The School of Engineering
HOWARD UNIVERSITY
WASHINGTON, D. C. 20001

Department of Mechanical Engineering

December 21, 1973

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Dear Sirs:

In accordance with the NASA Provisions for Research Grants, I am submitting five copies of a brief Semi - Annual Status Report of the research completed in connection with NASA Grant: NGR - 09-011-053 during the period June 16, 1973 to December 15, 1973. Copies are also being sent to the NASA Technical Officer for this grant.

Thank you for your kind attention to this matter.

Sincerely yours,

Peter M. Bainum

Peter M. Bainum
Associate Professor of
Aerospace Engineering,
Principal Investigator

PMB:mat

cc: Mr. Vearl Huff, NASA-HQ
Mr. J. G. Pohly, NASA-HQ

NASA-CR-137146) THE THREE DIMENSIONAL MOTION AND STABILITY OF A ROTATING SPACE STATION-CABLE - COUNTERWEIGHT CONFIGURATION (Howard, Needles, Tammen and Bergendoff) 17 p HC $4.00 CSCL 20K G3/32

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HOWARD UNIVERSITY

SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

WASHINGTON, D. C. 20001

SEMI - ANNUAL STATUS REPORT

(JUNE 16, 1973 - DEC. 15, 1973)

NASA - NGR - 09-011-053

THE THREE DIMENSIONAL MOTION AND STABILITY OF A ROTATING SPACE STATION-CABLE - COUNTERWEIGHT CONFIGURATION

by

Peter M. Bainum

Associate Professor of Aerospace Engineering

Principal Investigator

December 18, 1973

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CHEMICAL ENGINEERING — CIVIL ENGINEERING — ELECTRICAL ENGINEERING — MECHANICAL ENGINEERING

I. SUMMARY OF RESULTS ACCOMPLISHED DURING PERIOD JUNE 16, 1973 - DEC. 15, 1973

The three dimensional equations of motion for a cable connected space station - counterweight system are developed using a Lagrangian formulation. The system model employed allows for cable and end body damping and restoring effects. The equations are then linearized about the equilibrium motion and nondimensionalized.

To first degree, the out-of-plane equations uncouple from the in-plane equations. Therefore, the characteristic polynomials for the in-plane and out-of-plane equations are developed and treated separately. From the general in-plane characteristic equation, necessary conditions for stability are obtained. The Routh-Hurwitz necessary and sufficient conditions for stability are derived for the general out-of-plane characteristic equation. Special cases of the in-plane and out-of-plane equations (such as identical end masses, and when the cable is attached to the centers of mass of the two end bodies) are then examined for stability critria.

Time constants for the least damped mode are obtained for a range of system parameters by numerical examination of the roots of the in-plane and out-of-plane characteristic polynomials. For the in-plane case, a comparison with results previously obtained in a two dimensional treatment (but with a different damping scheme) is made. Transient responses are simulated for special cases by numerical integration of the linear system equations.

II. METHOD OF ANALYSIS

The literature in the area of the dynamics of two body cable (or tether) connected spacecraft systems was reviewed [1 - 7]. The previous three dimensional analyses of such systems are limited to: the paper by Robe [3] who conducted a stability analysis of a tethered connected gravitationally stabilized system whose end masses were unsymmetrical (but identical); the treatment by Beletskii and Novikova [4] which considered domains of possible three dimensional motion for a gravitationally stabilized point-mass system connected by a flexible, massless tether which can become slack (as in the case of deployment); and the recent paper by Nixon [7] which dealt with the determination of the dynamic equilibrium states of a completely undamped system with an arbitrary number of cables.

Of interest in this study was an examination of the three dimensional motion of a rotating cable-connected system for the general case where the end bodies have a distributed mass (finite, unequal moments of inertia) and are not assumed to be identical, and energy dissipation is included both in the cable and on the end bodies. This represents a direct extension to the problems treated in References 6 and 7.

The three dimensional nonlinear torque-free equations of motion were developed for this nine - degree - of freedom system (Figs. 1, 2) using Lagrange's general equations. Included within the mathematical model was cable damping proportional to the rate of cable extension and also rotational damping on the motions of both the space station and counter weight about their own mass centers. For the special case of planar motion it was seen that the expressions for the system kinetic and potential energy could be reduced to those previously developed in Ref. 6.

The nonlinear equations were then linearized about the equilibrium motion where the tensile force in the elastic cable is balanced by the centrifugal force associated with the system rotation.

The linearized out-of-plane equations uncoupled completely from the four linearized in-plane equations. Therefore, stability criteria resulting from the distinct factors of the over-all system characteristic equation could be treated separately. Furthermore it was observed that the equation corresponding to the coordinate which describes the displacement of the cable line from the original plane of rotation completely uncoupled from all the other equations and indicated simple harmonic motion. This can be interpreted to mean that, for small perturbations on the cable's initial orientation out of the nominal plane of rotation, the system will tend to rotate in a plane inclined to the original plane of rotation without affecting the spin rate or equilibrium cable length.

