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FINAL REPORT

HYDRODYNAMIC

WATER

IMPACT

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SUMMARY

The hydrodynamic impact of a falling body upon a viscous incompressible fluid is investigated by numerically solving the equations of motion. Initially the mathematical model simulated the axisymmetric impact of a rigid right circular cylinder upon the initially quiescent free surface of a fluid. A compressible air layer exists between the falling cylinder and the liquid free surface.

The mathematical model was developed by applying the Navier-Stokes equations to the incompressible air layer and the incompressible fluid. Assuming the flow to be one dimensional within the air layer, the average velocity, pressure and density distributions were calculated. The liquid free surface was allowed to deform as the air pressure acting on it increases. For the liquid the normalized equations were expressed in two-dimensional cylindrical coordinates.

The governing equations for the air layer and the liquid were expressed in finite difference form and solved numerically. For the liquid a modified version of the Marker-and-Cell method was used. This method, developed by the Los Alamos Scientific Laboratory, is especially suitable for solving time-dependent fluid flow problems involving a free surface.

The mathematical model has been reexamined and a new approach has recently been initiated. Essentially this consists of examining the impact of an inclined plate onto a quiesent water surface with the equations now formulated in cartesian coordinates.

INTRODUCTION

One of man's earliest serious investigations of hydrodynamic impact was with the problem of ship slamming. He learned throughout the centuries by experimentation that he could minimize the damage due to slamming by varying the size, shape, supporting structure and mass of the hull. The ship slamming phase of hydrodynamic impact is still being studied in great detail today. Advances in technology ultimately led to a study of the impact of seaplane floats during landing, the water impact connected with the laying of mines, the dropping of depth charges, the launching of torpedos, the dropping of oceanographic instrument packages and more recently the water landing of returned manned spacecraft. A knowledge of the pressure distribution during impact is necessary to be able to design a spacecraft for landing.

The hydrodynamic impact problem is a very difficult one because the physics of what actually happens during the instant of impacts is not understood. Many theoretical studies have been reported and in general did not closely agree with experimental results.

Part I describes the approach used in the initial stages of the investigation using a cylindrical coordinate system with a cylindrical body falling vertically downward along the Z axis with the lower surface of the body perpendicular to the Z axis and parallel to the water surface. Even with a deflecting water surface the singularities associated with high pressures at impact were always present.

Part II describes a reevaluation of the problem which took place in January 1972 with a view to understanding the basic impact phenomena. The main changes are (1) the problem is set up in cartesian coordinates instead of cylindrical coordinates; this removes problems originating at the origin where the radius is zero (2) the infinite flat plate approaches the water surface at a small angle. Thus the impact conditions are not as severe as in the original case. It is hoped that the results can be extrapolated to the case where the angle is zero.

LITERATURE SURVEY

Von Karmen (1) was one of the first to investigate impact in his 1929 study of the impact on seaplane floats during landing. When conservation of momentum is applied at impact, the body velocity is found to decrease and the total mass to increase due to water set in motion by the body. This increase in mass was called the "added apparent mass." Chu and Abramson (2) provided an excellent review and bibliography on the theories of hydrodynamic impact through 1961. They suggested consideration of compressibility effects during the initial stage of impact and the use of numerical techniques. Jensen and Rosenbaum (3) developed a mathematical model to study water impact of the Mercury Space Capsule by modifying Von Karmen's theory. The spherical bottom of the capsule was represented by a series of wedges with a 10° deadrise angle. It was found that accelerations obtained from this mathematical model were initially less than those obtained experimentally for vertical impact.

Moran (4) published a detailed survey of hydrodynamic impact theories through 1964. He stated that the inclusion of compressibility effects removed some of the glaring defects of the earlier theories but compressibility of the water is not very important for the water-entry of a blunt - or round-nosed body at low speeds. He recommended the consideration of a finite air layer between the body and the water. Because the free surface at the impact point has already accelerated to the body speed the inclusion of air density effects eliminates the abrupt velocity change at impact. This velocity discontinuity is responsible for the infinite pressures found in earlier impact theories.

Li and Sugimura (5) presented an analysis of the water impact of the Apollo Command Module in 1967. The impact of a rigid sphere upon a quiescent incompressible inviscid sea was assumed. A compression wave was considered at the first point of impact to prevent an infinite initial impact pressure.

The answer was given in the form of an infinite power series. Verhagen (6) presented an excellent publication on the investigation of the impact of an infinite flat plate when dropped vertically on an undisturbed water surface. A compressible inviscid air layer is assumed to exist between the plate and water surface. The water is considered incompressible. A deformable free surface is considered and impact is said to take place when the deformed free surface touches the edge of the plate. Verhagen found that impact appears to take place when the air velocity at the edge of the plate almost reaches acoustic velocity. The pressure distribution was smoothed to prevent singularities in the mathematical evaluation of the problem.

Kurland (7) in 1968, presented a review and comparison of all model and theoretical studies involving water landing loads on the Apollo Command Module. Lewison and Maclean (8) investigated the impact between a rigid flat plate and a free water surface with a compressible air layer. They postulated that the compression of the air under the plate sets the water surface in motion downwards and eventual impact occurs with vanishing relative velocity and hence infinite pressures do not occur.

