NASA TECHNICAL NOTE

NASA TN D-7666

NASA TN D-7666

(NASA-TN-D-7666) LANDING RATES FOR MIXED STOL AND CTOL TRAFFIC (NASA) 76 p HC $4.00 CSCL 17G

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LANDING RATES FOR MIXED STOL AND CTOL TRAFFIC

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1974

1. Report No. D- 7666	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle LANDING RATES FOR MIXED STOL AND CTOL TRAFFIC		5. Report Date APRIL 1974
		6. Performing Organization Code
7. Author(s) John S. White		8. Performing Organization Report No. A-5044
9. Performing Organization Name and Address NASA Ames Research Center Moffett Field, Calif. 94035		10. Work Unit No. 501-03-11
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Air Traffic Control Terminal Area Guidance Aircraft Operations	Unclassified – Unlimited CAT. 21

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	76	$4.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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NOTATION

Fortran Name	Symbol	
ACC	a	aircraft acceleration capability, m/sec^2
DA	D_a	distance from runway threshold at which velocity is changed, km
SIG	K	constant used to calculate ΔT
M or ZM	m	distance from runway threshold to outer marker, km
	N_{12}	number of takeoffs between successive landings
RD...	$r(\cdot)$	range or expected variation of stochastic quantity
	S	required separation distance, km
SF	S_f	required separation at touchdown, km
SO	S_o	required separation at outer marker, km
	t	time, sec
	$\overline{T}$	average time between landings, sec
	T_a	time before touchdown at which velocity is changed, sec
T2ACT	T_{act}	actual time from outer marker to touchdown, sec
To	T_c	runway clearance time, sec
	T_{m_a}	time when $x_1 = m_2$, sec
	T_m	time when $x = m$, sec
TNOM	T_{nom}	nominal time from outer marker to touchdown, sec
TTO	T_{to}	time required for takeoff, sec
TVF	T_{vf}	time at which aircraft must be at final velocity, sec
T12A	$T_{12_{act}}$	actual interarrival time, sec
T12M(I,J)	$T_{12_{nom}}$	nominal interarrival time, sec
VA or VB	V	aircraft velocity, m/sec

	x	aircraft location, km
	x'	location of separation boundary, km
DELP	δp	error in position at time of nominal passage over outer marker, km
DELT 1, DELT 2	δT	landing time deviation, sec
DEL...	$\delta(\dot{\ })$	error in $(\dot{\ })$
TDL 1, TDL 2	ΔT	allowance for errors in landing time, sec
ZLR	λ_L	landing rate, aircraft/hr
ZNOPR	λ_o	operation rate (landings plus takeoffs), aircraft/hr
	θ	generalized error
TDEL	τ_c	interarrival time required for runway clearance, sec
TFIN	τ_{fin}	interarrival time required for airborne spacing at touchdown, sec
TIN	τ_{in}	interarrival time required for airborne spacing at merge point, sec
TMED	τ_{med}	interarrival time required for airborne spacing at velocity change point, sec

Subscripts

1,2	refers to first or second aircraft in the landing sequence

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SUMMARY

A study was made to determine the expected landing rate for STOL-only traffic and mixed STOL-CTOL traffic. The conditions used vary from present day standards to an optimistic estimate of possible 1985 conditions. A computer program was used to determine the maximum landing rate for the specified conditions and aircraft mix. The results show that the addition of STOL on a CTOL runway increases the total landing rate if the STOL airborne spacing can be reduced by use of improved navigation equipment. Further, if both takeoff and landings are performed on the same runway, the addition of STOL traffic will allow an increase in the total operation rate, even with existing spacing requirements.

INTRODUCTION

Numerous studies have been made to determine the landing rate of aircraft on a single runway. In one of the early studies, Blumstein (ref. 1) considered a traffic mix of various commercial transports arriving at an airport and computed the landing rate, assuming that aircraft were always available at the gate. Simpson (ref. 2) and Odoni (ref. 3) describe theoretical equations for landing rate, including the effects of terminal area operations beyond the gate, and assume that the aircraft arrivals have a Poisson distribution. Both give some numerical results, but for CTOL vehicles only. An additional study by the National Bureau of Standards (ref. 4) used a different concept of landing capacity, which involved a facility serving sequential customers of various types. Again, however, the results presented are for CTOL vehicles only. The capacity of a STOL runway was considered by Rinker (ref. 5), but numerical results were presented for only two cases.

The purpose of this report is to supply additional information for STOL-only runways and to consider the effect of mixed STOL-CTOL traffic. Although the National Aviation System Plan (ref. 6) implies that STOL and CTOL traffic will use separate runways, there may be times when a common runway is required on a temporary basis. Thus this report develops a general theory of landing rates, which can be applied to separated or mixed traffic, and from which expected landing rates can be determined. Further, it indicates the accuracies in control and navigation required to achieve various levels of landing rates. Finally, previous studies have assumed that all aircraft fly at constant airspeed all the way from the instrument landing system (ILS) gate to touchdown, a distance of 6 to 10 miles. An alternate approach is for an aircraft to fly at 160 to 180 knots at the ILS gate and then reduce speed gradually to the final touchdown speed. The effect of such a speed change on the landing rate is also considered.

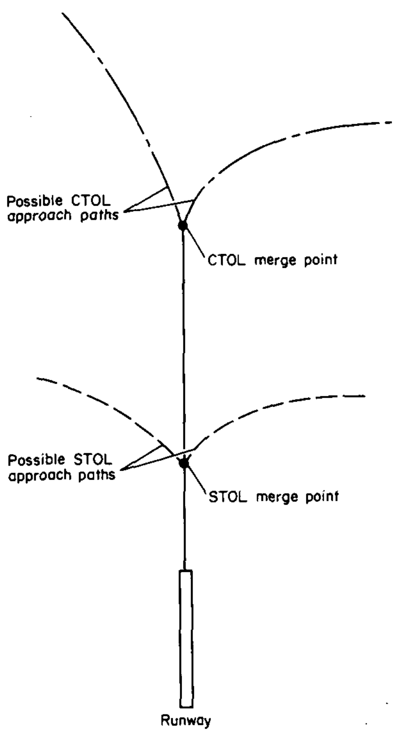
The general approach used here parallels that of Blumstein, but it is considerably more complex to account for the variable speed approach and different outer markers for CTOL and STOL. Also, Blumstein had only two separation criteria — airborne spacing at the outer marker and runway clearance. This study adds another criterion: adequate airborne spacing must be maintained until touchdown.

In this study, it is assumed that landings have priority, and that the aircraft land on a "first come, first serve" basis. We are concerned with maximum capacity, and assume that the aircraft are immediately available to start a landing approach whenever desired. This approximates the situation in which a good 4-D guidance system controls the aircraft in the terminal area to arrive within a few seconds of the desired time. If these arrival errors become large, they will approach a Poisson distribution, and the landing capacity will be reduced.

Takeoffs are also considered, but only when the sequence of landings is such that a takeoff can be inserted without affecting the landing rate.

Sketch (a) shows some possible approach paths. The various CTOL approaches merge fairly far out at the CTOL merge point, while STOL vehicles make curved approaches and arrive on the runway centerline at the STOL merge point.

The results of this study indicate the effect on airport capacity of varying the separation requirements and navigation accuracy. To fully utilize the increased runway capacity suggested herein will probably require a great deal of additional automation, both on the ground and in the air. This automation will probably not be available before about 1985. Further, in determining the actual separation criteria, additional factors, such as trailing vortices, pilot and controller reaction times, accuracy of Air Traffic Control (ATC) surveillance radars must be considered. However, the optimistic viewpoint taken here does provide useful information to aid in the determination of desirable trends in separation requirements, and indicates what portion of the approach system is presently acting as the limiting factor.



Sketch (a)

The remainder of this report first develops the theory of landing-rate calculations, then briefly describes a computer program to solve these equations, and presents the inputs and results obtained.

THEORY OF LANDING-RATE CALCULATIONS

To determine landing rates, some of the characteristics of a single aircraft approaching the runway are first considered, then a technique is developed for determining the minimum spacing between successive aircraft, and, finally, the maximum landing rate associated with a specific mix of aircraft is determined.

Airborne Spacing

The approach conditions for a single aircraft are shown in figure 1. The figure is a plot of distance from touchdown versus time and shows the aircraft initially at its approach velocity V_a. The aircraft will continue at this velocity for a while, and then it will decelerate at a maximum rate a to the final velocity V_b. The aircraft must reach the final velocity at time T_{vf} before touchdown so that the pilot can be sure that conditions have stabilized before touchdown. The aircraft continues at velocity V_b, touches down at time $t = 0$, and rolls down the runway, and clears the runway at time T_c after touchdown. The merge point may occur anywhere along this velocity time profile; two possible locations are shown in the figure.

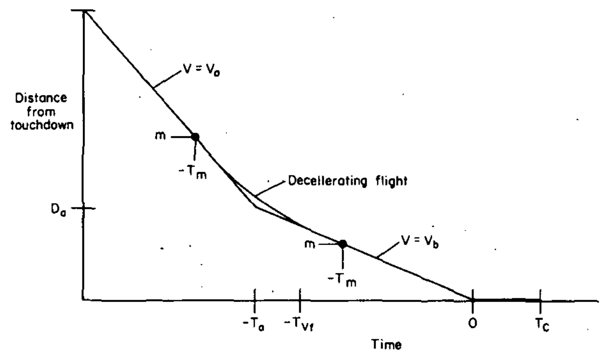


Figure 1.— Definition of approach quantities.

To simplify the equations associated with changing velocities, it was assumed that the aircraft would fly at constant speed V_a until it was a distance D_a from the end of the runway, at which time it would instantaneously change velocity to V_b. This step change in velocity simplifies the calculation and, for reasonable values of separation, decreases the separation by less than 10 percent.

To determine D_a, first locate the time T_a at which the two constant-velocity segments intersect, that is, $T_a = T_{vf} + (V_a - V_b)/2a$. Since $D_a = T_aV_b$ (fig. 1),

$$D_a = T_{vf}V_a + \frac{V_a - V_b}{2a} V_b \tag{1}$$

The location of the aircraft, x, at any time t is specified by

$$x = \begin{Bmatrix} -V_b t & , & x \leq D_a \\ D_a \left(1 - \dfrac{V_a}{V_b}\right) - V_a t \ , & & x > D_a \end{Bmatrix} \tag{2}$$

which can be solved for the time the aircraft is over the merge point:

$$T_m = \begin{Bmatrix} -\dfrac{m}{V_b} & , & m \leq D_a \\ -\dfrac{m}{V_a} + \dfrac{D_a}{V_a}\left(1 - \dfrac{V_a}{V_b}\right) & , & m > D_a \end{Bmatrix} \tag{3}$$

Consider two aircraft approaching a given runway in sequence. A minimum separation distance is required between the two aircraft at all times during the approach and, further, the second aircraft must not touch down until the first aircraft is off the runway. The problem is to determine the minimum time between touchdowns that meets the separation criteria and, from this, to determine the maximum landing rate. The two aircraft may be of different types, with different approach paths, velocities, time-to-clear runway, separation distances, etc. The separation requirements are stated in terms of in-trail distances and must be met only while the two aircraft are on a common path; before that, the vehicles are considered to have satisfactory lateral separation.

A few terms should be defined more explicitly. First, the merge point is that point where the aircraft arrives on the extended runway centerline, and is stabilized on the runway heading. The location of the merge point, m, varies from one type of aircraft to another. The common path is the section of the flight path that will be used by two successive aircraft; therefore, it occurs between the minimum of m_1 and m_2 and the runway and its length depends on the types of the two aircraft.

The separation requirements are defined so that, while the first aircraft is on the common path, the second aircraft must be far enough behind. The present-day separation requirement during approach is 6 km (~3 miles). Since the navigation systems of STOL vehicles may be more accurate, we wish to consider the possibility of reduced distance separations at the merge point, and also a smaller separation requirement at touchdown than at the merge point.

Separation is defined as that portion of airspace reserved for a given aircraft and is assumed to be half before and half behind the nominal aircraft position. The separation criterion for two aircraft is shown in figure 2, and the requirement is that the reserved airspaces do not overlap, or

$$x_1' = x_1 + S_1/2 < x_2' = x_2 - S_2/2 \tag{4}$$

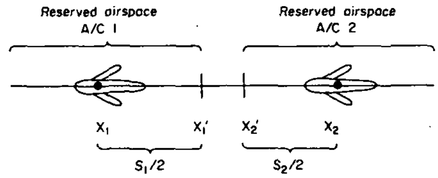


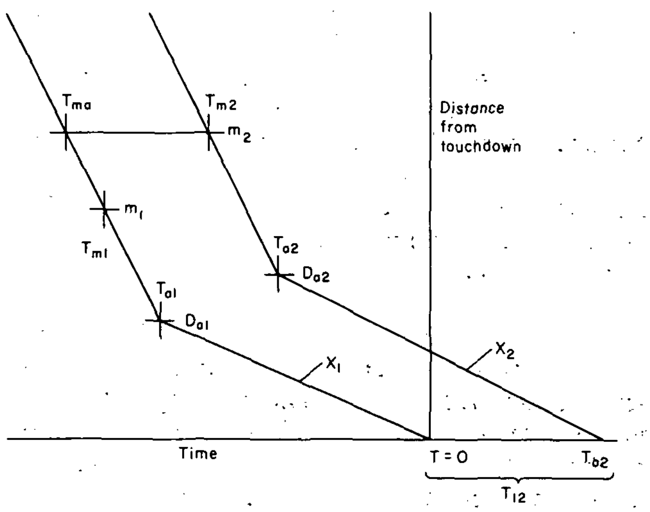
Figure 2.— Separation criteria.

where S_1 and S_2, the separation distances associated with each aircraft, need not be constant with time. For this study, define a separation requirement, S_{o_i}, at the merge point, and S_{f_i}, at touchdown, and assume that it varies linearly between those two points, that is,

$$S_i(x)_i = \begin{cases} S_{f_i} + \left(\dfrac{S_{o_i} - S_{f_i}}{m_i}\right) x_i \ , & \text{for } x_i \le m_i \\ S_{o_i} \ , & x_i > m_i \end{cases} \tag{5}$$

This form for $i = 1,2$ was chosen to allow for a possible reduction in spacing as the aircraft approaches the threshold while maintaining relatively simple equations.

To visualize the separation requirement, figure 3(a) shows a specific example of two approaching aircraft. For each aircraft, its merge point and velocity change point are shown. To guarantee separation at all times, checks must be made at three places, namely, at the end of the common path, at the beginning, and at the point where the velocity of aircraft 2 changes. In the example shown, these tests occur at $t = 0$, $t = T_{m_1}$ (since the common path starts at the closest merge point, m_1, as drawn), and at $t = T_{a_2}$. At these points, we want to guarantee that



(a) Generalized situation.

Figure 3.— Separation between successive aircraft.

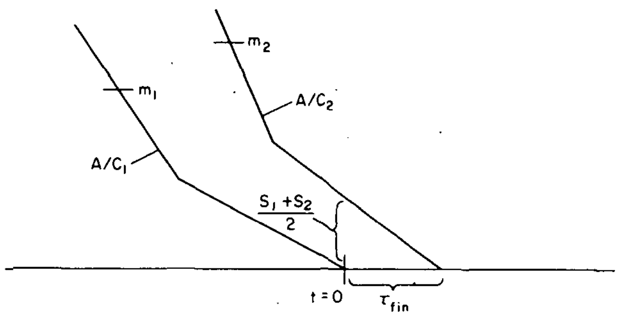
$$x_2' - x_1' = x_2 - \frac{S_2}{2} - \left(x_1 + \frac{S_1}{2}\right) > 0 \tag{6}$$

For different types of approaching aircraft, the relative locations of m_1, m_2, D_{a_1}, and D_{a_2}, vary, as does the time of the tests, resulting in a variety of situations, each of which must be considered individually.

