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ON THE INFLUENCE OF THE SURFACE AND BODY TIDES ON THE MOTION OF A SATELLITE

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INTRODUCTION

In the present article we investigate some geophysical aspects of the tidal perturbations in the motion of artificial satellites and develop a system of formulas convenient for computation of the tidal effects in the elements using a step-by-step numerical integration.

Recently the tidal perturbations in the motion of artificial satellites captured the imagination of theoreticians and observers. The significance of these perturbations lies in the fact that they are intimately connected with the elastic properties of the Earth and with the distribution of density inside the Earth. Thus, the comparison between the observed and computed tidal perturbations of the satellite permits one to obtain information about the average elastic properties of the Earth and to check the assumed model of the Earth.

The attraction of the Earth by the Moon and the Sun produces elastic displacements inside the Earth body (bodily tides) and the tidal buldge on its surface (surface tides). The result is a small change in the distribution of mass and, consequently, small variations in the exterior geopotential. These variations constitute the exterior perturbing tidal potential, and they produce the perturbations in the motion of an artificial satellite.

The tidal perturbations in the motion of the satellite are not the result of the gravitational attraction by the surface buldge only. The bodily tides contribute their share of influence (approximately 40%).

The exterior tidal potential can be expanded into a series of products of spherical harmonics. The first factor in each product is a polynomial in the components of the unit vector directed from the center of the Earth toward the Moon (the Sun). The second factor, of the same degree and order as the first, is a similar polynomial in the components of the unit vector directed toward the satellite. This form of the tidal potential is convenient for the computation of perturbations by means of numerical step by step integration. If we prefer an analytical expansion, then, to facilitate the integration, the products of spherical harmonics are expanded into Fourier series with arguments which are linear combinations of the arguments ℓ, ℓ', F, D and Γ of the lunar theory and of the mean angular elements of the satellite (Musen and Estes, 1971). All these arguments are either linear or nearly linear in time and, consequently, the integration in analytical form does not present any difficulty. The program for manipulating the Fourier series which appear in the theory of tidal perturbations was developed by C. Hipkins and R. Estes of BTS Company.

Each term of the Fourier expansion contains a factor, i.e. the Love number, which represents a measure of the elastic response of the Earth to the given tidal frequency. These numbers depend on the laws of change of Lamé elastic parameters and the density inside the Earth, on the degree of the spherical harmonics, and,to a lesser extent, on its order and on the frequency itself (Alterman et al., 1959).

EXPANSION OF THE EXTERIOR TIDAL POTENTIAL

The tidal oscillations of the point $\mathbf{r}''$ in the earth are governed by the partial differential equation (Alterman et al., 1959):

$$\rho \frac{\partial^2 \mathbf{s}}{\partial t^2} = \rho \nabla \psi + \mathbf{g} \nabla \cdot (\rho \mathbf{s}) - \nabla(\rho \mathbf{s} \cdot \mathbf{g}) + \nabla \cdot \sigma, \tag{1}$$

where ∇ is the del-operator relative to $\mathbf{r}''$, σ is the stress tensor

$$\sigma = \mu(\nabla \mathbf{s} + \mathbf{s} \nabla) + \lambda \mathrm{I} \nabla \cdot \mathbf{s}, \tag{2}$$

I is the idemfactor, $\mathbf{g}$ is the undisturbed acceleration of gravity, and ψ is the sum of the direct tidal potential acting on $\mathbf{r}''$ and the potential due to the tidal disturbance in the interior geopotential, as caused by the presence of elastic displacements inside the Earth and on its surface. The density ρ and the Lame elastic parameters, λ and μ, are considered in the frame of the present work as functions of $\nu'' = r''/R$ only, where R is the mean radius of the Earth. The potential ψ satisfies Poisson's equation

$$\nabla^2 \psi = + 4\pi G \nabla \cdot (\rho \mathbf{s}). \tag{3}$$

The particular solution of (3)

$$\Omega = - G \int_w \frac{\nabla \cdot (\rho \mathbf{s})}{|\mathbf{r} - \mathbf{r}''|} \, dv, \tag{4}$$

represents the exterior tidal potential acting on the satellite, where $\mathbf{r}$ is the position vector of the satellite. The integral in (4) is taken over the volume of the whole tidally disturbed Earth. In our approximation we assume that the tidally undisturbed Earth is a sphere of the radius R and neglect the influence of the Coriolis force. This assumption, however, must be removed in the investigations about the response of the satellite to the tidal forces over a very long interval of time. Under these assumptions the potential (4) represents the effects of the body and surface "solid" Earth tides on the motion of the satellite. The effects of the oceanic tides are not considered in the present paper.

