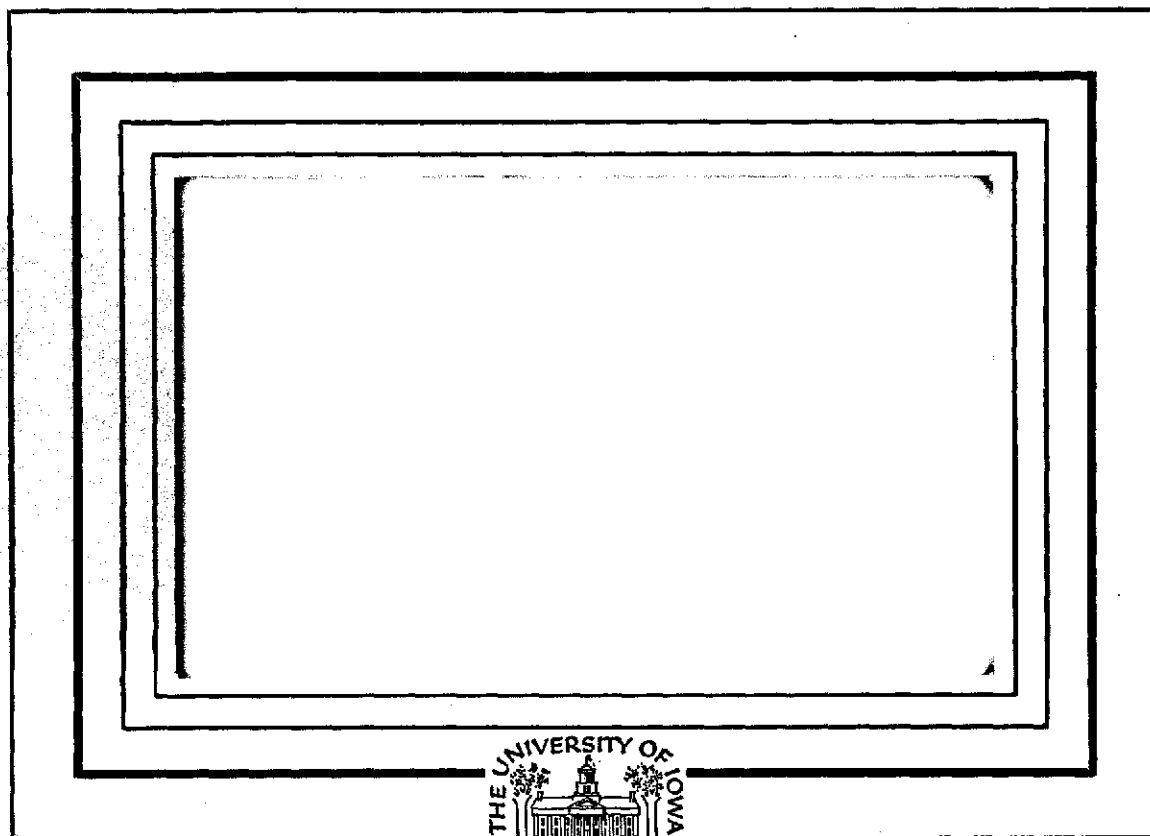


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Thermal Relaxation of a Two Dimensional  
Plasma in a dc Magnetic Field  
Part II: Numerical Simulation

by

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## ABSTRACT

The thermal relaxation process for a spatially uniform two dimensional plasma in a uniform dc magnetic field is simulated numerically. Thermal relaxation times are defined in terms of the time necessary for the numerically computed Boltzmann H-function to decrease through a given part of the distance to its minimum value. Dependence of relaxation time on two parameters is studied: number of particles per Debye square  $n_o \lambda_D^2$  and ratio of gyrofrequency to plasma frequency  $\Omega/\omega_p$ . When  $\Omega^2/\omega_p^2$  becomes  $\gg [\ln(L/2\pi\lambda_D)]^{1/2}$  where L is the linear dimension of the system, it is found that the relaxation time varies to a good approximation as  $(n_o \lambda_D^2)^{1/2}$  and  $\Omega/\omega_p$ .

