NASA TECHNICAL NOTE

NASA TN D-7543

(NASA-TN-D-7543) NBODY - A MULTIPURPOSE TRAJECTORY OPTIMIZATION COMPUTER PROGRAM (NASA) 123 p HC $3.00 CSCL 09B

N74-19835

Unclas

H1/08 33770

NASA TN D-7543

NBODY - A MULTIPURPOSE TRAJECTORY OPTIMIZATION COMPUTER PROGRAM

by William C. Strack

Lewis Research Center

Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1974

1. Report No. NASA TN D-7543	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle NBODY - A MULTIPURPOSE TRAJECTORY OPTIMIZATION COMPUTER PROGRAM		5. Report Date February 1974
		6. Performing Organization Code
7. Author(s) William C. Strack		8. Performing Organization Report No. E-6998
		10. Work Unit No. 502-04
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Documentation of the NBODY trajectory optimization program is presented in the form of a mathematical development plus a user's manual. Optimal multistage-launch-vehicle ascent trajectories may be determined by variational thrust steering during the upper phase. Optimal low-thrust interplanetary spacecraft trajectories may also be calculated with solar power or constant power, all-propulsion or embedded coast arcs, fixed or optimal thrust angles, and a variety of terminal end conditions. A hybrid iteration scheme solves the boundary-value problem, while either transversality conditions or a univariate search scheme optimize vehicle or trajectory parameters.

17. Key Words (Suggested by Author(s))		18. Distribution Statement	
Computer program Trajectory simulation Trajectory optimization		Unclassified - unlimited Cat. 08	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 122	22. Price* $3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

CONTENTS

Page intentionally left blank

Page

NBODY - A MULTIPURPOSE TRAJECTORY OPTIMIZATION COMPUTER PROGRAM

by William C. Strack

Lewis Research Center

SUMMARY

This report documents the NBODY computer program. NBODY calculates the performance and trajectories for a variety of space vehicles such as low-thrust electric spacecraft and multistage launch vehicles (three degrees of freedom). Thrust, n-body, and aerodynamic forces may be simulated through flexible vehicle and solar system models. The thrust steering program, for example, may be specified or optimized for maximum performance by using variational techniques. If coast arcs are permitted, the engine on-off times may be optimized also. The low-thrust spacecraft options include solar or nuclear power, two-body or n-body simulation, fixed or optimum thrust angles, analytic spiral escape or high-thrust departure and/or capture, and fixed or optimized vehicle parameters such as specific impulse (constant), initial acceleration, and launch speed. Parameter optimization is done with transversality conditions or a search procedure, depending on the particular set of parameters.

The trajectory integration is carried out with a variable-step-size, fourth-order Runge-Kutta technique with double-precision accumulation but single-precision derivative evaluation. Boundary-value problems are solved with a general-purpose iterator using a hybrid univariate search and linear correction scheme (modified multivariable Newton-Raphson scheme). The program is written FORTRAN IV and occupies about 20 K of IBM 7094 core storage exclusive of standard library routines. Both the mathematical description of the program and detailed operating instructions with examples are included in this report.

INTRODUCTION

The computer program NBODY is used to generate trajectories for low-thrust interplanetary spacecraft and high-thrust launch vehicles. It was originally developed as a general-purpose program for a wide variety of space mechanics problems (ref. 1). During the mid-1960's its evolution was directed toward the optimum low-thrust problem as the potential of electric rockets came to be more widely recognized. Today NBODY is used mainly to calculate trajectories for electrically propelled spacecraft although it still retains its earlier multipurpose capability, such as the calculation of multistage-launch-vehicle trajectories. Extending NBODY's capability to cover optimal low-thrust trajectories required a considerable increase in the size and complexity of the program and, although the revised program has been extensively exercised, no formal documentation was made available. This report provides such documentation in the form of a user's manual. It describes the assumed vehicle and solar system models, summarizes the basic equations and program logic, and provides operating instructions.

The solar system model may be specified during input. It could be a simple two-dimensional model with only a single point-mass gravitational body. Or it could be a more accurate but more complicated model involving three dimensions, n bodies, and a realistic Earth (atmosphere, rotation, and oblateness). The planetary positions are determined by analytical time-dependent orbital elements.

The vehicle model is also specified during input and is generally either a low-thrust electric spacecraft or a multistage launch vehicle, although any thrust level and as many as 10 stages are permitted in either case. All vehicles are assumed to be point masses and to operate at constant specific impulse. Either a constant or solar-electric power profile may be chosen. The thrust direction program may be defined by the user or optimized by the program to yield maximum net mass. The thrust attitude may be optimized over an infinite set of angles (a continuous thrust program) or over a finite set of specified angles. In either case, the engine operation mode may be selected as continuously on or on-off with optimal switch times.

The NBODY program numerically integrates trajectories by using a fourth-order Runge-Kutta scheme with automatic step-size control. The only exceptions occur when the user wishes to calculate the planetary phases of an interplanetary trajectory for an electric spacecraft with approximate closed-form solutions. This is common practice when either a high-thrust or low-thrust Earth departure maneuver is part of a problem since significant simplification results with only a relatively small sacrifice in accuracy (refs. 2 to 4). For example, an electric spacecraft may be assumed to be boosted to at least escape velocity by a launch vehicle of known performance. The numerical trajectory integration begins in heliocentric space just outside the Earth's sphere of influence. Alternatively, a closed-form low-thrust spiral may be assumed for the Earth-escape maneuver, with numerical integration again commencing just outside the Earth's sphere of influence.

Either flyby or orbiter trajectory modes may be selected for interplanetary problems. For orbiters, some or all of the arrival hyperbolic excess speed may be removed with a closed-form, high-thrust retromaneuver.

For launch vehicle studies, the trajectory simulation consists of a zero angle-of-attack phase followed by an optimally steered upper phase. This simulation is in accord with the usual practice of limiting the angle of attack during atmospheric flight to reduce structural and heating loads.

Many trajectory problems require finding a set of initial conditions that permit a terminal set of conditions to be satisfied. This troublesome nonlinear two-point boundary-value problem is normally solved with an iterative linear correction scheme. Past experience here at the Lewis Research Center has shown that it is often helpful to program several iteration schemes to improve the chances of obtaining a solution. NBODY uses a univariate search scheme when the terminal set of conditions are far from being satisfied and a Newton-Raphson scheme when the solution is not far away. If either scheme fails to converge rapidly, an automatic shift to the other scheme takes place. The partial derivatives needed by the Newton-Raphson scheme may always be generated in NBODY by finite differencing. However, it is faster and more accurate to generate these partial derivatives by numerical integration. This option is available in NBODY for many typical problems but not all, since this method cannot be programmed to accommodate arbitrary end conditions as can the finite differencing method. Therefore, if the user selects a set of end conditions different from those for which the numerically integrated partials have already been programmed, he must either reprogram several sections of NBODY or resort to the finite difference method.

The problem of optimizing the thrust program to maximize gross payload is solved by using variational calculus. This method requires guessing a set of initial values for the adjoint variables (equivalent to guessing the initial thrust direction and its first derivative) that yield a trajectory not too far from the solution trajectory that satisfies certain end conditions. The user may also choose to optimize the central travel angle, the magnitude and direction of any hyperbolic excess speeds due to high-thrust launch or retrobraking of an electric spacecraft, the spacecraft specific impulse (assumed to be constant), and the initial mass flow rate. These options are also handled by variational calculus through transversality conditions. Or, if the user wishes, he may choose to optimize these or any other arbitrary variables with a simple search scheme. Transversality conditions are preferred whenever possible because their use has marked convergence speed and accuracy advantages.

It is often desirable to generate many solutions over a range of some arbitrary parameter, and provision has been made in NBODY to automatically "sweep" from one solution to others. Since problems often arise for which good starting guesses of the adjoint variables are lacking, a feature has been provided that sweeps a known solution of a related problem to the solution of the sought problem by a continuous transformation.

The NBODY program is written in many different subprograms in an effort to retain as much flexibility as possible. It is therefore possible to modify the program (the solar system model, vehicle model, etc.) with a minimum amount of difficulty. Its primary advantages compared to other trajectory programs are its relatively small size and broad capability. It is not specifically tailored to two-body, low-thrust interplanetary trajectories as are HILTOP (ref. 5) and CHEBYTOP II (ref. 6) or to launch vehicle trajectories such as the program reported in reference 7. Still, even as a general-purpose tool, it has proven itself capable of handling most of the problems for which such special-purpose programs are designed. It is sized for running on a computer having 32 000 words of core storage.

SOLAR SYSTEM MODEL

EPHEMERIDES

Ephemeris data are needed in two-body problems if the user instructs the program to calculate initial and final end conditions to be identical with those of specified gravitational bodies. Ephemeris data are also needed in n-body problems where the perturbing bodies' positions need to be known at each point along the spacecraft's trajectory. Elliptic orbits are used to approximate the true paths. To increase the accuracy of this approximation, the orbital elements are computed as a function of the departure Julian date in accordance with the relations presented in reference 8. Prestored elliptic data for the solar system planets are referenced to the mean equinox and ecliptic of date. Data for bodies in addition to the planets (e.g., the Moon) may be added by amending subroutines WORDER and WORBEL. Also, for interplanetary problems involving an Earth departure specified in equatorial coordinates, the prestored elliptic ephemerides would have to be converted to a consistent equatorial framework by amending subroutine WORBEL.

PHYSICAL DATA

The assumed values of several astronomical constants and the planetary masses and sphere-of-influence radii are given in table I. These values are consistent with the Jet Propulsion Laboratory values given in reference 8. The 1962 U.S. Standard Atmosphere model (ref. 9) is programmed for the Earth in subroutine WICAO. Other atmosphere models may be simulated by altering this subroutine.

VEHICLE MODELS

The discussion of vehicle models is separated into two major parts: (1) electrically propelled low-thrust spacecraft and (2) non-electric-type vehicles including launch vehicles, high-thrust spacecraft, and ballistic spacecraft. Actually, while it is convenient and logical to separate the discussion in this way, it should be noted that the program makes no internal distinction between these two types of vehicles. The inputs for mass, specific impulse, and so forth, are loaded into the same storage locations; and the integrated equations are identical. Thus, the user never inputs a single indicator that "tells" the program which vehicle model to use; instead, he supplies only the type of input data applicable to a particular vehicle model. For example, one normally does not think in terms of a multistage low-thrust electric vehicle; however, if one inputs three stage times, specific impulses, and so forth (as one might do in the case of a launch vehicle), the program will calculate a three-phase low-thrust trajectory. In general, then, all the features discussed in this section apply to either vehicle model. It is clearer, however, to discuss them in two, logically distinct sections.

ELECTRICALLY PROPELLED VEHICLES

The electric spacecraft is assumed to be composed of the following components:

(1) Electric propulsion system, m_{ps}
(2) Propellant mass, m_p
(3) Tankage mass, m_t
(4) Structure mass, m_s
(5) Retropropulsion mass, m_r
(6) Net spacecraft mass (gross payload), m_n

The net spacecraft mass refers to everything aboard the spacecraft not specified in this list. It includes the scientific instruments, communications equipment, control system, and so forth. The spacecraft mass at deparature m_0 is just the sum of all these components,

$$m_0 = m_{ps} + m_p + m_t + m_s + m_r + m_n \tag{1}$$

which can also be written in a form that facilitates scaling,

$$\frac{m_n}{m_0} = 1 - \frac{m_{ps}}{m_0} - \frac{m_p}{m_0} - \frac{m_t}{m_0} - \frac{m_s}{m_0} - \frac{m_r}{m_0} \tag{2}$$

(All symbols are defined in appendix A.) This is the form actually programmed since the net mass ratio is usually the criterion to be maximized. The propulsion system mass ratio is computed from the electrical power available at 1 AU from the Sun P_r and the specific powerplant mass α_{ps}:

$$\frac{m_{ps}}{m_0} \equiv \frac{\alpha_{ps}P_r}{m_0} = -\frac{\dot{m}_0 c^2}{2\eta \frac{P_0}{P_r}} \frac{\alpha_{ps}}{m_0} \tag{3}$$

Here P_0/P_r is the ratio of initial power to the 1-AU power; $\dot{m}_0$ is the initial flow rate; c is the input jet exhaust speed; and η is the overall propulsion system conversion efficiency, assumed to be a function of the jet exhaust speed,

$$\eta = \frac{bc^2}{c^2 + d^2} \tag{4}$$

where b and d are input constants that reflect the assumed technology level. If the efficiency is constant, for example, $b = \eta$ and $d = 0$.

The propellant mass is determined by integrating the mass flow rate $\dot{m}$ over the entire trajectory

$$m_p = -\int_{t_0}^{t_a} \dot{m}\, dt = -\int_{t_0}^{t_a} \epsilon \left(\frac{P}{P_r}\right) \dot{m}_0\, dt \tag{5}$$

where ϵ is a step function equal to unity if the engines are on and equal to zero if they are off. The initial flow rate $\dot{m}_0$ may be inputted directly or, if the user prefers, computed from an input value of the initial thrust-weight ratio f/m_0g

$$\dot{m}_0 = -a_0 \frac{m_0}{c} = -\left(\frac{f}{m_0 g}\right)\frac{m_0 g}{c} \tag{6}$$

or from an input value of the initial power P_0

$$\dot{m}_0 = -2\eta P_r \frac{\left(\frac{P}{P_r}\right)_0}{c^2} \tag{7}$$

The power ratio P/P_r may be chosen at input to simulate nuclear electric propulsion, in which case $P/P_r = 1$; or it may be chosen to simulate solar electric propulsion, in which case it is a function of the distance r from the Sun

$$\frac{P}{P_r} = \begin{cases} 0 & r < 0.13 \\ 1.33 & 0.13 \leq r \leq 0.652 \\ \frac{2.825}{r^2} - \frac{1.825}{r^{2.5}} & 0.652 < r \end{cases} \tag{8}$$

This relation is derived in reference 10; however, any other preferred model may be substituted by altering subroutine WPOWER.

The tankage mass is assumed to be proportional to the propellant mass

$$\frac{m_t}{m_0} = k_t\left(\frac{m_p}{m_0}\right) \tag{9}$$

and the structure mass is assumed to be proportional to the initial mass

$$\frac{m_s}{m_0} = k_s \tag{10}$$

Both k_t and k_s are input constants.

The retropropulsion mass component is really two components, one representing the retropropellant m_{rp} and the other representing tankage, engine, and other retropropulsion structure m_{rt} (assumed proportional to m_{rp}). Hence,

$$\frac{m_{rp}}{m_0} = \left[1 - \frac{m_p}{m_0} - j\left(\frac{m_{ps}}{m_0} + \frac{m_t}{m_0}\right)\right]\left(1 - e^{-\Delta v_r/c_r}\right) \tag{11}$$

$$\frac{m_{rt}}{m_0} = k_{rt}\frac{m_{rp}}{m_0} \tag{12}$$

$$\frac{m_r}{m_0} = \frac{m_{rp}}{m_0} + \frac{m_{rt}}{m_0} \tag{13}$$

where j is a jettison indicator equal to unity if the electric propulsion system and tankage mass components are to be jettisoned prior to the retromaneuver and equal to zero if they are not, c_r is the retropropulsion jet exhaust speed (input), and Δv_r is the magnitude of the retropropulsion velocity increment. The latter is assumed to be an impulsive velocity change,

$$\Delta v_r = v_r - v_{c,r} \sqrt{1 + e_r} \tag{14}$$

where v_r is the planetocentric velocity at periapsis before the retrofire (input), $v_{c,r}$ is the planetocentric circular orbit velocity at periapsis (input), and e_r is the eccentricity of the planetocentric elliptic orbit (input). Note that $v_{c,r}$ and e_r specify the desired planetary orbit, while v_r controls the amount of high-thrust braking and is usually free for optimization.

It often happens that a user wishes to include the Earth escape phase as part of the overall optimization of the net spacecraft mass m_n. However, it is exceedingly difficult to obtain solutions to such problems because of the extreme sensitivity of the associated two-point boundary-value problem. To avoid this difficulty, two options are available in NBODY that involve departure-phase approximations that are generally regarded as sufficiently accurate for preliminary analysis. The first option is a high-thrust launch to at least escape energy, and the second option is a low-thrust escape spiral. If the high-thrust option is chosen, the net spacecraft mass ratio is redefined in terms of the launch vehicle's payload capability in a low Earth circular parking orbit m_{ref}

$$\frac{m_n}{m_{ref}} = \frac{m_0}{m_{ref}} \frac{m_n}{m_0} \tag{15}$$

The launch vehicle's mass ratio is assumed to obey the following relation:

$$\frac{m_0}{m_{ref}} = (1 + k_l)e^{-(v_l - v_{c,l})/c_l} - k_l \tag{16}$$

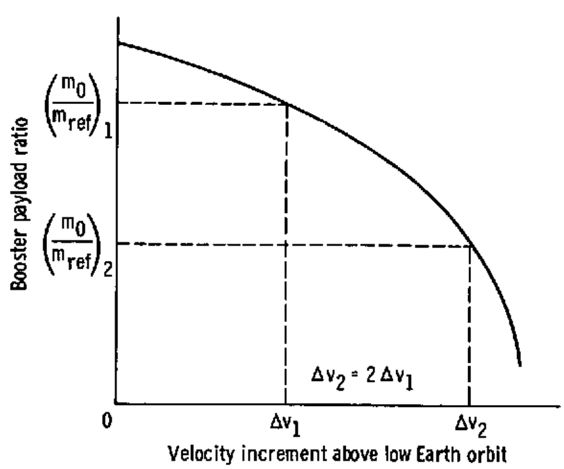
where v_l is the launch velocity relative to the Earth, $v_{c,l}$ is the circular orbit velocity of the low Earth parking orbit (e.g., at 185-km altitude), and c_l and k_l are input constants characterizing the launch vehicle performance. This equation is a curve fit to published launch vehicle performance curves, assuming impulsive velocity addition beyond the initial low Earth orbit. While c_l and k_l appear to be the launch vehicle's exhaust speed and propellant-sensitive hardware fraction, they are in fact, merely curve-fit parameters that only coincidently may be close to the actual values for these parameters. To obtain their values from a specified performance curve, select two v_l

values, note the corresponding m_0/m_{ref} values, and solve equation (16) for c_l and k_l. A particularly simple solution exists if the velocity increments $\Delta v = v_l - v_{c,l}$ are chosen such that $\Delta v_2 = 2\,\Delta v_1$, as illustrated in sketch (a), then

$$k_l = \frac{\left(\frac{m_0}{m_{ref}}\right)_1^2 - \left(\frac{m_0}{m_{ref}}\right)_2}{1 + \left(\frac{m_0}{m_{ref}}\right)_2 - 2\left(\frac{m_0}{m_{ref}}\right)_1} \tag{17}$$

and

$$c_l = \frac{\Delta v_1}{\ln\left[\frac{1 + k_l}{\left(\frac{m_0}{m_{ref}}\right)_1 + k_l}\right]} \tag{18}$$



(a)

Using this simple scheme ordinarily results in an adequate representation of launch vehicle performance for preliminary design analyses. The launch velocity v_l is an input variable subject to internal change if it is selected as an optimization variable, as is usually the case. In effect, choosing v_l is equivalent to choosing the initial spacecraft mass m_0. Note that m_{ref} does not need to be specified before trajectories are integrated if the initial acceleration a_0 is input directly or if $\dot{m}_0$ and m_0 are input together. In such cases the net mass ratios are evaluated and the absolute net mass calculated afterwards, if so desired, by multiplying by any m_{ref}. On the other hand, if the initial power P_0 is input, it is also necessary to input m_{ref} to determine $\dot{m}_0$ and m_0 (eqs. (7) and (16)). Solutions obtained with this option may be scaled by keeping P_0/m_{ref} constant.

If the closed-form tangential, low-thrust spiral escape option is chosen, equation (15) is still used but with a different form for m_0/m_{ref}:

$$\frac{m_0}{m_{ref}} = 1 - \xi\left(1 - e^{-v_{c,l}/c}\right) \tag{19}$$

where ξ is an empirical correction factor dependent on the thrust-weight ratio a_0/g, curve fitted from reference 11:

$$\xi = 0.28988 - 0.14084\left(\frac{a_0}{g}\right) - 0.010483\left(\frac{a_0}{g}\right)^2 - 0.00028355\left(\frac{a_0}{g}\right)^3 \tag{20}$$

The low-thrust spiral escape option is permitted when inputting either the initial flow rate $\dot{m}_0$ or the initial thrust-weight ratio a_0/g, but not when inputting the initial power P_0.

LAUNCH VEHICLES AND HIGH-THRUST OR BALLISTIC SPACECRAFT

This section discusses features normally associated with non-electric-type vehicles such as launch vehicles and ballistic or high-thrust spacecraft. As many as 10 trajectory phases are permitted, each specified by its flight time t_f, initial mass m_0, vacuum specific impulse I, and mass flow rate $\dot{m}_0$. These phases may be defined by actual vehicle staging or by a change in thrust vector control. Atmospheric flight may be simulated by also including the aerodynamic reference area S_{ref}, the engine exit area A_e, and lift and drag coefficient tabular data.

Between phases the vehicle mass may remain unchanged, be set to a new value, or decremented a fixed amount. The payload ratio is the same as that defined as net spacecraft mass for electric vehicles (eq. (2)) with the absence of the electric powerplant and planetary retropropulsion terms.

The thrust magnitude of each phase is assumed to be

$$f = -\dot{m}_0 Ig - pA_e \tag{21}$$

where p is the atmospheric pressure and A_e is the input engine exit area. Instead of inputting the mass flow rate $\dot{m}_0$, the user may input the initial vacuum thrust-weight ratio $f/m_0 g$, in which case $\dot{m}_0$ is calculated internally from

$$\dot{m}_0 = -\frac{\left(\frac{f}{m_0 g}\right) m_0 g + pA_e}{Ig} \tag{22}$$

The vehicle drag coefficient is composed of a parasitic component C_{D0} and an induced component C_{DI}. These coefficients are assumed to be quadratic functions of Mach number M,

$$C_D = C_{D0} + C_{DI} \tag{23}$$

$$C_{D0} = a_1 + a_2 M + a_3 M^2 \tag{24}$$

$$C_{DI} = \left(a_4 + a_5 M + a_6 M^2\right) C_L^2 \tag{25}$$

where the a_i coefficients are input in sets that apply to specific intervals of M. The lift coefficient C_L is determined in a similar manner

$$C_L = \left(a_7 + a_8 M + a_9 M^2\right) \sin \alpha \tag{26}$$

Here α is the vehicle angle of attack, which is identical to the angle between the thrust and relative velocity vectors because of the implied assumption that the engine thrust is always alined along the vehicle longitudinal axis.

PROGRAM LOGICAL STRUCTURE

The program NBODY is structured logically in what may be called three levels of operation. By analogy, these levels may be thought of as a set of three nested DO loops in FORTRAN programming. In the first level (analogous to an innermost DO loop), trajectories are integrated and an iteration scheme is available to solve two-point boundary-value problems. Thus, trajectories are found that satisfy specified terminal constraints such as fixed position and velocity. Every trajectory is integrated, including those for purely ballistic spacecraft. In addition to finding trajectory solutions, four vehicle-related parameters - specific impulse, initial mass flow rate, launch hyperbolic excess speed (equivalent to initial spacecraft mass), and high-thrust retropropulsion velocity increment (equivalent to retropropellant) - may be optimized in level 1 by incorporating the required transversality conditions into the two-point boundary-value problem.

Level 2 (analogous to a middle DO loop) permits direct optimization of either trajectory- or vehicle-related variables. The user selects the optimization criterion and the set of independent variables from among all variables computed by the program. If the user alters the net mass equation already programmed without also altering the vehicle-related transversality conditions, he must use level 2 to optimize specific impulse, acceleration, and so forth. Each time a level 2 independent variable is changed, the level 1 trajectory calculations are repeated, including the two-point boundary-value iteration.

Level 3 (analogous to an outermost DO loop) involves running several cases in succession with parameter sweep capability. Level 1 and level 2 calculations are repeated each time a level 3 variable is altered. Thus, variables that are optimized in level 2 may be reoptimized during a sweep on, for example, mission time.

LEVEL 1 - TRAJECTORIES AND VARIATIONAL NECESSARY CONDITIONS

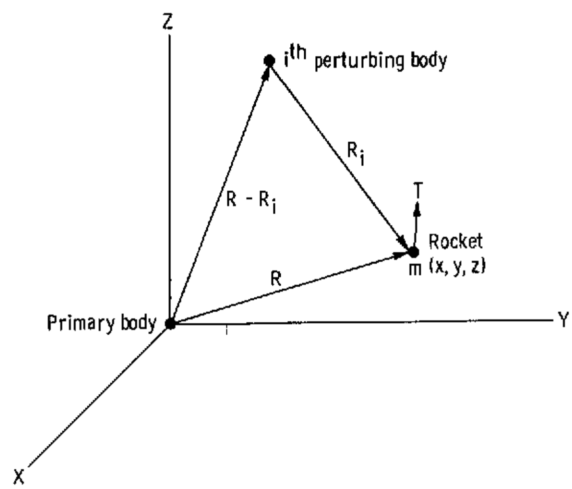
Trajectory Equations

The forces included in the trajectory simulation are gravitational forces of the Sun and the planets, thrust forces, and aerodynamic forces. These forces are vectorially summed as a resultant total force on the assumed point-mass vehicle relative to a primary center of attraction. The vector equation of motion is

$$\ddot{R} = -\nabla u - \sum_{i=2}^{n} \mu_i \left(\frac{R - R_i}{\left| R - R_i \right|^3} + \frac{R_i}{r_i^3} \right) + \frac{D}{m} + \frac{L}{m} + aT \qquad (27)$$

(Total) = (Primary body) + (Perturbing bodies) + (Drag) + (Lift) + (Thrust)

The convention of using capital letters to denote vectors and lower case letters to denote scalars is adopted in this report except where it interferes with well-known symbols. Here R is the vehicle's position vector (and r is the magnitude of R) relative to the origin of the coordinate system - located at the center of the primary body as shown in sketch (b); R_i is the position vector of the i^{th} perturbing body; μ_i is the body's gravitational constant; m is the vehicle mass; and T is a unit vector in the thrust direction.



(b)

Primary gravitational body attraction. - The first term ∇u in the acceleration equation denotes the gradient of the gravitational potential function $u = u(x, y, z)$ of the primary body. A point-mass body may be selected, in which case $u = -\mu/r$ and

$$\nabla u \equiv \left[\frac{\partial u}{\partial x} \; \frac{\partial u}{\partial y} \; \frac{\partial u}{\partial z}\right] = \frac{\mu}{r^3} R \tag{28}$$

Or, in the case of the Earth, an oblate potential function may be selected (from ref. 8)

$$u = \frac{-\mu}{r}\left[1 - \frac{J_2}{2}\left(\frac{a_e}{r}\right)^2 (3 \sin^2\varphi - 1) - \frac{J_3}{2}\left(\frac{a_e}{r}\right)^3 (5 \sin^3\varphi - 3 \sin \varphi) - \frac{J_4}{8}\left(\frac{a_e}{r}\right)^4 (35 \sin^4\varphi - 30 \sin^2\varphi + 3)\right] \tag{29}$$

where a_e is the equatorial radius of the Earth; φ is the vehicle's geocentric latitude relative to the Earth's equatorial plane; and J_2, J_3, and J_4 are zonal harmonic coefficients whose values are given in table I (from ref. 8). In this case,

$$\frac{\partial u}{\partial x} = \frac{\mu}{r^2}\left[1 - \frac{3}{2} J_2\left(\frac{a_e}{r}\right)^2 (5 \sin^2\varphi - 1) - \frac{5}{2} J_3\left(\frac{a_e}{r}\right)^3 (7 \sin^2\varphi - 3)\sin \varphi - \frac{35}{8} J_4\left(\frac{a_e}{r}\right)^4 \left(9 \sin^4\varphi - 6 \sin^2\varphi + \frac{3}{7}\right)\right]\frac{x}{r} \qquad x \rightarrow y \tag{30}$$

$$\frac{\partial u}{\partial z} = \frac{\mu}{r^2}\left[1 - \frac{3}{2} J_2\left(\frac{a_e}{r}\right)^2 (5 \sin^2\varphi - 3) - \frac{5}{2} J_3\left(\frac{a_e}{r}\right)^3 \left(6 - 7 \sin^2\varphi - \frac{3}{5}\frac{1}{\sin^2\varphi}\right)\sin \varphi - \frac{35}{8} J_4\left(\frac{a_e}{r}\right)^4 \left(9 \sin^4\varphi - 10 \sin^2\varphi + \frac{15}{7}\right)\right]\frac{z}{r} \tag{31}$$

Note that the oblateness forces are referenced to the Earth's equatorial plane. Thus, if oblateness terms are to be considered, the integrating inertial coordinate frame must be chosen as an equatorial frame or the user must program a coordinate transformation.

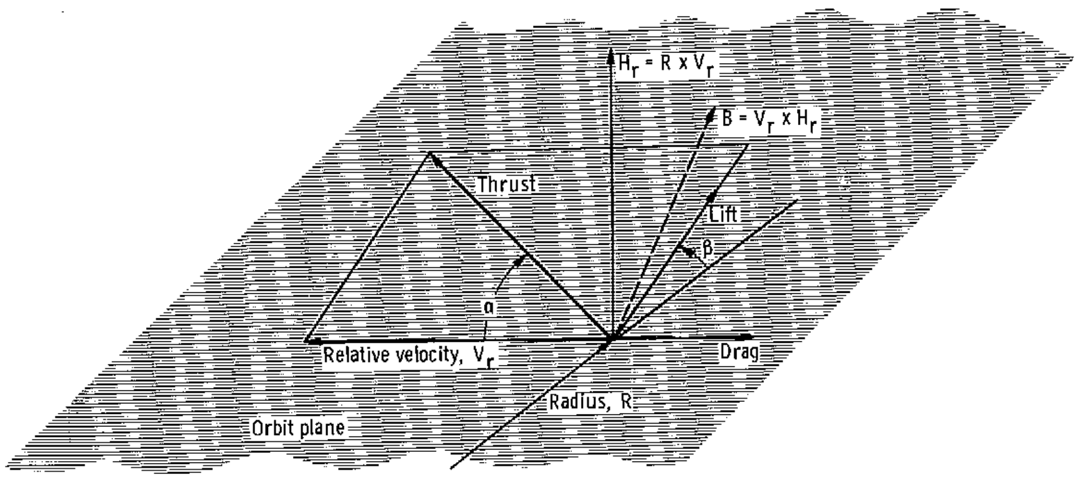
Perturbing body gravitational attraction. - The second term in equation (27) represents the acceleration caused by $n - 1$ perturbing bodies. The bodies are all assumed to be point masses, and their positions are determined by ephemerides as explained previously in the section SOLAR SYSTEM MODEL.

Aerodynamic forces. - Aerodynamic forces are split into drag and lift components with the usual definitions. Drag is opposite the relative wind vector and lift is perpendicular to the relative wind as shown in sketch (c). Since drag is opposite the relative wind velocity V_r

$$D = -(C_D q S_{ref}) \frac{V_r}{v_r} \tag{32}$$

where v_r is the magnitude of V_r and the dynamic pressure q is a function of atmospheric density ρ:

$$q = \frac{1}{2} \rho v_r^2 \tag{33}$$



(c)

The relative velocity V_r is referenced to the primary body, which is assumed to rotate counterclockwise about the z-axis. Hence,

$$v_{r,x} = v_x + \omega_r y \tag{34a}$$

$$v_{r,y} = v_y - \omega_r x \tag{34b}$$

$$v_{r,z} = v_z \tag{34c}$$

where the subscripts refer to the x, y, z components and ω_r is the rotation rate of the primary body and its atmosphere.

To compute the lift vector in the x, y, z inertial frame, it is convenient to first define the relative angular momentum vector per unit mass H_r

$$H_r = R \times V_r \tag{35}$$

and then define another vector B such that

$$B = V_r \times H_r \tag{36}$$

Note that H_r is normal to the $R \times V_r$ plane; B is within the $R \times V_r$ plane; and V_r, H_r, and B form an orthogonal set as shown in sketch (c). The lift vector L can be resolved along V_r, H_r, and B as follows:

$$L \cdot V_r = 0 \tag{37a}$$

$$L \cdot H_r = (l \sin \beta)H_r \tag{37b}$$

$$L \cdot B = (l \cos \beta)B \tag{37c}$$

where β is the out-of-orbit (relative orbit) thrust angle (sketch (c)) and the lift magnitude l is

$$l = C_L q S_{ref} \tag{38}$$

Solving these equations for L yields

$$L = l \sin \beta \frac{H_r}{|H_r|} + l \cos \beta \frac{B}{|B|} \tag{39}$$

The lift and drag coefficients (C_D and C_L) are tabular input data as explained in the section VEHICLE MODELS.

Thrust acceleration. - The fourth term in equation (27) is the thrust acceleration aT. The thrust acceleration magnitude is, in general,

$$a = -\epsilon\left(\frac{c\dot{m}_0}{m}\right)\left(\frac{P}{P_r}\right) - \frac{pA_e}{m} \tag{40}$$

where the second term is absent for exoatmospheric flight and the power ratio P/P_r is unity except for solar electric propulsion, as explained earlier in the section VEHICLE MODELS. The engine on-off switch parameter ϵ is unity for engine-on operation and zero for engine-off operation. It is needed in this equation only if the user selects the optimum thrust-coast profile option, in which case ϵ is calculated internally by the program.

The unit thrust vector T determines the thrust direction, and the user selects either an optimum T program or a specified T program. For a specified T program, the angle between the thrust force and the relative velocity (sketch (c)) is assumed to be a quadratic function of time

$$\alpha(t) = a_{10} + a_{11}t + a_{12}t^2 \tag{41}$$

where the a_i coefficients are input in sets that apply to specific time intervals. The out-of-plane thrust component is determined by the angle β, previously defined in the discussion of the lift force as an input constant (sketch (c)). The unit thrust vector can be resolved along the V_r, H_r, and B axes similar to the resolution of the lift force along these axes

$$\mathbf{T} \cdot \mathbf{V}_r = V_r \cos \alpha \tag{42a}$$

$$\mathbf{T} \cdot \mathbf{H}_r = H_r \sin \alpha \sin \beta \tag{42b}$$

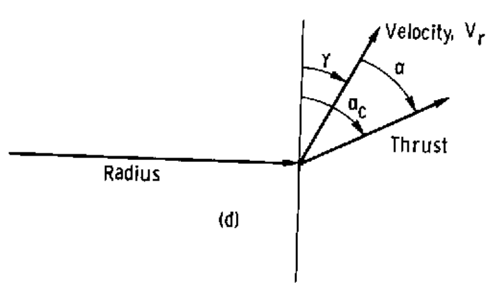
$$\mathbf{T} \cdot \mathbf{B} = B \sin \alpha \cos \beta \tag{42c}$$

Thus,

$$\mathbf{T} = \cos \alpha \frac{\mathbf{V}_r}{|\mathbf{V}_r|} + \sin \alpha \sin \beta \frac{\mathbf{H}_r}{|\mathbf{H}_r|} + \sin \alpha \cos \beta \frac{\mathbf{B}}{|\mathbf{B}|} \tag{43}$$

Instead of referencing the thrust angle to the relative velocity vector, the user may alternatively select to reference it to the circumferential direction as shown in sketch (d). In this case he specifies α_c - the angle from the forward circumferential direction to the thrust vector - as a function of time as in equation (41). The program then subtracts the path angle γ from α_c to determine α:

$$\alpha = \alpha_c - \gamma \tag{44}$$



(d)

and resolves the thrust, as before, using equation (43). This option is most useful for interplanetary missions where $\omega_r = 0$ (hence, V replaces V_r in sketch (d)) and the thrust orientation is often conveniently given in terms of the circumferential direction. A brief summary of frequently encountered thrust programs is given in the following table:

Thrust program	Required angles, deg
Tangential thrust (forward)	$\alpha = 0,\ \beta = 0$
Tangential thrust (rearward)	$\alpha = 180,\ \beta = 0$
Circumferential thrust (foward)	$\alpha_c = 0,\ \beta = 0$
Radial thrust (outward)	$\alpha_c = 90,\ \beta = 0$
Radial thrust (inward)	$\alpha_c = -90,\ \beta = 0$
Normal thrust (upward)	$\alpha_c = 0,\ \beta = 90$
Normal thrust (downward)	$\alpha_c = 0,\ \beta = -90$

If the user selects an optimum T program, variational calculus is employed to determine T(t). This is rather involved and is the subject of the following section. The user could, of course, closely approximate the optimum T(t) without variational calculus by using the built-in generalized search procedure to optimize β and the a_i coefficients in the $\alpha(t)$ equation. Since β is programmed as a constant, this could only be done for two-dimensional cases. Moreover, such a direct optimization procedure is generally inadequate unless the independent function (e. g., $\alpha(t)$) is subdivided many times, which in turn greatly increases the number of independent variables and slows down the search procedure significantly. Nevertheless, it may be applicable whenever the optimum thrust angle is fairly constant or when the thrust angle is constrained.

Optimal Thrust Control

Since the application of optimal control theory is especially complicated if oblateness and aerodynamic forces are present, these effects are not included in the optimal control formulation. This is quite acceptable for interplanetary transfers, of course, and usually so for preliminary launch vehicle studies. If the user should request optimal thrust control with oblateness or aerodynamic forces present, the equations of motion will account for such forces but the optimal control law will not. Hence, the trajectory will not be truly optimal.

With these restrictions the optimal control law formation is based on a simplified version of the equations of motion, namely

$$\dot{V} = -\frac{\mu}{r^3} R - \sum_{i=2}^{n} \mu_i \left(\frac{R - R_i}{|R - R_i|^3} + \frac{R_i}{r_i^3} \right) - \frac{c}{m} \dot{m} T \tag{45a}$$

$$\dot{R} = V \tag{45b}$$

where V is the vehicle's absolute velocity. The mass equation is

$$\dot{m} = \epsilon \dot{m}_0 \zeta \tag{45c}$$

where ζ is the power ratio P/P_r. These three equations define seven state variables - the three components of position and velocity, and the vehicle mass. The four parameters $\dot{m}_0$, c, v_l, and v_r may also be treated as state variables in order to optimize them with the variational method. To do this, four more state equations are appended to the preceding set.

$$(\dot{m}_0)^{\cdot} \equiv \frac{d}{dt} (\dot{m}_0) = 0 \tag{45d}$$

$$\dot{c} = 0 \tag{45e}$$

$$\dot{v}_l = 0 \tag{45f}$$

$$\dot{v}_r = 0 \tag{45g}$$

The necessary conditions for maximizing the net spacecraft mass are determined by variational principles (ref. 12). These conditions determine the optimum thrust orientation; engine on-off switch times; and values of $\dot{m}_0$, c, v_l, and v_r. Part of these conditions are differential equations known as Euler-Lagrange or adjoint equations which are

$$\dot{\Lambda} = -\Lambda_r \tag{46a}$$

$$\dot{\Lambda}_r = \sum_{i=1}^{n} \frac{\mu_i}{r_i^3} \left[\Lambda - \frac{3}{r_i^2} (\Lambda \cdot R_i) R_i \right] + \left(\frac{\epsilon \dot{m}_0 \zeta' \kappa}{r} \right) R \qquad (R_1 = R) \tag{46b}$$

$$\dot{\lambda}_m = -\frac{\dot{m}c\lambda}{m^2} \tag{46c}$$

$$\dot{\lambda}_{\dot{m}_0} = \frac{\dot{m}\kappa}{\dot{m}_0} \tag{46d}$$

$$\dot{\lambda}_c = \frac{\dot{m}\lambda}{m} \tag{46e}$$

$$\dot{\lambda}_{v_l} = 0 \tag{46f}$$

$$\dot{\lambda}_{v_r} = 0 \tag{46g}$$

where

$$\kappa \equiv \frac{c\lambda}{m} - \lambda_m \tag{47}$$

The subscripts here denote which state variable the adjoint variables λ_i are associated with. The term Λ_r is a vector with components λ_x, λ_y, and λ_z. Likewise, Λ is a three-component vector associated with velocity (it is not subscripted in keeping with its usual notation) and is customarily referred to as the primer vector.

Optimum continuously variable thrust angle. - Variational theory shows that the optimum value of T is determined by the primer vector:

$$T = \frac{\Lambda}{\lambda} \tag{48}$$

where λ is the magnitude of Λ. The engine on-off indicator ϵ is determined as follows:

$$\epsilon = 0 \qquad \text{if } \kappa < 0 \tag{49a}$$

$$\epsilon = 1 \qquad \text{if } \kappa > 0 \tag{49b}$$

The theoretical possibility of $\kappa = 0$ over a finite time interval very seldom occurs in practice, and this case presents no problems. If $\kappa = 0$ at the initial time, $\dot{\kappa}$ is interrogated to determine the proper initial value of ϵ. The adjoint equations must be integrated along with the equations of motion to determine the optimum values of T and ϵ. So far, we have not discussed how the optimization of $\dot{m}_0$, c, v_l, and v_r is accomplished, nor how the initial values of the adjoint variables λ_i are determined. Values for these variables are determined by the transversality conditions, which are dependent on the form of the net mass equation and the desired end conditions. Hence, a separate discussion of these conditions is presented later. It is sufficient for the moment to note that the adjoint equations are independent of these conditions. If any of the variables $\dot{m}_0$, c, v_l, or v_r are fixed instead of free for optimization, its corresponding state equation (eq. (45)) and adjoint equation (eq. (46)) are deleted. Conversely, the adjoint equations for any free variables are retained as they are needed in the evaluation of the transversality conditions.

Optimum choice of fixed thrust angles. - For two-dimensional problems, the user may choose to restrict the thrust angle to a finite set of fixed input values α_i - referenced either to the velocity vector or to the circumferential direction, as explained earlier. To determine which of the input angles should be used at any instant of time, the variational equations just presented are employed, with the exceptions that $T \cdot \Lambda$ is substituted at every occurrence of the scalar λ (eqs. (46c), (47), and several subsequent equations) and that equation (48) is replaced with

$$T \cdot \Lambda = \max_i(T_i \cdot \Lambda) \tag{50}$$

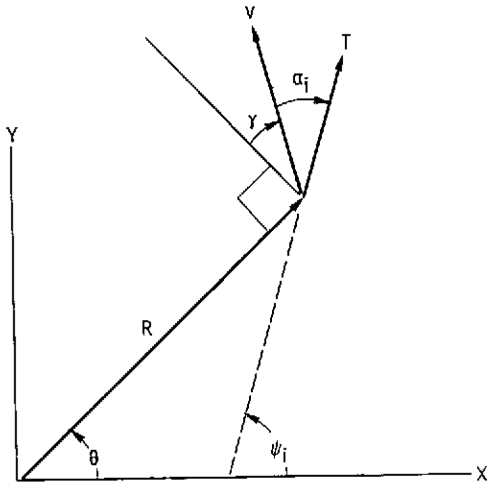
When this criterion is used, the program automatically switches the thrust angle whenever the difference between the two largest values of $T_i \cdot \Lambda$ reaches zero. The value of T_i depends on α_i; and since this option is programmed only for the two-dimensional case of trajectories in the x-y plane,

$$\Lambda \cdot T_i = \lambda_x \cos \psi_i + \lambda_y \sin \psi_i \tag{51}$$

where

$$\psi_i = \frac{\pi}{2} + \theta - (\alpha_i + \gamma) \tag{52}$$

Here ψ_i is the thrust angle referenced to the x-axis and θ is the travel angle as shown in sketch (e). If the thrust angle is referenced to the forward circumferential direction (α_c in sketch (d)), then $(\alpha_c)_i$ replaces $\alpha_i + \gamma$ in equation (52).



(e)

Boundary Conditions

Every problem involves certain boundary conditions that must be satisfied to obtain the desired solution. In some cases these are fixed, such as the initial position and velocity vectors. But in other cases, some of the boundary conditions are in the form of transversality conditions. These conditions arise whenever an end condition is not fixed but left open for optimization. The transversality equations are derived from variational theory (ref. 12) and are presented herein without proof.

Flight time. - The time at departure t_0 is an input constant $\bar{t}_0$

$$t_0 = \bar{t}_0 \qquad \text{(Departure time)} \tag{53}$$

The departure time is usually zero except for interplanetary problems involving actual planetary positions as a function of date. The arrival (or final) time is also a constant

$$t_a = t_0 + t_f \qquad \text{(Arrival time)} \tag{54}$$

where t_f is an input mission time. It should be noted at this point that although the boundary conditions are often stated herein in terms of input constants, such as t_f, the user always has the power to override this choice and declare such boundary conditions to be variable. For example, in typical launch vehicle problems the flight time t_f is not a fixed constant but an independent variable used in an iteration scheme to obtain certain orbit insertion conditions. How this is done is explained in the section The Two-Point Boundary Value Problem.

The flight time for low-thrust interplanetary missions must be defined in detail if any of the closed-form planetocentric simulations are involved. The planetocentric flight times are ignored for both high-thrust closed-form simulations; that is, the launch vehicle boost phase and the planetary capture phase (if any). In these cases, t_0 and t_a refer to the heliocentric portion of the flight, only. In the case of the closed-form low-thrust tangential escape spiral, t_0 and t_a again refer only to the heliocentric portion of the mission. But the program also calculates the spiral time required for the vehicle to reach escape velocity

$$t_s = \xi\left(\frac{c}{a_0}\right)\left(1 - e^{-v_{c,l}/c}\right) \tag{55}$$

where ξ is an empirical function of a_0 previously defined in equation (20). This time is added to the user-supplied heliocentric flight time t_f and printed out as total mission time. This method of handling the low-thrust spiral is acceptable since the known optimum trajectory for low-thrust escape from a circular orbit approximates a tangential thrust spiral (ref. 2). The main drawback in this method is that the total mission time $(t_f + t_s)$ is a dependent rather than an independent variable. (The user selects t_f, but the program calculates t_s.) This is because t_s depends on c and a_0 and these two variables are frequently changing during the course of an optimization iteration. In a typical case, t_s will be between 50 and 350 days.

