L^p-STABILITE ($1 \leq p \leq \infty$) DE SYSTEMES ASSERVIS NON LINEAIRES MULTIVARIABLES NON INVARIANTS QUI SONT INSTABLES EN BOUCLE OUVERTE

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RESUME :

Cette communication envisage une classe de systèmes multivariables non linéaires, variables avec le temps, avec comme boucle de prédiction un sous système de convolution instable, et comme boucle de retour un gain non linéaire variable avec le temps. La réponse impulsionnelle du sous système de convolution est la somme de :

i) un nombre fini d'exponentielles croissantes multipliées par des puissances non négatives du temps,

ii) un terme absolument intégrale

iii) une série infinie d'impulsions retardées.

Le résultat essentiel de cette communication est le théorème 1. Ce théorème montre essentiellement que si :

i) le sous système de convolution instable peut être stabilisé par un gain de retour constant F

ii) le gain incrémental de la différence entre la fonction de gain non linéaire et F est suffisamment faible,

alors le système non linéaire est L^p-stable, pour tout $p \in [1, \infty]$; de plus les solutions du système non linéaire dépendent continuement des entrées pour toute norme L^p. Le théorème du point fixe est fondamental pour élaborer le théorème 1.

L^p-STABILITY ($1 \leq p \leq \infty$) OF MULTIVARIABLE NONLINEAR TIME-VARYING FEEDBACK SYSTEMS THAT ARE OPEN-LOOP UNSTABLE.

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Abstract

This paper considers a class of multivariable, nonlinear time-varying feedback systems with an unstable convolution subsystem as feedforward and a time-varying nonlinear gain as feedback. The impulse response of the convolution subsystem is the sum of i) a finite number of increasing exponentials multiplied by nonnegative powers of the time t, ii) a term that is absolutely integrable and iii) a infinite series of delayed impulses. The main result of the paper is theorem 1. It essentially states that i) if the unstable convolution subsystem can be stabilized by a constant feedback gain F and ii) if the incremental gain of the difference between the nonlinear gain function and F is sufficiently small, then the nonlinear system is L^p-stable for any $p \in [1,\infty]$; furthermore the solutions of the nonlinear system depend continuously on the inputs in any L^p-norm. The fixed point theorem is crucial in deriving the above theorem.

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1. Introduction. In the past few years the L^2-stability [1], [2] the L^∞- stability [3], [4] of certain classes of nonlinear and time-varying feedback systems have been extensively studied. Desoer and Wu [5], [6] obtained L^p-stability conditions for a broad class of linear time-invariant feedback systems whose open-loop impulse responses may include an integration and an infinite series of delayed impulses. They also obtained L^p-stability conditions for a related class of nonlinear time-varying systems in [7]. Recently Callier and Desoer [8], [9], [10] derived necessary and sufficient conditions for stability of a very broad class of linear time-invariant feedback systems whose open-loop impulse responses may include increasing exponentials multiplied by nonnegative powers of time and an infinite series of delayed impulses. These conditions imply L^p-stability for any $p \in [1,\infty]$, [6]. In this paper the loop transformation technique [12], the fixed point theorem [16], and a generalized version of some results of Callier, Desoer and Wu [10], [7] are used to derive the L^p-stability for a related class of nonlinear time-varying feedback systems which are open-loop unstable. The application of the fixed point theorem in L^p shows that the nonlinear feedback system has one and only one solution for any pair of inputs in L^p, that the solutions are continuously dependent on the inputs and that closed loop system is L^p-stable for any $p \in [1,\infty]$.

2. Notations. In this paper we shall encounter real numbers (elements of $\mathbb{R}$), vectors (in $\mathbb{R}^n$), matrices (in $\mathbb{R}^{n\times n}$), elements in function spaces and operators acting on elements of function spaces. Lower-case letters denote numbers or vectors, upper-case letters denote matrices. Bold-face letters (indicated by a tilde under the symbol) denote operators. The symbol $|\cdot|$ denotes both the magnitude of a number and the norm of a vector in $\mathbb{R}^n$ or a matrix in $\mathbb{R}^{n\times n}$. In function spaces, we use the following norms: Let $x: \mathbb{R}_+ \to \mathbb{R}^n$, then by definition

$$\|x\|_p \triangleq \left[\int_0^\infty |x(t)|^p \, dt\right]^{1/p}, \quad 1 \le p < \infty,$$

