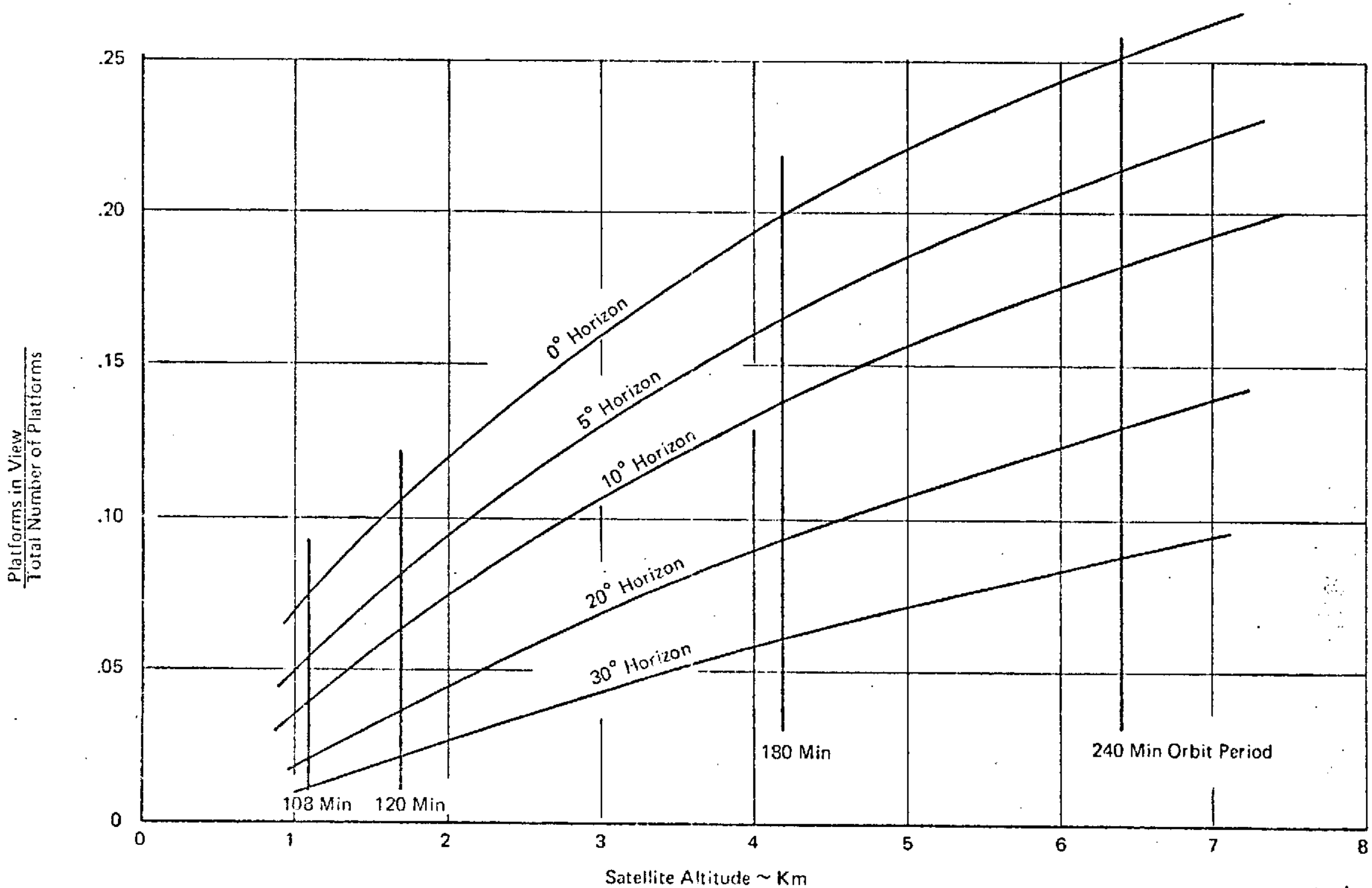

FIGURE 4.33. NUMBER OF PLATFORMS IN VIEW

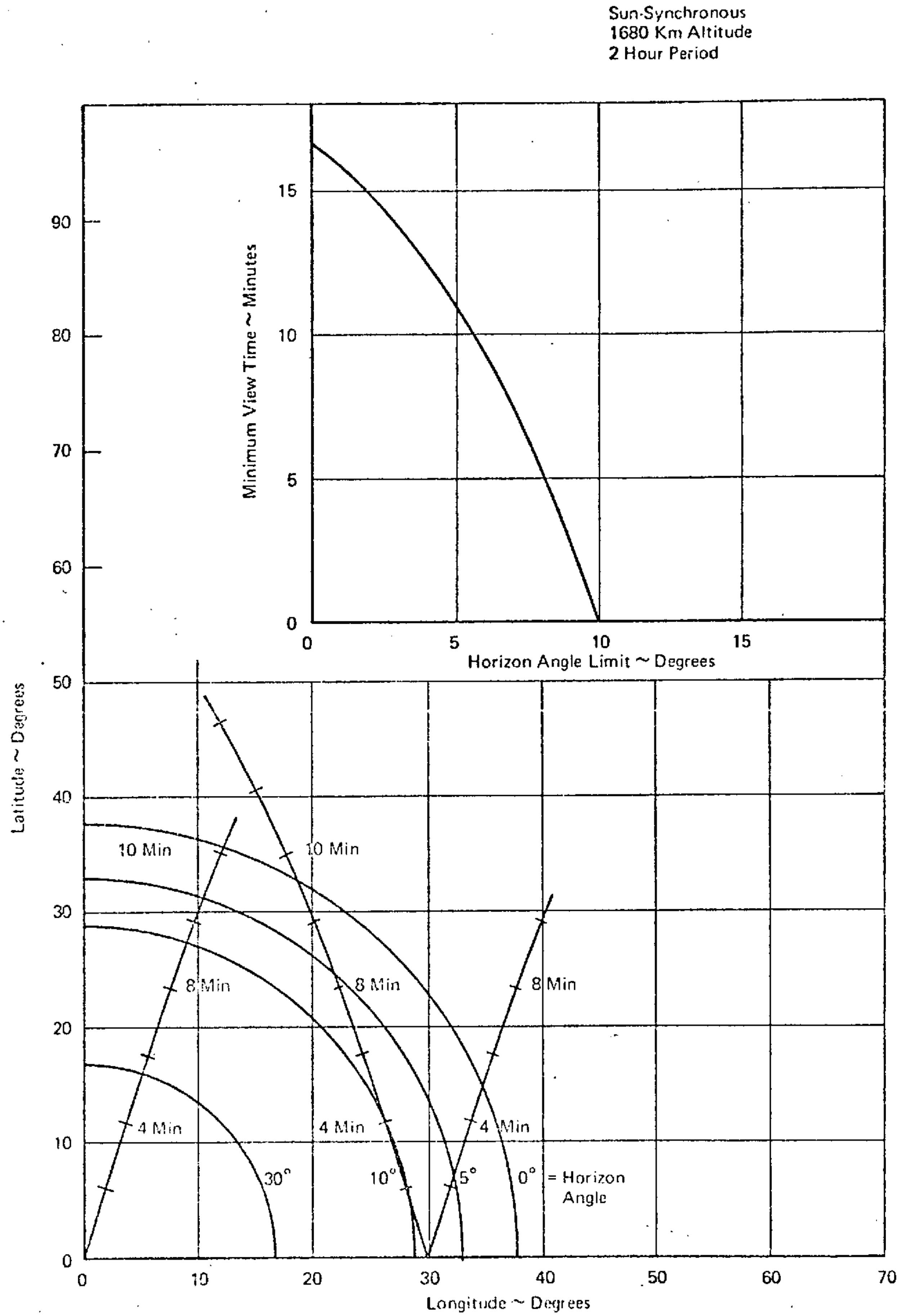

FIGURE 4.34 OVERPASS GEOMETRY

Sun-Synchronous
4180 Km Altitude
3 Hour Period

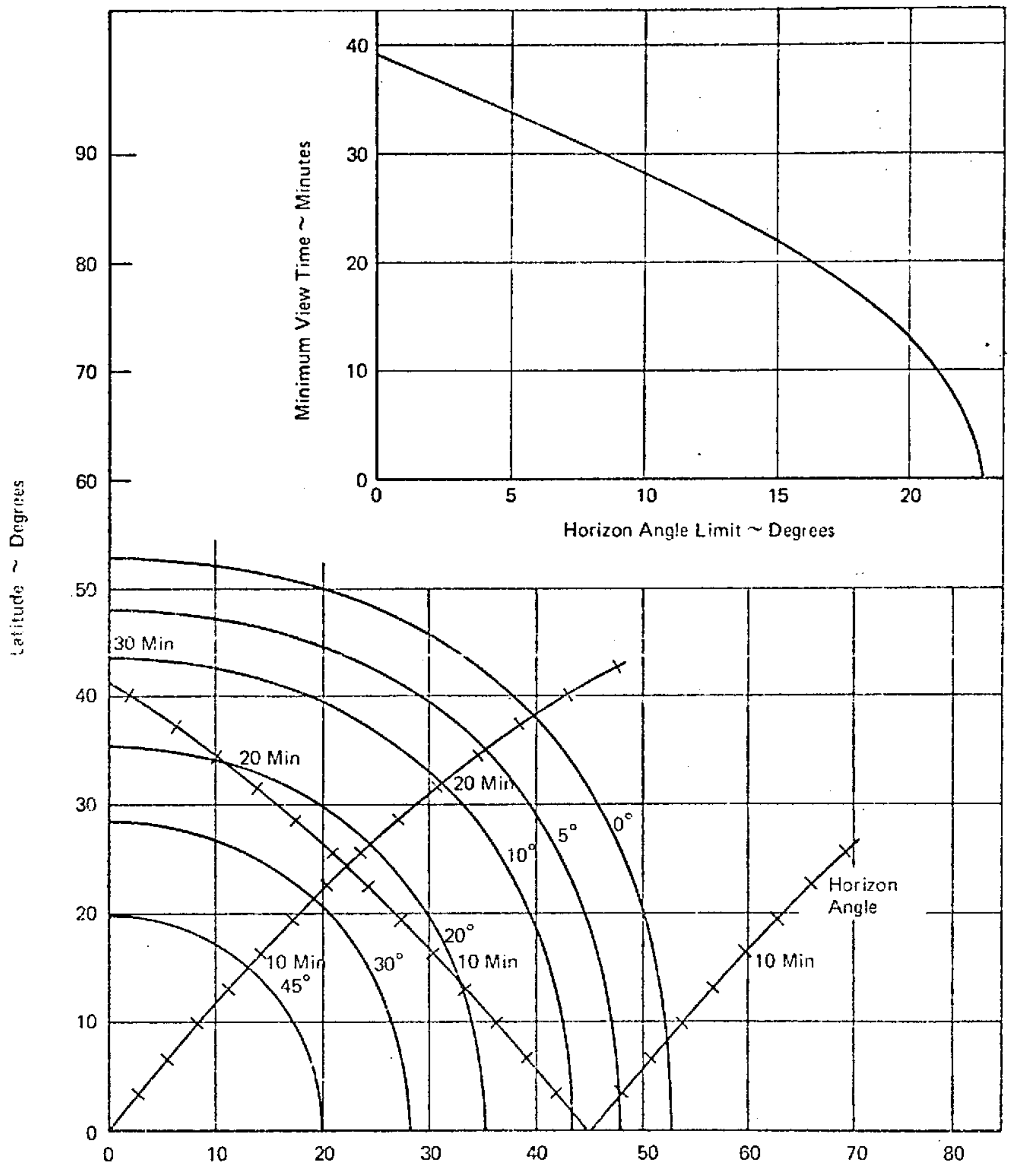

FIGURE 4.35 OVERPASS GEOMETRY

With these data, equivalent values of the (N/T) ratio for the two and three hour orbits can be determined as follows. From Figure 4.33 at 5° horizon angle, the ratio between number of platforms in view for a three hour orbit compared to a two hour orbit is 2.04—i.e., the ratio between .165 and .081. The ratio between minimum viewtimes is 28 minutes to 11 minutes or about 2.54. Therefore, the (N/T) ratio for the three hour orbit is about .8 of the (N/T) ratio for the two hour orbit i.e., $2.04/2.54 \sim .8$.

With this information, the system performance comparison between two and three hour orbits can be accomplished for the data presented in Figures 4.33 and 4.29. For example, at probability of signal acquisition and bit error rate less than 10^{-4} equal to .9, the system data rate is about 210 bits/second for a two hour orbit (Figure 4.29) and about 155 bits/second for a three hour orbit (Figure 4.32). However, (N/T) for the three hour orbit is .8 of the (N/T) for the two hour orbit. Therefore, the number of bits per transmission from a platform (τR) could be 1/.8 or 25% higher for the three hour orbit if system data rates ($N\tau R/T$) were equal. However, as noted above, the system data rate for the three hour orbit is only 155 compared to 210 bits/second for the two hour orbit. Therefore, the number of bits per transmission for a three hour orbit is about .92 (i.e., 1.25 x 155/210) that of a two hour orbit—a decrease of about 8%. In effect then, there is little difference between the amount of data transferred from platforms when the altitude of the satellite is increased from a two to a three hour orbit. However, one other type of comparison should be performed.

Instead of increasing the visibility circle as the altitude of the satellite is increased, the circle can be maintained by increasing the horizon angle limit. This can be accomplished by providing directional antennas for either or both the satellite and platforms. This concept introduces two considerations

- By utilizing more directional antennas for the higher altitude orbits, platform power may not increase to maintain a given signal level at the satellite
- Restricting the visibility circle for the higher altitude orbit will tend to keep platform signal levels at the satellite more equal which improves performance as shown previously.

While there are several ways in which the reduction in visibility can be established, one way makes the comparison between the two and three hour orbits particularly straightforward is to make the minimum viewtime equal for the two orbits. Using the eleven minute minimum viewtime of the two hour orbit as a standard, Figure 4.35 indicates that the horizon angle limit should be about 20 degrees. If the area (and therefore the number of uniformly distributed platforms) of the 3 hour 20° visibility circle is compared to that of the two hour 5° visibility circle, then there are approximately 84% less platforms in view for the two hour orbit. In this case then, the number of bits that can be transmitted during each platform's transmission for the three hour orbit is 62% of that of the two hour orbit—i.e., .84 x 155/210. This reduction in performance would have to be balanced against possible advantages accruing from the gain of the directional antennas employed.

V. ON-BOARD PROCESSING CONCEPTS

In Section 4, the fundamental limit of mutual interference is described for random time and frequency access to the satellite. This limit is presented as being comprised of two factors. One is the limit of statistical saturation of the time-frequency space within which platform transmissions arrive at the satellite. The second limit is established by statistical saturation of the signal processor on-board the satellite. This section of the report presents a spectrum of on-board processing concepts that establish this second limit as a function of the complexity of equipment.