## I. INTRODUCTION

In a companion paper (Part I), predictions were derived for the rate of thermal relaxation of a two dimensional plasma model in a dc magnetic field. The principal prediction which is numerically testable is that so long as  $\Omega/\omega_p$ , the ratio of gyrofrequency to plasma frequency, satisfies the inequality  $(\Omega/\omega_p)^2 \gg (\ln(L/2\pi\lambda_D))^{-1/2}$ , where  $\lambda_D$  is the Debye length and  $L$  is the linear dimension of the system, then the relaxation time should be proportional to  $\Omega/\omega_p$  and to  $(n_0 \lambda_D^2)^{1/2}$ , where  $n_0$  is the number density of the charged rods.

It will be seen below that both of these predictions appear to be well fulfilled. In Section 2, the method of simulation is briefly described. In Section 3, the results are described and compared with theory.

## 2. Method of Simulation

The dynamical evolution of the plasma was simulated by following the trajectories of a large number of negatively charged rods immersed in a positive neutralizing background. Standard particle-in-cell techniques with area weighting were employed (Morse 1970). The electric field was determined through Poisson's equation by fast Fourier transforms on a  $32 \times 32$  or  $64 \times 64$  grid system. Periodic boundary conditions were used throughout. The number of rods ranged from 4,000 to 16,000.

Periodically a histogram representing the spatially averaged distribution function was calculated. The value of the Boltzmann H function was determined by summing the values of  $2\pi v_{1i} f(v_{1i}) \cdot \ln f(v_{1i}) \Delta v_{1i}$  obtained from the histogram. The optimum value of  $\Delta v_{1i}$  was found empirically, and for a range of  $\Delta v_{1i}$  (simply labeled  $\Delta V$  in the figures) neither too small nor too large,  $H(t)$  was found not to depend upon  $\Delta v_{1i}$  (see Figure 1 ).

### 3. Results

Figure 1 shows a typical plot of  $H(t)$  vs  $t$  for zero magnetic field, for the initial distribution:  $f(\underline{v}) = 0$  for  $|\underline{v}| > 2$ ,  $f(\underline{v}) = (4\pi)^{-1}$  for  $|\underline{v}| < 2$ . Times are always expressed in units of the reciprocal plasma frequency  $\omega_p^{-1}$ , and velocities are in units of the thermal velocity  $V_{th}$  in the final expected Maxwellian  $f(\underline{v}) \xrightarrow{t \rightarrow \infty} (2\pi V_{th}^2)^{-1} \exp(-\underline{v}^2/2V_{th}^2)$ . This makes the natural unit of length  $V_{th}/\omega_p \equiv \lambda_D$ , the Debye length. The particles are loaded randomly in position space and in a spiral fashion in velocity.  $\Delta T$  is the size of the time step.  $\Delta V$  is the dimension of a cell size in velocity space used in computing the sum which approximates  $\int d\underline{v} f \otimes f$ .  $\alpha \equiv \lambda_D/m_s$  is the ratio of Debye length to the cell size  $m_s$  used in the particle-in-cell computations. Roughly speaking, larger values of  $\alpha$  mean that the short range part of the potential is represented more accurately. This accuracy is desirable in situations in which short-range interactions become dominant in the relaxation process, as is the case when  $\Omega/\omega_p$  becomes  $\gg 1$ .