and, for $p = \infty$,

$$\|x\|_\infty \triangleq \operatorname*{ess\,sup}_{t \ge 0} |x(t)|.$$

The resulting normed spaces are denoted by L_n^p, $1 \le p \le \infty$. (If $n = 1$ (scalar case) we write L^p.) When the symbols $|\cdot|$ and $\|\cdot\|$ are applied to a matrix or a matrix-valued function or an operator acting on function spaces, they denote the induced operator norms. Note that in defining the L_n^p norms above we may use any vector norm in $\mathbb{R}^n$ because all norms in $\mathbb{R}^n$ are equivalent. Following Sandberg [11] and Zames [2], the space L_{ne}^p, the extension of L_n^p space, is defined as follows:

$$L_{ne}^p \triangleq \{x(\cdot) \,|\, \int_0^T |x(t)|^p \, dt < \infty, \ \forall T \in [0,\infty), \ 1 \le p < \infty\}$$

and

$$L_{ne}^\infty \triangleq \{x(\cdot) \,|\, \operatorname*{ess\,sup}_{t \in [0,T]} |x(t)| < \infty, \ \forall T \in [0,\infty)\}.$$

Roughly speaking, if $x \in L_{ne}^\infty$, then x does not have a finite escape time. In order to allow us to consider a larger class of linear subsystems whose impulse responses may include an infinite series of impulses, we introduce the Banach Algebra $\mathcal{A}^{n\times n}$ (see [6]). Let A be a distribution whose support is in $[0,\infty)$. We say that A is an element of $\mathcal{A}^{n\times n}$ if

$$A(t) = A_a(t) + \sum_{i=0}^{\infty} A_i \, \delta(t-t_i)$$

where $A_a: [0,\infty) \to \mathbb{R}^{n\times n}$ is in $L_{n\times n}^1$, the sequence $\{t_i\}_0^\infty$ is in $[0,\infty)$ with $t_0 = 0$, $t_i > 0$ for $i \ge 1$ and $\{A_i\}_{i=0}^\infty$ is a sequence of matrices in $\mathbb{R}^{n\times n}$ subject to $\sum_{i=0}^\infty |A_i| < \infty$ and δ is the Dirac "function." The set of elements in $\mathcal{A}^{n\times n}$ constitute a non-commutative Banach algebra with a unit, with the usual definition for addition, the product defined by convolution, and the norm defined by

$$\|A\|_a \triangleq \int_0^\infty |A_a(t)|dt + \sum_{i=0}^\infty |A_i|.$$

These facts are well-known [6,15].

The symbol "^" over a function, such as $\hat{f}$, denotes the Laplace transform of f: it is defined by

$$\hat{f}(s) \triangleq \int_0^\infty f(t)\, e^{-st}dt.$$

For distributions, it is defined according to L. Schwartz [13] or, by using Stieltjes integrals, according to Widder [14]. The subscript T, as in f_T, denotes the truncation of the function f at time T, namely

$$f_T(t) = \begin{cases} f(t) & \text{for } 0 \le t \le T \\ 0 & \text{for } t > T \end{cases}$$

Finally $\hat{\mathcal{A}}^{n\times n}$ denotes the algebra of Laplace transforms of elements in $\mathcal{A}^{n\times n}$ (with pointwise product).

3. System Description and Assumptions.

We consider a 2n-input 2n-output nonlinear time-varying feedback system S as shown in Fig. 1. The inputs u_1, u_2, errors e_1, e_2, outputs y_1, y_2 are functions of time mapping $\mathbb{R}_+$ into $\mathbb{R}^n$. The block labeled ϕ is a memoryless, time-varying nonlinearity whose input-output relation is defined in terms of a nonlinear function ϕ: $\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ by

$$y_2(t) = \phi[e_2(t),t]. \tag{1}$$

The nonlinear function $\phi(\cdot,\cdot)$ satisfies the following assumptions:

(N.1) $\phi(\cdot,\cdot)$: $\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and ϕ is a continuous function with respect to its first argument and is a regulated function(†) with respect to its second argument.

(†) $\phi(x,t)$: $\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is called regulated in t iff for all fixed $x \in \mathbb{R}^n$, $t \mapsto \phi(x,t)$ has finite one-sided limits at every $t \in \mathbb{R}_+$.