From the equilibrium condition and the necessary condition for stability indicated by the constant coefficient of the general in-plane characteristic polynomial, the cable restoring constant must be greater than the value of the reduced mass of the system multiplied by the square of the system's inertial spin rate. From the out-of-plane general stability, positive damping is necessary about at least one principal axis in the plane of nominal rotation on both end bodies. From the necessary condition for in-plane stability, rotational restoring capability about an axis perpendicular to the nominal spin plane and on at least one end body is necessary for stability in the coordinates selected.

For the case of identical end masses, positive damping and restoring torques about this same axis are necessary for stability. In contrast to the general in-plane criteria for stability, for the special case in which the cable is attached at the centers of mass of the end bodies, damping and restoring effects must be provided on both end masses about an axis normal to the plane of rotation.

III. NUMERICAL RESULTS

The numerical integration of the first-order nonlinear and linear equations of motion was performed using the IBM 1130 digital computer at Howard University. A typical transient response for a system with identical end masses and with damping and restoring effects on both the cable and end masses is illustrated in Figs. 3 - 8. Figs. 3 - 6 represent the response of the in-plane variational coordinates where it is seen that the high frequency motion associated with cable oscillation is damped within the first 100 seconds leaving a longer period motion associated with the other in-plane modes having longer time constants. "$\dot{\chi}$" is the variational coordinate associated with the system spin whereas α_1, α_2 are associated with the in-plane rotations of the end bodies. In Figs. 7, 8 for one of the end body rotations it is seen that the out - of - plane coordinates are more rapidly damped than the in-plane coordinates for the choice of system damping parameters selected. Other transient responses have also been obtained for the case of non-identical end masses and will be reported subsequently.

In addition, computer subroutines have been employed to numerically determine the roots of the system characteristic equation and the time constants associated with the least damped modes of oscillation.

IV. PRESENTATION AND PUBLICATION OF RESULTS

On July 26, 1973 at NASA Headquarters an oral presentation was given to Mr. Vearl Huff, the Technical Monitor, of the preliminary progress to that date. On Nov. 9, 1973 a seminar was presented at Howard University by Mr. Keith Evans, the Graduate Research Assistant on the Grant, at which time Mr. Huff was also present. A Master's dissertation prepared by Mr. Evans has been approved by the Department of Mechanical Engineering and final publication of this document is expected during Dec. 1973. On Dec. 17, 1973 another presentation was given at NASA - Headquarters. In attendance in addition to Mr. Huff were: Dr. G. C. Deutsch, Director, Materials & Structures Division, ART, NASA - HQ; Dr. L. Harris, Manager, Structures and Dynamics Programs, ART; Mr. D. Michel, Manager, Dynamics Programs, ART; and Dr. H. Schaeffer, MTE.

V. PLANS FOR THE NEXT REPORTING PERIOD

Additional transient responses will be simulated for different sets of initial conditions and the more general, non-identical configuration. It is also planned to compare the results of the integration of the first - order nonlinear equations with those of the linear equations for special cases.

Time constants of the least damped mode will be obtained for a range of system parameters by numerical examination of the roots of the in-plane and out - of - plane characteristic polynomials and compared with the results of Ref. 6 for planar motion.

In addition, first order gravity - gradient terms have been derived and incorporated into the linear equations. The effect of gravity - gradient torques on both the steady - state and selected transient motions will be numerically evaluated.

It is planned to prepare at least one technical paper based on this research for open publication and/or presentation at a professional society meeting. Close liason with NASA - HQ will be maintained in an effort to provide the most useful information possible. A comprehensive final report will be prepared at the end of the contractual period.

VI. POSSIBLE EXTENSION OF THE CURRENT RESEARCH

In the development of the equations of motion in the current investigation it was assumed that the cable attachment points on both of the end bodies lie along one of the principal axes of inertia of each end mass. A possible experiment mentioned in connection with the Skylab mission would involve the extension of a small mass at the end of a relatively short tether in an effort to stabilize the spinning station about the desired axis of rotation. For this application the tether may not be attached precisely along one of the principal axis of the Skylab configuration.

It is therefore suggested that the current development be made more general (and thus more complex algebraically) by assuming that the attachment arm vectors have components along all three principal body axes. The stability analysis would be repeated to evaluate the effect of this generalization on the small amplitude motion of the system.

The development of first - order approximate gravity - gradient generalized force terms could be reconsidered by first deriving a completely general three dimensional gravitational potential expression and then evaluating the resulting generalized forces resulting from this potential. A more accurate representation of the gravity - gradient effects should be considered for applications where the system would rotate at very slow rates when compared with the orbital angular velocity.