Experimental tests on hydrodynamic impact are reported in references 9 through 14. The impact phase of the Apollo Command Module were studied in 1964 by Herting, Pollack and Pohlen (9). Full scale boiler plates were used to investigate the pressure loads on the craft during impact and the flotation characteristics after impact. Benson (10) in 1965 compared full-scale and model data obtained on the Apollo Module during water impact using a modified von Kārman analysis. Theoretical and model studies compared favorably. In 1966 Chuang (11) investigated rigid flat-bottom body slamming by dropping steel plates from various heights above a calm water surface. He found that because of the effect of the trapped air between the falling body and the water, maximum impact pressure was much lower than expected if the generally accepted acoustic pressure formula were applied. Baker and Westine (12) in 1966 studied water impact of the Apollo Module using 1/4.5 scale models. Data obtained from heavily instrumented models was compared with results of full scale experiments yielding good predictions of pressures, acceleration, displacements and impact velocity. In 1967 Chuang (13) dropped flat-bottomed and wedge-shaped bodies from various heights and found that maximum pressures occurred before the water came in contact with the impact surface of the flat bottom. For models with a deadrise angle of 3° or greater, most of the air escaped at the moment of impact. With a smaller deadrise angle, relatively large amounts of air were trapped to give a cushioning effect.

Thompson (14) investigated the rough water landing characteristics of a Gemini-type spacecraft in 1967 using a 1/6 scale model. Gerlach (15) in late 1967, experimentally investigated the importance of air density and other real fluid properties with respect to the water impact of small blunt rigid bodies. He found that the restriction of airflow reduced the peak impact pressure and that a small amount of air is actually trapped under the model.

Part I
THEORY

The case under consideration is that of a rigid right circular cylinder falling axisymmetrically toward the quiescent free surface of a viscous incompressible liquid. The cylinder of radius R_p is initially a distance h above the free surface and is falling with a current downward velocity of $v_p(t)$ as shown in figure 1. A compressible air layer exists between the falling cylinder and the free surface. The behavior of the compressible air layer is studied as the cylinder approaches the free surface. The effect of the air layer on the liquid free surface and pressure and velocity fields is determined. The problem is simulated by holding the cylinder stationary and moving the entire mass of liquid toward the cylinder.

Since the system is axisymmetric a two-dimensional system is utilized. The governing equations are

$$\frac{1}{r_s}\frac{\partial u_s r_s}{\partial r_s} + \frac{\partial v_s}{\partial z_s} = 0 \tag{1}$$

$$\rho\left\{\frac{\partial u_s}{\partial t_s} + \frac{1}{r_s}\frac{\partial u_s^2 r_s}{\partial r_s} + \frac{\partial u_s v_s}{\partial z_s}\right\} = -\frac{\partial p_s}{\partial r_s} + \mu\frac{\partial}{\partial z_s}\left[\frac{\partial u_s}{\partial z_s} - \frac{\partial v_s}{\partial r_s}\right] \tag{2}$$

$$\rho\left\{\frac{\partial v_s}{\partial t_s} + \frac{1}{r_s}\frac{\partial u_s v_s r_s}{\partial r_s} + \frac{\partial v_s^2}{\partial z_s}\right\} = -g_z - \frac{\partial p_s}{\partial z_s} - \frac{\mu}{r_s}\frac{\partial}{\partial r_s}\left[r_s\left(\frac{\partial u_s}{\partial z_s} - \frac{\partial v_s}{\partial r_s}\right)\right] \tag{3}$$

The origin of the moving coordinate system is attached to the center of the base of the falling projectile (see figure 2) and hence

$$r = r_s \quad ; \quad z = z_s - \int_0^{t_s} v_p \, dt_s \tag{4}$$

$$v = v_s - v_p \quad ; \quad u = u_s \quad ; \quad p = p_s \tag{5}$$

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial t_s} + v_p\frac{\partial F}{\partial z} \quad ; \quad \frac{\partial F}{\partial r} = \frac{\partial F}{\partial r_s} \quad ; \quad \frac{\partial F}{\partial z} = \frac{\partial F}{\partial z_s} \tag{6}$$

Variables without subscripts are variables in the moving coordinate system,

The equations (1), (2) and (3) become

$$\frac{1}{r}\frac{\partial}{\partial r}ur + \frac{\partial v}{\partial z} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}u^2 r + \frac{\partial}{\partial z}uv + v_p\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right] \tag{8}$$

$$\frac{\partial v}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(uvr) + \frac{\partial}{\partial z}v^2 = -g_z - \frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\mu}{\rho}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right] - \frac{\partial v_p}{\partial t} \tag{9}$$

Defining dimensionless parameters as

$$R = r/\alpha R_p \;,\quad Z = z/\beta R_p \;,\quad L_r = \alpha/\beta$$

$$U = u/V_o \;,\quad V = v/V_o \;,\quad V_p = v_p/V_o$$

$$P = p/\rho V_o^2 \;,\quad T = V_o t/\alpha R_p \tag{10}$$

$$G_z = g_z/G_s \;,\quad R_e = V_o \alpha R_p \rho/\mu$$

$$F_r = V_o^2/\alpha R_p G_s$$

equations (7), (8) and (9) become

$$\frac{1}{R}\frac{\partial}{\partial R}UR + L_r\frac{\partial V}{\partial Z} = 0 \tag{11}$$

$$\frac{\partial U}{\partial T}+\frac{1}{R}\frac{\partial U^2R}{\partial R}+L_r\frac{\partial UV}{\partial Z}+L_rV_p\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial R}+\frac{L_r}{R_e}\frac{\partial}{\partial Z}\left[L_r\frac{\partial U}{\partial Z}-\frac{\partial V}{\partial R}\right] \tag{12}$$

$$\frac{\partial V}{\partial T}+\frac{1}{R}\frac{\partial}{\partial R}UVR+L_r\frac{\partial V^2}{\partial Z^2}=-L_r\frac{\partial P}{\partial Z}-\frac{1}{R_e}\cdot\frac{1}{R}\frac{\partial}{\partial R}\left[R\left(L_r\frac{\partial U}{\partial Z}-\frac{\partial V}{\partial R}\right)\right]-\frac{G_z}{F_r}-\frac{\partial V_p}{\partial T} \tag{13}$$