Basically, the signal processor on-board the satellite can perform two distinctly different functions, both of which serve to decrease the quantity of data that must be stored and relayed to a ground station. One function is the elimination of those regions of time and frequency wherein no platform transmissions are present—i.e., the processor scans the arriving time-frequency space for platform transmissions and stores only the data contained therein. The second function, if performed, follows the detection, demodulation, and storage of platform data. In particular, logical operations to eliminate any superfluous data are performed.

The degree to which on-board processors perform one or both of these functions will characterize the degree to which the time-frequency space arriving at the satellite is compressed prior to re-transmission to the ground. Before presenting these concepts, however, the results of Section 4 can be used to establish the degree of compression possible regarding the elimination of time and frequency space wherein no platform transmission are present.

5.1 ELIMINATION OF NOISE

Elimination of those regions of time and frequency wherein no platform transmissions are present is basically an elimination of noise. The degree of compression accomplished in the process can be quantitatively described by equating the total "data rate" arriving at the satellite, including noise, to the total frequency band passed by the satellites' receiver, and comparing this to the effective system data rate ($N\tau R/T$) or system capacity as derived in Section 4. For example, the receiver bandwidth for a 2 hour satellite orbit is approximately 17 kilohertz. Therefore, if the system data rate is 170 bits per second, elimination of noise in the received bandwidth provides an effective noise compression ratio of 100 to 1.

By referring to the analysis of Section 4.2.2, this noise compression ratio can be seen to be directly related to probability of interference. Taking the expression for NRZ signaling in the case where the probability of interference (P_I) is small,

$$P_I \sim K \frac{N\tau}{T} \frac{R}{F_T}$$

However, the product ($N\tau R/T$) is merely system data rate while F_T is approximately the receiver bandwidth. Therefore,

$$P_I \sim \frac{\text{K x System Data Rate}}{\text{Receiver Bandwidth}} = \frac{K}{\text{Noise Compression Ratio}}$$

This relationship shows a fundamental tradeoff which must be made for random access systems. Namely, a system with high probability of signals being acquired with low error rates necessarily means a system with poor utilization of spectrum—i.e., high noise compression ratio. Therefore, in order to efficiently use a given spectrum within a random access system for data transfer, error detection and correction must be implemented either by after-the-fact correlation of redundant platform transmissions or coding.

From another viewpoint, noise compression ratio is an indication of the penalty paid to take advantage of the low cost aspects of a random access system compared to an ordered or interrogated system. If the system were completely time and frequency ordered, then there are no regions in time-frequency space without platform transmissions. Therefore, system capacity of a time and frequency ordered system will be greater than random access system utilizing the same spectrum by a factor which is at least as large as the noise compression ratio.

5.2 PROCESSOR CONCEPTS

From the above discussions, the most valuable function of an onboard processor is the elimination of the noise-only portion of the received time-frequency space. Therefore, the first and minimal type of on-board processor is one which is limited to the functions of detection and storage of platform transmissions with demodulation taking place after transmission to a ground processing center.