It is seen from Fig. 1, that while there is a substantial change in the relaxation rate as  $\alpha^{-1}$  decreases from 1.2732 to 0.6366, there is no appreciable change upon going to  $\alpha^{-1} = 0.3183$ . It may be safely assumed, then, that the interactions with impact parameters  $\geq 0.5\lambda_D$  are primarily responsible for the decrease of  $H(t)$ . Figure 1 should also inspire confidence that in the range in which the computations are performed, the decrease of  $H(t)$  is independent of  $\Delta T$

and  $\Delta V$ , for all practical purposes. The results shown in Fig. 1 are in good agreement with Montgomery and Nielson (1970).

Figure 2 shows that for finite magnetic field, increasing  $\alpha$  continues to increase the speed of thermal relaxation. This reflects the fact that in a magnetic field, the thermal relaxation is determined by the close encounters (over-lapping gyroradii). Since particle-in-cell computations necessarily "soften" the coulomb repulsion at distances  $\leq$  a cell size, a decrease in cell size is equivalent to a "hardening" of the coulomb repulsion at short distances, and thus results in an increased relaxation rate. It is this dependence upon the (inaccurately represented) short range part of the potential that has, for example, precluded our achieving better than a factor of two in absolute agreement with numerical calculations of energy exchange times for "test" particles and the analytical expressions for them which can be derived from the kinetic equation.

Figure 3 shows one of our two main results, the variation of the relaxation time with  $\Omega/\omega_p$ . The relaxation time is defined by setting  $H = -2.70$  (its initial value is  $-2.531$  and its  $t = \infty$  value is  $-2.837$ ). The  $\Omega = 0$  result is close to the value obtained previously (Montgomery and Nielson, 1970). Above about  $\Omega/\omega_p \approx 4$ , we may assert that  $\Omega^2/\omega_p^2 \gg [\ln(L/2\pi\lambda_D)]^{-1/2}$  (our system is 16 units of  $\lambda_D$  in linear dimension), so that the linear prediction of Part I is appropriate, and is seen to be well fulfilled, being accurately fit by  $d\tau_{REL}/d\Omega = 1.5$  for the lower curve and  $d\tau_{REL}/d\Omega = 2.9$  for the upper curve.

Note that the coefficients stand in about the same ratios as  $\sqrt{n_o \lambda_D^2}$ . Between  $\Omega/\omega_p = 0$  and  $\Omega/\omega_p = 4$ , we have no satisfactory theory.

Figure 4 shows the relaxation time  $\tau_{REL}$  vs the square root of the number of particles per Debye square for two different magnetic field strengths. Both sets of data points are seen to be well fit by straight lines.

#### 4. Discussion

The predictions that the thermal relaxation times of a two dimensional magnetized plasma model vary proportionally to  $\Omega/\omega_p$  and  $(n_o \lambda_D^2)^{1/2}$  have been verified by simulation methods.



## ACKNOWLEDGMENTS

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## FIGURE CAPTIONS

Figure 1 H-value as a function of time varying  $\Delta T$  (time step),  $\Delta V$  (velocity interval), and  $\alpha$ . The magnetic field is zero.

SYMBOL	$\Delta T$	$\Delta V$	$\alpha^{-1}$
x	0.25	1/55	1.2732
o	0.50	1/44	1.2732
•	0.50	1/55	1.2732
+	0.25	1/55	0.6366
□	0.50	1/33	0.6366
	0.50	1/44	0.6366
▲	0.25	1/55	0.3183
Δ	1.0	1/55	0.3183
■	0.50	1/44	0.3183

Figure 2 H-value as a function of time.