The equation for the pressure distribution is determined from equations (12) and (13) and becomes

$$\frac{1}{R}\cdot\frac{\partial^2 U^2R}{\partial R^2}+\frac{2L_r}{R}\frac{\partial^2 RUV}{\partial R\partial Z}+\frac{L_rV_p}{R}\frac{\partial^2 RU}{\partial R\partial Z}+L_r^2\frac{\partial^2 V^2}{\partial Z^2}= -\left[L_r^2\frac{\partial^2 P}{\partial Z^2}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial P}{\partial R}\right)\right] \tag{14}$$

Initial and boundary conditions

The following conditions exist at time zero. A thin layer of compressible air of constant thickness h is between the clindrical body and the liquid. The entire mass of liquid is moving upwards with a velocity v_p. Hydrostatic pressures exist throughout the liquid.

The boundary conditions prior to impact are shown in figure 3. During and after impact the boundary conditions are shown in figure 4.

Properties of the air layer

As the cylinder falls, the air rushes from beneath it. The outward air velocities become quite large as the cylinder approaches the free surface and the thickness of the layer becomes small. It is assumed that the compressible air behaves such that $pv^n = k$ where $1 \leq n \leq \gamma$.

The continuity equation for a compressible air layer is

$$\frac{1}{R}\frac{\partial}{\partial R}\left(\rho_a U_a R\right)+\rho_a L_r\frac{\partial V}{\partial Z}=-\frac{\partial}{\partial T}\rho_a \tag{15}$$

Substituting

$$\frac{p}{\rho^n} = \left(\frac{p}{\rho^n}\right)_{ambient} = K \tag{16}$$

and expressing the time derivative in explicit form

$$P_{air\atop t+\Delta t} = \left\{ P_{air\atop t} - \Delta T\, K^{\frac{1}{n}} \left[\frac{1}{R}\frac{\partial}{\partial R}\rho_a R U_a + \rho_a L_r \frac{\partial V_a}{\partial Z} \right]_t \right\}^n \tag{17}$$

The average radial component of air velocity U_a is obtained by applying the R-direction momentum equation to the air layer which after simplification becomes

$$\rho_a\left[\frac{\partial U_a}{\partial T} + U_a\frac{\partial U_a}{\partial R}\right] = -\frac{\partial P_a}{\partial R} + \frac{L_r^2}{R_a}\cdot\frac{\partial^2 U_a}{\partial Z^2} \tag{18}$$

Solving for the term $\partial U_a/\partial T$

$$\frac{\partial U_a}{\partial T} = -\frac{1}{2}\frac{\partial U_a^2}{\partial R} - \frac{1}{\rho_a}\frac{\partial P_a}{\partial R} + \frac{L_r^2}{\rho_a R_a}\frac{\partial^2 U_a}{\partial Z^2} \tag{19}$$

The air velocity distribution is obtained by integrating equation (19) explicitly.

The deceleration of the projectile is found by applying Newton's law of motion. Before impact the deceleration force is that due to the air pressure in the air layer. After impact the retarding force is that due to water pressure in contact with the bottom of the projectile and the viscous drag of the liquid surrounding the projectile as it penetrates below the water surface. The velocity of the projectile is then found from

$$V_p = V_i - \int a\,dt \tag{20}$$

and the change in air layer thickness is

$$\Delta h = -V_p \Delta t \tag{21}$$

COMPUTATIONAL DETAILS

The equations for the air layer and the water are set up in finite difference form as described in the MAC report (reference (16)). The two components of velocity, u and v, are determined explicitly. It was found that, for any cell, the components of velocity did not satisfy the continuity equation. In order to ensure that this equation was satisfied the v component was found from equation (11) and the u component from equation (12).

The order of calculations, stated briefly, are

Define air and liquid properties

P_a from equation (17)
↓
U_a from equation (19)
↓
ρ_a from equation (16)
↓
Determine deceleration of projectile
↓
Determine velocity of projectile
↓
Determine air layer thickness
↓
Determine pressure distribution in water, equation (14)
↓
Determine v component of velocity-equation (11)
↓
Determine u from equation (12)
↓
Determine if impact has taken place
↓
Move liquid markers and find new free surface of fluid

Many details involving free surface treatment, marker movement, cell flagging, velocity reflections are found in the basic MAC Report.

The compressible air layer is used to generate pressure and velocity boundary conditions in the free surface before impact and used in the calculation of the free surface velocity and deformation.

It is to be noted that all variables are calculated explicitly which automatically means that the time step of integration is very small. In the initial stages of the problem solution several hundred time cycles may be executed in the air layer before the air pressures increase enough above atmospheric pressure to influence the behavior of the liquid free surface. During this part of the problem solution it is not necessary to enter the liquid calculations until many cycles in the air layer are completed. Once the pressures, densities and velocities in the air layer begin to build up, they increase very rapidly with time. This rapid buildup limits the magnitude of the time step.