The next logical extension to processor complexity is the insertion of the demodulation function between detection and storage. This will represent the minimal processor wherein actual data compression takes place instead of the compression achieved by elimination of noise only regions of the uplink time-frequency space.

The last and most complex type of processor corresponds to the insertion of logical operations on the demodulated platform data prior to storage. These can comprise elimination of superfluous or redundant data and/or preliminary data reduction typified by whole or partial solution of platform location algorithms.

Before these different types of processors are discussed, however, it is necessary to describe, in general, the format of the platform transmissions. This is pictorially shown in Figure 5.1. The modulation is assumed to be digital (e.g., NRZ or split phase) with six basically different types of data as follows:

- C.W. tone—a sequence of 1's or 0's which are used for the purpose of signal detection
- Bit timing recovery—a sequence of alternating 1's and 0's
- Start data indicator—a number of bits establishing the beginning of actual data transfer (frame sync)
- Platform ID—a series of bits uniquely identifying the platform from which the transmission originated
- Sensor data—the data to be transferred which may be for example, a number of separate sensor readings
- End-of-transmission—a unique series of bits indicating termination of the platform's transmission.

In the analyses of Section 4, system data rate is based upon the sum total of all of these six different types of bits. Note, therefore, detection followed by demodulation can achieve meaningful data compression compared to noise-elimination only if for no other reason than the overhead bits can be eliminated with demodulation. This can be significant for those systems wherein the quantity of sensor data is small.

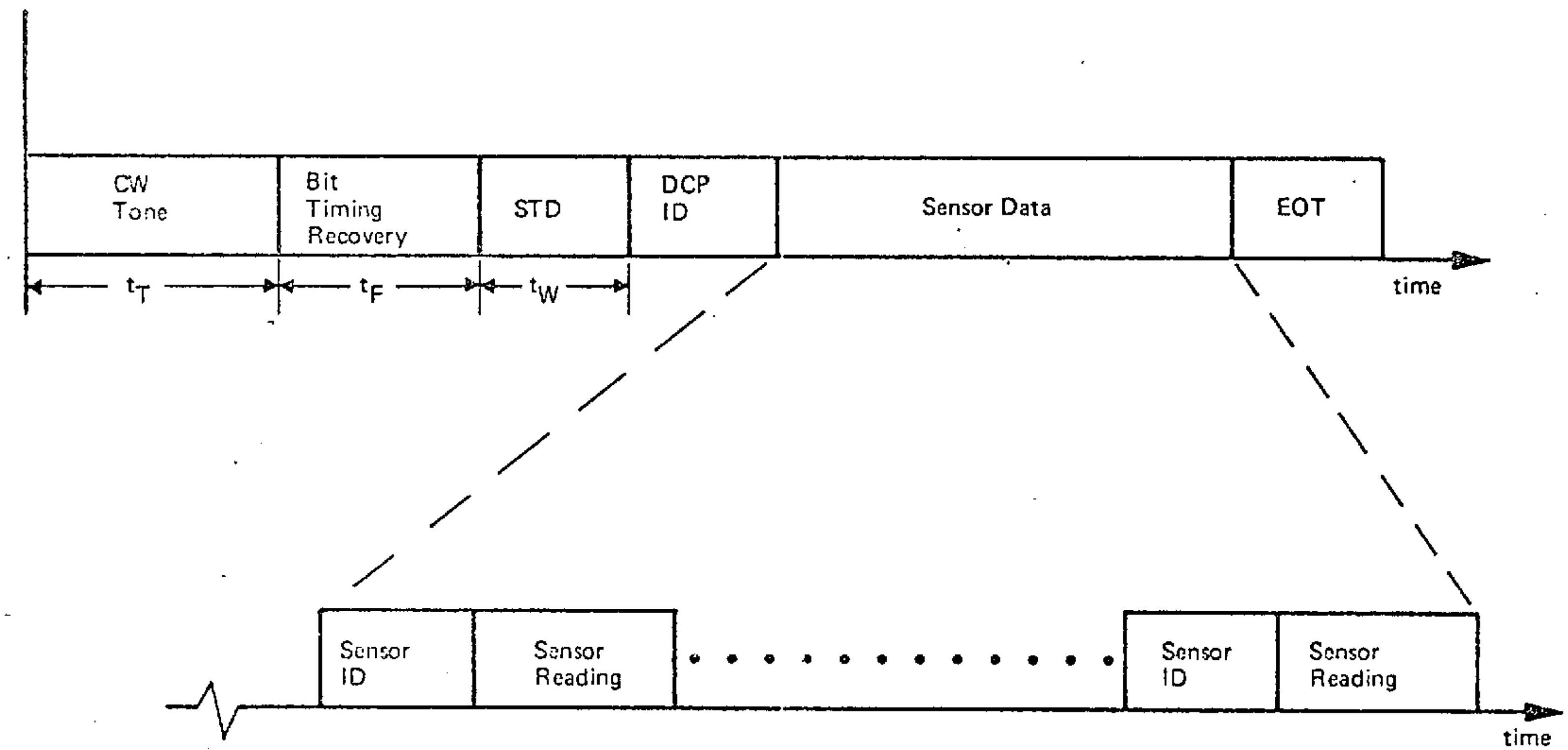

FIGURE 5.1. DCP TRANSMISSION BURST FORMAT

5.2.1 Noise-Elimination Processors

The concept of a processor which eliminates those regions of the uplink time-frequency space not containing platform transmissions is shown in Figures 5.2 and 5.3 — the difference between them being whether storage is analog or digital. In both these processors, the satellite receiver provides the down-converted baseband containing platform signals at frequencies from several kilohertz to a value somewhat larger than the total possible doppler shift anticipated. For a two hour orbit, this band might be, for example, from two to twenty kilohertz.

Following filtering and amplification, the detection function is performed. This can be accomplished with any of several detection mechanisms including comb filters, sweeping filters, digital filters, fast fourier transform, etc. The function of the detector is to provide an indication that a signal is present and if so, its "approximate" frequency. The number of frequency cells will be determined by the received carrier-to-noise density ratio, the data rate, and the required probability of detection.

To evaluate the number of detectors, assume no residual carrier power during data demodulation and a given energy per bit to noise density ratio (E_B/η_o). The carrier to noise density ratio during the CW portion of the message must be $(E_B/\eta_o)R$ - where R is the platform data rate. Finally, the signal-to-noise ratio is $(E_B/\eta_o)R/\Delta f)$ where Δf is the frequency width of the detector.

Assuming equal width detectors across the received band, the signal-to-noise ratio may be put in terms of the number of detectors. In particular, if n_d is the number of detectors, then

Relate to P(I) $$(S/N)_d = \frac{(E_B/\eta_o)}{F_T/n_d} = \text{signal-to-noise power ratio during detection}$$

From this expression,

$$n_d = \frac{(S/N)_d}{(F_B/\eta_o)} \left(\frac{F_T}{R}\right) = \text{number of detectors required}$$

This expression emphasizes the general conclusion reached in Section 4. The on-board processor will be less complex with higher platform data rates. Therefore a maximum platform power limitation is the major factor in configuring the processor.

Returning to Figures 5.2 and 5.3, the next function after signal detection is storage. However, whereas the detection function for both analog and digital processors may be the same, there can be a significant difference between the two regarding data storage.