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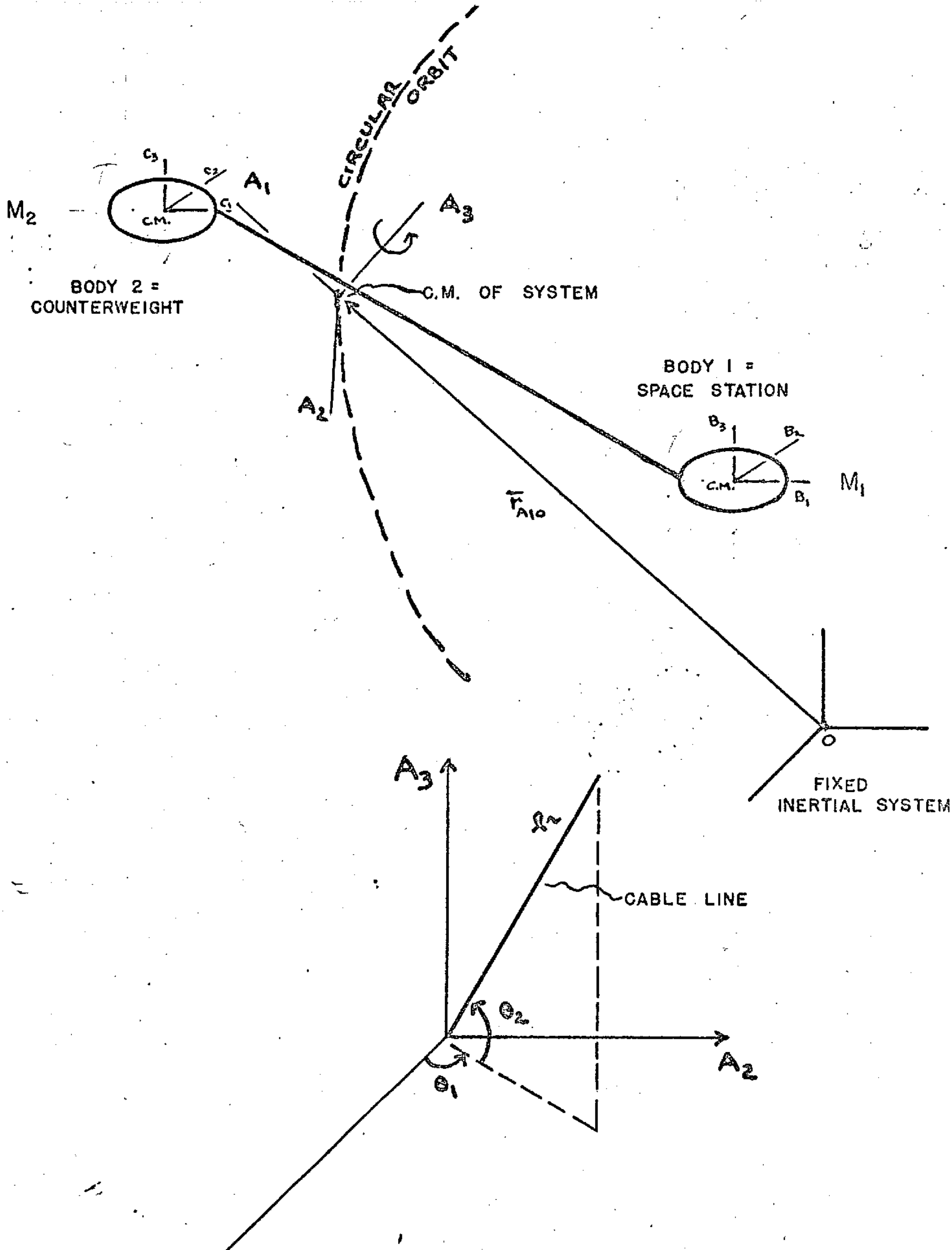


FIG. 1. SYSTEM GEOMETRY

$$\bar{\ell} + \bar{r}_{1|P} - \bar{r}_{1|A} + \bar{r}_{2|A} - \bar{r}_{2|P} = 0$$