As the projectile became close to the water surface the velocity of the outflow of the air became large resulting in a relatively large velocity gradient at the water surface. This was allowed to act on the water surface in order to produce radial motion of the water at the surface.

Because the area of most interest is that adjacent to the impact zone, experiments were performed with a variable mesh, using a fine mesh near the origin of coordinates and an increasingly coarse mesh as the spatial variables moved away from the origin. However as the stable time limit is decided by the smallest mesh size this procedure was abandoned and a constant mesh size in each direction was used.

IMPACT CONDITIONS

The conditions that prevail immediately before or at impact is not known. The problems of velocity discontinuities and infinite pressures at the impact surface have always been present in hydrodynamic investigations. The pressures in the air layer increases as the projectile approaches the water surface and the water surface is depressed. However, none of the existing theories prevented the velocity singularity occuring, i.e., at the position where contact between the projectile and the water took place the vertical component of velocity had two different values at the same time. Lewison and Maclean (8) were the first to state that the condition of impact was when the relative velocity between the cylinder and free surface becomes zero. However inspite of numerous numerical experiments involving grid size and time interval, the velocity discontinuity always existed. Because of the entrapped air, contact initially took place around the periphery of the projectile, the water surface below the projectile being concave up. The air layer pressures increased rapidly as the body became nearer the surface being always a maximum on the centerline and decreasing radially outwards. In spite of many numerical experiments there was always a relative velocity between the body and the adjacent water surface. If when initial contact took place the velocity of the water surface was instantaneously made equal to the projectile velocity and continuity satisfied by recalculating the radial velocity in the surface cell the pressures near the projectile became negative!

It is felt that future work should be concentrated on ascertaining the conditions prevailing immediately before and at impact.

EXPERIMENTAL WORK

Experiments were initiated to investigate the motion of the water during impact and hence derive a distribution of pressure by numerically integrating the equations of motion.

A large rectangular tank was constructed from lucite and filled with water to a depth of three feet. A right circular cylinder was used as the falling object with its axis parallel to the water surface. It was sufficiently long so that at the vertical center line plane two dimensional motion would be accurately obtained. A number of colored beads was suspended in the water at this centerline plane. The impact of the falling cylinder impacted on the water surface causing the water to be displaced together with the suspended colored particles. The motion of these particles was recorded by high speed motion film photography. Frame by frame examination of this film would enable the velocity components to be determined.

Experimental tests did not prove satisfactory for the following reasons.

(a) There was always a small movement of the suspended particles. It proved impossible to find particles with exactly the same density as water.

(b) The particles could not be constrained to move in one vertical plane.

(c) The particles had to be sufficiently large for photographic purposes and consequently did not behave as a water particle. This was evident when the projectile hit a particle at or near impact. The particle was projected through the water.

(d) The projectile could not be dropped so that the axis was perpendicular to the plane containing the particles. Thus three dimensional particle motion was obtained.

Part II

The configuration now being investigated is that of an infinite flat plate as shown in Figure 5. There is symmetry about the center line and hence only one half of the field need be investigated. The shape of the lower surface of the projectile can be any defined mathematical function including a flat plate for which experimental results are readily available. A moving coordinate system is no longer being used and the origin of the coordinate surface is on the original water surface.

It is absolutely essential to consider a deflecting water surface. A configuration as shown in figure 6 with two perfectly parallel surfaces and a compressible air layer will not impact. As the surfaces approach each other, the air pressure, (which is proportional to $1/h^2$), increases sufficiently so that the projectile is slowly decelerated and brought to rest.

By making the following assumptions (a) the pressure does not vary across the air gap (b) there is no flow in the Z direction i.e. an infinitely wide plate and (c) inertia terms can be neglected as compared with the viscous terms, the equations for the compressible air layer are very similar to those used in squeeze film air lubricated bearings. These have been extensively studied-see, for example, the book by W.A. Gross, "Gas Film Lubrication", John Wiley and Sons. The equations for the pressure in the air film becomes

$$\frac{\partial}{\partial x}\left[\rho\frac{h^3}{12\mu}\cdot\frac{\partial p}{\partial x}\right] = \rho(V_0 - V_w) + h\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho\frac{U_w h}{2}\right) - \rho U_w\frac{\partial y_2}{\partial x} \qquad (22)$$

$$\left(P/\rho^n\right)_{air} = K$$

The equations for the water are the Navier-Stokes equations in carterian coordinates. The Marker -and- Cell method was found to be time consuming. The velocities at the free surface will be calculated from Navier-

Stokes equation so that the air gap can be continously calculated.

To date numerical experiments have confirmed the fact that two perfectly parallel surfaces will not impact. Numerical results are currently being obtained using equation (22) for the case of a deformable lower surface i.e. for a water surface.

CONCLUDING REMARKS

No consistent results were obtained due to the velocity singularity which occurs at impact conditions. Before further progress can be made it is absolutely necessary to determine the basic physical phenomena of what happens at conditions of hydrodynamic impact. Is there a velocity singularity? If so, how can this be handled mathematically? Does the relative velocity go to zero? Should the compressibility of the water be considered? Does contact take place at a certain point and surface tension become important? What happens to any air trapped in the center because contact takes place first near the edge of the projectile?