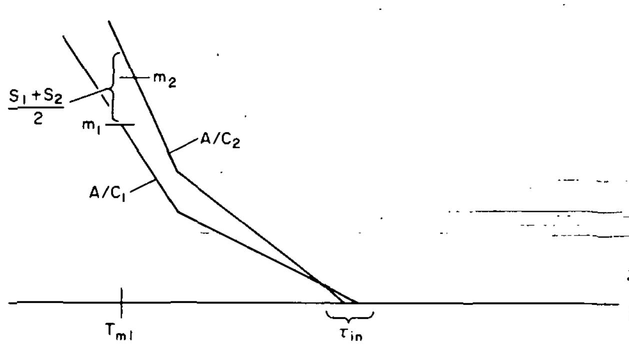
To determine the closest possible spacing, three interarrival times are calculated by selecting x_2, in inequality (6), so that the difference is zero at each of the three test times. The largest of these times guarantees adequate separation throughout the airborne portion of the flight. The effect of runway clearance requirements is considered in the next section.

Considering the example of figure 3(a) again, first calculate τ_{fin}, which assumes that minimum separation occurs at $t = 0$. This is shown in figure 3(b), where the trajectory for aircraft 2 is moved closer to that for aircraft 1, and the minimum separation, $(S_1 + S_2)/2$, occurs at $t = 0$. Next, calculate τ_{in}, which assumes minimum separation at $t = T_{m_1}$ (fig. 3(c)). Finally, calculate τ_{med}, which assumes minimum separation at $t = T_{a_2}$ (fig. 3(d)). For this example, the critical test is the calculation for τ_{fin}, and an interarrival time of $T_{12} = \tau_{fin}$, guarantees adequate airborne separation throughout the flight.

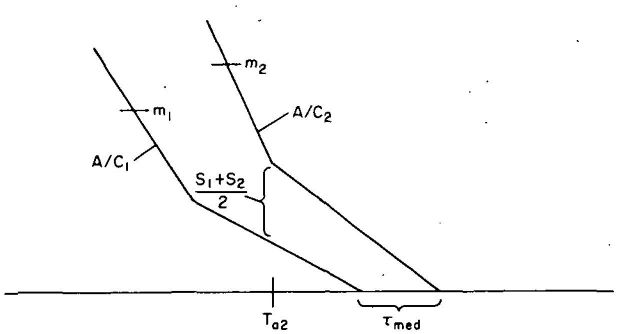
The various cases that can occur, depending on the locations of m_1, m_2, D_{a_1}, and D_{a_2}, are described in appendix A, along with the equations for calculating the various τ.



(b) Minimum separation at $t = 0$.



(c) Minimum separation at $t = T_{m_1}$.



(d) Minimum separation at $t = T_{a_2}$.

Figure 3.— Concluded.

Runway Clearance

Having computed the τ value that guarantees satisfactory separation everywhere on the flight path, we now consider separation on the runway. The FAA rule is that a following aircraft may not touch down until the preceding aircraft clears the runway. The time a given type of aircraft needs on the runway before turning off, T_c, is known, but, in scheduling aircraft arrivals, some time must be added to allow for the possibility of a late landing of the first aircraft or an early landing of the second aircraft. Four sources of error in landing time are considered: (1) a position error at the merge point, δp, (2) an error in the approach velocity, δV_a, (3) an error in the touchdown velocity, δV_b, and (4) an error in the time at which the aircraft reaches the final velocity, δT_{vf}. For a given aircraft, the nominal time, T_{nom}, from its merge point to touchdown is given by the negative of equation (3), and the actual time is

$$T_{act} = \left\{ \begin{array}{ll} \dfrac{m + \delta p}{V_b + \delta V_b} , & m \le D_a \\ \dfrac{m + \delta p}{V_a + \delta V_a} - \dfrac{D_a + \delta D_a}{V_a + \delta V_a}\left(1 - \dfrac{V_a + \delta V_a}{V_b + \delta V_b}\right), & m > D_a \end{array} \right\} \tag{7}$$

where

$$D_a + \delta D_a = (T_{vf} + \delta T_{vf})(V_b + \delta V_b) + \frac{[(V_a + \delta V_a) - (V_b + \delta V_b)](V_b + \delta V_b)}{2a}$$

which results in a landing time error of

$$\delta T = T_{act} - T_{nom} \tag{8}$$

From this, the actual time between touchdowns can be calculated, given the nominal time, by

$$T_{12_{act}} = T_{12_{nom}} - \delta T_1 + \delta T_2 \tag{9}$$

and if $T_{12_{act}}$ is less than the runway clearance time, T_c, the second aircraft must abort its landing. This implies that an allowance must be made for δT_1 and δT_2 to minimize aborts. Assuming the four error sources are independent, one can take the partials of $T_{12_{act}}$ with respect to the various error sources, multiply them by the expected range of the error, and add them on an RSS basis to obtain a time allowance, ΔT:

$$\Delta T = K\left[\sum_{i=1}^{2} \sum_{\text{all } \theta} \left(\frac{\partial T_{12_{act}}}{\partial \theta_i} r_{\theta_i}\right)^2\right]^{1/2} \tag{10}$$

where θ represents one of the four error sources, r_{θ_i} is the range of probability distribution of that error source, and K is chosen so that ΔT will be large enough to minimize the probability of abort. The planned separation for runway clearance, τ_c, should then be

$$\tau_c = T_c + \Delta T \tag{11}$$

and the nominal interarrival time can be obtained from the various τ values as

$$T_{12_{nom}} = max(\tau_{in}, \tau_{med}, \tau_{fin}, \tau_c) \tag{12}$$

where $T_{12_{nom}}$ depends on the types of aircraft 1 and 2. (The equations for the various partials used in equations (10) are given in appendix A.)

Probability of Abort

Since we are trying to maximize the number of aircraft to be landed, the aircraft will be flying close together, and if one separation criterion is violated, the second aircraft should abort the landing. It is assumed that airborne separation has been determined from a consideration of the likely errors in position and velocity of the aircraft; therefore, aborts due to violation of the airborne separation will not occur (at least from the expected errors). However, the runway occupancy rule does not consider possible errors; any violation of this rule will therefore cause an abort, as implied by equation (9). The quantity ΔT (eq. (11)) is designed to consider the effect of errors on the runway occupancy rule, and K (eq. (10)) is chosen so that the probability of abort because of a violation of the runway occupancy rule will be sufficiently small. As shown in appendix A, if $K = 1$, the probability of abort is less than 0.006.

Landing Rate

To determine the average interarrival time $\overline{T}$, calculate $T_{12_{nom}}$ for all possible sequences of aircraft types, and then obtain an average, weighted according to the relative probabilities of arrival associated with each type of aircraft. It is assumed that there are n types of aircraft to be considered, and the probability that the next aircraft is of type i is P_i and is independent of the type of the previous aircraft. One then obtains

$$\overline{T} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_i P_j T_{ij_{nom}} \tag{13}$$

where $T_{ij_{nom}}$, obtained from equation (12), depends on the types of aircraft i and j, and

$$\sum_{i=1}^{n} P_i = 1$$

The nominal maximum landing rate, λ_L, is then

$$\lambda_L = 3600/\overline{T} \tag{14}$$

which assumes that aircraft are continually available to start their approach as soon as needed, and is thus a true measure of runway capacity. To achieve this capacity the terminal area controller must be told when an aircraft should be at the outer marker, and a time-controlled guidance scheme, such as suggested by Erzberger and Pecsvaradi (ref. 7), must be available.

Total Operation Rate

Another factor of interest is the total operation rate, λ_o, of a runway which allows takeoffs and landings to be mixed. To determine λ_o, consider a specific interarrival time and determine if it is long enough to allow a takeoff to occur without delaying the landing of the second aircraft (therefore, landings have priority over takeoffs). Assume that the time for takeoff T_{to} is constant for all aircraft, and that the types of aircraft 1 and 2 are specified. We can determine the largest integer N_{12} that satisfies

$$N_{12}T_{to} < (T_{12_{nom}} - T_c) \tag{15}$$

which specifies that N_{12} aircraft can take off in the available interarrival time.[1] From the weighted sum of N_{ij} over all types of aircraft, as in equation (13), and the landing rate, λ_L, the total operation rate, λ_o, is obtained:

$$\lambda_o = \lambda_L \left(1 + \sum_{i,j} P_i P_j N_{ij}\right) \tag{16}$$

COMPUTER PROGRAM

A computer program was written to solve the various equations given above so that the landing rate and total operation rate could be determined as a function of the various parameters of the system. The program is written in Fortran IV, and a brief description and listing are included in appendix B.

For each type of aircraft (the program handles up to 12 types), the following input quantities are specified:

m — distance of the outer marker from runway threshold

S_o, S_f — airspace required for separation at the outer marker and at touchdown

[1]This takeoff criterion is not strictly compatible with the present FAA requirements that specify 1 minute between successive takeoffs and 2 miles separation between a takeoff and the following landing. However, it suggests how arrivals can be interspersed with landings.

T_c runway clearance time

V_a, V_b approach velocity and touchdown velocity

T_{vf} time before touchdown when the aircraft should be at velocity V_b

$\left.\begin{matrix} r\delta p, r\delta V_a \\ r\delta V_b, r\delta t_{Vf} \end{matrix}\right\}$ range of uncertainties of the various quantities

P probability of arrival of that aircraft type

The principal outputs were calculated from deterministic data using weighted averaging techniques. These outputs were λ_L, the landing rate, and λ_o, the operations rate (calculated for T_{to} = 36 and 48 sec). Additional output was obtained from Monte Carlo type runs in which 300 landings were made to determine the number of landing aborts that occurred for a given set of conditions. For these runs, the random variables δp, δV_a, etc., were selected from a triangular distribution with the width equal to the corresponding range. Also, to control the number of aborts, the value K in equation (10) was variable.

PROGRAM INPUTS

To obtain average landing rates, the program referred to in the previous section was used over a range of the various input parameters. The specific values used, along with a justification for the choice, are indicated in this section. The parameters are the aircraft mix, and the quantities m, S_o, S_f, T_c, V_a, V_b, T_{vf}, a, $r\delta p$, $r\delta V_a$, $r\delta V_b$, and $r\delta t_{vf}$ selected separately for each type of aircraft.

For CTOL vehicles, two values of distance to the merge point, m, were used — 19 km (≈10 miles) and 7 km (≈4 miles), which represent fairly long and fairly short approach paths presently in use. For STOL vehicles, 7 km (≈4 miles) and 2 km (≈1 mile) were used since it is unlikely that STOL vehicles will use the long approaches of the CTOL vehicles and, when a curved approach is used may turn onto the final heading fairly close to the runway.

The nominal values of separation, S_o, are 6 km (≈3 miles) for CTOL vehicles and 2 km (≈1 mile) for STOL vehicles. The CTOL separation is representative of present standards, but STOL vehicles may have more precise navigation and control systems so that reduced separations might be possible.

The airborne separation at touchdown, S_f, was usually set equal to S_o, the separation at the outer marker, but some runs were made with S_f essentially zero. In this case, the actual separation at touchdown is held to reasonable values by both S_o and T_c. It is not clear whether present practice is to have $S_f = S_o$, or to ignore it.

The runway clearance time was determined by computing braking time and adding to that an allowance for taxi time to the turn off. Braking time to a dead stop, using 0.25 g deceleration, is roughly 30 sec for CTOL and 15 sec for STOL. The assumption of an additional 30 and 15 sec, respectively, for taxi time gives the runway clearance time used, namely, 60 sec for CTOL vehicles and 30 sec for STOL. Data were also obtained for high-speed turnoffs that are immediately available giving runway clearance times of 30 sec for CTOL and 15 sec for STOL.

Two different situations were assumed for the approach velocity. First, the aircraft is assumed to fly at touchdown velocity from its merge point and, second, the aircraft is assumed to be at a velocity commanded by ATC when it passes its merge point, and to slow down during the approach. The touchdown speeds, V_b, used were 60, 65, and 70 m/sec (≈120, 130, and 140 knots) for CTOL and 30, 35, and 40 m/sec (≈60, 70, and 80 knots) for STOL. It is felt that these represent present-day CTOL touchdown speeds and proposed STOL touchdown speeds. Also, it was assumed that, for the dual-velocity case, the initial approach velocity, V_a, was 90 m/sec (≈180 knots) for all CTOL vehicles and 60 m/sec (≈120 knots) for all STOL vehicles, which represent terminal area cruise speeds.

In any case, a CTOL vehicle was required to reach its touchdown velocity 1-1/2 min before touchdown and a STOL vehicle, 1 min before touchdown. These are felt to be the current requirements set forth by pilots, and were the values used for T_{vf}. The deceleration, a, used during the change of speed is 0.6 m/sec^2 (≈0.06 g) for all vehicles.

As mentioned earlier, all random variables are assumed to be independent and to have a triangular distribution, centered about the nominal value, with a specified range. Thus, for a generalized random variable, θ:

$$\theta_{nom} - r\theta/2 \leq \theta_{act} \leq \theta_{nom} + r\theta/2$$

It is assumed that the pilot will maintain velocity within ±1-1/4 m/sec (±2.5 knots) so that the range for both δV_a and δV_b is $r\delta V_a = r\delta V_b = 2.5$ m/sec. The position at the merge point is assumed to be ±1 km (±1/2 mile), $r\delta p = 2$ km, and the variation on T_{vf} is assumed to be $r\delta T_{vf}$ = 30 sec for CTOL vehicles and 10 sec for STOL vehicles.

A brief study of the actual mix of CTOL traffic at the San Francisco Airport was the basis for the CTOL traffic mix used here. Three different landing speeds were assumed and their relative percentages are shown in table 1. The same percentages were used for STOL traffic. For mixed STOL and CTOL traffic, the percentage of aircraft with each landing speed is shown in table 1. The mix of aircraft shown is fairly representative and all results presented here use this mix. Data were obtained for a mix with somewhat higher average landing speed, which resulted in a slight increase in landing rate, but the trends were not affected.

TABLE 1.— AIRCRAFT MIX FOR VARIOUS PERCENTAGES OF STOL TRAFFIC

Type	Landing speed, m/sec \ STOL, percent	0	25	50	75	100
STOL	30	0	11	22	34	45
	35	0	9	17	25	34
	40	0	5	11	16	21
CTOL	60	45	34	22	11	0
	65	34	25	17	9	0
	70	21	16	11	5	0

RESULTS

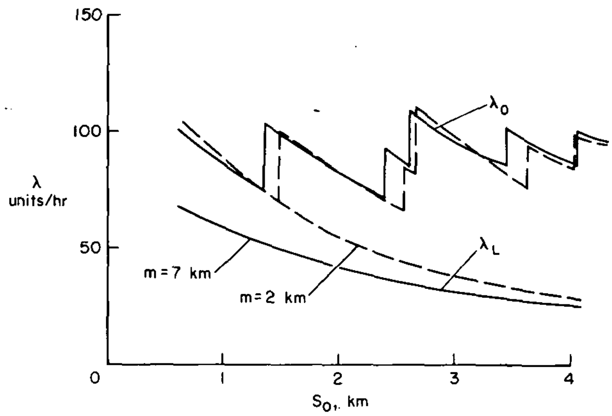
The computer program was used with the input data just described for a number of cases. The results are presented in a series of plots, following a brief discussion of the effects of some of the uncertainties.