$$\Omega/\omega_p = 0.5.$$

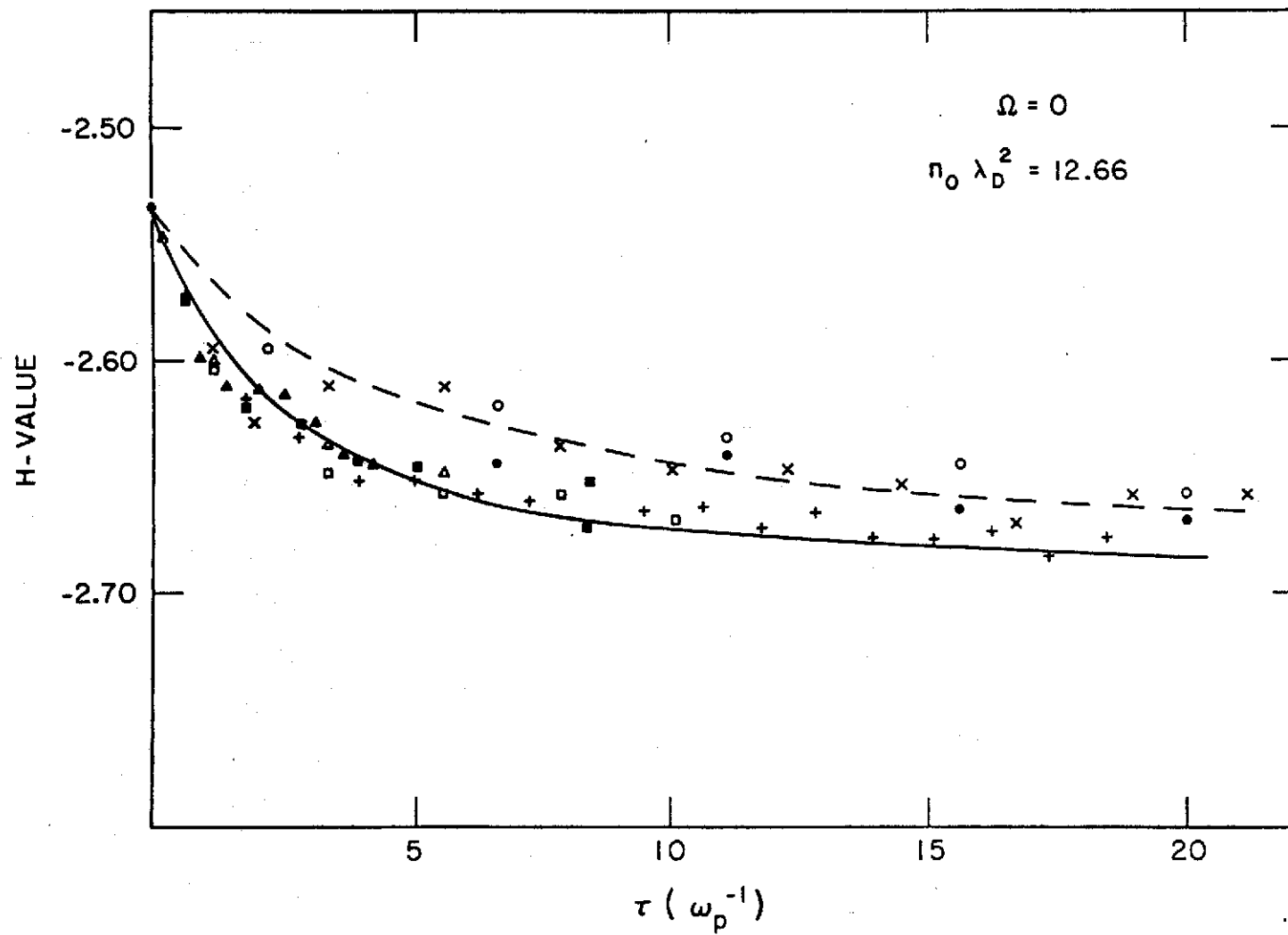
Figure 3 Relaxation time vs magnetic field strength for constant  $n_o \lambda_D^2$ .

Figure 4 Relaxation time vs  $(n_o \lambda_D^2)^{1/2}$  for constant magnetic field strength.