Initial conditions. - At departure the vehicle position and velocity may be assumed fixed

$$R_0 = \overline{R}_0 \qquad \text{(Initial position)} \tag{56}$$

$$V_0 = \overline{V}_0 \qquad \text{(Initial velocity)} \tag{57}$$

where $\overline{R}_0$ and $\overline{V}_0$ are constants that are inputted directly or calculated by the program using an ephemeris. In the latter option, $\overline{R}_0$ and $\overline{V}_0$ are identical with a specified planet's position and velocity at time t_0.

For interplanetary low-thrust problems that begin in heliocentric space with the closed-form launch vehicle simulation, the velocity equation is modified to read

$$V_0 = \overline{V}_0 + v_{s,d}\left(\frac{\Lambda}{\lambda}\right)_0 \qquad \text{(Optimum launch orientation)} \tag{58}$$

where

$$v_{s,d}^2 = v_l^2 - 2v_{c,l}^2\left(1 - \frac{r_l}{r_{s,d}}\right) \tag{59}$$

Here $v_{s,d}$ is the spacecraft speed relative to the departure planet as it passes through a sphere of influence of radius $r_{s,d}$. If $r_{s,d} = \infty$, $v_{s,d}$ is identical to the often used hyperbolic excess speed. The launch vehicle burnout velocity v_l is assumed to occur at radius r_l, where the circular orbit speed is $v_{c,l}$. Both $v_{c,l}$ and the ratio $r_l/r_{s,d}$ are input constants. The launch vehicle burnout velocity v_l may be fixed or, as is often the case, optimized to yield maximum net spacecraft mass. If v_l is optimized, a transversality condition can be used for this purpose; and one is given later (eq. (72)) in the section Transversality conditions for optimum spacecraft parameters. The spacecraft velocity at the sphere of influence $v_{s,d}$ is added to the planet's velocity $\overline{V}_0$ in the primer direction Λ/λ in order to minimize propellant expenditure (ref. 13).

Arrival end conditions. - There are several sets of programmed arrival end conditions. The simplest of these is requiring the vehicle position and velocity to match those of a given target

$$R_a = \overline{R}_a \qquad \text{(Arrival position)} \tag{60}$$

$$V_a = \overline{V}_a \qquad \text{(Arrival velocity)} \tag{61}$$

Here $\overline{R}_a$ and $\overline{V}_a$ are the target position and velocity vectors. They are inputted directly or calculated by the program using an ephemeris for a specified target body.

For problems involving the analytic high-thrust capture maneuver, the arrival velocity equation must be modified to account for the braking maneuver,

$$\widetilde{V}_a \equiv V_a + v_{s,a}\left(\frac{\Lambda}{\lambda}\right)_a = \overline{V}_a \qquad \text{(Optimum retromaneuver orientation)} \tag{62}$$

where

$$v_{s,a}^2 = v_r^2 - 2v_{c,r}^2\left(1 - \frac{r_r}{r_{s,a}}\right) \tag{63}$$

Here $v_{s,a}$ is the vehicle speed relative to the target planet as the vehicle passes through the sphere of influence of radius $r_{s,a}$. Orienting the planetocentric path so that $v_{s,a}$ is directed in the primer direction at arrival $(\Lambda/\lambda)_a$ is a transversality result (ref. 13). The retrofire is assumed to take place at radius r_r, where the circular orbit speed is $v_{c,r}$. Both $v_{c,r}$ and the ratio $r_r/r_{s,a}$ are input constants. The velocity just prior to retrofire v_r is either fixed or open for optimization. Optimizing v_r effectively optimizes the amount of high-thrust braking into the specified capture orbit. The transversality condition for optimum v_r is given in the next section.

For two-dimensional trajectories in the x-y plane, the user may elect to specify the polar coordinates of a target point,

$$r_a = \bar{r}_a \qquad \text{(Arrival radius)} \tag{64}$$

$$\tilde{v}_a = \bar{v}_a \qquad \text{(Arrival velocity)} \tag{65}$$

$$\tilde{\gamma}_a = \bar{\gamma}_a \qquad \text{(Arrival path angle)} \tag{66}$$

$$\theta_a = \bar{\theta}_a \qquad \text{(Central travel angle)} \tag{67}$$

where $\bar{r}_a$, $\bar{v}_a$, $\bar{\gamma}_a$, and $\bar{\theta}_a$ are input constants. The tilde on the arrival velocity $\tilde{v}_a$ and path angle $\tilde{\gamma}_a$ indicates that these variables are not necessarily the arrival values. They are the arrival values if an analytic braking maneuver is not used. But if it is used, $\tilde{v}_a$ and $\tilde{\gamma}_a$ refer to values evaluated after the $v_{s,a}$ term is added to v_a in equation (62).

A frequently used variation of the polar set for launch vehicle and parametric interplanetary problems is leaving the travel angle θ open for optimization since θ is seldom constrained in these cases. In this case the θ equation (eq. (67)) is replaced with the transversality equation

$$\left(\lambda_1 v_y - \lambda_2 v_x + \lambda_4 y - \lambda_5 x\right)_a = 0 \qquad \text{(Optimum travel angle)} \tag{68a}$$

where λ_1 and λ_2 are the components of Λ, and λ_4 and λ_5 are the components of Λ_r. Fortunately, this expression is also a constant of the motion (for two-body problems only, strictly speaking). It therefore can be invoked at the departure point to solve for λ_5 (or any other λ_i) directly instead of solving equation (68a) at the arrival point by iteration. Also, this transversality condition may be generalized to three-dimensional problems to optimize the arrival latitude and longitude,

$$\left(\Lambda \times V + \Lambda_r \times R\right)_a = 0 \tag{68b}$$

In this case it is used to determine the initial values of λ_3, λ_5, and λ_6.

In some problems, such as interplanetary flybys, the arrival velocity vector is left open for optimization, which leads to the transversality condition

$$\left(\frac{\Lambda}{\lambda_m}\right)_a = 0 \qquad \text{(Optimum arrival velocity)} \tag{69}$$

This equation replaces the V_a equation if the arrival end conditions are specified in rectangular coordinates (eqs. (60) to (62)) or the $\tilde{v}_a$ and $\tilde{\gamma}_a$ equations if polar coordinates are specified (eqs. (64) to (67)).

Transversality conditions for optimum spacecraft parameters. - There are four variables associated with low-thrust spacecraft that may be open for optimization: The initial mass flow rate $\dot{m}_0$; the exhaust speed c; the launch vehicle burnout speed v_l; and for capture missions, the spacecraft speed just prior to the braking retrofire v_r (equivalent to the amount of retrofire propellant). The transversality conditions for these variables are

$$T_{\dot{m}_0} \equiv \frac{(\lambda_m)_a}{(\lambda_m)_0}\left(\frac{A_2}{A_1}\frac{m_{ps}}{m_0} - \frac{m_a}{m_0}\right) + 1 = 0 \qquad (\text{Optimum } \dot{m}_0) \tag{70a}$$

$$T_c \equiv \left(\frac{\lambda_m}{\lambda_c}\right)_a m_{ps}\frac{A_2}{A_1}\left(\frac{2}{c} - \frac{1}{\eta}\frac{d\eta}{dc}\right) + 1 = 0 \qquad (\text{Optimum } c) \tag{70b}$$

$$T_{v_l} \equiv \frac{m_n}{A_1}\left(\frac{v_{s,d}}{v_l}\right)\frac{(\lambda_m)_a}{\lambda_0}\frac{\left(1 + k_l\dfrac{m_{ref}}{m_0}\right)}{c_l} - 1 = 0 \qquad (\text{Optimum } v_l) \tag{70c}$$

$$T_{v_r} = \left(\frac{\lambda_m}{\lambda}\right)_a\left(\frac{v_{s,a}}{v_r}\right)\frac{1}{A_1 c_r}\left\{(1 + k_{rt})\left[m_a - j(m_{ps} + m_t)\right] - m_r\right\} - 1 = 0 \qquad (\text{Optimum } v_r) \tag{70d}$$

where

$$A_1 = 1 + k_t - \frac{m_r(1 + jk_t)}{m_a - j(m_{ps} + m_t)} \tag{71}$$

$$A_2 = 1 - \frac{jm_r}{m_a - j(m_{ps} + m_t)} \tag{72}$$

and m_a is the arrival mass at the target before any retrofire (i.e., $m_a = m_0 - m_p$). It has been assumed in these equations that

$$\left(\lambda_c\right)_0 = \left(\lambda_{\dot{m}_0}\right)_a = \lambda_{v_l} = \lambda_{v_r} = 0 \tag{73}$$

since they are arbitrary and choosing the value zero yields the simplest expressions. Unlike the trajectory transversality conditions, these four spacecraft transversality conditions are dependent on the particular definition of net spacecraft mass. Hence, the user is cautioned that any modification he makes to the net mass equations (eqs. (2), (3), and (9) to (16)) invalidates these four transversality equations.

The Two-Point Boundary-Value Problem

In some nonvariational and in nearly all variational trajectory problems, a two-point boundary-value problem arises that must be solved with iterative methods. In variational problems, for example, it is necessary to guess values for the initial adjoint variables λ_i and then use an iterative scheme that adjusts the λ_i until a given set of end conditions are satisfied. The NBODY user has the power to create any boundary-value problem he chooses. He does this at input time by specifying a particular set of independent variables x_i and a particular set of dependent variables y_i, where $i = 1, 2, \ldots, n$ ($n \leq 10$). He must be careful, of course, to create a well-defined boundary-value problem by selecting y_i that really do depend on x_i. He must also input guesses for the x_i and specify his desired values $\bar{y}_i$ for y_i. The program then adjusts the x_i until $y_i = \bar{y}_i$ by using techniques discussed in the section Iterator for Boundary-Value Problems.

Users of NBODY specify x_i, y_i, and $\bar{y}_i$ by loading values into the program arrays IA, IB, and DESIRE. The array DESIRE is simply $\bar{y}_i$. The array IA is a list of program locations relative to the beginning of COMMON where the x_i are stored. The array IB is a similar list for the dependent variables y_i. Table II is intended to assist users in this task by giving the COMMON locations of the anticipated candidates for x_i and y_i. If the user does not find his selections for x_i or y_i in table II, he must consult the complete COMMON map given in table III. To save the user the trouble of looking up the IA and IB indexes for frequently encountered problems, the program will fill these arrays automatically if the user selects special values for the program control variable NOPT. Since the description of how to use NOPT is too lengthy to include as a part of the input instructions given later, its use is detailed here and summarized in table IV.

NOPT=0. - NOPT=0 is the option for nonoptimal control - only the equations of motion are integrated, not the adjoint equations. The user must fill the IA, IB, and DESIRE arrays himself through input. If the program finds IA empty, it assumes that a boundary-value problem does not exist and calculates only a single trajectory.

NOPT=1. - All nonzero values for NOPT specify a variational problem involving the adjoint equations. The NOPT=1 option is useful for rendezvous problems where the vehicle is required to match a target's velocity $\overline{V}_a$ and position $\overline{R}_a$. The IA array is automatically filled by the program with the COMMON locations of the initial values for the adjoint variables λ_i (i = 1, 2, . . ., 6), where λ_1, λ_2, and λ_3 are the components of the primer vector Λ and λ_4, λ_5, and λ_6 are the components of the position adjoint vector Λ_r. The IB array is automatically filled with the locations of the components of the modified arrival velocity $\widetilde{V}_a$ and position R_a. The modified arrival velocity (eq. (62)) is used here so that cases involving the analytic high-thrust braking maneuver may be included as a generalization. If this braking maneuver is not used, $\widetilde{V}_a = V_a$. The user must fill the DESIRE list with the components of the target velocity and position ($\overline{V}_a$ and $\overline{R}_a$) unless he selects the ephemeris option. In this case, DESIRE is filled by the program using the selected target body's ephemeris. For two-dimensional problems in the x-y plane, only the x and y components of Λ, Λ_r, $\widetilde{V}_a$, R_a, $\overline{V}_a$, and $\overline{R}_a$ are used.

The preceeding discussion assumes that the engines may be shut down and restarted whenever a zero is attained by the on-off switching function κ. Thus, coast arcs will occur in the optimum trajectory. The user is cautioned here that he must select a value of initial mass flow rate $\dot{m}_0$ (or initial thrust-weight ratio a_0/g) large enough to permit a solution. If he selects too small a value of $\dot{m}_0$, not enough propulsive effort can be expended within the allotted mission time to accomplish the mission. In this case, the program will eliminate all coast arcs and fail to converge to a solution. Simultaneously, the adjoint variables will tend toward infinity - a sure signal to the user that $\dot{m}_0$ is too small.

If the user selects the all-propulsion constraint (COAST=F), the initial mass flow rate $\dot{m}_0$ (or a_0/g if inputted) is treated as an independent variable and replaces λ_1 in the IA list. Thus, in this case, the input value of $\dot{m}_0$ is merely a first guess and will be changed by the program in the process of iterating to a solution trajectory. The converged value of $\dot{m}_0$ will be the smallest possible value that may be used to reach the target at the specified mission time. The use of a larger value of $\dot{m}_0$ (or a_0/g) would result in a lower payload all-propulsion trajectory.

NOPT=2. - Valid only for two-dimensional trajectories in the x-y plane, NOPT=2 is a polar coordinate option that utilizes the end conditions defined by equations (64) to (67). The user must fill the DESIRE list with values for the target's radius $\overline{r}_a$, speed $\overline{v}_a$, path angle $\overline{\gamma}_a$, and the central travel angle $\overline{\theta}_a$. The IA list is automatically filled with the locations of λ_1, λ_2, λ_4, and λ_5 (the initial values of the adjoint variables). The IB list is automatically filled with the locations of the modified arrival conditions r_a, $\tilde{v}_a$, $\tilde{\gamma}_a$, and θ_a. Again, $\tilde{v}_a$ and $\tilde{\gamma}_a$ differ from v_a and γ_a only in cases involving the analytic high-thrust braking maneuver. And as explained in the NOPT=1 option, the initial mass flow rate or initial thrust-weight ratio is substituted for λ_1 in the IA list for all-propulsion missions.

NOPT=3. - The NOPT=3 option is identical to the NOPT=2 option with the exception that the optimum-travel-angle transversality condition (eq. (68a)) replaces the fixed-travel-angle condition (eq. (67)). Actually, as explained earlier, equation (68a) also applies at the departure point and may be used to solve for λ_5. Hence, only three end conditions require iteration (eqs. (64) to (66)). The user must load only the target values $\bar{r}_a$, $\bar{v}_a$, and $\bar{\gamma}_a$ into the DESIRE list and also guess values for λ_1, λ_2, and λ_4. For all-propulsion missions, the initial propellant flow rate (or the initial thrust-weight ratio) replaces λ_1 in the IA list as previously explained.

NOPT=4. - The NOPT=4 option is useful for two-dimensional flybys and involves end conditions, equations (64), (67), and (69). The user must fill the DESIRE list with a target radius $\bar{r}_a$, two consecutive zeros for the two components of $\left(\bar{\Lambda}/\lambda_m\right)_a$, and the central travel angle $\bar{\theta}_a$. The two zeros do not have to be loaded since the program default values are set to zero, but the loading order must be maintained (i.e., $\bar{\theta}_a$ must be loaded as the fourth element of DESIRE). The IA list is automatically filled with the λ_1, λ_2, λ_4, and λ_5 locations, while the IB list is filled with the location of r_a, $\left(\lambda_1/\lambda_m\right)_a$, $\left(\lambda_2/\lambda_m\right)_a$, and θ_a. The optimum arrival velocity end condition $\Lambda_a = 0$ is modified here by scaling with the arrival value of the mass adjoint variable λ_m. One of the transversality conditions requires $\left(\lambda_m\right)_a = 1$ and, because of the homogeneity of the adjoint equations, we are free to scale all the λ_i without changing the trajectory. In effect then, the user need only supply a target radius $\bar{r}_a$; the desired travel angle $\bar{\theta}_a$; and guesses for λ_1, λ_2, λ_4, and λ_5. For all-propulsion trajectories, the comments under the NOPT=1 option apply here also.

NOPT=5. - NOPT=5 is the optimum-travel-angle flyby option and is similar to the NOPT=4 option except that the optimum-travel-angle end condition is invoked to calculate λ_5 in the manner described for the NOPT=3 option. The user is required only to load a target radius $\bar{r}_a$ and guess the initial values of λ_1, λ_2, and λ_4. The all-propulsion mission comments of the NOPT=1 option also apply here.

NOPT=6. - In the NOPT=6 option the user must fill the IA and IB lists himself. The only difference between this option and the NOPT=0 option is that with this option the thrust control is optimal. Thus, the adjoint equations are integrated, and the user must load initial values for the adjoint variables λ_i.

NOPT=7. - The NOPT=7 option is identical to the NOPT=6 option with the addition of the appropriate optimum angle condition (eq. (68a) or (68b)). Thus, the user must fill the IA, IB, and initial adjoint variable lists. However, he need not furnish a value for λ_5 for two-dimensional problems (or λ_3, λ_5, and λ_6 for three-dimensional problems) since equation (68) is used at the departure point to determine them.

Additional features of the NOPT options. - Since choosing reasonable initial values for the adjoint variables λ_i is often a difficult task, a somewhat simpler scheme is provided for the common two-dimensional case. The user may elect instead to input variables that have more physical significance; namely, the departure thrust angle ψ_0, its derivative $\dot{\psi}_0$, the engine on-off switch function at departure κ_0, its derivative $\dot{\kappa}_0$, and the magnitude of the primer vector at departure λ_0. Sketch (e) defines ψ, and equation (47) defines κ. Under this option, the initial values of the adjoint variables are calculated with the following equations:

$$\left(\lambda_1\right)_0 = (\lambda \cos \psi)_0 \tag{74}$$

$$\left(\lambda_2\right)_0 = (\lambda \sin \psi)_0 \tag{75}$$

$$\left(\lambda_4\right)_0 = \left(\lambda_2\dot{\psi} - \frac{\lambda_1 m\dot{\kappa}}{c\lambda}\right)_0 \tag{76}$$

$$\left(\lambda_5\right)_0 = \left(-\lambda_1\dot{\psi} - \frac{\lambda_2 m\dot{\kappa}}{c\lambda}\right)_0 \tag{77}$$

$$\left(\lambda_7\right)_0 = \left(\frac{c\lambda}{m} - \kappa\right)_0 \tag{78}$$

Here $\left(\lambda_7\right)_0$ is the initial value of the mass adjoint variable λ_m. It is usually convenient to let the program set $\lambda_0 = 1$ by default since it represents a scale factor here and since all values of λ_0 result in identical trajectories. It is useful to set $\lambda_0 \neq 1$ when attempting to reproduce a previously computed trajectory for which the initial value of λ is not unity. (It avoids scaling by the user in such a situation.) If one of the optimum-travel-angle options is selected (NOPT=3, 5, or 7), it is unnecessary for the user to load $\dot{\kappa}_0$ since the program will compute its value using a variation of the transversality condition,

$$\dot{\kappa}_0 = \left[\frac{\lambda_2(v_x - y\dot{\psi}) - \lambda_1(v_y + x\dot{\psi})}{\frac{m}{c\lambda}(\lambda_2 x - \lambda_1 y)} \right]_0 \qquad (79)$$

In most NOPT options, inputting the alternative set ψ_0, $\dot{\psi}_0$, κ_0, $\dot{\kappa}_0$, and λ_0 instead of λ_1, λ_2, λ_4, λ_5, and λ_7 will result in its use for only the first trajectory of the boundary-value-problem iteration sequence. The remaining trajectories are begun by the program using the adjoint variables directly. However, the NOPT=6 and 7 options permit using the alternative set through the iteration sequence, if preferred, simply by filling the IA list with the locations of whichever members of this set are chosen.

Partial Derivatives

The boundary-value problem consists of driving a set of n dependent variables y_i to desired values $\bar{y}_i$ by adjusting a set of n independent variables x_i. The iterator that does this needs a partial derivative matrix G whose elements are $\partial y_i/\partial x_j$ evaluated for the current approximate solution set of x_i. There are two methods used in NBODY to generate G:

(1) A finite difference method

(2) An analytical method

Finite difference method. - This method consists of computing n perturbation trajectories about a reference trajectory - one for each x_i. Then the elements of G are formed in an approximate way by differencing the results of the perturbation trajectories with the reference trajectory.

$$\frac{\partial y_i}{\partial x_j} \cong \frac{\Delta y_i}{\Delta x_j} \equiv \frac{y_i - y_i^0}{x_j - x_j^0} \qquad (80)$$

The superscript zero denotes the reference trajectory values. This method has the advantage of being quite general and straightforward. It does suffer, however, from two standpoints: (1) it is relatively slow in comparison with the analytical method, and (2) it is often not easy to select appropriate perturbation sizes Δx_j. The latter difficulty manifests itself in highly nonlinear, sensitive problems, where a large Δx_j results in an excessively large error in Δy_i and where a small Δx_j results in too much numerical noise in Δy_i. To help alleviate this difficulty, NBODY is programmed to monitor Δy_i and to adjust Δx_j accordingly. If Δy_i is judged to be too small or too large, the perturbation trajectory is repeated; therefore, more than n perturbation trajectories are frequently necessary.

Analytical method. - The analytical method of generating the partial derivatives is faster and more accurate than the finite difference method. The partial derivatives are generated by integrating an additional set of differential equations along with the state and adjoint equations. Thus, the problem of choosing perturbation sizes is avoided. However, the method has the serious disadvantage of not being general - that is, a change in the definition of the end conditions or payoff criterion generally requires deriving and programming new partial derivative equations. In the NBODY program these equations are currently programmed for a limited set of options; namely, variational problems not involving any of the transversality equations for optimum $\dot{m}_0$, c, v_l, or v_r. Thus, the user must resort to the finite difference method if his problem is nonvariational or if any of these four variables are to be optimized by using transversality conditions. (They may also be optimized by using an ordinary search scheme, as discussed later.) In particular, the program will use the analytical scheme only if $1 \leq \text{NOPT} \leq 5$. (The NOPT operations are discussed in detail in the preceding section and are summarized in table IV.) If the user wishes to include analytical partial derivatives for $\dot{m}_0$, c, v_l, or v_r (or any others), he may do so by amending subroutines WDERIV, WALTER, WLOOK, WBEGIN, and WOUT.

The differential equations used to generate the partials are derived by differentiating the state and adjoint equations (eqs. (45) and (46)) with respect to an arbitrary variable. When δ is used to denote a partial with respect to the arbitrary variable, the resulting equations are

$$\frac{d}{dt}(\delta V) = -\sum_{j=1}^{n-1} \frac{\mu_j}{r_j^3}\left[\delta R - \frac{3}{r_j^2}(R_j \cdot \delta R_j)R_j\right]$$

$$- \frac{c\dot{m}_0}{m\lambda}\zeta\epsilon\left\{\delta\Lambda + \left[\frac{\delta c}{c} + \frac{\delta\dot{m}_0}{\dot{m}_0} - \frac{\delta m}{m} + \frac{\zeta'(R \cdot \delta R)}{\zeta r} - \frac{\Lambda \cdot \delta\Lambda}{\lambda^2}\right]\Lambda\right\} \tag{81a}$$

$$\frac{d}{dt}(\delta R) = \delta V \tag{81b}$$

$$\frac{d}{dt}(\delta m) = \epsilon\zeta\delta\dot{m}_0 + \epsilon\dot{m}_0\zeta' \frac{R \cdot \delta R}{r} \tag{81c}$$

$$\frac{d}{dt}(\delta\Lambda) = -\delta\Lambda_r \tag{81d}$$

$$\frac{d}{dt}(\delta\Lambda_r) = \sum_{j=1}^{n-1} \frac{\mu_j}{r_j^3}\left(\delta\Lambda - \frac{3}{r_j^2}\left\{(R_j \cdot \delta R)\Lambda + (\Lambda \cdot R_j)\delta R + \left[(R_j \cdot \delta\Lambda) + (\Lambda \cdot \delta R) - \frac{5}{r_j^2}(\Lambda \cdot R_j)(R_j \cdot \delta R\right] R_j\right\}\right) \tag{81e}$$

$$+ \frac{\dot{m}_0\zeta'}{r}\left\{\left[\frac{\delta\dot{m}_0}{\dot{m}_0} + \frac{R \cdot \delta R}{r}\left(\frac{\zeta''}{\zeta'} - \frac{1}{r}\right)\right]\kappa R + \left[\frac{\lambda}{m}\delta c + \frac{c(\Lambda \cdot \delta\Lambda)}{m\lambda} - \frac{c\lambda}{m^2}\delta m - \delta\lambda_m\right] R + \delta R\right\}$$

$$\frac{d}{dt}(\delta\lambda_m) \equiv -\frac{\epsilon\dot{m}_0\zeta c\lambda}{m^2}\left[\frac{\delta\dot{m}_0}{\dot{m}_0} + \frac{\delta c}{c} + \frac{\zeta'}{\zeta}\left(\frac{R \cdot \delta R}{r}\right) + \frac{\Lambda \cdot \delta\Lambda}{\lambda} - 2\,\frac{\delta m}{m}\right] \tag{81f}$$

$$\frac{d}{dt}(\delta\dot{m}_0) = \frac{d}{dt}(\delta\dot{a}_0) = 0 \tag{81g}$$

$$\frac{d}{dt}(\delta c) = 0 \tag{81h}$$

$$\frac{d}{dt}(\delta\lambda_c) = \frac{\epsilon\dot{m}_0\zeta\lambda}{m}\left(\frac{\delta\dot{m}_0}{\dot{m}_0} - \frac{\delta m}{m} + \frac{\Lambda \cdot \delta\Lambda}{\lambda} + \frac{R \cdot \delta R}{r}\,\frac{\zeta'}{\zeta}\right) \tag{81i}$$

$$\Big[\delta\lambda_{\dot{m}_0}\Big]_0^a = -\frac{1}{\dot{m}_0}\Big[\lambda_m\delta m + m\delta\lambda_m + \lambda\dot{m}_0\delta\dot{m}_0\Big]_0^a \tag{81j}$$

Each time the engine is switched on or off ($\kappa = 0$), discontinuities appear in several of these partials due to the presence of the factor ϵ in the state and adjoint equations. In general, the jumps are given by

$$\Delta\delta y \equiv (\delta y)^+ - (\delta y)^- = \frac{(\dot{y})_{\text{engines off}} - (\dot{y})_{\text{engines on}}}{\dot{\kappa}}\,\delta\kappa \tag{82}$$

where

$$\delta\kappa = \frac{c}{m\lambda}(\Lambda \cdot \delta\Lambda) - \delta\lambda_m - \frac{c\lambda}{m^2}\delta m + \frac{\lambda}{m}\delta c \tag{83}$$

$$\dot{\kappa} = -\frac{c(\Lambda \cdot \Lambda_r)}{m\lambda} \tag{84}$$

Specifically, the nonzero jumps are

$$\Delta\delta V = -\left(\frac{\dot{m}_0 \zeta \delta\kappa}{\Lambda \cdot \Lambda_r}\right)\Lambda \tag{85a}$$

$$\Delta\delta m = \frac{\dot{m}_0 \zeta m \lambda \delta\kappa}{c(\Lambda \cdot \Lambda_r)} \tag{85b}$$

$$\Delta\delta\lambda_m = -\frac{\dot{m}_0 \zeta \lambda^2 \delta\kappa}{m(\Lambda \cdot \Lambda_r)} \tag{85c}$$

$$\Delta\delta\lambda_c = \frac{\dot{m}_0 \zeta \lambda^2 \delta\kappa}{c(\Lambda \cdot \Lambda_r)} \tag{85d}$$

A jump discontinuity also occurs in δV at the arrival point if the analytic high-thrust braking maneuver option is selected,

$$\Delta\delta V = \frac{v_{s,a}}{\lambda_a}\left(\delta\Lambda - \frac{\Lambda \cdot \delta\Lambda}{\lambda^2}\Lambda\right)_a \tag{86}$$

Recall that δ denotes a partial derivative with respect to any variable x. The x_i of main interest, of course, are those variables that are defined as independent variables in the two-point boundary-value problem - namely, the initial values of the adjoint variables (plus $\dot{m}_0$ for all-propulsion cases). Actually, there are nine x_i programmed in NBODY:

$$x_1 = (\lambda_1)_0 \qquad \text{(Initial value of x-component of } \Lambda) \tag{87a}$$

$$x_2 = \left(\lambda_2\right)_0 \qquad \text{(Initial value of y-component of } \Lambda\text{)} \tag{87b}$$

$$x_3 = \left(\lambda_3\right)_0 \qquad \text{(Initial value of x-component of } \Lambda\text{)} \tag{87c}$$

$$x_4 = \left(\lambda_4\right)_0 \qquad \text{(Initial value of x-component of } \Lambda_r\text{)} \tag{87d}$$

$$x_5 = \left(\lambda_5\right)_0 \qquad \text{(Initial value of y-component of } \Lambda_r\text{)} \tag{87e}$$

$$x_6 = \left(\lambda_6\right)_0 \qquad \text{(Initial value of x-component of } \Lambda_r\text{)} \tag{87f}$$

$$x_7 = \left(\lambda_m\right)_0 \qquad \text{(Initial value of } \lambda_m\text{)} \tag{87g}$$

$$x_8 = c \qquad \text{(Specific impulse)} \tag{87h}$$

$$x_9 = \dot{m}_0 \qquad \text{(Initial mass flow rate)} \tag{87i}$$

The initial value of λ_m is included in this list since it is needed for the evaluation of $\delta\,(\Lambda/\lambda_m)$ used in the flyby end condition $\left(\Lambda/\lambda_m\right)_a = 0$. The specific impulse c is also included but is not required in the present version of NBODY. If the user prefers other x_i (such as ψ_0, $\dot{\psi}_0$, κ_0, $\dot{\kappa}_0$), he must alter subroutines WDERIV, WBEGIN, WOUT, WLOOK, and WINTEG.

Having defined the list of x_i, it is now possible to calculate the initial values of the partials - that is, the partial derivatives have values at the departure point according to the conditions imposed at departure. For example, the value of δV at departure depends on the magnitude of the boost velocity supplied by the launch vehicle (if any) and the boost velocity orientation. Thus, by differentiating equation (58), the first four of the following set of initial-value equations may be derived. The others are derived similarly. The subscripts on the partials denote which x_i in the preceding list is the independent variable $\left(\text{e.g., } \delta V_1 \equiv \text{partial derivative of } V \text{ with respect to } \left(\lambda_1\right)_0\right)$. Also, $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the x, y, z axes, respectively. The following, then, are the initial values of the partial derivatives:

$$\delta V_1 = \left[\frac{v_{s,d}}{\lambda}\left(\hat{i} - \frac{\lambda_x}{\lambda^2}\Lambda\right)\right]_0 \tag{88a}$$

$$\delta V_2 = \left[\frac{v_{s,d}}{\lambda}\left(\hat{j} - \frac{\lambda_y}{\lambda^2}\Lambda\right)\right]_0 \tag{88b}$$

$$\delta V_3 = \left[\frac{v_{s,d}}{\lambda}\left(\hat{k} - \frac{\lambda_z}{\lambda^2}\Lambda\right)\right]_0 \tag{88c}$$

$$\delta V_i = 0 \qquad i = 4, \ldots, 9 \tag{88d}$$

$$\delta R_i = 0 \qquad i = 1, \ldots, 9 \tag{88e}$$

$$\delta m_i = 0 \qquad i = 1, \ldots, 9 \tag{88f}$$

$$\delta \Lambda_1 = \hat{i} \tag{88g}$$

$$\delta \Lambda_2 = \hat{j} \tag{88h}$$

$$\delta \Lambda_3 = \hat{k} \tag{88i}$$

$$\delta \Lambda_i = 0 \qquad i = 4, \ldots, 9 \tag{88j}$$

$$\delta\left(\Lambda_r\right)_1 = \begin{cases} 0 & \text{for fixed travel angle} \\ \left(\dfrac{v_y + \lambda_1 \delta v_{y1} - \lambda_2 \delta v_{x1}}{x}\right)_0 \hat{j} & \text{for optimum travel angle (two dimensions only)} \end{cases} \tag{88k}$$

$$\delta\left(\Lambda_r\right)_2 = \begin{cases} 0 & \text{for fixed travel angle} \\ \left(\dfrac{-v_x + \lambda_1 \delta v_{y2} - \lambda_2 \delta v_{x2}}{x}\right)_0 \hat{j} & \text{for optimum travel angle (two dimensions only)} \end{cases}$$

(88l)

$$\delta\left(\Lambda_r\right)_3 = 0 \tag{88m}$$

$$\delta\left(\Lambda_r\right)_4 = \begin{cases} \hat{i} & \text{for fixed travel angle} \\ \hat{i} + \left(\dfrac{y}{x}\right)_0 \hat{j} & \text{for optimum travel angle (two dimensions only)} \end{cases} \tag{88n}$$

$$\delta\left(\Lambda_r\right)_5 = \hat{j} \tag{88o}$$

$$\delta\left(\Lambda_r\right)_6 = \hat{k} \tag{88p}$$

$$\delta\left(\Lambda_r\right)_i = 0 \qquad i = 7, 8, 9 \tag{88q}$$

$$\delta\left(\lambda_m\right)_i = 0 \qquad i = 1, \ldots, 9 \tag{88r}$$

$$\delta(c)_8 = 1 \tag{88s}$$

$$\delta(c)_i = 0 \qquad i = 1, \ldots, 7, 9 \tag{88t}$$

$$\delta\left(\dot{m}_0\right)_9 = 1 \tag{88u}$$

$$\delta\left(\dot{m}_0\right)_i = 0 \qquad i = 1, \ldots, 8 \tag{88v}$$

$$\delta\left(\lambda_c\right)_i = 0 \qquad i = 1, \ldots, 9 \tag{88w}$$

$$\delta\left(\lambda_{\dot{m}_0}\right)_i = 0 \qquad i = 1, \ldots, 9 \tag{88x}$$

After the partial derivative equations are integrated, it is necessary to transform them into another set if the end conditions are in terms of polar coordinates (two dimensions only):

$$\delta r = \frac{R \cdot \delta R}{r} \qquad \text{(Radius magnitude)} \tag{89a}$$

$$\delta v = \frac{V \cdot \delta V}{v} \qquad \text{(Velocity magnitude)} \tag{89b}$$

$$\delta\theta = \frac{x\delta y - y\delta x}{r^2} \qquad \text{(Polar travel angle)} \tag{89c}$$

$$\delta\gamma = \delta\theta - \frac{v_x \delta v_y - v_y \delta v_x}{v^2} \qquad \text{(Path angle)} \tag{89d}$$

Also, for the flyby end condition $\left(\Lambda/\lambda_m\right)_a = 0$ we need the following transformation equation:

$$\delta\left(\frac{\Lambda}{\lambda_m}\right) = \left(\frac{1}{\lambda_m}\right)\delta\Lambda - \left(\frac{\delta\lambda_m}{\lambda_m^2}\right)\Lambda \tag{90}$$

Iterator for Boundary-Value Problems

Convergence criterion. - Before each trajectory integration, an N-vector of independent variables X is selected that yields, after the integration, a particular N-vector of dependent variables Y. That is, the end condition vector Y is a function of X,

$$Y = f(X) \tag{91}$$

The boundary-value problem is to determine the solution to this equation if the Y vector is known - that is, given $\bar{Y}$ find the $\bar{X}$ that satisfies

$$\bar{Y} = f(\bar{X}) \tag{92}$$

A solution is judged to be found if the square root of the sum of the squares of the weighted residuals $\Delta Y = \bar{Y} - Y$ in less than a tolerance criterion $\bar{\tau}$:

$$\tau \equiv \sqrt{\Delta Y^T W^2 \, \Delta Y} < \bar{\tau} \tag{93}$$

The weighting matrix W is diagonal and positive definite. The diagonal elements of W consist of the weighting factors $1/w_i$ either selected by the user, or by default, calculated by the program as follows:

$$w_i = \begin{cases} \bar{y}_i & \text{if } \bar{y}_i \neq 0 \\ 1 & \text{if } \bar{y}_i = 0 \\ 360 & \text{if } \bar{y}_i = 0 \text{ and } y_i \text{ is path angle} \end{cases} \tag{94}$$

In the majority of cases the default weighting factors will result in approximately equal emphasis on all residuals, and the error τ will be of the order

$$\tau \approx \max_i \left| 1 - \frac{y_i}{\bar{y}_i} \right| \tag{95}$$

The convergence criterion $\bar{\tau}$ may be selected by the user or defaulted to 10^{-4}.

Linear correction scheme (Newton-Raphson). - Starting with a guess X_i that yields an error τ_i, the problem is to choose a new value X_{i+1} such that $\tau_{i+1} < \tau_i$. In general, the end condition vectors are related by

$$Y_{i+1} = Y_i + G \, \Delta X + (\text{Higher order terms}) \tag{96}$$

where $\Delta X = X_{i+1} - X_i$ and G is the partial derivative matrix $\partial Y/\partial X$. By ignoring the higher order nonlinear terms, we may estimate ΔX by setting $Y_{i+1} = \bar{Y}$ as follows:

$$\Delta X = G^{-1}(\bar{Y} - Y_i) \tag{97}$$

If X_i is close to the solution value $\overline{X}$, this estimate will usually result in $\tau_{i+1} < \tau_i$ and the process is repeated until convergence is obtained ($\tau < \overline{\tau}$). However, ΔX may be too large if X_i is not close to $\overline{X}$, and the new error value may exceed the old value. When this occurs, the trajectory is repeated using a smaller value of ΔX. In particular,

$$\Delta X = \chi G^{-1}(\overline{Y} - Y_i) \tag{98}$$

where χ is an inhibitor whose value lies between zero and unity ($0 < \chi \leq 1$). Several cutbacks in the size of χ may be necessary before $\tau_{i+1} < \tau_i$. The program reduces χ by a factor of 2 for each cutback and restores its value to unity upon satisfying $\tau_{i+1} < \tau_i$. Thus, each iteration cycle is initially attempted with $\chi = 1$. This method of controlling χ has proven to be just as effective as more elaborate inhibitor controllers in terms of reducing overall computation time.

The partial derivative matrix G is generated either by finite differencing or by numerical integration, as explained in the section Partial Derivatives, and is updated each time X_i is improved.

This convergence scheme is generally quite satisfactory providing the initial estimate of X is reasonably close to $\overline{X}$. If X is not close to $\overline{X}$, however, the inhibitor χ may be forced to very small values in order to improve X. This situation is undesirable since X is improved very slowly. To alleviate this difficulty, another error-reducing scheme is programmed to handle cases of large errors.

Univariate search scheme. - This scheme is called upon when the initial guess X is poor. In particular, it is used if $\tau > \tau^*$, where the value of τ^* is selected by the user or defaulted to unity (experience is the best guide for selecting τ^*). It is also called upon if the linear correction scheme bogs down because of inaccurate partial derivatives or highly nonlinear behavior. Each member of X is varied - one at a time - to reduce τ. The individual searches are conducted by increasing the step increments until a minimum in τ is detected. Rather than attempting to pinpoint the minimum, the search proceeds to vary the next variable as soon as τ begins to increase after having been in a downward trend. After the search cycles through all the variables, it begins over again with reduced initial step increments.

Although this technique has the capability to reduce large errors quickly, it is unacceptably slow in the neighborhood of the solution. Thus, whenever $\tau < \tau^*$, the univariate scheme is abandoned in favor of the linear correction scheme. This switch also occurs if the univariate scheme fails to halve the error τ within 15N trajectory simulations. Actually, control may be passed between these two schemes several times in difficult problems. If it is determined that neither scheme is working well, the linear correction scheme is activated without inhibitor control ($\chi = 1$) in a final effort to salvage the iteration. With reasonable first guesses, however, this hybrid technique is a powerful iterator that combines the advantages of both schemes.

Integration Method

Runge-Kutta scheme. - All the state equations, adjoint equations, and partial derivative equations (if used) are numerically integrated simultaneously by using a fourth-order Runge-Kutta method. This method for a single equation of the form $\dot{y} = f(t, y)$ may be described as follows:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{99}$$

where

$$k_1 = hf(t_n, y_n) \tag{100a}$$

$$k_2 = hf\left(t_n + \frac{h}{2}, \; y_n + \frac{k_1}{2}\right) \tag{100b}$$

$$k_3 = hf\left(t_n + \frac{h}{2}, \; y_n + \frac{k_2}{2}\right) \tag{100c}$$

$$k_4 = hf(t_n + h, \; y_n + k_3) \tag{100d}$$

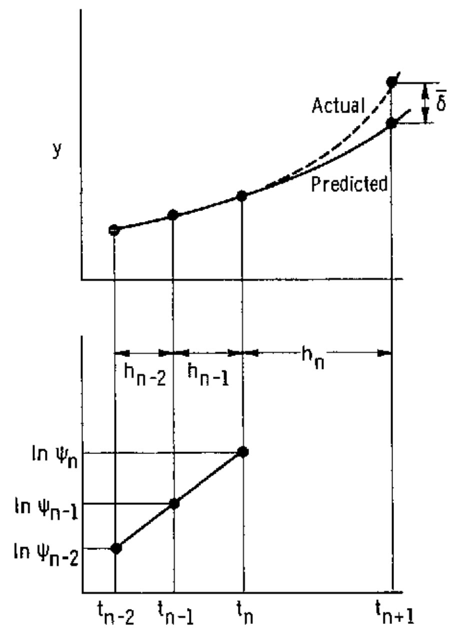
Four function evaluations are required for each integration step of size h. One of the disadvantages of this method is the absence of a simple yet accurate method to estimate the truncation error propagated along the trajectory. The truncation error is the difference between the true value of y and the value obtained with the integration formulas. If an accurate estimate of the truncation error were available, one could use it to control the step size in a manner that would maintain a specified accuracy level. In the absence of a rigorous and efficient step-size controller, an approximate but very efficient step-size controller is programmed that experience has shown to be stable and well-behaved in difficult situations. In particular, it reduces the step size in regions where the time derivative of the f function is changing rapidly.

The basis of the technique is the assumption that the truncation error δ is proportional to the fifth power of h,

$$\delta(t, h) = \Psi(t)h^5 \tag{101}$$

In general Ψ varies with time t in some unknown fashion. We further assume that, over the time span of two steps, the logarithm of Ψ varies linearly with t. This assumption allows us to predict a value of h that will result in a desired error $\bar{\delta}$ if we know the values of $\ln \Psi$ at the two previous time points (sketch f),

$$\ln h_n = \frac{1}{5} (\ln \bar{\delta} - \ln \Psi_n) \tag{102}$$



(f)

where

$$\ln \Psi_n = \ln \Psi_{n-1} + (\ln \Psi_{n-1} - \ln \Psi_{n-2}) \frac{h_{n-1}}{h_{n-2}} \tag{103}$$

The values of $\ln \Psi_{n-1}$ and $\ln \Psi_{n-2}$ are computed with the same basic formula except that the computed error δ is used in place of the desired error $\bar{\delta}$,

$$\ln \Psi = \ln \delta - 5 \ln h \tag{104}$$

Evaluating this equation at times t_{n-1} and t_{n-2} requires determining the error δ. Instead of attempting to calculate δ precisely, a very simple estimate is generated with the following Lagrangian interpolation formula:

$$\dot{y} = \dot{y}_{n-2} \frac{(t - t_{n-1})(t - t_n)}{h_{n-2}(h_{n-2} + h_{n-1})} - \dot{y}_{n-1} \frac{(t - t_{n-2})(t - t_n)}{h_{n-2}h_{n-1}} + \dot{y}_n \frac{(t - t_{n-2})(t - t_{n-1})}{h_{n-1}(h_{n-2} + h_{n-1})} \tag{105}$$

Integrating this equation between t_{n-1} and t_n yields

$$\Delta y \equiv y_n - y_{n-1}$$

$$= \frac{1}{6}\left[- \frac{h_{n-1}^3 \dot{y}_{n-2}}{h_{n-2}(h_{n-2} + h_{n-1})} + \frac{h_{n-1}(h_{n-1} + 3h_{n-2})\dot{y}_{n-1}}{h_{n-2}} + \left(2h_{n-1} + \frac{h_{n-2}h_{n-1}}{h_{n-2} + h_{n-1}}\right)\dot{y}_n\right] \tag{106}$$

This low-order integration formula may be evaluated quite efficiently since all the required data (the derivatives in particular) are already available from the Runge-Kutta integration. Thus, at each integration step the error δ may be estimated by differencing the value of Δy obtained by the Runge-Kutta formulas with the value obtained with this low-order method.

$$\delta_r = \left| \frac{(\Delta y)_{\text{Runge-Kutta}} - (\Delta y)_{\text{Low-order scheme}}}{y_n} \right| \tag{107}$$

This definition of error δ_r is not the same as the previous definition of δ: (1) because δ_r represents the difference in answers between two integration schemes instead of the true error δ and (2) because δ_r is a relative error since the Δy increments are divided by a normalization factor y_n.