The uncertainties in position and velocity affect the landing rate only through the calculations to guarantee adequate spacing on the runway (airborne spacing was controlled by the required separation). As a result, reducing these uncertainties had an effect very similar to reducing the time on the runway.

The nominal position uncertainty of 2 km at the merge point increased the required time spacing between touchdowns by between 15 and 30 sec, whereas each of the other uncertainties increased it by about 1 or 2 sec. If these numbers are compared with the nominal runway occupancy time of 30 to 60 sec, it is apparent that reducing the position uncertainty (or, equivalently, increasing the accuracy of time of arrival at the merge point) might change the landing rate significantly, but reducing the other uncertainties would have virtually no effect. Therefore, only the results for position uncertainty are presented here.

The basic results are presented in figures 4 through 9 for single-velocity cases and in figures 10 through 15 for dual-velocity cases. In each figure, the landing rate is plotted against the percentage of STOL traffic. For simplicity, only two landing rate curves are shown which represent merge point distances of 19 km for CTOL and 7 km for STOL, and 7 km and 2 km, respectively. Other combinations of merge point distances fall between the curves given.

The figures also show the total operation rate (sum of takeoffs and landings) for takeoff times, T_{to}, of 36 and 48 sec. These times approximate minimum and maximum takeoffs for CTOL jets and, although a STOL vehicle could take off in less time, it was felt that an allowance of less than 36 sec for takeoff would be unrealistic. These curves are shown as bands in the figures rather than as individual curves because of the step-wise fashion in which takeoffs are added. This procedure is explained in sketch (b), where landing and total operation rate are plotted versus separation for two values of *m*. The conditions for sketch (b) are: two types of STOL vehicles, each using a single-velocity approach, one at 40 m/sec and the other at 30 m/sec, with 50 percent probability of each type, T_c = 30, T_{to} = 36. As the landing rate decreases, because of an increase in separation, the number of takeoffs that can be inserted between landings increases in a step-wise fashion, one takeoff at a time. These steps occur at different values of S_o for m = 2 and m = 7 km, so that the total operation rate is sometimes larger for one case and sometimes for the other. Other values of *m* would cause the steps to change at other locations. As a result, it is very difficult to determine the effect of *m* on the total operation rate and, as indicated previously, the total operation is shown as a band in figures 4 through 15. One noticeable trend is that the total operation rate tends to increase as the landing rate decreases.



Sketch (b)

Table 2 presents the values of the parameters used for each case. Six cases are listed and each was run for both the single-velocity and dual-velocity approach.

TABLE 2.— DATA FOR EACH CASE

Case	STOL S_o, km	CTOL S_o, km	STOL T_c, sec	CTOL T_c, sec	STOL $r\delta p$, km	CTOL $r\delta p$, km
Present	6	6	30	60	2	2
Nominal	2	6	30	60	2	2
Reduce runway time	2	6	15	30	2	2
Improve STOL navigation	2	6	30	60	1	2
Minimal navigation errors	2	6	30	60	.1	.1
Minimal spacing	.1	.1	30	60	.1	.1

The present case used the current FAA rules for both STOL and CTOL. For the nominal case, since it is assumed that the STOL vehicles will use more accurate path guidance, the STOL spacing can be reduced to 2 km. Next is a nominal case with the runway occupancy time cut in half, then the case for which the STOL navigation and guidance errors were reduced from the nominal. To show the maximum effect of these errors, they were essentially eliminated for both CTOL and STOL for the next case. For the final case, with the spacing and errors reduced to nearly zero, the aircraft are not allowed to pass and must obey the runway occupancy rule, but otherwise they may be extremely close. Separation requirements for trailing vortices are not considered.

The results are now discussed, case by case, to show the changes to the landing rate and total operation rate, as STOL traffic is added to existing CTOL traffic. Also, the factor that controls the landing rate will be described, that is, whether the controlling factor is the airborne spacing at the merge point, at the touchdown point, or, for the dual-velocity case, at the midpoint (where the velocity changes), or whether it is the runway occupancy time.

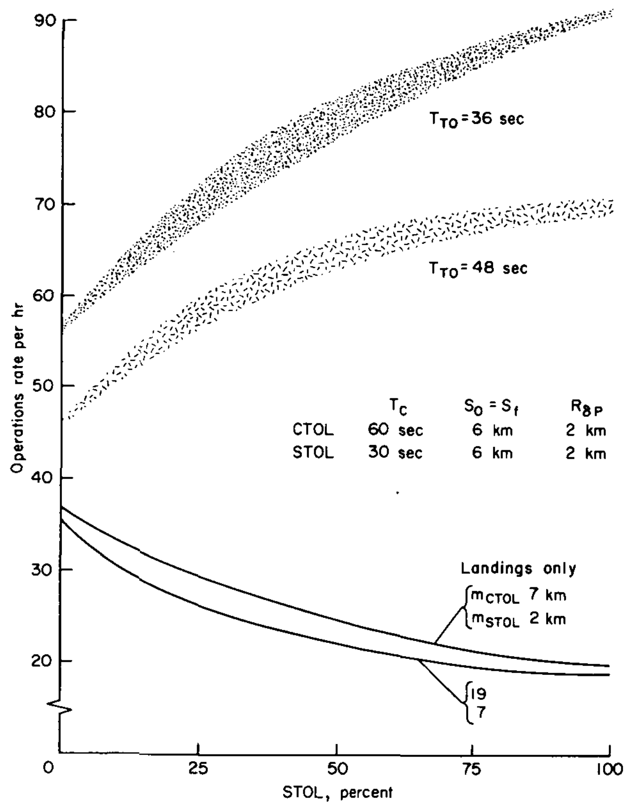


Figure 4.— Single velocity, present standards.

Single-Velocity Approach

Present standards— The situation using present standards is shown in figure 4. As expected, increasing the percentage of STOL traffic reduces the landing rate even when the merge point for CTOL and STOL traffic is fairly close in. Also, for fixed percentages of STOL traffic, reducing m_{STOL} and m_{CTOL} increases the landing rate. The factors that control the landing rate are airborne spacing at the merge point when the second aircraft is slower than the first and the spacing at touchdown when the second aircraft is faster. In either case, the spacing is such that runway occupancy time does not affect the landing rate.

When total operations are considered, the addition of STOL traffic may allow an increase in the rate, assuming aircraft are waiting to take off, since there are more gaps in the arriving traffic which can be used for takeoffs.

Nominal— The nominal case, with improved STOL navigation accuracy and therefore reduced STOL separation, is shown in figure 5. Here, adding STOL traffic decreases the landing rate only slightly and, with the higher percentages of STOL traffic, the landing rate actually increases. Also, an all-STOL runway has a higher landing rate than a CTOL runway. The controlling factor is again the spacing at the merge point or at touchdown when the first aircraft is STOL.

When total operations are considered, the addition of STOL vehicles increases the total operation rate.

Reduced runway time— Next consider the case with reduced time on the runway for both CTOL and STOL (fig. 6). Since the landing rate for CTOL vehicles is controlled by airborne spacing, reducing the runway time does not affect the landing rate although it allows a considerable increase in the total operations. The landing rate for STOL vehicles was controlled by runway time so that the STOL landing rate increased considerably. A further reduction in the runway time below 15 sec would increase landing rate only slightly. Again, for this case, introducing STOL vehicles to the traffic mix decreases the landing rate only slightly, with a considerable increase for larger percentages of STOL. The trend is for increasing total operations as STOL aircraft are added to the mix.

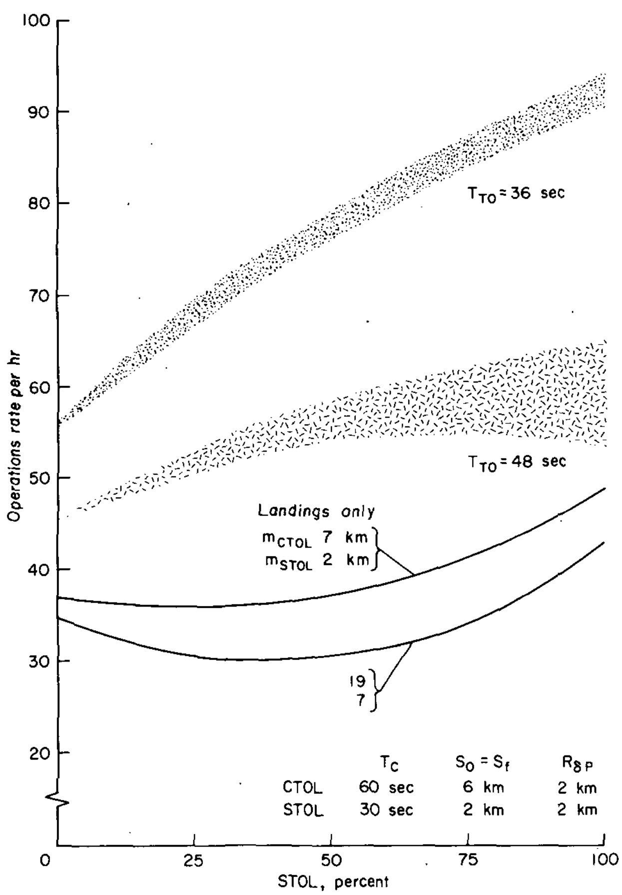


Figure 5.— Single velocity nominal case.

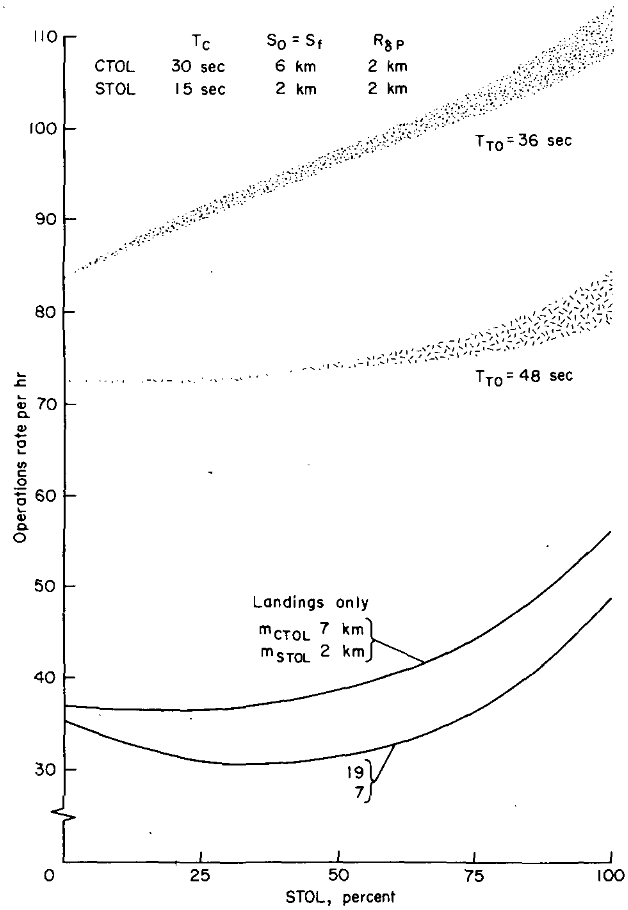


Figure 6.— Single velocity reduced runway time.

Improved STOL navigation— In figure 7, the nominal case is considered with improved guidance and navigation for the STOL vehicles, which reduces the initial position error from 2 to 1 km, with a corresponding reduction in the uncertainty of the landing time. Since this uncertainty is directly added to runway time (eq. 11), the landing rates here and for the previous case (fig. 6) are essentially the same. The effect on total operations is quite different, however. For CTOL only, the total operation rate is the same as for the nominal case (fig. 5), and the increase with increasing STOL traffic is similar to that for the nominal case.

Minimal navigation error— In figure 8, the navigation errors for both STOL and CTOL aircraft were set to essentially zero. The resulting landing rate and total operation rate are essentially the same as for the previous case (fig. 7), indicating that an extreme reduction in the navigation error alone is not useful.

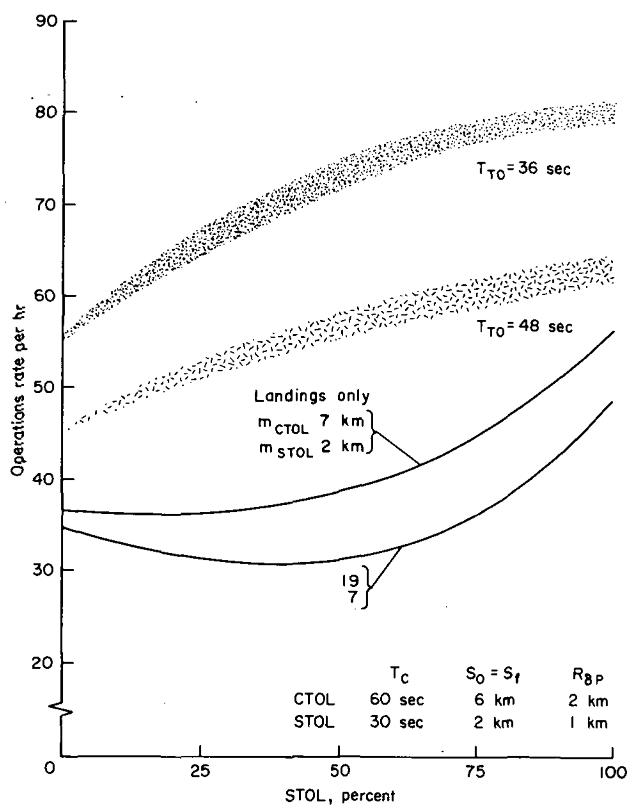


Figure 7.— Single velocity improved STOL navigation.

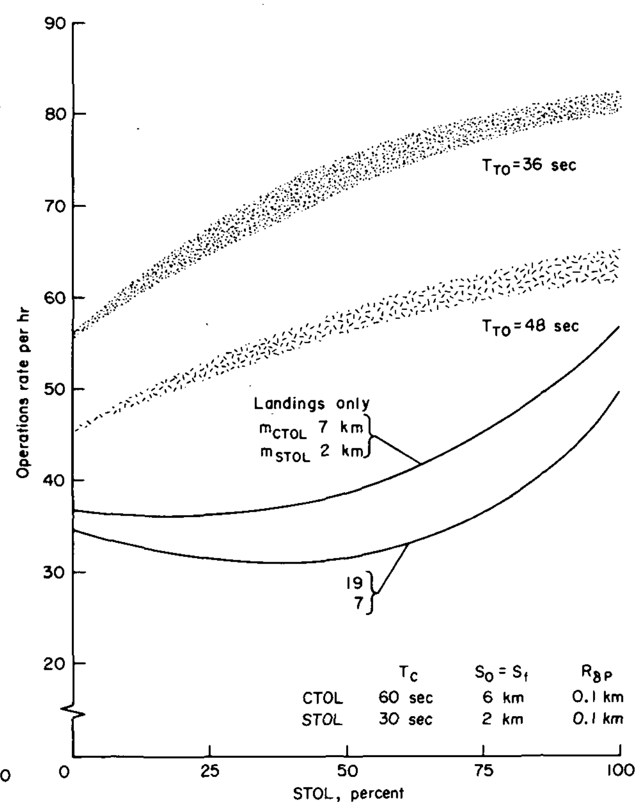


Figure 8.— Single velocity minimal navigation error.