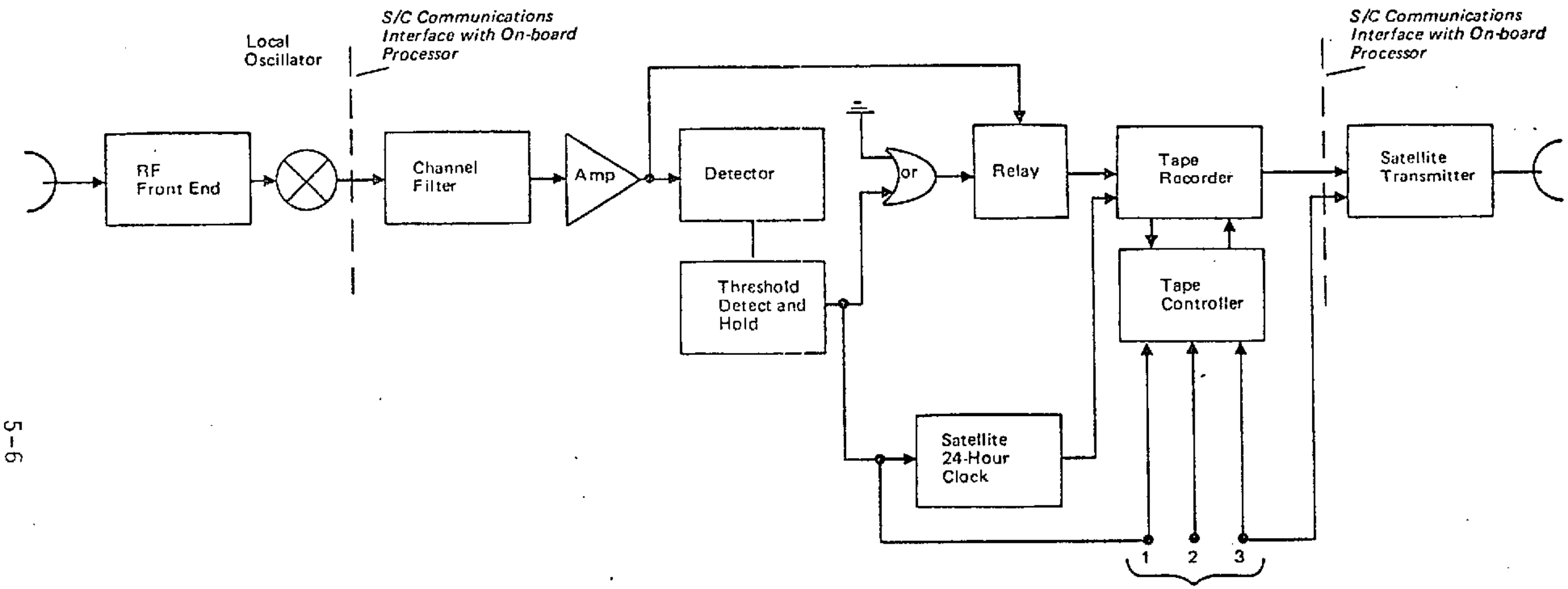

FIGURE 5.2. ON-BOARD PROCESSOR DETECTION AND TAPE RECORDING

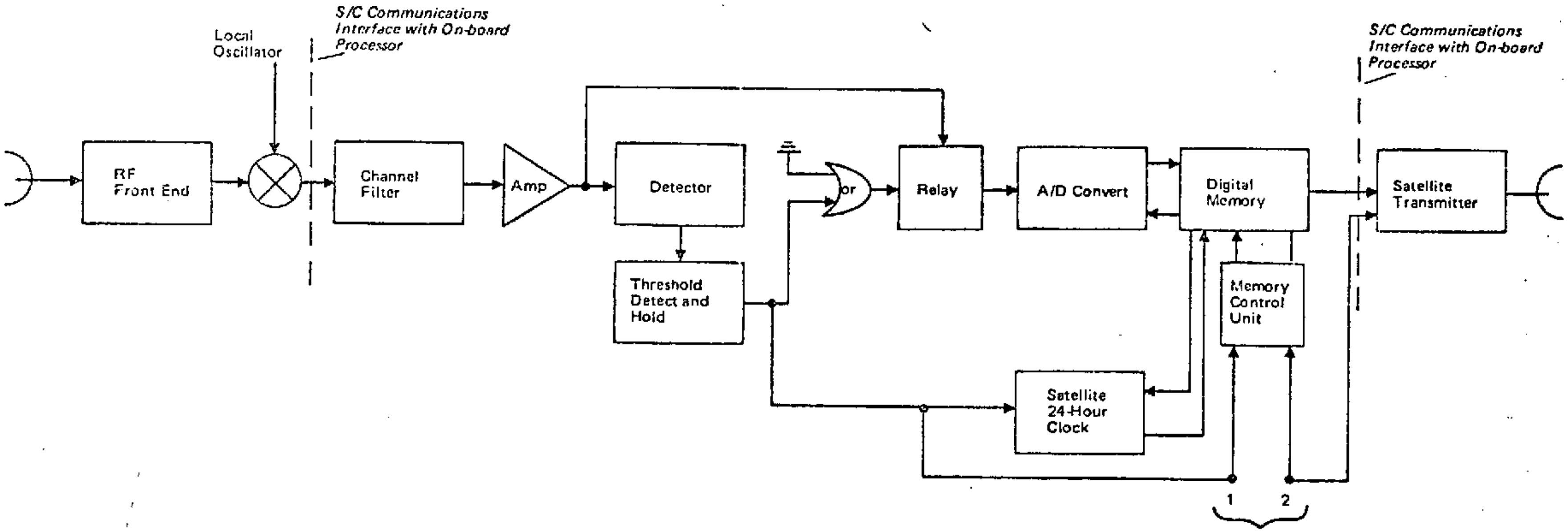

FIGURE 5.3. ON-BOARD PROCESSOR DETECTION AND DIGITAL RECORDING

In Figure 5.2, the analog processor is shown to have a multiple channel tape recorder for data storage. Between this and the detectors, two additional functions are performed. One function is assigning the output of a frequency cell wherein a signal has been detected to one of the multiple channels of the tape recorder. The other function is to reduce the required bandwidth of the recorder. In particular, prior to recording the output from a frequency cell, the filtered output of that cell is mixed with a local reference such that the maximum frequency reaching the tape recorder is on the order of a small multiple of the platform's data rate. To preserve the ability to measure the frequency of the received signal, the frequency of the local reference is stored on the tape along with the timing data required to solve location algorithms.

The performance of an analog processor of this type can be evaluated by means of the results of Section 4 in terms of the data compression accomplished.

Assume that through correlation or error coding, the probability of bit error rate being less than 10^{-4} is determined on a per transmission basis. If NRZ modulation is employed and a two hour sub-synchronous orbit is utilized, then the system data rate ($N_T R/T$) received at the satellite can be determined from Figure 4.16 for signal amplitudes established by free space losses. Also, assume a .95 probability of having a tape (storage) channel available when a signal arrives is required—the value for this probability should also be determined from error correction considerations and from requirements for platform location.

With these assumptions, the data shown in Figure 5.4 can be derived. This figure indicates the number of tape (storage) channels required as a function of platform data rate where the number of detection cells is established by assuming the width of each cell is equal to twice the platform data rate. The parameter of the curves is the system data rate and the corresponding probability of bit error rate being less than 10^{-4} (Figure 4.16).

By assuming the bandwidth for each channel of the recorder is three times the platform data rate, then the data of Figure 5.4 can provide the noise compression effected by an analog, tape-recorder processor. For example, if platform power limits platform data rate to be 600 bits/second and error correction considerations permit a .9 probability of bit error rate being less than 10^{-4}, then three parallel tape channels are adequate. Assuming the tape runs continuously, then with three channels at a frequency response of 1800 Hertz (i.e., 3 x platform data rate), the tape recorder is storing equivalent bits of data at a rate of 5400 Hertz. Comparing this to the received bandwidth of about 17,000 Hertz, this processor would effect a noise compression ratio of better than three to one.

