TWO-PHASE CHOKED FLOW OF SUBCOOLED NITROGEN THROUGH A SLIT

by Robert J. Simoneau
Lewis Research Center
Cleveland, Ohio 44135

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Robert J. Simoneau
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

Two-phase choked flow rate and pressure distribution data are reported for subcooled nitrogen flowing through a slit. The slit was a narrow rectangular passage of equal length and width, \((L/D_H=43.5)\). The inlet stagnation pressure ranged from slightly above saturation to twice the thermodynamic critical pressure. Four stagnation isotherms were investigated covering a range which spanned the critical temperature.

The results suggested a uniform two-phase flow pattern with vaporization occurring at or near the exit in most cases. The results compared favorably with the theory of Henry for nonequilibrium subcooled two-phase choked flow in long tubes.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>B</td>
<td>Constant in quality relaxation expression, eq. (6)</td>
</tr>
<tr>
<td>CD</td>
<td>Orifice (or entrance) coefficient</td>
</tr>
<tr>
<td>DH</td>
<td>Hydraulic diameter, (4\pi A/W)</td>
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<tr>
<td>G</td>
<td>Mass flux, gm/cm²·sec</td>
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<td>L</td>
<td>Flow passage length, cm</td>
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<tr>
<td>N</td>
<td>Nonequilibrium coefficient, eq. (3)</td>
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<tr>
<td>P</td>
<td>Pressure, N/cm²</td>
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<td>S</td>
<td>Entropy, J/gm·K</td>
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<td>T</td>
<td>Temperature, K</td>
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<tr>
<td>v</td>
<td>Specific volume, cm³/gm</td>
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<tr>
<td>x</td>
<td>Quality</td>
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<tr>
<td>Z</td>
<td>Distance from flow passage entrance, cm</td>
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Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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<tbody>
<tr>
<td>B</td>
<td>Back pressure</td>
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<tr>
<td>c</td>
<td>Thermodynamic critical point</td>
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<tr>
<td>E</td>
<td>Exit</td>
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<tr>
<td>equil</td>
<td>Thermodynamic equilibrium</td>
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<tr>
<td>g</td>
<td>Gas or vapor condition</td>
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<tr>
<td>max</td>
<td>Maximum or choked condition</td>
</tr>
<tr>
<td>l</td>
<td>Liquid condition</td>
</tr>
<tr>
<td>sat</td>
<td>Saturation condition</td>
</tr>
<tr>
<td>o</td>
<td>Stagnation condition</td>
</tr>
<tr>
<td>10</td>
<td>Pressure measuring station number 10</td>
</tr>
<tr>
<td>11</td>
<td>Pressure measuring station number 11</td>
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INTRODUCTION

The present paper is part of a series of work (1-4) directed at the study of two-phase choked flow of cryogens discharging under high pressure through various openings from storage vessels. In the early work Hendricks and Simoneau, et al., (1, 2) mapped an extensive range of choked flows of cryogenic nitrogen in a converging-diverging nozzle with subcooled inlets at stagnation temperatures and pressures both above and below the thermodynamic critical point. Subsequently liquid methane (3) was added to the data and a flow normalizing parameter was introduced. In the most recent work Hendricks, et al., (4) turned their attention to flow through long, narrow passages. Reference 4 and also the present study were motivated by an interest in being able to predict the two-phase leak rate and flow distribution of pressurized cryogens discharging into space through a long, narrow rotating seal passage. A description of these fluid film seals can be found in ref. 5. A special feature of these seals, in addition to the small gap heights (0.008 - 0.0013 cm) and long passage length (L/D_h=50-200), is that they present a very wide front or aspect ratio to the flow. It is of concern whether vaporization will occur locally or uniformly, also whether the flow behaves similarly to conventional tube and nozzle geometry, and finally whether it can be predicted. Insight to the first part of this was provided in the visual work of ref. 4. The visual results indicate that, if the passage inlet is subcooled and no bubbles are generated near it, the vaporization will be uniform and will occur near the exit plane. On the other hand, if the inlet is saturated or nearly-saturated or if bubbles are generated near the inlet, then localized vaporization may occur and the vaporization plane will move upstream from the exit plane. In the present study the inlet was always subcooled with no bubble source and the purpose of the study was to gather data relevant to this situation and compare it with theory.