Since many independent variables are integrated simultaneously, there are many values of δ_r calculated at each step (one for each state and adjoint variable). Only the maximum value of δ_r is used to calculate the next step size. Obviously, inaccurate predictions of step size can occur - particularly when the maximum value of δ_r shifts from one variable to another or when any sudden change occurs. Whenever the error is excessive ($\delta_r > \delta_{limit}$), the step is recomputed with a smaller value of h, which is calculated by updating the $\ln \Psi$ data (using the excessive error in eq. (104)). Two consecutive failures at satisfying $\delta_r > \delta_{limit}$ result in a restart of the integration procedure at the time of failure. The start (and restart) procedure is to take two identical sized steps before checking the relative error δ_r. This is necessary because no values of $\ln \Psi$ are yet available. In this procedure the value of $\ln \Psi_{n-2}$ is set equal to $\ln \Psi_{n-1}$ and the low-order integration formula (eq. (106)) is replaced by a simplified form (Simpson's Rule) because h_{n-2} equals h_{n-1},

$$\Delta y = \frac{h_{n-1}}{3} (\dot{y}_{n-2} + 4\dot{y}_{n-1} + \dot{y}_n) \tag{108}$$

The program user selects the level of accuracy and initial step size as follows:

Parameter	FORTRAN name	Default value
Reference relative error, $\bar{\delta}_r$	EREF	10^{-4}
Limit relative error, δ_{limit}	ERLIMT	3×10^{-4}
Initial step size, h_1	STEP	$t_f/100$

The default initial step size is $1/100^{th}$ of the mission time t_f. If the estimate of h_1 is too large, the program automatically reduces it until the limit error criterion is satisfied. If h_1 is grossly underestimated ($\delta_r << \delta_{limit}$), the step is accepted and the next step size is increased substantially, but as a precaution no step is permitted to be greater than three times the size of the previous step.

Comparisons with exact solutions have shown that $\bar{\delta}_r = 10^{-4}$ is sufficiently accurate for most parametric studies requiring only three or four figures of accuracy. With a 36-bit word length computer, roundoff error will ordinarily exceed truncation error if $\bar{\delta}_r < 10^{-7}$. Furthermore, very little accuracy difference exists between $\bar{\delta}_r = 10^{-6}$ and $\bar{\delta}_r = 10^{-7}$. Hence, as a general guideline, setting $\delta_r < 10^{-6}$ is not recommended since little improvement in accuracy can be gained at the expense of much greater computer execution time. Decreasing $\bar{\delta}_r$ by an order of magnitude will result roughly in doubling the number of integration steps and execution time.

Most of the integration process is computed in single precision on 36-bit word length computers ($8\frac{1}{3}$ significant figures), although the variables being integrated are accumulated in double precision. That is, the derivatives $\dot{y}$ are evaluated in single precision, but the integration variables y are accumulated in double precision. This is nearly as fast and compact as complete single-precision integration and approaches the accuracy afforded by complete double-precision integration since usually $\Delta y << y$.

Trajectory interrupt. - It is often necessary to interrupt the integration process before the trajectory terminates to allow some specific action to be taken. Interrupts for printouts at selected time or step intervals are an obvious example. Figure 1 illustrates several other interrupt situations. For problems involving more than one phase (fig. 1(a)), the phase-defining data are changed at each phasing point. When phases are identical to physical vehicle stages, this amounts to reinitializing the mass, specific impulse, propellant flow rate, and so forth. These data are read in during a single input at the beginning of a case and stored in arrays. Interrupts also occur whenever a trajectory passes through a sphere of influence in order to translate the coordinate system origin to the center of another body (fig. 1(b)). A third type of interrupt occurs under the optimal thrust option and involves switching the engines either on or off when $\kappa = 0$ (fig. 1(c)). Several of the partial derivatives (if integrated) are discontinuous whenever the engines shut down or start up. Yet a fourth type of interrupt occurs under the optimum-fixed-thrust-angle option (fig. 1(d)). In this case the thrust angle changes discontinuously whenever the difference between the two largest values of $\Lambda \cdot T_i$ vanishes. The unit thrust vector T_i is dependent on the thrust angle α_i according to equations (50) to (52).

The last type of interrupt provides the user with a means to force phase points or printouts to occur whenever a user-selected variable attains a user-selected target value (fig. 1(e)). This flexibility circumvents many awkward situations such as trying to guess the firing time of a launch vehicle's first stage so that the first-stage burnout occurs at some desired altitude. In this example the user can specify directly that the first stage should cease when the vehicle's altitude reaches some input value. The user commands the program to search for an interrupt by loading the COMMON location of the specified variable into LOOKX and the target values into XLOOK. Table III is a map of COMMON which the user refers to in order to determine LOOKX. If desired, the search for an interrupt may be delayed until some side condition is met; namely, C(LOOKSW) > SWLOOK, where C refers to COMMON, and LOOKSW and SWLOOK are both input parameters. Whenever an interrupt does occur (i.e., when C(LOOKX)=XLOOK), a printout is issued and the program interrogates the input parameter ENDX to determine whether to continue the current phase (ENDX=0), terminate the current phase and begin the next (ENDX=1), or terminate the entire trajectory (ENDX=-1). After the first interrupt, the search will continue for more interrupts of the same kind unless a minus sign was attached to the LOOKX entry as a trigger to cease searching.

The LOOKX interrupt search feature is programmed to accommodate five simultaneous searches. Thus, LOOKX, XLOOK, ENDX, LOOKSW, and SWLOOK are actually five-element arrays whose values may be set either by the user or the program as follows:

LOOKX (1) always available to the user

LOOKX (2) always available to the user

LOOKX (3) available to the user unless the option of finding the best thrust angle from a set of fixed angles is selected

LOOKX (4) available to the user unless perturbing bodies are involved (n-body problems)

LOOKX (5) available to the user unless the optimal engine on-off timing option is selected

Elements 3, 4, and 5 of these arrays may be filled by the program if certain options are selected so that the user must be careful to avoid interferring with preprogrammed searches when he sets up his input. Ordinarily, no more than two user-selected searches are required, and it is always permissible to use the first two elements of these arrays. The interrupts for normal printout and time-specified phase points do not require the use of the LOOKX search scheme.

Choice of coordinate systems. - The basic coordinate system is a Cartesian inertial system with its origin at the center of the primary gravitational body. It has nó specific reference axis or reference plane. This system is useful for problems that do not refer to NBODY's built-in ephemeris data. For example, if the user wishes to input the departure and arrival points directly without reference to NBODY's built-in ephemeris, this coordinate system is ideal. However, if the user calls on NBODY to supply ephemeris date, the coordinate system is defined by the mean equinox and ecliptic of date - the x-y plane lies in the ecliptic plane of date and the x-axis points toward the mean equinox of date. By modifying subroutine WORBEL, however, the user may redefine the coordinate system. If, for example, he wishes to use the 1950 mean equatorial system, he would simply supply elliptic ephemeris data in that system instead of the system just defined.

Origin shift. - To minimize integration error in n-body problems, it is necessary to shift the origin of the coordinate system occasionally. These shifts take place whenever the vehicle penetrates a body's sphere of influence. Values of the programmed sphere-of-influence radii are given in table I. These shifts translate the origin to the center of the dominant gravitating body while keeping the coordinate axes alined. (There is no rotation of axes.) A printout message is issued each time the origin is shifted, and the integration procedure is restarted.

Orbit element integration. - In many problems where the perturbation forces are relatively small it is advantageous to integrate a set of six orbit elements instead of the rectangular coordinates because fewer integration steps are required for the same accuracy. The NBODY user may select orbit element integration as an option only for nonvariational problems (problems not involving the adjoint equations for optimal thrust control). Instead of integrating the $\dot{R}$ and $\dot{V}$ equations the following set of equations is integrated (ref. 14):

$$\dot{e} = \sqrt{\frac{p}{\mu}} \left[(\sin \nu)\mathscr{R} + \frac{1}{e}\left(\frac{p}{r} - \frac{1 - e^2}{\frac{p}{r}} \right)\mathscr{C} \right] \quad \text{(Eccentricity)} \tag{109a}$$

$$\dot{\omega} = \sqrt{\frac{p}{\mu}} \left[\frac{\sin \nu}{e}\left(1 + \frac{r}{p}\right)\mathscr{C} - \left(\frac{\cos \nu}{e}\right)\mathscr{R} - \left(\frac{r}{p} \sin u \cot i\right)\mathscr{N} \right] \quad \text{(Argument of pericenter)} \tag{109b}$$

$$\dot{\Omega} = \left(\frac{r}{\sqrt{p\mu}} \frac{\sin u}{\sin i} \right)\mathscr{N} \quad \text{(Longitude of ascending node)} \tag{109c}$$

$$\dot{i} = \left(\frac{r}{\sqrt{p\mu}} \cos u \right)\mathscr{N} \quad \text{(Inclination)} \tag{109d}$$

$$\dot{M} = n + \sqrt{\frac{p}{\mu} \left|1 - e^2\right|} \left[\left(\frac{\cos \nu}{e} - 2\frac{r}{p} \right)\mathscr{R} - \frac{\sin \nu}{e}\left(1 + \frac{r}{p}\right)\mathscr{C} \right] \quad \text{(Mean anomaly)} \tag{109e}$$

$$\dot{p} = \left(2r\sqrt{\frac{p}{\mu}} \right)\mathscr{C} \quad \text{(Semilatus rectum)} \tag{109f}$$

where

$$u = \omega + \nu \quad \text{(Argument of latitude)} \tag{110}$$

$$n = \pm\sqrt{\frac{\mu}{p^3} \left|1 - e^2\right|^3} \quad + \text{ if } e < 1, \ - \text{ if } e > 1 \tag{111}$$

Here ν is the true anomaly; n is the mean angular motion; and $\mathscr{R}$, $\mathscr{C}$, and $\mathscr{N}$ are the radial, circumferential, and normal perturbative acceleration components, respectively:

$$\mathscr{R} = a_x(\cos u \cos \Omega - \sin u \sin \Omega \cos i) + a_y(\cos u \sin \Omega + \sin u \cos \Omega \cos i) + a_z(\sin u \sin i) \tag{112a}$$

$$\mathscr{C} = a_x(-\sin u \cos \Omega - \cos u \sin \Omega \cos i) + a_y(-\sin u \sin \Omega + \cos u \cos \Omega \cos i) + a_z(\cos u \sin i) \tag{112b}$$

$$\mathcal{N} = a_x \sin \Omega \sin i - a_y \cos \Omega \sin i + a_z \cos i \tag{112c}$$

Here a_x, a_y, and a_z are the components of the perturbative acceleration along the x, y, and z axes (i. e., the sum of the four rightmost terms of eq. (27)). The true anomaly ν requires solving Kepler's equation iteratively for the eccentric anomaly E (or F)

$$M = E - e \sin E \qquad (e < 1) \tag{113a}$$

$$M = -F + e \sinh F \qquad (e > 1) \tag{113b}$$

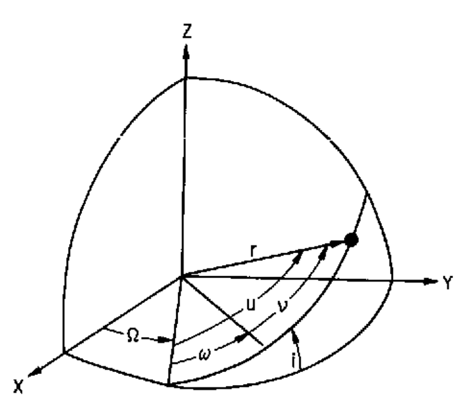
before substituting E (or F) into

$$\cos \nu = \frac{\cos E - e}{1 - e \cos E} \qquad (e < 1) \tag{114a}$$

$$\cos \nu = \frac{\cosh F - e}{1 - e \cosh F} \qquad (e > 1) \tag{114b}$$

This set of orbit element equations experiences numerical difficulties under any of the following conditions: (1) $e \sim 1$; (2) $e \sim 0$, $\mathcal{R} \neq 0$, $\mathcal{C} \neq 0$; (3) $i \sim 0$ and $\mathcal{N} \neq 0$; and (4) whenever a vehicle approaches an asymptote while on a hyperbolic orbit. In these situations the program will temporarily shift from orbit element integration to rectangular coordinate integration until the difficulty subsides. A printout message is issued whenever such an integration shift takes place.

Coordinate transformations. - A transformation from orbit element to rectangular coordinates is sometimes required for numerical reasons and also because the user may wish to input orbit elements but to integrate rectangular coordinates. The transformation equations are given following sketch (g), which illustrates the geometry.



(g)

$$x = r(\cos \Omega \cos u - \sin \Omega \sin u \cos i) \tag{115a}$$

$$y = r(\sin \Omega \cos u + \cos \Omega \sin u \cos i) \tag{115b}$$

$$z = r(\sin u \sin i) \tag{115c}$$

$$\dot{x} = -\sqrt{\frac{\mu}{p}}\,(N \cos i \sin \Omega + Q \cos \Omega) \tag{115d}$$

$$\dot{y} = \sqrt{\frac{\mu}{p}}\,(N \cos i \cos \Omega - Q \sin \Omega) \tag{115e}$$

$$z = \sqrt{\frac{\mu}{p}}\,(N \sin i) \tag{115f}$$

where

$$r = \frac{p}{1 + e \cos \nu} \tag{116}$$

$$N \equiv e \cos \omega + \cos u \tag{117}$$

$$Q \equiv e \sin \omega + \sin u \tag{118}$$

The inverse transformation equations are

$$p = \frac{h^2}{\mu} \tag{119a}$$

$$i = \tan^{-1}\left(\frac{\sqrt{h_x^2 + h_y^2}}{h_z}\right) \tag{119b}$$

$$\Omega = \tan^{-1}\left(\frac{h_x}{-h_y}\right) \tag{119c}$$

$$\omega = \tan^{-1}\left[\frac{z \sin i + (y \cos \Omega - x \sin \Omega)\cos i}{x \cos \Omega + y \sin \Omega}\right] - \nu \tag{119d}$$

$$e = \sqrt{1 + p\left(\frac{V^2}{\mu} - \frac{2}{r}\right)} \tag{119e}$$

$$M = \tan^{-1}\left(\frac{\sin E}{\cos E}\right) - e \sin E \qquad (e < 1) \tag{119f}$$

$$M = -\ln\left(-\sin E + \sqrt{\sin^2 E + 1}\right) - e \sin E \qquad (e > 1) \tag{119g}$$

where

$$h_x = y\dot{z} - z\dot{y} \tag{120a}$$

$$h_y = z\dot{x} - x\dot{z} \tag{120b}$$

$$h_z = x\dot{y} - y\dot{x} \tag{120c}$$

$$h^2 = h_x^2 + h_y^2 + h_z^2 \qquad \text{(Unit angular momentum)} \tag{121}$$

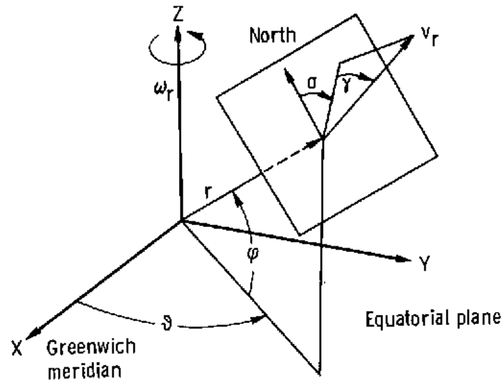
$$\sin E = \frac{\sqrt{1 - e^2}\sin \nu}{1 + e \cos \nu} \tag{122}$$

$$\cos E = \frac{e + \cos \nu}{1 + e \cos \nu} \tag{123}$$

$$\nu = \tan^{-1}\left[\frac{hR \cdot V}{\mu(p - r)}\right] \tag{124}$$

$$r^2 = x^2 + y^2 + z^2 \tag{125}$$

Earth-fixed coordinate frame. - For many launch vehicle problems it is convenient to specify the departure conditions in terms of an Earth-fixed frame of reference. The Earth-fixed equatorial frame is related to a space-fixed frame as shown in sketch (h). The Earth-fixed position vector is specified by the radius r, north latitude φ, and east longitude ϑ.



(h)

It is convenient to translate the Earth-fixed velocity vector V_r to the end of the position vector and project it on the local horizontal. Then it is specified by its magnitude v_r, path angle or elevation angle above the local horizontal γ, and the north azimuth σ as shown in the sketch. The transformation between the Earth-fixed spherical coordinates and the space-fixed Cartesian coordinates is

$$x = r \cos \varphi \cos \vartheta \tag{126a}$$

$$y = r \cos \varphi \sin \vartheta \tag{126b}$$

$$z = r \sin \varphi \tag{126c}$$

$$\dot{x} = v_r(\Phi \cos \vartheta - \cos \gamma \sin \sigma \sin \vartheta) - y\omega_r \tag{126d}$$

$$\dot{y} = v_r(\Phi \sin \vartheta + \cos \gamma \sin \sigma \cos \vartheta) + x\omega_r \tag{126e}$$

$$\dot{z} = v_r(\sin \varphi \sin \gamma + \cos \varphi \cos \sigma \cos \gamma) \tag{126f}$$

where ω_r is the Earth's rotation rate and

$$\Phi \equiv \cos \varphi \sin \gamma - \sin \varphi \cos \gamma \cos \sigma \tag{127}$$

Since this transformation is not the mean-ecliptic and equinox-of-date system, the inclusion of n-body effects is not permitted for launch vehicle problems which use the Earth-centered coordinates for input unless the user alters subroutine WORBEL to redefine the coordinate system, as explained in the previous section Choice of coordinate systems.

Two problems emerge if one attempts to use these Earth-fixed coordinates for a launch vehicle starting from rest and aimed straight vertically. First, if $v_r = 0$, defining the thrust direction relative to the velocity vector results in an undefined thrust direction at lift-off. And, secondly, the lift-off thrust should be alined with the sensible gravity direction, which is not identical to the radial direction ($\gamma = 90^{\circ}$) in the case of an oblate or rotating Earth. To avoid the first difficulty, the launch vehicle is assumed to rise vertically for a short time t_v and atmospheric forces are ignored, which leads to a closed-form solution for the changes in relative velocity Δv_r and radius Δr,

$$\Delta v_r = c_0 \ln\left(\frac{m_0}{m}\right) - gt_v \tag{128}$$

$$\Delta r = v_0 t_v + c_0 \frac{m_0}{\dot{m}_0}\left[1 - \frac{m}{m_0}\left(1 + \ln \frac{m_0}{m}\right)\right] - \frac{1}{2} g t_v^2 \tag{129}$$

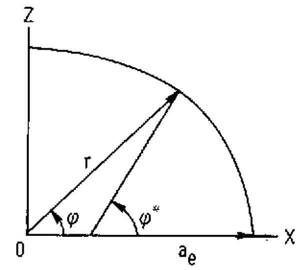
where

$$m = m_0 + \dot{m}_0 t_v \tag{130}$$

$$c_0 = c + \frac{pA_e}{\dot{m}_0} \tag{131}$$

The subscript 0 refers to values at lift-off. The numerical integration is begun just after this short vertical rise with an instantaneous tilt of the velocity vector to the desired path angle γ (generally between 85° and 89.5°) and azimuth σ.

To avoid the second difficulty (vertical direction not identical to sensible gravity direction), small corrections are made to the latitude φ used in the preceding transformation equations so that the rocket will be alined with the sensible gravity direction when $\gamma = 90^{\circ}$. In effect, this helps avoid the problem of having a low-acceleration ($1 \leq a_0/g \leq 1.2$) launch vehicle turn quickly and crash into the ground just because the vehicle's velocity and thrust vectors are not properly alined to the net external force field. The correction for an oblate Earth model is to replace the geocentric latitude φ with a simple approximation to the geodetic latitude φ^* as illustrated in sketch (i).



(i)

$$\varphi^* \cong \tan^{-1}\left\{\left[\frac{\frac{15}{2} J_2\left(\frac{a_e}{r}\right)^2 (\sin^2\varphi - 0.6) - 1}{\frac{15}{2} J_2\left(\frac{a_e}{r}\right)^2 (\sin^2\varphi - 0.2) - 1}\right]\tan\varphi\right\} \tag{132}$$

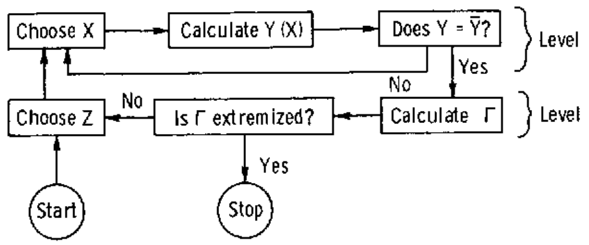
This equation is derived by comparison of the oblate potential function written in the geocentric framework (eq. (29)) with a similar function written in the geodetic system, ignoring terms higher than J_2. Here J_2 is the second zonal harmonic coefficient and a_e is the Earth's mean equatorial radius. The correction for centrifugal force is

$$\Delta\varphi \cong \frac{\omega_r^2 r \cos\varphi \sin\varphi}{g} \tag{133}$$

If both effects are present, φ is replaced by $\varphi^* + \Delta\varphi$ when applying the transformation equations.

LEVEL 2 - DIRECT OPTIMIZATION OF VEHICLE AND MISSION PARAMETERS

After the solution to the boundary-value problem (assuming there is one) is obtained $(Y = \overline{Y})$, the program control passes to the level 2 optimizer if there are any additional vehicle or mission parameters to be optimized. The level 2 optimizer is a general-purpose iterator that extremizes a user-specified payoff criterion Γ over a field of user-specified variables Z. It operates on one variable at a time in the same fashion as the univariate search scheme (in fact, the same subroutine is utilized for both schemes). Each time it changes one of the Z variables, the boundary-value problem of level 1 must be resolved, as illustrated in sketch (j).



(j)

The level 2 search cycles over each Z variable in sequence, records the extremum of Γ for the first complete iteration, and then repeats the complete iteration a second time. The Γ value at the end of the second complete iteration is compared with the value obtained from the first iteration and the entire process repeated until

$$\left| \frac{\Gamma_{i+1} - \Gamma_i}{\Gamma_{i+1}} \right| < 0.001 \tag{134}$$

where i refers to values obtained after a complete iteration cycle on all variables.

Each time Z is changed in level 2, the level 1 independent variable vector $\bar{X}$ is also estimated with the functional form $\bar{X} = \bar{X}(Z)$ by using a linear extrapolation. In this regard, it is desirable to avoid such large changes in Z that the resulting $\bar{X}$ estimate is so poor that it hinders convergence of the level 1 boundary-value problem. Thus, ΔZ is constrained in the sense that if ΔZ produces an initial level 1 error τ greater than 0.3, Z is reduced until this constraint is satisfied.

The user specifies the payoff criterion Γ by loading its COMMON location into IBB. The most frequently used criterion are given in the following table:

Typical level 2 payoff criterion, Γ	COMMON location (for IBB)
Final mass, m_f	2159
Net spacecraft mass, m_n/m_{ref}	437
Payload ratio for launch vehicle problems, m_n/m_0	437

The latter two criterion have the same IBB location since they are both calculated with equations (2) and (15), although $m_0/m_{ref} = 1$ for launch vehicles.

The user selects the level 2 independent variable list Z by loading the COMMON locations of his chosen set into the IAA array during input. Thus, if C denotes COMMON storage, Z = C(IAA). A list of likely candidates for Z is given in the following table, along with their COMMON locations:

Typical level 2 independent variable, Z	COMMON location (for IAA)
Electric vehicles:	
Specific impulse, I	418
Initial mass flow rate, $\dot{m}_0$ } Only one of	383
Initial thrust-weight ratio, a_0/g } these may	408
Initial electric power level, P_0 } be selected	397
Launch vehicle burnout velocity, v_l	429
Vehicle velocity just prior to capture retro-fire, v_r	430
Departure date, t_0	11
Mission time, t_f	1
Launch vehicles:	
Stage firing times, $(t_f)_i$	1, 2, . . ., 10
Elevation angle at launch, γ	48
Either vehicle: Desired final conditions in level 1, $\overline{Y}$	866, . . ., 875

Any or all of the set I, $\dot{m}_0$, v_l, v_r may be optimized either in level 2 or in level 1 if the payoff criterion is net spacecraft mass ratio. Level 1 is recommended in this case because it is faster and more accurate. Choosing any other independent variables or payoff criterion requires the use of the level 2 optimizer.

SWEEP SCHEMES FOR RUNNING SIMILAR CASES

Studies frequently require a set of answers over a range of some parameter s. The basic problem that arises in generating such a set of answers is to make reasonable estimates of the independent variables (X and Z) as s is "swept" from s_1 to s_n. If s is varied too quickly, the successive boundary-value solutions $\overline{X}$ cannot be estimated with sufficient accuracy to avoid nonconvergence or unacceptably slow convergence. And if s is varied too slowly, much computer time will be wasted solving intermediate problems. Thus, the successive values of s must be chosen carefully to avoid both computational difficulties. Two sweeping schemes are programmed which have the following descriptions:

Manual sweeping scheme	Automatic sweeping scheme
User must guess each new s.	Program guesses each new s.
Sweep is terminated if Δs is oversize.	Sweep recovers from oversize Δs.
Output occurs for every value of s.	Output occurs on selected values of s.
Scope includes levels 1 and 2.	Scope includes level 1 only.
Program estimates $\bar{X}$ and $\bar{Z}$.	Program estimates $\bar{X}$.

The main advantage of the manual scheme is that it covers both level 1 and level 2, instead of just level 1 as is the case with the automatic scheme. However, the automatic scheme is much more convenient to use and is recommended in all cases except those involving level 2 optimization.

Manual Sweeps

In this method a group of cases are executed sequentially and the user selects each new value of the sweeping parameter s. This is implemented by submitting a separate data set for each case, as illustrated in figure 2. The first case consists of all the usual data plus a value for NSWEEP which is the COMMON location of the sweep parameter s. The only data entered for the subsequent cases are new values of s. The program will execute the first case and then the second case starting with the converged solution ($\bar{X}_1$ and $\bar{Z}_1$) of the first case. Since the manual sweep cannot recover if any of the cases fails to converge, the best policy is to select a small increment in s for the second case (e.g., $s_2 = 1.01\ s_1$). The third and remaining cases are started with a linear extrapolation of the two previous solutions for $\bar{X}$ and $\bar{Z}$. The user increases Δs to whatever value he believes is satisfactory and may expect several trial-and-error attempts in sensitive problems if he chooses large steps in s. It is often more productive to pick small increments in s and accept the extra output and computer execution time.

If the entire sweep is executed to completion, a second sweep may be initiated where the first sweep terminated by resetting the value of NSWEEP in the first data set of the second sweep. Additional sweeps may be appended to the second sweep in a similar fashion.

Automatic Sweeps

The automatic sweep scheme eliminates the guesswork in selecting the Δs increments. The user only needs to specify which variable is to be "swept" and the particular values of s for which he desires a full trajectory printout. Only a single data set is required for this scheme, as shown in figure 3. The program will execute the first problem and then proceed to sweep s toward the first printout value s_1. All intermediate s-values will be noted in the output, but only s_1 (and subsequently $s_2, s_3, \ldots, s_n$) will trigger a full trajectory printout. The program attempts to select Δs increments that will result in an initial boundary-value problem error of 0.1 each time s is changed. If the error is greater than 0.3 or the boundary-value problem iteration falters, the iteration is terminated and a smaller Δs is selected before reiteration. Thus, this scheme recovers from poor estimates of Δs. As with the manual scheme, the $\overline{X}$ estimates are produced with a linear extrapolation of the previous two solutions. As an additional option, the user may override the linear extrapolation with an n^{th}-order least-squares curve-fit extrapolation of the previous m solutions by also inputting

MORDER order of the curve fit, n

MAXPTS number of points used in the curve fit, m

Experience has shown that the linear extrapolation (and sometimes a quadratic) usually is more productive overall than high-order extrapolations. Finally, the automatic sweep scheme cannot be used in cases of level 2 optimization - only the manual scheme can.

Multidimensional Sweeps

The sweeping method is also very useful in obtaining the solution of a boundary-value problem when one does not have a reasonable estimate of the solution vector $\overline{X}$, but does have the solution to a related problem. In such a case, the related problem can often be transformed into the sought problem by a continuous transformation of the set of independent variables S that differ between the two problems.

The manual sweeping scheme can handle this situation, albeit rather awkwardly, simply by performing multiple sweeps in tandem. Alternatively, the user may vary the entire set of parameters S in parallel by changing each element of S in each data set of a single sweep. The extrapolation of $\overline{X}$ and $\overline{Z}$ will be based on only one element (specified by NSWEEP), however, so this method is prone to failure unless the user is very careful in choosing successive values of S.

The automatic sweeping scheme is better equipped to handle a multidimensional sweep. The user loads the COMMON locations of all elements of S into the IAA vector and the corresponding sought values of S into the SVALUE vector. This is a simple extension of the definitions given in figure 3 for a single-dimensional sweep except that SVALUE represents a set of single values for n parameters instead of n values of a single parameter. The program automatically sweeps all elements of S simultaneously to the target values loaded in SVALUE. It does this by the linear transformation

$$S = S_1 + l_s(S_2 - S_1) \qquad 0 \leq l_s \leq 1 \tag{135}$$

where S_1 is the initial value of S loaded as normal input, S_2 is the sought value of S loaded in SVALUE, and l_s is a scalar. The program solves the initial problem for S_1 and then sweeps l_s from 0 to 1, which completes the multidimensional parallel sweep since $S = S_2$ when $l_s = 1$. Other transformations may be better than this linear one in situations where a constraint (such as maintaining a circular orbit) preserves the similarity of the problems, but this one is general and works reasonably well in most cases.

PROGRAM DESCRIPTION

PROGRAM STRUCTURE

The entire NBODY program is written in the FORTRAN IV (7090 compiler, version 5) language and occupies about 22 000 core storage locations on an IBM 7094. Peripheral equipment is assigned as follows:

(1) Logical unit 5: All input is taken from this unit from a single READ command in the main program.

(2) Logical unit 6: All output is written on this unit from many of the subroutines.

Most of the variables that are transferred between NBODY's 35 subroutines are located in "labeled COMMON blocks" as follows:

COMMON block name	Description of variables in block
TIME	time-related variables such as departure time, mission time, and integration step size
FIXED	fixed physical constants such as π and g
ENTER	input variables assigned either true or false values
LAT	input variables for the Earth-fixed spherical coordinate system option

COMMON block name	Description of variables in block
LOOK	variables related to the interrupt search scheme
CASES	bookkeeping variables for running successive cases
OUTPUT	output option variables
LOCATE	indexes that give locations of variables relative to the beginning of COMMON
IGRATE	integration scheme controls and the increments Δy
COFV	variables associated with the optimal thrust control formulation
ROCKET	variables that describe the vehicle
TRAJEC	variables that describe the trajectory
ITERAT	iteration scheme control variables
BODIES	variables that describe the gravitational bodies
AERODY	variables associated with aerodynamics
SAVE	bookkeeping variables that must be saved each time a trajectory is repeated
HD	values of the integration variables at the current time (at time t_n in eq. (99)) in double precision
H	values of the integration variables at the current time plus any Runge-Kutta subinterval increment (at time t_n, $t_n + (h/2)$, or $t_n + h$ in eq. (100))
HDOT	derivatives of the integration variables

The program's built-in flexibility regarding free choices of optimization variables, criterion of merit, interrupt parameters, sweep parameters, and so forth, is implemented by specifying the COMMON locations of these variables. Hence, it is important that these labeled COMMON blocks be loaded in the same sequence as just given. This loading will be handled automatically by many computer software packages, but in others it is necessary to always load the main program (or block data subprogram) first just to ensure the proper loading sequence. The user is also cautioned against changing these COMMON blocks without also changing the prestored indexes in LOCATE. It is generally recommended that any user-supplied additional COMMON be appended after the last block (HDOT) or defined as "unlabeled COMMON." Appendix B is a glossary of the variables appearing in COMMON, along with their relative locations.

An overall flow diagram of the program is given in figure 4. The user's input data set is read by a NAMELIST-type read command in the main program. Control is then passed to subroutine WSTAGE, which initiates phase 1 (same as vehicle stage 1 in many cases) by supplying the appropriate phase data such as initial mass, specific impulse, and so forth, to the integrator. After more initializing in subroutines WORDER and WBEGIN, the trajectory integration is carried out by subroutine WINTEG. The derivatives of the integration variables are computed in WDERIV, the time step sizes are calculated in WSTEP, and the relative integration error is evaluated in WERROR. After the phase 1 trajectory arc is computed, control is returned to WSTAGE for the initialization of phase 2 and it, and any remaining trajectory phases, are computed in a similar manner. After the last trajectory phase is computed, control is passed to subroutine WOPT, which controls the iteration of the boundary-value problem, the level 2 optimization schemes, and the automatic sweep scheme. The program control is passed back to WSTAGE each time a new trajectory must be computed during these processes. When level 1, level 2, and any automatic sweep are all completed, control is finally sent back to the main program for the next case's input data set (if any). The main program also performs the extrapolation on the level 1 ($\overline{X}$) and level 2 ($\overline{Z}$) independent variables if the manual parameter sweep option is selected.

There are many other subprograms that perform specific tasks, and appendix B provides a definition of every subprogram's function. The small TIMLFT routine is of particular concern since it would probably be deleted or rewritten at other installations. It is a convenience routine for batch sequence operation that warns the program when its allotted execution time is almost over. Thus, some useful information can be extracted before an imminent termination by triggering a final trajectory printout. A complete subprogram call sequence diagram is given in figure 5.

INPUT

The input data sets are read by a single NAMELIST read command in the main program, and successive cases may be stacked in tandem indefinitely. All variables are input in SI units using floating-point, single-precision format unless otherwise noted. In the list of operating instructions that follows, the input variable names are written entirely in capital letters. The default value of all variables is zero (or F, false, for logical variables) unless otherwise noted. The dimensionality of the coordinate system is specified as follows:

NDEM=2 two-dimensional model (default value)

=3 three-dimensional model

The set of gravitational bodies is specified by a list of indexes:

NUMBOD=index of the origin body, index of the first perturbing body, . . ., index of n^{th} perturbing body ($0 \le n \le 6$); default value: 1, 6*0

The first index refers to the origin body at the departure date, and the remaining indexes are all of perturbing bodies in random order. The vehicles initial coordinates are referenced to the origin body. The permissible indexes and corresponding body names are

1	Sun	7	Saturn
2	Mercury	8	Uranus
3	Venus	9	Neptune
4	Earth-Moon	10	Pluto
5	Mars	11	Earth
6	Jupiter		

The physical model for the Earth may be selected as follows:

OBLATE=T oblate Earth model

=F spherical Earth model (default value)

ROTATE=T rotating Earth

=F nonrotating Earth (default value)

The atmospheric Earth model is automatically programmed for the 1962 U.S. Standard Atmosphere. Altering this model or adding another planet's atmosphere requires reprogramming subroutine WICAO.

Vehicle Model

The program provides the capability to simulate an n-stage vehicle ($1 \le n \le 10$). The term "stage" really refers to "trajectory phase" since a "stage" change does not necessarily mean that a vehicle stage is discarded. It may only mean that the thrust steering control is switched from a tangential program to an optimal program, for example. The vehicle related inputs are as follows ($1 \le i \le 10$):

VMASS(i)>0 initial mass of stage i, m_0, kg (default value: 1, 9*0)

VMASS(i) =0	vehicle mass is continuous between stage i - 1 and stage i
<0	vehicle mass decreases between stage i - 1 and stage i by the amount specified for stage i, m_0, kg
ISP(i)	vacuum specific impulse, I, sec
TB(i)>0	stage flight time, t_f, sec
<0	total flight time of i stages, $\sum_{j=1}^{i} \left(t_f\right)_j$, sec
NOPT(i)	preprogrammed optimal-thrust-control end condition options (see table IV for a summary or preceding text for complete discussion)
Choose only one: PFLOW(i)	propellant flow rate at 1 AU, $-\dot{m}_0$, kg/sec
Choose only one: TW(i)	initial thrust-weight ratio (at 1 AU), a_0/g
Choose only one: POWER(i)[1]	initial electric power, P_0, kW
SOLAR=T	propulsion power depends on solar distance (eq. (8))
=F	propulsion power is constant (default value)
KE	propellant tankage factor, k_t
STRUCT	structural mass factor, k_s
ALFPOW	specific mass of electric propulsion system, α_{ps}, kg/kW
BE, DE	overall powerplant efficiency η factors, b and d (default value: BE=0.75, DE=14 350.)
DISPO=T	electric propulsion system and tankage mass are jettisoned just prior to high-thrust retromaneuver (j = 1)
=F	electric propulsion system and tankage mass are not jettisoned prior to high-thrust retromaneuver (j = 0) (default value)

The last six entries are normally used only for single-stage electric vehicles. For n-stage vehicles, they are applicable to the entire flight as a whole (e.g., the tankage factor k_t is applied to all stages taken together). The number of stages is taken to be the number of nonzero flight times that are inputted.

---

[1]This option is valid only when using the analytical launch vehicle simulation and requires that NOPT be equal to 0, 6, or 7. Also instead of inputting VMASS, the reference mass m_{ref} must be loaded into BOOSTM.

The following group of inputs is required only if aerodynamic forces are to be included in the simulation:

REFA(i) aerodynamic reference area of i^{th} stage, S_{ref}, m^2

AEXIT(i) engine exit area, A_e, m^2

CD0C set of parasite drag coefficient data (eq. (24)); M_1, $(a_1, a_2, a_3)_1$, M_2, $(a_1, a_2, a_3)_2$, M_3, . . ., M_n; the coefficients $(a_1, a_2, a_3)_i$ apply to the Mach number interval (M_i, M_{i+1})

CDIC set of induced drag coefficient data (eq. (25)); M_1, $(a_4, a_5, a_6)_1$, M_2, $(a_4, a_5, a_6)_2$, M_3, . . ., M_n

CLC set of lift coefficient data (eq. (26)); M_1, $(a_7, a_8, a_9)_1$, M_2, $(a_7, a_8, a_9)_2$, M_3, . . ., M_n

The user may install his own method of handling the aerodynamic data by modifying subroutine WAERO.

Analytic Spiral Escape Maneuver at Departure

A tangential-thrust spiral escape from a departure planet circular orbit will be simulated for electrically propelled vehicles (eqs. (19) and (55)) if the following are input:

SPIR=T

VC1 speed in initial circular orbit, $v_{c,l}$, m/sec

Analytic High-Thrust Departure of Electric Vehicle

The launch vehicle is assumed to impart a speed v_l to the electric vehicle at a distance r_l from the departure planet's center. The inputs are

VB1 launch vehicle's burnout speed, v_l, m/sec

RRAT1 departure planet's sphere-of-influence radius ratio, $r_l/r_{s,d}$

VC1 circular orbit speed at r_l, $v_{c,l}$, m/sec

VJET1, K1 curve-fit parameters defining launch vehicle's performance (eq. (16)), c_l, m/sec, and k_l

Analytic High-Thrust Capture Retromaneuver of Electric Vehicle

If an electric vehicle is to be braked into a planetary capture orbit at the arrival planet with a high-thrust retrorocket, input the following:

VB2	planetocentric vehicle speed just before retrofire at periapsis radius r_r, v_r, m/sec
RRAT2	arrival planet's sphere-of-influence radius ratio, $r_r/r_{s,a}$
VC2	circular orbit speed at r_r, $v_{c,r}$, m/sec
VJET2	retrojet exhaust speed, c_r, m/sec
K2	retropropulsion tankage factor, k_{rt}
ECC2	eccentricity of capture ellipse, e_r

Departure Time

The departure time t_d need only be specified in problems involving ephemerides. It is input as a Julian date in Greenwich time as follows:

DTOFFJ	whole Julian day number (default value: 2 440 000.)
TOFFT	fraction of Julian day

Initial Position and Velocity

The vehicle coordinates at departure may be specified in any of these sets:

(1) Rectangular coordinates (double-precision variables):

R	x, y, z components of position vector R_0, m
V	x, y, z components of velocity vector V_0, m/sec

(2) Orbit elements (double-precision variables, sketch (g):

E	eccentricity, e
OMEGA	argument of pericenter, ω, rad
NODE	longitude of ascending node, Ω, rad
INCL	orbit inclination to reference plane, i, rad
MA	mean anomaly, M, rad

RECTUM semilatus rectum, p, m

(3) Earth-fixed spherical coordinates (sketch (h)):

LAT northern latitude, φ, deg

LONG eastern longitude from Greenwich, ϑ, deg

ALT0 altitude above sea level, $r_0 - r_e$, m

VEL0 relative velocity, v_r, m/sec

ELEV elevation angle, γ, deg

AZI azimuth, eastward from north, σ, deg

TKICK duration of short, vertical, nondrag ascent to facilitate starting (eq. (129)), t_v, sec

Alternatively, the user may instruct the program to use ephemerides to compute the departure (and desired arrival) coordinates. This option is intended for Sun-centered two-body problems only where the departure coordinates are taken to be identical to the specified departure planet coordinates (and likewise for the arrival conditions). The option is invoked by setting

EPHEM=T

NUMBOD=1, index of departure planet, index of arrival planet

The program will not consider this an n-body problem even though the two planets would normally be considered perturbing bodies by the previous definition of NUMBOD. The program will compute rectangular coordinates for both end points (i. e., values of R and V for departure option 1 and similar values for DESIRE as defined later).

Thrust Program Options

The previously defined array NOPT determines whether a variational thrust program (NOPT(i)≠0) or a nonvariational thrust program (NOPT(i)=0) applies to the i^{th} stage. The inputs depend on which of these two types is selected:

(1) Nonvariational thrust program (NOPT(i)=0):

ALFCOE a set of thrust angle coefficient data (eq. (42)); t_1, $(a_{10}, a_{11}, a_{12})_1$, t_2, $(a_{10}, a_{11}, a_{12})_2$, . . ., t_n; the coefficients $(a_{10}, a_{11}, a_{12})_i$ apply to the time interval (t_i, t_{i+1}); if $a_{11} = a_{12} = 0$, the thrust angle α equals a_{10}.

ALPHAC=T thrust angle α is referenced to local horizontal

=F thrust angle α is referenced to velocity vector

BETA out-of-orbit-plane thrust angle (sketch (c)), β, deg

If the user prefers another method of specifying the thrust program, he may do so simply by modifying subroutine WVREL.

(2) Variational thrust program (NOPT(i)≠0):

COAST=T coast arcs permitted (default)

=F coast arcs not permitted

KBODYS number of gravitating bodies included in variational equations (It may be desirable to limit this number to 1 even though n bodies affect the equations of motion; default value: 1.)

LAMDA seven element array of initial values of the adjoint variables (Lagrange multipliers): the three components of Λ ((kg)(sec)/m), the three components of Λ_r (kg/m), and λ_m

As an alternative to inputting LAMDA, the following set of variables may be input for two-dimensional problems only (sketch (d) and eqs. (74) to (79)):

PS initial thrust direction relative to x-axis, ψ_0, deg

DPS time derivative of thrust angle, $\dot{\psi}_0$, deg/sec

KAPPA thrust on-off switching function, κ_0

DKAPPA time derivative of on-off switching function, $\dot{\kappa}_0$, sec^{-1}

LAM scale factor, λ_0, (kg)(sec)/m (default value: 1.)

The program will always use the latter set if PS≠0. If the thrust angle at any given moment is to be picked from a specified set of angles α_i instead of varying continuously, input

ALF set of angles α_i ($i \leq 5$) referenced according to the value of ALPHAC, deg

Trajectory Integration Controls

The input initial coordinates for any problem may be (1) rectangular coordinates, (2) orbit elements, or (3) Earth-fixed spherical coordinates as previously explained. Regardless of which set of input coordinates is selected, the user may also choose between rectangular coordinate integration or orbit element integration for nonvariational problems. Only rectangular coordinates may be integrated for variational problems. The input coordinates - integration coordinates option is defined by MODEI as follows:

Trajectory integration coordinates	Input coordinates		
	Rectangular	Orbit elements	Earth fixed
Rectangular	MODEI=2 (default)	MODEI=-1	MODEI=4
Orbit elements (nonvariational only)	MODEI=-2	MODEI=1	MODEI=-4

The following controls affect the accuracy and efficiency of the trajectory integration:

EREF reference relative error, $\bar{\delta}_r$ (default value: 10^{-4})

ERLIMT limit relative error, δ_{limit} (default value: 3×10^{-4})

STEP(i) initial step size for i^{th} stage, $\left(h_1\right)_i$, sec (default value: $t_f/100$)

Output Controls

For problems that involve neither a level 1 boundary-value problem iteration nor a level 2 optimization search, the user selects the frequency of trajectory printout as follows:

Select only one:

- STEPS number of trajectory integration steps between printouts
- DELMAX time interval between printouts, sec (default value: DELMAX = 50 days)

By default, the first and last trajectories of level 1 and level 2 iteration sequences will be printed out in full, and a one-line summary will be printed out for each intermediate trajectory. After inspecting a computer run, it is occasionally desirable to repeat the run with a request for more trajectories to be printed out in full (to examine odd behavior, for example). This request will be fulfilled if the following is input:

NOUT = $n_1, n_2, \ldots, n_l$ ($l \le 5$), where each n_i is the sequence number of the specific trajectory for which printout is desired. These sequence numbers appear as the leftmost entry in the one-line summary printouts (default value: NOUT = 1, 4*0).