Minimal spacing— Finally, consider the case (fig. 9) in which the spacing is reduced to correspond to the increased navigation accuracy. To determine the extreme situation, a minimal spacing value of 0.1 km was used. With this spacing, the landing rate is determined almost exclusively by the runway occupancy time and, to a lesser degree, by the error in touchdown velocity. The addition of STOL traffic increases the landing rate, and a STOL-only runway again has a higher landing capacity than the CTOL-only runway. Moving the STOL merge point closer also increases the landing rate by decreasing the allowance required to compensate for the uncertainty in final velocity. For this case, times are available for only a few takeoffs, and then only with mixed traffic, when the distant merge points are used.

Dual-Velocity Approach

For a dual-velocity approach, all CTOL aircraft use the same constant speed from the CTOL merge point until time to change velocity, about 5 or 6 km from the threshold. As a result, the location of the merge point for CTOL vehicles has very little effect on the landing or takeoff rate; therefore, it is not shown as a parameter in the figures.

The landing rates and the total operation rates for the dual-velocity approach are shown in figures 10 through 15.

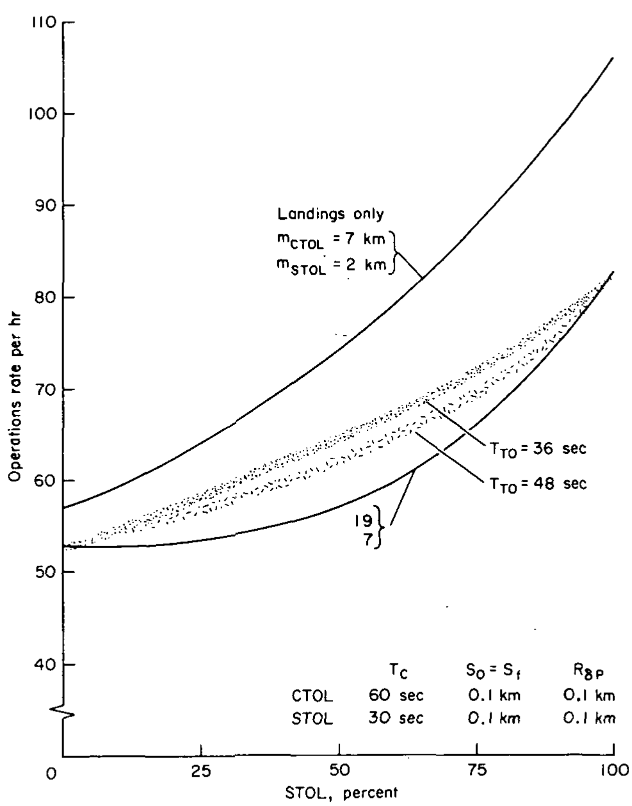


Figure 9.— Single velocity minimal spacing.

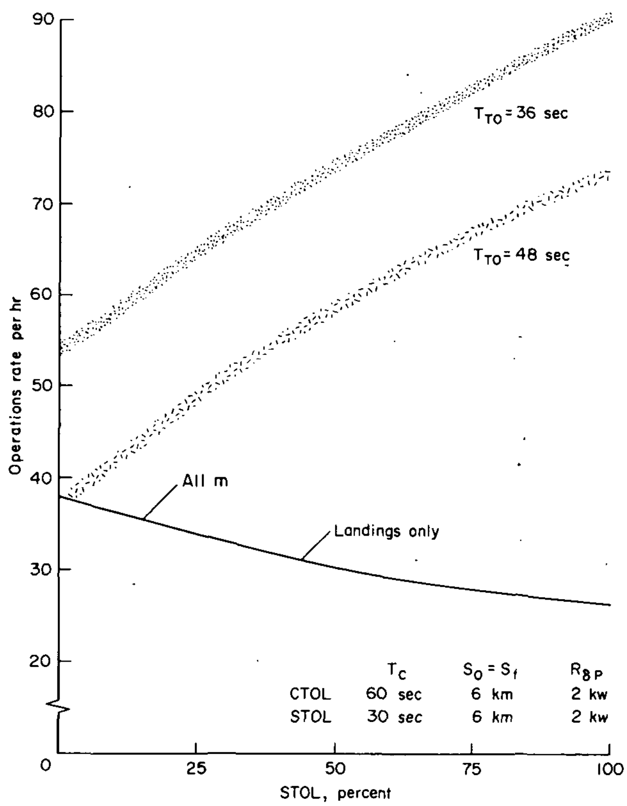


Figure 10.— Dual velocity present standards.

Present standards— The results for this case are shown in figure 10, and the trends are generally the same as for the constant velocity approach. Adding STOL to the traffic mix reduces the landing rate but increases the total operation rate. The landing rate is controlled strictly by airborne spacing, usually at touchdown but, in some cases, at the velocity change point.

Nominal— The reduced spacing for STOL vehicles (fig. 11) now results in a strictly increasing landing rate as the proportion of STOL vehicles increases and in an increasing total operation rate. The controlling factor is the airborne spacing for a CTOL first aircraft and either airborne spacing or runway clearance, depending on m_{STOL}, for a STOL first aircraft. The landing rate for this case is somewhat larger than for the corresponding single-velocity case (fig. 5), but the total operation rate is less.

Reduced runway time— When the runway occupancy time was reduced (fig. 12), the landing rate increased compared to the nominal (fig. 11), for m_{STOL} = 7 km,

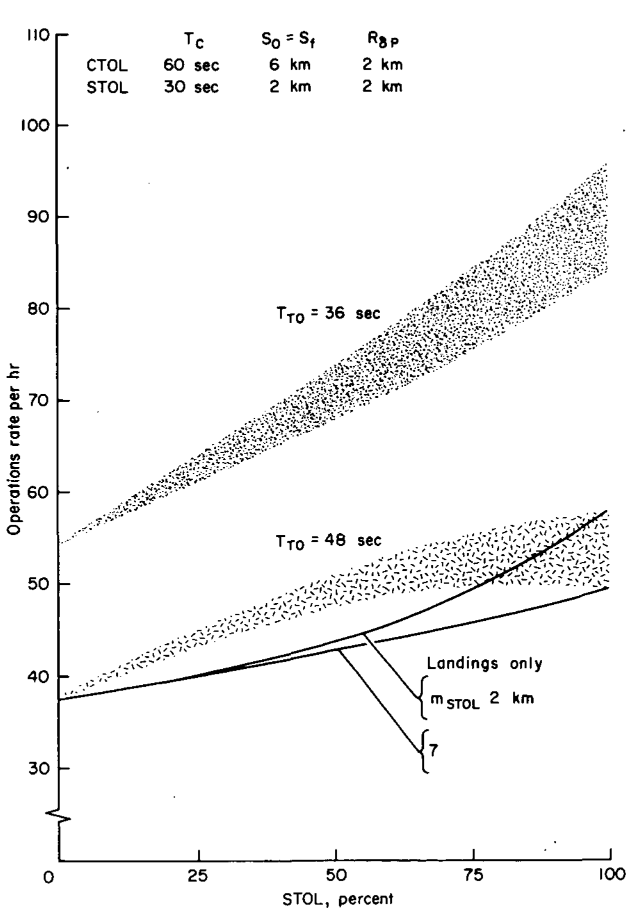


Figure 11.— Dual velocity nominal case.

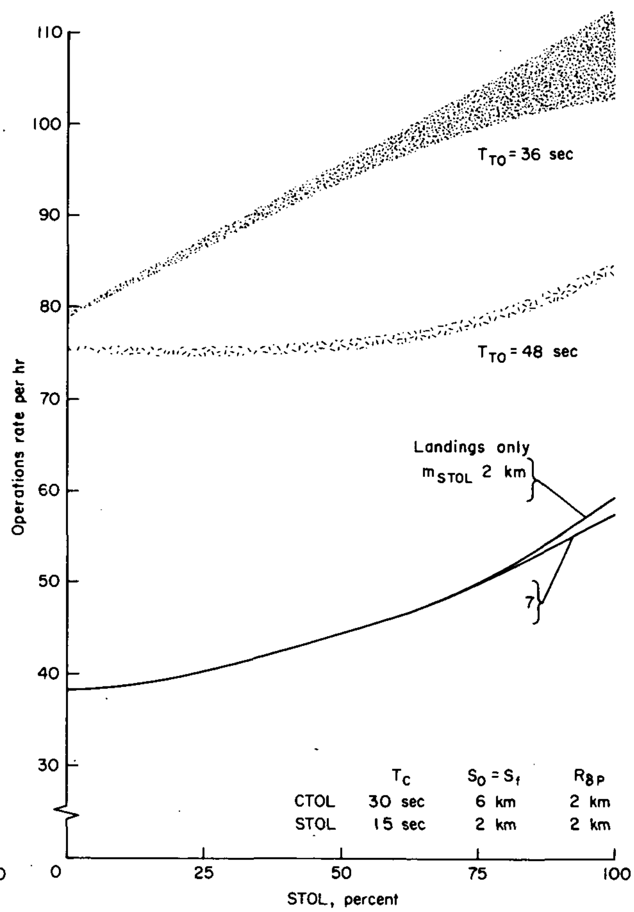


Figure 12.— Dual velocity reduced runway time.

where runway occupancy was the controlling factor, but otherwise was unaffected. The total operation rate, however, increased considerably compared to the nominal. For this case, increasing the STOL traffic increased the landing rate and also increased the total operation rate.

Improved STOL navigation— Again, as in the single-velocity case, improving the STOL navigation and guidance (fig. 13) had the same effect on the landing rate as reducing the runway occupancy time (fig. 12). The total operation rate, however, is essentially the same as for the nominal case (fig. 11). Both landing rate and total operation rate increase with increasing STOL traffic.

Minimal navigation errors— Further reductions in the navigation and guidance errors (fig. 14) had no effect on either the landing rate or the total operation rate.

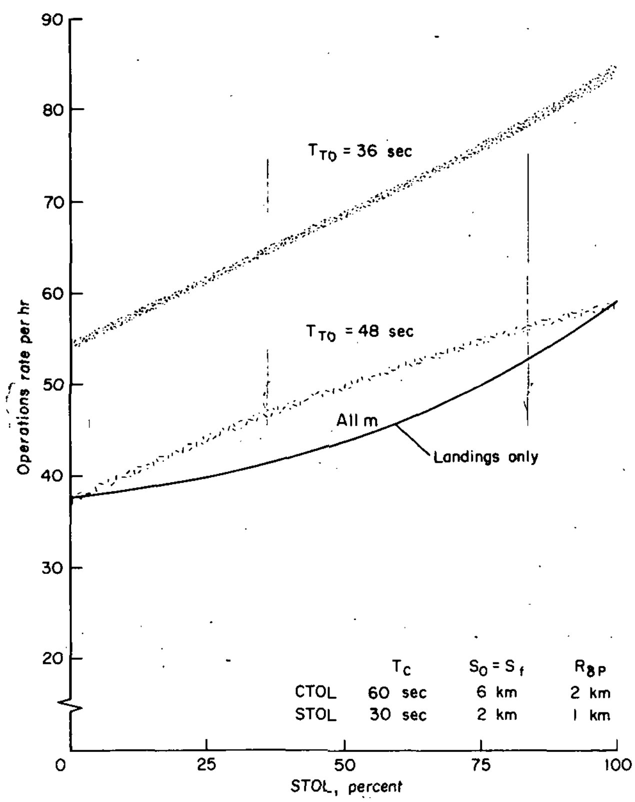


Figure 13.— Dual velocity improved STOL navigation.

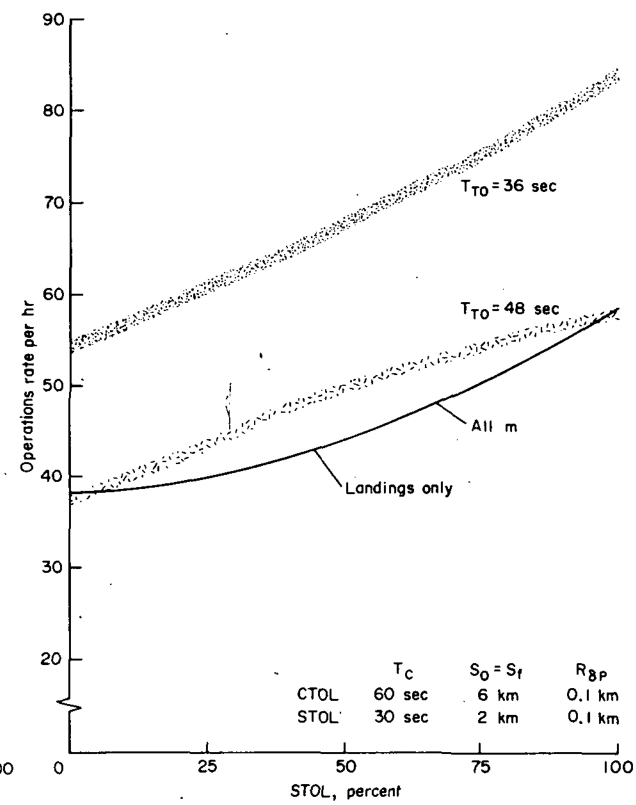


Figure 14.— Dual velocity minimal navigation errors.

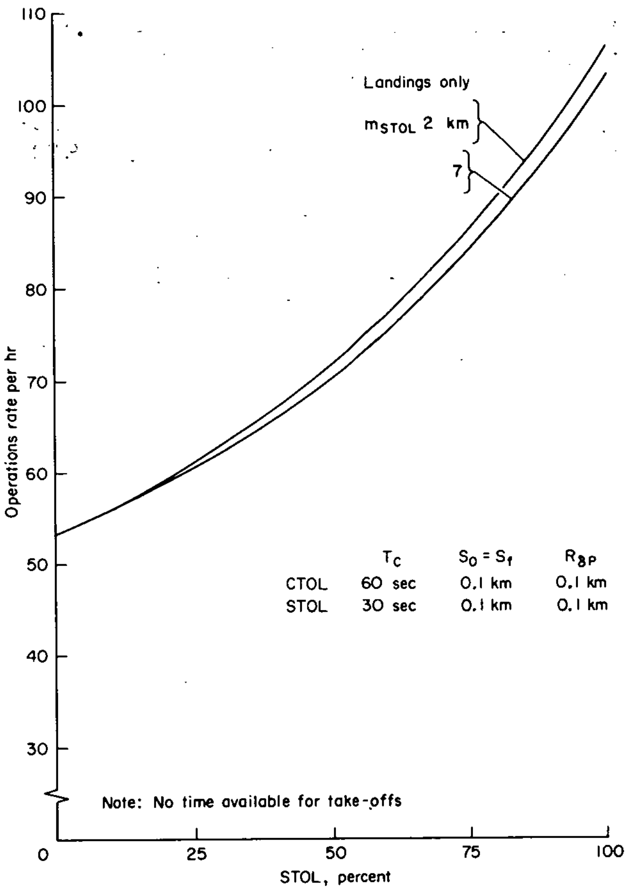


Figure 15.— Dual velocity minimal spacing.

Minimal spacing— Reducing the spacing to a minimal value (fig. 15), however, increased the landing rate dramatically over the nominal, but left no time for takeoffs.