A very large and sustained contribution to this whole field, and especially to the area of two-phase choked flow in tubes, has been that of Henry and Fauske and co-workers (6-12). Their work, which focuses on an attempt to account for the thermodynamic nonequilibrium that can occur in this very rapid two-phase process, is thoroughly described in refs. 6 and 7. Of most significance to the present work is the paper by Henry (8) in which he presents the analysis for tubes with subcooled inlets. The model assumes an entrance pressure drop to L/D_h=12 and then a non-equilibrium flashing flow momentum pressure drop from L/D_h=12 to exit. The visual observations of ref. 4 suggest that at high pressure the L/D=12 demarcation not be a fixed number. However, this author agrees with Henry that the two essential components are momentum drop due to the entrance and due to the nonequilibrium flashing flow. Since the analysis neglects friction, the precise location of flashing may not have that great a bearing and Henry's analysis (8) was taken over directly in this study. The analysis was performed using the same computer program as Henry, et al. (9) and was furnished to the author by M. N. Hutcherson. Only the essential equations are presented herein; the reader is directed to ref. 8 for detail. The basic equation for subcooled homogeneous choked flow is:
\[ G_{\text{max}}^2 = - \left[ x \frac{dv_g}{dP} + (v_g - v_l) \frac{dx}{dP} \right]^{-1} \] (1)

The nonequilibrium is introduced empirically through the quality derivative

\[ \frac{dx}{dP} = N \frac{dx_{\text{equil}}}{dP} \] (2)

where

\[ N = \begin{cases} 20 x_{\text{equil}} & x < 0.05 \\ 1.0 & x \geq 0.05 \end{cases} \]

\[ x_{\text{equil}} = \left( \frac{S_0 - S_l}{S_g - S_l} \right)_{\text{equil}} \] (3)

This eventually yields

\[ \frac{P_E}{P_o} = 1 - G_{\text{max}}^2 \left[ \frac{v_{l,0}}{2C^2 D} + x_E (v_g, E - v_{l,0}) \right] \] (4)

and

\[ G_{\text{max}}^2 = \left[ \frac{x v_g}{P} - (v_g - v_{l,0}) N \frac{dx_{\text{equil}}}{dP} \right]^{-1} \] (5)

where

\[ x_E = N x_{\text{equil}} \left[ 1 - e^{-B(L/D_H)^{-1/2}} \right] \] (6)
represents the relaxation in quality over the tube length.

Henry successfully applied this analysis to the data of refs. 12-16. In addition to the analysis, Henry (10) and Prisco (11) explored extensively the validity of the pressure measurement near the exit. They found that for rapidly expanding exits the wall pressure measurement was not a true representation of the centerline and could not be called the "exit" pressure. If a 70° diffuser was attached to the exit this same pressure was much higher and more representative.

In addition to the above work, there are many significant papers in two-phase choked flow. These are surveyed by Henry, et al. (7) and also by Hsu (17). Two papers, Ryley and Parker (18) and Agostinelli and Salemann (19) deal directly with slots and should be mentioned. Ryley and Parker used a visual test section 3.56 cm long by 2.54 cm wide by .127 cm high (L/DH ≈ 15) and observed steam flow with water injection at the inlet. They measured flow rates but not exit pressure. They visually observed separated flow. They found that the equilibrium model substantially underpredicted flow rates and the other current models overpredicted. This was, of course, not a subcooled inlet. Agostinelli and Salemann (19), on the other hand, did deal with subcooled inlets. Their test section was a long concentric annulus 2.54 cm in diameter and gap heights from .015 to .043 cm. The test sections were very long, L/DH of 174-830. Their analysis was kind of a clever graphical (pre-computer days) solution which defined the choked flow rate as the average of two values. The first (lower limit) value was the flow rate corresponding to frictional pressure drop from stagnation to saturation pressure. The second (upper limit) value was obtained by an imaginary extrapolation of the liquid friction flow to a sub-saturation pressure where the flow intercepted the homogeneous, isentropic equilibrium nozzle expansion. This is an intriguing analysis because it does have kind of a nonequilibrium element to it; however, it does require a situation where friction is dominant. It was not pursued in the present study.