## REFERENCES

- Montgomery, D. and Nielson, C. W. 1970 Phys. Fluids 13, 1405.
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Figure 1



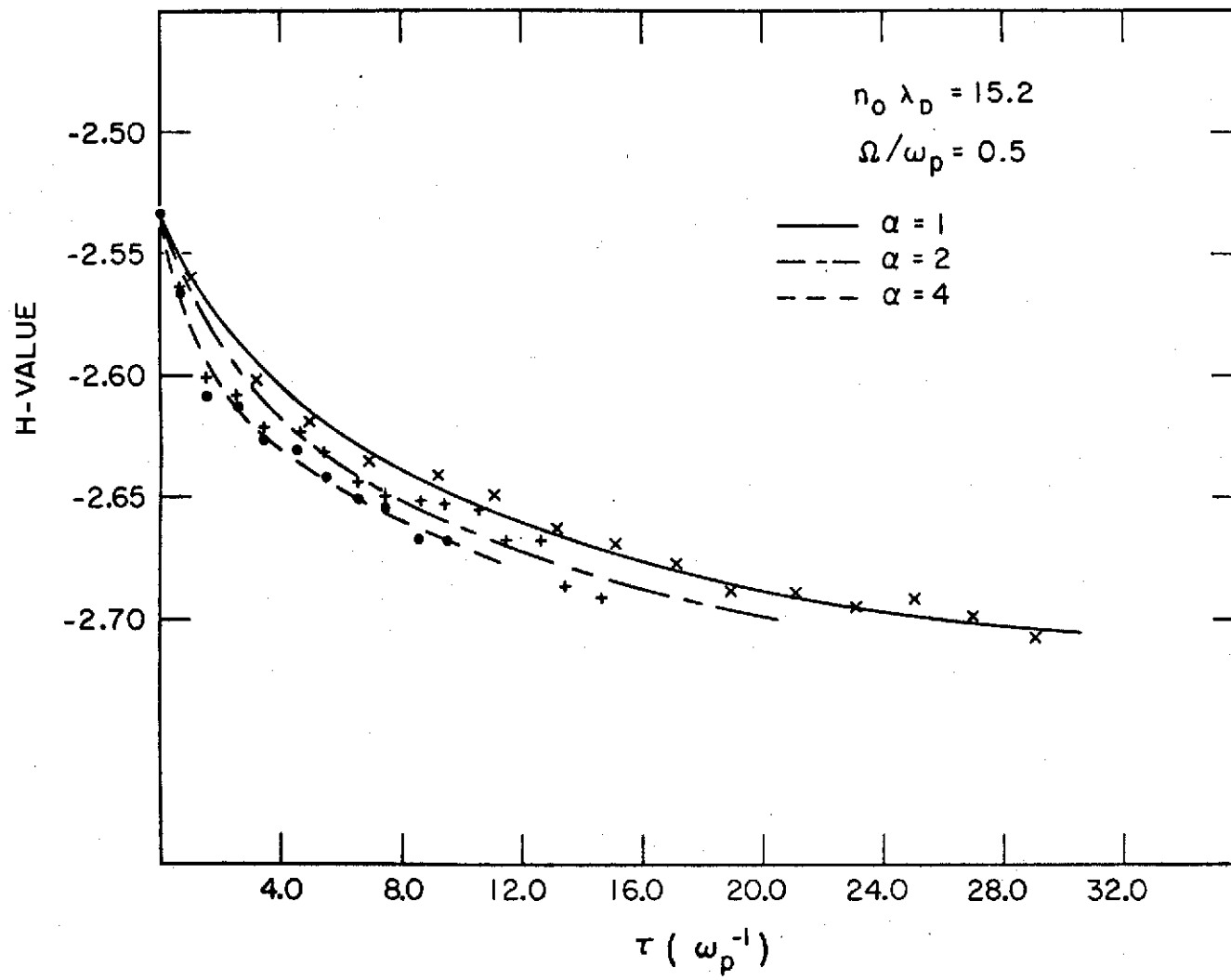
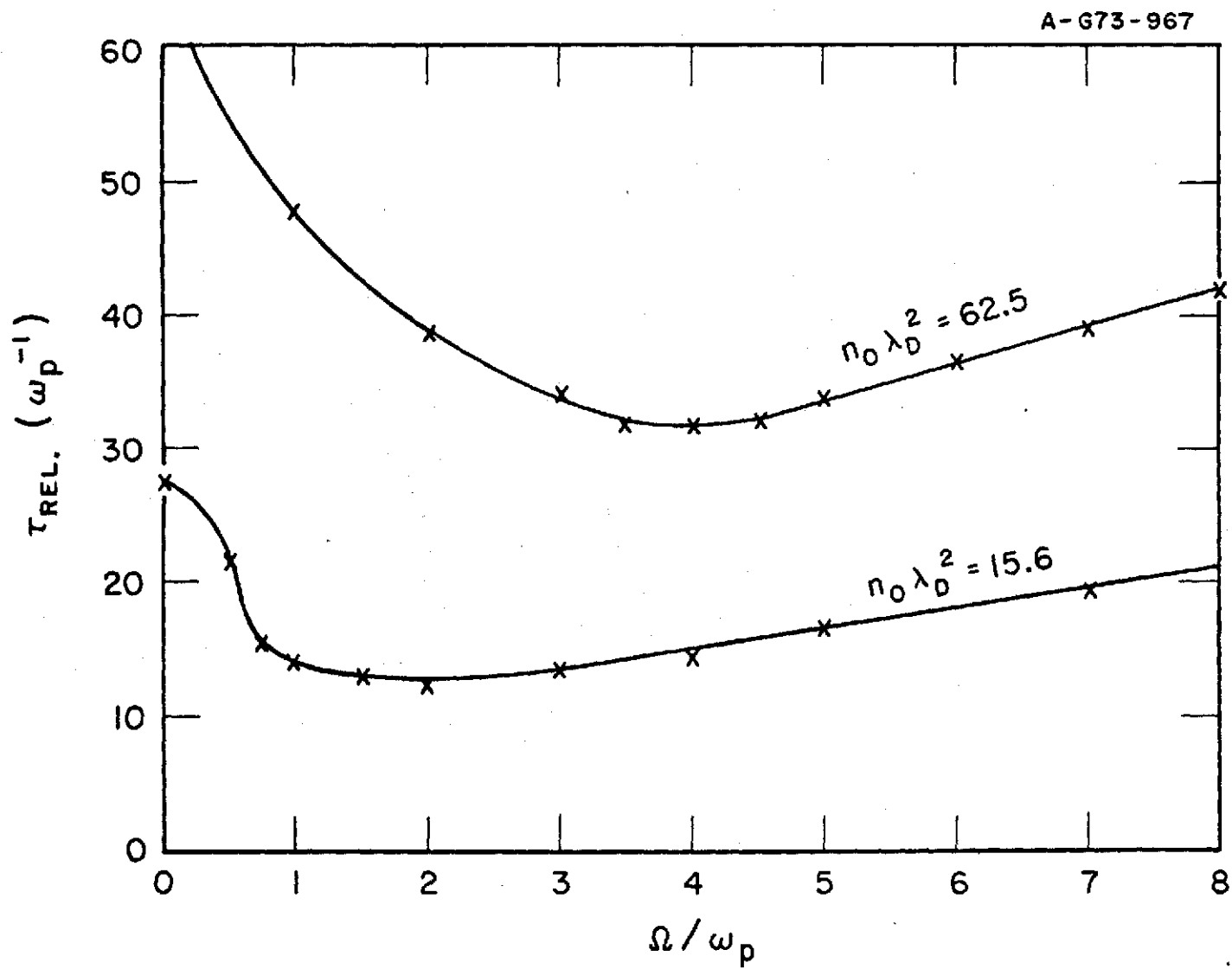