We split the integral (4) into the sum of the integral over the volume of the sphere $r'' = R$ and the integral over the surfaces of density discontinuities,

$$\Omega = -G \int_v \frac{\nabla\cdot(\rho \mathbf{s})}{|\mathbf{r} - \mathbf{r}''|}\, dv + G \int_S \frac{\rho' \mathbf{s}\cdot\mathbf{r}''^0}{|\mathbf{r} - \mathbf{r}''|}\, dS, \tag{5}$$

where $\mathbf{r}''^0$ is the unit vector in the direction of $\mathbf{r}''$,

$$dv = r''^2 dr'' d\sigma'',$$

$$dS = r''^2 d\sigma'',$$

$$d\sigma'' = \cos\beta'' d\beta'' d\lambda'',$$

and β'' is the latitude and λ'' the east longitude, and ρ' is the density discontinuity, $\rho' = \rho$ on the outer surface.

The first term in (5) represents the disturbing influence of the internal elastic tidal displacements inside the Earth's body; the second term represents the perturbative influence of the tidal buldge. The displacement s, as given by the differential equation (1), is a combination of the spheroidal and torsional oscillations of the Earth.

In the investigations of the influence of tides on the motion of a satellite it is convenient to represent the components of the spheroidal oscillations in the form:

$$s_{nm}(\kappa) = \frac{4\pi}{2n+1} \frac{m'}{M} p'^{n+1} R\{H_{nm}(\nu'') Y_{nm}(\beta'', \lambda'') \mathbf{r}''^{0}$$

$$+ r'' L_{nm}(\nu'') \operatorname{grad} Y_{nm}(\beta'', \lambda'')\} e^{i(\kappa t + m\theta)} \cdot K_{nm}(\kappa) \tag{6}$$

This representation of s_{nm} differs in form only slightly from the corresponding representation by Alterman et al (1959). In (6) m'/M is the ratio of the mass of the Moon (Sun) to the mass of the Earth, and p' is the lunar (solar) parallactic factor R/a', where a' is the mean distance of the Moon (Sun) from the Earth. $H_{nm}(\nu'')$ and $L_{nm}(\nu'')$ can be termed the generalized Love and Shida "numbers", with $\nu'' = r''/R$. They satisfy a system of ordinary differential equations (Alterman, et al., 1959), (Takeuchi, 1950) which can be integrated only numerically. The functions $Y_{nm}(\beta'', \lambda'')$ are the normalized spherical harmonics,

$$Y_{nm}(\beta'', \lambda'') = \sqrt{\frac{2n+1}{4\pi} \cdot \frac{(n-|m|)!}{(n+|m|)!}} P_n^m(\sin\beta'') e^{+im\lambda''}; \tag{7}$$

$$P_n^m(x) = \frac{(-1)^m}{n!\,2^n} (1 - x^2)^{m/2} \frac{d^{n+m}(x^2 - 1)^n}{dx^{n+m}},$$

with the normalization and orthogonality conditions

$$\int_0^{2\pi} d\lambda'' \int_{-\pi/2}^{+\pi/2} Y_{nm}(\beta'', \lambda'')\, Y^*_{n'm'}(\beta'', \lambda'') \cos\beta'' d\beta'' = \delta_{nn'} \delta_{mm'}. \tag{8}$$