It is also considered that only a carefully controlled experimental investigation will yield a physical picture which can then be used in a numerical solution. Certain experimental difficulties have already been discussed. It is to be noted that the mathematical model was an extreme case in which the bottom of the projectile was parallel to the initially calm water surface. Any real body would fall at an angle with contact occurring on a finite single area.

By considering an inclined plate it is expected that these questions will be answered.

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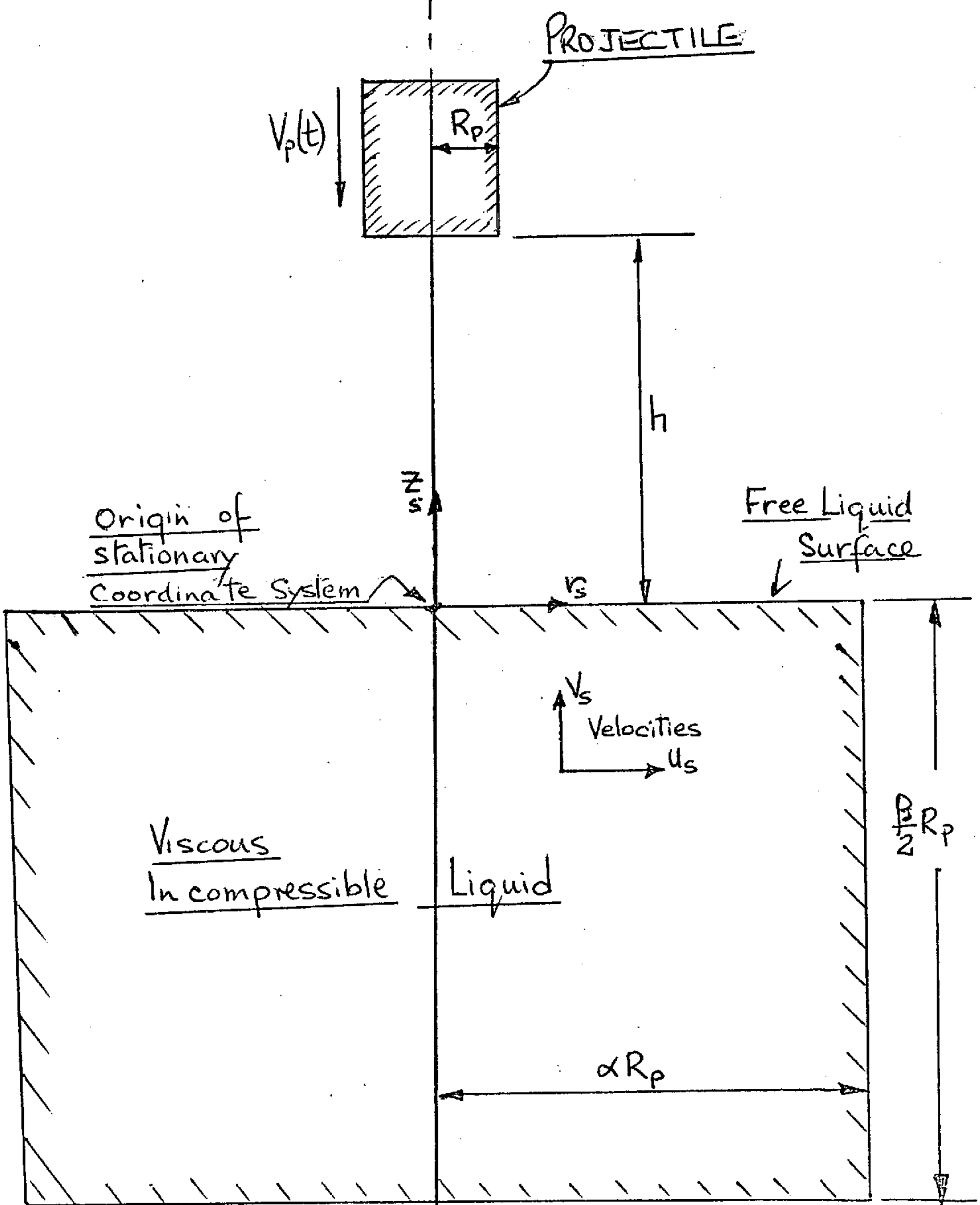