(N.2) There exists a nonsingular matrix $F \in \mathbb{R}^{n\times n}$ and a positive real number μ such that

$$|\phi(x,t) - \phi(x',t) - F(x-x')| \le \mu|x-x'| \tag{2}$$

for all $t \in \mathbb{R}_+$ and all x, $x' \in \mathbb{R}^n$; moreover

$$\phi(0,t) = 0 \quad \text{for all } t \in \mathbb{R}_+. \tag{3}$$

The block labeled G is a linear time-invariant subsystem whose input-output relation is defined in terms of its impulse response matrix G by convolution, i.e.

$$y_1(t) = (G*e_1)(t) \text{ for all } t \in \mathbb{R}_+ \tag{4}$$

G is a matrix valued distribution on $[0,\infty)$ whose Laplace transform $\hat{G}$ satisfies the assumption (G):

$$\hat{G}(s) = \sum_{k=1}^{\ell} \sum_{\alpha=0}^{m_k-1} R_{k\alpha}\,(s-p_k)^{-m_k+\alpha} + \hat{G}_\rho(s) \quad \text{for Re } s \ge 0, \tag{5}$$

where Re $p_k \ge 0$ for k = 1, 2, ..., ℓ; the poles p_k and the coefficient matrices $R_{k\alpha}$ are either real or occur in complex conjugate pairs; $\hat{G}_\rho(s) \in \hat{\mathcal{A}}^{n\times n}$. The system equations are (1), (4) and

$$e_1 = u_1 - y_2 \tag{6}$$

$$e_2 = u_2 + y_1. \tag{7}$$

Definition: Let $p \in [1,\infty]$; the system S (Fig. 1) defined by (1) - (7) is said to be L^p-stable iff the maps $(u_1,u_2) \mapsto (e_1,e_2)$ and $(u_1,u_2) \mapsto (y_1,y_2)$ are L^p-stable i.e. to any input pair (u_1,u_2) belonging to L^p_{2n} corresponds an error pair (e_1,e_2) and an output pair (y_1,y_2) both belonging to L^p_{2n} and there is a number $k \in \mathbb{R}_+$ such that

$$\|e_1\|_p + \|e_2\|_p \leq k\,[\|u_1\|_p + \|u_2\|_p]$$

$$\|y_1\|_p + \|y_2\|_p \leq k\,[\|u_1\|_p + \|u_2\|_p]$$

for all $(u_1,u_2) \in L^p_{2n}$.

4. Main Result.

Theorem 1. Consider the system S described by (1), (4), (6) and (7), where the assumptions (G), (N.1) and (N.2) are satisfied. Let H_F be the closed-loop impulse response of the n-input n-output convolution feedback system $u_1 \mapsto y_1$ with G as open-loop impulse response and F as constant feedback matrix, i.e.

$$\hat{H}_F = \hat{G}[I+F\hat{G}]^{-1}. \tag{8}$$

In (5) for k = 1, 2, ..., ℓ set

$$R_k(s) \triangleq \sum_{\alpha=0}^{m_k-1} R_{k\alpha}(s-p_k)^{-m_k+\alpha}. \tag{9}$$

At each pole p_k for k = 1, 2, ..., ℓ consider the Laurent expansion of $I + F\hat{G}(s)$ up to and including the constant term. This proper rational function can be represented as the product $N_k(s)\,D_k(s)^{-1}$ where N_k and D_k are right-coprime polynomial matrices [18-21], i.e. for k = 1, 2, ..., ℓ

$$N_k(s)\,D_k(s)^{-1} = I + F[R_k(s) + \sum_{\substack{\beta=1\\ \beta\neq k}}^{\ell} R_\beta(p_k) + \hat{G}_\rho(p_k)]. \tag{10}$$

Under these conditions, if

$$\inf_{\mathrm{Re}\, s \geq 0} |\det[I + F\hat{G}(s)]| > 0 \tag{11}$$

$$\det N_k(p_k) \neq 0 \quad \text{for } k = 1, 2, \ldots, \ell \tag{12}$$

and

$$\|H_F\|_{a}\,\mu < 1 \tag{13}$$

then,

(i) for any $p \in [1,\infty]$, the maps $(u_1,u_2) \mapsto (e_1,e_2)$ and $(u_1,u_2) \mapsto (y_1,y_2)$ are well-defined maps sending L^p_{2n} into L^p_{2n};

(ii) for any $p \in [1,\infty]$, these maps are uniformly continuous on L^p_{2n};

(iii) for any $p \in [1,\infty]$, the system S is L^p-stable.

5. Proof.

To prove Theorem 1, we need two lemmas.