In spherical coordinates we have:

$$\nabla Y_{nm} = + \frac{1}{r''} \frac{\partial Y_{nm}}{\partial \beta''} \mathbf{e}_{\beta''} + \frac{1}{r'' \cos\beta''} \frac{\partial Y_{nm}}{\partial \lambda''} \mathbf{e}_{\lambda''}, \tag{9}$$

where $\mathbf{e}_{\beta''}$ and $\mathbf{e}_{\lambda''}$ are the corresponding unit vectors. The factor K_{nm} · exp $i(\kappa t + m\theta)$ in (6) is inherited by $\mathbf{s}_{nm}$ in (6) from the "lunar" ("solar") factors $(a'/r')^n$ $Y^*_{nm}(\beta', \lambda')$ in the expansion of the static tidal potential acting on each point of the Earth. The part κt is a linear combination of ℓ, ℓ', F, D and Γ, and θ is the sidereal time. Besides (6) there is a second geophysically important solution of (1). It is of the form

$$\mathbf{s}_{nm} = \frac{4\pi}{2n+1} \cdot \frac{m'}{M} R p'^{\,n+1} W_{nm}(\nu'') (\mathbf{r}'' \times \mathrm{grad}\, Y_{nm})\, e^{i\varkappa_1 t}$$

and represents the torsional oscillations of the Earth (Alterman et al., 1959), Jeffreys (1967). However, it is not of any importance in our problem. Its radial component is zero and, because

$$\mathrm{div}(W_{nm}\mathbf{r}'' \times \mathrm{grad}\ Y_{nm}) = 0,$$

it does not produce any dilatation. Consequently, it will not produce any disturbances in the exterior geopotential at all, and will not influence the motion of an artificial satellite. Thus, only the spheroidal oscillations of the Earth as given by (6) can produce perturbations in the motion of a satellite.

The tidal potential (5) can be represented as a sum of particular tidal potentials of the form:

$$\Omega_{nm} = -G\int_v \frac{\nabla\cdot(\rho\mathbf{s}_{nm})}{|\mathbf{r}-\mathbf{r}''|}\,dv + G\int_S \frac{\rho\mathbf{s}_{nm}\cdot\mathbf{r}''^0}{|\mathbf{r}-\mathbf{r}''|}\,dS. \tag{10}$$

We have

$$\nabla\cdot(\rho\mathbf{s}_{nm}) = \frac{1}{R}\,\frac{d\rho}{d\nu''}\,\mathbf{r}''^0\cdot\mathbf{s}_{nm} + \rho\nabla\cdot\mathbf{s}_{nm}. \tag{11}$$

From the last equation, taking into account

$$\mathbf{r}''^0\cdot\nabla Y_{nm} = 0,$$

$$\nabla^2 Y_{nm} = -\frac{n(n+1)}{r''^2}\,Y_{nm},$$

and (6), we obtain:

$$\nabla \cdot s_{nm} = \frac{4\pi}{2n+1} \cdot \frac{m'}{M} p'^{\,n+1} \left[\frac{dH_{nm}}{d\nu''} + \frac{2}{\nu''} H_{nm}\right. \tag{12}$$

$$\left. - \frac{n(n+1)}{\nu''} L_{nm}\right] Y_{nm}(\beta'', \lambda'') e^{i(\kappa t + m\theta)}$$

Substituting (12) into (11), expanding $|r'' - r|^{-1}$ in terms of spherical harmonics, and taking (6) into consideration we obtain:

$$\Omega_{nm}(\kappa) = \frac{4\pi}{2n+1} \cdot \frac{Gm'}{R} (pp')^{n+1} e^{i\kappa t} \; K_{nm}(\kappa) \tag{13}$$

$$\cdot\, k_{nm}(\kappa) \left(\frac{a}{r}\right)^{n+1} Y_{nm}(\delta, \alpha),$$

and

$$k_{nm} = \frac{3}{2n+1} \left\{ \sum \frac{\rho'}{\rho_0} \nu''^{\,n+2} H_{nm}(\nu'') - \int_0^1 \nu''^{\,n+2} Q_{nm}(\nu'')\, d\nu'' \right\}, \tag{14}$$

where k_{nm} is the Love number. The summation symbol relates to the surfaces of the density discontinuity and we set

$$Q_{nm}(\nu'') = \frac{\rho(\nu'')}{\rho_0} \left[H'_{nm}(\nu'') + \frac{2}{\nu''} H_{nm}(\nu'') \right. \tag{16}$$

$$\left. - \frac{n(n+1)}{\nu''} L_{nm}(\nu'') + H_{nm} \frac{d \log \rho(\nu'')}{d\nu''} \right],$$

p is the parallactic factor R/a, and a is the semi-major axis of the satellite's orbit, ρ_0 is the mean density of the Earth. The numerical value of the first term in (14) gives a general idea about the order of magnitude of the perturbing by the tidal buldge alone. The value of the second term gives information about the magnitude of the disturbing influence of bodily tides. In (13) all short period terms depending on the Earth's rotation and involving the sidereal time have disappeared automatically.