**Description of the Experiment**

The flow passage used in this experiment was a narrow rectangular channel of equal length and width as illustrated in figure 1. This test section assembly was placed in a liquid nitrogen blowdown facility designed for use in choked flow experiments. This rig is illustrated in figure 2.

Only the essential features of the flow facility are illustrated in figure 2. The flow facility is probably best described by going through a normal operating sequence describing the components as we go along. After some preliminary gas flow checks, a normal test is begun by filling the high pressure vessel with liquid nitrogen from a large low pressure supply. The pressure vessel has a capacity of 0.11 m³ (30 gal) and is capable of pressures up to
1000 N/cm$^2$ (1500 psia). At present the system is being operated at pressures up to 680 N/cm$^2$ based on the choice of pressure transducers. This allows stagnation pressures from near saturation to twice the thermodynamic critical pressure, ($P_c = 341.7$ N/cm$^2$). The stagnation temperature of the fluid nitrogen is controlled by bubbling ambient gas nitrogen into the bottom of the pressure vessel. What is really desired is not the static tank temperature, but rather the temperature of the flowing system at the inlet plenum of the test section. With a little practice using this procedure, the inlet stagnation temperature can be set to within a spread of about 1 K. Tank temperatures from near the normal boiling point (77.4 K) to ambient temperatures are possible. In this experiment $T_o$ ranged from 100-130 K ($T_c = 126.2$ K).

Upon completion of the filling and warming operations, the flow sequence is initiated by supplying the pressurized gas nitrogen to the top of the tank and opening the back pressure control valve. The liquid nitrogen flows from the tank, through the primary orifice flowmeter, through the test section and back pressure control valve. It is then completely vaporized in a steam heat exchanger and passed through a backup orifice flowmeter before venting to the atmosphere. The tank, test section, and primary flowmeter were contained in a vacuum chamber to reduce heat losses. Choking is demonstrated by making a substantial change in the back pressure and observing whether or not the flow rate and pressure profile in the test section are affected. All of the data presented herein have this type of choking confirmation. Temperatures were measured at appropriate points throughout the system with platinum resistance thermometers accurate to $\pm 0.1$ K. The temperature profile in the tank was monitored with a rake of chromel-constantan thermocouples referenced to one of the platinum thermometers. This gave an indication of the liquid level and the temperature stratification in the tank. A typical temperature variation from just below the liquid-gas interface to the bottom of the tank was 1 K. All of the pressures were measured with strain gage transducers rated accurate to $\pm 0.5$ % of full scale. They were independently calibrated in a standards laboratory and before each day's run were statically checked against one another. They consistently remained with $\pm 0.25$ % of each other on all crucial measurements. The flowmeters were calibrated in a standards laboratory; however, examining the various measurements required for flow rates it is felt that they cannot be known to better than $\pm 2$ %. All of the analog data signals were digitized with a scanning digital voltmeter and transmitted to a central data acquisition and computing facility. It required 15 seconds to acquire all the data and this represented 3 separate samples of each point.