Trajectory Interrupt Controls

As explained in the section Trajectory interrupt (p. 44), the trajectory may be interrupted occasionally in order to take some specific action. The program may do this automatically in some cases (such as when the engine is turned on or off), but the user may also cause this to happened by inputting the following:

LOOKX(i) location relative to COMMON of interrupt parameter i entered in fixed point format; table III contains a map of these locations; a minus sign on LOOKX(i) will cause the interrupt search to terminate after the first interrupt, otherwise interrupts will continue to occur each time XLOOK(i)=C(LOOKX(i)), where C=COMMON (default value: consult text)

XLOOK(i) value that interrupt parameter i must attain to trigger an interrupt

ENDX(i)=-1 flight is terminated at interrupt

=0 flight continues after interrupt (default value)

=1 stage is terminated, but flight continues

If the interrupt search is to be delayed until an arbitrary criterion $y > \bar{y}$ is satisfied, input

LOOKSW(i) location relative to COMMON of the delay parameter y entered in fixed-point format (default value: location of time, t)

SWLOOK(i) value that the delay parameter must exceed before interrupt may occur, $\bar{y}$

All these interrupt inputs are five-element arrays. The first two elements are always available to the user, but the latter three may not be, as explained in the Trajectory interrupt section.

Level 1 Boundary-Value Problem

The program will recognize that a two-point boundary-value problem exists if the following are input:

IA(i) COMMON location of the i^{th} independent variable x_i in fixed-point format $(0 \leq i \leq 10)$

IB(i) COMMON location of the i^{th} dependent variable y_i in fixed-point format $(0 \leq i \leq 10)$

DESIRE(i) desired value of the i^{th} dependent variable $\bar{y}_i$ $(0 \le i \le 10)$

WEIGHT(i) weighting factor of the i^{th} residual, w_i $(0 \le i \le 10)$; (default value: $\bar{y}_i$ if $\bar{y}_i \neq 0$, 1.0 if $\bar{y}_i = 0$, 360 if $\bar{y}_i = 0$ and y_i is path angle)

TOLER convergence criterion, $\bar{\tau}$ (default value: 10^{-4})

ERSTAR relative error value above which the univariate search scheme is used and below which the linear corrective scheme is used, τ^* (default value: 1.0)

NBVP trajectory phase number where boundary-value problem begins in fixed-point format (default value: j, where j is the number of the first stage having NOPT(j)≠0)

MAXNUM maximum number of trajectories allowed before execution is terminated (default value: 500.)

The IA and IB vectors are filled automatically by the program if $1 \le |NOPT| \le 5$, as indicated in table IV. For other cases, the COMMON locations may be selected from table II or table III. Also, DESIRE is calculated by the program as the arrival planet's velocity and position if EPHEM=T, as explained in the section Initial Position and Velocity.

Occasionally, the situation arises that successive iterations fluctuate between n coast phases and m coast phases. Convergence difficulty is often experienced in the region of such a boundary, especially when finite difference partials are used. This type of difficulty is avoided if solutions are sought away from such a boundary and an extrapolation is accepted in the boundary's immediate vicinity. An alternative method that sometimes works is to ignore phase shifts near the boundary by setting TSKIP equal to t_1, t_2 (phase shifts are ignored in the time interval (t_1, t_2), sec) until convergence is obtained and then releasing this constraint (TSKIP=0, 0) to determine whether n or m phases are optimal.

If any of the vehicle-related variables $\dot{m}_0$, c, v_l, or v_r are to be optimized in level 1, the appropriate COMMON locations are automatically loaded into IA and IB vectors simply by inputting

OPTA=T for optimum $\dot{m}_0$ (or its equivalent, f/m_0g)

OPTC=T for optimum c

OPTVB1=T for optimum v_l

OPTVB2=T for optimum v_r

Level 2 Optimization

User-specified variables z_i will be optimized in level 2 if the following are input:

IAA(i) COMMON location of the i^{th} optimization variable z_i ($0 \leq i \leq 10$)

IBB COMMON location of the extermal criterion Γ (default value: location of payload)

TOL2 relative tolerance on Γ to be satisfied for convergence: positive for a maximization problem, negative for a minimization problem (default value: 0.001)

MAXNUM maximum number of iteration trajectories allowed before execution is terminated (total of level 2 and level 1, if any; default value: 500)

PERT2(i) initial perturbation size for z_i ($0 \leq i \leq 10$), expressed as a fraction of z_i (default value: 0.001)

Parameter Sweeps

For manual sweeps the user simply inputs successive data sets in tandem, as shown in figure 2, and identifies the sweep parameter s in the first data set:

NSWEEP COMMON location of the sweep parameter s (see table IV for likely candidates)

For automatic sweeps on a single parameter s, the user inputs

IAA COMMON location of the sweep parameter s

SVALUE(i) sequential set of values of s for which a full trajectory printout is desired ($1 \leq i \leq 10$)

MAXPTS=2

For multidimensional automatic sweeps on n sweep parameters, input the following:

IAA(i) COMMON locations of the sweep parameters s_i ($1 \leq i \leq 10$)

SVALUE(i) desired set of s_i values; each SVALUE(i) corresponds to IAA(i)

MAXPTS=2

The automatic sweep schemes may be applied only to level 1 (not level 2). The estimation procedure of the level 1 independent variable array X is defaulted to a linear extrapolation of the previous two solutions. The order and number of data points used in this procedure may be changed as explained in the section Automatic Sweeps.

PROGRAM OUTPUT

The frequency of output is controlled by the input variables NOUT, STEPS, and DELMAX, as explained in the input instructions. Each trajectory is noted on the print-out in either (1) a full output mode or (2) a one-line summary mode.

Full Output Mode

The full output mode produces information blocks at specified intervals of flight time (DELMAX) or integration step number (STEPS) with the following format:

STEP= +	ECCENTRICITY=	OMEGA=	V=	R=	REFER=
TIME=	SEMILATUS R. =	TRU A=	VX=	X=	RMASS=
DAYS=	MEAN ANOMALY=	NODE=	VY=	Y=	REVS. =
ALFA=	PATH ANGLE=	INCL=	VZ=	Z=	DELT=

STEP	current integration step number to the left of the plus sign and a count of the step-size cutbacks on the right
TIME	current flight time, t, sec
DAYS	flight time, t, days
ALFA	angle between thrust and velocity vectors, α, deg
ECCENTRICITY	orbit eccentricity, e
SEMILATUS R.	semilatus rectum of orbit, p, m
MEAN ANOMALY	mean anomaly, M, rad
PATH ANGLE	path angle, γ, deg
OMEGA	argument of pericenter, ω, rad
TRU A	true anomaly, ν, rad
NODE	longitude of ascending node, Ω, rad
INCL	orbit inclination, i, rad
V, VX, VY, VZ	velocity and its components, V, m/sec

R, X, Y, Z	radius and its components, R, m
REFER	name of reference body followed by integration mode
RMASS	vehicle mass, kg
REVS.	revolutions past x-axis
DELT	current integration step size, h, sec

In the case of atmospheric flight the following two lines are appended to the preceding:

BETA=	R PATH ANGLE=	DRAG=	VR=	G=	PUSH=
ALT. =	MACH NUMBER=	LIFT=	CD=	Q=	HEAT=

BETA	out-of-plane thrust angle, β, deg
ALT.	altitude, m
R PATH ANGLE	path angle relative to Earth (may be rotating), deg
MACH NUMBER	Mach number, M
DRAG, LIFT	drag and lift acceleration magnitudes, $\lvert D \rvert/m$, l/m; m/sec^2
VR	velocity relative to rotating Earth, V_r, m/sec
CD	drag coefficient
G	net force acting along longitudinal axis of vehicle, Earth g's
Q	dynamic pressure, q, N/m^2
PUSH	thrust acceleration magnitude, a, m/sec^2
HEAT	heating rate per unit mass, $W/m^2/sec$

In the case of an n-body problem, additional lines of printout give the vehicle-to-perturbing-body position vectors in terms of their magnitudes in meters, followed by the three x, y, z direction cosines (e.g., EARTH R=9.25E8 0.580 0.743 0.335). In the case of variational thrust steering programs, the following two lines are added to the basic output block:

PSI=	DPSI=	THETA=	DK=	K=	L7=
L1=	L2=	L3=	L4=	L5=	L6=

PSI, DPSI	thrust angle relative to x-axis, ψ, deg; and its derivative, $\dot{\psi}$, deg/sec
K, DK	engine on-off switch function, κ, and its derivative, $\dot{\kappa}$, sec^{-1}
THETA	central travel angle, θ, deg

L1, . . ., L7 Lagrange multipliers (adjoint variables): the components of Λ and Λ_r, and also λ_m

The full trajectory output occurs for the first trajectory and the converged solution trajectory. During automatic sweeps, full output will occur for each converged solution that corresponds to the SVALUE list.

One-Line Summary

Each trajectory generated during the level 1 boundary-value iteration is noted in a one-line summary table. The table heading is

RUN N   ERROR   TIME   INDEPENDENT VARIABLES -- DEPENDENT VARIABLES

The trajectory number is listed under RUN, the number of engine on-off switch points under N, the level 1 boundary-value error τ under ERROR, and flight time in seconds under TIME. The remaining columns list the values of the independent variable vector X and the dependent variable vector Y. During sweeps or level 2 optimizations, this table is interrupted each time a solution is attained with a one-line notation of the current IAA and IBB values. Finally, the following letters may appear between the RUN and N columns:

E indicates the beginning of a univariate search sequence

N indicates that a new partial derivative matrix G is being generated for the linear correction scheme

P indicates that both search schemes have bogged down and that control will now pass to the linear correction scheme without inhibitor ($\chi = 1.0$) in a last-ditch effort to achieve convergence

There are other printout messages that are intended to be self-explanatory.

EXAMPLE PROBLEMS

EXAMPLE 1 - JUPITER RENDEZVOUS USING THE MULTIDIMENSIONAL SWEEP FEATURE

This example illustrates how the multidimensional sweep method can be used to assist in finding the solution of a problem. The mission is a 500-day Jupiter rendezvous commencing in a circular orbit at 1 AU. The heliocentric travel angle is fixed at 133°. The spacecraft's initial thrust-weight ratio is 2×10^{-4}, the specific impulse is 5000 seconds, coasting is permitted, and the thrust is constant. The final conditions being sought are

(1) Radius, 7.778×10^{11} meters (Jupiter's distance from Sun)

(2) Velocity, 13 062.5 meters per second (Jupiter's circular orbit speed)

(3) Path angle, 0° (rendezvous condition)

(4) Travel angle, 133° (assumed)

Suppose we already know the optimum-thrust-angle solution to a similar problem; namely, one that has the same vehicle parameters but different target conditions:

(1) Radius, 7.0528×10^{11} meters

(2) Velocity, 16 570 meters per second

(3) Path angle, 30°

(4) Travel angle, 138°

Since these conditions (especially the path angle) are significantly different than the sought conditions, we may expect trouble if we straightforwardly attempt to begin our search with the same set of adjoint variables. Therefore, we use the multidimensional sweep scheme to gradually transform the known solution to the sought solution. The input is as follows:

TB=4.32E7	mission time, sec
R=1.49597893E11, 0, 0	initial position vector on x-axis, m
V=0, 29784.7, 0	initial position vector in y-direction
VMASS=1000	initial vehicle mass, kg
TW=2.E-4	initial thrust-weight ratio
ISP=5000	specific impulse, sec
COAST=T	coast arcs permitted
STEPS=100	output every 100th integration step
NOPT=2	fixed-travel-angle rendezvous option
SVALUE=7.778E11, 13062.5, 0, 133	sought values of final conditions
DESIRE=7.0528E11, 16570, 30, 138	current values of final conditions
WEIGHT(3)=365	better weighting factor for γ than the default value of 30

LAMDA=3.3431, 4.40876, 0, 1.072E-6, 3.70E-7, 0, 63.3065	correct solution values of the adjoint variables for DESIRE
IAA=866, 867, 868, 869	COMMON locations of the sweep variables (the DESIRE array)
MAXPTS=2	number of points used to extrapolate $\overline{X}$

For simplicity, this input list contains no vehicle model variables other than those necessary to generate a trajectory. High-thrust chemical propulsion options have also been ignored - for the same reason.

The output is shown below. Since a whole sequence of problems are actually solved in the process of transforming the given solution into the sought solution, the first trajectory printout is followed by the one-line trajectory summaries and, finally, the complete trajectory printout of the sought solution. The one-line trajectory summaries are interrupted each time an intermediate solution is found to present the value of the first sweep variable (final radius in this case) and several other parameters. The computer execution time on the IBM 7094II is 1.1 minutes.

EXAMPLE 1 - JUPITER RENDEZVOUS

```
SAVED INITIAL DATA FOR STAGE 1 OF CASE   1.

REFERENCE BODY IS SUN

2 DIMENSIONS 150 DIFF.EQNS.  T/W= 2.0000000E-04  ISP= 5000.0000     PFLOW= 4.0000000E-05  REFA= 0         AEXIT= 0

STEP=    0. +  0.   ECCENTRICITY= 1.7263349E-04   OMEGA= 0             V= 29784.700       R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 1.4559793E+11   TRU A= 0            VX= 0               X= 1.4959789E+11   RMASS= 1000.0000
DAYS=    0.         MEAN ANOMALY= 0                NODE= 0            VY= 29784.700       Y= 0               REVS.= 0
ALFA= 37.172525       PATH ANGLE= 0                INCL= 0            VZ= 0               Z= 0                DELT= 432000.00
 PSI= 52.827475             DPSI= 6.5304213E-06   THETA= 0            DK=-4.6215996E-05   K= 207.99178          L7= 63.306500
  L1= 3.3431000               L2= 4.4087600          L3= 0            L4= 1.0720000E-06  L5= 3.7000000E-07     L6= 0

STEP=   23. +  3.   ECCENTRICITY= 0.7846867       OMEGA=-0.2690612     V= 31937.052       R= 2.1119842E+11   REFER=SUN  RECT 2
TIME= 6955146.4     SEMILATUS R.= 2.1537830E+11   TRU A= 1.5455718    VX=-18578.035       X= 6.1259438E+10   RMASS= 721.79414
DAYS=   80.4554     MEAN ANOMALY= 0.1762923        NODE= 0            VY= 25977.576       Y= 2.0211891E+11   REVS.= 0.2031630
ALFA= 62.369410       PATH ANGLE= 37.567967        INCL= 0            VZ= 0               Z= 0                DELT= 14.753720
 PSI= 63.201293             DPSI=-2.0086438E-06   THETA= 73.138671    DK=-1.3444954E-05   K= 0                  L7= 119.70815
  L1= 0.7544848               L2= 1.5729008          L3= 0            L4= 3.4050458E-08  L5= 2.0451261E-07     L6= 0
TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  0

STEP=   23. +  3.   ECCENTRICITY= 0.7846867       OMEGA=-0.2690612     V= 31937.052       R= 2.1119842E+11   REFER=SUN  RECT 2
TIME= 6955146.4     SEMILATUS R.= 2.1537830E+11   TRU A= 1.5455718    VX=-18578.035       X= 6.1259438E+10   RMASS= 721.79414
DAYS=   80.4554     MEAN ANOMALY= 0.1762923        NODE= 0            VY= 25977.576       Y= 2.0211891E+11   REVS.= 0.2031630
ALFA= 62.369410       PATH ANGLE= 37.567967        INCL= 0            VZ= 0               Z= 0                DELT= 172861.56
 PSI= 63.201293             DPSI=-2.0086438E-06   THETA= 73.138671    DK=-1.3444954E-05   K= 0                  L7= 119.70815
  L1= 0.7944848               L2= 1.5729008          L3= 0            L4= 3.4090496E-08  L5= 2.0451261E-07     L6= 0

STEP=   51. +  6.   ECCENTRICITY= 0.7846864       OMEGA=-0.2690611     V= 16367.572       R= 5.2593102E+11   REFER=SUN  RECT 2
TIME= 2.6390800E+07 SEMILATUS R.= 2.1537830E+11   TRU A= 2.4226565    VX=-15547.528       X=-2.8945300E+11   RMASS= 721.79414
DAYS=  305.4491     MEAN ANOMALY= 0.7098759        NODE= 0            VY= 5115.8064       Y= 4.3911315E+11   REVS.= 0.3427553
ALFA= 288.84640       PATH ANGLE= 51.605344        INCL= 0            VZ= 0               Z= 0                DELT= 2285934.5
 PSI=-127.05982             DPSI=-1.3714007E-07   THETA= 123.39192    DK= 6.7890353E-06   K=-95.775606          L7= 119.70815
  L1=-0.2123126               L2=-0.2811371          L3= 0            L4= 6.0900455E-08  L5= 7.9243080E-08     L6= 0

STEP=   60. +  7.   ECCENTRICITY= 0.7846862       OMEGA=-0.2690613     V= 12242.311       R= 6.8646743E+11   REFER=SUN  RECT 2
TIME= 4.0946728E+07 SEMILATUS R.= 2.1537829E+11   TRU A= 2.6353138    VX=-12197.260       X=-4.9026337E+11   RMASS= 721.79414
DAYS=  473.9205     MEAN ANOMALY= 1.1094937        NODE= 0            VY= 1045.2904       Y= 4.8049908E+11   REVS.= 0.3766008
ALFA= 301.23928       PATH ANGLE= 50.493146        INCL= 0            VZ= 0               Z= 0                DELT=-0.1514472
 PSI=-126.15615             DPSI= 1.0022469E-07   THETA= 135.57628    DK= 6.2970745E-06   K= 0                  L7= 119.70815
  L1=-1.0396555               L2=-1.4227928          L3= 0            L4= 5.2200761E-08  L5= 7.6662588E-08     L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  0

STEP=   60. +  7.   ECCENTRICITY= 0.7846862       OMEGA=-0.2690613     V= 12242.311       R= 6.8646743E+11   REFER=SUN  RECT 2
TIME= 4.0946728E+07 SEMILATUS R.= 2.1537829E+11   TRU A= 2.6353138    VX=-12197.260       X=-4.9026337E+11   RMASS= 721.79414
DAYS=  473.9205     MEAN ANOMALY= 1.1094937        NODE= 0            VY= 1045.2904       Y= 4.8049908E+11   REVS.= 0.3766008
ALFA= 301.23928       PATH ANGLE= 50.493146        INCL= 0            VZ= 0               Z= 0                DELT= 1739147.6
 PSI=-126.15615             DPSI= 1.0022469E-07   THETA= 135.57628    DK= 6.2970745E-06   K= 0                  L7= 119.70815
  L1=-1.0396555               L2=-1.4227928          L3= 0            L4= 5.2200761E-08  L5= 7.6662588E-08     L6= 0

STEP=   62. +  7.   ECCENTRICITY= 0.6232751       OMEGA= 0.8915956     V= 16280.684       R= 7.0643525E+11   REFER=SUN  RECT 2
TIME= 4.3199999E+07 SEMILATUS R.= 7.3336940E+11   TRU A= 1.5095863    VX=-15598.487       X=-5.2148454E+11   RMASS= 631.66329
DAYS=  500.0000     MEAN ANOMALY= 0.3822734        NODE= 0            VY=-4663.4608       Y= 4.7655497E+11   REVS.= 0.3821600
ALFA= 322.57753       PATH ANGLE= 30.932568        INCL= 0            VZ= 0               Z= 0                DELT= 514123.65
 PSI=-125.93251             DPSI= 5.7792500E-08   THETA= 137.57759    DK= 7.0750244E-06   K= 15.027596          L7= 137.84041
  L1=-1.1556472               L2=-1.5945583          L3= 0            L4= 5.0763977E-08  L5= 7.5771679E-08     L6= 0

PHASE 1 COMPLETED.  DELV=  22526.  MASS RATIO= 0.63166  ***  TOTAL DELV=  22526.  TOTAL MASS RATIO= 0.63166  PAYLOAD RATIO= 0.63166
```

```
NOPT= 2, CCAST=T, EPHEM=F, NBVP=1, NBODYS=1, ERSTAR=  1.00000, NSWEEP=   0, IBR= 437

IAA        IA        IB           DESIRE             WEIGHT          PERTEW            PERTNR

866        347       1255      7.0528000E+11      7.053E+11       -1.000E-02        -1.000E-04
867        348        493      16570.000          1.657E+04       -1.000E-02        -1.000E-04
868        350        479      30.000000          365.0           -1.000E-02        -1.000E-04
869        351        485      138.00000          138.0           -1.000E-02        -1.000E-04

RUN N  ERROR     TIME                     4 INDEPENDENT VARIABLES --  4 DEPENDENT VARIABLES

  1N2 0.017584  4.3200E+07  3.34310    4.40876    1.07200E-06  3.70000E-07  7.06435E+11  16280.7     30.9326      137.578
  2N2 0.000148  4.3200E+07  3.30510    4.37732    1.06222E-06  3.65892E-07  7.05278E+11  16567.8     30.0230      138.000
  3N2 0.000001  4.3200E+07  3.30499    4.37739    1.06218E-06  3.65895E-07  7.05280E+11  16570.0     30.0000      138.000
      C( 866) =  7.0528000E+11   YIELDS  C( 437) =  0.6285279   DELV= 22769.807     PAY= 0.6285279
  4N2 0.002518  4.3200E+07  3.30499    4.37739    1.06218E-06  3.65855E-07  7.05280E+11  16570.0     30.0000      138.000
  5N2 0.000308  4.3200E+07  3.30233    4.38132    1.06161E-06  3.65964E-07  7.06001E+11  16540.0     29.6858      137.951
  6N2 0.000003  4.3200E+07  3.30228    4.38126    1.06159E-06  3.65958E-07  7.06005E+11  16535.0     29.6999      137.950
      C( 866) =  7.0600519E+11   YIELDS  C( 437) =  0.6278256   DELV= 22824.624     PAY= 0.6278256
  7N2 0.001760  4.3200E+07  3.29686    4.38902    1.06042E-06  3.66082E-07  7.07430E+11  16493.7     29.0141      137.857
  8N2 0.000001  4.3200E+07  3.29660    4.38870    1.06034E-06  3.66044E-07  7.07456E+11  16464.8     29.1001      137.850
      C( 866) =  7.0745559E+11   YIELDS  C( 437) =  0.6263676   DELV= 22938.628     PAY= 0.6263676
  9N2 0.012288  4.3200E+07  3.27958    4.41102    1.05658E-06  3.66302E-07  7.11535E+11  16455.2     26.6033      137.606
 10N2 0.000063  4.3200E+07  3.27809    4.40886    1.05610E-06  3.66037E-07  7.11802E+11  16255.3     27.3076      137.551
      C( 866) =  7.1180679E+11   YIELDS  C( 437) =  0.6215873   DELV= 23314.276     PAY= 0.6215873
 11N2 0.026657  4.3200E+07  3.25032    4.43910    1.04975E-06  3.66026E-07  7.17357E+11  16374.6     22.7355      137.236
 12N2 0.000427  4.3200E+07  3.24866    4.43500    1.04925E-06  3.65509E-07  7.18294E+11  15945.4     24.6402      137.105
 13N2 0.000003  4.3200E+07  3.24809    4.43503    1.04908E-06  3.65489E-07  7.18334E+11  15938.7     24.5998      137.100
      C( 866) =  7.1833360E+11   YIELDS  C( 437) =  0.6133820   DELV= 23965.852     PAY= 0.6133820
 14N2 0.035568  4.3200E+07  3.20309    4.47428    1.03853E-06  3.64667E-07  7.25578E+11  16100.6     16.7025      136.646
 15N2 0.001693  4.3200E+07  3.20718    4.47113    1.03990E-06  3.64255E-07  7.27930E+11  15492.6     20.6251      136.437
 16N2 0.000074  4.3200E+07  3.20474    4.47064    1.03924E-06  3.64079E-07  7.28129E+11  15463.9     20.5556      136.424
      C( 866) =  7.2812379E+11   YIELDS  C( 437) =  0.5995240   DELV= 25086.358     PAY= 0.5995240
 17N2 0.050554  4.3200E+07  3.13971    4.52406    1.02448E-06  3.61965E-07  7.36868E+11  15522.5     7.36966      135.701
 18N2 0.005738  4.3200E+07  3.16141    4.53115    1.03144E-06  3.62718E-07  7.42132E+11  14848.6     14.4581      135.433
 19N2 0.000071  4.3200E+07  3.15330    4.52787    1.02936E-06  3.61939E-07  7.42805E+11  14756.0     14.4698      135.413
      C( 866) =  7.4280909E+11   YIELDS  C( 437) =  0.5767678   DELV= 26983.755     PAY= 0.5767678
 20N2 0.031440  4.3200E+07  3.10186    4.58509    1.01949E-06  3.59798E-07  7.51721E+11  14454.4     1.98638      134.549
 21N2 0.004413  4.3200E+07  3.12684    4.59983    1.02777E-06  3.61167E-07  7.56593E+11  14117.4     8.24628      134.406
 22N2 0.000021  4.3200E+07  3.12101    4.59659    1.02628E-06  3.60542E-07  7.57494E+11  14045.0     8.40022      134.400
      C( 866) =  7.5749439E+11   YIELDS  C( 437) =  0.5535626   DELV= 28997.301     PAY= 0.5535626
 23N2 0.031649  4.3200E+07  3.07636    4.69161    1.02201E-06  3.58611E-07  7.70339E+11  13403.4     -7.88106     133.063
 24N2 0.006690  4.3200E+07  3.11103    4.71721    1.03350E-06  3.61015E-07  7.77119E+11  13171.1     -0.33983     132.998
 25 2 0.000050  4.3200E+07  3.10287    4.71142    1.03136E-06  3.60093E-07  7.77798E+11  13063.3     1.70330E-03  133.000

REFERENCE BODY IS SUN

2 DIMENSIONS 15C DIFF.EQNS.  T/W= 2.0000000E-04  ISP= 5000.0000      PFLOW= 4.0000000E-05  REFA= 0          AEXIT= 0

STEP=    0. +  0.   ECCENTRICITY= 1.7263349E-04   OMEGA= 0              V= 29784.700        R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 1.4959793E+11   TRU A= 0             VX= 0                X= 1.4959789E+11   RMASS= 1000.0000
DAYS=    0.         MEAN ANOMALY= 0                NODE= 0             VY= 29784.700        Y= 0               REVS.= 0
ALFA= 33.368304       PATH ANGLE= 0                INCL= 0             VZ= 0                Z= 0                DELT= 432000.00
 PSI= 56.631696             DPSI= 6.7365258E-06   THETA= 0             DK=-4.2560814E-05   K= 213.30895          L7= 63.306500
  L1= 3.1028686               L2= 4.7114164          L3= 0             L4= 1.0313585E-06  L5= 3.6009298E-07     L6= 0

STEP=   25. +  2.   ECCENTRICITY= 0.8819417       OMEGA=-0.2376959      V= 32301.512        R= 2.2522151E+11   REFER=SUN  RECT 2
TIME= 7617579.6     SEMILATUS R.= 2.1822231E+11   TRU A= 1.6060406     VX=-19035.910        X= 4.5285619E+10   RMASS= 695.29681
DAYS=   88.1664     MEAN ANOMALY= 7.8929201E-02    NODE= 0             VY= 26096.395        Y= 2.2062172E+11   REVS.= 0.2177788
ALFA= 63.610723       PATH ANGLE= 42.291648        INCL= 0             VZ= 0                Z= 0                DELT= 10.383288
 PSI= 62.498007             DPSI=-3.4471007E-06   THETA= 78.400377     DK=-1.1113466E-05   K= 0                 L7= 127.81130
  L1= 0.8369190               L2= 1.6075700          L3= 0             L4=-2.3944742E-08  L5= 1.9013343E-07     L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  0

STEP=   25. +  2.   ECCENTRICITY= 0.8819417       OMEGA=-0.2376959      V= 32301.512        R= 2.2522151E+11   REFER=SUN  RECT 2
TIME= 7617579.6     SEMILATUS R.= 2.1822231E+11   TRU A= 1.6060406     VX=-19035.910        X= 4.5285619E+10   RMASS= 695.29681
DAYS=   88.1664     MEAN ANOMALY= 7.8929201E-02    NODE= 0             VY= 26096.395        Y= 2.2062172E+11   REVS.= 0.2177788
ALFA= 63.610723       PATH ANGLE= 42.291648        INCL= 0             VZ= 0                Z= 0                DELT= 181419.05
 PSI= 62.498007             DPSI=-3.4471007E-06   THETA= 78.400377     DK=-1.1113466E-05   K= 0                 L7= 127.81130
  L1= 0.8369190               L2= 1.6075700          L3= 0             L4=-2.3944742E-08  L5= 1.9013343E-07     L6= 0

STEP=   58. +  2.   ECCENTRICITY= 0.8819413       OMEGA=-0.2376959      V= 14805.135        R= 7.4912962E+11   REFER=SUN  RECT 2
TIME= 3.8884524E+07 SEMILATUS R.= 2.1822231E+11   TRU A= 2.5040603     VX=-13810.624        X=-4.8005846E+11   RMASS= 695.29681
DAYS=  450.0524     MEAN ANOMALY= 0.4489699        NODE= 0             VY= 5334.6675        Y= 5.7509918E+11   REVS.= 0.3607031
ALFA= 231.31045       PATH ANGLE= 60.973300        INCL= 0             VZ= 0                Z= 0                DELT=-0.1302527
 PSI=-72.430640             DPSI=-1.3682199E-06   THETA= 129.85311     DK= 7.3217223E-06   K= 0                 L7= 127.81130
  L1= 0.5470647               L2=-1.7278348          L3= 0             L4= 9.9206284E-09  L5= 1.1204407E-07     L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  0

STEP=   58. +  2.   ECCENTRICITY= 0.8819413       OMEGA=-0.2376959      V= 14805.135        R= 7.4912962E+11   REFER=SUN  RECT 2
TIME= 3.8884524E+07 SEMILATUS R.= 2.1822231E+11   TRU A= 2.5040603     VX=-13810.624        X=-4.8005846E+11   RMASS= 695.29681
DAYS=  450.0524     MEAN ANOMALY= 0.4489699        NODE= 0             VY= 5334.6675        Y= 5.7509918E+11   REVS.= 0.3607031
ALFA= 231.31045       PATH ANGLE= 60.973300        INCL= 0             VZ= 0                Z= 0                DELT= 1390852.4
 PSI=-72.430640             DPSI=-1.3682199E-06   THETA= 129.85311     DK= 7.3217223E-06   K= 0                 L7= 127.81130
  L1= 0.5470647               L2=-1.7278348          L3= 0             L4= 9.9206284E-09  L5= 1.1204407E-07     L6= 0

STEP=   63. +  3.   ECCENTRICITY= 2.4414063E-04   OMEGA= 2.1172107      V= 13063.328        R= 7.7779753E+11   REFER=SUN  RECT 2
TIME= 4.3199999E+07 SEMILATUS R.= 7.7790927E+11   TRU A= 0.2040819     VX=-9554.1361        X=-5.3045932E+11   RMASS= 522.67779
DAYS=  500.0000     MEAN ANOMALY= 0.2039829        NODE= 0             VY=-8908.9252        Y= 5.6884261E+11   REVS.= 0.3694452
ALFA= 299.98503       PATH ANGLE= 1.7033000E-03    INCL= 0             VZ= 0                Z= 0                DELT= 933913.67
 PSI=-76.586474             DPSI=-7.9824046E-07   THETA= 133.00027     DK= 1.0362181E-05   K= 37.619324         L7= 175.87607
  L1= 0.5124645               L2=-2.2173392          L3= 0             L4= 6.0189019E-09  L5= 1.1476006E-07     L6= 0

PHASE 1 COMPLETED.  DELV=  31812.  MASS RATIO= 0.52268  ***  TOTAL DELV=  31812.  TOTAL MASS RATIO= 0.52268  PAYLOAD RATIO= 0.52268

     C( 866) =  7.7779999E+11   YIELDS  C( 437) =  0.5226778     DELV= 31812.287     PAY= 0.5226778
```

EXAMPLE 2 - 0.1-AU SOLAR PROBE WITH A SWEEP ON SPECIFIC IMPULSE

This example considers a mission to 0.1 AU using a Titan IIID/Centaur to launch a 10-kilowatt solar-electric spacecraft. A sequence of solutions are sought for specific-impulse values from 2600 to 4000 seconds. From previous experience (ref. 10), it is known that permitting coast flight adds very little to the performance but much to the convergence difficulty for these missions. Hence, we assume optimum thrust steering with the no-coast constraint for simplicity. The Earth-escape phase is simulated analytically, and the electric spacecraft betins its heliocentric flight on the x-axis. The level 1 boundary-value problem is set up such that ψ_0, $\dot{\psi}_0$, and v_l will be iterated to satisfy the optimum flyby conditions at 0.1 AU: $r_a = 0.1$ AU and $\left(\Lambda/\lambda_m\right)_a = 0$. The launch velocity v_l is used here instead of κ_0 because the power level is fixed at 10 kilowatts. Technically, this leaves κ_0 open for optimization; however, we shall ignore this optimization since the payoff criterion m_n/m_{ref} is quite insensitive to κ_0. The input required for this example is as follows:

ISP=2600	spacecraft specific impulse, I, sec
TB=4.32E7	mission time, t_f, sec; equal to 500 days
NOPT=7	manual specification of level 1 boundary-value problem with optimum-travel-angle option
POWER=10	initial spacecraft electric power, P_0, kW
SOLAR=T	solar power option
KE=0.03	low-thrust tankage factor, k_t
STRUCT=0	low-thrust structure factor, k_s
ALFPOW=30	specific powerplant mass, α_{ps}, kg/kW
BE=0.8, DE=15700	overall powerplant efficiency factors, b and d
BOOSTM=15500	reference mass of launch vehicle, m_{ref}, kg
VB1=13375	launch velocity, v_l, m/sec
RRAT1=150	sphere-of-influence radius ratio, $r_{s,a}/r_l$
VC1=7810	circular orbit speed at 160-n mi launch altitude, v_c, m/sec
VJET1=3811	launch vehicle performance parameter, c_l, m/sec
K1=0.129	launch vehicle performance parameter, k_l

R=1.49597893D11, 0, 0	initial heliocentric position vector, R_0, m
V=0, 29765.2, 0	heliocentric velocity of Earth, V_0, m/sec
COAST=F	coast arcs not permitted
PS=-88.27	initial thrust angle, ψ_0, deg
DPS=5.58E-6	initial thrust angle rate, $\dot{\psi}_0$, deg/sec
KAPPA=28.3	initial engine on-off switch function, κ_0
LAM=4.19145	initial magnitude of primer, λ_0, (kg)(sec)/m
EREF=1.E-3	integration scheme relative error control, $\bar{\delta}_r$
ERLIMT=3.E-3	limit relative integration error, δ_{limit}
DELMAX=8640000	output frequency, sec; every 100 days
IA=343, 344, 429	COMMON locations of ψ_0, $\dot{\psi}_0$, v_l
IB=480, 363, 364	COMMON locations of r, λ_1/λ_m, λ_2/λ_m
DESIRE=1.49597893E10, 0, 0	desired values of arrival conditions, $\bar{y}$
WEIGHT=1.496E11	weighting factor w_1 for radius
TOLER=0.001	convergence tolerance, $\bar{\tau}$
IAA=418	COMMON location of sweep variable, specific impulse
SVALUE=3000, 3500, 4000	values of specific impulse for which full trajectory printout is desired
MAXPTS=2	number of points used in extrapolation of $\bar{X}$

The output of this example follows. Note that the $2\frac{1}{2}$-revolution solution was found. (There are also solutions for 1/2, $1\frac{1}{2}$, $3\frac{1}{2}$, etc., revolutions.) Full printouts occur for the first trajectory and for the solutions with I of 3000, 3500, and 4000 seconds. The computer execution time on the IBM 7094II is 0.9 minute.