Comparison with single-velocity case— Generally, the landing rate for the dual-velocity case is somewhat higher than that for the corresponding single-velocity case. This higher landing rate tends to accompany a somewhat lower total operation rate.

The trends for both the dual-velocity and single-velocity cases are very similar with changes in spacing, navigation, and runway occupancy time. However, as the percentage of STOL traffic increases, the landing rate always increases in the dual-velocity case; in the single-velocity case, the landing rate first dips and then increases.

CONCLUSIONS

The following conclusions are based on the philosophy that landings have priority over takeoffs.

Adding STOL traffic to a CTOL runway causes only a minimal decrease in the landing rate for a constant-velocity approach, while for the dual-velocity approach, there is an actual increase in the landing rate. This assumes that the STOL navigation is sufficiently better than existing CTOL navigation so that the STOL spacing can be reduced from 6 to 2 km (3 to 1 mile). With this spacing, the landing rate on a STOL-only runway will be greater than on existing CTOL runways.

Under all circumstances, including the use of present standards, the addition of STOL traffic to a CTOL runway will allow an increase in the total operation rate (i.e., landings plus takeoffs). Thus, with mixed STOL-CTOL operations, there is frequently time for a takeoff without disturbing the landing sequence so that mixing takeoffs and landings on the same runway is feasible and perhaps desirable.

To increase landing rates (from present values), airborne spacing must be reduced, which, in turn, requires improvements in navigational accuracy.

Reducing runway occupancy time has little effect on the landing rate but does allow for an increased total operation rate.

Generally, the landing rate is higher if the STOL merge point is as close as possible to the runway threshold, thus taking maximum advantage of the STOL maneuvering capability.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif. 94035, October 29, 1973

APPENDIX A

DERIVATION OF EQUATIONS FOR T_{12}

The kinematic equation of motion of an aircraft approaching the runway is given by equation (2), assuming that the aircraft touches down at $t = 0$. This is true for the first aircraft, but the second aircraft touches down at time T_{12} after the first. The equations of motion for the two aircraft are then:

$$x_1 = \begin{cases} -V_{b_1} t & , \quad x_1 \le D_{a_1} \\ D_{a_1}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right) - V_{a_1} t & , \quad x_1 > D_{a_1} \end{cases} \tag{A1}$$

$$x_2 = \begin{cases} -V_{b_2}(t - T_{12}) & , \quad x_2 \le D_{a_2} \\ D_{a_2}\left(1 - \dfrac{V_{a_2}}{V_{b_2}}\right) - V_{a_2}(t - T_{12}) & , \quad x_2 > D_{a_2} \end{cases} \tag{A2}$$

From these equations, one can calculate the times when the aircraft are at specific points. These times, and their equations, are: the time the first aircraft is at its merge point, m_1,

$$T_{m_1} = \begin{cases} -m_1/V_{b_1} & , \quad m_1 \le D_{a_1} \\ -\dfrac{m_1}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right) & , \quad m_1 > D_{a_1} \end{cases} \tag{A3}$$

the time the first aircraft is at the merge point for the second aircraft, m_2,

$$T_{m_a} = \begin{cases} -m_2/V_{b_1} & , \quad m_2 \le D_{a_1} \\ -\dfrac{m_2}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right) & , \quad m_2 > D_{a_1} \end{cases} \tag{A4}$$

and, finally, the time at which the second aircraft changes its speed,

$$T_{a_2} = T_{12} - \frac{D_{a_2}}{V_{b_2}} \tag{A5}$$

The equation that defines the reserved airspace for the first aircraft is obtained by combining equations (4) and (5):

$$x_1' = x_1 + \frac{S_1(x_1)}{2} = \frac{S_{f_1}}{2} + \left(1 + \frac{S_{o_1} - S_{f_1}}{2m_1}\right)x_1 = \frac{S_{f_1}}{2} + K_1 x_1 \ , \qquad x_1 \leq m_1 \tag{A6}$$

where

$$K_1 = \left(1 + \frac{S_{o_1} - S_{f_1}}{2m_1}\right)$$

Note that since no tests are needed at times before the first aircraft arrives at its merge point, x_1' is not needed for $x_1 > m_1$. Similarly, the reserved airspace for the second aircraft is

$$x_2' = x_2 - \frac{S_2(x_2)}{2} = \begin{cases} -\dfrac{S_{f_2}}{2} + \left(1 - \dfrac{S_{o_2} - S_{f_2}}{2m_2}\right)x_2 \ , & x_2 \leq m_2 \\ -\dfrac{S_{o_2}}{2} + x_2 \ , & x_2 > m_2 \end{cases}$$

$$= \begin{cases} -\dfrac{S_{f_2}}{2} + K_2 x_2 \ , & x_2 \leq m_2 \\ -\dfrac{S_{o_2}}{2} + x_2 \ , & x_2 > m_2 \end{cases}$$

where

$$K_2 = \left(1 - \frac{S_{o_2} - S_{f_2}}{2m_2}\right)$$

Assume that $S_o \geq S_f$, that is, the required separation at touchdown may be less than (or equal to) that at the outer marker, but never greater; thus, $K_1 > 0$. However, if

$$\frac{S_{o_2} - S_{f_2}}{2} > m_2 \quad ,$$

then $K_2 < 0$. Consider first the situation when $K_2 > 0$ (fig. 16). When $x_2 > m_2$, the separation is $S_{o_2}/2$; when $x_2 \leq m_2$, the separation is $(S_{f_2}/2) + K_2 x_2$. Next, figure 17 shows the situation for $K_2 < 0$, when the

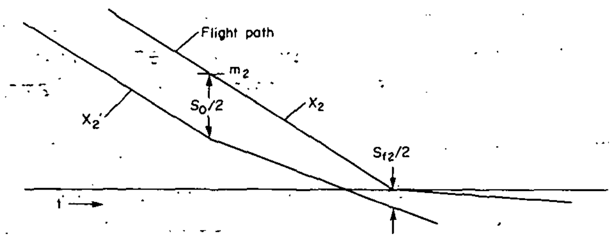


Figure 16.— Reserved space before second aircraft, $K_2 > 0$.

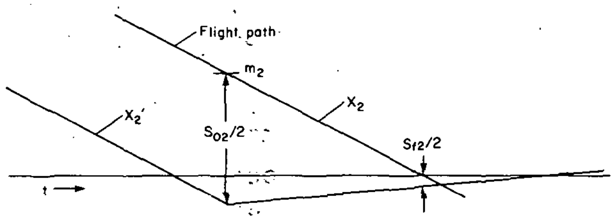


Figure 17.— Reserved space before second aircraft, $K_2 < 0$.

first aircraft will land before the second aircraft reaches its merge point, so that airborne separation after the merge point is not required. Thus, for $K_2 < 0$, the second part of the above equation for x_2 should be used. With these restrictions,

$$x_2' = \left\{ \begin{array}{lll} -\dfrac{S_{f_2}}{2} + K_2 x_2 \; , & x_2 \leq m_2 \text{ and } K_2 \geq 0 \\ \\ -\dfrac{S_{o_2}}{2} + x_2 \; , & x_2 > m_2 \text{ or } K_2 < 0 \end{array} \right\} \tag{A7}$$

With the equation of the boundaries, the next step is to calculate T_{12} so that the boundaries do not cross but touch at some point. This will clearly determine the minimum T_{12} since, if it were smaller, the separation criteria would not be met at some points along the trajectory. Thus $x_1'(t) - x_2'(t) = 0$, is solved at selected values of time, for T_{12}. This equation must be solved for several different values of t, namely, at the final point, $t = 0$, at the merge point, $t = \max(T_{m_a}, T_{m_1})$, and for the case when the second aircraft slows down during the approach, at the velocity change point $t = T_{a_2}$. To minimize confusion, the time difference from each equation is labeled τ, and $T_{12} \triangleq \max\ (\tau_{in}, \tau_{fin}, \tau_{med}, \tau_c)$. (The value of τ_c is determined later.) Thus, the following three cases are obtained:

$$\left.\begin{array}{lll}
\text{I:} & x_1'(0) - x_2'(0) = 0 \ , & \text{solve for } \tau_{fin} \\
\text{IIA:} & x_1'(T_{ma}) - x_2'(T_{ma}) = 0 \ , \quad m_2 \le m_1 & \multirow{2}{*}{\text{solve for } \tau_{in}} \\
\text{IIB:} & x_1'(T_{m1}) - x_2'(T_{m1}) = 0 \ , \quad m_2 > m_1 & \\
\text{III:} & x_1'\left(T_{a_2}\right) - x_2'\left(T_{a_2}\right) = 0 \ , \quad \max(T_{ma},T_{m1}) < T_{a_2} < 0 \ , & \text{solve for } \tau_{med}
\end{array}\right\} \tag{A8}$$

If the restriction for case III test is not met, then a test for case III is not needed; for convenience, we set $\tau_{med} = 0$ so that it cannot be selected as the maximum in determining T_{12}. To solve equation (A8) for τ, use equations (A1) through (A7). Since these are conditional equations, depending on the relationships between $x_2(t)$, D_{a_2}, m_2, m_1, etc., many subcases arise. Each must be solved for and then checked to ensure that all conditional relationships are met, and, finally, the defining conditions for each subcase must be determined in terms of the input quantities, m_1, D_{a_1}, S_{o_1}, S_{f_1}, V_{a_1}, V_{b_1}, and $i = 1,2$. This procedure is followed for each subcase in the following sections. A small sketch shows position versus time and indicates the location of the input quantities for each subcase.

CASE I

In this case, $t = 0$, so the following equations are needed

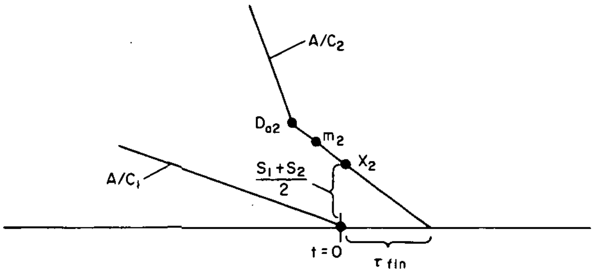
$$x_1 = 0 \ , \qquad x_1' = \frac{S_{f_1}}{2} \tag{A9}$$

$$x_2 = \begin{cases} V_{b_2} T_{12} & , \quad x_2 \le D_{a_2} \\ D_{a_2}\left(1 - \dfrac{V_{a_2}}{V_{b_2}}\right) + V_{a_2} T_{12} & , \quad x_2 > D_{a_2} \end{cases} \tag{A10}$$

$$x_2' = \begin{cases} -\dfrac{S_{f_2}}{2} + K_2 x_2 & , \quad x_2 \le m_2 \text{ and } K_2 \ge 0 \\ -\dfrac{S_{o_2}}{2} + x_2 & , \quad x_2 > m_2 \text{ or } K_2 < 0 \end{cases} \tag{A11}$$

These equations lead to four subcases, each of which is considered separately.

Case Ia: $x_2 \leq D_{a_2}$ and ($x_2 \leq m_2$ and $K_2 \geq 0$)



Sketch (c)

Combining the appropriate equations from (A9) to (A11) gives

$$\tau_{fin} = \frac{S_{f_1} + S_{f_2}}{2K_2 V_{b_2}}$$

If this value for T_{12} is used in equation (A10),

$$x_2 = \frac{S_{f_1} + S_{f_2}}{2K_2}$$

To meet the conditions for this case,

$$x_2 \leq D_{a_2} \longrightarrow \frac{S_{f_1} + S_{f_2}}{2K_2} \leq D_{a_2}$$

$$x_2 \leq m_2 \longrightarrow \frac{S_{f_1} + S_{f_2}}{2K_2} \leq m_2$$

$$K_2 \geq 0$$

These three tests can be combined into two, since $K_2 \geq 0$, if both sides are multiplied by K_2:

$$x_2 \leq D_{a_2} \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} \leq K_2 D_{a_2}$$

$$x_2 \leq m_2 \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} \leq K_2 m_2$$

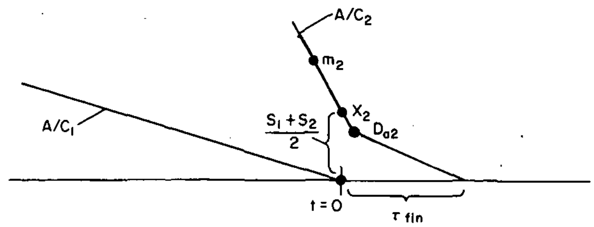
which are automatically not satisfied if $K_2 < 0$.

Case Ib: $x_2 > D_{a_2}$ and ($x_2 \leq m_2$ and $K_2 \geq 0$) (sketch (d))

If these conditions are used in equations (A9) to (A11),

$$\tau_{fin} = \frac{S_{f_1} + S_{f_2}}{2K_2V_{a_2}} - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$

$$x_2 = D_{a_2}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$



Sketch (d)

$$+ V_{a_2}\left(\frac{S_{f_1} + S_{f_2}}{2K_2V_{a_2}} - \frac{D_{a_2}}{V_{a_2}}\right)\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) = \frac{S_{f_1} + S_{f_2}}{2K_2}$$

To meet the necessary conditions, with $K_2 \geq 0$, one must have

$$x_2 > D_{a_2} \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} > K_2D_{a_2}$$

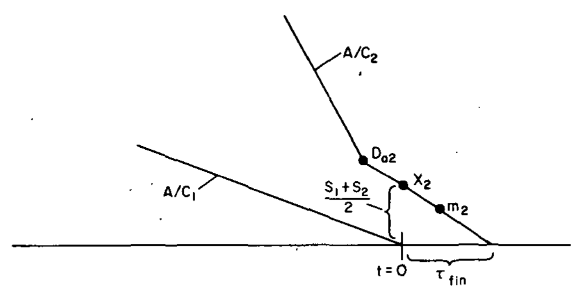
$$x_2 \leq m_2 \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} \leq K_2m_2$$

Case Ic: $x_2 \leq D_{a_2}$ and ($x_2 > m_2$ or $K_2 < 0$) (sketch (e))

Again, solving equations (A9) to (A11) gives

$$\tau_{fin} = \frac{S_{f_1} + S_{o_2}}{2V_{b_2}}$$

$$x_2 = \frac{S_{f_1} + S_{o_2}}{2}$$



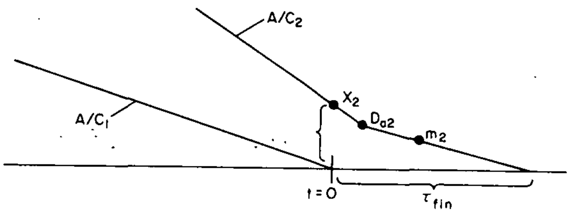
Sketch (e)

The condition tests are

$$x_2 \le D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} \le D_{a_2}$$

$$x_2 > m_2 \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} > m_2$$

Case Id: $x_2 > D_{a_2}$ and ($x_2 > m_2$ or $K_2 < 0$) (sketch (f))



Sketch (f)

$$\tau_{fin} = \frac{S_{f_1} + S_{o_2}}{2V_{a_2}} - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$

$$x_2 = D_{a_2}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) + \frac{S_{f_1} + S_{o_2}}{2}$$

$$- D_{a_2}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) = \frac{S_{f_1} + S_{o_2}}{2}$$

The condition tests are

$$x_2 > D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} > D_{a_2}$$

$$x_2 > m_2 \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} > m_2$$

Remarks

Although all four cases have been discussed, the tests do not appear to guarantee that all possible conditions were included. However, if the value of K_2 is substituted into the second test for case Ia, one obtains

$$\frac{S_{f_1} + S_{f_2}}{2} \le \left(1 - \frac{S_{o_2} - S_{f_2}}{2m_2}\right) m_2 = m_2 - \frac{S_{o_2} - S_{f_2}}{2}$$

or

$$\frac{S_{f_1} + S_{f_2}}{2} + \frac{S_{o_2} - S_{f_2}}{2} = \frac{S_{f_1} + S_{o_2}}{2} \le m_2$$

which is the identical test used in cases Ic and Id, thus showing that all possible conditions were included. Figure 18 is a flow chart of the case I tests. The test used in the figure is that described under cases Ia and Ib.