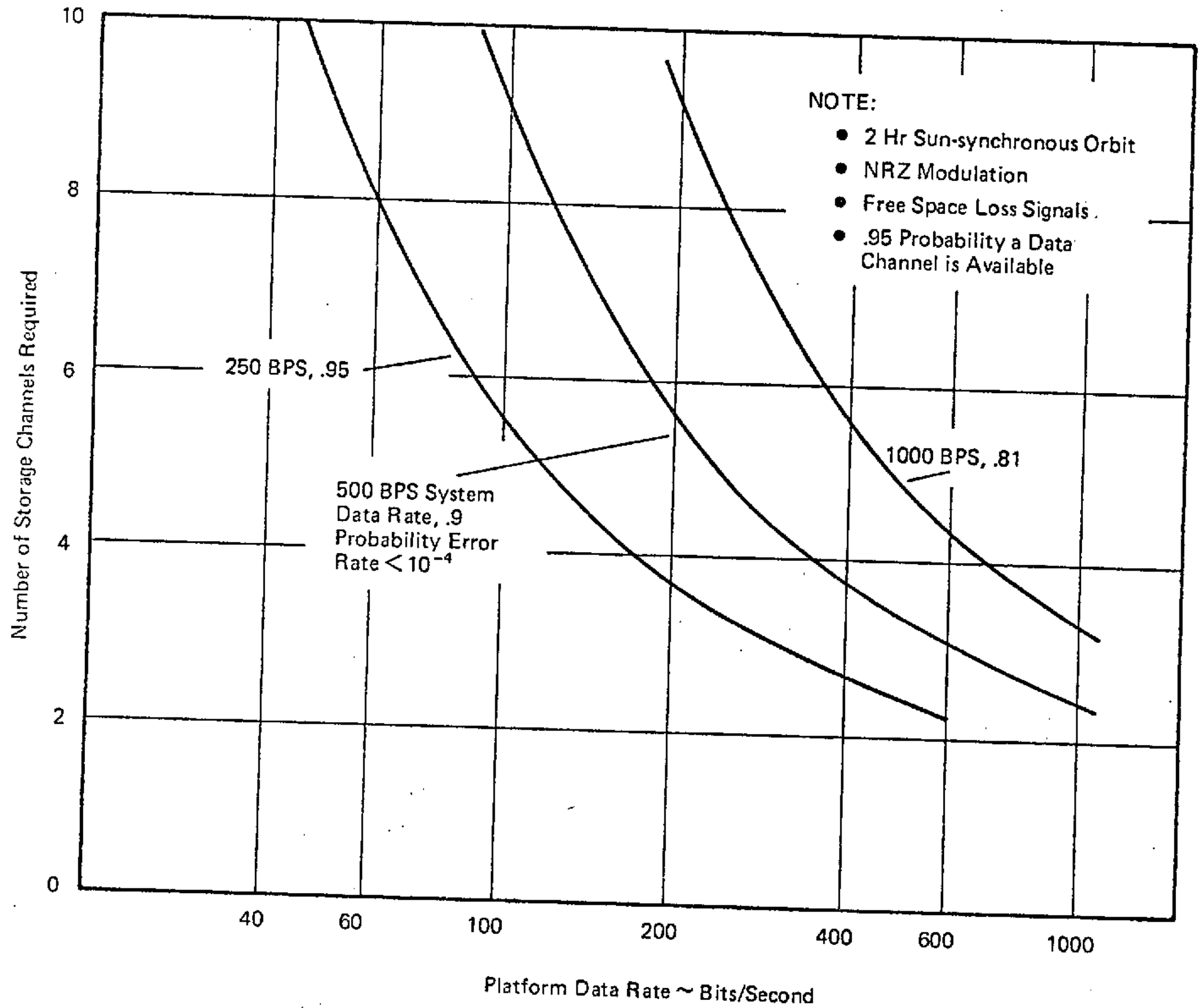

FIGURE 5.4. NUMBER OF STORAGE CHANNELS

If platform data rate is constrained to be lower than 600 bits/second, then different compression ratios will result. For comparison, these ratios are noted in the following table along with the number of parallel tape channels.

SYSTEM DATA RATE

Platform Data Rate	250 BPS	500 BPS	1000 BPS
600	3.1:1, 3 channels	3.1:1, 3 channels	1.9:1, 5 channels
400	4.7:1, 3 channels	3.5:1, 4 channels	2.4:1, 6 channels
200	7.1:1, 4 channels	4.7:1, 6 channels	3.2:1, 9 channels
100	9.4:1, 6 channels	5.7:1, 10 channels	

Note, on a comparative basis that there is always a greater quantity of data stored at the higher system data rates even though the compression ratios are invariably smaller. For example, at a platform data rate of 200 bits/second, a 250 bit/second system data rate achieves a 7.1:1 compression ratio. However, compared to the 500 bit/second system data rate, there is only one half the amount of data present. Therefore, on an equal amount of data basis, the equivalent compression ratio for the 500 bit/second system is twice 4.7:1 or 9.4:1 compared to 7.1:1 for the 250 bit/second system.

A major reason why the analog storage system does not achieve high compression ratios is the assumption that the multiple channel recorder must run continuously as opposed to recording only when a signal is present. A means to avoid this problem is to store the data digitally. In particular, as shown in Figure 5.3, a digital noise-elimination processor can be configured to replace the multiple channel analog recorder with parallel channel analog-to digital converters followed by digital storage. If the assumption is made that eight bit precision is required and that the sampling rate is three times the platform data rate, then the rate of bit storage is the product of the system data rate and the factors of three and eight for sampling rate and precision. For example, from Figure 5.4, a 600 bit/second platform rate and a 500 bit/second system data rate requires three channels. The rate of bit storage (average) would then be 500 x 3 x 8 or 12,000 bits/second. More simply, the rate of bit storage is just 24 times the system data rate. This means the noise compression ratio for a digital noise-elimination processor is the received bandwidth divided by the product of system data rate and the factor of 24. Relative to the quantity of data present, the compression ratio is then independent of system data rate.

In summary, noise elimination processors are not able to approach the limit value of noise compression ratio for random access systems. In the case of analog processors, the requirement to continuously run a tape recorder even when signals are not present results in storage of noise only regions of the time-frequency space. Digital systems, on the other hand, can be configured to eliminate most noise-only regions but suffer from the requirement to sample at higher than the platform data rates and to store many (eight) bits for each sample. In this regard a hybrid processor which periodically re-converts digitally stored data to a time smoothed series of analog data for tape recorder storage might be advantageous—i.e., the factor of eight could be eliminated.

5.2.2 Processors Using Demodulation

A logical extension of the On-Board Processor from the noise elimination processor is to add demodulation of the signals. An example of such a processor is shown in Figure 5.5. As with the noise elimination processor, this processor interfaces with the spacecraft electronics at the output of a down converter. The detection mechanism is a Fast Fourier Transform Unit operating on the incoming spectrum to determine signal presence and to estimate frequency. This is under control of the central processing unit that also controls and coordinates all the processor functions. The overall architecture of the processor is like that of a small computer with peripherals and special purpose processing devices interfacing with it. The modulation is assumed to be digital and coherent. The processor has several demodulators; the number of which is determined by specifying the probability that a demodulator is available when a transmission arrives at the spacecraft. The detection processor (Fast Fourier Transform (FFT) Unit) obtains a spectrum estimate on a periodic basis of the entire system bandwidth. The period of the FFT is less than the time interval allowed for carrier synchronization i.e., the CW tone indicated in Figure 5.1. The spectrum estimates are stored and processed to determine signal presence and a coarse estimate of the frequency of each signal. This information is forwarded to a buffer which contains a record of signals present at any instant of time. The coarse frequency estimates are then used to drive the demodulators to the approximate frequency of the incoming signals. This then is the mechanism by which demodulators are assigned to incoming transmissions. This process should place the arriving signal within the acquisition bandwidth of the coherent demodulator.

The signal is then demodulated and only the sensor data and the platform identification are stored in the data buffer shown. At the same time a "fine" estimate of the received carrier frequency is obtained for position location purposes.

The data processing unit processes the sensor and frequency data. The specific functions performed are:

- Formating of Downlink Data
- Obtain time information from Satellite clock.

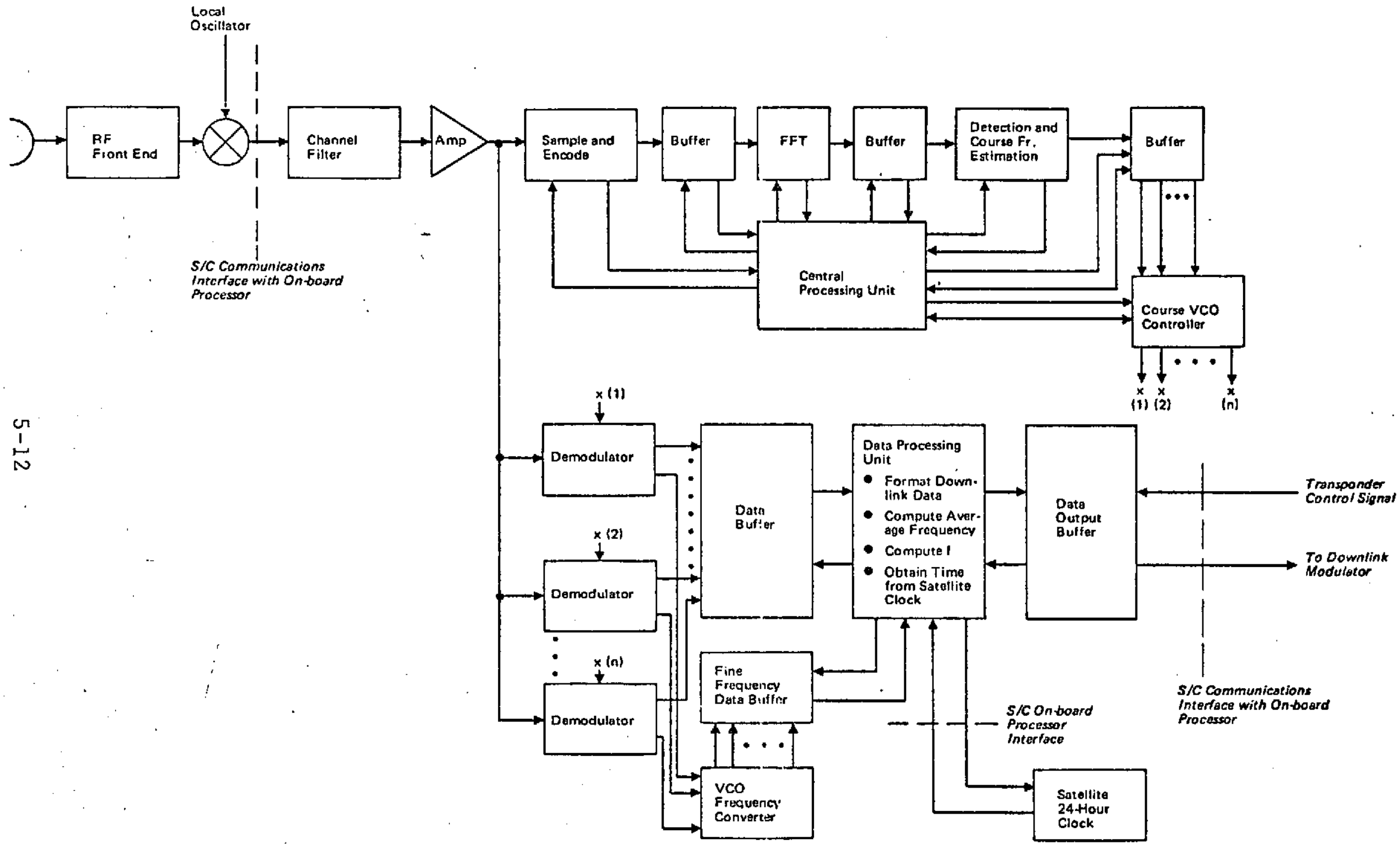

FIGURE 5.5. ON-BOARD PROCESSOR USING DEMODULATION

The downlink data format would be as shown in Figure 5.6. Note that since the processor does not eliminate redundant transmissions, the entire dump to the Data Collection Earth Station may contain several identical data blocks—although the received frequency data will differ. As shown, the entire dump is prefaced by a standard preamble and ends with an end of transmission character. Further, each data block contains platform identification, sensor data, frequency data, and time of arrival of the transmission.