EXAMPLE 2 - SOLAR PROBE

```
SAVED INITIAL DATA FOR STAGE 1 OF CASE   1.

REFERENCE BODY IS SUN

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 2.2484632E-05  ISP= 2600.0000         PFLOW= 1.7845157E-05  REFA= 0            AEXIT= 0

STEP=    0. +  0.  ECCENTRICITY= 0.4458999       OMEGA=-3.1287374     V= 22173.026       R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0            SEMILATUS R.= 8.2857723E+10   TRU A= 3.1287374    VX= 229.34507       X= 1.4959789E+11   RMASS= 2063.5164
DAYS=    0.        MEAN ANOMALY= 3.1115682        NODE= 0            VY= 22171.840       Y= 0               REVS.= 0
ALFA= 177.67735      PATH ANGLE= 0.5926453        INCL= 0            VZ= 0               Z= 0                DELT= 432000.00
 PSI=-88.270000              DPSI= 5.5799999E-06 THETA= 0            DK= 4.6357672E-07   K= 28.299998          L7= 23.490534
  L1= 0.1265382                L2=-4.1895394        L3= 0            L4=-4.0914925E-07  L5= 2.5177062E-08     L6= 0

STEP=   32. +  2.  ECCENTRICITY= 0.5281270       OMEGA= 3.2329176     V= 64545.622       R= 4.8502337E+10   REFER=SUN  RECT 2
TIME= 8639995.6    SEMILATUS R.= 7.3287868E+10   TRU A=-0.2552384    VX=-4894.3926       X=-4.7852224E+10   RMASS= 1881.0954
DAYS=  100.0000    MEAN ANOMALY=-6.7433164E-02    NODE= 0            VY=-64363.799       Y= 7.9146290E+09   REVS.= 0.4739124
ALFA= 356.12258      PATH ANGLE=-5.0429915        INCL= 0            VZ= 0               Z= 0                DELT= 77067.925
 PSI=-90.471545              DPSI= 2.7892346E-05 THETA= 170.60845    DK= 4.7716681E-06   K= 1.6793537          L7= 26.809296
  L1=-1.7297547E-02            L2=-2.1017157        L3= 0            L4=-1.0202459E-06  L5= 3.6044472E-07     L6= 0

STEP=   68. +  2.  ECCENTRICITY= 0.6113584       OMEGA=-3.1243871     V= 17928.566       R= 1.6352188E+11   REFER=SUN  RECT 2
TIME= 1.7279598E+07 SEMILATUS R.= 6.3845129E+10  TRU A= 3.0649180    VX= 3189.2918       X= 1.6323281E+11   RMASS= 1702.8326
DAYS=  200.0000    MEAN ANOMALY= 2.8907908        NODE= 0            VY= 17642.616       Y=-9.7187752E+09   REVS.= 0.9905352
ALFA= 170.55678      PATH ANGLE= 6.8394836        INCL= 0            VZ= 0               Z= 0                DELT= 594779.63
 PSI=-90.803607              DPSI= 4.4576789E-06 THETA= 356.59266    DK= 1.3671519E-06   K= 32.797728          L7= 30.657357
  L1=-5.9436254E-02            L2=-4.2374214        L3= 0            L4=-3.2839580E-07  L5= 9.5920259E-08     L6= 0

STEP=   93. +  2.  ECCENTRICITY= 0.6888826       OMEGA= 3.2426031     V= 69550.428       R= 4.2758698E+10   REFER=SUN  RECT 2
TIME= 2.5919958E+07 SEMILATUS R.= 5.0890502E+10  TRU A=-1.2910949    VX=-43862.914       X=-1.5888340E+10   RMASS= 1534.2834
DAYS=  300.0000    MEAN ANOMALY=-0.2220886        NODE= 0            VY=-53975.057       Y= 3.9697190E+10   REVS.= 1.3105922
ALFA= 324.72015      PATH ANGLE=-29.087727        INCL= 0            VZ= 0               Z= 0                DELT= 77962.497
 PSI=-93.819648              DPSI=-4.2896791E-05 THETA= 471.81318    DK= 1.0212851E-05   K=-18.902946          L7= 35.610293
  L1=-6.6976246E-02            L2=-1.0031207        L3= 0            L4= 7.9156759E-07  L5= 5.6304457E-07     L6= 0

STEP=  145. +  2.  ECCENTRICITY= 0.7689342       OMEGA=-3.1231938     V= 13475.256       R= 1.7279529E+11   REFER=SUN  RECT 2
TIME= 3.4559956E+07 SEMILATUS R.= 4.0048448E+10  TRU A= 3.0988463    VX= 2215.8046       X= 1.7274408E+11   RMASS= 1361.0252
DAYS=  400.0000    MEAN ANOMALY= 2.9327975        NODE= 0            VY= 13291.870       Y=-4.2067218E+09   REVS.= 1.9961250
ALFA= 170.12782      PATH ANGLE= 8.0653787        INCL= 0            VZ= 0               Z= 0                DELT= 532501.57
 PSI=-89.592216              DPSI= 3.7053917E-06 THETA= 718.60499    DK= 1.2242662E-06   K= 38.184099          L7= 41.249037
  L1= 3.0177056E-02            L2=-4.2399709        L3= 0            L4=-2.7466947E-07  L5= 6.3397122E-08     L6= 0

STEP=  181. +  2.  ECCENTRICITY= 0.8339287       OMEGA=-3.0331403     V= 127540.65       R= 1.4962139E+10   REFER=SUN  RECT 2
TIME= 4.3199956E+07 SEMILATUS R.= 2.7437360E+10  TRU A= 1.8506017E-02 VX= 15083.700      X=-1.4841718E+10   RMASS= 1200.7586
DAYS=  500.0000    MEAN ANOMALY= 9.2488130E-04    NODE= 0            VY=-126645.60       Y=-1.8944690E+09   REVS.= 2.5202060
ALFA= 158.94537      PATH ANGLE= 0.4821485        INCL= 0            VZ= 0               Z= 0                DELT= 11453.155
 PSI= 117.84664              DPSI= 5.0644076E-03 THETA= 907.27417    DK= 1.2101470E-05   K=-48.425145          L7= 48.753842
  L1=-7.2996536E-03            L2= 1.3817826E-02    L3= 0            L4= 1.4874144E-06  L5= 1.4161210E-07     L6= 0

STEP=  182. +  2.  ECCENTRICITY= 0.8339287       OMEGA=-3.0331404     V= 127540.67       R= 1.4962143E+10   REFER=SUN  RECT 2
TIME= 4.3200000E+07 SEMILATUS R.= 2.7437360E+10  TRU A= 1.8534423E-02 VX= 15085.659      X=-1.4841668E+10   RMASS= 1200.7586
DAYS=  500.0000    MEAN ANOMALY= 9.2630111E-04    NODE= 0            VY=-126645.35       Y=-1.8948911E+09   REVS.= 2.5202105
ALFA= 158.92537      PATH ANGLE= 0.4828886        INCL= 0            VZ= 0               Z= 0                DELT= 3.3325195
 PSI= 117.86351              DPSI= 5.0631846E-03 THETA= 907.27579    DK= 1.2110086E-05   K=-48.425105          L7= 48.753842
  L1=-7.3046505E-03            L2= 1.3817354E-02    L3= 0            L4= 1.4874156E-06  L5= 1.4161420E-07     L6= 0

PHASE 1 COMPLETED.  DELV=  13806.  MASS RATIO= 0.58190  ***  TOTAL DELV=  13806.  TOTAL MASS RATIO= 0.58190  PAYLOAD RATIO= 0.05644

KE=.030 STRUCT=.       ALFPOW= 30.000  PJ/M0= 3.7423699E-04 PJ/M1= 2.8110625E-03 MPP/M1= 0.145 ETAPOW=0.580      ML/M1= 0.42397
K1=.129  VJET1= 3811.0    VC1= 7810.0     VB1= 13375.000      VSPH1= 7596.8221       M1/M0= 0.133  TSPIR=  0.    TMISSION= 500.000

  NOPT= 7, COAST=F, EPHEM=F, NBVP=1, KBODYS=1, ERSTAR=  1.00000, NSWEEP=   0, IBB= 437

 IAA        IA         IB          DESIRE              WEIGHT            PERTEW            PERTNR

 418       343        480      1.4959789E+10         1.496E+11         -1.000E-02        -1.000E-04
   0       344        363      0                     1.000             -1.000E-02        -1.000E-04
   0       429        364      0                     1.000             -1.000E-02        -1.000E-04

RUN N  ERROR     TIME                        3 INDEPENDENT VARIABLES --  3 DEPENDENT VARIABLES

  1NO 0.000321  4.3200E+07 -88.2700       5.58000E-06  13375.0       1.49621E+10 -1.49827E-04  2.83411E-04
      C( 418) =  2600.0000        YIELDS  C( 437) =  5.6443601E-02  DELV= 13805.712      PAY= 5.6443601E-02
  2NO 0.000357  4.3200E+07 -88.2700       5.58000E-06  13375.0       1.49869E+10 -1.60803E-04  2.62608E-04
      C( 418) =  2602.6000        YIELDS  C( 437) =  5.6523452E-02  DELV= 13793.950      PAY= 5.6523452E-02
  3NO 0.000590  4.3200E+07 -88.2700       5.58000E-06  13375.0       1.50366E+10 -1.84803E-04  2.24355E-04
      C( 418) =  2607.8000        YIELDS  C( 437) =  5.6682658E-02  DELV= 13770.625      PAY= 5.6682658E-02
  4NO 0.001541  4.3200E+07 -88.2700       5.58000E-06  13375.0       1.51868E+10 -2.46422E-04  1.08948E-04
  5 0 0.002257  4.3200E+07 -88.2612       5.58000E-06  13375.0       1.52803E+10 -7.27573E-04  3.58885E-04
  6 0 0.001486  4.3200E+07 -88.2700       5.57944E-06  13375.0       1.51801E+10 -1.71973E-04  9.83249E-05
  7 0 0.001644  4.3200E+07 -88.2700       5.58000E-06  13373.7       1.52055E+10  2.95332E-05  4.48317E-05
  8NO 0.000700  4.3200E+07 -88.2765       5.57267E-06  13380.1       1.50641E+10 -5.72901E-05  3.83709E-05
      C( 418) =  2623.3999        YIELDS  C( 437) =  5.6790720E-02  DELV= 13754.237      PAY= 5.6790720E-02
  9NO 0.001604  4.3200E+07 -88.2959       5.55070E-06  13395.3       1.51827E+10  2.65535E-04 -5.20816E-04
 10 0 0.001375  4.3200E+07 -88.2870       5.55070E-06  13395.3       1.51620E+10 -1.66104E-04 -2.27933E-04
 11 0 0.001695  4.3200E+07 -88.2959       5.55014E-06  13395.3       1.51950E+10  3.47372E-04 -5.29973E-04
 12 0 0.002264  4.3200E+07 -88.2959       5.55070E-06  13393.9       1.52772E+10  5.27663E-04 -5.88961E-04
 13NO 0.000310  4.3200E+07 -88.2842       5.53730E-06  13402.2       1.50061E+10 -5.55725E-06 -1.30302E-06
      C( 418) =  2670.1999        YIELDS  C( 437) =  5.6760501E-02  DELV= 13723.155      PAY= 5.6760501E-02
 14NO 0.000910  4.3200E+07 -88.3074       5.43119E-06  13468.8       1.48295E+10  2.17174E-04 -1.48983E-04
      C( 418) =  2810.5959        YIELDS  C( 437) =  5.6417645E-02  DELV= 13647.547      PAY= 5.6417645E-02
 15NO 0.002020  4.3200E+07 -88.3387       5.28804E-06  13558.5       1.46748E+10  5.77067E-04 -3.39895E-04
 16 0 0.002857  4.3200E+07 -88.3298       5.28804E-06  13558.5       1.45349E+10  2.89141E-04 -1.08321E-04
 17 0 0.001849  4.3200E+07 -88.3387       5.28751E-06  13558.5       1.47045E+10  6.20853E-04 -3.45994E-04
 18 0 0.001172  4.3200E+07 -88.3287       5.28804E-06  13557.1       1.48399E+10  7.56641E-04 -4.00448E-04
 19 0 0.001848  4.3200E+07 -88.3387       5.28804E-06  13558.2       1.47042E+10  6.12374E-04 -3.51068E-04
 20 0 0.000661  4.3200E+07 -88.3030       5.31594E-06  13536.3       1.49644E+10  6.79796E-05  3.07944E-05

REFERENCE BODY IS SUN

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 2.2776023E-05  [ISP= 3000.0000]     PFLOW= 1.4388166E-05  REFA= 0            AEXIT= 0
```

```
STEP=    0. +  0.   ECCENTRICITY= 0.4598296        OMEGA=-3.1290736       V= 21892.619        R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 8.0813749E+10    TRU A= 3.1290736       VX= 233.27533       X= 1.4959789E+11   RMASS= 1895.1753
DAYS=    0.         MEAN ANOMALY= 3.1115499         NODE= 0               VY= 21891.376       Y= 0               REVS.= 0
ALFA= 177.69249       PATH ANGLE= 0.6105229         INCL= 0               VZ= 0               Z= 0                DELT= 432000.00
 PSI=-88.303012              DPSI= 5.3159420E-06   THETA= 0               DK= 5.6240465E-07   K= 28.299999          L7= 36.766409
  L1= 0.1241243                L2=-4.1896117          L3= 0               L4=-3.8578755E-07   L5= 2.4696773E-08     L6= 0

STEP=   34. +  2.   ECCENTRICITY= 0.5429763        OMEGA= 3.2239206       V= 66389.050        R= 4.6441442E+10   REFER=SUN  RECT 2
TIME= 8639999.6     SEMILATUS R.= 7.1562116E+10    TRU A=-8.7253034E-02   VX= 1710.7850       X=-4.6440875E+10   RMASS= 1747.8114
DAYS=  100.0000     MEAN ANOMALY=-2.1721855E-02     NODE= 0               VY=-66367.004       Y= 2.2872727E+08   REVS.= 0.4992161
ALFA= 358.90166       PATH ANGLE=-1.7588099         INCL= 0               VZ= 0               Z= 0                DELT= 75514.208
 PSI=-87.425044              DPSI= 3.3440147E-05   THETA= 179.71781       DK= 2.2557642E-06   K=-0.5580359          L7= 40.438375
  L1= 0.1064419                L2=-2.3668611          L3= 0               L4=-1.3874171E-06   L5= 7.1753713E-08     L6= 0

STEP=   69. +  2.   ECCENTRICITY= 0.6243998        OMEGA=-3.1259579       V= 17501.177        R= 1.6477836E+11   REFER=SUN  RECT 2
TIME= 1.7279999E+07 SEMILATUS R.= 6.2070258E+10    TRU A= 3.0825195       VX= 2459.3305       X= 1.6462293E+11   RMASS= 1605.4229
DAYS=  200.0000     MEAN ANOMALY= 2.9424206         NODE= 0               VY= 17327.518       Y=-7.1554568E+09   REVS.= 0.9930866
ALFA= 172.04311       PATH ANGLE= 5.5893162         INCL= 0               VZ= 0               Z= 0                DELT= 662280.34
 PSI=-90.121272              DPSI= 4.2591450E-06   THETA= 357.51116       DK= 1.3544592E-06   K= 33.782144          L7= 44.609245
  L1=-9.0543172E-03            L2=-4.2777451          L3= 0               L4=-3.1782985E-07   L5= 7.6767408E-08     L6= 0

STEP=   98. +  2.   ECCENTRICITY= 0.6956600        OMEGA= 3.2348388       V= 75632.248        R= 3.7492514E+10   REFER=SUN  RECT 2
TIME= 2.5919958E+07 SEMILATUS R.= 4.9846592E+10    TRU A=-1.0804471       VX=-39696.930       X=-2.0659639E+10   RMASS= 1469.1925
DAYS=  300.0000     MEAN ANOMALY=-0.1622246         NODE= 0               VY=-64376.943       Y= 3.1287346E+10   REVS.= 1.3428821
ALFA= 338.72279       PATH ANGLE=-24.903105         INCL= 0               VZ= 0               Z= 0                DELT= 32116.280
 PSI=-100.38214              DPSI=-8.5869136E-06   THETA= 483.43755       DK= 2.1812458E-05   K=-25.471545          L7= 49.679219
  L1=-0.2178588                L2=-1.1891060          L3= 0               L4= 3.7451408E-07   L5= 1.0388001E-06     L6= 0

STEP=  148. +  2.   ECCENTRICITY= 0.7753833        OMEGA=-3.1244797       V= 13178.016        R= 1.7455829E+11   REFER=SUN  RECT 2
TIME= 3.4559957E+07 SEMILATUS R.= 3.9292828E+10    TRU A= 3.1063362       VX= 1825.4927       X= 1.7452956E+11   RMASS= 1331.1315
DAYS=  400.0000     MEAN ANOMALY= 2.9658669         NODE= 0               VY= 13050.967       Y=-3.1669253E+09   REVS.= 1.9971124
ALFA= 171.34917       PATH ANGLE= 6.9229913         INCL= 0               VZ= 0               Z= 0                DELT= 521642.52
 PSI=-89.311723              DPSI= 3.5844693E-06   THETA= 718.96045       DK= 1.2555903E-06   K= 39.414749          L7= 55.365194
  L1= 5.1514076E-02            L2=-4.2880924          L3= 0               L4=-2.6894885E-07   L5= 5.3583431E-08     L6= 0

STEP=  184. +  2.   ECCENTRICITY= 0.8358092        OMEGA=-3.0395966       V= 127594.67        R= 1.4964411E+10   REFER=SUN  RECT 2
TIME= 4.3199956E+07 SEMILATUS R.= 2.7461456E+10    TRU A=-4.0679919E-02   VX= 10175.907       X=-1.4936290E+10   RMASS= 1201.8968
DAYS=  500.0000     MEAN ANOMALY=-1.9980140E-03     NODE= 0               VY=-127188.25       Y=-9.1698376E+08   REVS.= 2.5097587
ALFA= 250.22177       PATH ANGLE=-1.0611493         INCL= 0               VZ= 0               Z= 0                DELT= 10770.641
 PSI= 24.352512              DPSI= 5.8410155E-03   THETA= 903.51315       DK=-3.3207338E-05   K=-62.423087          L7= 62.536300
  L1= 4.2555378E-03            L2= 1.9261463E-03      L3= 0               L4= 1.4320416E-06   L5= 1.2546459E-07     L6= 0

STEP=  185. +  2.   ECCENTRICITY= 0.8358093        OMEGA=-3.0395967       V= 127594.71        R= 1.4964404E+10   REFER=SUN  RECT 2
TIME= 4.3200000E+07 SEMILATUS R.= 2.7461456E+10    TRU A=-4.0654063E-02   VX= 10177.701       X=-1.4936259E+10   RMASS= 1201.8968
DAYS=  500.0000     MEAN ANOMALY=-1.9967428E-03     NODE= 0               VY=-127188.14       Y=-9.1736951E+08   REVS.= 2.5097629
ALFA= 250.20485       PATH ANGLE=-1.0604748         INCL= 0               VZ= 0               Z= 0                DELT= 3.0329590
 PSI= 24.270243              DPSI= 5.8513098E-03   THETA= 903.51463       DK=-3.3203707E-05   K=-62.423187          L7= 62.536300
  L1= 4.2511945E-03            L2= 1.9257658E-03      L3= 0               L4= 1.4320405E-06   L5= 1.2546473E-07     L6= 0

PHASE 1 COMPLETED.  DELV=  13398.  MASS RATIO= 0.63419  ***  TOTAL DELV=  13398.  TOTAL MASS RATIO= 0.63419  PAYLOAD RATIO= 0.05685

 KE=.030 STRUCT=.       ALFPOW= 30.000  PJ/M0= 4.0172437E-04 PJ/M1= 3.2855684E-03 MPP/M1= 0.158 ETAPOW=0.623     ML/M1= 0.46492
 K1=.129  VJET1= 3811.0    VC1= 7810.0     VB1= 13536.264      VSPH1= 7877.2786      M1/M0= 0.122  TSPIR=  0.   TMISSION= 500.000
       C( 418) =  3000.0000        YIELDS  C( 437) =  5.6845060E-02  DELV= 13398.150      PAY= 5.6845060E-02
  22 0 0.000404  4.3200E+07 -88.2915        5.01170E-06  13714.5        1.50197E+10  4.08761E-05  3.60450E-05
--------------------------------------------------------------------------------------------------------------------------------
REFERENCE BODY IS SUN

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 2.2891123E-05  |ISP= 3500.0000|    PFLOW= 1.1231402E-05  REFA= 0              AEXIT= 0

STEP=    0. +  0.   ECCENTRICITY= 0.4746445        OMEGA=-3.1290883       V= 21590.508        R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 7.8557628E+10    TRU A= 3.1290883       VX= 243.87569       X= 1.4959789E+11   RMASS= 1717.2555
DAYS=    0.         MEAN ANOMALY= 3.1107007         NODE= 0               VY= 21589.131       Y= 0               REVS.= 0
ALFA= 177.64429       PATH ANGLE= 0.6471985         INCL= 0               VZ= 0               Z= 0                DELT= 432000.00
 PSI=-88.291489              DPSI= 5.0117046E-06   THETA= 0               DK= 7.1577063E-07   K= 28.299998          L7= 55.475706
  L1= 0.1245669                L2=-4.1895866          L3= 0               L4=-3.6753398E-07   L5= 2.4864427E-08     L6= 0

STEP=   36. +  2.   ECCENTRICITY= 0.5578642        OMEGA=-3.0679704       V= 67861.237        R= 4.4871897E+10   REFER=SUN  RECT 2
TIME= 8639999.6     SEMILATUS R.= 6.9777862E+10    TRU A= 0.1005587       VX= 9347.4139       X=-4.4192932E+10   RMASS= 1602.0116
DAYS=  100.0000     MEAN ANOMALY= 2.3714441E-02     NODE= 0               VY=-67214.384       Y=-7.7763686E+09   REVS.= 0.5277218
ALFA= 1.2463493       PATH ANGLE= 2.0625656         INCL= 0               VZ= 0               Z= 0                DELT= 75015.062
 PSI=-83.329068              DPSI= 3.7315241E-05   THETA= 189.97983       DK=-4.0116795E-06   K=-2.7671399          L7= 59.586010
  L1= 0.3080707                L2=-2.6340212          L3= 0               L4=-1.6937178E-06   L5=-3.8661270E-07     L6= 0

STEP=   70. +  2.   ECCENTRICITY= 0.6368262        OMEGA=-3.1270948       V= 17105.012        R= 1.6587940E+11   REFER=SUN  RECT 2
TIME= 1.7279959E+07 SEMILATUS R.= 6.0336249E+10    TRU A= 3.0995842       VX= 1723.0544       X= 1.6581663E+11   RMASS= 1491.9086
DAYS=  200.0000     MEAN ANOMALY= 2.9957663         NODE= 0               VY= 17018.006       Y=-4.5628679E+09   REVS.= 0.9956215
ALFA= 173.63239       PATH ANGLE= 4.2051909         INCL= 0               VZ= 0               Z= 0                DELT= 701021.13
 PSI=-89.413830              DPSI= 4.0552393E-06   THETA= 358.42375       DK= 1.4061767E-06   K= 35.442089          L7= 64.148172
  L1= 4.4285768E-02            L2=-4.3286018          L3= 0               L4=-3.0729415E-07   L5= 5.7980697E-08     L6= 0

STEP=  102. +  2.   ECCENTRICITY= 0.7097314        OMEGA= 3.2271324       V= 82615.622        R= 3.2479934E+10   REFER=SUN  RECT 2
TIME= 2.5919958E+07 SEMILATUS R.= 4.8909410E+10    TRU A=-0.7774381       VX=-30075.257       X=-2.5010726E+10   RMASS= 1385.2128
DAYS=  300.0000     MEAN ANOMALY=-0.1014500         NODE= 0               VY=-76946.865       Y= 2.0722202E+10   REVS.= 1.3898809
ALFA= 349.23223       PATH ANGLE=-18.294441         INCL= 0               VZ= 0               Z= 0                DELT= 44941.064
 PSI=-100.58066              DPSI= 3.2164838E-05   THETA= 500.35714       DK= 3.2119139E-05   K=-31.877349          L7= 69.415372
  L1=-0.2781751                L2=-1.4891948          L3= 0               L4=-5.9798917E-07   L5= 1.4303814E-06     L6= 0

STEP=  149. +  2.   ECCENTRICITY= 0.7808711        OMEGA=-3.1254842       V= 12918.150        R= 1.7620714E+11   REFER=SUN  RECT 2
TIME= 3.4559957E+07 SEMILATUS R.= 3.8665098E+10    TRU A= 3.1138327       VX= 1419.5102       X= 1.7619518E+11   RMASS= 1278.7433
DAYS=  400.0000     MEAN ANOMALY= 3.0007866         NODE= 0               VY= 12839.921       Y=-2.0530482E+09   REVS.= 1.9981456
ALFA= 172.65871       PATH ANGLE= 5.6410958         INCL= 0               VZ= 0               Z= 0                DELT= 606791.25
 PSI=-88.967410              DPSI= 3.4758360E-06   THETA= 719.33241       DK= 1.3073705E-06   K= 41.771118          L7= 75.245789
  L1= 7.8564220E-02            L2=-4.3588583          L3= 0               L4=-2.6530695E-07   L5= 4.3933229E-08     L6= 0
```

```
STEP=  185. + 2.   ECCENTRICITY= 0.8369020          OMEGA=-3.0454146         V= 127398.25         R= 1.5019665E+10    REFER=SUN  RECT 2
TIME= 4.3199956E+07 SEMILATUS R.= 2.7582031E+10     TRU A=-3.4824706E-02  VX= 9821.6571          X=-1.4991405E+10    RMASS= 1178.2644
DAYS=  500.0000     MEAN ANOMALY=-1.6927677E-03      NODE= 0              VY=-127018.61          Y=-9.2092884E+08    REVS.= 2.5097647
ALFA= 233.05221        PATH ANGLE=-0.9090640          INCL= 0             VZ= 0                  Z= 0                 DELT= 10852.285
 PSI= 41.372021              DPSI= 1.1069430E-02     THETA= 903.51528     DK=-3.3932340E-05     K=-82.123343            L7= 82.252939
  L1= 3.3666455E-03            L2= 2.9651766E-03        L3= 0             L4= 1.4466656E-06  L5= 1.1935029E-07          L6= 0

STEP=  186. + 2.   ECCENTRICITY= 0.8369020          OMEGA=-3.0454145         V= 127398.27         R= 1.5019659E+10    REFER=SUN  RECT 2
TIME= 4.3200000E+07 SEMILATUS R.= 2.7582031E+10     TRU A=-3.4798535E-02  VX= 9825.6687          X=-1.4991375E+10    RMASS= 1178.2644
DAYS=  500.0000     MEAN ANOMALY=-1.6914955E-03      NODE= 0              VY=-127018.50          Y=-9.2132075E+08    REVS.= 2.5097688
ALFA= 233.01855        PATH ANGLE=-0.9083809          INCL= 0             VZ= 0                  Z= 0                 DELT= 3.0854492
 PSI= 41.406202              DPSI= 1.1087171E-02     THETA= 903.51678     DK=-3.3917255E-05     K=-82.123446            L7= 82.252939
  L1= 3.3621812E-03            L2= 2.9648084E-03        L3= 0             L4= 1.4468688E-06  L5= 1.1935057E-07          L6= 0

PHASE 1 COMPLETED.  DELV=  12929.  MASS RATIO= 0.68613  ***  TOTAL DELV=  12929.  TOTAL MASS RATIO= 0.68613  PAYLOAD RATIO= 0.05562

 KE=.030 STRUCT=.        ALFPOW= 30.000  PJ/MO= 4.2682486E-04 PJ/M1= 3.8525341E-03 MPP/M1= 0.175 ETAPOW=0.662     ML/M1= 0.50202
 K1=.129  VJET1= 3811.0    VC1= 7810.0     VB1= 13714.463      VSPH1= 8179.7054      M1/MO= 0.111  TSPIR=  0.   TMISSION= 500.000
       C( 418) =  3500.0000          YIELDS  C( 437) =  5.5619008E-02  DELV= 12929.059          PAY= 5.5619008E-02
  24NO 0.002166  4.3200E+07 -88.2800          4.70747E-06  13892.7         1.46362E+10  8.01513E-05 -6.85868E-05
  25 0 0.002157  4.3200E+07 -88.2711          4.70747E-06  13892.7         1.46316E+10 -8.83281E-05  7.86366E-05
  26 0 0.002128  4.3200E+07 -88.2800          4.70700E-06  13892.7         1.46419E+10  1.00861E-04 -7.26786E-05
  27 0 0.002055  4.2200E+07 -88.2800          4.70747E-06  13892.4         1.46469E+10  5.82538E-05 -7.49556E-05
  28 0 0.000331  4.3200E+07 -88.2653          4.72301E-06  13879.1         1.49128E+10  6.53366E-05 -8.29893E-05
-------------------------------------------------------------------------------------------------------------------------------
REFERENCE BODY IS SUN

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 2.2979227E-05  |ISP= 4000.0000|      PFLOW= 8.9625001E-06  REFA= 0            AEXIT= 0

STEP=    0. + 0.   ECCENTRICITY= 0.4878450          OMEGA=-3.1289892         V= 21317.762          R= 1.4959789E+11    REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 7.6623113E+10     TRU A= 3.1289892       VX= 255.88020          X= 1.4959789E+11    RMASS= 1560.1047
DAYS=    0.         MEAN ANOMALY= 3.1096328          NODE= 0              VY= 21316.227          Y= 0                REVS.= 0
ALFA= 177.57756        PATH ANGLE= 0.6877461          INCL= 0             VZ= 0                  Z= 0                 DELT= 432000.00
 PSI=-88.265306              DPSI= 4.7230087E-06     THETA= 0             DK= 8.9814856E-07     K= 28.299997            L7= 77.088012
  L1= 0.1268814                L2=-4.1895291            L3= 0             L4=-3.4643280E-07  L5= 2.5245343E-08          L6= 0

STEP=   38. + 2.   ECCENTRICITY= 0.5709333          OMEGA=-3.0747913         V= 68696.152          R= 4.4007767E+10    REFER=SUN  RECT 2
TIME= 8639955.6     SEMILATUS R.= 6.8172519E+10     TRU A= 0.2774313       VX= 16571.422          X=-4.1426037E+10    RMASS= 1468.0002
DAYS=  100.0000     MEAN ANOMALY= 6.2795700E-02      NODE= 0              VY=-66667.494          Y=-1.4851499E+10    REVS.= 0.5547863
ALFA= 3.0411243        PATH ANGLE= 5.7640718          INCL= 0             VZ= 0                  Z= 0                 DELT= 66513.282
 PSI=-79.082122              DPSI= 3.8912090E-05     THETA= 199.72308     DK=-1.4614214E-05     K=-4.3699532            L7= 81.692156
  L1= 0.5480672                L2=-2.8412979            L3= 0             L4=-1.8260637E-06  L5=-9.0923359E-07          L6= 0

STEP=   72. + 2.   ECCENTRICITY= 0.6478109          OMEGA=-3.1279183         V= 16768.765          R= 1.6677164E+11    REFER=SUN  RECT 2
TIME= 1.7279999E+07 SEMILATUS R.= 5.8776782E+10     TRU A= 3.1138332       VX= 1090.1793          X= 1.6675510E+11    RMASS= 1380.8755
DAYS=  200.0000     MEAN ANOMALY= 3.0426967          NODE= 0              VY= 16733.290          Y=-2.3489121E+09    REVS.= 0.9977583
ALFA= 175.09080        PATH ANGLE= 2.9205559          INCL= 0             VZ= 0                  Z= 0                 DELT= 562978.62
 PSI=-88.818385              DPSI= 3.8755323E-06     THETA= 359.19298     DK= 1.3627841E-06     K= 37.660158            L7= 86.699344
  L1= 9.0276589E-02            L2=-4.3768380            L3= 0             L4=-2.9704212E-07  L5= 4.1856863E-08          L6= 0

STEP=  108. + 2.   ECCENTRICITY= 0.7191506          OMEGA= 3.2209582         V= 88525.743          R= 2.8939045E+10    REFER=SUN  RECT 2
TIME= 2.5919999E+07 SEMILATUS R.= 4.7955389E+10     TRU A=-0.4136425       VX=-14254.044          X=-2.7337209E+10    RMASS= 1295.4601
DAYS=  300.0000     MEAN ANOMALY=-4.8101936E-02      NODE= 0              VY=-87370.645          Y= 9.4945017E+09    REVS.= 1.4467982
ALFA= 355.70179        PATH ANGLE=-9.8867954          INCL= 0             VZ= 0                  Z= 0                 DELT= 29935.598
 PSI=-94.967666              DPSI= 6.4856442E-05     THETA= 520.84734     DK= 2.9004976E-05     K=-36.770218            L7= 92.272442
  L1=-0.1587228                L2=-1.8260781            L3= 0             L4=-1.9840975E-06  L5= 1.1339600E-06          L6= 0

STEP=  152. + 2.   ECCENTRICITY= 0.7866527          OMEGA=-3.1264217         V= 12664.981          R= 1.7777790E+11    REFER=SUN  RECT 2
TIME= 3.4559997E+07 SEMILATUS R.= 3.7960789E+10     TRU A= 3.1200851       VX= 1080.2763          X= 1.7777434E+11    RMASS= 1211.4817
DAYS=  400.0000     MEAN ANOMALY= 3.0304552          NODE= 0              VY= 12618.825          Y=-1.1265010E+09    REVS.= 1.9989915
ALFA= 173.76490        PATH ANGLE= 4.5300036          INCL= 0             VZ= 0                  Z= 0                 DELT= 648749.16
 PSI=-88.657576              DPSI= 3.3628694E-06     THETA= 719.63693     DK= 1.3623856E-06     K= 45.206621            L7= 98.386829
  L1= 0.1038648                L2=-4.4335508            L3= 0             L4=-2.6120445E-07  L5= 3.5968601E-08          L6= 0

STEP=  188. + 2.   ECCENTRICITY= 0.8395701          OMEGA=-3.0498740         V= 127944.87          R= 1.4912852E+10    REFER=SUN  RECT 2
TIME= 4.3199957E+07 SEMILATUS R.= 2.7417549E+10     TRU A=-5.0064442E-02  VX= 8247.0884          X=-1.4899917E+10    RMASS= 1131.5998
DAYS=  500.0000     MEAN ANOMALY=-2.3728205E-03      NODE= 0              VY=-127678.80          Y=-6.2100276E+08    REVS.= 2.5066294
ALFA= 325.46619        PATH ANGLE=-1.3091338          INCL= 0             VZ= 0                  Z= 0                 DELT= 10662.935
 PSI=-51.770464              DPSI=-6.0710085E-03     THETA= 902.38660     DK=-2.7253494E-05     K=-105.10148            L7= 105.48676
  L1= 6.8960022E-03            L2=-8.7539663E-03        L3= 0             L4= 1.4140135E-06  L5= 1.1318018E-07          L6= 0

STEP=  189. + 2.   ECCENTRICITY= 0.8395701          OMEGA=-3.0498740         V= 127944.91          R= 1.4912844E+10    REFER=SUN  RECT 2
TIME= 4.3200000E+07 SEMILATUS R.= 2.7417548E+10     TRU A=-5.0041054E-02  VX= 8248.7141          X=-1.4899894E+10    RMASS= 1131.5998
DAYS=  500.0000     MEAN ANOMALY=-2.3717119E-03      NODE= 0              VY=-127678.73          Y=-6.2135091E+08    REVS.= 2.5066332
ALFA= 325.48346        PATH ANGLE=-1.3085222          INCL= 0             VZ= 0                  Z= 0                 DELT= 2.7268066
 PSI=-51.787022              DPSI=-6.0733353E-03     THETA= 902.38793     DK=-2.7241661E-05     K=-105.10156            L7= 105.48676
  L1= 6.8921465E-03            L2=-8.7542750E-03        L3= 0             L4= 1.4140121E-06  L5= 1.1317913E-07          L6= 0

PHASE 1 COMPLETED.  DELV=  12596.  MASS RATIO= 0.72534  ***  TOTAL DELV=  12596.  TOTAL MASS RATIO= 0.72534  PAYLOAD RATIO= 0.05232

 KE=.030 STRUCT=.        ALFPOW= 30.000  PJ/MO= 4.4486557E-04 PJ/M1= 4.4198422E-03 MPP/M1= 0.192 ETAPOW=0.690     ML/M1= 0.52480
 K1=.129  VJET1= 3811.0    VC1= 7810.0     VB1= 13879.104      VSPH1= 8452.8469      M1/MO= 0.101  TSPIR=  0.   TMISSION= 500.000
       C( 418) =  4000.0000          YIELDS  C( 437) =  5.2822234E-02  DELV= 12596.468          PAY= 5.2822234E-02
```

EXAMPLE 3 - JUPITER CAPTURE MISSION WITH HIGH-THRUST DEPARTURE AND CAPTURE AND OPTIMUM VEHICLE PARAMETERS

This example illustrates (1) the analytic high-thrust approximations for the departure and capture phases; (2) the use of transversality conditions to optimize the initial mass flow rate, specific impulse, high-thrust launch velocity, and high-thrust retrobraking; and (3) the alternate method of specifying the initial values of the adjoint variables. We will specify a two-dimensional solar system model with only the Sun's gravitational force acting on the spacecraft. The spacecraft will start its heliocentric path on the x-axis at 1 AU with a velocity equal to Earth's circular velocity plus an incremental velocity from the high-thrust launch vehicle. The launch vehicle is assumed to inject the electric spacecraft at 185-kilometer altitude, after which it coasts to a sphere of influence of radius 150 times the launch radius. Hence,

$$R_0 = (1.49597893\times10^{11},\ 0,\ 0)\ \text{m}$$

$$V_0 = (0,\ 29765.2,\ 0)\ \text{m/sec}$$

$$v_{c,l} = 7795 \quad \text{m/sec (circular speed at 185 km)}$$

$$r_{s,d}/r_l = 150$$

The launch vehicle performance simulates the Atlas/Centaur/SLV-3C:

$$k_l = 0.369$$

$$c_l = 4001\ \text{m/sec}$$

Instead of specifying the reference mass of the launch vehicle in low Earth orbit, a nondimensional approach will be used to permit simple scaling to any reference mass. This is done by specifying that the initial heliocentric mass of the electric vehicle is some convenient number - 1000 kilograms - and letting the program print out the appropriate mass ratios. The electric vehicle assumptions are

(1) Specific powerplant mass, α_{ps}, 34 kg/kW
(2) Structure mass factor, k_s, 0.1
(3) Tankage mass factor, k_t, 0.1
(4) Specific impulse, I, 3650 sec
(5) Initial thrust-weight ratio, f/m_0g, 3.73×10^{-5}
(6) Powerplant efficiency, the default efficiency curve

(7) Type of power source, solar panels using built-in power profile

(8) Thrust program, optimum angle with coast arcs permitted

Since the specific impulse and initial thrust-weight ratio are to be optimized, the values for I and f/m_0g quoted simply serve as first estimates. Likewise, the launch velocity v_l and the spacecraft velocity just prior to retrofire must also be estimated, although both will be optimized:

(1) Launch velocity, v_l, 11540 m/sec

(2) Velocity just before retrofire, v_r, 43200 m/sec

After 1200 days of flight time, the high-thrust retropropulsion unit is assumed to brake the entire spacecraft into a parabolic orbit about Jupiter with a periapsis of 2 Jupiter radii. The Jovian sphere of influence for this maneuver is assumed to be 345 times the periapse radius. Hence,

$$t_0 = 0$$

$$t_f = 1.368\times10^8 \text{ sec (1200 days)}$$

$$e_r = 1$$

$$v_{c,r} = 30500 \text{ m/sec (circular speed at 2 Jupiter radii)}$$

$$r_{s,a}/r_r = 345$$

The retropropulsion unit parameters are

$$c_r = 2940$$

$$k_{rt} = 0.2$$

Instead of guessing initial values of the adjoint variables Λ, Λ_r, and λ_m, we will use the alternate set of thrust program variables; namely, the thrust angle ψ_0, its derivative $\dot{\psi}_0$, and the engine on-off switch function κ_0:

$$\psi_0 = 103^\circ$$

$$\dot{\psi}_0 = 8.97\times10^{-6} \text{ deg/sec}$$

$$\kappa_0 = 29$$

A value of $\dot{\kappa}_0$ is not required because the central travel angle θ_a is left open for optimization. The other desired target conditions are assumed to be

(1) Jupiter's heliocentric radius, $\bar{r}_a$, 7.778×10^{11} m

(2) Jupiter's orbit speed, $\bar{v}_a$, 13050 m/sec

(3) Jupiter's path angle, $\bar{\gamma}_a$, 0^o

These three conditions plus the four transversality conditions for optimum c, f/m_0g, v_l, and v_r comprise a seven-variable level 1 boundary-value problem. The presence of the vehicle-related transversality conditions requires generating the partial derivative matrix G with the finite difference method. Hence, the NOPT=7 option must be used. In this option the COMMON locations of r_a, $\tilde{v}_a$, $\tilde{\gamma}_a$ must be loaded into the IB vector and the locations of ψ_0, $\dot{\psi}_0$, and κ_0 into the IA vector. (The locations of the four vehicle-related variables and their transversality conditions are set by the program by inputting OPTA=T, etc.)

The nondefault input values are given here in the same order as presented in the input instructions:

VMASS=1000, ISP=3650, TB=1.0368E8, NOPT=7,

TW=3.73E-5, SOLAR=T, KE=0.1, STRUCT=0.1,

ALFPOW=34, VB1=11540, RRAT1=150, VC1=7795

VJET1=4001, K1=0.369, VB2=43200, RRAT2=345

VC2=30500, VJET2=2940, K2=0.2, ECC2=1,

R=1.49597893D11, 0, 0, V=0, 29765.2, 0,

PS=103, DPS=8.97E-6, KAPPA=29, EREF=1.E-3,

ERLIMT=3.E-3, DELMAX=3.456E7, IA=343, 344, 345,

IB=480, 493, 479, DESIRE=7.778E11, 13050, 5*0,

OPTA=T, OPTC=T, OPTVB1=T, OPTVB2=T

The output for this example is reproduced on the following pages.

EXAMPLE 3 - JUPITER ORBITER

```
SAVED INITIAL DATA FOR STAGE 1 OF CASE   1.

REFERENCE BODY IS SUN

2 DIMENSIONS  15 DIFF.EQNS.  T/W= 3.7259999E-05  ISP= 3650.0000     PFLOW= 1.0219178E-05  REFA= 0          AEXIT= 0

STEP=    0. +  0.   ECCENTRICITY= 0.2446139     OMEGA= 0.1217890      V= 33213.777        R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0             SEMILATUS R.= 1.8592056E+11 TRU A=-0.1217890     VX=-793.57553       X= 1.4959789E+11   RMASS= 1000.0000
DAYS=    0.         MEAN ANOMALY=-7.1740789E-02  NODE= 0             VY= 33204.286       Y= 0               REVS.= 0
ALFA=-11.630213       PATH ANGLE=-1.3697864      INCL= 0             VZ= 0               Z= 0                DELT= 1036800.0
 PSI= 103.00000              DPSI= 8.9699996E-06 THETA= 0            DK= 2.9379649E-06   K= 29.000000           L7= 6.7942726
  L1=-0.2245510                L2= 0.9743701        L3= 0            L4= 1.7100721E-07 L5=-4.4758067E-08      L6= 0

STEP=   29. +  2.   ECCENTRICITY= 0.5250132     OMEGA=-5.1901622      V= 16429.182        R= 4.6346922E+11   REFER=SUN  RECT 2
TIME= 3.4559557E+07 SEMILATUS R.= 3.1750373E+11 TRU A= 2.2141383     VX=-6162.2346       X=-4.5713118E+11   RMASS= 825.80685
DAYS=  400.0000     MEAN ANOMALY= 1.1584204      NODE= 0             VY=-15229.737       Y=-7.6385905E+10   REVS.= 0.5263511
ALFA= 4.2471085       PATH ANGLE= 31.515560      INCL= 0             VZ= 0               Z= 0                DELT= 2162021.2
 PSI=-116.27629              DPSI= 1.5164135E-06 THETA= 189.48639    DK=-6.5042652E-07   K= 14.303463           L7= 14.867560
  L1=-0.2975382                L2=-0.6034609        L3= 0            L4=-2.2614573E-08 L5=-5.5700164E-09      L6= 0

STEP=   40. +  2.   ECCENTRICITY= 0.4799415     OMEGA=-4.8343219      V= 11989.573        R= 6.7336884E+11   REFER=SUN  RECT 2
TIME= 6.6285345E+07 SEMILATUS R.= 4.0787876E+11 TRU A= 2.5348311     VX= 4864.2631       X=-4.4839376E+11   RMASS= 783.51517
DAYS=  767.1515     MEAN ANOMALY= 1.7735078      NODE= 0             VY=-10958.504       Y=-5.0236305E+11   REVS.= 0.6340247
ALFA= 7.6532669       PATH ANGLE= 24.313395      INCL= 0             VZ= 0               Z= 0                DELT= 29.034986
 PSI=-73.717786              DPSI= 1.4344809E-06 THETA= 228.24888    DK=-2.9561101E-07   K= 0                   L7= 16.049241
  L1= 9.8495858E-02            L2=-0.3372181        L3= 0            L4=-6.6285374E-09 L5=-8.6772047E-09      L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  0

STEP=   40. +  2.   ECCENTRICITY= 0.4799415     OMEGA=-4.8343219      V= 11989.573        R= 6.7336884E+11   REFER=SUN  RECT 2
TIME= 6.6285345E+07 SEMILATUS R.= 4.0787876E+11 TRU A= 2.5348311     VX= 4864.2631       X=-4.4839376E+11   RMASS= 783.51517
DAYS=  767.1515     MEAN ANOMALY= 1.7735078      NODE= 0             VY=-10958.504       Y=-5.0236305E+11   REVS.= 0.6340247
ALFA= 7.6532669       PATH ANGLE= 24.313395      INCL= 0             VZ= 0               Z= 0                DELT= 2326153.7
 PSI=-73.717786              DPSI= 1.4344809E-06 THETA= 228.24888    DK=-2.9561101E-07   K= 0                   L7= 16.049241
  L1= 9.8495858E-02            L2=-0.3372181        L3= 0            L4=-6.6285374E-09 L5=-8.6772047E-09      L6= 0

STEP=   42. +  2.   ECCENTRICITY= 0.4799415     OMEGA=-4.8343219      V= 11661.268        R= 6.8689979E+11   REFER=SUN  RECT 2
TIME= 6.9119996E+07 SEMILATUS R.= 4.0787876E+11 TRU A= 2.5799103     VX= 5391.8772       X=-4.3384592E+11   RMASS= 783.51517
DAYS=  800.0000     MEAN ANOMALY= 1.8581534      NODE= 0             VY=-10339.866       Y=-5.3254957E+11   REVS.= 0.6411992
ALFA= 7.1104088       PATH ANGLE= 23.291273      INCL= 0             VZ= 0               Z= 0                DELT= 508496.94
 PSI=-69.569558              DPSI= 1.4534311E-06 THETA= 230.83173    DK=-2.7032387E-07   K=-0.8020886           L7= 16.049241
  L1= 0.1165003                L2=-0.3127578        L3= 0            L4=-6.0866344E-09 L5=-8.5816424E-09      L6= 0

STEP=   48. +  2.   ECCENTRICITY= 0.4799396     OMEGA=-4.8343267      V= 9465.3947        R= 7.8061689E+11   REFER=SUN  RECT 2
TIME= 1.0368000E+08 SEMILATUS R.= 4.0787816E+11 TRU A= 3.0405672     VX= 8998.6744       X=-1.7261039E+11   RMASS= 783.51517
DAYS= 1200.0000     MEAN ANOMALY= 2.8901662      NODE= 0             VY=-2935.5672       Y=-7.6129389E+11   REVS.= 0.7145143
ALFA= 347.46035       PATH ANGLE= 5.2526217      INCL= 0             VZ= 0               Z= 0                DELT= 4820869.6
 PSI=-5.5278316              DPSI= 1.9336885E-06 THETA= 257.22514    DK= 8.1699706E-08   K=-4.2870117           L7= 16.049241
  L1= 0.2562708                L2=-2.4801715E-02    L3= 0            L4=-2.6170798E-09 L5=-8.4766711E-09      L6= 0

PHASE 1 COMPLETED.  DELV=   8733.  MASS RATIO= 0.78352  ***  TOTAL DELV=   8733.  TOTAL MASS RATIO= 0.78352  PAYLOAD RATIO= 0.04976

 KE=.100 STRUCT=.100    ALFPOW= 34.000  PJ/MO= 1.0991951E-03 PJ/M1= 6.5465583E-03 MPP/M1= 0.244 ETAPOW=0.646      ML/ML= 0.29637
 K1=.369  VJET1= 4001.0     VC1= 7795.0     VB1= 11540.000      VSPH1= 3529.5483       M1/MO= 0.168  TSPIR=  0.   TMISSION=1200.000
 K2=.200  VJET2= 2940.0     VC2=30500.0     VB2= 43200.000      VSPH2= 3336.5803                     ECC2=1.000 CAPSML/MO= 0.05141

   NOPT= 7, COAST=T, EPHEM=F, NBVP=1, KBODYS=1, ERSTAR=  1.00000, NSWEEP=   0, IBB= 437

  IAA        IA        IB          DESIRE                 WEIGHT             PERTEW              PERTNR

   0        343       480       7.7779999E+11          7.778E+11          -1.000E-02          -1.000E-04
   0        344       493       13050.000              1.305E+04          -1.000E-02          -1.000E-04
   0        345       479       0                      360.0              -1.000E-02          -1.000E-04
   0        418       359       0                      1.000              -1.000E-02          -1.000E-04
   0        408       360       0                      1.000              -1.000E-02          -1.000E-04
   0        429       361       0                      1.000              -1.000E-02          -1.000E-04
   0        430       362       0                      1.000              -1.000E-02          -1.000E-04

 RUN N  ERROR     TIME                       7 INDEPENDENT VARIABLES --  7 DEPENDENT VARIABLES

   1 1 0.427300  1.0368E+08  103.000       8.97000E-06  29.0000       3650.00       3.73000E-05  11540.0        43200.0       7.80617E+11
   N                         12743.0       2.03368     -6.59810E-02 -9.25637E-02  8.34189E-02  0.40262
   2 1 0.410784  1.0368E+08  102.990       8.97000E-06  29.0000       3650.00       3.73000E-05  11540.0        43200.0       7.80653E+11
                             12744.3       1.96880     -6.61078E-02 -9.22259E-02  8.25550E-02  0.38522
   3 1 0.423958  1.0368E+08  102.998       8.97000E-06  29.0000       3650.00       3.73000E-05  11540.0        43200.0       7.80624E+11
                             12743.2       2.02047     -6.60065E-02 -9.24963E-02  8.33273E-02  0.39911
   4 1 0.426631  1.0368E+08  103.000       8.97000E-06  29.0000       3650.00       3.73000E-05  11540.0        43200.0       7.80618E+11

                             12743.0       2.03103     -6.59861E-02 -9.25502E-02  8.34006E-02  0.40192
   5 1 0.428223  1.0368E+08  103.000       8.96910E-06  29.0000       3650.00       3.73000E-05  11540.0        43200.0       7.80529E+11
                             12740.8       1.99699     -6.61314E-02 -9.24503E-02  8.24056E-02  0.40360
   6 1 0.429579  1.0368E+08  103.000       8.97000E-06  28.9971       3650.00       3.73000E-05  11540.0        43200.0       7.80582E+11
                             12742.4       2.03351     -6.62041E-02 -9.23026E-02  8.36665E-02  0.40501
   7 1 0.427759  1.0368E+08  103.000       8.97000E-06  28.9994       3650.00       3.73000E-05  11540.0        43200.0       7.80610E+11
                             12742.9       2.03366     -6.60257E-02 -9.25115E-02  8.34685E-02  0.40311
   8 1 0.424539  1.0368E+08  103.000       8.97000E-06  29.0000       3649.63       3.73000E-05  11540.0        43200.0       7.80664E+11
                             12743.4       2.03521     -6.56097E-02 -9.29087E-02  8.31514E-02  0.40016
   9 1 0.426829  1.0368E+08  103.000       8.97000E-06  29.0000       3649.93       3.73000E-05  11540.0        43200.0       7.80626E+11
                             12743.1       2.03400     -6.59067E-02 -9.26327E-02  8.33654E-02  0.40213
  10 1 0.429435  1.0368E+08  103.000       8.97000E-06  29.0000       3650.00       3.72563E-05  11540.0        43200.0       7.80439E+11
                             12743.4       2.01464     -6.59499E-02 -9.26060E-02  8.35823E-02  0.40486
  11 1 0.427126  1.0368E+08  103.000       8.97000E-06  29.0000       3650.00       3.72993E-05  11540.0        43200.0       7.80581E+11
                             12743.1       2.02987     -6.59748E-02 -9.25722E-02  8.24516E-02  0.40307
  12 1 0.407168  1.0368E+08  103.000       8.97000E-06  29.0000       3650.00       3.73000E-05  11538.8        43200.0       7.80102E+11
                             12747.7       1.87888     -6.58740E-02 -9.25024E-02  8.14292E-02  0.38169
```