$$M_1 \bar{r}_{1|A} + M_2 \bar{r}_{2|A} = 0$$

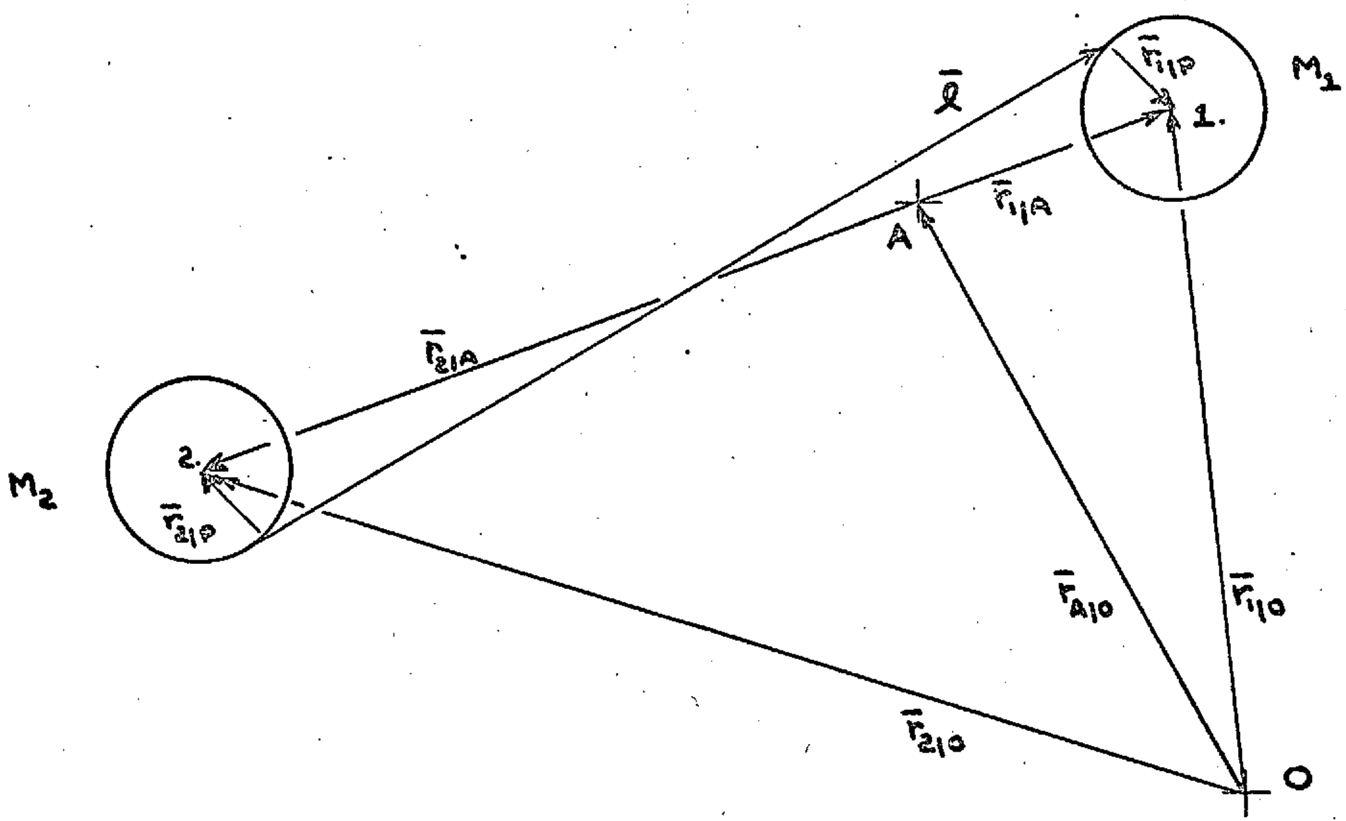
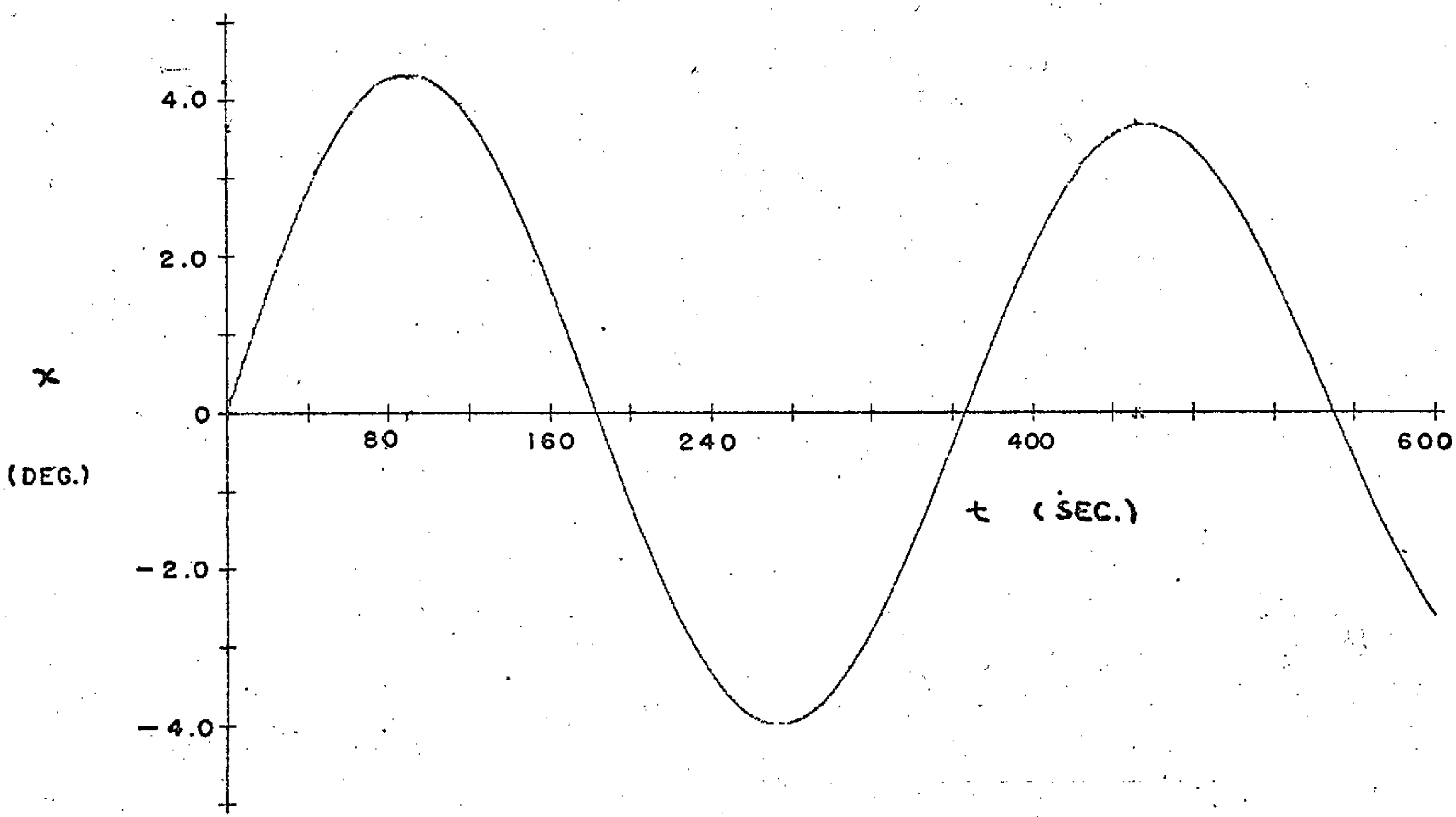