Assuming a high probability of access, essentially all Platform transmissions will be recorded. Thus the effect of probability of access on the processor compression ratio is assumed neglibible. Improved compression over the previously mentioned noise elimination processor is achieved because the processor operates only when platofrm transmission are present.

The compression ratio for this type of processor is a function of the system data rate and the data format associated with the uplink transmission as follows:

$$\text{compression ratio} = \frac{B}{\eta_u R \frac{N}{T} \tau}$$

where,

B = system bandwidth

R = platform data rate

$\frac{N \tau R}{T}$ = system data rate

η_u = frame efficiency of uplink transmission burst

The frame efficiency can be derived from Figure 5.6 as the ratio of the number of information bits to the total number of bits in a transmission. The overhead bits include:

- carrier recovery
- bit timing recovery
- start of data code word
- end of transmission word

The data includes:

- sensor identification word (if necessary)
- platform identification
- sensor data.

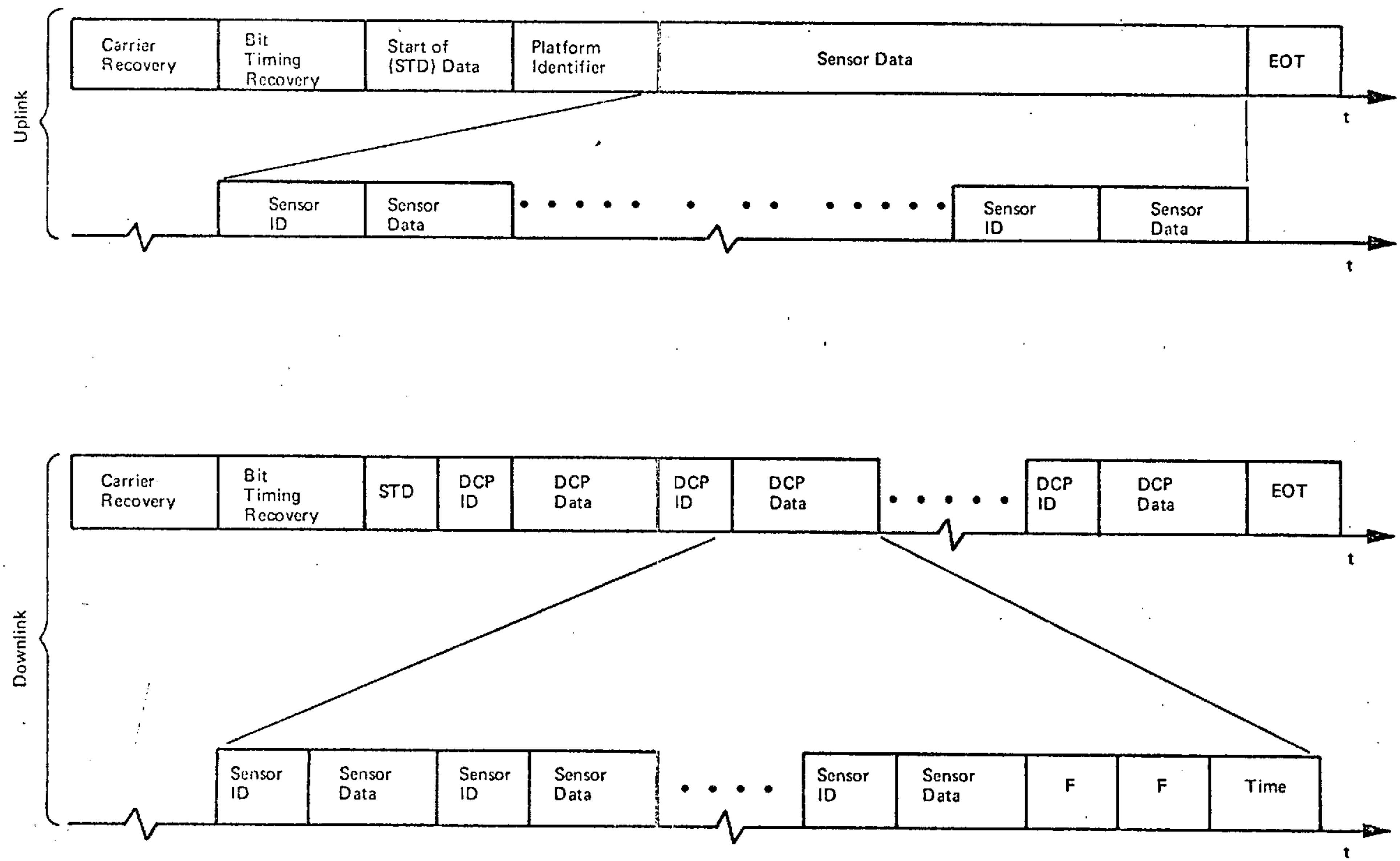

FIGURE 5.6. UPLINK AND DOWNLINK DATA FORMATS

The uplink efficiency can be expressed as:

$$\eta_u = \frac{n_D}{n_D + n_{OH}}$$

where;

n_D = number of data bits

n_{OH} = number of overhead bits.

The frame efficiency then, accounts for the fact that a demodulating processor will discard all uplink overhead bits—i.e., they will not be stored for retransmission.

It is of interest to compare the compression ratios of the processor with the noise elimination processor wherein the compression ratio was given by:

$$C_R = \frac{B}{3nR}$$

where;

n = number of data channels in processor

R = platform data rate

Upon examination of the two expressions, a major difference is evident. That is, the compression ratio for the noise elimination processor is inversly proportional to the number of data channels whereas for the demodulating processor it is not. Also, the compression ratio for the demodulating processor depends on frame efficiency whereas for the noise elimination processor it does not, since this latter processor does not discard overhead bits.

Typical values for compression ratio for a processor using demodulation can be obtained using Figure 4.26. For a probability of 0.9 that the signal is acquired and the bit error rate is less than 10^{4}, the compression ratios shown in the following Table are possible. These calculations assume a 60% uplink frame efficiency and a system bandwidth (B) of 18 kHz.

DEMODULATION PROCESSOR COMPRESSION RATIOS

System Data Rate (bps)	Platform Data Rate (bps)	Compression Ratio
140	100	150.5:1
220	200	96:1
270	400	78:1
280	1000	75.5:1

The difference between the noise elimination processor and the demodulation processor is significant. The noise elimination processor yields compression ratios from 1.9:1 to 9.4:1 whereas the demodulation processor yields values ranging from 75.5:1 to 150.5:1. It is evident then that using the demodulation processor will yield significant benefits if the price payed for additional processor complexity is acceptable.

While the demodulating processor achieves significantly better compression ratios compared to the noise elimination processor, a significant disadvantage is present in the processor concept as presented above. There is no means to assess the presence or absence of interfering transmissions throughout the demodulation process. To be able to obtain this interference information, a second demodulating processor concept is indicated in Figure 5.7. Furthermore, this processor can also be shown to inherently decrease the level of interference within a random access system requiring location and velocity estimation.*

The fundamental differences of this second processor are in two areas, the spectrum of platform transmissions and the parameters measured to provide location/velocity estimation. The spectrum of platform signals is indicated in Figure 5.8 with typical values of the indicated parameters. In essence, instead of the data being modulated directly onto the carrier, the data is moduled onto one or another side band. This modulation provides the two desired advantages:

- By maintaining a clean carrier tone during demodulation and storing its value and time of occurrance, a record is obtained of the occurrance of all platform transmissions. This enables after-the-fact determination of whether interference occurred during demodulation. Furthermore, separation of data from the carrier enhances the ability to precisely measure the frequency of the carrier for location purposes.

- By adding a side tone, phase changes from one platform transmission to another provides an independent measurement of range difference between satellite and platform between two transmission. Therefore, there are fewer transmissions required per overpass to obtain requisite data for location purpose and consequently the level of mutual interference can be less with the same number of platforms present.