13	1	0.423230	1.0368E+08	103.000	8.97000E-06	29.0000	3650.00	3.73000E-05	11539.8	43200.0	7.80514E+11
				12743.9	2.00228	-6.59598E-02	-9.25516E-02	8.20231E-02	0.39840		
14	1	0.426488	1.0368E+08	103.000	8.97000E-06	29.0000	3650.00	3.73000E-05	11540.0	43200.0	7.80596E+11
				12743.2	2.02739	-6.59768E-02	-9.25613E-02	8.33357E-02	0.40178		
15	1	0.405932	1.0368E+08	103.000	8.97000E-06	29.0000	3650.00	3.73000E-05	11540.0	43195.7	7.80617E+11
				12687.3	2.07476	-6.42701E-02	-9.37807E-02	8.66136E-02	0.37886		
16	1	0.423028	1.0368E+08	103.000	8.97000E-06	29.0000	3650.00	3.73000E-05	11540.0	43199.1	7.80617E+11
				12731.9	2.04181	-6.56385E-02	-9.28073E-02	8.40584E-02	0.39790		
17	1	0.426445	1.0368E+08	103.000	8.97000E-06	29.0000	3650.00	3.73000E-05	11540.0	43195.8	7.80617E+11
				12740.8	2.03530	-6.59125E-02	-9.26125E-02	8.35466E-02	0.40168		
18	1	0.241393	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74965E+11
N				12906.4	-0.19497	1.50560E-02	-5.07052E-03	-3.31684E-02	0.23829		
19	1	0.240561	1.0368E+08	103.373	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74967E+11
				12906.4	-0.19643	1.50500E-02	-5.05899E-03	-3.31824E-02	0.23785		
20	1	0.240534	1.0368E+08	103.373	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74968E+11
				12906.4	-0.19787	1.50439E-02	-5.04748E-03	-3.31964E-02	0.23742		
21	1	0.239665	1.0368E+08	103.372	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74971E+11
				12906.3	-0.20075	1.50317E-02	-5.02452E-03	-3.32241E-02	0.23656		
22	1	0.241610	1.0368E+08	103.374	9.10142E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74818E+11
				12905.3	-0.23376	1.49110E-02	-4.98987E-03	-3.31317E-02	0.23872		
23	1	0.241668	1.0368E+08	103.374	9.10233E-06	21.5486	3040.10	3.84458E-05	11410.8	43226.2	7.74955E+11
				12906.4	-0.19586	1.50280E-02	-5.04318E-03	-3.31360E-02	0.23858		
24	1	0.241948	1.0368E+08	103.374	9.10233E-06	21.5481	3040.10	3.84458E-05	11410.8	43226.2	7.74945E+11
				12906.4	-0.19673	1.50000E-02	-5.01590E-03	-3.31036E-02	0.23887		
25	1	0.242501	1.0368E+08	103.374	9.10233E-06	21.5473	3040.10	3.84458E-05	11410.8	43226.2	7.74924E+11
				12906.4	-0.19850	1.49439E-02	-4.96129E-03	-3.30387E-02	0.23944		
26	1	0.241102	1.0368E+08	103.374	9.10233E-06	21.5490	3040.04	3.84458E-05	11410.8	43226.2	7.74978E+11
				12906.4	-0.19376	1.51117E-02	-5.10959E-03	-3.32106E-02	0.23799		
27	1	0.240809	1.0368E+08	103.374	9.10233E-06	21.5490	3039.98	3.84458E-05	11410.8	43226.2	7.74992E+11
				12906.4	-0.19255	1.51674E-02	-5.14862E-03	-3.32527E-02	0.23768		
28	1	0.240228	1.0368E+08	103.374	9.10233E-06	21.5490	3039.86	3.84458E-05	11410.8	43226.2	7.75018E+11
				12906.3	-0.19014	1.52787E-02	-5.22677E-03	-3.33369E-02	0.23707		
29	1	0.241641	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84450E-05	11410.8	43226.2	7.74923E+11
				12906.6	-0.19974	1.50618E-02	-5.08047E-03	-3.31342E-02	0.23855		
30	1	0.241893	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84443E-05	11410.8	43226.2	7.74881E+11
				12906.9	-0.20450	1.50675E-02	-5.09041E-03	-3.31000E-02	0.23881		
31	1	0.242397	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84427E-05	11410.8	43226.2	7.74797E+11
				12907.3	-0.21403	1.50790E-02	-5.11032E-03	-3.30315E-02	0.23933		
32	1	0.240725	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.2	7.74946E+11
				12906.5	-0.19981	1.50605E-02	-5.06815E-03	-3.32495E-02	0.23760		
33	1	0.240062	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.7	43226.2	7.74928E+11
				12906.6	-0.20465	1.50650E-02	-5.06580E-03	-3.33306E-02	0.23692		
34	1	0.240703	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43226.1	7.74965E+11
				12904.4	-0.19316	1.51196E-02	-5.11355E-03	-3.30510E-02	0.23760		
35	1	0.240016	1.0368E+08	103.374	9.10233E-06	21.5490	3040.10	3.84458E-05	11410.8	43225.9	7.74965E+11
				12902.4	-0.19136	1.51829E-02	-5.15645E-03	-3.29339E-02	0.23691		
36	1	0.018825	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.9	7.79475E+11
N				13086.9	0.40537	1.58752E-03	7.31613E-04	-4.00831E-03	1.79265E-02		
37	1	0.017654	1.0368E+08	102.729	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.9	7.79496E+11
				13087.2	0.40494	1.57436E-03	7.84735E-04	-4.09436E-03	1.67060E-02		
38	1	0.018558	1.0368E+08	102.730	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.9	7.79479E+11
				13087.0	0.40528	1.58485E-03	7.42250E-04	-4.02550E-03	1.76821E-02		
39	1	0.018989	1.0368E+08	102.731	9.13335E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.9	7.79324E+11
				13084.8	0.36543	1.41719E-03	8.01760E-04	-3.92595E-03	1.81800E-02		
40	1	0.019543	1.0368E+08	102.731	9.13426E-06	22.1675	3093.34	3.80806E-05	11440.9	43231.9	7.79427E+11
				13086.5	0.39870	1.46030E-03	8.36909E-04	-3.84573E-03	1.87320E-02		
41	1	0.020262	1.0368E+08	102.731	9.13426E-06	22.1657	3093.34	3.80806E-05	11440.9	43231.9	7.79381E+11
				13086.1	0.39209	1.33319E-03	9.42169E-04	-3.68339E-03	1.95282E-02		
42	1	0.018083	1.0368E+08	102.731	9.13426E-06	22.1692	3093.09	3.80806E-05	11440.9	43231.9	7.79535E+11
				13087.2	0.41315	1.82697E-03	5.78800E-04	-4.20860E-03	1.70648E-02		
43	1	0.017367	1.0368E+08	102.731	9.13426E-06	22.1692	3092.84	3.80806E-05	11440.9	43231.9	7.79595E+11
				13087.4	0.42090	2.06628E-03	4.25989E-04	-4.40905E-03	1.62110E-02		
44	1	0.019411	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80776E-05	11440.9	43231.9	7.79304E+11
				13087.4	0.38476	1.60069E-03	6.84898E-04	-3.84274E-03	1.85990E-02		
45	1	0.020016	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80745E-05	11440.9	43231.9	7.79132E+11
				13087.8	0.36415	1.61394E-03	6.38271E-04	-3.67751E-03	1.92801E-02		
46	1	0.018036	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.8	43231.9	7.79445E+11
				13087.3	0.39985	1.60141E-03	7.41071E-04	-4.18438E-03	1.70523E-02		
47	1	0.017263	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.7	43231.9	7.79416E+11
				13087.7	0.39434	1.61520E-03	7.50492E-04	-4.36026E-03	1.61846E-02		
48	1	0.017652	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.5	7.79475E+11
				13083.0	0.40950	1.71618E-03	6.46042E-04	-3.76645E-03	1.68261E-02		
49	1	0.018598	1.0368E+08	102.731	9.13426E-06	22.1692	3093.34	3.80806E-05	11440.9	43231.8	7.79475E+11
				13086.1	0.40620	1.61329E-03	7.14473E-04	-3.95587E-03	1.77065E-02		
50	1	0.000119	1.0368E+08	102.699	9.12338E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77825E+11
N				13049.7	1.64015E-03	-5.10127E-06	-1.80237E-06	-2.22572E-05	-1.08915E-04		
51	1	0.000355	1.0368E+08	102.699	9.12338E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77829E+11
				13049.7	1.60595E-03	-7.81297E-06	8.82265E-06	-3.93663E-05	-3.49909E-04		
52	1	0.000594	1.0368E+08	102.699	9.12338E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77833E+11
				13049.8	1.57506E-03	-1.05088E-05	1.93695E-05	-5.63744E-05	-5.88881E-04		
53	1	0.001074	1.0368E+08	102.698	9.12338E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77842E+11
				13049.9	1.52437E-03	-1.58530E-05	4.05027E-05	-9.04926E-05	-1.06808E-03		
54	1	0.002032	1.0368E+08	102.697	9.12338E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77859E+11
				13050.1	1.40641E-03	-2.66938E-05	8.28153E-05	-1.58632E-04	-2.02213E-03		
55	1	0.000360	1.0368E+08	102.699	9.12247E-06	22.2629	3100.27	3.80487E-05	11443.1	43229.5	7.77670E+11
				13047.7	-3.86772E-02	-1.76644E-04	6.80621E-05	5.97778E-05	1.37918E-04		
56	1	0.001546	1.0368E+08	102.699	9.12338E-06	22.2594	3100.27	3.80487E-05	11443.1	43229.5	7.77728E+11
				13048.9	-1.21853E-02	-2.61956E-04	2.10391E-04	3.04464E-04	1.47251E-03		
57	1	0.000228	1.0368E+08	102.699	9.12338E-06	22.2622	3100.27	3.80487E-05	11443.1	43229.5	7.77806E+11
				13049.5	-1.13903E-03	-5.65576E-05	4.06223E-05	4.32902E-05	2.09734E-04		
58	1	0.001927	1.0368E+08	102.699	9.12338E-06	22.2629	3099.78	3.80487E-05	11443.1	43229.5	7.77945E+11
				13050.2	1.76393E-02	4.76756E-04	-3.09359E-04	-4.24249E-04	-1.78126E-03		
59	1	0.000476	1.0368E+08	102.699	9.12338E-06	22.2629	3100.18	3.80487E-05	11443.1	43229.5	7.77849E+11
				13049.8	4.84545E-03	9.13297E-05	-6.32625E-05	-1.02738E-04	-4.45911E-04		
60	1	0.001222	1.0368E+08	102.695	9.12338E-06	22.2629	3100.27	3.80426E-05	11443.1	43229.5	7.77480E+11
				13050.7	-3.98722E-02	2.12194E-05	-9.42529E-05	3.05346E-04	1.20923E-03		

```
61 1 0.000173  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80475E-05  11443.1      43225.5      7.77756E+11
                           13049.9    -6.66687E-03  1.34564E-07 -2.03334E-05  4.23817E-05  1.55336E-04
62 1 0.001654  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11442.9      43229.5      7.77766E+11
                           13050.5    -8.90933E-03  2.23536E-05  1.66951E-05 -3.73235E-04 -1.81439E-03
63 1 0.000461  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11443.1      43229.5      7.77813E+11
                           13049.8    -4.74839E-04  4.05643E-07  1.89779E-06 -9.25768E-05 -4.50924E-04
64 1 0.000343  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11443.1      43225.4      7.77825E+11
                           13048.9     2.49726E-03  2.07008E-05 -1.88890E-05  2.60656E-05 -3.28779E-04
65 1 0.000576  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11443.1      43225.4      7.77825E+11
                           13048.1     3.35259E-03  4.65157E-05 -3.59840E-05  7.43580E-05 -5.48129E-04
66 1 0.001043  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11443.1      43225.2      7.77825E+11
                           13046.5     5.06155E-03  9.79307E-05 -7.00319E-05  1.70632E-04 -9.86352E-04
67 1 0.001582  1.0368E+08  102.699     9.12338E-06  22.2629      3100.27      3.80487E-05  11443.1      43228.9      7.77825E+11
                           13043.4     8.48763E-03  2.00956E-04 -1.38257E-04  3.62482E-04 -1.86457E-03
68 1 0.000002  1.0368E+08  102.699     9.12346E-06  22.2625      3100.26      3.80481E-05  11443.1      43229.5      7.77800E+11
 N                         13050.0    -1.79936E-06  1.47359E-07 -1.01805E-07  3.10153E-07 -1.83255E-06

REFERENCE BODY IS SUN

2 DIMENSIONS  15 DIFF.EQNS.  T/W= 3.8048110E-05  ISP= 3100.2576      PFLOW= 1.2272564E-05  REFA= 0             AEXIT= 0

STEP=    0. +  0.  ECCENTRICITY= 0.2206041        OMEGA= 0.1184343       V= 32893.094         R= 1.4959789E+11   REFER=SUN  RECT 2
TIME= 0            SEMILATUS R.= 1.8236860E+11   TRU A=-0.1184343      VX=-703.16301         X= 1.4959789E+11   RMASS= 1000.0000
DAYS=    0.        MEAN ANOMALY=-7.3823441E-02     NODE= 0             VY= 32885.578         Y= 0               REVS.= 0
ALFA=-11.474305      PATH ANGLE=-1.2249179         INCL= 0             VZ= 0                 Z= 0                DELT= 1036800.0
 PSI= 102.69922             DPSI= 9.1234558E-06  THETA= 0              DK= 2.4541203E-06    K= 22.262549          L7= 8.1405917
  L1=-0.2198330               L2= 0.9755375         L3= 0              L4= 1.7308391E-07  L5=-4.3739735E-08      L6= 0

STEP=   29. +  2.  ECCENTRICITY= 0.5154938        OMEGA=-5.0881774       V= 16726.532         R= 4.5646380E+11   REFER=SUN  RECT 2
TIME= 3.4559997E+07 SEMILATUS R.= 3.2298634E+11  TRU A= 2.1739649      VX=-5150.6483         X=-4.4471458E+11   RMASS= 786.96352
DAYS=  400.0000    MEAN ANOMALY= 1.1299006         NODE= 0             VY=-15913.758         Y=-1.0289871E+11   REVS.= 0.5361887
ALFA= 1.5062675      PATH ANGLE= 30.962614         INCL= 0             VZ= 0                 Z= 0                DELT= 1984997.1
 PSI=-109.44097             DPSI= 1.7669130E-06  THETA= 193.02791      DK=-5.5033957E-07    K= 9.5635965          L7= 16.940683
  L1=-0.2283295               L2=-0.6469294         L3= 0              L4=-2.4691590E-08  L5=-6.3913075E-09      L6= 0

STEP=   38. +  2.  ECCENTRICITY= 0.4781054        OMEGA=-4.7409714       V= 12938.977         R= 6.3439650E+11   REFER=SUN  RECT 2
TIME= 5.9690872E+07 SEMILATUS R.= 4.0791872E+11  TRU A= 2.4138703      VX= 4499.6794         X=-4.3534718E+11   RMASS= 743.53381
DAYS=  690.8666    MEAN ANOMALY= 1.5673169         NODE= 0             VY=-12131.364         Y=-4.6144529E+11   REVS.= 0.6296304
ALFA= 358.63235      PATH ANGLE= 26.316463         INCL= 0             VZ= 0                 Z= 0                DELT= 4688.1303
 PSI=-68.282867             DPSI= 1.6631173E-06  THETA= 226.66693      DK=-2.3602332E-07    K= 4.7683716E-07      L7= 18.198613
  L1= 0.1646838               L2=-0.4134725         L3= 0              L4=-9.8674233E-09  L5=-1.0143279E-08      L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(5)) =  4.7683716E-07

STEP=   38. +  2.  ECCENTRICITY= 0.4781054        OMEGA=-4.7409714       V= 12938.977         R= 6.3439650E+11   REFER=SUN  RECT 2
TIME= 5.9690872E+07 SEMILATUS R.= 4.0791872E+11  TRU A= 2.4138703      VX= 4499.6794         X=-4.3534718E+11   RMASS= 743.53381
DAYS=  690.8666    MEAN ANOMALY= 1.5673169         NODE= 0             VY=-12131.364         Y=-4.6144529E+11   REVS.= 0.6296304
ALFA= 358.63235      PATH ANGLE= 26.316463         INCL= 0             VZ= 0                 Z= 0                DELT= 1805142.8
 PSI=-68.282867             DPSI= 1.6631173E-06  THETA= 226.66693      DK=-2.3602332E-07    K= 4.7683716E-07      L7= 18.198613
  L1= 0.1646838               L2=-0.4134725         L3= 0              L4=-9.8674233E-09  L5=-1.0143279E-08      L6= 0

STEP=   42. +  2.  ECCENTRICITY= 0.4781054        OMEGA=-4.7409714       V= 11725.115         R= 6.8329083E+11   REFER=SUN  RECT 2
TIME= 6.9119996E+07 SEMILATUS R.= 4.0791872E+11  TRU A= 2.5734996      VX= 6300.3589         X=-3.8393809E+11   RMASS= 743.53381
DAYS=  800.0000    MEAN ANOMALY= 1.8498045         NODE= 0             VY=-9888.5656         Y=-5.6522376E+11   REVS.= 0.6550361
ALFA= 354.66752      PATH ANGLE= 23.310349         INCL= 0             VZ= 0                 Z= 0                DELT= 1505995.1
 PSI=-52.184871             DPSI= 1.7494427E-06  THETA= 235.81301      DK=-1.3569002E-07    K=-1.7490680          L7= 18.198613
  L1= 0.2466486               L2=-0.3178041         L3= 0              L4=-7.6691130E-09  L5=-1.0152575E-08      L6= 0

STEP=   46. +  3.  ECCENTRICITY= 0.4781029        OMEGA=-4.7409784       V= 9501.4211         R= 7.7779596E+11   REFER=SUN  RECT 2
TIME= 1.0368000E+08 SEMILATUS R.= 4.0791782E+11  TRU A= 3.0381891      VX= 9260.2002         X=-1.0226635E+11   RMASS= 743.53381
DAYS= 1200.0000    MEAN ANOMALY= 2.8852111         NODE= 0             VY=-2127.3683         Y=-7.7103431E+11   REVS.= 0.7289926
ALFA= 340.70033      PATH ANGLE= 5.3755587         INCL= 0             VZ= 0                 Z= 0                DELT= 1.0060053E+07
 PSI= 6.3614607             DPSI= 1.4599399E-06  THETA= 262.43735      DK= 1.8461425E-07    K=-0.5779645          L7= 18.198613
  L1= 0.4282741               L2= 4.7746962E-02     L3= 0              L4=-3.2704649E-09  L5=-1.1413001E-08      L6= 0

PHASE 1 COMPLETED.  DELV=    9010.  MASS RATIO= 0.74353  ***  TOTAL DELV=    9010.  TOTAL MASS RATIO= 0.74353  PAYLOAD RATIO= 0.04976


KE=.100 STRUCT=.100    ALFPOW= 34.000  PJ/MO= 1.0270034E-03 PJ/M1= 5.6720784E-03 MPP/M1= 0.314 ETAPOW=0.613      ML/M1= 0.27481
K1=.369  VJET1= 4001.0     VC1= 7795.0     VB1= 11443.124      VSPH1= 3198.6239      M1/MO= 0.181  TSPIR=  0.   TMISSION=1200.000
K2=.200  VJET2= 2940.0     VC2=30500.0     VB2= 43229.507      VSPH2= 3699.0572                      ECC2=1.000 CAPSML/MO= 0.05213
```

The optimization of f/m_0g, c, v_l, and v_r could also have been accomplished with the level 2 optimization scheme instead of with transversality conditions. In this case, the OPTA, OPTC, OPTVB1, and OPTVB2=T statements would be deleted from the input list and, instead, the following would be needed:

IAA = 408, 418, 429, 430

These numbers are the COMMON locations of the four vehicle-related variables to be optimized. By default, the optimization criterion is payload ratio m_n/m_{ref} (IBB=437). Also, it would be more economical in this case to change NOPT=7 to NOPT=3 so that the partial derivative matrix G would be integrated rather than computed by finite differencing. The computer execution time on the IBM 7094II is 0.9 minute.

SINGLE-STAGE LAUNCH VEHICLE WITH CHEMICAL AND NUCLEAR PROPULSION

The performance of an advanced, hypothetical single-stage Earth shuttle is sought. The shuttle uses conventional chemical propulsion during lift-off and ascent through the atmosphere but switches to nuclear propulsion for the upper trajectory phase. This upper phase terminates with injection into a 444 165-meter (240-n mi) circular parking orbit. A zero angle-of-attack thrust program is assumed for the chemical boost phase and an optimal steering program for the nuclear upper phase. To be specific, assume that the shuttle has the following description:

Vehicle:

(1) Gross lift-off mass, 2×10^6 kg
(2) Maximum cross-sectional area, 100 m^2
(3) Drag coefficient, C_D, $0.4 + 0.6\ M^2$ $(0 \le M \le 1)$;
$1.15306 - 0.16326\ M + 0.010204\ M^2$ $(1 < M \le 8)$; 0.5 $(M > 8)$

Chemical engine:

(1) Vacuum specific impulse, 425 sec
(2) Ratio of thrust to lift-off weight, 1.25
(3) Exit area, 40 m^2
(4) Specific weight, 0.02

Nuclear engine:

(1) Vacuum specific impulse, 1200 sec
(2) Propellant flow rate, 140 kg/sec
(3) Specific weight, 1/3

The payoff criterion is payload delivered into orbit, and for this calculation it will be assumed that the tankage factor k_t is 0.1 and the structure factor k_s is 0.053. This particular value of structure factor is simply the total engine mass divided by the gross lift-off mass:

$$\frac{\text{(Chemical engine mass)} + \text{(Nuclear engine mass)}}{\text{Gross lift-off mass}} = \frac{0.02\, m_0\left(\frac{f_0}{m_0 g}\right) + \frac{1}{3}(\dot{m}I)_{nuclear}}{m_0}$$

$$= \frac{0.02(2\times10^6)(1.25) + \frac{1}{3}(140)(1200)}{2\times10^6}$$

$$= 0.053$$

Of course, in any real shuttle there would be many additional items (such as radiation shielding, reentry structure, landing engines, and fuel) that should also be subtracted from the injected weight to calculate net payload, but these items are simply lumped together with net payload and called gross payload in this illustration.

A rotating, spherical Earth model is assumed and, for convenience, a due-eastward launch from an equatorial site is assumed so that the calculations need be done in only two dimensions. The launch site is also assumed to be at the Greenwich meridian (zero longitude) and 10 meters above mean sea level. The short vertical rise segment t_v is assumed to be 20 seconds. After 20 seconds, the vehicle is instantaneously tilted to 90° azimuth (eastward launch) and to an elevation angle initially assumed to be 89.4° but later optimized for maximum gross payload. The elevation angle γ strongly affects the amount of trajectory lofting and must be carefully chosen to avoid paths that go straight up (γ too close to 90°) and paths that fall back to Earth (γ too far from 90°). Lift-off acceleration and vertical rise duration strongly affect the proper choice of γ, and experience dictated the choice of $t_v = 20$ seconds and $\gamma = 89.4^\circ$.

The level 1 boundary-value problem is set up as follows:

Independent variables	Dependent variables (at burnout)
Thrust angle at start of optimal steering, ψ_0 Thrust angle rate at start of optimal steering, $\dot{\psi}_0$ Nuclear engine firing time, $\left(t_f\right)_2$	Altitude, $r_a - r_0$, 444 165 m Velocity, v_a, 7643.8 m/sec Path angle, γ_a, 0

With this set of conditions plus the usual optimal-travel-angle assumption, the NOPT(2)=7 option is needed for the second stage. (Option NOPT(1)=0 is required for the first stage.)

A level 2 optimization scheme is set up so that the initial elevation angle γ and the amount of chemical propulsion $\left(t_f\right)_1$ are optimized to yield maximum gross payload. The initial guesses for the level 1 and level 2 independent variables are

Level 1:

$$\psi_0 = 54^\circ$$

$$\dot{\psi}_0 = 0.053 \text{ deg/sec}$$

$$\left(t_f\right)_2 = 1100 \text{ sec}$$

Level 2:

$$\gamma = 89.4^\circ$$

$$\left(t_f\right)_1 = 220 \text{ sec}$$

Finally, the use of the trajectory interrupt feature is illustrated by requiring a trajectory step printout to occur if the path angle attains zero before orbit injection. This occurs whenever the acceleration level is low enough to produce lob-type trajectories:

The input for this case is given here:

```
NUMBOD=11, ROTATE=T, VMASS=2. E6, ISP=425, 1200,
TB=220, 1100, NOPT=0, 7, TW=1. 25, PFLOW(2)=140,
KE=0. 1, STRUCT=0. 053, REFA=100, AEXIT=40,
CD0C=0, 0. 4, 0, 0. 6, 1, 1. 15306, -0. 16326, 0. 010204, 8, 0. 5, 0, 0, 100,
LAT=0, LONG=0, ALT0=10, ELEV=89. 4, AZI=90, TKICK=20,
```

COAST=F, PS=54, DPS=0.053, MODEI=4, DELMAX=100,

LOOKX=-479, IA=343, 344, 2, IB=1263, 493, 479,

DESIRE=444165, 7643.8, 0, IAA=48, 1, PERT2=0.0003

The small perturbation size (0.0003) for the elevation angle is necessary to prevent non-convergence difficulties during the level 2 search procedure. The output for this case is presented below. Note that two trajectory phases are indeed listed: the zero angle-of-attack, chemical propulsion, atmospheric phase; and the optimum steering, nuclear propulsion vacuum phase. The level 1 boundary-value problem concerns only phase 2, while the level 2 optimization is over both phases. The IBM 7094II computer execution time is 4.5 minutes.