Lemma 1. Consider a special case of the system S (Fig. 1), where for all $e_2 \in \mathbb{R}^n$, all $t \in \mathbb{R}_+$, $\phi(e_2,t) = Fe_2$, with F a nonsingular element of $\mathbb{R}^{n\times n}$. Let the open-loop transfer function matrix $\hat{G}$ be defined by (5). Let N_k and D_k be the right-coprime polynomial matrices defined by (10). Under these conditions

$$[I + F\hat{G}]^{-1} \in \hat{\mathcal{A}}^{n\times n}$$

and

$$\hat{H}_F \triangleq \hat{G}[I+F\hat{G}]^{-1} \in \hat{\mathcal{A}}^{n\times n}$$

if and only if

$$\inf_{\mathrm{Re}\, s \geq 0} |\det[I+F\hat{G}(s)]| > 0 \tag{11}$$

and

$$\det N_k(p_k) \neq 0 \quad \text{for } k = 1, 2, \ldots, \ell. \tag{12}$$

This is a generalized version of a result of [10].

Lemma 2. Consider a more general system than the one shown in Fig. 1, in that G and ϕ are replaced by H_1 and H_2 respectively. Let p be fixed and $p \in [1,\infty]$. Let H_1 and H_2 be nonanticipative maps of L^p_{ne} into L^p_{ne}. Let H_1 be linear, thus $H_1 0 = 0$. Let $H_2 0 = 0$. Let e_1, e_2 and u_1, u_2 be defined by the system equations. Under these conditions if

(a) for some $F \in \mathcal{R}^{n\times n}$, F nonsingular, $(\underline{I}+\underline{F}\underline{H}_1)^{-1}$ maps L^p_{ne} into L^p_{ne} and is nonanticipative;

(b) there exists some positive real number μ such that

$$\|(\underline{H}_2 e_2)_T - (\underline{H}_2 e_2')_T - \underline{F}(e_2 - e_2')_T\|_p \le \mu \|e_{2T} - e_{2T}'\|_p$$

for all $T \in [0,\infty)$ and for all $e_2, e_2' \in L^p_{ne}$;

(c) $$\|\underline{H}_1(\underline{I}+\underline{F}\underline{H}_1)^{-1}\| < \infty;$$

(d) $$\gamma = \|\underline{H}_1(\underline{I}+\underline{F}\underline{H}_1)^{-1}\|\mu < 1,$$

then: (i) given any input pair (u_1,u_2) in L^p_{2ne}, a unique error e_2 in L^p_{ne} is obtained by a fixed point iteration starting from an arbitrary point;

(ii) if u_1 and u_2 are the zero elements in L^p_{ne}, then e_2 is the zero element in L^p_{ne};

(iii) to any two input pairs, say (u_1,u_2), (u_1',u_2') in L^p_{2ne}, there correspond two errors e_2 and e_2' in L^p_{ne} such that

$$\|e_{2T} - e_{2T}'\|_p \le (1-\gamma)^{-1}\|\underline{F}^{-1}(\underline{I}+\underline{F}\underline{H}_1)^{-1}\underline{F}(u_{2T}-u_{2T}')\|_p + \|\underline{H}_1(\underline{I}+\underline{F}\underline{H}_1)^{-1}(u_{1T}-u_{1T}')\|_p \quad \forall T \in [0,\infty).$$

Therefore the map $(u_1,u_2) \mapsto e_2$ is a well-defined L^p-stable map sending L^p_{2n} into L^p_n which is uniformly continuous on L^p_{2n}.

This Lemma is a consequence of the loop transformation technique [12] and the fixed point Theorem [16].

Proof of Theorem 1. Let F be the nonsingular n×n constant matrix of assumption (N.2). Make the system transformation such that the block in the forward path becomes

$$\underline{H}_F = \underline{G}(\underline{I}+\underline{F}\underline{G})^{-1} \tag{14}$$

and the block in the feedback path becomes

$$\underline{\psi} = \underline{\phi} - \underline{F}\underline{I}. \tag{15}$$

Let $\hat{H}_F(s)$ be the transfer function matrix of $\underline{H}_F$, then

$$\hat{H}_F(s) = \hat{G}(s)(I+F\hat{G}(s))^{-1}. \tag{16}$$

By assumptions (11) and (12) of Theorem 1, Lemma 1 implies that $(I+F\hat{G}(s))^{-1}$ and $\hat{H}_F(s)$ are in $\hat{\mathcal{A}}^{n\times n}$; since they are the transfer functions of the operators $(\underline{I}+\underline{F}\underline{G})^{-1}$ and $\underline{H}_F$, these operators are nonanticipative, send L^p_n into L^p_n for any $p \in [1,\infty)$, and are L^p-stable for all $p \in [1,\infty]$, [6]. Thus the impulse response matrix H_F is in $\mathcal{A}^{n\times n}$ and is of the form