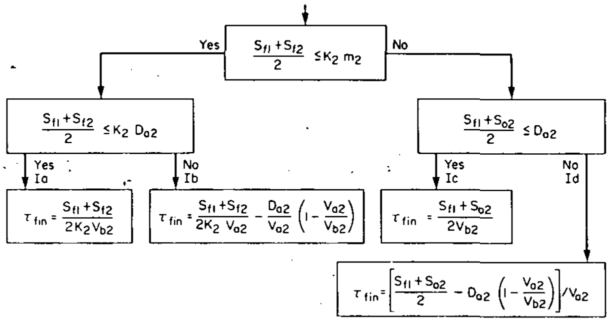


Figure 18.— Calculation of τ_{fin}.

CASE IIA

Here, the minimum separation is determined when the first aircraft is at the merge point. The merge point is assumed to be m_2, so that $x_1 = m_2$, $m_2 \le m_1$ and $t = T_{m_a}$. The equations needed obtained from equations (A1) to (A7), are

$$x_1 = m_2 \tag{A12}$$

$$x_2 = \left\{ \begin{array}{ll} -V_{b_2}\left(T_{m_a} - T_{12}\right) & , \quad x_2 \le D_{a_2} \\ D_{a_2}\left(1 - \dfrac{V_{a_2}}{V_{b_2}}\right) - V_{a_2}\left(T_{m_a} - T_{12}\right) & , \quad x_2 > D_{a_2} \end{array} \right\} \tag{A13}$$

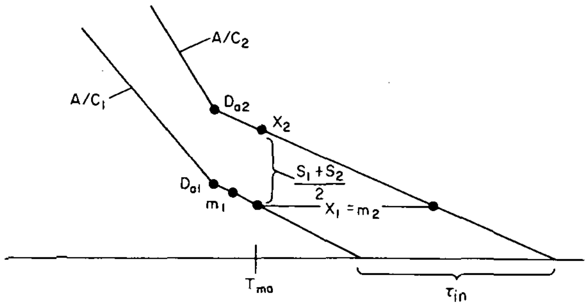
$$\left. \begin{array}{l} x_1' = \dfrac{S_{f_1}}{2} + K_1 x_1 \\ x_2' = -\dfrac{S_{o_2}}{2} + x_2 \end{array} \right\} \tag{A14}$$

Note that only one equation for x_2' is needed here since $x_2 > x_1 = m_2$. Further, since K_2 does not enter the equation, its value need not be considered.

$$T_{m_a} = \left\{ \begin{array}{ll} -m_2/V_{b_1} & , \quad x_1 = m_2 \le D_{a1} \\ -\dfrac{m_2}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right), & \quad x_1 = m_2 > D_{a_1} \end{array} \right\} \tag{A15}$$

For each of the four cases here, solve equations (A12) to (A15) for $T_{12} = \tau_{in}$. The solutions and the required tests are as follows:

Case IIAa: $x_1 \le D_{a_1}$, $x_2 \le D_{a_2}$ (sketch (g))



Note: m_1 shown $< D_{a1}$, but may be $> D_{a1}$

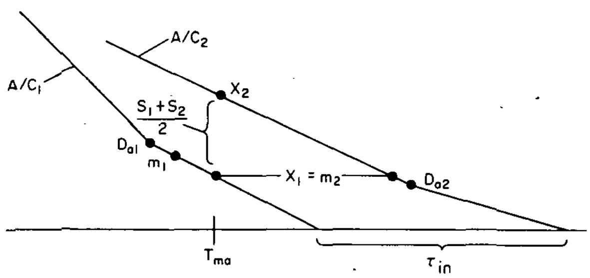
Sketch (g)

$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{b_2}} + \frac{m_2}{V_{b_2}}\left(K_1 - \frac{V_{b_2}}{V_{b_1}}\right)$$

$$x_1 \le D_{a_1} \longrightarrow m_2 \le D_{a_1}$$

$$x_2 \le D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_2 \le D_{a_2}$$

Case IIAb: $x_1 \le D_{a_1}$, $x_2 > D_{a_2}$ (sketch (h))



Note: m_1 shown $< D_{a1}$, but may be $> D_{a1}$
D_{a2} shown $< m_2$, but may be $> m_2$

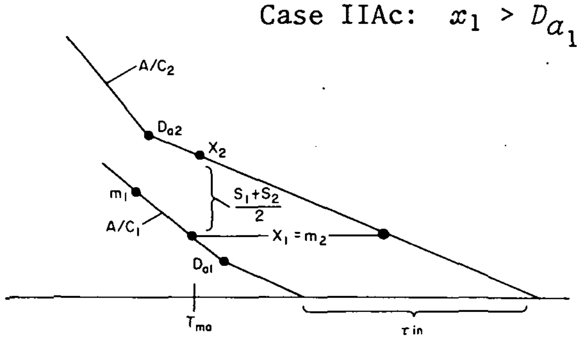
Sketch (h)

$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{a_2}} - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) + \frac{m_2}{V_{a_2}}\left(K_1 - \frac{V_{a_2}}{V_{b_1}}\right)$$

$$x_1 \le D_{a_1} \longrightarrow m_2 \le D_{a_1}$$

$$x_2 > D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_2 > D_{a_2}$$

Case IIAc: $x_1 > D_{a_1}$, $x_2 \le D_{a_2}$ (sketch (i))



Sketch (i)

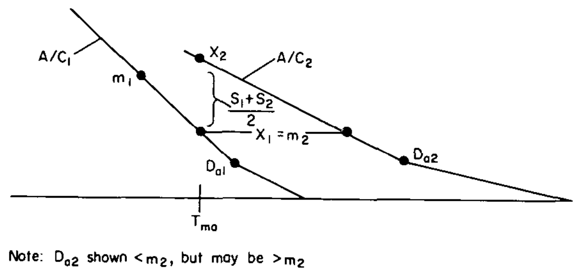
$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{b_2}} + \frac{m_2}{V_{b_2}}\left(K_1 - \frac{V_{b_2}}{V_{a_1}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right)$$

$$x_1 > D_{a_1} \longrightarrow m_2 > D_{a_1}$$

$$x_2 \leq D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_2 \leq D_{a_2}$$

Case IIAd: $x_1 > D_{a_1}$, $x_2 > D_{a_2}$ (sketch (j))

$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{a_2}} + \frac{m_2}{V_{a_2}}\left(K_1 - \frac{V_{a_2}}{V_{a_1}}\right) - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right)$$



Sketch (j)

$$x_1 > D_{a_1} \longrightarrow m_2 > D_{a_1}$$

$$x_2 > D_{a_2} \longrightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_2 > D_{a_2}$$

Figure 19 is a flow chart of the case IIA tests, A4.

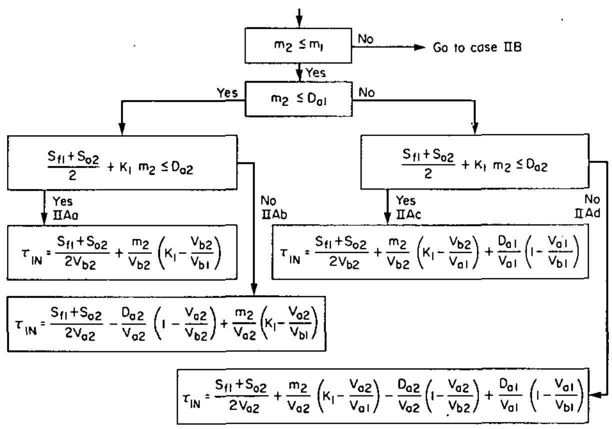


Figure 19.

CASE IIB

Again, the minimum separation is determined when the first aircraft is at the merge point. Here, however, the merge point is at m_1, so $x_1 = m_1$ and $m_2 > m_1$, with $t = T_{m_1}$. The equations needed are

$$x_2 = \left\{ \begin{array}{ll} -V_{b_2}\left(T_{m_1} - T_{12}\right) & , \quad x_2 \leq D_{a_2} \\ D_{a_2}\left(1 - \dfrac{V_{a_2}}{V_{b_2}}\right) - V_{a_2}\left(T_{m_1} - T_{12}\right) & , \quad x_2 > D_{a_2} \end{array} \right\} \tag{A16}$$

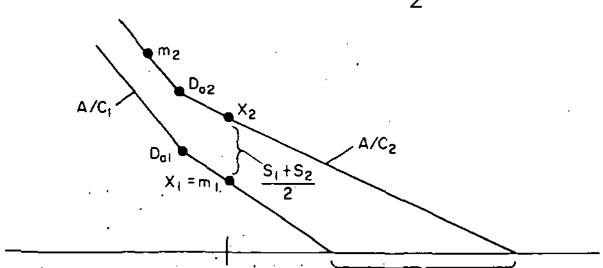
$$x_1' = \frac{S_{f_1}}{2} + K_1 x_1 \tag{A17}$$

$$x_2' = \left\{ \begin{array}{ll} -\dfrac{S_{f_2}}{2} + K_2 x_2 & , \quad x_2 \leq m_2 \text{ and } K_2 \geq 0 \\ -\dfrac{S_{o_2}}{2} + x_2 & , \quad x_2 > m_2 \text{ or } K_2 < 0 \end{array} \right\} \tag{A18}$$

$$T_{m_1} = \left\{ \begin{array}{ll} -m_1/V_{b_1} & , \quad x_1 = m_1 \leq D_{a_1} \\ -\dfrac{m_1}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right) & , \quad x_1 = m_1 > D_{a_1} \end{array} \right\} \tag{A19}$$

Here three conditions must be considered, which lead to eight cases. For each case solve equations A(16) to A(19) for $T_{12} = \tau_{in}$.

Case IIBa: $x_2 \leq D_{a_2}$, ($x_2 \leq m_2$ and $K_2 \geq 0$), $x_1 \leq D_{a_1}$ (sketch (k))



Note: m_2 shown $> D_{a2}$, but may be $< D_{a2}$

Sketch (k)

$$\tau_{in} = \frac{S_{f_1} + S_{f_2}}{2K_2 V_{b_2}} + \frac{m_1}{K_2 V_{b_2}}\left(K_1 - \frac{K_2 V_{b_2}}{V_{b_1}}\right)$$

$$x_2 \leq D_{a_2} \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \leq K_2 D_{a_2}$$

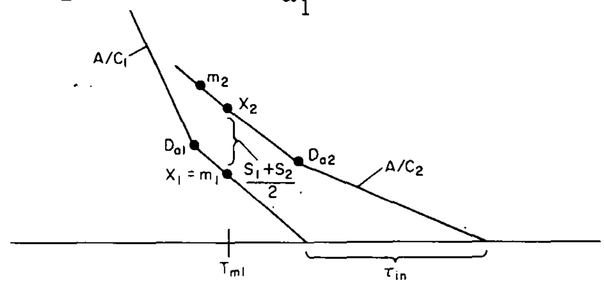
Again, as in case I, since $K_2 \geq 0$, both sides have been multiplied by K_2. The resulting test will fail if $K_2 < 0$:

$$x_2 \le m_2 \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \le K_2 m_2$$

$$x_1 \le D_{a_1} \rightarrow m_1 \le D_{a_1}$$

Case IIBb: $x_2 > D_{a_2}$, ($x_2 \le m_2$ and $K_2 \ge 0$), $x_1 \le D_{a_1}$ (sketch (ℓ))

$$\tau_{in} = \frac{S_{f_1} + S_{f_2}}{2K_2 V_{a_2}} + \frac{m_1}{K_2 V_{a_2}}\left(K_1 - \frac{K_2 V_{a_2}}{V_{b_1}}\right) - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$



Sketch (ℓ)

$$x_2 > D_{a_2} \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 > K_2 D_{a_2}$$

$$x_2 \le m_2 \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \le K_2 m_2$$

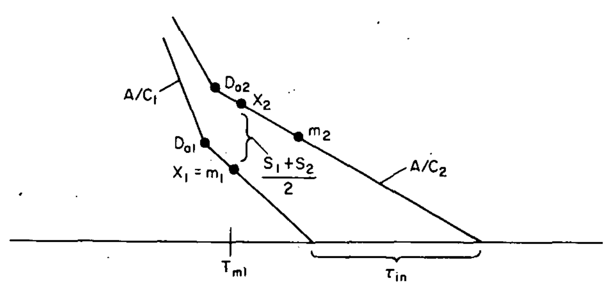
$$x_2 \le D_{a_1} \rightarrow m_1 \le D_{a_1}$$

Case IIBc: $x_2 \le D_{a_2}$, ($x_2 > m_2$ or $K_2 < 0$), $x_1 < D_{a_1}$ (sketch (m))

$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{b_2}} + \frac{m_1}{V_{b_2}}\left(K_1 - \frac{V_{b_2}}{V_{b_1}}\right)$$

$$x_2 \le D_{a_2} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 \le D_{a_2}$$

$$x_2 > m_2 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > m_2$$

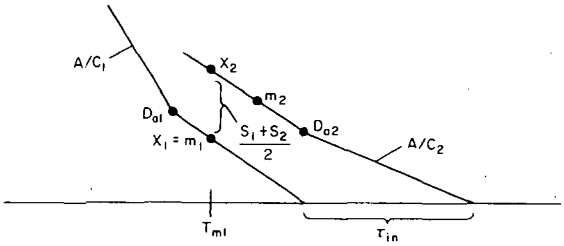


Sketch (m)

Note that, as in case I, substituting the value of K_2 into the test for $x_2 \le m_2$ of case IIBa or IIBb gives the tests determined above.

$$x_1 \le D_{a_1} \rightarrow m_1 \le D_{a_1}$$

Case IIBd: $x_2 > D_{a_2}$, $(x_2 > m_2$ or $K_2 < 0)$, $x_1 \leq D_{a_1}$ (sketch (n))



Note: m_2 shown $>D_{a2}$, but may be $<D_{a2}$

Sketch (n)

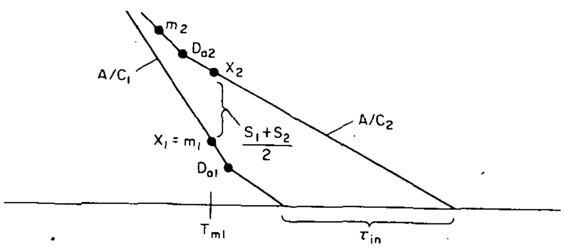
$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{a_2}} + \frac{m_1}{V_{a_2}}\left(K_1 - \frac{V_{a_2}}{V_{b_1}}\right) - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$

$$x_2 > D_{a_2} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > D_{a_2}$$

$$x_2 > m_2 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > m_2$$

$$x_1 \leq D_{a_1} \rightarrow m_1 \leq D_{a_1}$$

Case IIBe: $x_2 \leq D_{a_2}$, $(x_2 \leq m_2$ and $K_2 \geq 0)$, $x_1 > D_{a_1}$ (sketch (o))