EXAMPLE 4 - NUCLEAR BOOSTER

```
SAVED INITIAL DATA FOR STAGE 1 OF CASE   1.

REFERENCE BODY IS EART

STEP=   0. +  0.     LAT.= 0                  LONG.= 0                AZI.= 90.0000000     ELEV.= 89.3999996     ALT.= 10.0000000
TIME= 0              VEL.= 0                  RMASS= 2000000.00        X= 6378170.00          Y= 0                  Z= 0

2 DIMENSIONS   7 DIFF.EQNS.  T/W= 1.2500000      ISP= 425.00000       PFLOW= 6854.8022      REFA= 100.00000      AEXIT= 40.000000

STEP=    0. +  0.   ECCENTRICITY= 0.9965287     OMEGA=-3.1397000       V= 469.35613      R= 6378720.4       REFER=EART RECT 2
TIME= 20.000000     SEMILATUS R.= 22143.050     TRU A= 3.1411584      VX= 57.377169      X= 6378713.7       RMASS= 1862904.0
DAYS=    0.0002     MEAN ANOMALY= 3.1208004      NODE= 0              VY= 465.83565      Y= 9302.8701       REVS.= 2.3211524E-04
ALFA= 0               PATH ANGLE= 7.1053368      INCL= 0              VZ= 0              Z= 0                DELT= 2.2000000
BETA= 0             R PATH ANGLE= 89.399998      DRAG= 4.3850195E-02  VR= 58.055677      G= 1.3518681        PUSH= 13.301148
ALT.= 560.43750       MACH NUMBER= 0.1715502     LIFT= 0              CD= 0.4116577      Q= 1955.8770        HEAT= 0.1219146

STEP=   13. +  2.   ECCENTRICITY= 0.9897790     OMEGA=-3.1265241       V= 948.46774      R= 6399349.9       REFER=EART RECT 2
TIME= 99.999957     SEMILATUS R.= 65546.738     TRU A= 3.1349661      VX= 504.70255      X= 6399121.9       RMASS= 1314519.8
DAYS=    0.0012     MEAN ANOMALY= 2.9578824      NODE= 0              VY= 803.02574      Y= 54022.475       REVS.= 1.3435812E-03
ALFA= 0               PATH ANGLE= 32.632774      INCL= 0              VZ= 0              Z= 0                DELT= 12.443851
BETA= 0             R PATH ANGLE= 57.003933      DRAG= 0.8935276      VR= 609.82334      G= 2.1108796        PUSH= 21.594186
ALT.= 21189.875       MACH NUMBER= 2.0595339     LIFT= 0              CD= 0.8601026      Q= 13656.043        HEAT= 12.670442

STEP=   13. +  4.   ECCENTRICITY= 0.9897790     OMEGA=-3.1265241       V= 948.46774      R= 6399349.9       REFER=EART RECT 2
TIME= 99.595957     SEMILATUS R.= 65546.738     TRU A= 3.1349661      VX= 504.70255      X= 6399121.9       RMASS= 1314519.8
DAYS=    0.0012     MEAN ANOMALY= 2.9578824      NODE= 0              VY= 803.03574      Y= 54022.475       REVS.= 1.3435812E-03
ALFA= 0               PATH ANGLE= 32.632774      INCL= 0              VZ= 0              Z= 0                DELT= 12.445473
BETA= 0             R PATH ANGLE= 57.003933      DRAG= 0.8935276      VR= 609.82334      G= 2.1108796        PUSH= 21.594186
ALT.= 21189.875       MACH NUMBER= 2.0555339     LIFT= 0              CD= 0.8601026      Q= 13656.043        HEAT= 12.670442

STEP=   24. +  4.   ECCENTRICITY= 0.8416291     OMEGA=-3.0159442       V= 3478.6038      R= 6494296.7       REFER=EART RECT 2
TIME= 199.99599     SEMILATUS R.= 1050632.5     TRU A= 3.0515858      VX= 1360.2888      X= 6490172.2       RMASS= 629039.59
DAYS=    0.0023     MEAN ANOMALY= 2.5842967      NODE= 0              VY= 3201.6088      Y= 231418.35       REVS.= 5.6725424E-03
ALFA= 0               PATH ANGLE= 25.061620      INCL= 0              VZ= 0              Z= 0                DELT= 8.6473604
BETA= 0             R PATH ANGLE= 28.825070      DRAG= 1.4705086E-05  VR= 3056.2083      G= 4.6313300        PUSH= 45.417847
ALT.= 116136.69       MACH NUMBER= 8.4880325     LIFT= 0              CD= 0.5000000      Q= 0.1850016        HEAT= 1.7976722E-03

STEP=   27. +  4.   ECCENTRICITY= 0.7423421     OMEGA=-2.9337468       V= 4423.5327      R= 6526866.4       REFER=EART RECT 2
TIME= 220.00000     SEMILATUS R.= 1744528.5     TRU A= 2.9803745      VX= 1610.5387      X= 6519772.6       RMASS= 491943.52
DAYS=    0.0025     MEAN ANOMALY= 2.4287575      NODE= 0              VY= 4119.7716      Y= 304222.80       REVS.= 7.4210361E-03
ALFA= 0               PATH ANGLE= 24.028374      INCL= 0              VZ= 0              Z= 0                DELT= 4.0277823
BETA= 0             R PATH ANGLE= 26.809944      DRAG= 1.6706407E-06  VR= 3993.5349      G= 5.9220025        PUSH= 58.075008
ALT.= 148706.44       MACH NUMBER= 6.5097824     LIFT= 0              CD= 0.5226906      Q= 1.5723660E-02    HEAT= 2.5528534E-04

PHASE 1 COMPLETED.  DEL V=    5846.  MASS RATIO= 0.24597

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 0.3415026      ISP= 1200.0000       PFLOW= 140.00000      REFA= 0              AEXIT= 0

STEP=   27. +  4.   ECCENTRICITY= 0.7423421     OMEGA=-2.9337468       V= 4423.5327      R= 6526866.4       REFER=EART RECT 2
TIME= 220.00000     SEMILATUS R.= 1744528.5     TRU A= 2.9803745      VX= 1610.9387      X= 6519772.6       RMASS= 491943.52
DAYS=    0.0025     MEAN ANOMALY= 2.4287575      NODE= 0              VY= 4119.7716      Y= 304222.80       REVS.= 7.4210361E-03
ALFA= 14.643199       PATH ANGLE= 24.028374      INCL= 0              VZ= 0              Z= 0                DELT= 11.000000
BETA= 0             R PATH ANGLE= 26.809944      DRAG= 0              VR= 3993.5345      G= 0.3415026        PUSH= 3.3489966
ALT.= 148706.44       MACH NUMBER= 6.5097824     LIFT= 0              CD= 0.5226906      Q= 1.5723660E-02    HEAT= 2.5528534E-04
 PSI= 54.000000               DPSI= 5.2999998E-02  THETA= 2.6715730   DK=-2.2959281E-05  K= 0                  L7= 2.3921404E-02
  L1= 0.5877853                 L2= 0.8090170      L3= 0             L4= 1.3125049E-03  L5= 2.3276243E-04      L6= 0

STEP=   30. +  4.   ECCENTRICITY= 0.7162203     OMEGA=-2.9046912       V= 4414.4061      R= 6656227.8       REFER=EART RECT 2
TIME= 299.99595     SEMILATUS R.= 1935863.0     TRU A= 3.0011162      VX= 1030.5033      X= 6625307.8       RMASS= 480743.52
DAYS=    0.0035     MEAN ANOMALY= 2.5583230      NODE= 0              VY= 4292.4404      Y= 640832.31       REVS.= 1.5346507E-02
ALFA= 18.314229       PATH ANGLE= 19.024485      INCL= 0              VZ= 0              Z= 0                DELT= 45.999996
BETA= 0             R PATH ANGLE= 21.315077      DRAG= 0              VR= 3958.7011      G= 0.3494587        PUSH= 3.4270190
ALT.= 278067.81       MACH NUMBER= 4.7258207     LIFT= 0              CD= 0.6094123      Q= 4.3293290E-04    HEAT= 7.1300054E-06
 PSI= 58.186029               DPSI= 5.1714303E-02  THETA= 5.5247426   DK=-2.1560393E-05  K=-1.7784238E-03      L7= 2.4457774E-02
  L1= 0.4884128                 L2= 0.7873005      L3= 0             L4= 1.1749210E-03  L5= 3.0762257E-04      L6= 0
```

```
STEP=   32. + 4.   ECCENTRICITY= 0.6789390     OMEGA=-2.8679470     V= 4495.1187        R= 6779503.8      REFER=EART RECT 2
TIME= 399.99559    SEMILATUS R.= 2206345.5     TRU A= 3.0279098    VX= 325.23558       X= 6692951.4      RMASS= 466743.52
DAYS=    0.0046    MEAN ANOMALY= 2.7090487      NODE= 0            VY= 4483.3373       Y= 1079849.2      REVS.= 2.5458864E-02
ALFA= 22.552283      PATH ANGLE= 13.314341      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE= 14.538983      DRAG= 0            VR= 4015.6516       G= 0.3599407       PUSH= 3.5298127
ALT.= 401343.81     MACH NUMBER= 4.3020591      LIFT= 0            CD= 0.6355585       Q= 5.1335688E-05   HEAT= 8.8333839E-07
 PSI= 63.298566             DPSI= 5.0632794E-02 THETA= 9.1651911   DK=-2.0236996E-05  K=-3.8649293E-03     L7= 2.5106420E-02
  L1= 0.3785626               L2= 0.7526411       L3= 0            L4= 1.0257764E-03  L5= 3.8251008E-04     L6= 0

STEP=   34. + 4.   ECCENTRICITY= 0.6361366     OMEGA=-2.8306689     V= 4661.0650        R= 6864933.8      REFER=EART RECT 2
TIME= 499.99559    SEMILATUS R.= 2513732.2     TRU A= 3.0564113    VX=-367.84452       X= 6690758.1      RMASS= 452743.52
DAYS=    0.0058    MEAN ANOMALY= 2.8473027      NODE= 0            VY= 4646.5275       Y= 1536578.3      REVS.= 3.5928024E-02
ALFA= 26.194279      PATH ANGLE= 8.4076816      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE= 9.4142852      DRAG= 0            VR= 4166.4891       G= 0.3710710       PUSH= 3.6389636
ALT.= 486773.81     MACH NUMBER= 4.2508385      LIFT= 0            CD= 0.6434506       Q= 1.6314481E-05   HEAT= 3.0027643E-07
 PSI= 68.332027             DPSI= 5.0136012E-02 THETA= 12.934089   DK=-1.9254909E-05  K=-5.8371550E-03     L7= 2.5732515E-02
  L1= 0.2826155               L2= 0.7113386       L3= 0            L4= 8.9596645E-04  L5= 4.4114044E-04     L6= 0

STEP=   36. + 4.   ECCENTRICITY= 0.5673783     OMEGA=-2.7927703     V= 4895.6850        R= 6917415.6      REFER=EART RECT 2
TIME= 599.99559    SEMILATUS R.= 2860259.0     TRU A= 3.0873177    VX=-1057.0914       X= 6619507.7      RMASS= 438743.52
DAYS=    0.0069    MEAN ANOMALY= 2.9728449      NODE= 0            VY= 4780.2018       Y= 2008172.4      REVS.= 4.6878669E-02
ALFA= 29.122110      PATH ANGLE= 4.4066427      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE= 4.9121516      DRAG= 0            VR= 4392.5252       G= 0.3829116       PUSH= 3.7550804
ALT.= 539255.63     MACH NUMBER= 4.3858102      LIFT= 0            CD= 0.6330151       Q= 9.2793775E-06   HEAT= 1.8581977E-07
 PSI= 73.346567             DPSI= 5.0264911E-02 THETA= 16.876321   DK=-1.8511265E-05  K=-7.7238322E-03     L7= 2.6337064E-02
  L1= 0.1988746               L2= 0.6648465       L3= 0            L4= 7.8104716E-04  L5= 4.8673408E-04     L6= 0

STEP=   38. + 4.   ECCENTRICITY= 0.5321913     OMEGA=-2.7541569     V= 5184.8259        R= 6941797.9      REFER=EART RECT 2
TIME= 699.99559    SEMILATUS R.= 3248196.8     TRU A= 3.1212638    VX=-1748.5706       X= 6479263.2      RMASS= 424743.52
DAYS=    0.0081    MEAN ANOMALY= 3.0852315      NODE= 0            VY= 4881.0820       Y= 2491526.7      REVS.= 5.8426882E-02
ALFA= 31.300487      PATH ANGLE= 1.3244159      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE= 1.4676941      DRAG= 0            VR= 4678.7758       G= 0.3955328       PUSH= 3.8788519
ALT.= 563637.88     MACH NUMBER= 4.6349823      LIFT= 0            CD= 0.6154287       Q= 7.8145064E-06   HEAT= 1.7216188E-07
 PSI= 78.408774             DPSI= 5.1108908E-02 THETA= 21.033677   DK=-1.7927276E-05  K=-9.5447153E-03     L7= 2.6920407E-02
  L1= 0.1260106               L2= 0.6143536       L3= 0            L4= 6.7802569E-04  L5= 5.2145217E-04     L6= 0

STEP=   43. + 4.   ECCENTRICITY= 0.4986825     OMEGA= 3.5507417     V= 5263.9946        R= 6945040.5      REFER=EART RECT 2
TIME= 755.36558    SEMILATUS R.= 3481670.6     TRU A=-3.1415926    VX=-2133.9515       X= 6371794.6      RMASS= 416992.34
DAYS=    0.0087    MEAN ANOMALY=-3.1415926      NODE= 0            VY= 4921.2487       Y= 2762937.1      REVS.= 6.5118099E-02
ALFA= 32.182456      PATH ANGLE=-1.9686544E-07  INCL= 0            VZ= 0               Z= 0               DELT= 2.7416115E-05
BETA= 0            R PATH ANGLE=-2.1739030E-07  DRAG= 0            VR= 4857.5541       G= 0.4028851       PUSH= 3.9509532
ALT.= 566880.50     MACH NUMBER= 4.8089253      LIFT= 0            CD= 0.6039301       Q= 8.1001096E-06   HEAT= 1.8871675E-07
 PSI= 81.260059             DPSI= 5.1936421E-02 THETA= 23.442516   DK=-1.7648346E-05  K=-1.0529439E-02     L7= 2.7234215E-02
  L1= 8.9942942E-02           L2= 0.5850519       L3= 0            L4= 6.2535042E-04  L5= 5.3656873E-04     L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(1)) = -1.9686544E-07

STEP=   43. + 4.   ECCENTRICITY= 0.4986825     OMEGA= 3.5507417     V= 5263.9946        R= 6945040.5      REFER=EART RECT 2
TIME= 755.36558    SEMILATUS R.= 3481670.6     TRU A=-3.1415926    VX=-2133.9515       X= 6371794.6      RMASS= 416992.34
DAYS=    0.0087    MEAN ANOMALY=-3.1415926      NODE= 0            VY= 4921.2487       Y= 2762937.1      REVS.= 6.5118099E-02
ALFA= 32.182456      PATH ANGLE=-1.9686544E-07  INCL= 0            VZ= 0               Z= 0               DELT= 25.000000
BETA= 0            R PATH ANGLE=-2.1739030E-07  DRAG= 0            VR= 4857.5541       G= 0.4028851       PUSH= 3.9509532
ALT.= 566880.50     MACH NUMBER= 4.8089253      LIFT= 0            CD= 0.6039301       Q= 8.1001096E-06   HEAT= 1.8871675E-07
 PSI= 81.260059             DPSI= 5.1936421E-02 THETA= 23.442516   DK=-1.7648346E-05  K=-1.0529439E-02     L7= 2.7234215E-02
  L1= 8.9942942E-02           L2= 0.5850519       L3= 0            L4= 6.2535042E-04  L5= 5.3656873E-04     L6= 0

STEP=   45. + 4.   ECCENTRICITY= 0.4700513     OMEGA= 3.5684226     V= 5516.8204        R= 6943112.8      REFER=EART RECT 2
TIME= 799.99559    SEMILATUS R.= 3679981.2     TRU A=-3.1243110    VX=-2446.3936       X= 6269580.3      RMASS= 410743.52
DAYS=    0.0093    MEAN ANOMALY=-3.0992840      NODE= 0            VY= 4944.7413       Y= 2983149.0      REVS.= 7.0682559E-02
ALFA= 32.726565      PATH ANGLE=-0.8780223      INCL= 0            VZ= 0               Z= 0               DELT= 19.634412
BETA= 0            R PATH ANGLE=-0.9667393      DRAG= 0            VR= 5010.5861       G= 0.4090144       PUSH= 4.0110607
ALT.= 564952.81     MACH NUMBER= 4.9636354      LIFT= 0            CD= 0.5540957       Q= 8.8210748E-06   HEAT= 2.1521340E-07
 PSI= 83.597178             DPSI= 5.2820664E-02 THETA= 25.445721   DK=-1.7437487E-05  K=-1.1312421E-02     L7= 2.7482384E-02
  L1= 6.2939364E-02           L2= 0.5608677       L3= 0            L4= 5.8493385E-04  L5= 5.4680927E-04     L6= 0

STEP=   47. + 4.   ECCENTRICITY= 0.4003594     OMEGA= 3.6085352     V= 5882.2321        R= 6926779.5      REFER=EART RECT 2
TIME= 899.99559    SEMILATUS R.= 4158448.3     TRU A=-3.0823225    VX=-3152.8935       X= 5989693.6      RMASS= 396743.52
DAYS=    0.0104    MEAN ANOMALY=-3.0148536      NODE= 0            VY= 4965.8753       Y= 3479058.1      REVS.= 8.3749348E-02
ALFA= 33.402239      PATH ANGLE=-2.2621854      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE=-2.4746155      DRAG= 0            VR= 5377.5538       G= 0.4234474       PUSH= 4.1526001
ALT.= 548619.50     MACH NUMBER= 5.3566276      LIFT= 0            CD= 0.5713250       Q= 1.2391962E-05   HEAT= 3.3592705E-07
 PSI= 89.009711             DPSI= 5.5640616E-02 THETA= 30.149765   DK=-1.6981771E-05  K=-1.3033350E-02     L7= 2.8022481E-02
  L1= 8.7337730E-03           L2= 0.5052653       L3= 0            L4= 5.0056404E-04  L5= 5.6395332E-04     L6= 0
STEP=   49. + 4.   ECCENTRICITY= 0.3224062     OMEGA= 3.6488572     V= 6272.4941        R= 6858777.1      REFER=EART RECT 2
TIME= 999.99559    SEMILATUS R.= 4687228.0     TRU A=-3.0348504    VX=-3868.7448       X= 5638688.3      RMASS= 382743.52
DAYS=    0.0116    MEAN ANOMALY=-2.9447514      NODE= 0            VY= 4938.5769       Y= 3974710.0      REVS.= 9.7722222E-02
ALFA= 33.300000      PATH ANGLE=-2.8941692      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE=-3.1463464      DRAG= 0            VR= 5771.1248       G= 0.4389362       PUSH= 4.3044940
ALT.= 520617.13     MACH NUMBER= 5.8041189      LIFT= 0            CD= 0.5492298       Q= 2.0206100E-05   HEAT= 6.0934761E-07
 PSI= 94.774168             DPSI= 5.9532123E-02 THETA= 35.180000   DK=-1.6497772E-05  K=-1.4707844E-02     L7= 2.8539982E-02
  L1=-3.7442742E-02           L2= 0.4483176       L3= 0            L4= 4.2428758E-04  L5= 5.7387993E-04     L6= 0

STEP=   51. + 4.   ECCENTRICITY= 0.2353214     OMEGA= 3.6871902     V= 6684.5807        R= 6865776.3      REFER=EART RECT 2
TIME= 1100.0000    SEMILATUS R.= 5271373.0     TRU A=-2.9791847    VX=-4593.1411       X= 5215658.6      RMASS= 368743.52
DAYS=    0.0127    MEAN ANOMALY=-2.8872805      NODE= 0            VY= 4856.6116       Y= 4464951.4      REVS.= 0.1126826
ALFA= 32.340276      PATH ANGLE=-2.8372087      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE=-3.0667960      DRAG= 0            VR= 6184.5837       G= 0.4556012       PUSH= 4.4679217
ALT.= 487616.31     MACH NUMBER= 6.3068948      LIFT= 0            CD= 0.5292801       Q= 3.5541523E-05   HEAT= 1.1922082E-06
 PSI= 101.06256             DPSI= 6.6223032E-02 THETA= 40.565729   DK=-1.5911779E-05  K=-1.6329531E-02     L7= 2.9034173E-02
  L1=-7.6386449E-02           L2= 0.3906961       L3= 0            L4= 3.5550119E-04  L5= 5.7761092E-04     L6= 0

STEP=   53. + 4.   ECCENTRICITY= 0.1380313     OMEGA= 3.7122355     V= 7110.8765        R= 6835202.9      REFER=EART RECT 2
TIME= 1200.0000    SEMILATUS R.= 5918314.5     TRU A=-2.9036459    VX=-5324.1437       X= 4719839.4      RMASS= 354743.52
DAYS=    0.0139    MEAN ANOMALY=-2.8313757      NODE= 0            VY= 4713.6066       Y= 4943997.9      REVS.= 0.1286910
ALFA= 30.371521      PATH ANGLE=-2.1519044      INCL= 0            VZ= 0               Z= 0               DELT= 50.000000
BETA= 0            R PATH ANGLE=-2.3140628      DRAG= 0            VR= 6612.8254       G= 0.4735816       PUSH= 4.6442488
ALT.= 457042.94     MACH NUMBER= 6.8587048      LIFT= 0            CD= 0.5133227       Q= 6.1805803E-05   HEAT= 2.3042618E-06
 PSI= 108.10915             DPSI= 7.5230240E-02 THETA= 46.328771   DK=-1.5125513E-05  K=-1.7883572E-02     L7= 2.9504641E-02
  L1=-0.1088878               L2= 0.3329625       L3= 0            L4= 2.9546051E-04  L5= 5.7634178E-04     L6= 0
```

```
STEP=   55. + 4.    ECCENTRICITY= 3.0035251E-02   OMEGA= 3.5215729      V= 7549.2360        R= 6815188.5        REFER=EART RECT 2
TIME= 1300.0000     SEMILATUS R.= 6639197.1       TRU A=-2.6056207     VX=-6059.2623       X= 4150693.0        RMASS= 340743.52
DAYS=    0.0150     MEAN ANOMALY=-2.5743408        NODE= 0             VY= 4502.9218       Y= 5405417.9        REVS.= 0.1457783
ALFA= 27.155118       PATH ANGLE=-0.9020390        INCL= 0             VZ= 0               Z= 0                 DELT= 50.000000
BETA= 0             R PATH ANGLE=-0.9656022        DRAG= 0             VR= 7052.3304       G= 0.4930394         PUSH= 4.8350653
ALT.= 437028.50      MACH NUMBER= 7.3583961        LIFT= 0             CD= 0.5037267       Q= 9.3415645E-05    HEAT= 3.8668261E-06
 PSI= 116.22711               DPSI= 8.7786118E-02 THETA= 52.480190     DK=-1.3996605E-05   K=-1.9343287E-02      L7= 2.9951738E-02
  L1=-0.1357474                 L2= 0.2755459       L3= 0             L4= 2.4307687E-04  L5= 5.7153681E-04      L6= 0

STEP=   56. + 4.    ECCENTRICITY= 1.0677141E-02   OMEGA= 2.7822057      V= 7638.1855        R= 6813206.7        REFER=EART RECT 2
TIME= 1320.0000     SEMILATUS R.= 6793582.2       TRU A=-1.8439675     VX=-6206.5614       X= 4028034.8        RMASS= 337943.52
DAYS=    0.0153     MEAN ANOMALY=-1.8233609        NODE= 0             VY= 4452.0191       Y= 5454972.4        REVS.= 0.1493253
ALFA= 26.335252       PATH ANGLE=-0.5907141        INCL= 0             VZ= 0               Z= 0                 DELT= 20.000006
BETA= 0             R PATH ANGLE=-0.6318097        DRAG= 0             VR= 7141.3868       G= 0.4971249         PUSH= 4.8751259
ALT.= 435046.75      MACH NUMBER= 7.5003893        LIFT= 0             CD= 0.5025810       Q= 9.8568712E-05    HEAT= 4.1658872E-06
 PSI= 118.01255               DPSI= 9.0786190E-02 THETA= 53.757091     DK=-1.3712645E-05   K=-1.9620418E-02      L7= 3.0038482E-02
  L1=-0.1405123                 L2= 0.2641275       L3= 0             L4= 2.3356582E-04  L5= 5.7020036E-04      L6= 0

PHASE 2 COMPLETED.  DELV=   4419.  MASS RATIO= 0.68696  ***  TOTAL DELV=  10264.  TOTAL MASS RATIO= 0.16897  PAYLOAD RATIO= 0.03287

 KE=.100 STRUCT=.053    ALFPOW=  0.     PJ/M0= 25.545258      PJ/M1= 25.545258      MPP/M1= 0.     ETAPOW=0.058        ML/M1= 0.03287

  NOPT= 7, COAST=F, EPHEM=F, NBVP=2, KBODYS=1, ERSTAR=  1.00000, NSWEEP=   0, I88= 437

 IAA        IA        IB              DESIRE                  WEIGHT              PERTEW              PERTNR

  48       343      1263          444165.00                 4.442E+05           -1.000E-02          -1.000E-04
   1       344       493          7643.8000                 7.644E+03           -1.000E-02          -1.000E-04
   0         2       479          0                         360.0               -1.000E-02          -1.000E-04

 RUN N  ERROR     TIME                        3 INDEPENDENT VARIABLES --  3 DEPENDENT VARIABLES

   1NO 0.020600  1320.0         54.0000     5.30000E-02  1100.00     4.35047E+05  7638.19      -0.59071
   2 0 0.020591  1320.0         53.9946     5.30000E-02  1100.00     4.35054E+05  7638.32      -0.59327
   3 0 0.020575  1320.0         53.9892     5.30000E-02  1100.00     4.35061E+05  7638.47      -0.59551
   4 0 0.020542  1320.0         53.9784     5.30000E-02  1100.00     4.35076E+05  7638.79      -0.60047
   5 0 0.020291  1320.0         54.0000     5.29947E-02  1100.00     4.35189E+05  7637.62      -0.58694
   6 0 0.020591  1319.9         54.0000     5.30000E-02  1099.89     4.35055E+05  7637.69      -0.59264
   7 0 0.020574  1319.8         54.0000     5.30000E-02  1099.78     4.35064E+05  7637.20      -0.59424
   8 0 0.020542  1319.6         54.0000     5.30000E-02  1099.56     4.35082E+05  7636.22      -0.59793
   9NO 0.001067  1332.0         54.2568     5.26112E-02  1112.04     4.44613E+05  7641.12       2.64017E-03
  10 0 0.001140  1332.0         54.2351     5.26112E-02  1112.04     4.44657E+05  7641.70      -6.71439E-03
  11 0 0.001397  1332.0         54.2568     5.26060E-02  1112.04     4.44755E+05  7640.52       6.39795E-03
  12 0 0.001172  1331.6         54.2568     5.26112E-02  1111.60     4.44612E+05  7639.22      -5.49723E-03
  13 0 0.001319  1331.2         54.2568     5.26112E-02  1111.15     4.44613E+05  7637.32      -1.32944E-02
  14 0 0.001687  1330.3         54.2568     5.26112E-02  1110.27     4.44615E+05  7633.51      -2.93550E-02
  15 0 0.002565  1328.5         54.2568     5.26112E-02  1108.49     4.44627E+05  7625.52      -6.08630E-02
  16NO 0.000002  1332.3         54.2659     5.26270E-02  1112.31     4.44164E+05  7643.80      -2.20233E-04
       C(  48) =  89.400000          YIELDS C( 437) =  3.1921132E-02  DELV= 10324.401      PAY= 3.1921132E-02
  17NO 0.194801  1332.3         54.2659     5.26270E-02  1112.31     5.30045E+05  7462.57       0.17808
  18 0 0.195012  1332.3         54.2442     5.26270E-02  1112.31     5.30143E+05  7463.00       0.17056
  19 0 0.195119  1332.3         54.2659     5.26218E-02  1112.31     5.30183E+05  7461.99       0.18173
  20 0 0.194892  1328.7         54.2659     5.26270E-02  1108.75     5.29978E+05  7447.96       0.11437
  21 0 0.194813  1331.6         54.2659     5.26270E-02  1111.60     5.30029E+05  7459.65       0.16528
  22NO 0.041744  1331.0         58.8181     5.51239E-02  1111.02     4.25935E+05  7701.18      -0.47112
  23 0 0.041046  1331.0         58.7946     5.51239E-02  1111.02     4.26247E+05  7700.85      -0.47333
  24 0 0.040341  1331.0         58.7711     5.51239E-02  1111.02     4.26562E+05  7700.53      -0.47547
  25 0 0.041444  1331.0         58.8181     5.51184E-02  1111.02     4.26065E+05  7700.68      -0.46802
  26 0 0.041576  1330.3         58.8181     5.51239E-02  1110.31     4.25981E+05  7698.23      -0.48599
  27NO 0.000113  1333.8         58.7599     5.43771E-02  1113.76     4.44468E+05  7642.20       5.99904E-03
  28 0 0.002160  1333.8         58.7128     5.43771E-02  1113.76     4.45115E+05  7641.50       2.44152E-03
  29 0 0.001004  1333.8         58.7505     5.43771E-02  1113.76     4.44599E+05  7642.06       5.34132E-03
  30 0 0.001016  1333.8         58.7599     5.43717E-02  1113.76     4.44599E+05  7641.68       9.10829E-03
  31 0 0.000853  1333.0         58.7599     5.43771E-02  1113.04     4.44468E+05  7639.40      -9.64560E-03
  32 0 0.001171  1332.3         58.7599     5.43771E-02  1112.33     4.44472E+05  7636.60      -2.52009E-02
  33 0 0.001824  1330.9         58.7599     5.43771E-02  1110.90     4.44477E+05  7630.55      -5.63734E-02
  34NO 0.000020  1333.8         58.7579     5.43907E-02  1113.84     4.44174E+05  7643.80       1.52669E-04
       C(  48) =  89.426819          YIELDS C( 437) =  3.1802968E-02  DELV= 10331.923      PAY= 3.1802968E-02
  35NO 0.027986  1331.8         52.9183     5.20979E-02  1111.85     4.31951E+05  7682.81      -0.36494
  36 0 0.028047  1331.8         52.9098     5.20979E-02  1111.85     4.31928E+05  7683.16      -0.36996
  37 0 0.028108  1331.8         52.9013     5.20979E-02  1111.85     4.31904E+05  7683.52      -0.37485
  38 0 0.027637  1331.8         52.9183     5.20927E-02  1111.85     4.32102E+05  7682.20      -0.36094
  39 0 0.027404  1329.0         52.9183     5.20979E-02  1109.00     4.32099E+05  7669.93      -0.40882
  40 0 0.027665  1331.3         52.9183     5.20979E-02  1111.28     4.31979E+05  7680.23      -0.37382
  41NO 0.000124  1334.3         52.9171     5.16720E-02  1114.31     4.44152E+05  7642.89       6.46827E-03
  42 0 0.000113  1334.3         52.9002     5.16720E-02  1114.31     4.44116E+05  7643.62       [illegible]588E-03
  43 0 [illegible]000366  1334.3         52.9171     5.16668E-02  1114.31     4.44[illegible]E+05  7642.26       [illegible]
  [illegible]47  133[illegible]         52.9171     5.[illegible]720E-02  1113.74     [illegible]  7640.3[illegible]
  [illegible]                   [illegible]171     [illegible]02  1113.17     [illegible]
  [illegible]

  [illegible]
                                            5.62[illegible]  979.006     4.54491E+05  7642.16       0.13569
                                            5.62018E-02  978.266     4.54476E+05  7638.93       0.12388
       [illegible].000[illegible]  1196.9         60.6056     5.63810E-02  974.494     4.44053E+05  7644.49      -5.64604E-03
 358NO 0.000069  1196.9         60.6013     5.63810E-02  974.494     4.44161E+05  7644.31      -5.74537E-03
       C(   1) =  222.43759          YIELDS C( 437) =  3.3342853E-02  DELV= 9971.6410       PAY= 3.3342853E-02
 359NO 0.006035  1186.6         61.7729     5.67395E-02  963.988     4.41517E+05  7650.84      -5.78132E-02
 360 0 0.005756  1186.6         61.7644     5.67395E-02  963.988     4.41640E+05  7650.63      -5.73873E-02
 361 0 0.005485  1186.6         61.7560     5.67395E-02  963.988     4.41760E+05  7650.41      -5.70860E-02
 362 0 0.005720  1186.6         61.7729     5.67304E-02  963.988     4.41654E+05  7650.38      -5.40042E-02
 363 0 0.005536  1185.2         61.7729     5.67395E-02  962.531     4.41531E+05  7644.47      -8.12563E-02
 364 0 0.005542  1183.7         61.7729     5.67395E-02  961.075     4.41550E+05  7638.10      -0.10450
 365NO 0.000043  1186.8         61.7219     5.66126E-02  964.138     4.44146E+05  7643.74      -8.08373E-04
       C(   1) =  222.64879          YIELDS C( 437) =  3.3343980E-02  DELV= 9948.3817       PAY= 3.3343980E-02
 366NO 0.010931  1166.5         63.9634     5.70757E-02  943.427     4.39343E+05  7653.23      -0.12577
 367 0 0.010275  1166.5         63.9458     5.70757E-02  943.427     4.39633E+05  7652.68      -0.12359
 368 0 0.010650  1166.5         63.9634     5.70666E-02  943.427     4.39466E+05  7652.83      -0.12238
 369 0 0.010143  1163.6         63.9634     5.70757E-02  940.576     4.39401E+05  7640.75      -0.17187
 370 0 0.010677  1165.9         63.9634     5.70757E-02  942.857     4.39353E+05  7650.73      -0.13505
 371NO 0.000030  1167.6         63.8881     5.68081E-02  944.490     4.44176E+05  7643.68       2.21924E-04
       C(   1) =  223.07119          YIELDS C( 437) =  3.3264410E-02  DELV= 9906.8184       PAY= 3.3264410E-02
```

```
REFERENCE BODY IS EART

STEP=   0. +  0.     LAT.= 0                     LONG.= 0                   AZI.= 90.0000000        ELEV.= 89.3642712        ALT.= 10.0000000
TIME= 0              VEL.= 0                     RMASS= 2000000.00            X= 6378170.00              Y= 0                    Z= 0

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 1.2500000      ISP= 425.00000         PFLOW= 6854.8022         REFA= 100.00000          AEXIT= 40.000000

STEP=    0. +  0.    ECCENTRICITY= 0.9965282     OMEGA=-3.1396999         V= 469.39201           R= 6378720.4           REFER=EART RECT 2
TIME= 20.000000      SEMILATUS R.= 22146.453     TRU A= 3.1411584        VX= 57.276726           X= 6378713.7           RMASS= 1862904.0
DAYS=    0.0002      MEAN ANOMALY= 3.1208006      NODE= 0                VY= 465.87206           Y= 9302.8701           REVS.= 2.3211524E-04
ALFA= 0                 PATH ANGLE= 7.1047428     INCL= 0                VZ= 0                   Z= 0                    DELT= 2.2264879
BETA= 0              R PATH ANGLE= 89.364269      DRAG= 4.3850194E-02    VR= 58.059676           G= 1.3518681            PUSH= 13.301148
ALT.= 560.43750        MACH NUMBER= 0.1715502     LIFT= 0                CD= 0.4176577           Q= 1955.8770            HEAT= 0.1219146

STEP=    8. +  3.    ECCENTRICITY= 0.9952450     OMEGA=-3.1342482         V= 610.74861           R= 6385855.6           REFER=EART RECT 2
TIME= 65.247620      SEMILATUS R.= 30383.557     TRU A= 3.1391738        VX= 273.04018           X= 6385778.2           RMASS= 1552740.5
DAYS=    0.0008      MEAN ANOMALY= 3.0427730      NODE= 0                VY= 546.21761           Y= 31454.266           REVS.= 7.8393911E-04
ALFA= 0                 PATH ANGLE= 26.837315     INCL= 0                VZ= 0                   Z= 0                    DELT= 2.8410634
BETA= 0              R PATH ANGLE= 73.954230      DRAG= 1.3212091        VR= 286.90530           G= 1.6436605            PUSH= 17.440013
ALT.= 7695.6250        MACH NUMBER= 0.9264910     LIFT= 0                CD= 0.9150313           Q= 22419.941            HEAT= 8.2852222

STEP=   17. +  4.    ECCENTRICITY= 0.9892652     OMEGA=-3.1264041         V= 960.44389           R= 6399203.9           REFER=EART RECT 2
TIME= 99.999996      SEMILATUS R.= 68835.183     TRU A= 3.1349186        VX= 495.42451           X= 6398972.0           RMASS= 1314519.8
DAYS=    0.0012      MEAN ANOMALY= 2.9611097      NODE= 0                VY= 822.80437           Y= 54485.336           REVS.= 1.3551240E-03
ALFA= 0                 PATH ANGLE= 31.540706     INCL= 0                VZ= 0                   Z= 0                    DELT= 5.9494073
BETA= 0              R PATH ANGLE= 54.590351      DRAG= 0.9238064        VR= 613.40442           G= 2.1074643            PUSH= 21.590971
ALT.= 21043.527        MACH NUMBER= 2.0723181     LIFT= 0                CD= 0.8585545           Q= 14144.261            HEAT= 13.200490

STEP=   28. +  4.    ECCENTRICITY= 0.8307139     OMEGA=-3.0185903         V= 3524.1483           R= 6488878.6           REFER=EART RECT 2
TIME= 199.99999      SEMILATUS R.= 1118575.5     TRU A= 3.0552106        VX= 1232.8708           X= 6484528.1           RMASS= 629039.59
DAYS=    0.0023      MEAN ANOMALY= 2.6277867      NODE= 0                VY= 3301.4619           Y= 237571.60           REVS.= 5.8283024E-03
ALFA= 0                 PATH ANGLE= 22.575419     INCL= 0                VZ= 0                   Z= 0                    DELT= 8.3550345
BETA= 0              R PATH ANGLE= 25.942812      DRAG= 3.3237581E-05    VR= 3092.5684           G= 4.6313280            PUSH= 45.417847
ALT.= 110718.56        MACH NUMBER= 9.4177803     LIFT= 0                CD= 0.5000000           Q= 0.4181551            HEAT= 4.1115797E-03

STEP=   32. +  4.    ECCENTRICITY= 0.7063571     OMEGA=-2.9232258         V= 4624.4761           R= 6523008.2           REFER=EART RECT 2
TIME= 222.64879      SEMILATUS R.= 1980825.8     TRU A= 2.9729171        VX= 1466.0930           X= 6514956.5           RMASS= 473786.58
DAYS=    0.0026      MEAN ANOMALY= 2.4631072      NODE= 0                VY= 4385.9265           Y= 324003.53           REVS.= 7.9086195E-03
ALFA= 0                 PATH ANGLE= 21.330449     INCL= 0                VZ= 0                   Z= 0                    DELT= 1.3015041
BETA= 0              R PATH ANGLE= 23.699933      DRAG= 2.3222290E-06    VR= 4184.5720           G= 6.1489516            PUSH= 60.300619
ALT.= 144848.19        MACH NUMBER= 7.1221350     LIFT= 0                CD= 0.5078962           Q= 2.1662714E-02        HEAT= 3.8269498E-04

PHASE 1 COMPLETED.  DELV=    6002.  MASS RATIO= 0.23689

2 DIMENSIONS  14 DIFF.EQNS.  T/W= 0.3545900      ISP= 1200.0000         PFLOW= 140.00000         REFA= 0                  AEXIT= 0

STEP=   32. +  4.    ECCENTRICITY= 0.7063571     OMEGA=-2.9232258         V= 4624.4761           R= 6523008.2           REFER=EART RECT 2
TIME= 222.64875      SEMILATUS R.= 1980825.8     TRU A= 2.9729171        VX= 1466.0930           X= 6514956.5           RMASS= 473786.58
DAYS=    0.0026      MEAN ANOMALY= 2.4631072      NODE= 0                VY= 4385.5265           Y= 324003.53           REVS.= 7.9086195E-03
ALFA= 9.7947056         PATH ANGLE= 21.330449     INCL= 0                VZ= 0                   Z= 0                    DELT= 9.6413822
BETA= 0              R PATH ANGLE= 23.699933      DRAG= 0                VR= 4184.9730           G= 0.3545900            PUSH= 3.4773403
ALT.= 144848.19        MACH NUMBER= 7.1221350     LIFT= 0                CD= 0.5078962           Q= 2.1662714E-02        HEAT= 3.8269498E-04
 PSI= 61.721948               DPSI= 5.6612598E-02  THETA= 2.8471030       DK=-1.8318748E-05       K= 0                      L7= 2.4838145E-02
  L1= 0.4737509                 L2= 0.8806589       L3= 0                 L4= 1.2195612E-03    L5= 1.8140578E-04          L6= 0

STEP=   35. +  4.    ECCENTRICITY= 0.6752868     OMEGA=-2.8929056         V= 4659.5218           R= 6639447.0           REFER=EART RECT 2
TIME= 299.99999      SEMILATUS R.= 2204591.0     TRU A= 2.9941062        VX= 876.45449           X= 6605476.9           RMASS= 462957.41
DAYS=    0.0035      MEAN ANOMALY= 2.5886544      NODE= 0                VY= 4576.7564           Y= 670769.59           REVS.= 1.6106573E-02
ALFA= 13.129520         PATH ANGLE= 16.639328     INCL= 0                VZ= 0                   Z= 0                    DELT= 45.139881
BETA= 0              R PATH ANGLE= 18.531679      DRAG= 0                VR= 4198.3287           G= 0.3628843            PUSH= 3.5586798
ALT.= 261287.00        MACH NUMBER= 5.1112722     LIFT= 0                CD= 0.5851743           Q= 6.8963647E-04        HEAT= 1.2507935E-05
 PSI= 66.029118               DPSI= 5.4818091E-02  THETA= 5.7983661       DK=-1.7554865E-05       K=-1.3856406E-03          L7= 2.5402820E-02
  L1= 0.3838649                 L2= 0.8633552       L3= 0                 L4= 1.1065986E-03    L5= 2.6378669E-04          L6= 0

STEP=   37. +  4.    ECCENTRICITY= 0.6256273     OMEGA=-2.8534567         V= 4795.6022           R= 6752440.9           REFER=EART RECT 2
TIME= 399.99999      SEMILATUS R.= 2530765.1     TRU A= 3.0230342        VX= 127.41247           X= 6655585.1           RMASS= 448957.41
DAYS=    0.0046      MEAN ANOMALY= 2.7395133      NODE= 0                VY= 4793.9094           Y= 1139582.0           REVS.= 2.6989097E-02
ALFA= 17.060435         PATH ANGLE= 11.238523     INCL= 0                VZ= 0                   Z= 0                    DELT= 50.000000
BETA= 0              R PATH ANGLE= 12.513257      DRAG= 0                VR= 4313.7159           G= 0.3742003            PUSH= 3.6696514
ALT.= 374280.94        MACH NUMBER= 4.7187242     LIFT= 0                CD= 0.6098870           Q= 9.0938311E-05        HEAT= 1.7475245E-06
 PSI= 71.417117               DPSI= 5.3039169E-02  THETA= 9.7160749       DK=-1.6919825E-05       K=-3.1065333E-03          L7= 2.6124760E-02
  L1= 0.2798496                 L2= 0.8323787       L3= 0                 L4= 9.7624658E-04    L5= 3.5279097E-04          L6= 0

STEP=   39. +  4.    ECCENTRICITY= 0.5773667     OMEGA=-2.8136790         V= 5014.5921           R= 6828166.9           REFER=EART RECT 2
TIME= 499.99999      SEMILATUS R.= 2900758.3     TRU A= 3.0544832        VX=-615.75679           X= 6631150.5           RMASS= 434957.41
DAYS=    0.0058      MEAN ANOMALY= 2.8770553      NODE= 0                VY= 4976.6393           Y= 1628406.1           REVS.= 3.8325168E-02
ALFA= 20.398170         PATH ANGLE= 6.7432796     INCL= 0                VZ= 0                   Z= 0                    DELT= 50.000000
BETA= 0              R PATH ANGLE= 7.4843367      DRAG= 0                VR= 4520.4978           G= 0.3862447            PUSH= 3.7877667
ALT.= 450006.88        MACH NUMBER= 4.7072646     LIFT= 0                CD= 0.6106557           Q= 3.1889510E-05        HEAT= 6.6285320E-07
 PSI= 76.655610               DPSI= 5.1820889E-02  THETA= 13.797060       DK=-1.6573198E-05       K=-4.7791821E-03          L7= 2.6838638E-02
  L1= 0.1881828                 L2= 0.7933277       L3= 0                 L4= 8.5890326E-04    L5= 4.2582259E-04          L6= 0

STEP=   41. +  4.    ECCENTRICITY= 0.5180467     OMEGA=-2.7734945         V= 5299.4982           R= 6872015.1           REFER=EART RECT 2
TIME= 599.99995      SEMILATUS R.= 3316875.7     TRU A= 3.0891989        VX=-1360.1168           X= 6532385.7           RMASS= 420957.41
DAYS=    0.0069      MEAN ANOMALY= 3.0005734      NODE= 0                VY= 5121.9882           Y= 2133665.3           REVS.= 5.0245912E-02
ALFA= 23.072453         PATH ANGLE= 3.2171446     INCL= 0                VZ= 0                   Z= 0                    DELT= 50.000000
BETA= 0              R PATH ANGLE= 3.5528888      DRAG= 0                VR= 4799.2545           G= 0.3950902            PUSH= 3.9137384
ALT.= 493855.06        MACH NUMBER= 4.8776339     LIFT= 0                CD= 0.5995041           Q= 1.9688891E-05        HEAT= 4.4893856E-07
 PSI= 81.798530               DPSI= 5.1132514E-02  THETA= 18.088528       DK=-1.6427906E-05       K=-6.4278618E-03          L7= 2.7544825E-02
  L1= 0.1077536                 L2= 0.7476590       L3= 0                 L4= 7.5106058E-04    L5= 4.8547750E-04          L6= 0

STEP=   43. +  4.    ECCENTRICITY= 0.4511888     OMEGA=-2.7327958         V= 5635.4598           R= 6889570.2           REFER=EART RECT 2
TIME= 699.99999      SEMILATUS R.= 3781367.6     TRU A= 3.1278353        VX=-2105.8189           X= 6358545.0           RMASS= 406957.41
DAYS=    0.0081      MEAN ANOMALY= 3.1091298      NODE= 0                VY= 5225.6169           Y= 2651414.1           REVS.= 6.2872484E-02
ALFA= 25.086653         PATH ANGLE= 0.6479276     INCL= 0                VZ= 0                   Z= 0                    DELT= 50.000000
BETA= 0              R PATH ANGLE= 0.7113410      DRAG= 0                VR= 5133.0997           G= 0.4128196            PUSH= 4.0483774
ALT.= 511410.19        MACH NUMBER= 5.1789770     LIFT= 0                CD= 0.5812259           Q= 1.7958143E-05        HEAT= 4.5302498E-07
 PSI= 86.899513               DPSI= 5.0967103E-02  THETA= 22.634094       DK=-1.6416752E-05       K=-8.0692221E-03          L7= 2.8243048E-02
  L1= 3.7733802E-02             L2= 0.6566251       L3= 0                 L4= 6.5038497E-04    L5= 5.3332222E-04          L6= 0
```

```
STEP=   47. + 4.   ECCENTRICITY= 0.4273031       OMEGA= 3.5640023      V= 5755.7643          R= 6890605.7          REFER=EART RECT 2
TIME= 733.1137C    SEMILATUS R.= 3946228.8       TRU A=-3.1415926      VX=-2359.6303         X= 6284947.2          RMASS= 402321.50
DAYS=    0.0085    MEAN ANOMALY=-3.1415926        NODE= 0              VY= 5249.8540         Y= 2824869.4          REVS.= 6.7228587E-02
ALFA= 25.614188      PATH ANGLE=-1.8491502E-07    INCL= 0              VZ= 0                 Z= 0                   DELT= 5.2192765E-03
BETA= 0            R PATH ANGLE=-2.0260192E-07    DRAG= 0              VR= 5253.2933         G= 0.4175765           PUSH= 4.0950265
ALT.= 512445.75     MACH NUMBER= 5.2983338        LIFT= 0              CD= 0.5745042         Q= 1.8563431E-05       HEAT= 4.8478217E-07
 PSI= 88.588102              DPSI= 5.1030205E-02 THETA= 24.202291      DK=-1.6432814E-05     K=-8.6130872E-03         L7= 2.8472376E-02
  L1= 1.6729C61E-02            L2= 0.6787395       L3= 0               L4= 6.1835871E-04  L5= 5.4673174E-04            L6= 0

TRAJECTORY INTERRUPT -- C(LOOKX(I)) = -1.8491502E-07

STEP=   47. + 4.   ECCENTRICITY= 0.4273031       OMEGA= 3.5640023      V= 5755.7643          R= 6890605.7          REFER=EART RECT 2
TIME= 733.1137C    SEMILATUS R.= 3946228.8       TRU A=-3.1415926      VX=-2359.6303         X= 6284947.2          RMASS= 402321.50
DAYS=    0.0085    MEAN ANOMALY=-3.1415926        NODE= 0              VY= 5249.8540         Y= 2824869.4          REVS.= 6.7228587E-02
ALFA= 25.614188      PATH ANGLE=-1.8491502E-07    INCL= 0              VZ= 0                 Z= 0                   DELT= 25.000000
BETA= 0            R PATH ANGLE=-2.0260192E-07    DRAG= 0              VR= 5253.2933         G= 0.4175765           PUSH= 4.0950265
ALT.= 512445.75     MACH NUMBER= 5.2983338        LIFT= 0              CD= 0.5745042         Q= 1.8563431E-05       HEAT= 4.8478217E-07
 PSI= 88.588102              DPSI= 5.1030205E-02 THETA= 24.202291      DK=-1.6432814E-05     K=-8.6130872E-03         L7= 2.8472376E-02
  L1= 1.6729C61E-02            L2= 0.6787395       L3= 0               L4= 6.1835871E-04  L5= 5.4673174E-04            L6= 0

STEP=   49. + 4.   ECCENTRICITY= 0.3762899       OMEGA= 3.5917404      V= 6005.9944          R= 6886876.4          REFER=EART RECT 2
TIME= 799.99558    SEMILATUS R.= 4296531.7       TRU A=-3.1122303      VX=-2866.6483         X= 6110181.6          RMASS= 392957.41
DAYS=    0.0093    MEAN ANOMALY=-3.0815736        NODE= 0              VY= 5282.2685         Y= 3177223.2          REVS.= 7.6316404E-02
ALFA= 26.477525      PATH ANGLE=-1.0144530        INCL= 0              VZ= 0                 Z= 0                   DELT= 41.886284
BETA= 0            R PATH ANGLE=-1.1069444        DRAG= 0              VR= 5507.8813         G= 0.4275272           PUSH= 4.1926100
ALT.= 508716.28     MACH NUMBER= 5.5623304        LIFT= 0              CD= 0.5606608         Q= 2.1396568E-05       HEAT= 5.9980929E-07
 PSI= 92.010432              DPSI= 5.1345364E-02 THETA= 27.473905      DK=-1.6483582E-05     K=-9.7138059E-03         L7= 2.8932510E-02
  L1=-2.2513627E-02            L2= 0.6413576       L3= 0               L4= 5.5544027E-04  L5= 5.7025790E-04            L6= 0

STEP=   50. + 4.   ECCENTRICITY= 0.2928106       OMEGA= 3.6339166      V= 6412.5180          R= 6870673.1          REFER=EART RECT 2
TIME= 899.99558    SEMILATUS R.= 4864892.7       TRU A=-3.0641798      VX=-3629.8087         X= 5785401.9          RMASS= 378957.41
DAYS=    0.0104    MEAN ANOMALY=-3.0063889        NODE= 0              VY= 5286.2914         Y= 3706113.0          REVS.= 9.0676432E-02
ALFA= 27.286717      PATH ANGLE=-1.8317517        INCL= 0              VZ= 0                 Z= 0                   DELT= 100.00000
BETA= 0            R PATH ANGLE=-1.9869646        DRAG= 0              VR= 5911.7783         G= 0.4433216           PUSH= 4.3474995
ALT.= 492513.13     MACH NUMBER= 6.0126899        LIFT= 0              CD= 0.5403278         Q= 3.0414418E-05       HEAT= 9.4893667E-07
 PSI= 97.188550              DPSI= 5.2324383E-02 THETA= 32.643516      DK=-1.6577251E-05     K=-1.1366827E-02         L7= 2.9612010E-02
  L1=-7.3521683E-02            L2= 0.5829208       L3= 0               L4= 4.6554190E-04  L5= 5.9677397E-04            L6= 0

STEP=   51. + 4.   ECCENTRICITY= 0.2001526       OMEGA= 3.6771355      V= 6823.8354          R= 6848591.6          REFER=EART RECT 2
TIME= 999.99558    SEMILATUS R.= 5489515.1       TRU A=-3.0109175      VX=-4395.9126         X= 5384120.2          RMASS= 364957.41
DAYS=    0.0116    MEAN ANOMALY=-2.9497580        NODE= 0              VY= 5232.2335         Y= 4232547.2          REVS.= 0.1060319
ALFA= 27.536231      PATH ANGLE=-1.8636071        INCL= 0              VZ= 0                 Z= 0                   DELT= 100.00000
BETA= 0            R PATH ANGLE=-2.0105018        DRAG= 0              VR= 6334.7171         G= 0.4603277           PUSH= 4.5142725
ALT.= 470431.56     MACH NUMBER= 6.5212815        LIFT= 0              CD= 0.5223422         Q= 4.7098085E-05       HEAT= 1.6350019E-06
 PSI= 102.49886              DPSI= 5.4012510E-02 THETA= 38.171482      DK=-1.6647290E-05     K=-1.3028460E-02         L7= 3.0280059E-02
  L1=-0.1157890                L2= 0.5223397       L3= 0               L4= 3.8067465E-04  L5= 6.1319647E-04            L6= 0

STEP=   53. + 4.   ECCENTRICITY= 9.7623438E-02   OMEGA= 3.7216460      V= 7265.8054          R= 6829292.1          REFER=EART RECT 2
TIME= 1100.0000    SEMILATUS R.= 6174492.6       TRU A=-2.9523812      VX=-5159.0928         X= 4906315.2          RMASS= 350957.41
DAYS=    0.0127    MEAN ANOMALY=-2.9128355        NODE= 0              VY= 5116.2183         Y= 4750505.3          REVS.= 0.1224323
ALFA= 27.218441      PATH ANGLE=-1.1634399        INCL= 0              VZ= 0                 Z= 0                   DELT= 50.000000
BETA= 0            R PATH ANGLE=-1.2490431        DRAG= 0              VR= 6767.9157         G= 0.4786505           PUSH= 4.6943507
ALT.= 451132.06     MACH NUMBER= 7.0430357        LIFT= 0              CD= 0.5093767         Q= 7.0352104E-05       HEAT= 2.7133611E-06
 PSI= 108.02063              DPSI= 5.6591278E-02 THETA= 44.075629      DK=-1.6640355E-05     K=-1.4693721E-02         L7= 3.0935024E-02
  L1=-0.1496431                L2= 0.4606053       L3= 0               L4= 3.0141662E-04  L5= 6.1992415E-04            L6= 0

STEP=   55. + 4.   ECCENTRICITY= 1.9301011E-04   OMEGA= 1.7499298      V= 7643.7448          R= 6822306.0          REFER=EART RECT 2
TIME= 1186.7870    SEMILATUS R.= 6822384.6       TRU A=-0.8858732      VX=-5813.0078         X= 4430118.1          RMASS= 338807.23
DAYS=    0.0137    MEAN ANOMALY=-0.8855743        NODE= 0              VY= 4963.4428         Y= 5188247.6          REVS.= 0.1375189
ALFA= 26.445008      PATH ANGLE=-8.0837334E-04    INCL= 0              VZ= 0                 Z= 0                   DELT= 36.787025
BETA= 0            R PATH ANGLE=-8.6464872E-04    DRAG= 0              VR= 7146.2543         G= 0.4958572           PUSH= 4.8626978
ALT.= 444146.00     MACH NUMBER= 7.4663814        LIFT= 0              CD= 0.5029354         Q= 8.6614951E-05       HEAT= 3.6538326E-06
 PSI= 113.06259              DPSI= 5.9766716E-02 THETA= 49.506793      DK=-1.6526171E-05     K=-1.6133830E-02         L7= 3.1491612E-02
  L1=-0.1732101                L2= 0.4068213       L3= 0               L4= 2.3797822E-04  L5= 6.1845142E-04            L6= 0

PHASE 2 COMPLETED.  DELV=   3946.  MASS RATIO= 0.71511  ***  TOTAL DELV=   9948.  TOTAL MASS RATIO= 0.16940  PAYLOAD RATIO= 0.03334

 KE=.100 STRUCT=.053    ALFPOW=  0.     PJ/MO= 25.545258     PJ/M1= 25.545258     MPP/M1= 0.     ETAPOW=0.058      ML/M1= 0.03334
```

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, September 11, 1973,

502-04.

APPENDIX A

SYMBOLS

A_e	engine exit area, m^2
a	thrust acceleration magnitude, m/sec^2
a_e	Earth's equatorial radius, m
a_x, a_y, a_z	components of total perturbating acceleration, m/sec^2
$a_1, \ldots, a_{12}$	curve-fit coefficients
B	$V_r \times H_r$
b	electric thruster efficiency parameter
C	vector constant of motion, kg
$\mathscr{C}$	perturbative acceleration in circumferential direction, m/sec^2
C_D	total drag coefficient
C_{DI}	induced drag coefficient
C_{D0}	parasite drag coefficient
C_L	lift coefficient
c	jet exhaust speed of vehicle, m/sec
c_l	launch vehicle performance parameter, m/sec
c_r	jet exhaust speed of high-thrust retroengine, m/sec
D	vehicle drag force vector, N
d	electric thruster efficiency parameter, m/sec
E	eccentric anomaly, rad
e	orbit eccentricity
e_r	eccentricity of planetary capture orbit
F	eccentric anomaly equivalent for hyperbolic orbits, rad
f	thrust force magnitude, N
G	partial derivative matrix for two-point boundary-value problem
g	universal gravitational constant, m/sec^2
H_r	relative angular momentum per unit mass vector, m^2/sec

h	absolute angular momentum magnitude, m^2/sec; and integration step size, sec
I	specific impulse, sec
i	orbit inclination, rad
$\hat{i}, \hat{j}, \hat{k}$	unit vectors along x, y, z axes
J_2, J_3, J_4	zonal harmonic oblateness coefficients
j	retrorocket jettison indicator
k_l	launch vehicle performance parameter
k_{rt}	retrosystem tankage factor
k_s	structure factor
k_t	tankage factor
k_1, k_2, k_3, k_4	Runge-Kutta subinterval increments
$\mathbf{L}$	lift force vector, N
l	lift force magnitude, N
l_s	transformation factor used in multidimensional sweeps
M	Mach number; and mean anomaly, rad
m	vehicle mass, kg
m_n	net spacecraft mass, kg
m_p	propellant mass, kg
m_{ps}	propulsion system mass, kg
m_r	retrosystem mass, kg
m_{rp}	retropropellant mass, kg
m_{rt}	retrosystem tankage mass, kg
m_{ref}	reference mass in planetary orbit, kg
m_s	structure mass, kg
m_t	tankage mass, kg
N	$e \cos \omega + \cos u$
$\mathcal{N}$	perturbative acceleration normal to orbit plane, m/sec^2
n	mean motion
P	instantaneous electric power available from power source, W

P_r	electric power available from power source at 1-AU distance from Sun, W
p	atmospheric pressure, N/m^2; and semilatus rectum, m
Q	$e \sin \omega + \sin u$
q	dynamic pressure, N/m^2
R	position vector of vehicle, m
$\mathcal{R}$	perturbative acceleration in outward radial direction, m/sec^2
r	distance from origin to vehicle, m
r_l	radius of launch vehicle at injection, m
r_r	radius of retrofire maneuver at arrival planet, m
$r_{s,a}$	sphere-of-influence radius of arrival planet, m
$r_{s,d}$	sphere-of-influence radius of departure planet, m
S	vector of sweep parameters
S_{ref}	aerodynamic reference area, m^2
s	sweep parameter
T	unit vector in thrust direction
$T_{\dot{m}_0}, T_c, T_{v_l}, T_{v_r}$	transversality conditions for $\dot{m}_0$, c, v_l, and v_r
t	time, sec
t_f	flight duration of a stage, sec
t_s	duration of low-thrust escape spiral maneuver, sec
t_v	time of short vertical rise for launch vehicles, ignoring atmosphere
u	gravitational potential function, m^2/sec^2; and argument of latitude, rad
V	absolute vehicle velocity vector, m/sec
V_r	vehicle speed relative to a planet, m/sec
v	vehicle speed, m/sec
$v_{c,l}$	circular orbit speed about departure planet at radius r_l, m/sec
$v_{c,r}$	circular orbit speed about arrival planet at radius r_r, m/sec
v_l	launch speed of spacecraft when analytic launch vehicle simulation is invoked, m/sec

v_r spacecraft speed just prior to an analytic high-thrust retrofire maneuver, m/sec; and relative spacecraft speed, m/sec

Δv_r retrofire speed increment, m/sec

$v_{s,a}, v_{s,d}$ vehicle speed as it passes through arrival (departure) planet's sphere of influence, m/sec

W weighting matrix for end condition residuals

w_i diagonal elements of W

X vector of level 1 independent variables

X, Y, Z inertial Cartesian coordinate axes

x, y, z components of vehicle position, m

x_i i^{th} element of X

Y vector of level 1 dependent variables

y_i i^{th} element of Y

y_n value of integration variable at n^{th} step

Z vector of level 2 optimization variables

z_i i^{th} element of Z

α angle between thrust vector and velocity vector (numerically identical with angle of attack), deg

α_c angle between thrust vector and circumferential direction, deg

α_{ps} specific weight of propulsion system, kg/kW

β out-of-orbit thrust angle, deg

Γ level 2 optimization criterion

γ vehicle path angle, rad

δ integration scheme truncation error

δ_{limit} acceptable limit value of δ_r

δ_r relative truncation error (between fourth-order Runge-Kutta scheme and lower order scheme)

$\delta(\ )$ partial derivative with respect to arbitrary variable

ϵ engine on-off indicator

ζ ratio P/P_r

η thruster efficiency

θ	central travel angle, rad
ϑ	east longitude relative to Greenwich, rad
κ	engine on-off switching function
Λ	vector of velocity-related adjoint variables (primer vector), (kg)(sec)/m
Λ_r	vector of position-related adjoint variables, kg/m
λ	magnitude of primer vector Λ, (kg)(sec)/m
λ_c	adjoint variable for engine exhaust speed c, (kg)(sec)/m
λ_m	adjoint variable for mass
$\lambda_{\dot{m}_0}$	adjoint variable for initial mass flow rate $\dot{m}_0$, sec
λ_{v_l}	adjoint variable for analytic launch vehicle speed v_l, (kg)(sec)/m
λ_{v_r}	adjoint variable for analytic retrofire speed v_r, (kg)(sec)/m
$\lambda_1, \ldots, \lambda_7$	components of Λ, components of Λ_r, and λ_m in that order
μ	gravitational constant, m^3/sec^2
ν	true anomaly, rad
ξ	empiral factor used in spiral escape equations
ρ	atmospheric density, kg/m^3
σ	azimuth measured eastward from north, rad
τ	boundary-value-problem error criterion
τ^*	value of τ separating univariate scheme domain from linear correction scheme domain
Φ	$\cos \varphi \sin \gamma - \sin \varphi \cos \gamma \cos \sigma$
φ	geocentric latitude, rad
φ^*	geodetic latitude, rad
χ	inhibitor for linear correction scheme
Ψ	time-dependent term of Runge-Kutta truncation error
ψ	angle between thrust vector and x-axis, rad
Ω	longitude of ascending node, rad
ω	argument of periapsis, rad
ω_r	rotation rate of Earth, rad/sec

Subscripts:

a	arrival value
x, y, z	x, y, z components of vector
0	departure value

Superscripts:

0	reference trajectory value
$\cdot$	derivative with respect to time t
$'$	derivative with respect to radius r
$-$	desired value
$\sim$	modified arrival planet value

APPENDIX B

SUBPROGRAM GLOSSARY

WAERO — computes lift and drag acceleration

WALSO — provides auxiliary computation after each integration step, such as a check for zero vehicle mass and determining the optimum fixed-thrust-angle switch function

WALTER — alters the independent variables X of level 1

WBEGIN — initializes variables and controls for the boundary-value problem involved with optimal thrust steering

WCREEP — univariate search scheme

WCURVE — least squares n-order curve fit to m points

WDERIV — evaluates derivatives of integration variables

WELIPS — computes position and velocity of a body in elliptic orbit

WEPHEM — computes n-body accelerations, and position and velocity of bodies

WELEM — transforms rectangular coordinates to orbit elements

WENDST — auxiliary computations between trajectory phases

WERROR — computes relative integration errors between fourth-order Runge-Kutta scheme and low-order scheme

WGAUSS — Gauss-Jordan elimination solution to a set of linear equations

WICAO — U. S. Standard Atmosphere 1962 model

WINTEG — Runge-Kutta (fourth order) integrator, also low-order integrator

WLOOK — computes jump discontinuities or takes other appropriate action at trajectory interrupt points

WMARCH — automatic parameter sweep scheme

WNR — finite difference method (generalized Newton-Raphson) of generating partial derivatives for boundary-value problem

WOBLAT — computes oblateness acceleration

WOPT — master controller of level 1 boundary-value-problem iteration, level 2 variable optimization, and the automatic parameter sweep scheme

WORBEL — computes analytic time-series approximate ephemerides

WORDER sorts gravitational body list so that a body's position from the reference is dependent upon positions already computed for other bodies

WOUT basic trajectory output routine

WPENAL computes boundary-value-problem error function

WPOWER computes the solar power ratio P/P_r and its derivatives with respect to distance

WPUSH computes thrust acceleration for nonoptimal steering

WQUAD curve-fit model based on n quadratic functions pieced together

WRXV computes unit angular momentum

WSTAGE prepares data for use in integrator by updating key variables (mass, specific impulse, etc.) with current trajectory phase data

WSTEP computes integration step size and searches for trajectory interrupts

WTUDES transforms Earth-fixed spherical coordinates to space-fixed inertial rectangular coordinates

WVREL computes velocity relative to a rotating planet and the nonoptimal thrust angle

WXFER tests for and translates the coordinate system origin when a sphere of influence is penetrated

TIMLFT calculates the amount of computer execution time remaining (in 1/60-sec units) before execution is terminated by the system monitor. This is a Lewis Research Center non-FORTRAN routine that uses the $IBFTC card time estimate for batch sequencing operation. A dummy FORTRAN version is substituted for other users unless otherwise requested. The function of this routine is to provide a warning that the job is about to be prematurely terminated, thus giving the program an opportunity to print out the best unconverged trajectory instead of being "thrown-off" without gaining any useful information.