FIG. 2 DEFINITION OF VECTORS



FIG, 3 TRANSIENT RESPONSE OF IDENTICAL SYSTEM

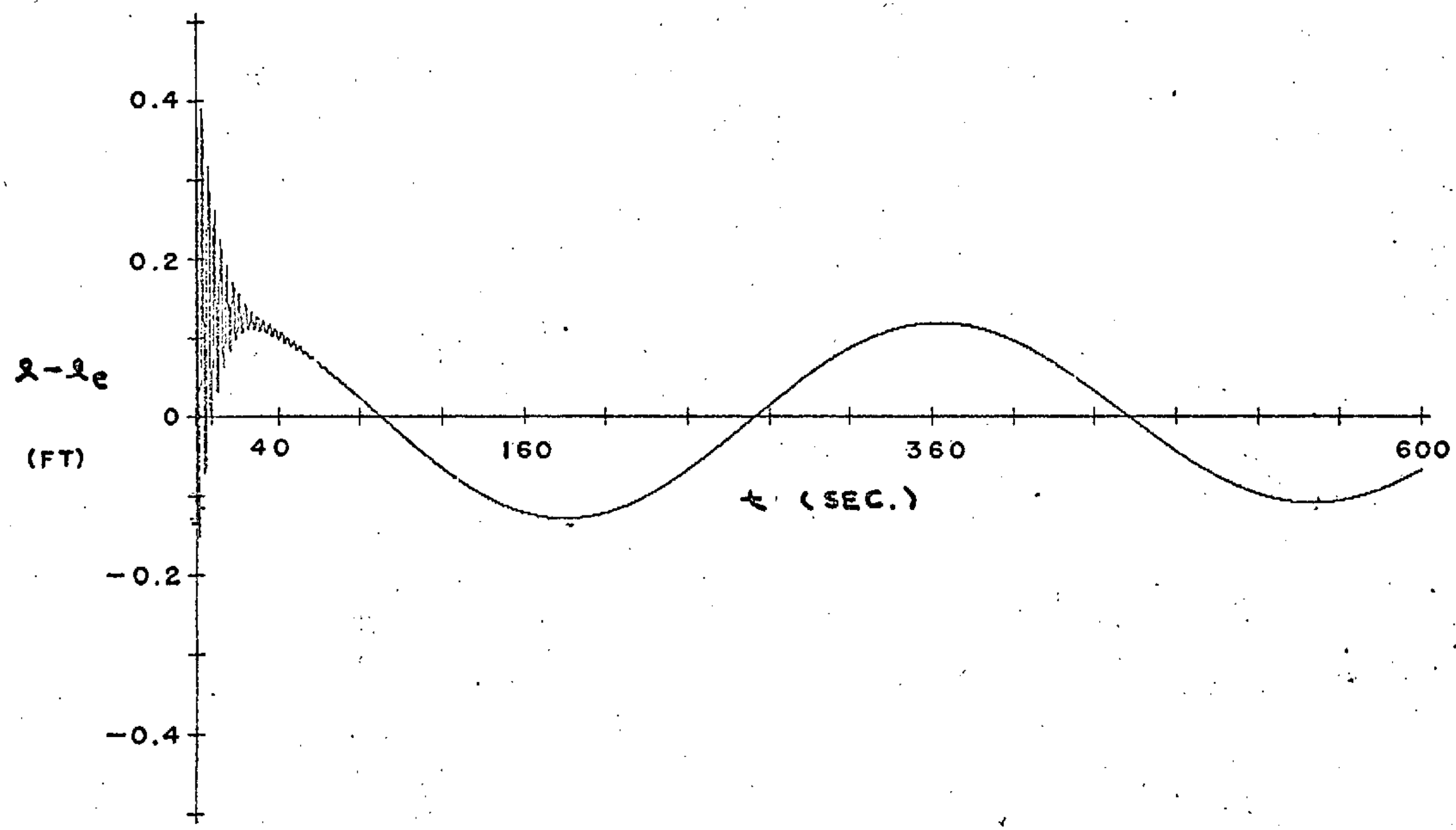