Figure 2

Figure 3



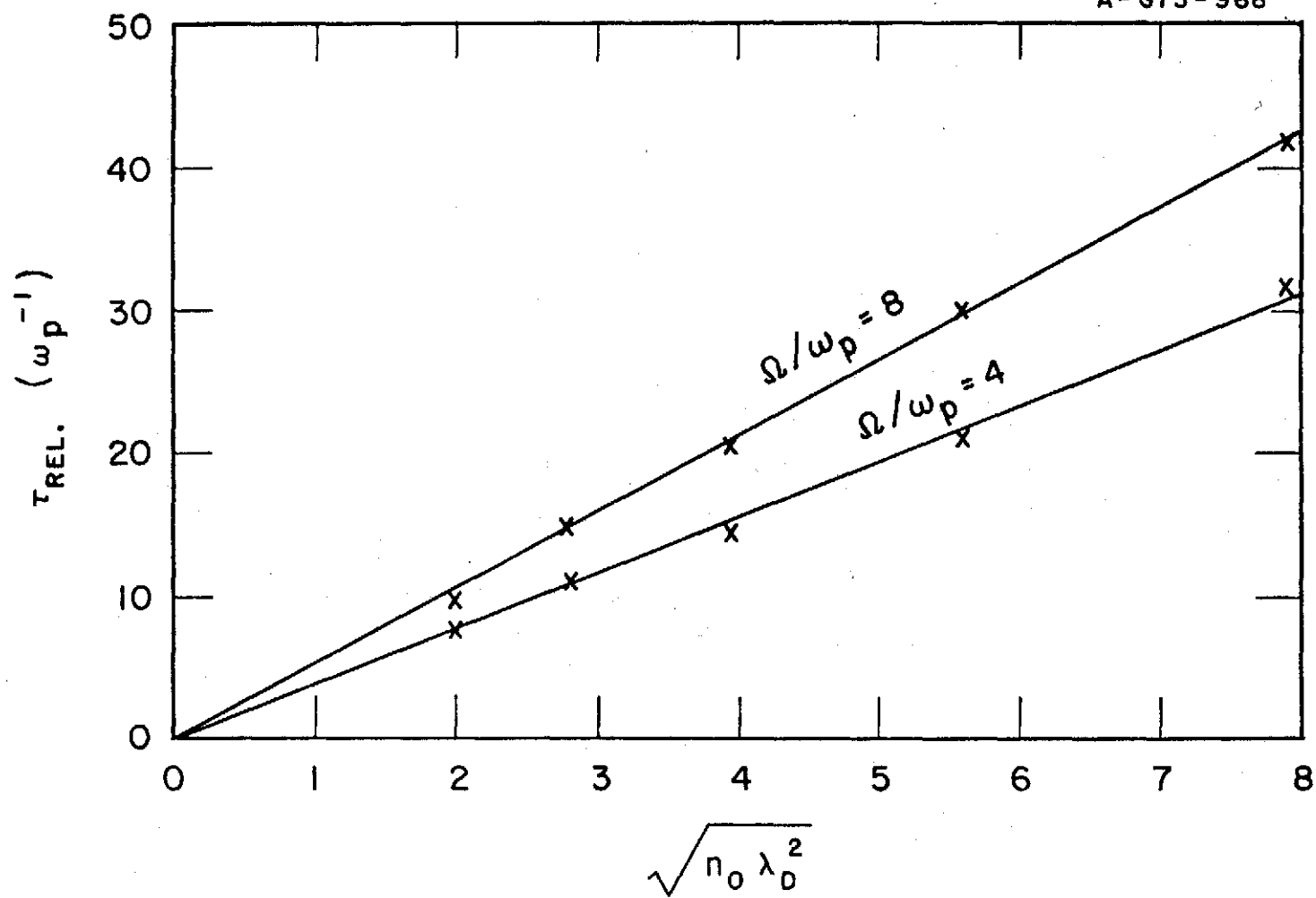


Figure 4