REFERENCES

1. Strack, William C.; and Huff, Vearl N.: The N-Body Code - A General FORTRAN Code for the Numerical Solution of Space Mechanics Problems on an IBM 7090 Computer. NASA TN D-1730, 1963.
2. Ledédev, V. N.: Variational Problem of Escape from Circular Orbit. Rep. FTD-TT-64-1200/1+2+4, Foreign Technology Div., Air Force Systems Command (AD-610208), Dec. 5, 1964.
3. MacKay, John S.; and Mascy, Alfred C.: Some Three-Body Numerical Solutions to Outer Planet Orbiter Missions Using Nuclear-Electric Propulsion. Paper 70-1040, AIAA, Aug. 1970.
4. Strack, William C.: Some Numerical Comparisons of Three-Body Trajectories with Patched Two-Body Trajectories for Low Thrust Rockets. NASA TN D-4559, 1968.
5. Flanagan, Paul F.; and Horsewood, Jerry L.: HILTOP, Heliocentric Interplanetary Low-Thrust Trajectory Optimization Program. Rep. 70-46, Analytical Mechanics Associates, Inc., NASA contract NAS5-20126, Dec. 1970.
6. Hahn, D. W.; and Johnson, F. T.: Chebychev Trajectory Optimization Program (CHEBYTOP II). Rep. D180-12916-1, Boeing Co. (NASA CR-114354), June 1971.
7. Spurlock, Omer F.; and Teren, Fred: A Trajectory Code for Maximizing the Payload of Multistage Launch Vehicles. NASA TN D-4729, 1968.
8. Melbourne, William G.; Mulholland, J. Derral; Sjogren, William L.; and Sturms, Francis M., Jr.: Astrodynamic Calculations, 1968. Rep. TR-32-1306, Jet Propulsion Lab. (NASA CR-97666), July 15, 1968.
9. Anon.: U.S. Standard Atmosphere, 1962. Prepared under sponsorship of NASA, USAF, and USWB, Dec. 1962.
10. Strack, William C.: Solar-Electric Propulsion System Performance for a Close Solar Probe Mission. J. Spacecraft Rockets, vol. 4, no. 4, Apr. 1967, pp. 469-475.
11. Melbourne, William G.: Interplanetary Trajectories and Payload Capabilities of Advanced Propulsion Vehicles. TR 32-68, Jet Prop. Lab., California Inst. Tech., Mar. 1961.
12. Leitmann, George, ed.: Optimization Techniques with Applications to Aerospace Systems. Academic Press, 1962.

13. Horsewood, Jerry L.: Interplanetary Trajectory Analysis for Combined High- and Low-Thrust Propulsion Systems. American Astronautical Society Space Flight Mechanics Specialists Symposium. Vol. 11 of the AAS Science and Technology Series. M. L. Anthony, ed., American Astronautical Society, 1967, pp. 457-476.

14. Dobson, Wilbur F.; Huff, Vearl N.; and Zimmerman, Arthur V.: Elements and Parameters of the Osculating Orbit and Their Derivatives. NASA TN D-1106, 1962.

TABLE I. - ASSUMED PHYSICAL DATA

Constant	Assumed value
Astronomical unit, m	1.495978730×10^{11}
Gravitational constant of the Sun, AU^3/day^2	2.959122083×10^{-4}
Earth's rotation rate, ω_r, rad/sec	7.29211515×10^{-5}
Equatorial Earth radius, a_e, m	6 378 160
J_2 zonal harmonic coefficient for Earth	1082.7×10^{-6}
J_3 zonal harmonic coefficient for Earth	-2.56×10^{-6}
J_4 zonal harmonic coefficient for Earth	-1.58×10^{-6}

Body	Reciprocal mass	Sphere-of-influence radius, m
Sun	1	1.0×10^{20}
Mercury	5 983 000	1.0×10^{8}
Venus	408 522	6.14×10^{8}
Earth-Moon	328 900.1	9.25×10^{8}
Mars	3 098 700	5.78×10^{8}
Jupiter	1 047.3908	4.81×10^{10}
Saturn	3 499.2	5.46×10^{10}
Uranus	22 930	5.17×10^{10}
Neptune	19 260	8.61×10^{10}
Pluto	1 812 000	3.81×10^{10}
Earth	332 945.6	9.25×10^{8}
Moon	[a]81.3010	1.60×10^{8}

[a]Earth reference.

TABLE II. - COMMON LOCATIONS OF ANTICIPATED CANDIDATES FOR BOUNDARY-VALUE VARIABLES

(a) Independent variables, X, for IA vector

Variable	FORTRAN name	COMMON location
Stage flight times, $(t_f)_i$, sec	TB	1, 2, . . ., 10
Elevation angle for launch vehicles, γ, deg	ELEV	48
Initial thrust angle relative to x-axis, ψ_0, deg	PS	343
Initial thrust-angle rate, $\dot{\psi}_0$, deg/sec	DPS	344
Initial value of engine on-off switch function, κ_0	KAPPA	345
Time derivative of κ_0, $\dot{\kappa}_0$, sec^{-1}	DKAPPA	346
Initial values of adjoint variables, Λ, $\dot{\Lambda}_r$, λ_m	LAMDA	347 to 353
Stage initial propellant flow rates, $(-\dot{m}_0)_i$, kg/sec	PFLOW	383 to 392
Initial power level, P_0, kW	POWER	397
Stage initial thrust-weight ratios, f/m_0g	TW	408 to 417
Launch speed of electric spacecraft, v_l, m/sec	VB1	429
Spacecraft speed just prior to high-thrust retromaneuver, v_r, m/sec	VB2	430
Nonvariational thrust program coefficients, a_{10}, a_{11}, a_{12}	ALFCOE	1458 to 1507
x, y, z components of initial velocity, V_0, m/sec	V	2161, 2163, 2165
x, y, z components of initial position, R_0, m	R	2167, 2169, 2171

(b) Dependent variables, Y, for IB vector

Variable	FORTRAN name	COMMON location
Orbit elements, e, ω, Ω, i, M, p	ORBELS	447 to 452
Energy per unit mass, J/kg	ENERGY	462
Path angle, γ, deg	PATH	479
Radius, r, m	RADIUS	480
Central travel angle, θ, deg	THETA	485
True anomaly, ν, deg	TRU	486
x, y, z components of position, R, m	X, Y, Z	487 to 489
x, y, z components of velocity, V, m/sec	VX, VY, VZ	490 to 492
Velocity magnitude, v, m/sec	VEL	493

TABLE III. - GLOSSARY OF COMMON VARIABLES

Block name	Variable name	Relative location	Absolute location	Definition
TIME	TB	1	1	Array of phase flight times, t_f, sec
	DTOFFJ	11	11	Julian departure date
	TOFFT	12	12	Fraction of day at departure
	TABLT	13	13	Time since takeoff, days
	TMAX	14	14	Total flight time, sec
	TTEST	15	15	Control used when switching from orbit element integration to rectangular coordinates, sec
	TTOL	16	16	Time tolerance used to terminate a trajectory, sec
	DELT	17	17	Integration step size, h, sec
	T0	18	18	Time at departure, t_0, sec
	STEP	19	19	Ten-element array of phase initial step size, sec
FIXED	AU	1	29	Astronomical unit, m
	SPD	2	30	Seconds per day
	G	3	31	Gravitational constant at Earth's surface, g, m/sec^2
	RE	4	32	Earth's equatorial radius, a_e, m
	RESQRD	5	33	RE squared, m^2
	SQRDK1	6	34	Gravitational constant of the Sun, AU^3/day^2
	DEGREE	7	35	Degrees per radian
	PI	8	36	π
	TWOPI	9	37	2π
	DUMMY1	10	38	Dummy variable causing an even number of locations in COMMON block (required for double-precision usage in some computers)
ENTER	END	1	39	Alphameric value 'END'
	T	2	40	Logical value .TRUE.
	F	3	41	Logical value .FALSE.
	EPHEM	4	42	Control causing ephemerides determination of departure and arrival conditions if .TRUE.
	OBLATE	5	43	Control causing oblateness effects to be considered if .TRUE.
	ROTATE	6	44	Control causing planetary rotation to be included if .TRUE.
LAT	LAT	1	45	Launch site latitude, φ, deg
	LONG	2	46	Launch site longitude, ϑ, deg
	AZI	3	47	Launch site azimuth, σ, deg
	ELEV	4	48	Launch elevation angle, γ, deg
	VEL0	5	49	Initial velocity at launch, v_0, m/sec
	ALT0	6	50	Initial altitude above mean sea level, m
	TKICK	7	51	Duration of short vertical rise for launch vehicles (ignoring atmosphere), t_v, sec
	DUMMY2	8	52	See DUMMY1 above
LOOK	XLOOK	1	53	Five-element array of trajectory interrupt values corresponding to LOOKX array
	LOOKSW	6	58	Five-element array of COMMON indices for interrupt delay
	SWLOOK	11	63	Five-element array of trajectory interrupt delay values corresponding to LOOKSW
	ENDX	16	68	Five-element array of control variables that determine post-interrupt action
	LOOKY	21	73	Has the value .TRUE. if trajectory interrupt feature is operative
	DUMMY3	22	74	See DUMMY 1 above
CASES	NSWEEP	1	75	COMMON location of the sweep variable S
	NSAVE	2	76	Stage number of saved set of initial conditions
	RECALL	3	77	Causes saved data to be recalled for succeeding cases if input .TRUE.
	KCASE	4	78	Counter on the case number used only for manual sweeps
	NCASE	5	79	Current case number
	NCASES	6	80	Case number of the saved initial data
OUTPUT	STEPS	1	81	Number of steps between printouts
	DELMAX	2	82	Trajectory time interval between printouts, sec
	ADDOUT	3	83	Internal control variable that indicates whether subroutine WOUT is to be called after every step
	NOUT	4	84	Five-element array specifying the trajectories to be printed out in full
	NBUG	9	89	Five-element array specifying the trajectories to be debugged
	STEP1	14	94	Integration step number where debug output is to begin
	STEP2	15	95	Integration step number where debug output is to cease
	DEBUG	16	96	Triggers debug output when .TRUE.
	OUTPOT	17	97	Triggers full trajectory printout when .TRUE.
	OUTEND	18	98	Triggers final (converged) trajectory printout when .TRUE.

TABLE III. - Continued. GLOSSARY OF COMMON VARIABLES

Block name	Variable name	Relative location	Absolute location	Definition
LOCATE	LCAPPA	1	99	COMMON location of thrust on-off switching function κ
	LLAMDA	2	100	COMMON location of primer vector Λ
	LV	3	101	COMMON location of vehicle velocity V
	LVEL	4	102	COMMON location of vehicle speed v
	LPATH	5	103	COMMON location of vehicle path angle γ
	LSIMP	6	104	COMMON location of specific impulse I
	LFLOW	7	105	COMMON location of propellant flow rate $\dot{m}_p$
	LVB1	8	106	COMMON location of launch speed of spacecraft (using analytic launch vehicle simulation) v_l
	LVB2	9	107	COMMON location of spacecraft speed just prior to analytic retrofire maneuver v_r
	LTC	10	108	COMMON location of transversality condition for optimum exhaust speed T_c
	LTA	11	109	COMMON location of transversality condition for optimum initial propellant flow rate $T_{\dot{m}_0}$
	LTVB1	12	110	COMMON location of transversality condition for optimum launch vehicle speed T_{v_l}
	LTVB2	13	111	COMMON location of transversality condition for optimum speed prior to retromaneuver T_{v_r}
	LALFSW	14	112	COMMON location of fixed-thrust-angle switching function $\Delta(\Lambda \cdot T_l)$
	LPSI	15	113	COMMON location of initial angle between thrust vector and X-axis ψ_0
	LRMAG	16	114	COMMON location of vehicle's position vector magnitude r
	LLAMF	17	115	COMMON location of final primer vector value Λ_a
	LTHETA	18	116	COMMON location of central travel angle θ
	JSLOPE	19	117	COMMON location of time derivatives of C(LOOKX)
	LTW	24	122	COMMON location of initial thrust-weight ratio $(f/mg)_0$
IGRATE	NDEM	1	123	Number of coordinate system dimensions
	A1	2	124	Factor used in determining integration step size
	A2	3	125	Factor used in determining integration step size
	ADJOIN	4	126	Triggers integration of partial deviative matrix G
	ADJOINT	5	127	Temporary value of ADJOIN
	DONE	6	128	Indicator for trajectory termination
	ERROR	7	129	Relative truncation error of integration scheme, δ_r
	EMONE	8	130	Eccentricity minus 1
	ERLOG	9	131	$\ln \bar{\delta}_r$
	H2	10	132	Step size for previous step
	ESTART	11	133	Control variable for starting procedure of integration scheme
	ETOL	12	134	If e is in region 1±ETOL, integration is always done in rectangular coordinates
	EXMODE	13	135	Eccentricity e, only calculated when in temporary rectangular coordinates
	KERROR	14	136	Index of integration variable producing maximum truncation error
	KSUB	15	137	Index of Runge-Kutta subinterval
	NSTAGE	16	138	Stage index
	LSTAGE	17	139	Number of stages
	MODEI	18	140	Input control indicating choice of set of integration variables
	EREF	19	141	Relative truncation error, δ_r
	ERLIMT	20	142	Maximum value of truncation error permitted, δ_{limit}
	STEPMX	21	143	Limit on number of trajectory integration steps
	NEQ	22	144	Number of differential equations being integrated
	NSTART	23	145	Control variable used in starting Runge-Kutta scheme
	RATIO	24	146	Ratio h_n/h_{n-1}
	XWHOLE	25	147	A saved set of integration variables used when translating the coordinate system origin
	HINC	31	153	Array of integration variable increments, Δy_n
COFV	ABS0	1	303	Initial magnitude of the primer vector, λ_0
	ABSLAM	2	304	Current magnitude of the primer vector, λ
	CAPPA	3	305	Engine on-off switching function, κ
	CV	4	306	Indicates a calculus-of-variations problem if .TRUE.
	DCAPPA	5	307	Derivative of engine on-off switching function, $\dot{\kappa}$
	FCV	6	308	Indicates optimal fixed-angle-thrust steering option if .TRUE.
	HAM	7	309	Five-element array of $\Lambda \cdot T_l$
	HAMMAX	12	314	$\max_i(\Lambda \cdot T_i)$
	ALF	13	315	Five-element array of thrust angles, either α or α_c
	NALF	18	320	Length of ALF array (NALF $\leq$ 5)

TABLE III. - Continued. GLOSSARY OF COMMON VARIABLES

Block name	Variable name	Relative location	Absolute location	Definition
COFV	NCAPPA	19	321	Number of times engine is turned on or off during a single trajectory ($\kappa = 0$ condition)
	NLAMDA	20	322	Element of NOPT corresponding to the current stage number NSTAGE
	NOPT	21	323	Ten-element array indicating boundary-value-problem end condition definition for each stage
	PLDOTL	31	333	$\delta\Lambda \cdot \Lambda$ array
	PSI	40	342	Angle between thrust vector and x-axis, ψ, deg
	PS	41	343	Initial value of angle between thrust vector and x-axis, ψ_0, deg
	DPS	42	344	$\dot{\psi}_0$, deg/sec
	KAPPA	43	345	Initial value of engine on-off switching function, κ_0
	DKAPPA	44	346	$\dot{\kappa}_0$, sec^{-1}
	LAMDA	45	347	Initial value of adjoint variables (7 total), Λ, Λ_r, and λ_m
	IMAX	52	354	Index of maximum ($\Lambda \cdot T_i$) value
	COAST	53	355	Input option that indicates coast arcs are permitted if .TRUE.
	IUSE	54	356	Index of which ALF value is current for optimal fixed-thrust-angle option
	LAM	55	357	Scale factor for adjoint variables used for alternate set of inputs involving ψ_0, $\dot{\psi}_0$, etc.
	ALFSWT	56	358	Fixed-thrust-angle switching function, $\Delta(\Lambda \cdot T_i)$
	TRAC	57	359	Transversality condition residual for optimum exhaust speed, T_c
	TRAA	58	360	Transversality condition residual for optimum initial propellant flow rate, $T_{\dot{m}_0}$
	TRAVB1	59	361	Transversality condition residual for optimum launch vehicle speed, T_{v_l}
	TRAVB2	60	362	Transversality condition residual for optimum retromaneuver vehicle speed, T_{v_r}
	LAMDAF	61	363	Arrival values of Λ and Λ_r vectors
	GETDOT	67	369	Six-element array indicating the need for certain partial derivatives G_i
	OPTA	73	375	Input option specifying optimum initial propellant flow rate $\dot{m}_0$ if .TRUE.
	OPTC	74	376	Input option specifying optimum jet exhaust speed c if .TRUE.
	OPTVB1	75	377	Input option specifying optimum launch vehicle speed v_l if .TRUE.
	OPTVB2	76	378	Input option specifying optimum retromaneuver vehicle speed v_r if .TRUE.
ROCKET	ALFPOW	1	379	Specific weight of propulsion system α_{ps}, kg/kW
	BE	2	380	Electric thruster efficiency parameter, b
	DE	3	381	Electric thruster efficiency parameter, d
	BOOSTM	4	382	Reference vehicle mass in planetary orbit, m_{ref}, kg
	PFLOW	5	383	Ten-element array of stage initial propellant flow rates, $\dot{m}_0$, kg/sec
	FW	15	393	Initial stage thrust-weight ratio
	K1	16	394	Launch vehicle performance factor, k_l
	K2	17	395	Retrosystem tankage factor, k_{rt}
	KE	18	396	Vehicle tankage factor, k_t
	POWER	19	397	Electric power available from power source at 1 AU, P_r, W
	VMASS	20	398	Ten-element array of stage initial mass, m_0, kg
	TW	30	408	Ten-element array of stage initial thrust-weight ratio, f/m_0g
	ISP	40	418	Ten-element array of stage specific impulses, I, sec
	SOLAR	50	428	Input option specifying solar-electric propulsion if .TRUE.
	VB1	51	429	Launch speed of spacecraft when analytic launch vehicle simulation is used, v_l, m/sec
	VB2	52	430	Spacecraft speed just prior to retrofire maneuver, v_r, m/sec
	VJET1	53	431	Launch vehicle performance parameter, c_l, m/sec
	VJET2	54	432	Jet exhaust speed of high-thrust retroengine, c_r, m/sec
	A0	55	433	Initial thrust acceleration magnitude, a_0, m/sec^2
	CPAR	56	434	The quantity c/m
	ETAPOW	57	435	Thruster efficiency, η
	FLOW	58	436	Current propellant flow rate, $\dot{m}_p$, kg/sec
	PAY	59	437	Net spacecraft mass ratio, m_n/m_{ref}
	PUSH	60	438	Engine thrust, f, N
	PUSH0	61	439	Engine thrust in vacuum, N
	RMASS0	62	440	Initial spacecraft mass, m_0, kg
	SIMP	63	441	Current specific impulse, I, sec
	TMAG	64	442	Thrust acceleration, a, m/sec^2
	VJET	65	443	Jet exhaust speed, c, m/sec
	FLOWX	66	444	Stage initial propellant flow rate at 1 AU, kg/sec
	DISPO	67	445	Electric propulsion system jettison indicator
	STRUCT	68	446	Structure factor, k_s

TABLE III. - Continued. GLOSSARY OF COMMON VARIABLES

Block name	Variable name	Relative location	Absolute location	Definition
TRAJEC	ORBELS	1	447	Six-element array of orbit elements e, ω, Ω, i, M, p (in radians and meters)
	U	7	453	Eccentric anomaly, E, rad
	RAMC	8	454	Three components of relative angular momentum, H_r, m^2/sec
	RAM	11	457	Relative angular momentum magnitude, $\|H_r\|$, m^2/sec
	RAMSRD	12	458	Square of $\|H_r\|$, m^4/sec^2
	ALPHAC	13	459	Input option indicating circumferential thrust angle reference (using ALF or ALFCOE) if . TRUE.
	BETA	14	460	Out-of-orbit plane thrust angle, β, deg
	ECC2	15	461	Eccentricity of planetary capture orbit, e_r
	ENERGY	16	462	Vehicle energy per unit mass, m^2/sec^2
	SPIR	17	463	Input option indicating an analytic Earth escape spiral if . TRUE.
	VC1	18	464	Circular orbit speed about departure planet (at radius r_l), $v_{c,l}$, m/sec
	VC2	19	465	Circular orbit speed about arrival planet (at radius r_r), $v_{c,r}$, m/sec
	RRAT1	20	466	Radius ratio $r_{s,d}/r_l$ at departure planet
	RRAT2	21	467	Radius ratio $r_{s,a}/r_r$ at arrival planet
	ALPHA	22	468	Angle between thrust and velocity vectors, α, deg
	AMC	23	469	Angular momentum components, m^2/sec
	AM	26	472	Magnitude of angular momentum, h, m^2/sec
	AMSQRD	27	473	Square of h, m^4/sec^2
	COSALF	28	474	cosine α
	COSBET	29	475	cosine β
	COSTRU	30	476	cosine ν
	DELV	31	477	Change in vehicle velocity, Δv, m/sec
	DPSI	32	478	Derivative of thrust angle, $\dot{\psi}$, rad/sec
	PATH	33	479	Path angle of vehicle, γ, deg
	RADIUS	34	480	Vehicle's position vector magnitude, r, m
	RSQRD	35	481	Square of r, m^2
	SINALF	36	482	sine α
	SINBET	37	483	sine β
	SINTRU	38	484	sine ν
	THETA	39	485	Central travel angle, θ, deg
	TRU	40	486	True anomaly, ν, rad
	X	41	487	x-component of position vector R, m
	Y	42	488	y-component of position vector R, m
	Z	43	489	z-component of position vector R, m
	VX	44	490	x-component of velocity vector V, m/sec
	VY	45	491	y-component of velocity vector V, m/sec
	VZ	46	492	z-component of velocity vector V, m/sec
	VEL	47	493	Vehicle speed, v, m/sec
	VSQRD	48	494	Speed squared, v^2, m^2/sec^2
ITERAT	IA	1	495	Ten-element array of COMMON locations of the level 1 independent variables X
	IAA	11	505	Ten-element array of COMMON locations of the level 2 optimization variables Z
	IB	21	515	Ten-element array of COMMON locations of the level 1 dependent variables Y
	IBB	31	525	COMMON location of the level 2 criterion of merit Γ
	MAXNUM	32	526	Maximum number of trajectories allowed for a particular case
	WEIGHT	33	527	Ten-element array of level 1 weighting factors for boundary-value problems w_i
	NUM	43	537	Length of IA array (dimensionality of level 1 boundary-value problem)
	NUM2	44	548	Length of IAA array (dimensionality of level optimization problem)
	JNUM	45	539	IA+IAA
	DAMP	46	540	Inhibitor for level 1 linear correction scheme, χ
	CHANGE	47	541	Array increments in the level 1 independent variable vector ΔX
	XIA	67	561	Reference value of the level 1 independent variation vector X
	XIB	87	581	Reference value of the level 2 optimization variable vector Z
	NRUNB	107	601	Run number of best trajectory yet calculated
	TOLER	108	602	Level 1 iteration convergence tolerance, $\bar{\tau}$
	ELEMEN	109	603	10 × 11 Element (double precision) partial derivative matrix G
	PERTEW	329	823	Ten-element array of perturbation factors for univariate search scheme

TABLE III. - Continued. GLOSSARY OF COMMON VARIABLES

Block name	Variable name	Relative location	Absolute location	Definition
ITERAT	PERTNR	339	833	Ten-element array of perturbation factors for linear correction scheme
	PERT2	349	843	Ten-element array of perturbation factors for level 2 search scheme
	SVALUE	359	853	Set of desired sweep variable values, s_i
	MAXPTS	369	863	Maximum number of points to be used in the sweep extrapolation of X
	MORDER	370	864	Order of curve fit to be used in the sweep extrapolation of X
	KOUNT	371	865	Iteration counter for level 1 boundary-value problems
	DESIRE	372	866	Ten-element array of desired values of the dependent vector $\overline{Y}$
	TSKIP	382	876	Two-element array specifying a time interval in which trajectory interrupts are inhibited, sec
	ERRBAR	384	878	Preferred value of initial level 1 error for each step of an automatic sweep
	ERSTAR	385	879	Value of level 1 boundary-value error separating univariate and linear correction schemes, τ^*
	NBVP	386	880	Stage number defining beginning of level 1 boundary-value problem
	NRUN	387	881	Trajectory counter
	RETURN	388	882	Internal control used to indicate boundary-value-problem termination
	ATOE	389	883	Internal control indicating which stage data are saved for boundary-value problems
	XMISSL	390	884	Current lowest error δ_r obtained during the level 1 iteration process
	TOL2	391	885	Tolerance criterion used between level 2 loops to indicate convergence
	DUMMY4	392	886	See DUMMY1 above
BOD	PNAME	1	887	Alphameric list of gravitational body names defining permissible body choices
	REFER	31	917	Alphameric list of reference body names corresponding to PNAME list
	AMASS	61	947	List of body masses corresponding to PNAME list, Sun mass units
	RCRIT	91	977	List of body sphere-of-influence radii corresponding to PNAME list, m
	NUMBOD	121	1007	Input list of selected body numbers, corresponds to PNAME list
BODIES	BODIES	1	1017	Alphameric list of body names corresponding to NUMBOD list of indexes
	BNAME	11	1027	Same list as BODIES but reordered for computational purposes
	BMASS	19	1035	List of body masses corresponding to BNAME list, Sun mass units
	EFMRS	27	1043	Alphameric list of ephemerides needed for a particular case
	GK2M	34	1050	Gravitational constant of the origin body, μ, m^3/sec^2
	GKM	35	1051	Square root of GK2M
	IBODY	36	1052	Index list corresponding to BNAME that indicates position of reference body (also in BNAME)
	MBODYS	44	1060	Number of perturbating bodies selected
	NBODYS	45	1061	Total number of bodies selected
	KBODYS	46	1062	Number of bodies selected for inclusion in the variational equations
	XMASS	47	1063	Mass scaling factor (usually from 0 to 1) that may be varied to smoothly include n-body effects
	NCHAMP	48	1064	BNAME index of the dominant gravitational body
	NEFMRS	49	1065	Index list indicating location of EFMRS bodies in PNAME list
	TRSFER	57	1073	Control whose value is . TRUE. when an origin shift is required
	LBODY	58	1074	An unconditional origin shift to LBODY will take place at trajectory termination if LBODY is loaded (alphameric)
	RBCRIT	59	1075	List of body sphere of influences corresponding to BNAME list, m
	VEFM	66	1082	3 × 8 Array of velocity components of vehicle relative to all bodies, m/sec
	SQRDK	90	1106	Gravitational constant of the Sun, m^3/sec^2
	XFER	91	1107	Control indicating an origin transfer is in progress
	TDAT	92	1108	14 × 7 Array of ephemerides data
	XP	190	1206	3 × 8 Array of perturbating body position components relative to the origin, m
	OBLA	214	1230	Control whose value is . TRUE. if oblateness effects are being included
	RB	215	1231	3 × 8 Array of position components of the vehicle to all bodies, m
	RMAG	239	1255	List of distances of the vehicle to all bodies, m
AERODY	ALT	1	1263	Vehicle altitude above ground, m
	AREA	2	1264	Vehicle's aerodynamic reference area for current stage S_{ref}, m^2
	REFA	3	1265	Ten-element array of stage aerodynamic reference areas, m^2
	CD	13	1275	Total vehicle drag coefficient, C_D
	CL	14	1276	Vehicle lift coefficient, C_L
	DNSITY	15	1277	Atmospheric density, ρ, kg/m^3
	PRESS	16	1278	Atmospheric pressure, p, N/m^3
	TM	17	1279	Atmospheric molecular scale temperature, K
	EXITA	18	1280	Engine exit area, A_e, m^2
	Q	19	1281	Dynamic pressure, q, N/m^2

TABLE III. - Concluded. GLOSSARY OF COMMON VARIABLES

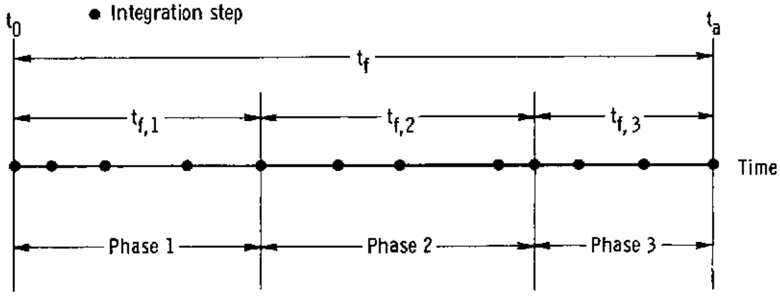
Block name	Variable name	Relative location	Absolute location	Definition
AERODY	AEXIT	20	1282	Ten-element array of stage engine exit areas, m^2
	REVOLV	30	1292	Earth's rotation rate, ω_r, rad/sec
	HEATR	31	1293	Vehicle heating rate, $W/m^2/kg$
	P	32	1294	The vector $B = V_r \times H_r$
	PMAGN	35	1297	Magnitude of the vector B
	RATMOS	36	1298	Radius of the outer limit of the sensible atmosphere, m
	TDRAG	37	1299	Magnitude of vehicle drag acceleration, $\lvert D/m \rvert$, N/kg
	TLIFT	38	1300	Magnitude of vehicle lift acceleration, $\lvert L/m \rvert$, N/kg
	VATM	39	1301	Components of vehicle velocity relative to planet, V_r, m/sec
	VQ	42	1304	Vehicle relative speed, $\lvert V_r \rvert$, m/sec
	VQSQRD	43	1305	Square of VQ, m^2/sec^2
	VMACH	44	1306	Vehicle Mach number, M
	ICD0	45	1307	Index that points at current position in CD0C array
	CD0C	46	1308	Array of parasite drag coefficient data, C_{D0} vs. M
	ICDI	95	1357	Index that points at current position in CDIC array
	CDIC	96	1358	Array of induced drag coefficient data, C_{DI}/C_L^2 vs. M
	ICL	145	1407	Index that points at current position in CLC array
	CLC	146	1408	Array of lift coefficient data, $C_L/\sin\alpha$ vs. M
	IALF	195	1457	Index that points at current position in ALFCOE array
	ALFCOE	196	1458	Array of angle of attack (thrust angle α) data, α vs. t, deg and sec
SAVE	TIME	1	1507	Time (double precision), t, sec
	STEPGO	3	1509	Number of successful integration steps
	STEPNO	4	1510	Number of unsuccessful integration steps
	REVS	5	1511	Number of complete revolutions around x-axis
	DEL	6	1512	Time increment to next output point, sec
	IMODE	7	1513	Current indicator of the integration mode (see MODEI)
	ASYMPT	8	1514	Control set to . TRUE. when path lies too close to an asymptote to use orbit element integration
	LOOKX	9	1515	Five-element array defining COMMON locations of trajectory interrupt variables
	NLOOK	14	1520	Five-element array of counters for each trajectory interrupt, corresponds to LOOKX
	SAVE1	19	1525	Seventeen-element array of initial-value variables saved during level 2 optimization
	SAVE2	36	1542	Seventeen-element array of initial-value variables saved during level 1 iterations
	HDS1	53	1559	One-hundred-fifty-array element of initial integration values saved during level 2 optimization (double precision)
	HDS2	353	1859	One-hundred-fifty-array element of initial integration values saved during level 1 iterations (double precision)
HD	RMASS	1	2159	Vehicle mass (double precision), m, kg
	V	3	2161	Vehicle velocity (double precision), V, m/sec
	R	9	2167	Vehicle position (double precision), R, m
	L	15	2173	Adjoint variables (double precision), Λ, Λ_r, λ_m
	PV	31	2189	Velocity partial derivatives (double precision), δV
	PR	85	2243	Position partial derivatives (double precision), δR
	PL	139	2297	Adjoint partial derivatives (double precision), $\delta\Lambda$
	PJ	193	2351	Adjoint partial derivatives (double precision), $\delta\Lambda_r$
	PM	247	2405	Adjoint partial derivatives (double precision), $\delta\lambda_m$
	P7	265	2423	Adjoint partial derivatives (double precision), δc
	P8	283	2441	Adjoint partial derivatives (double precision), $\delta\dot{m}_0$
H	H	1	2459	Array of current values of the integration variables y_n
HDOT	HDOT	1	2609	Array of current values of the derivatives of the integration variables $\dot{y}_n$

TABLE IV. - SUMMARY OF NOPT OPTIONS[a, b]

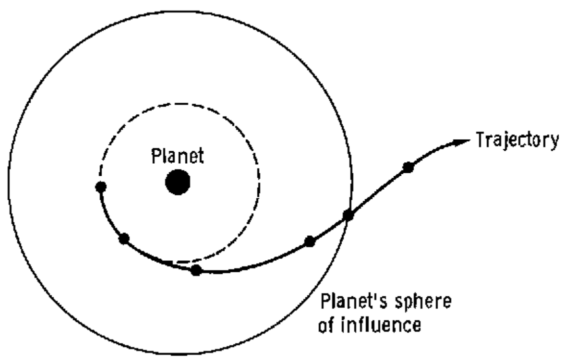
Value of NOPT	Dependent variables (at arrival)	Independent variables (at departure)	Dimensionality of coordinate system	Typical usage
0	Input	Input	2 or 3	Nonvariational problems
1	v_x, v_y, v_z, x, y, z	$\lambda_1, \ldots, \lambda_6$	2 or 3	Cartesian rendezvous
2	r, v, γ, θ	$\lambda_1, \lambda_2, \lambda_4, \lambda_5$	2	Polar rendezvous
3	r, v, γ	$\lambda_1, \lambda_2, \lambda_4$	↓	Optimum-angle rendezvous
4	r, θ, Λ/λ_m	$\lambda_1, \lambda_2, \lambda_4, \lambda_5$	↓	Flybys
5	r, Λ/λ_m	$\lambda_1, \lambda_2, \lambda_4$	↓	Optimum-angle flybys
6	Input	Input	2 or 3	Any variational case not specified above
7	Input	Input	2 or 3	Same as option 6 but with optimum travel angle

[a]By default, the program will generate the partial derivatives by numerical integration if $1 \leq$ NOPT ≤ 5 and by finite differencing otherwise. If the user prefers the finite difference scheme even if $1 \leq$ NOPT ≤ 5, he should attach a minus sign to his NOPT entry.

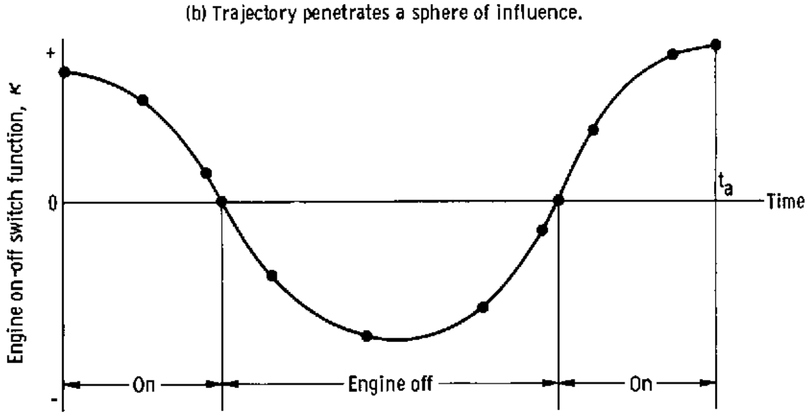
[b]For all propulsion cases, λ_1 in this table is replaced by $\dot{m}_0$ (or by a_0/g if initial thrust-weight ratio was input).



(a) Phases or staging.

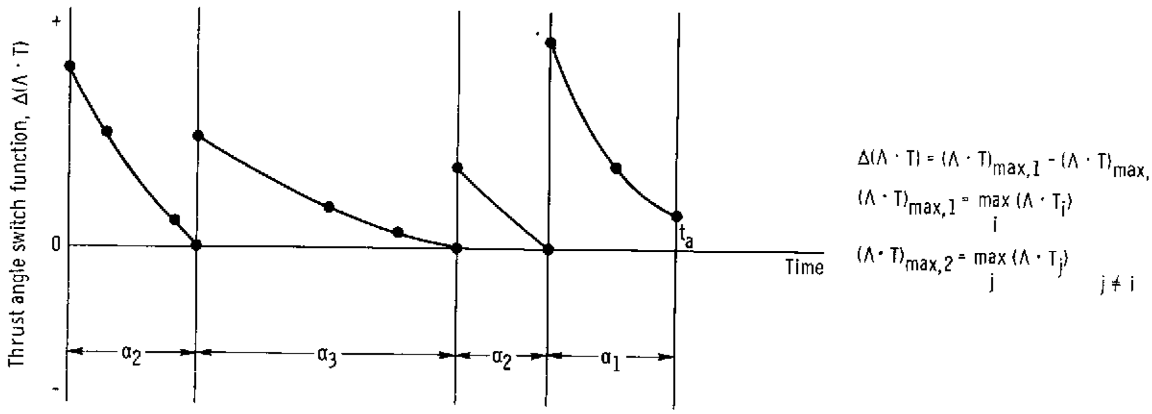


(b) Trajectory penetrates a sphere of influence.

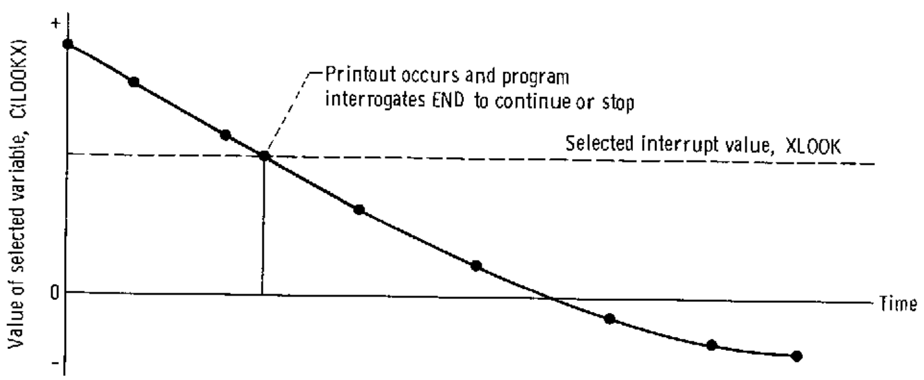


(c) Optimal engine on-off times.

Figure 1. - Trajectory interrupt situations.



(d) Optimum fixed-thrust-angle selection from a set of angles α_i.



(e) User-selected interrupt on arbitrary variable.

Figure I. - Concluded.

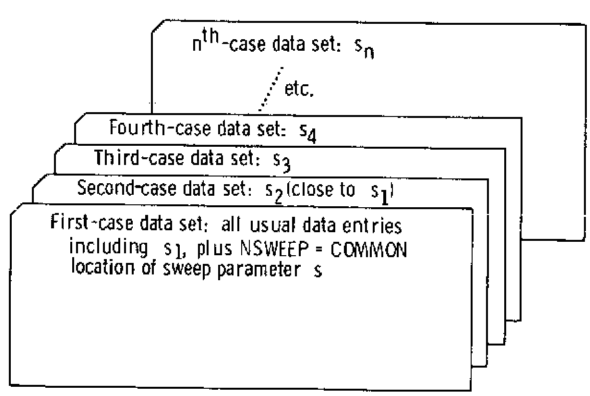


Figure 2. - Data deck setup for manual sweep.

All usual data entries, plus IAA = COMMON location of sweep parameter s
SVALUE = $s_1, s_2, \cdots, s_n$; $n \leq 10$ (the values of s where a full trajectory printout will occur)
MAXPTS = 2

Figure 3. - Data deck setup for automatic sweep.

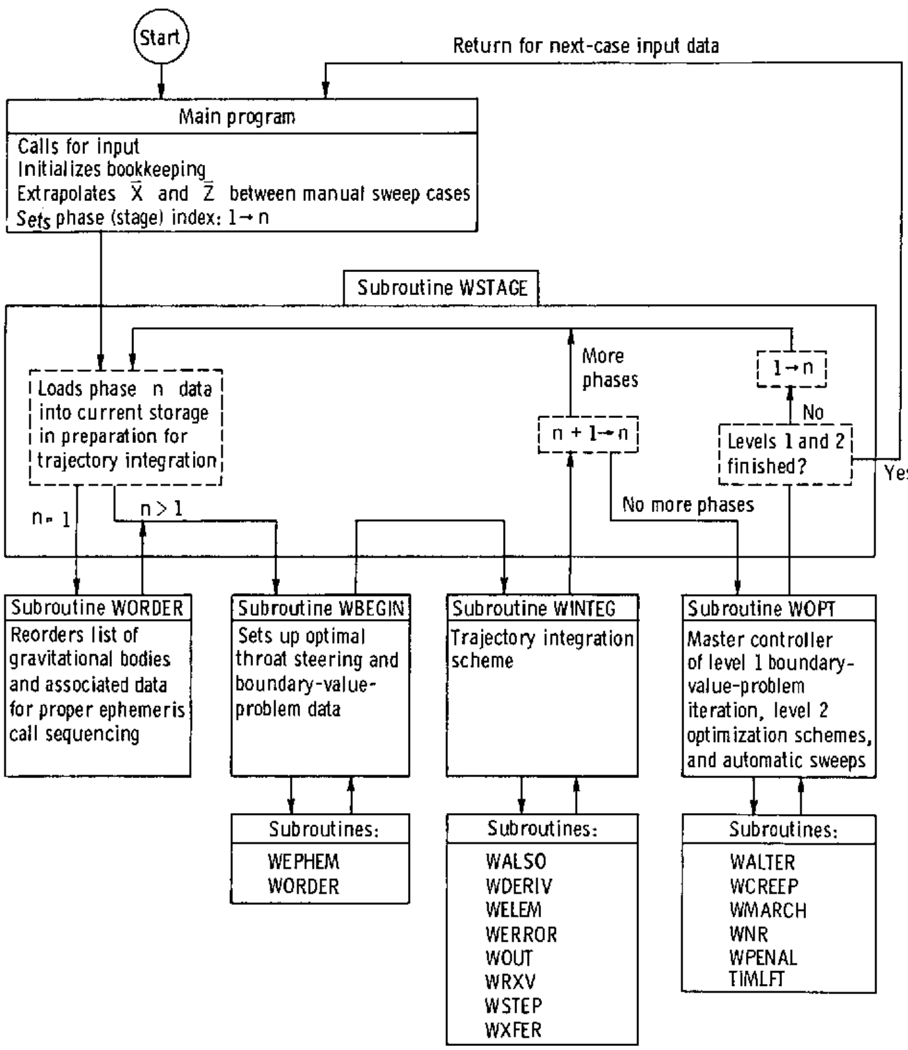


Figure 4. - NBODY flow diagram.

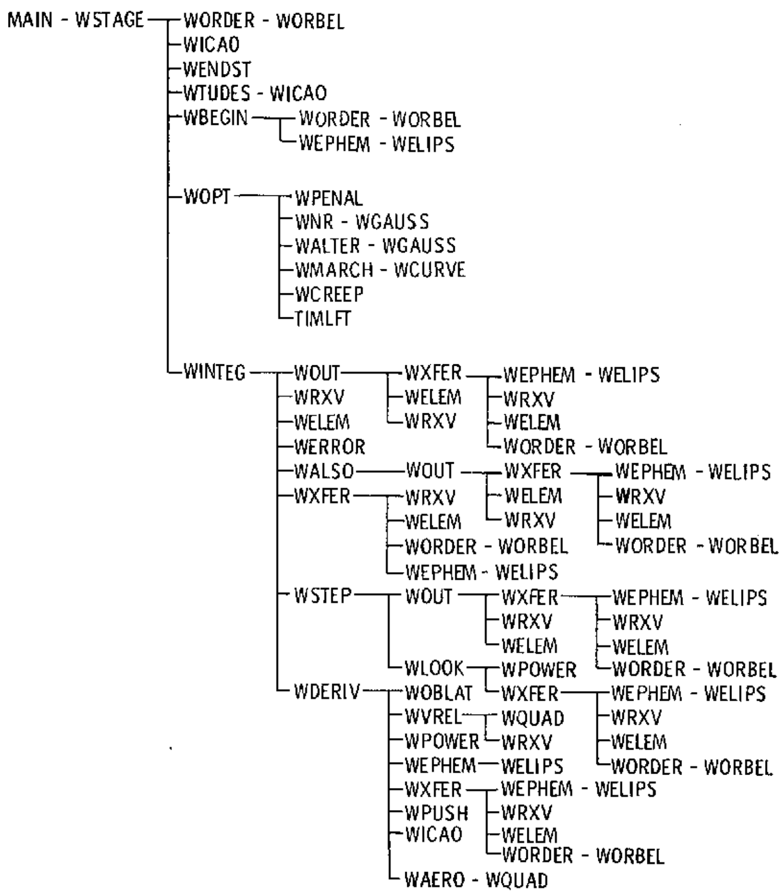


Figure 5. - Subprogram call sequence.

NASA-Langley, 1974 E-6998