FIG. 1. PHYSICAL SYSTEM

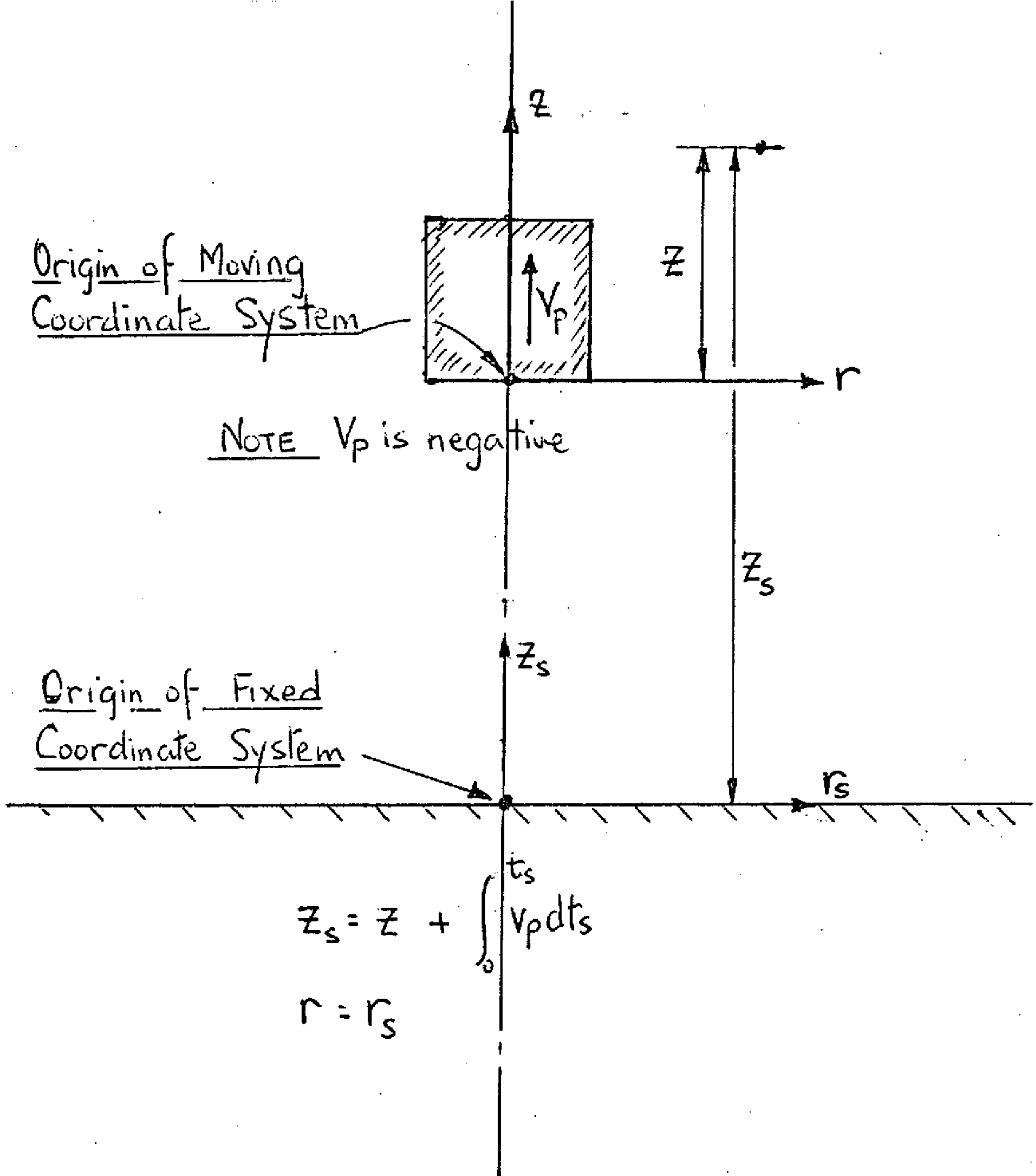

FIG. 2. RELATIONSHIP BETWEEN COORDINATE SYSTEMS.