The most significant tidal perturbations in the motion of a satellite are those of long period. We obtain the disturbing potentials Ω_{nm} with long period terms only, by averaging the expressions (13) over the orbit of the satellite. Thus:

$$\Omega_{nm} = \frac{4\pi}{2n+1} \frac{Gm'}{R} (pp')^{n+1} k_{nm} e^{i\kappa t} W_{nm},$$

where

$$W_{nm} = \frac{1}{2\pi} \int_0^{\pi} \left(\frac{a}{r}\right)^{n+1} Y_{nm}(\delta, \alpha)\, dg \tag{17}$$

and g is the mean anomaly of the satellite. The results of this averaging in terms of the orbital elements of the satellite are given by several authors (Kaula, 1969), (Kozai, 1965), (Newton, 1968), (Musen and Estes, 1972), (Musen and Felsentreger, 1973) and therefore are omitted in the present exposition.

The differential equation (1) indicates the dependence of Love numbers k_{nm} on the tidal frequency κ.

The dependence of k_{nm} on n is much stronger than the dependence on m and κ. As a consequence the satellite will have some "difficulties" distinguishing between different k_{nm} for a given n. For this reason static model of the Earth is being used at the present time in the computation of the tidal effects in the motion of satellites and the dependence of Love numbers on the tidal frequencies is suppressed.

At present the value k_n is used for all m and all κ associated with the Fourier expansions of

$$\left(\frac{a'}{r'}\right)^n Y_{nm}(\delta', \alpha') \quad \text{and} \quad \left(\frac{a'}{r'}\right)^n Y^*_{nm}(\delta', \alpha').$$

in terms of the arguments ℓ, ℓ', F, D and Γ of the lunar theory. Under these limitations, at the present time the disturbing function of the form

$$\Omega_{nm} = \frac{4\pi}{2n+1} k_n \frac{Gm'}{R} (pp')^{n+1} \tag{13'}$$

$$\cdot \left(\frac{a'}{r'}\right)^{n+1} Y^*_{nm}(\delta', \alpha')$$

$$\cdot \left(\frac{a}{r}\right)^{n+1} Y_{nm}(\delta, \alpha),$$

is being used,

and we deduce for the total tidal potential

$$\Omega = \sum_{n=2}^{+\infty} \sum_{m=-n}^{m=+n} \Omega_{nm}, \tag{18}$$

by taking the addition theorem

$$P_n(\cos S) = \frac{4\pi}{2n+1} \sum_{m=-n}^{m=+n} Y^*_{nm}(\delta', \alpha')\, Y_{nm}(\delta, \alpha), \tag{19}$$

$$\cos S = \mathbf{r}^0 \cdot \mathbf{r}'^0,$$

into consideration,

$$\Omega = \frac{Gm'}{R} \sum_{m=2}^{+\infty} k_n (pp')^{n+1} \left(\frac{a'}{r'}\right)^{n+1} \left(\frac{a}{r}\right)^{n+1} P_n(\cos S) \tag{20}$$

The expansion (20) is the standard one and presently serves as the basis for the computation of the tidal perturbations in the motion of the artificial satellite as caused by the tides of the "solid Earth".

However, with improved observational techniques and the use of satellite altimetry the dependence of Love numbers on frequency must be considered for accurate geoid determination from satellite observations. In the present exposition we develop a set of differential equations for the long period tidal perturbations in satellite elements in a form suitable for numerical step-by-step integration. Having this goal in mind we re-write the disturbing function (13) as

$$\Omega_{nm} = \frac{4\pi}{2n + 1} n^2 a^2 q_{nm} e^{i\kappa t} \left(\frac{a}{r}\right)^{n+1} Y_{nm}(\delta, \alpha), \tag{21}$$

where we set

$$q_{nm}(\kappa) = \frac{m'}{M} p^n p'^{n+1} k_{nm} K_{nm}$$

and take the relation

$$GM = n^2 a^3$$

into account.