FIG. 4 TRANSIENT RESPONSE OF IDENTICAL SYSTEM

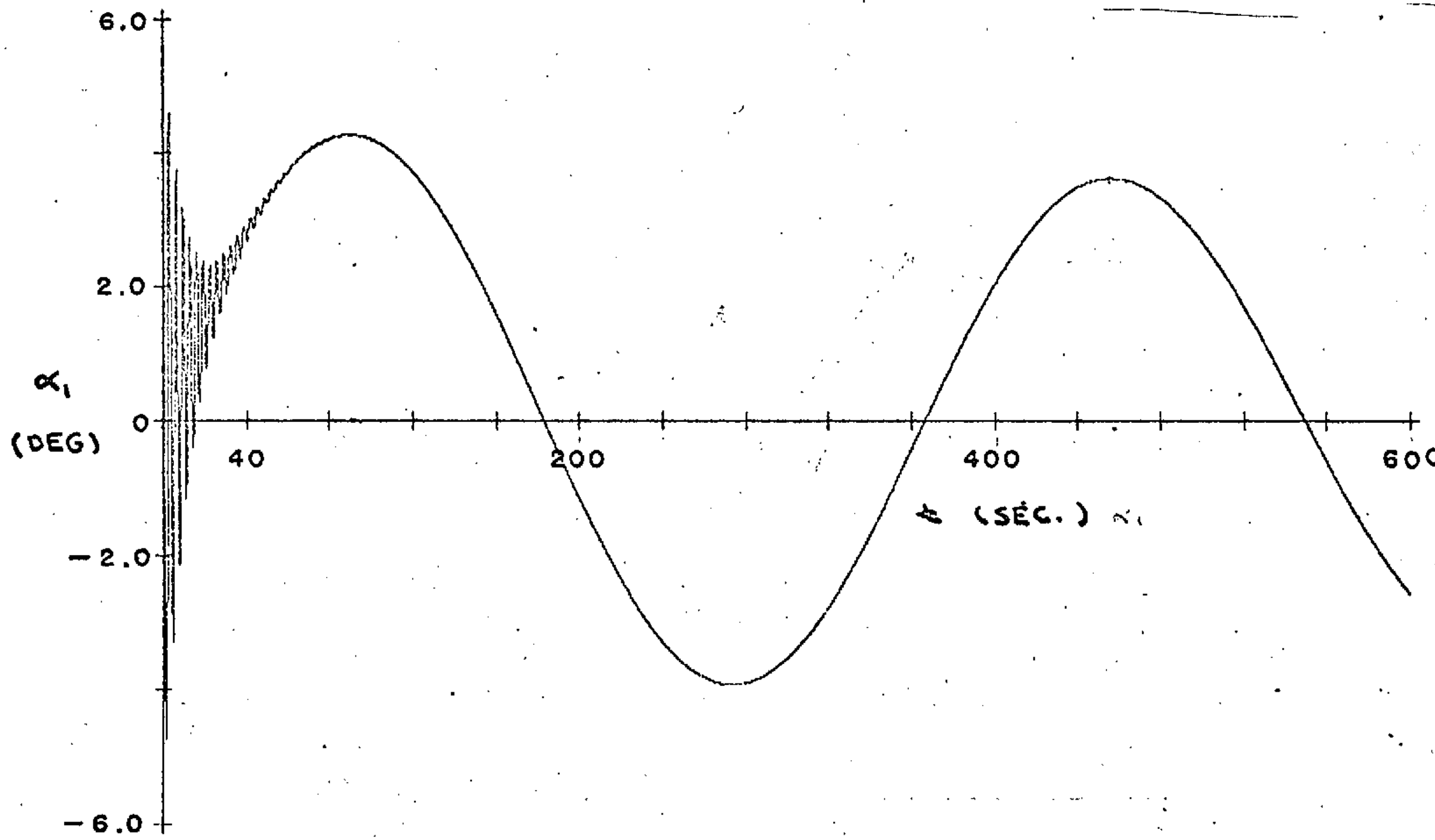


FIG. 5 TRANSIENT RESPONSE OF IDENTICAL SYSTEM

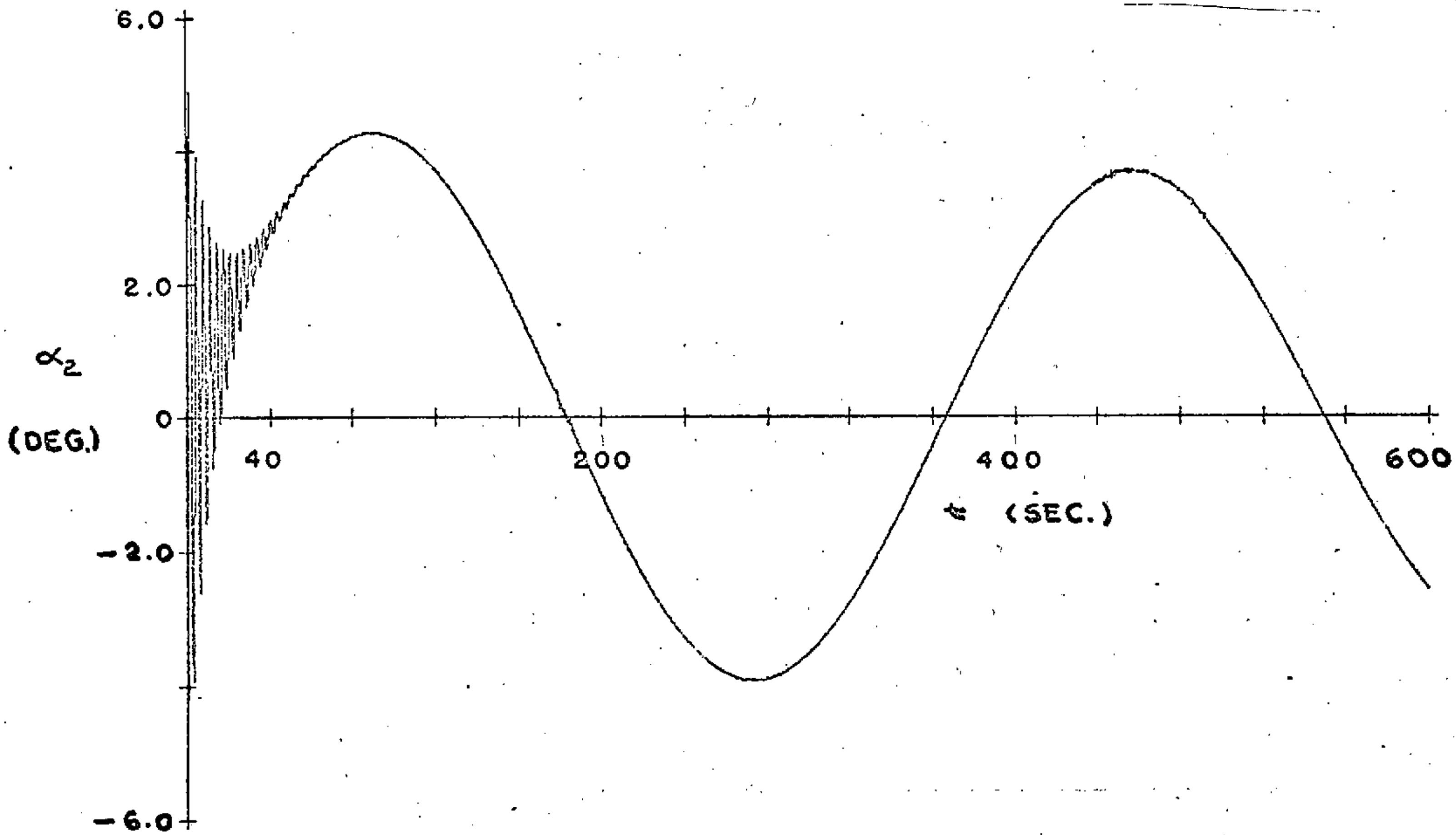