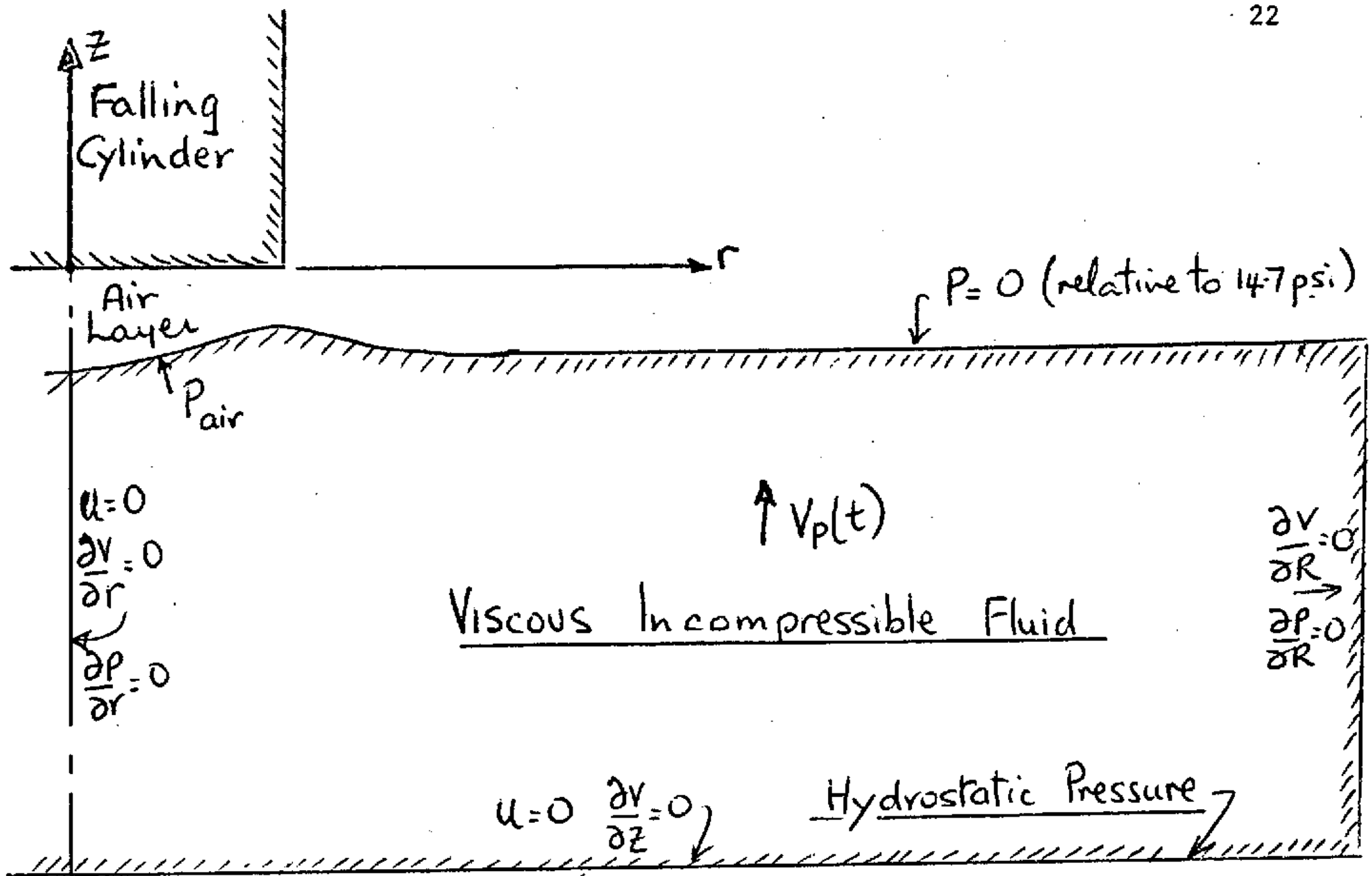

FIG. 3. BOUNDARY CONDITIONS BEFORE IMPACT

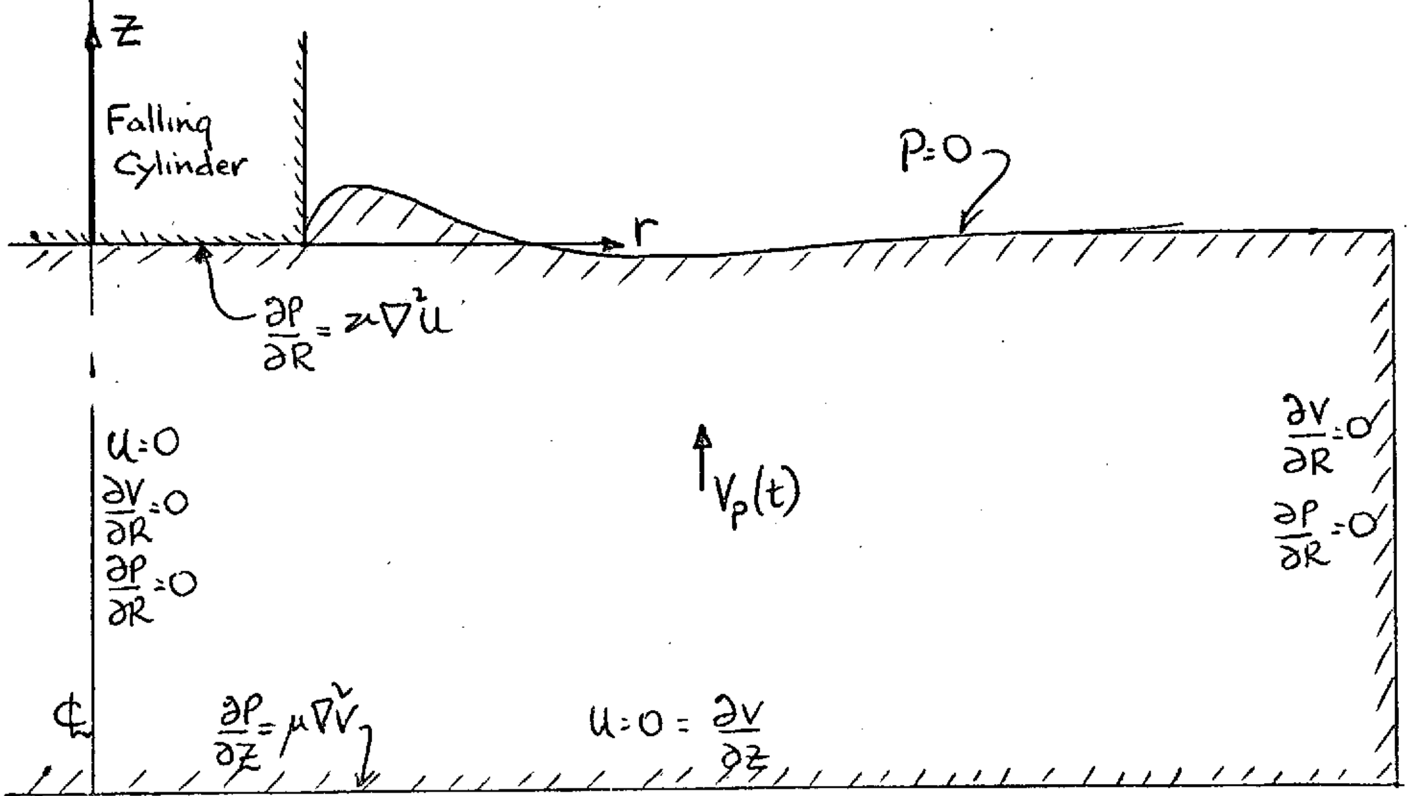

FIG. 4. BOUNDARY CONDITIONS AFTER IMPACT