Note: m_2 shown $>D_{a2}$, but may be $<D_{a2}$

Sketch (o)

$$\tau_{in} = \frac{S_{f_1} + S_{f_2}}{2K_2 V_{b_2}} + \frac{m_1}{V_{b_2}}\left(\frac{K_1}{K_2} - \frac{V_{b_2}}{V_{a_1}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right)$$

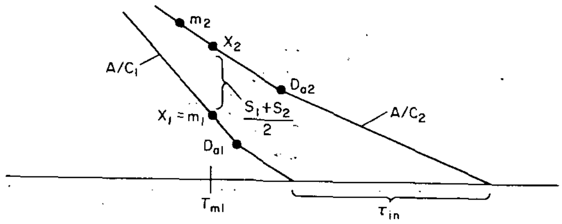
$$x_2 \leq D_{a_2} \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \leq K_2 D_{a_2}$$

$$x_2 \leq m_2 \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \leq K_2 m_2$$

$$x_1 > D_{a_1} \rightarrow m_1 > D_{a_1}$$

Case IIBf: $x_2 > D_{a_2}$, ($x_2 \leq m_2$ and $K_2 \geq 0$), $x_1 > D_{a_1}$ (sketch (p))

$$\tau_{in} = \frac{S_{f_1} + S_{f_2}}{2K_2 V_{a_2}} + \frac{m_1}{V_{a_2}}\left(\frac{K_1}{K_2} - \frac{V_{a_2}}{V_{a_1}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right) - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right)$$



Sketch (p)

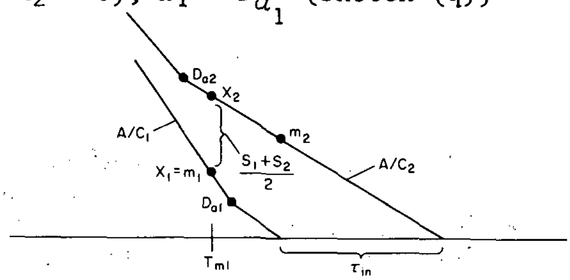
$$x_2 > D_{a_2} \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 > K_2 D_{a_2}$$

$$x_2 \leq m_2 \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 m_1 \leq K_2 m_2$$

$$x_1 > D_{a_1} \rightarrow m_1 > D_{a_1}$$

Case IIBg: $x_2 \leq D_{a_2}$, ($x_2 > m_2$ or $K_2 < 0$), $x_1 > D_{a_1}$ (sketch (q))

$$\tau_{fin} = \frac{S_{f_1} + S_{o_2}}{2V_{b_2}} + \frac{m_1}{V_{b_2}}\left(K_1 - \frac{V_{b_2}}{V_{a_1}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right)$$



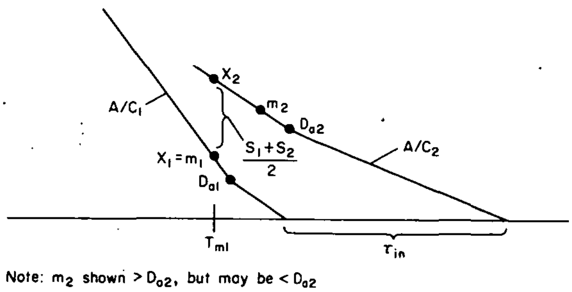
Sketch (q)

$$x_2 \leq D_{a_2} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 \leq D_{a_2}$$

$$x_2 > m_2 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > m_2$$

$$x_1 > D_{a_1} \rightarrow m_1 > D_{a_1}$$

Case IIBh: $x_2 > D_{a_2}$, ($x_2 > m_2$ or $K_2 < 0$), $x_1 > D_{a_1}$ (sketch (r))



Sketch (r)

$$\tau_{in} = \frac{S_{f_1} + S_{o_2}}{2V_{a_2}} + \frac{m_1}{V_{a_2}}\left(K_1 - \frac{V_{a_2}}{V_{a_1}}\right) - \frac{D_{a_2}}{V_{a_2}}\left(1 - \frac{V_{a_2}}{V_{b_2}}\right) + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right)$$

$$x_2 > D_{a_2} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > D_{a_2}$$

$$x_2 > m_2 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 m_1 > m_2$$

$$x_1 > D_{a_1} \rightarrow m_1 > D_{a_1}$$

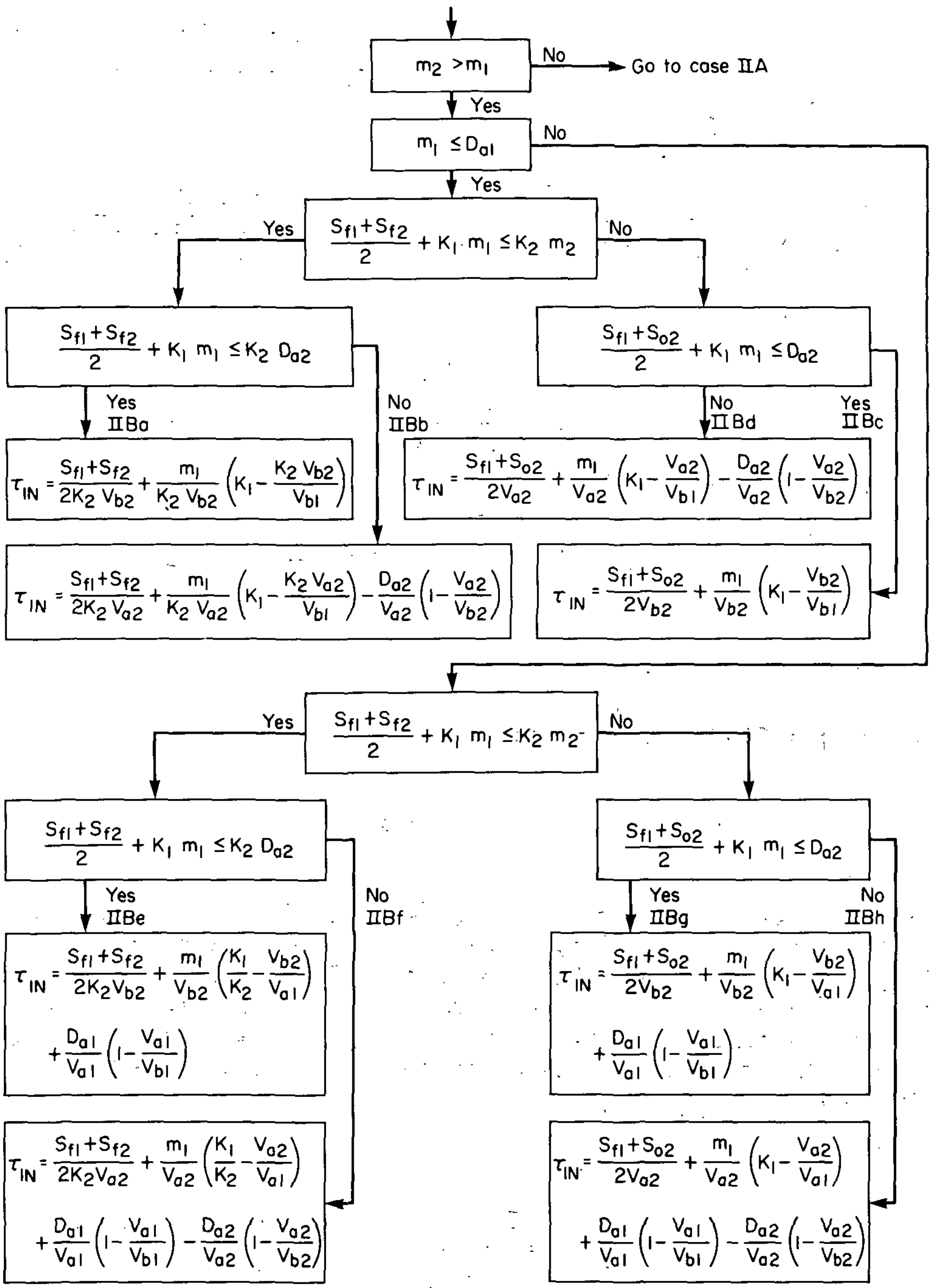
Figure 20 is a flow chart of case IIB.

CASE III

For this case, the minimum separation is determined when the second aircraft is at $t = T_{a_2}$. Note, however, that this test applies only if $\mathrm{Max}(T_{ma}, T_{m1}) < T_{a_2} < 0$. Since the value of T_{a_2} depends on the value calculated from $T_{12} = \tau_{med}$, the procedure is to first calculate τ_{med}, according to the conditions imposed by x_1, x_2, etc., and then, for these conditions, check that the value of T_{a_2} is acceptable. The equations used are again (A1) to (A7), with $t = T_{a_2}$:

$$x_1 = \begin{cases} -V_{b_1}T_{a_2} , & x_1 \le D_{a_1} \\ D_{a_1}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right) - V_{a_1}T_{a_2} , & x_1 > D_{a_1} \end{cases} \tag{A20}$$

$$x_2 = D_{a_2} \tag{A21}$$



$$\tau_{IN} = \frac{S_{f1}+S_{f2}}{2K_2 V_{b2}} + \frac{m_1}{K_2 V_{b2}}\left(K_1 - \frac{K_2 V_{b2}}{V_{b1}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{o2}}{2V_{a2}} + \frac{m_1}{V_{a2}}\left(K_1 - \frac{V_{a2}}{V_{b1}}\right) - \frac{D_{a2}}{V_{a2}}\left(1 - \frac{V_{a2}}{V_{b2}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{f2}}{2K_2 V_{a2}} + \frac{m_1}{K_2 V_{a2}}\left(K_1 - \frac{K_2 V_{a2}}{V_{b1}}\right) - \frac{D_{a2}}{V_{a2}}\left(1 - \frac{V_{a2}}{V_{b2}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{o2}}{2V_{b2}} + \frac{m_1}{V_{b2}}\left(K_1 - \frac{V_{b2}}{V_{b1}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{f2}}{2K_2 V_{b2}} + \frac{m_1}{V_{b2}}\left(\frac{K_1}{K_2} - \frac{V_{b2}}{V_{a1}}\right) + \frac{D_{a1}}{V_{a1}}\left(1 - \frac{V_{a1}}{V_{b1}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{o2}}{2V_{b2}} + \frac{m_1}{V_{b2}}\left(K_1 - \frac{V_{b2}}{V_{a1}}\right) + \frac{D_{a1}}{V_{a1}}\left(1 - \frac{V_{a1}}{V_{b1}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{f2}}{2K_2 V_{a2}} + \frac{m_1}{V_{a2}}\left(\frac{K_1}{K_2} - \frac{V_{a2}}{V_{a1}}\right) + \frac{D_{a1}}{V_{a1}}\left(1 - \frac{V_{a1}}{V_{b1}}\right) - \frac{D_{a2}}{V_{a2}}\left(1 - \frac{V_{a2}}{V_{b2}}\right)$$

$$\tau_{IN} = \frac{S_{f1}+S_{o2}}{2V_{a2}} + \frac{m_1}{V_{a2}}\left(K_1 - \frac{V_{a2}}{V_{a1}}\right) + \frac{D_{a1}}{V_{a1}}\left(1 - \frac{V_{a1}}{V_{b1}}\right) - \frac{D_{a2}}{V_{a2}}\left(1 - \frac{V_{a2}}{V_{b2}}\right)$$

Figure 20.— Calculation of τ_{in} for $m_2 < m_1$.

$$x_1' = \frac{S_{f_1}}{2} + K_1 x_1 \tag{A22}$$

$$x_2' = \begin{cases} -\dfrac{S_{f_2}}{2} + K_2 x_2 , & x_2 \le m_2 \text{ and } K_2 \ge 0 \\ -\dfrac{S_{o_2}}{2} + x_2 , & x_2 > m_2 \text{ or } K_2 < 0 \end{cases} \tag{A23}$$

$$T_{a_2} = T_{12} - \frac{D_{a_2}}{V_{b_2}} \tag{A24}$$

which leads to four subcases. Further, to test the acceptability of the value of T_{a_2}, one must also consider the equations for T_{m_1} if $m_2 > m_1$, and T_{a_2} if $m_2 \le m_1$. These equations are

$$\left.\begin{array}{l} T_{ma} = \left\{\begin{array}{ll} -\dfrac{m_2}{V_{b_1}} , & m_2 \le D_{a_1} \\ -\dfrac{m_2}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right), & m_2 > D_{a_1} \end{array}\right\} m_2 \le m_1 \\ \\ T_{m_1} = \left\{\begin{array}{ll} -\dfrac{m_1}{V_{b_1}} , & m_1 \le D_{a_1} \\ -\dfrac{m_1}{V_{a_1}} + \dfrac{D_{a_1}}{V_{a_1}}\left(1 - \dfrac{V_{a_1}}{V_{b_1}}\right), & m_1 > D_{a_1} \end{array}\right\} m_2 > m_1 \end{array}\right\} \tag{A25}$$

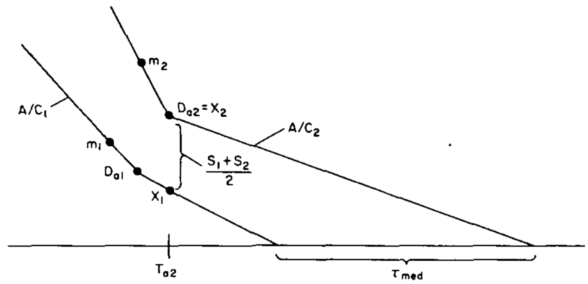
Note that the form of these equations is identical, the only change being in the choice of m_1 or m_2. In the sequel, these equations will be combined into one equation, with $m_{min} = \min(m_1, m_2)$.

Case IIIa: $(x_2 \le m_2$ and $K_2 \ge 0)$, $x_1 \le D_{a_1}$ (sketch (s))

$$\tau_{med} = \frac{S_{f_1} + S_{f_2}}{2K_1 V_{b_1}} - \frac{D_{a_2}}{V_{b_1}}\left(\frac{K_2}{K_1} - \frac{V_{b_1}}{V_{b_2}}\right)$$

$$x_2 \le m_2 \longrightarrow D_{a_2} \le m_2$$

$$x_1 \le D_{a_1} \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 D_{a_1} \ge K_2 D_{a_2}$$



Note: m_1 shown $> D_{a1}$, but maybe $< D_{a1}$

Sketch (s)

To check for applicability, use

$$T_{a_2} < 0 \longrightarrow \frac{S_{f_1} + S_{f_2}}{2} < K_2 D_{a_2}$$

Note that this will not be satisfied unless $K_2 > 0$, so a separate check for this is unnecessary:

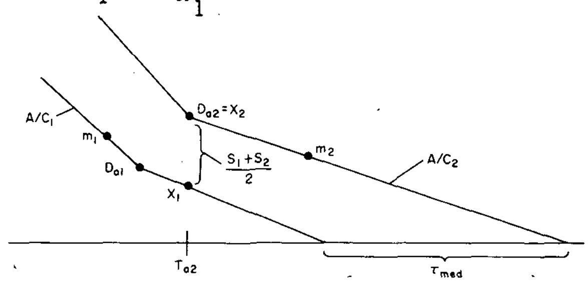
$$\max\left(T_{m_a}, T_{m_1}\right) < T_{a_2} \longrightarrow \begin{cases} \dfrac{S_{f_1} + S_{f_2}}{2} + K_1 m_{min} > K_2 D_{a_2} & , \quad m_{min} \le D_{a_1} \\ \dfrac{S_{f_1} + S_{f_2}}{2V_{b_1}} + \dfrac{K_1 D_{a_1}}{V_{b_1}} - \dfrac{K_2 D_{a_2}}{V_{b_1}} > \dfrac{K_1 D_{a_1}}{V_{a_1}} - \dfrac{K_1 m_{min}}{V_{a_1}} & , \quad m_{min} > D_{a_1} \end{cases}$$

Note that when $m_{min} > D_{a_1}$, the left side of the last part of the equation is positive by the test for $x_1 \le D_{a_1}$, and the right side is negative since $m_{min} > D_{a_1}$. Therefore, the inequality is automatically satisfied if $m_{min} > D_{a_1}$ and need not be checked.