FIG. 6 TRANSIENT RESPONSE OF IDENTICAL SYSTEM

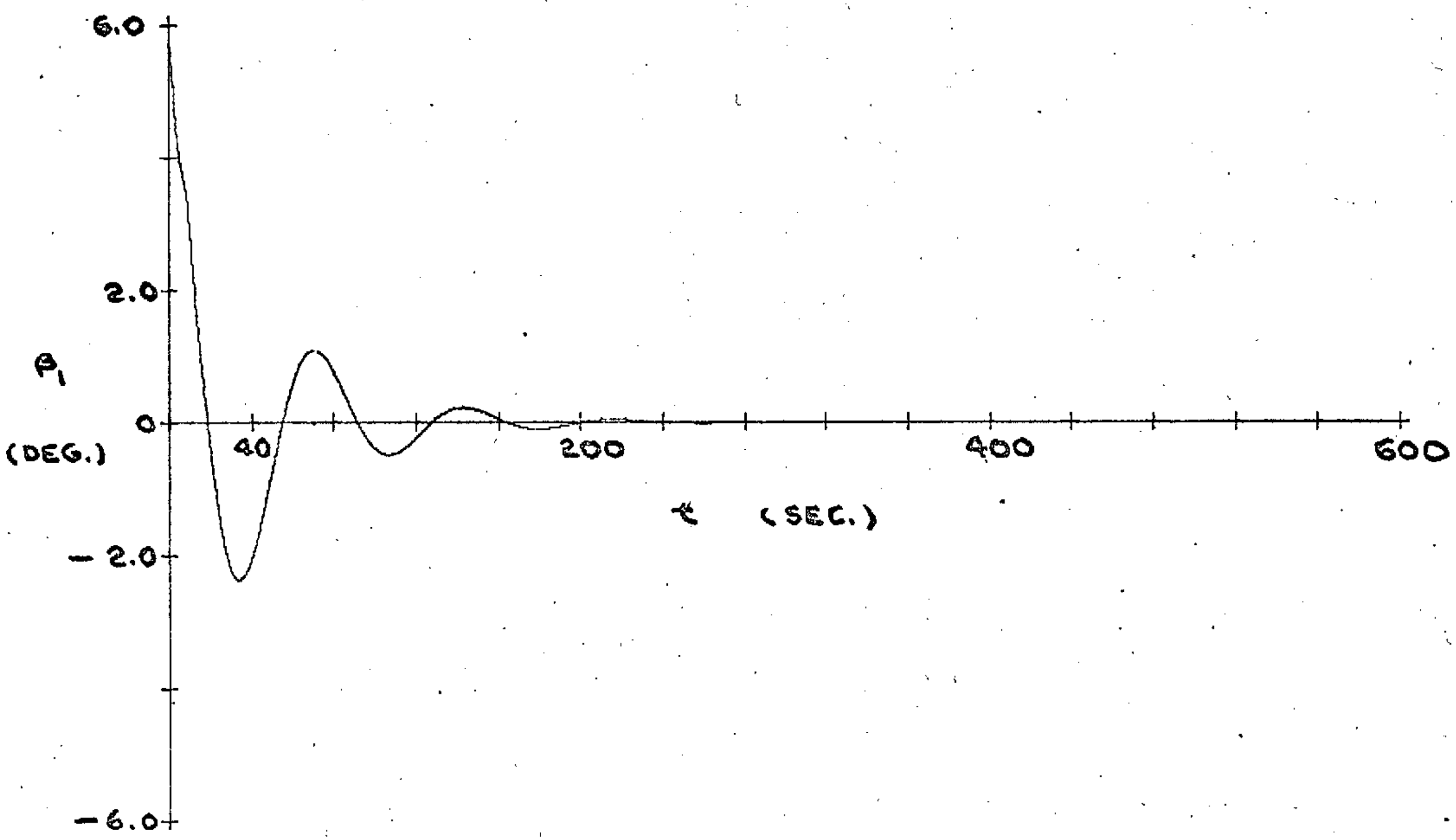


FIG. 7 TRANSIENT RESPONSE OF IDENTICAL