The test section was formed by placing two square 2.54 cm x 2.54 cm blocks within 0.0292 cm (0.0115 in.) of each other as illustrated in figure 1. In the flow direction the walls were parallel to within 0.0003 cm. Across the channel the passage height varied uniformly from 0.0284 cm on one side to 0.0300 cm on the other. The entrance and exit shapes were nominally "sharp edged";
however, careful examination indicates the machining process left an approximately 45° chamfer on the order of 0.006 cm. All dimensions were taken from the edge of the chamfer. Pressure profiles were measured using an array of 14 pressure taps, arranged as shown schematically in figure 1. The center of the "exit" tap was at 0.49 hydraulic diameters from the exit and the next tap upstream was at 3.11 hydraulic diameters from the exit. The overall L/DH was 43.47. The complete description of the tap locations appears in figure 3 which is a plot of pressure profiles. Eleven of the pressure taps were along the centerline of flow while 3 were located off center to detect flow maldistribution, if any. The hole diameters ranged from 0.020 cm for those near the exit to 0.033 cm for the upstream taps. Pressure taps and platinum resistance thermometers were located in the inlet and exit plenums of the flow passage.

RESULTS

The results of this experiment consist of flow rate and pressure drop data for choked flow of subcooled liquid nitrogen through a narrow channel as shown in figure 1. Recall a special feature of the test section is that it was as wide as it was long (43.5 hydraulic diameters). Data were acquired for a range inlet stagnation pressures from somewhat above saturation to twice the thermodynamic critical pressure. Four inlet stagnation temperatures were investigated over a range 0.84 < T0/Tc < 1.03.

The pressure distribution in the channel for a typical flow condition is shown in figure 3. The five profiles shown in figure 3 were all taken during the discharge of a single tank of nitrogen with nominally constant inlet stagnation conditions at five different back pressure levels. The 1 K temperature stratification of the tank is apparent in the series of runs recorded. As can be seen from the data, the first two runs represent unchoked conditions while the last three represent choked flows. A 60% change in back pressure produces virtually no change in flow rate or pressure profile. The slight changes can be accounted for by the slight rise in stagnation temperature.

A couple of observations on these profiles are in order. First, the entrance region. The fact that the pressure drops substantially in the first few Z/DH suggests that indeed a vena contracta does exist in the entrance as expected and any analysis will require some sort of orifice coefficient. It should be pointed out that the pressure dropping below downstream values at the first location, as shown in figure 3, was unusual. It was more normal to have a monotonic decrease in pressure with Z/DH. In any case it was common for the pressure to drop to 60% of the stagnation pressure before Z/DH = 3. The second area of importance is the profile behavior near the exit plane. A careful examination of the three choked flow profiles show that the pressures at taps 7-10 increase slightly as the temperature increases. This is consistent with normal data trends. This trend is not true at tap 11, (the pressure tap nearest the exit plane). At tap 11 the pressure drops slightly as the back
pressure is lowered even though the temperature is rising. This suggests that, even though the flow is choked, variations in back pressure do slightly influence the pressure at this point. The work of Henry and co-workers (10, 11) indicate quite conclusively that wall pressure taps near the exit are not indicative of the centerline pressure for rapidly diverging exits. They indicate that wall pressures will be lower and this appears consistent in the present data. To obtain a feel for the magnitude of this discrepancy in the present experiment we can linearly extrapolate these profiles to the exit plane. This intercept value in the case plotted is 6% below the value at tap 10 and about 10% above the value at the last tap, 11. It is not claimed that this intercept value is the true exit pressure; however, it appears reasonable that the true value falls between the values measured at the last two taps and the percentage figures provide an insight into the level of error involved.

The choked flow rates for inlet stagnation temperatures of 103.4, 110.0, 124.1, and 129.3 K are plotted on figures 4-7 respectively as a function of inlet stagnation pressure. Included on figures 4-7 are the mean lines of unpublished choked flow rate data taken in a two-dimensional converging-diverging nozzle in the same test facility, figure 2. These data are offered as a reference level for the narrow channel data of the present experiment. Finally, figures 4 and 5 include computations of choked flow rates based on Henry's model referred to in the introduction. This will be taken up a little later; first, the data. The data are probably best discussed in terms of the reference nozzle data. For the most part the choked flow rates in the narrow channel fall about 25% below those in a converging-diverging nozzle for the same stagnation conditions. In addition, the trend or slope of the data as the stagnation pressure increases is different for the two geometries, especially at the higher temperatures. It appears that the primary reason for the difference in flow rate is the large pressure drop experienced at the "sharp" entrance of the narrow channel. Friction should play some role and maybe the difference in trend, especially at the higher temperature, can be attributed to friction. It was observed that the inlet pressure drop was not as great at the higher temperatures.