$$H_F(t) = \begin{cases} H_a(t) + \sum_{i=0}^{\infty} H_i \delta(t-t_i) & \text{for } t \ge 0 \\ 0 & \text{for } t < 0, \end{cases}$$

where $H_a \in L^1_{n\times n}$, the H_i's are constant matrices such that $\sum_{i=0}^{\infty} |H_i| < \infty$ and $t_0 = 0$, $t_i > 0$ for $i \ge 1$. Also H_F has a well-defined norm in $\mathcal{A}^{n\times n}$

$$\|H_F\|_a \triangleq \int_0^{\infty} |H_a(t)|dt + \sum_{i=0}^{\infty} |H_i|.$$

Note that $\|H_F\|_a$ is the induced operator norm when $p = \infty$ and is an upper bound on the induced operator norm when $p \ne \infty$. By assumption (N.2) we have

$$\|(\underline{\phi} e_2)_T - (\underline{\phi} e_2')_T - \underline{F}(e_2-e_2')_T\|_p \le \mu\|e_{2T}-e_{2T}'\|_p$$

$$\text{for all } T \in [0,\infty), \text{ for all } e_2, e_2' \in L^p_{ne}.$$

Finally by assumption (13): $\|H_F\|_a\mu < 1$; furthermore $\underline{G}$ is linear so $\underline{G}\,0 = 0$ and, by assumption (N.2), $\underline{\phi}\,0 = 0$. So all the conditions of Lemma 2 are met for any $p \in [1,\infty]$ with $\underline{H}_1 = \underline{G}$ and $\underline{H}_2 = \underline{\phi}$. Hence, for any

$p \in [1,\infty]$, it follows that for the system S the map $(u_1,u_2) \to e_2$ is well defined sending L^p_{2n} into L^p_n, is L^p-stable and is uniformly continuous on L^p_{2n}. Since $y_2 = \underset{\sim}{\phi} e_2$,

$$\|(\underset{\sim}{\phi}e_2)_T - (\underset{\sim}{\phi}e_2')_T\|_p - \|\underset{\sim}{F}(e_{2T}-e_{2T}')\|_p \le$$

$$\|(\underset{\sim}{\phi}e_2)_T - (\underset{\sim}{\phi}e_2')_T - \underset{\sim}{F}(e_{2T}-e_{2T}')\|_p \le \mu \|e_{2T} - e_{2T}'\|_p,$$

and $\underset{\sim}{\phi}\, 0 = 0$,

it follows for any $p \in [1,\infty]$, that the map $e_2 \mapsto y_2$ is a well-defined map sending L^p_n into L^p_n which is L^p-stable and uniformly continuous on L^p_n. Finally since $e_1 = u_1 - y_2$ and $y_1 = e_2 - u_2$, the conclusion of the theorem follows.

6. Final Remark. If, instead of assuming that ϕ satisfies an incremental gain condition as in (2) of assumption (N.2), we had assumed that there exists a positive real number μ such that

$$|\phi(x,t) - Fx| \le \mu|x| \quad \text{for all } t \in \mathbb{R}_+, \quad \text{for all } x \in \mathbb{R}^n, \qquad (2')$$

then we would be able to use the small gain theorem to prove the following: suppose that for some $p \in [1,\infty]$ and for any input pair $(u_1,u_2) \in L^p_{2n}$ the error pair $(e_1,e_2) \in L^p_{2ne}$, then assumptions (N.1), (2'), (3), (G) and (11), (12), (13) imply that system S is L^p-stable. This result is easily obtained by standard techniques [1], [2], [11] and extends a recent result of Prada and Bickart [17]. Note that under the relaxed assumption (2') we do not guarantee existence, nor uniqueness, nor continuous dependence.

7. Conclusion. We have shown that if the given nonlinear time-varying feedback system S will be uniquely defined, stable and continuously dependent on its inputs in any L^p norm if eventually i) the unstable convolution subsystem can be stabilized by a constant feedback gain F and ii) if the incremental gain of the difference of the nonlinear gain function ϕ and F is sufficiently small.

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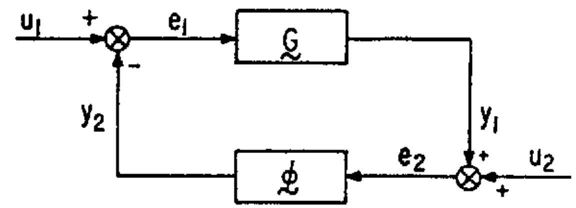

Fig. 1. The system S.