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* "Study of Advanced System for Position Location and Navigation", Operations Research, Inc., Prepared under Contract NAS5-21685 for NASA, August 1972

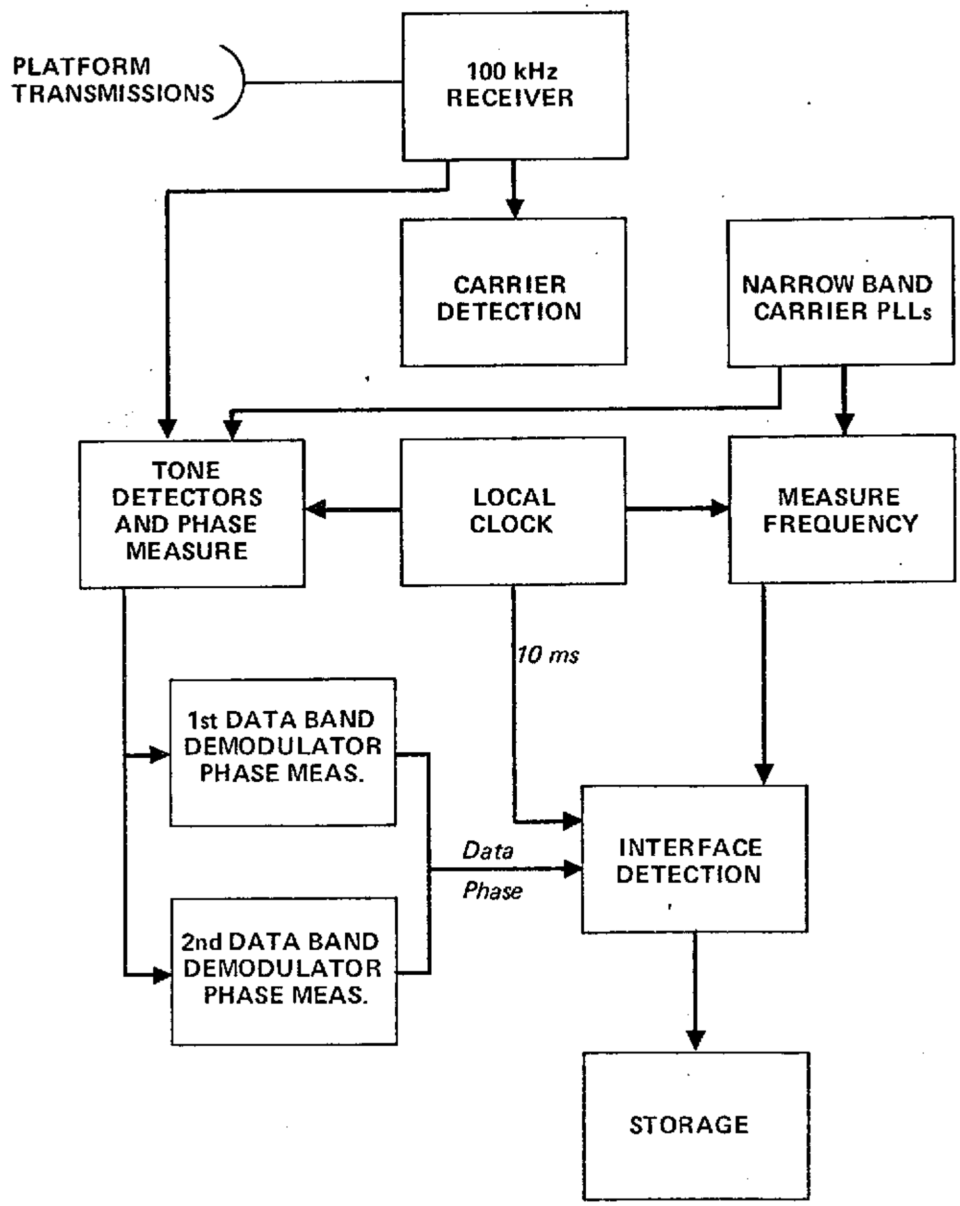

FIGURE 5.7. SIDEBAND SATELLITE PROCESSOR (TWO DATA BANDS)

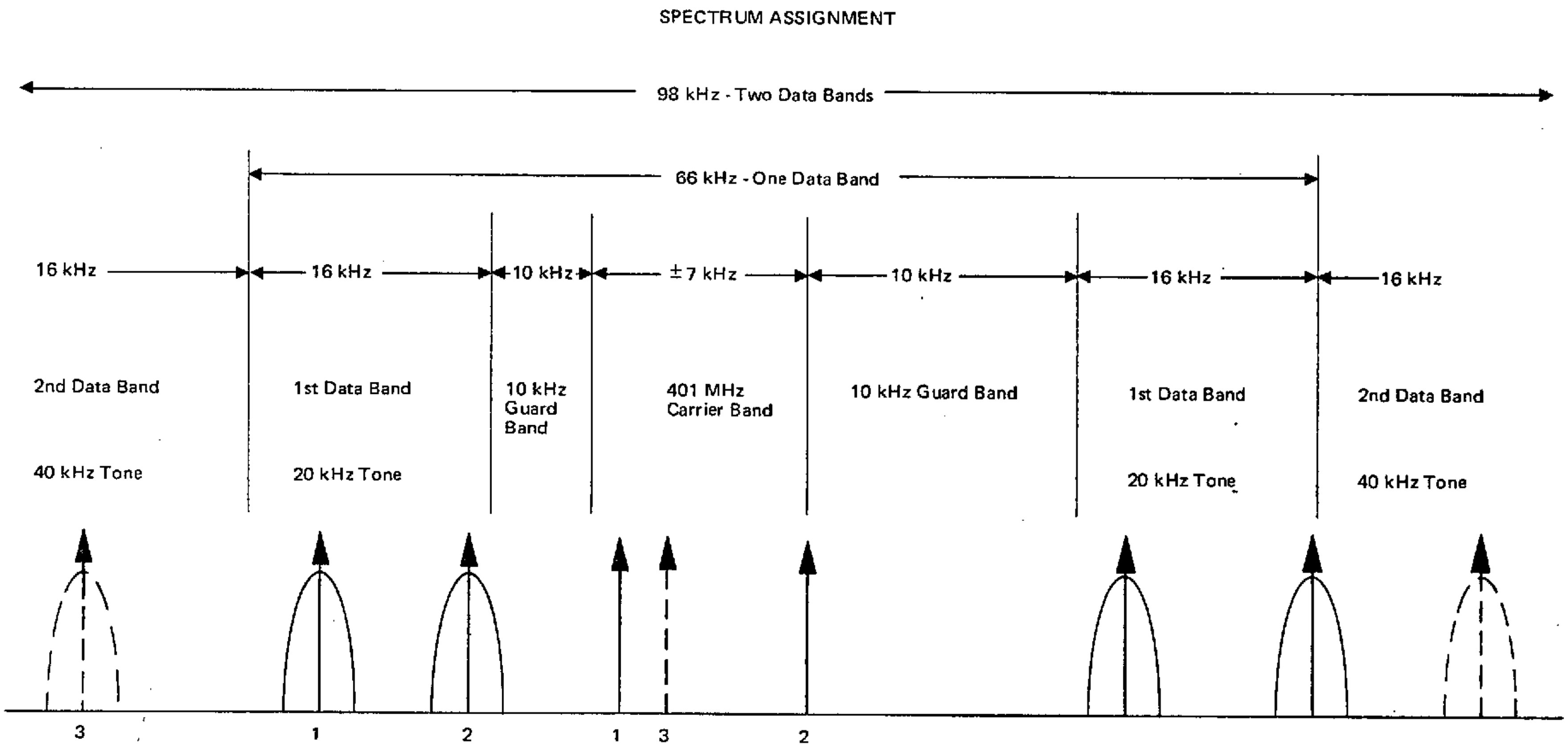

FIGURE 5.8. THREE TRANSMISSION EXAMPLE

To obtain these advantages, the demodulating procesor of Figure 5.7 is more complex as indicated by the addition of two new functions. One of these is the necessity to measure side-tone phase relative to a stable on-board clock of nominally the same frequency off-set from the carrier. The second additional function is the detection or recording of interference during demodulation of a signal. There is, of course, no advantage indicated for this processor concept in terms of data compression over and above that of the demodulating processor of Figure 5.7.

5.2.3 Logical Data Compression

With platform data in digital form, further data compression may be achieved by performing logical operations prior to storage. Several types of these are possible:

- For those platforms not requiring location/velocity estimation, delete storage of precise time of receipt and frequency measurement data
- Perform part or all of the location/velocity algorithm to preclude storage of precise time and measured frequency data
- Preclude repeated storage of a platform's identifier by co-locating, in storage, multiple transmission data
- Delete storage of repeated, identical sensor data received from a given platform.

The advantage of performing any one or all of these operations in terms of the additional data compression achieved depends upon a number of factors including number of platforms, amount of sensor data per transmission, etc. However, two considerations are perhaps most important.

- Is location only or location and velocity estimation necessary, and if so, to what precision?
- What level of bit error rate as seen by the user is acceptable?