The data for the two lower isotherms (103.4 and 110.0 K) were analyzed using the model proposed by Henry (8) for long tubes with subcooled inlets. This model attempts to account for the thermodynamic nonequilibrium associated with two-phase choked flow. It does not include friction. The analysis was performed using a computer program written by M. H. Hutcherson of Argonne National Laboratories and is the same program used in reference 9. It was modified by the author for use with nitrogen. The results using orifice entrance, coefficients of 0.611 and 0.820 are shown in figures 4 and 5. The data fall between these two curves, slightly favoring the $C_p=0.820$ curve, and agreeing very well in trend. The importance of the entrance is emphasized in these curves. The edge at the entrance was nominally "sharp" but because the walls are so close the slight chamfer becomes large. In a fluid film seal
application the walls will be over an order of magnitude closer which even further highlights the entrance. Unfortunately, it was not possible to carry out the analysis for $T_o = 124.3$ and 129.5 K. This was because of the way the thermodynamic properties are handled in the computer program. The program was written for relatively low stagnation temperature. Some of the approximations used, while perfectly valid at low temperature, cause problems near the critical temperature, $T_c = 126.2$ K. The program would have to be completely rewritten, which was not possible at this time. Also it is not clear that the Henry model would be valid in this region. The nonequilibrium relationship given in eq. (2) was formulated from "flashing" data at low pressures ($P/P_c \approx 0.05$). Near the critical point, where density differences, surface tension, and the heat of vaporization are tending to zero, such nonequilibrium "flashing" considerations may be of little importance. Also, the compressibility of the liquid phase becomes important in the proximity of the critical point, and this was not included in the model presented in ref. 8.

The pressure distribution at the last two stations corresponding to the above choked flow rates are shown in figures 8-11. Recall from the discussion of figure 3 that it is the author's opinion that the true exit pressure lies between these two values. The corresponding two-dimensional nozzle throat pressures were not plotted because, unlike flow rates, the pressures are very sensitive to geometry and comparisons may not be valid. Again the Henry model is shown for $T_o = 103.4$ and 110.0 K on figures 8 and 9. The agreement is very good. If one accepts the contention that the exit pressure falls somewhere between the two values plotted, the prediction is probably slightly high—maybe about 10%. A curious peak occurs in the calculated $P_E/P_o$ using the Henry model at low stagnation pressure along the $T_o = 110.0$ K isotherm. This is presented cautiously for a couple of reasons. First, Henry and co-workers have never reported similar calculated results. Secondly, the data at this temperature level do not show a peak. This may be due to a subtle computational difficulty; however, nothing appears obvious. On the other hand, the equilibrium calculations of Hendricks and Simoneau (3) show similar peaks. Also the data at higher temperatures, both in nozzles (2, 3) and the slit (figures 10 and 11), do exhibit peaks. Again it is emphasized that this result is presented cautiously, but it is interesting. One thing is sure; the pressure behavior at temperatures near the thermodynamic critical in both nozzles (2, 3) and long passages is quite different from what one is used to seeing at more conventional low temperatures.

Finally, in a test section, such as this, with a wide flow aspect ratio the question of flow maldistribution arises. The results of this test, coupled with the visual observations of Hendricks, et al. (4) suggest that for subcooled inlets no significant flow maldistribution exists. The pressures measured at the off-centerline taps were well within the range of their centerline
counterparts. Most important, the choked flow rate and pressure measurements were consistent with round tube and nozzle experience and agreed with accepted theory. In making these remarks, however, it should be pointed out that this was a high inertia test facility and any small or short time perturbations would not be recorded.