SYSTEM - OUT-OF-PLANE MOTION

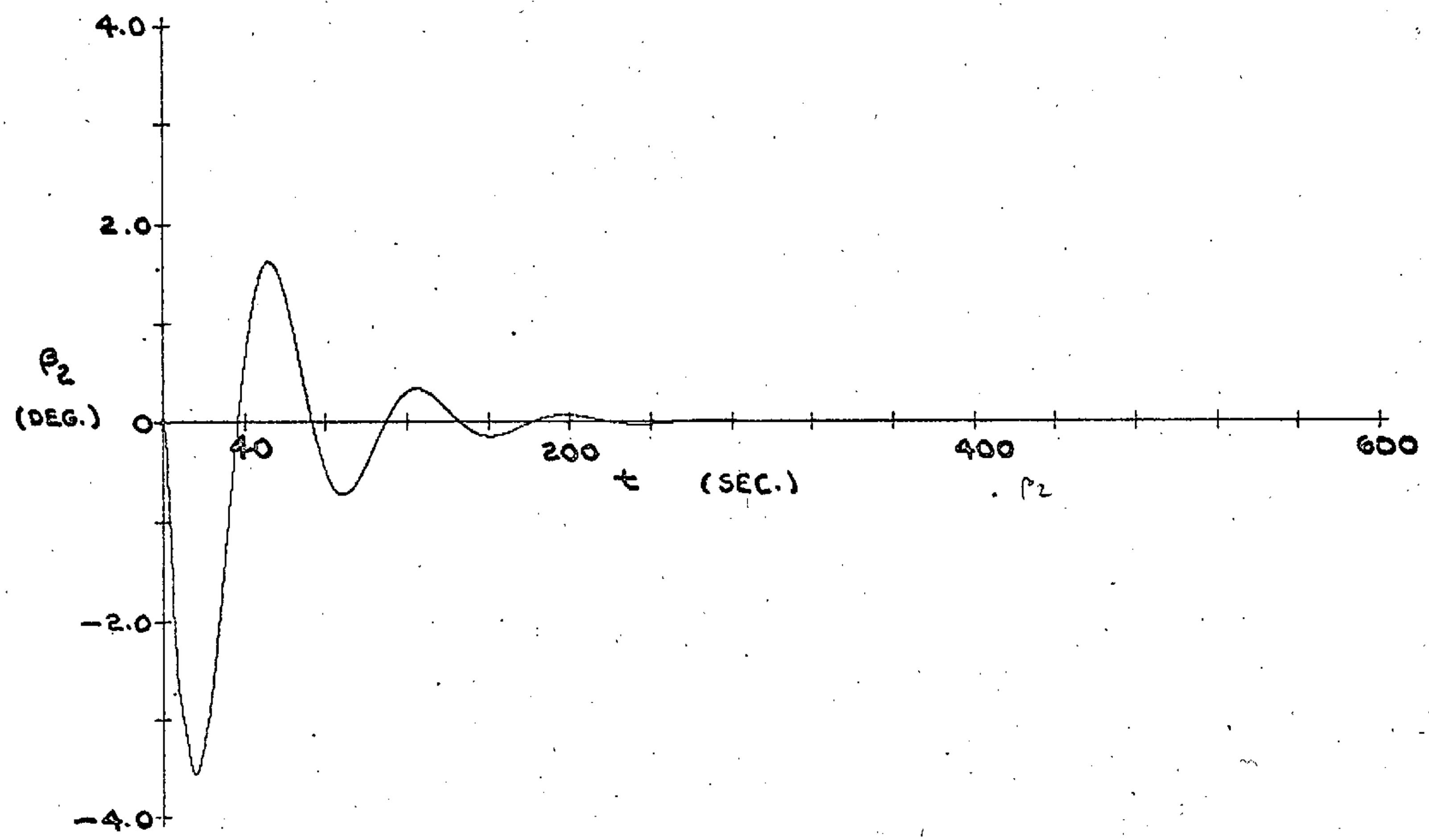


FIG. 8 TRANSIENT RESPONSE OF IDENTICAL SYSTEM OUT-OF-PLANE MOTION