The importance of location and/or velocity estimation is derived from the need for multiple transmissions during one or successive overpasses of the satellite. Without location being required, the number of transmissions from a given platform may be determined solely by the probability of acquiring at least one set of error free sensor data. Therefore, in this case, relatively little compression of data can be achieved by deleting identical data and/or repeated storage of platform identifiers. However, at the other extreme, the need to locate and estimate platform velocity necessitates receipt of at least three to four transmissions during each overpass and therefore, significant compression may be afforded by logically deleting repeated sensor and identifier data.

The effect of user bit error rate is more difficult to assess. From the results of the interference analyses of Section 4, the effective bit error rate of random access systems on a per transmission basis is high—i.e., on the order of the interference probability or near .10 or .05 if significant capacity is desired. Achieving a specified level of bit error rate then requires either error correction coding or correlation or as briefly discussed in the second demodulating processor concept, the ability to flag those transmissions during which significant interference occurred.

APPENDIX
DISTRIBUTION OF INTERFERENCE

The derivation of mutual interference statistics in Section 4 is based upon statistics of time and frequency overlap. The probability of time overlap is shown to be approximated by Poisson statistics but, the probability of a given level of interference is assumed to be provided as a function of interference level. This Appendix presents the derivation of this interference probability—i.e., the probability of a specified bit error rate for a given platform's transmission when another simultaneous transmission is present.

From the description of the gaussian noise model and its application to defining bit error rate in an interference environment, the problem of determining the probability of a given level of interference can be broken into two categories.

- Determination of the statistical distribution describing the frequency separation between two simultaneous transmissions
- Determination of the statistical distribution describing the relative amplitudes of the given and interferring signals as received at the satellite

To determine the frequency separation between two simultaneously signals at the satellite, two factors must be known. The transmission frequencies of both the given and interfering transmissions must be established. The doppler shift experienced by the individual transmissions must be established.

Regarding the frequency of transmissions from platforms, the assumption is made that these are normally distributed about the mean frequency of all platforms. Furthermore, the standard deviation of this distribution is held fixed in all analyses at a value of 2000 Hertz. At a nominal transmission frequency of 400 megahertz, this corresponds to platform equipment drifting on the order of .0005%.

Once the transmission frequency is established, the frequency of the transmission received at the satellite is determined by the doppler shift experienced. This is, in turn, the result of the relative kinematics existing between the platform and satellite at the moment of transmission. Therefore, the probability of a particular doppler shift is directly related to the statistics describing the location of the interfering platform relative to the satellite.

In order to analytically describe these relationships, a coordinate system is necessary to describe the location of platforms relative to each other and relative to the satellite. The satellite is always assumed to be at zero degrees latitude and longitude and the line between the satellite and the earth defines the x axis of the coordinate system. The origin of the coordinate system is put at the center of the earth and the z axis corresponds to the rotational or polar axis of the earth. With a right handed coordinate system, the y axis is then in the equatorial plane (as well as the x axis) pointing east as seen from the satellite.

The location of platforms within this coordinate system is accomplished by means of two angles. One or these is the earth central angle (E) between the x axis and the line between the earth center and the platform location on the earth's surface. The second angle describes the azimuth of the platform relative to the y axis. In particular, if the position of the platform is projected onto the y-x plane, the aximuth (α) is defined as the angle between the y axis and the line connecting the origin of the y-z axes and the projected position of the platform. With these coordinates, the statistics of received frequencies at the satellite can be analytically described.

The difference between the frequency of a transmission received at the satellite and the frequency transmitted can be determined from the doppler shift relationship

$$f_T = f_R \left(1 - V_{R/C}\right)$$

where;

f_T is the transmitted frequency

F_R is the frequency received at the satellite

V_R is the radial velocity component of the relative velocity between the satellite and transmitter

C is the signal propagation velocity

With platforms located on the earth's surface, the radial velocity component V_R is completely determined by the direction and magnitude of the satellite's velocity, vector the altitude of the satellite, and the two angles describing platform location—i.e., ϵ and α. In particular,

$$V_R = (V_S - V_P) \cdot (R_S - R_P)/R_{SP}$$

where;

V_S is the satellite's velocity vector relative to a non-rotating earth and assumed parallel

V_P is the platform's velocity vector as caused by earth rotation—for present analyses, velocities of the platforms relative to the earth are presumed to be negligible

R_S is the position vector of the satellite which is merely its x coordinate by definition of the coordinate axes

R_P is the position vector of the platform as defined by ϵ and α and the assumption that the platform is on the earth's surface

R_{SP} is the range between the satellite and platform—i.e., the magnitude of $(R_S - R_P)$

If these vector manipulations are performed, then V_R can be shown to be

$$R_{SP}V_R = V_P \text{SIN}\beta\ ({}^{R}S/R_E - \text{COS}\ \epsilon) - (V_S \text{COS}\ i - V_P \text{COS}\beta)\text{SIN}\epsilon\ \text{COS}\ \alpha - V_S \text{SIN} i \text{SIN}\epsilon \text{SIN}\alpha$$

where;

i is the inclination of the satellite's orbit

β is defined as $\text{TAN}^{-1}\ (\text{SIN}\ \epsilon\ \text{COS}\alpha/\text{COS}\ \epsilon)$

R_E is the earth's radius

V_S, V_P represent the magnitudes of the satellite and platform velocity vectors respectively.

With this relationship and the doppler shift equation, the distribution of received frequency can be determined once the geographic distribution of platforms is established assuming for the moment that the transmission frequency is fixed and known. For the present analyses, the distribution of platforms over the earth's surface is assumed to be uniform in a statistical sense. In particular, the probability of a platform being within an arbitrarily small area is assumed to be proportional to the area and the average density of platforms. Analytically, this may be stated as follows

$P_p(\alpha, \epsilon)$ = probability of a platform being at the coordinates α and ϵ

$$= \frac{\rho RE^2 \ SIN \ \epsilon \ d\epsilon \ d\alpha}{2\pi\rho R_E^2 \ [1-COS \ \epsilon_{MAX}]} = \frac{SIN\epsilon \ d\epsilon \ d\alpha}{2\pi[1 - COS\epsilon_{MAX}]}$$

where;

ρ is the average density of platforms within view of the satellite—e.g., number per square kilometer ϵ_{MAX} is the maximum value of the earth central angle separation between satellite and platforms such that communication (interference) can be established

This relationship is also the probability that a given radial velocity exists between the satellite and platform and therefore, the probability of receiving a particular frequency provided the transmitted frequency is known.

If instead of equal amplitude signals being received at the satellite the received signals have amplitude variations corresponding to free-space losses, the density function of interference level defined in Section 4.2.3 must be determined. The relationship required to accomplish this is the equation defining relative power levels at the satellite. From geometry

$$\left(\frac{S_0}{S_1}\right) = \left(\frac{R_0}{R_1}\right)^2 = \left\{\frac{x^2 - 2xCOS\epsilon_0 + 1}{x^2 - 2xCOS \ \epsilon_1 + 1}\right\}$$

where;

(S_0/S_1) is the ratio between received signal power of the given platform at $\epsilon_0 \, \alpha_0$ and the received signal power of the interfering platform located at ϵ_1 , α_1

$(R_0 \, R_1)$ is the ratio of the range between the satellite and given platform (R_0) and the satellite to interfering platform range (R_1)

x is defined as (R_S/R_E)

Returning to the assumption of normally distributed transmission frequencies, the probability of a specific frequency being transmitted is

$$P_f = \frac{1}{\sqrt{2\pi\sigma_f}} \quad e^{-\left(\frac{f_T - f_{T_o}}{\sigma_f}\right)^2} \, df_T$$

where

σ_f is the standard deviation—i.e., 2000 Hertz

f_{T_o} is the mean transmission frequency for all platforms

and

P_f is the probability that a transmission frequency is f_T

If the platform for which interference is to be described is located at given coordinates $(\epsilon_o \; \alpha_o)$, then the probability density function of the difference between its received frequency at the satellite and the received frequency of an interfering transmission can be shown to be

$$p(b_I | \; \epsilon_1, \alpha_1, \epsilon_o, \alpha_o) = \frac{1}{2\sigma_f\sqrt{\pi}} \, e^{-\left\{\frac{b_I - f_{T_o}(V_{R_o} - V_{R_1})/C}{2\,\sigma_f}\right\}^2}$$

where;

b_I is the frequency difference between the carriers of the given and interfering platform

ϵ_1, α_1 are the coordinates of the interfering platforms

V_{R_o}, V_{R_1} is the radial velocity between the staellite and given or interfering platform

Multiplication of the relationship by the probability of the interfering platforms position—namely, P_p (ϵ_1, α_1)—gives the density function of b_I. If amplitude of all received signals at the satellite are equal, then integration over ϵ_1 and α_1 within the limits of the visibility circle gives the density function of b_I including all possible locations of the interfering platform. For equal amplitude signals, this is identical with the density function of interference level as defined in Section 4.2.3.