CONCLUSIONS

An experiment has been conducted and data acquired for two-phase choked flow of cryogenic nitrogen through a narrow slit. A special characteristic of the slit was the wide front or aspect ratio presented to the flow. The resulting data analysis and comparison with theory yields the following concluding observations.

First, for stagnation temperatures that are subcooled and significantly below the thermodynamic critical temperature \( T_0/T_c = 0.820 \) and 0.871 in the present case) several remarks can be made. The fact that the flow rates in the slit exhibit the same trend but are substantially lower than a two-dimensional converging-diverging nozzle suggests the primary pressure drop is momentum and that the entrance geometry is significant. The agreement with Henry's nonequilibrium subcooled inlet theory, which assumes momentum dominates and includes an entrance coefficient, reinforces this observation. The agreement with Henry's theory also reinforces that thermodynamic nonequilibrium is important in the two-phase expansion. The two-phase expansion appears to be uniform with no flow maldistribution and this slit geometry, with its wide front, can be treated analytically as a long tube with a theory such as Henry's.

For inlet stagnation temperatures near the thermodynamic critical temperature \( T_0/T_c = 0.985 \) and 1.022 in the present case) some different observations are in order. The exit pressure ratio, \( P_E/P_0 \), does not decrease monotonically with increased stagnation pressure as appears to be the case at lowest stagnation temperatures. This also occurs in nozzles. The choked flow rates, while still lower than the two-dimensional nozzle, no longer exhibit the same trend as the nozzle flow rates. This suggests maybe other factors, primarily friction and the influence of the critical point on thermo-physical properties, would have to enter into the analysis. Since it was not possible to compute this close to the critical point with the present program, the applicability of Henry's model in this region is an open question.

Acknowledgement

The author wishes to thank Mr. Michael N. Hutcherson of Argonne National Laboratory for writing and furnishing the computer program used herein for analyzing the data.
References


Figure 1. - Flow passage illustration. (All dimensions in cm.)

Figure 2. - Schematic illustration of the flow system.

Figure 3. - Typical axial pressure distribution for both choked and unchoked flow of subcooled nitrogen.
TWO-DIMENSIONAL 9000 - 9000 NOZZLE DATA

$T_0 = 95.3$ K
$T_0 = 109.9$ K
$C_D = 0.820$
$C_D = 0.611$
HENRY MODEL, $T_0 = 103.4$ K

Figure 4. - Choked flow rate data at $T_0 = 103.4$ K, including comparison with two-dimensional nozzle data and Henry's theory.

$P_0/P_C$ FOR $T = 103.4$ K

$P_0/P_C$ FOR $T = 109.9$ K

$C_D = 0.820$
$C_D = 0.611$
HENRY MODEL, $T_0 = 110.0$ K

Figure 5. - Choked flow rate data at $T_0 = 110.0$ K, including comparison with two-dimensional nozzle data and Henry's theory.

$P_0/P_C$ FOR $T = 103.4$ K

$P_0/P_C$ FOR $T = 110.0$ K

$C_D = 0.820$
$C_D = 0.611$
HENRY MODEL, $T_0 = 124.3$ K

Figure 6. - Choked flow rate data at $T_0 = 124.3$ K, including comparison with the two-dimensional nozzle data.

$P_0/P_C$ FOR $T = 124.3$ K

$P_0/P_C$ FOR $T = 129.3$ K

$C_D = 0.820$
$C_D = 0.611$
HENRY MODEL, $T_0 = 129.3$ K

Figure 7. - Choked flow rate data at $T_0 = 129.3$ K, including comparison with two-dimensional nozzle data.
Figure 8. Pressure behavior near the exit during choked flow at $T_0 = 103.4$ K, including comparison with Henry's theory.

Figure 9. Pressure behavior near the exit during choked flow at $T_0 = 110.0$ K, including comparison with Henry's theory.

Figure 10. Pressure behavior near the exit during choked flow at $T_0 = 124.3$ K.

Figure 11. Pressure behavior near the exit during choked flow at $T_0 = 129.3$ K.