If for given n the Love numbers can be replaced by their average values, then instead of (21) we can use the "combined" disturbing function (13'), which we re-write in the form:

$$\Omega_{nm} = \frac{4\pi}{2n + 1} n^2 a A'^{*}_{nm} \left(\frac{a}{r}\right)^{n+1} Y_{nm}(\delta, \alpha), \tag{22}$$

where

$$A'_{nm} = q_n \left(\frac{a'}{r'}\right)^{n+1} Y_{nm}(\delta', \alpha'),$$

$$\tag{23}$$

$$q_n = \frac{m'}{M} p^n p'^{n+1} k_n .$$

Representations (21) and (22) lead to a compact form of the differential equations for the perturbations in elements.

We assume that the spherical harmonics $Y_{nm}(\delta, \alpha)$, $Y^*_{nm}(\delta', \alpha')$ are represented as polynomials in the equatorial components

$$\lambda = \cos\delta \cos\alpha, \qquad \mu = \cos\delta \sin\alpha, \qquad \nu = \sin\delta,$$

$$\lambda' = \cos\delta' \cos\alpha', \quad \mu' = \cos\delta' \sin\alpha', \quad \nu' = \sin\delta'$$

of the unit-vectors $\mathbf{r}^0$ and $\mathbf{r}'^0$, respectively.

DIFFERENTIAL EQUATIONS FOR THE VARIATION OF ELEMENTS

At the present time only the long period tidal perturbations in elements can be easily observed. It is convenient to compute them by means of a step-by-step numerical integration making use of the Gaussian form of the differential equations for the variation of elliptic elements:

$$\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{n\,ae} \cdot \frac{1}{2\pi} \int_0^{2\pi} \frac{r}{a} T\, dg, \tag{24}$$

$$\frac{d\pi}{dt} = +\frac{\sqrt{1-e^2}}{n\,ae} \cdot \frac{1}{2\pi} \int_0^{2\pi} \left\{ -S\cos f + \left(1 + \frac{1}{1-e^2}\,\frac{r}{a}\right) T \sin f \right\} dg$$
$$+ 2\sin^2\frac{i}{2}\,\frac{d\Omega}{dt} \tag{25}$$

$$\frac{d\Omega}{dt} = + \frac{1}{n a\sqrt{1 - e^2}\,\sin i} \cdot \frac{1}{2\pi} \int_0^{2\pi} \frac{r}{a} Z \sin(f + \omega)\, dg, \tag{26}$$

$$\frac{di}{dt} = + \frac{1}{n\, a\sqrt{1 - e^2}} \cdot \frac{1}{2\pi} \int_0^{2\pi} \frac{r}{a} Z \cos(f + \omega)\, dg \tag{27}$$

$$\frac{dL}{dt} = - \frac{2}{n\,a} \cdot \frac{1}{2\pi} \int_0^{2\pi} \frac{r}{a} S\, dg + \frac{e}{1 + \sqrt{1 - e^2}} \left(e\, \frac{d\pi}{dt}\right) + 2\sqrt{1 - e^2}\, \sin^2 \frac{i}{2}\, \frac{d\Omega}{dt}, \tag{28}$$

where

$$S = \mathbf{r}^0 \cdot \mathbf{F}, \quad T = \mathbf{n}^0 \cdot \mathbf{F}, \quad Z = \mathbf{R} \cdot \mathbf{F} \tag{29}$$

are the projections of the disturbing force

$$\mathbf{F} = \nabla \Omega$$

on the directions

$$\mathbf{r}^0 = \mathbf{P} \cos f + \mathbf{Q} \sin f, \quad \mathbf{n}^0 = \mathbf{R} \times \mathbf{r}^0 = - \mathbf{P} \sin f + \mathbf{Q} \cos f, \quad \text{and } \mathbf{R}, \tag{30}$$

$\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are the Gibbsian vectors associated with the instantaneous set of elliptic elements. The values of the components S, T and Z are computed for a set of points along the instantaneous orbit as defined by this set.