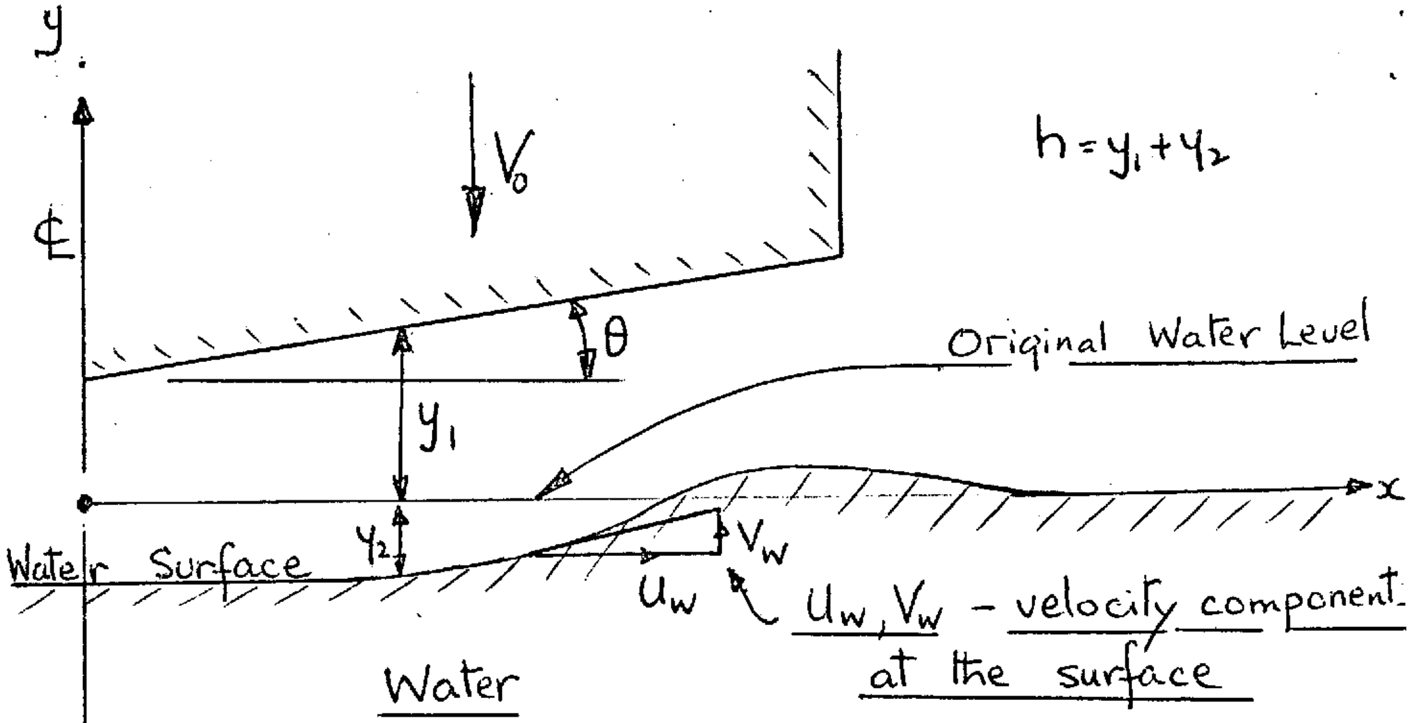

FIG. 5 IMPACT OF A FLAT PLATE

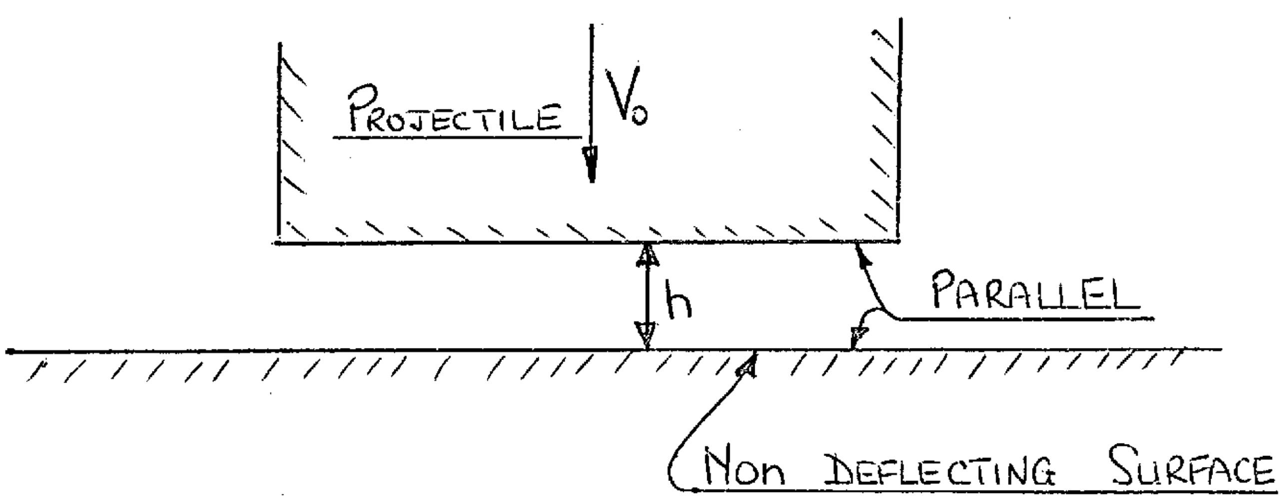

FIG. 6. NON-IMPACTING SURFACES