We have:

$$\nabla = \mathbf{r}^0 \frac{\partial}{\partial \mathbf{r}} + \frac{1}{\mathbf{r}} (\mathbf{I} - \mathbf{r}^0 \mathbf{r}^0) \cdot \nabla_0, \tag{31}$$

where ∇_0 is the del-operator relative to $\mathbf{r}^0$,

$$\nabla_0 = \mathbf{i} \frac{\partial}{\partial \lambda} + \mathbf{j} \frac{\partial}{\partial \mu} + \mathbf{k} \frac{\partial}{\partial \nu} \cdot$$

Making use of (21) we obtain for the disturbing forces

$$\mathbf{F}_{nm} = \nabla \Omega_{nm} = \frac{4\pi}{2n + 1} n^2 a q_{nm} e^{i\kappa t} \left(\frac{a}{r}\right)^{n+2} \{-(n + 1) \mathbf{r}^0 + (\mathbf{I} - \mathbf{r}^0 \mathbf{r}^0) \cdot \nabla_0\} Y_{nm} \tag{32}$$

We deduce from (24) – (28), (30) and (32) a set of formulas convenient for programming:

$$\frac{de}{dt} = - \frac{4\pi n}{2n + 1} \frac{1}{e} q_{nm} \cdot e^{i\kappa t} \cdot \frac{1}{2\pi} \int_0^{2\pi} B_{nm}\, df, \tag{33}$$

$$\frac{d\pi}{dt} = + \frac{4\pi n}{2n + 1} \frac{1}{e} q_{nm} \cdot e^{i\kappa t} \tag{34}$$

$$\cdot \frac{1}{2\pi} \int_0^{2\pi} \left\{ + A_{nm} \cos f + \left(\frac{a}{r} + \frac{1}{1 - e^2}\right) B_{nm} \sin f \right\} df + 2 \sin^2 \frac{i}{2} \frac{d\Omega}{dt},$$

$$\frac{d\Omega}{dt} = + \frac{4\pi n}{2n+1} \cdot \frac{1}{(1-e^2)\sin i} q_{nm} \cdot e^{i\kappa t} \tag{35}$$

$$\cdot \frac{1}{2\pi} \int_0^{2\pi} C_{nm} \sin(f + \pi - \Omega)\, df,$$

$$\frac{di}{dt} = + \frac{4\pi n}{2n+1} \cdot \frac{1}{1-e^2} q_{nm} e^{i\kappa t} \tag{36}$$

$$\cdot \frac{1}{2\pi} \int_0^{2\pi} C_{nm} \cos(f + \pi - \Omega)\, df.$$

$$\frac{dL}{dt} = + \frac{4\pi n}{2n+1} \cdot \frac{2}{\sqrt{1-e^2}} q_{nm} \cdot e^{i\kappa t} \cdot \frac{1}{2\pi} \int_0^{2\pi} D_{nm}\, df \tag{37}$$

$$+ \frac{e}{1+\sqrt{1-e^2}} \left(e \frac{d\pi}{dt} \right) + 2\sqrt{1-e^2} \sin^2 \frac{i}{2} \frac{d\Omega}{dt},$$

where we set for brevity:

$$A_{nm} = (n+1)\left(\frac{a}{r}\right)^n Y_{nm},$$

$$B_{nm} = \left(\frac{a}{r}\right)^{n-1} \mathbf{n}^0 \cdot \nabla_0 Y_{nm},$$

$$C_{nm} = \left(\frac{a}{r}\right)^{n-1} \mathbf{R} \cdot \nabla_0 Y_{nm},$$

and

$$D_{nm} = \left(\frac{a}{r}\right)^{n-1} Y_{nm}$$

$$\mathbf{n}^0 \cdot \nabla_0 = \mathbf{R} \times \mathbf{r}^0 \cdot \nabla_0$$

$$= (\nu R_y - \mu R_z)\frac{\partial}{\partial\lambda} + (\lambda R_z - \nu R_x)\frac{\partial}{\partial\mu}$$

$$+ (\mu R_x - \lambda R_y)\frac{\partial}{\partial\nu},$$

$$\mathbf{R} \cdot \nabla_0 = R_x \frac{\partial}{\partial\lambda} + R_y \frac{\partial}{\partial\mu} + R_z \frac{\partial}{\partial\nu}.$$

If instead of particular values of Love parameters k_{km} (κ) we decide to use the average value k_n, which assumes k_{nm} (κ) to be the same for a given n and for all admissible m and κ, then we can also use a more compact form of the disturbing function:

$$\Omega_{nm} = \frac{4\pi}{2n + 1} \, n^2 a A'^{*}_{nm} \left(\frac{a}{r}\right)^{n+1} Y_{nm}, \tag{22}$$

where

$$A'_{nm} = q_n \left(\frac{a'}{r'}\right)^{n+1} Y'_{nm}$$

(23)

$$q_n = \frac{m'}{M} \, p^n p'^{n+1} k_n$$

and the differential equations (33) – (37) become:

$$\frac{de}{dt} = - \frac{4\pi}{2n + 1} \cdot \frac{n}{e} A'^{*}_{nm} \frac{1}{2\pi} \int_0^{2\pi} B_{nm} df, \tag{33'}$$

$$\frac{d\pi}{dt} = + \frac{4\pi n}{2n + 1} \cdot \frac{1}{e} A'^{*}_{nm} \tag{34'}$$

$$\cdot \frac{1}{2\pi} \int_0^{2\pi} \left\{ + A_{nm} \cos f + \left(\frac{a}{r} + \frac{1}{1 - e^2}\right) B_{nm} \sin f \right\} df$$

$$+ \, 2 \sin^2 \frac{i}{2} \, \frac{d\Omega}{dt},$$

$$\frac{d\Omega}{dt} = + \frac{4\pi}{2n+1} \cdot \frac{n}{(1-e^2)\sin i} A'^{*}_{nm} \tag{35'}$$

$$\cdot \frac{1}{2\pi} \int_0^{2\pi} C_{nm} \sin(f + \pi - \Omega)\, df,$$

$$\frac{di}{dt} = + \frac{4\pi}{2n+1} \cdot \frac{n}{1-e^2} A'^{*}_{nm} \cdot \frac{1}{2\pi} \int_0^{2\pi} C_{nm} \cos(f + \pi - \Omega)\, df, \tag{36'}$$

$$\frac{dL}{dt} = + \frac{4\pi}{2n+1} \cdot \frac{2n}{\sqrt{1-e^2}} A'^{*}_{nm} \cdot \frac{1}{2\pi} \int_0^{2\pi} D_{nm}\, df \tag{37'}$$

$$+ \frac{e}{1+\sqrt{1-e^2}} \left(e \frac{d\pi}{dt}\right) + 2\sqrt{1-e^2} \sin^2 \frac{i}{2} \frac{d\Omega}{dt}.$$

Conclusions

In expanding the exterior tidal potential in the present work we assumed spherical symmetry of the Earth's elastic properties and density. The Love number for a given frequency can be computed (Alterman et al., 1957) on the basis of the assumed laws (from seismic evidence) of variation of Lame elastic parameters,

λ and μ, and of the density with depth. At the moment these hypotheses, commonly used in the theory of oscillations of the Earth, seem to be plausible.

At the present time, however, an even more simplified model of the exterior tidal potential, which is implicitly based on a static model of the Earth, is being widely used in the computations of satellite tidal perturbations. In the frame of this model the dependence of Love numbers on the order of the spherical harmonics and frequency is supressed in the expansion of the tidal potential and the same Love number is assigned to all spherical harmonics of the same degree.

With the accumulation of a long series of observations and a further improvement in observational techniques, consideration should be given to the dependence of Love numbers on frequency, at least for the few most important lunar and solar tidal constituents.

Only the investigations based on the use of an analytical (or semi-analytical) theory can easily provide information on the significance of this dependence. These investigations of the dependence of Love numbers on frequencies are necessary in connection with the determination of an accurate geoid.

There are other un-resolved problems which await solution. The influence of the core is one such problem. The speed of rotation of the core is different from the speed of rotation of the mantle and crust.

As a consequence resonances and variability of amplitudes and phases appear in the expansion of the tidal effects in the motion of a satellite. The determination of lags of tidal constituents in the perturbations satellites will permit one to understand the process of dissipation of energy. The problem of the influences of oceanic tides on satellite motion still is not solved numerically, although it is satisfactorily understood theoretically. The numerical solution of this problem is tied with the integration of the Laplace tidal differential equations over the global ocean for different tidal constituents.

Our final conclusion is that in parallel with attempts to determine the average elastic properties of the Earth from satellite observations it is necessary to approach the problem from the geophysical side and integrate the differential equations for Love parameters, at least for the most important tidal frequencies in the motion of the satellite.

These are some problems which constitute topics for a long term future work.

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