Case IIIb: $(x_2 > m_2$ or $K_2 < 0)$, $x_1 \le D_{a_1}$ (sketch (t))

$$\tau_{med} = \frac{S_{f_1} + S_{o_2}}{2K_1 V_{b_1}} - \frac{D_{a_2}}{V_{b_1}}\left(\frac{1}{K_1} - \frac{V_{b_1}}{V_{b_2}}\right)$$

$$x_2 > m_2 \longrightarrow D_{a_2} > m_2$$



Note: m_1 shown $< D_{a1}$, but maybe $> D_{a1}$

Sketch (t)

$$x_1 \le D_{a_1} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 D_{a_1} \ge D_{a_2}$$

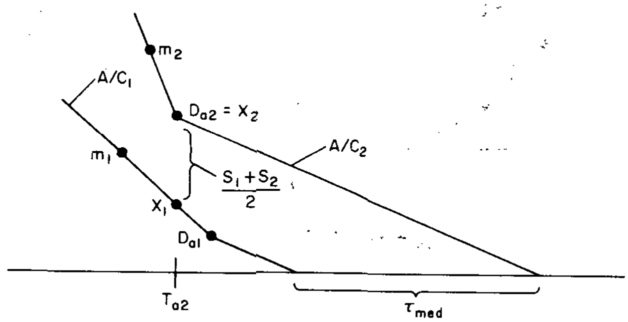
Note that K_2 does not enter these equations so its sign is not important:

$$T_{a_2} < 0 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} < D_{a_2}$$

$$\max\left(T_{m_a}, T_{m_1}\right) < T_{a_2} \rightarrow \begin{cases} \dfrac{S_{f_1} + S_{o_2}}{2} + K_1 m_{\min} > D_{a_2} \,, & m_{\min} < D_{a_1} \\ \\ (\cdot) \,, & m_{\min} > D_{a_1} \end{cases}$$

where the second test is automatically satisfied as in case IIIa.

Case IIIc: $(x_2 \le m_2$ and $K_2 \ge 0)$, $x_1 > D_{a_1}$ (sketch (u))



Sketch (u)

$$\tau_{med} = \frac{S_{f_1} + S_{f_2}}{2K_1 V_{a_1}} + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right) - \frac{D_{a_2}}{V_{a_1}}\left(\frac{K_2}{K_1} - \frac{V_{a_1}}{V_{b_2}}\right)$$

$$x_2 \le m_2 \rightarrow D_{a_2} \le m_2$$

$$x_1 \ge D_{a_1} \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 D_{a_1} < K_2 D_{a_2}$$

This test will also not be satisfied if $K_2 < 0$:

$$T_{a_2} < 0 \rightarrow \frac{S_{f_1} + S_{f_2}}{2} + K_1 D_{a_1}\left(1 - \frac{V_{a_1}}{V_{a_2}}\right) < K_2 D_{a_2}$$

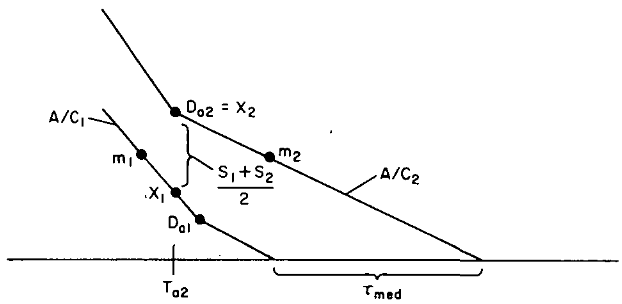
This test is satisfied since the test for $x_1 > D_{a_1}$ is satisfied, and therefore need not be considered:

$$\max\left(T_{m_a}, T_{m_1}\right) < T_{a_2} \rightarrow \begin{cases} (\cdot) & , \quad m_{min} \le D_{a_1} \\ \dfrac{S_{f_1} + S_{f_2}}{2} + K_1 m_{min} > K_2 D_{a_2} & , \quad m_{min} > D_{a_1} \end{cases}$$

Again, as in the previous two cases, the first of these two tests is included in the previous tests, but now the test can never be satisfied; therefore, it need not be tested. Also, if the second test is satisfied, then $m_{min} > D_{a_1}$; if this test is not satisfied, case III is not needed (so tests for $m_{min} > D_{a_1}$ are also not needed).

Case IIId: $(x_2 > m_2$ or $K_2 < 0)$, $x_1 > D_{a_1}$ (sketch (v))

$$\tau_{med} = \frac{S_{f_1} + S_{o_2}}{2K_1 V_{a_1}} + \frac{D_{a_1}}{V_{a_1}}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right) - \frac{D_{a_2}}{V_{a_1}}\left(\frac{1}{K_1} - \frac{V_{a_1}}{V_{b_2}}\right)$$



Sketch (v)

$$x_2 > m_2 \rightarrow D_{a_2} > m_2$$

$$x_1 > D_{a_1} \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 D_{a_1} < D_{a_2}$$

$$T_{a_2} < 0 \rightarrow \frac{S_{f_1} + S_{o_2}}{2} + K_1 D_{a_1}\left(1 - \frac{V_{a_1}}{V_{b_1}}\right) < D_{a_2}$$

This test will always be satisfied since $x_1 > D_{a_1}$ and therefore need not be checked:

$$\max\left(T_{m_a}, T_{m_1}\right) < T_{a_2} \rightarrow \begin{cases} (\cdot) & , \quad m_{min} < D_{a_1} \\ \dfrac{S_{f_1} + S_{o_2}}{2} + K_1 m_{min} > D_{a_2} & , \quad m_{min} > D_{a_1} \end{cases}$$

As in case IIIc, the first part of this test can never be satisfied and so need not be tested. Also, the test for $m_{min} > D_{a1}$ can be omitted. As shown in figure 21, where the tests for $T_{a_2} < 0$ or $\max(T_{ma}, T_{m_1}) < T_{a_2}$ are not satisfied, the value of τ_{med} is set to zero, so that it is ignored in the maximization process for T_{12}.

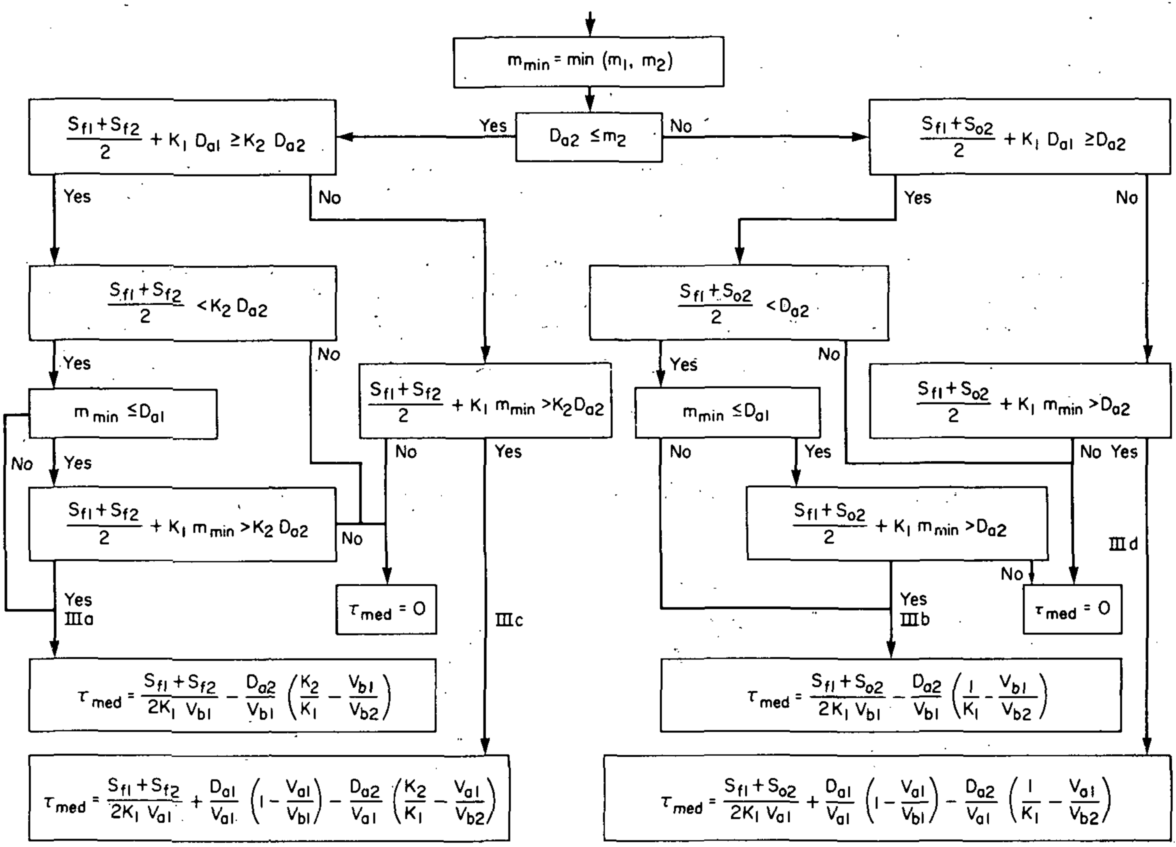


Figure 21.— Calculation of τ_{med}.

If $T_{12} = \max(\tau_{in}, \tau_{fin}, \tau_{med})$, adequate separation will be maintained while the aircraft are airborne, but the second aircraft may still land before the first aircraft is off the runway. To prevent this, an additional quantity, τ_o, is computed which controls the spacing between the aircraft so that the first aircraft will be off the runway before the second lands despite errors in the aircraft's maneuvers.

RUNWAY SEPARATION

The interarrival time required for runway separation is defined by equations (10) and (11). The partial derivatives used in equation (10) are defined as

$$\frac{\partial T_{12_{act}}}{\partial \theta_i} = \frac{\partial T_{nom}}{\partial \theta_i}$$

where T_{nom} is the negative of equation (3) or (A3):

$$T_{nom} = \begin{cases} \dfrac{m}{V_b} & , \quad m \le D_a \\[2ex] \dfrac{m}{V_a} + \dfrac{D_a}{V_a}\left(1 - \dfrac{V_a}{V_b}\right), & m > D_a \end{cases}$$

With this equation, along with equation (1), the various derivatives can be calculated as follows:

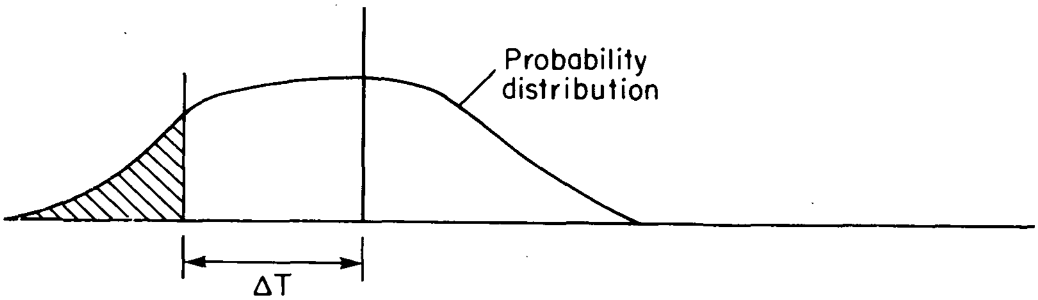
	$m \le D_a$	$m > D_a$
$\dfrac{\partial T_{12_{act}}}{\partial m}$	$\dfrac{1}{V_b}$	$\dfrac{1}{V_a}$
$\dfrac{\partial T_{12_{act}}}{\partial V_a}$	0	$\dfrac{\dfrac{V_a^2-V_b^2}{2a} + T_{vf}V_b - m}{V_a^2}$
$\dfrac{\partial T_{12_{act}}}{\partial V_b}$	$-\dfrac{m}{V_b^2}$	$\dfrac{\dfrac{V_b-V_a}{a} - T_{vf}}{V_a}$
$\dfrac{\partial T_{12_{act}}}{\partial T_{vf}}$	0	$1 - \dfrac{V_b}{V_a}$

All the quantities needed to calculate $T_{12_{nom}}$ in equation (12) are now available.

PROBABILITY OF ABORT

In equation (10), a value for ΔT is determined. This quantity is an allowance to be added to the runway occupancy time to reduce the probability that errors in arrival position and velocity will cause the second aircraft to abort, that is, if the first aircraft is not off the runway when the second aircraft wants to touch down. Thus, it is desirable to determine analytically the probability of abort so that a desirable value for K is obtained.

The probability of abort is a function of four random variables (δV_a, δV_b, δp, and δT_{vf}) for each aircraft (a total of eight variables), each with an assumed triangular distribution. The resultant distribution of deviations of interarrival times (sketch (w)) is essentially Gaussian. Also shown is



Sketch (w)

ΔT, K times the root sum square of half the ranges of the n individual triangular error sources. The probability of an abort is shown by the shaded area. To estimate the probability of abort, first assume that all variables have been normalized and have uniform ranges. Also, note that a triangular distribution function is the convolution of two identical uniform distributions. Therefore, the probability of abort of the n triangular distributions is the same as that of $2n$ uniform distributions. Thus, given the density function for $2n$ uniform identical random variables,

$$f_{x_i}(\alpha) = \begin{cases} 1 \ , & 0 < \alpha < 1 \\ 0 \ , & \text{else} \end{cases} \tag{A26}$$

determine the probability that

$$Z = \sum_{i=1}^{2n} x_i < \bar{x} - T_s$$

where $T_s = \sqrt{n}$. Thus, T_s corresponds to ΔT, with $K = 1$. Using Laplace transforms, the moment generating function ϕ_{x_i} for a single uniform variable is

$$\phi_{x_i} = \phi_x = \int_{-\infty}^{\infty} f_{x_i} e^{-s\alpha} \, d\alpha = \int_0^1 e^{-s\alpha} \, d\alpha = \frac{1 - e^{-s}}{s} \tag{A27}$$

From this, the overall function ϕ_z is

$$\phi_z = \prod_{i=1}^{2n} \phi_{x_i} = \left(\frac{1 - e^{-s}}{s}\right)^{2n}$$

$$= \sum_{k=0}^{2n} \binom{2n}{k} (-1)^k \frac{e^{-ks}}{s^{2n}} \, . \tag{A28}$$

Taking the inverse Laplace transform gives the density function,

$$f_z = \sum_{k=0}^{2n} \binom{2n}{k} (-1)^k \begin{cases} 0 & , \quad t \leq k \\ \dfrac{(t - k)^{2n-1}}{(2n - 1)!} & , \quad t > k \end{cases} \tag{A29}$$

which can be integrated to give the probability distribution function:

$$\left. \begin{aligned} F_z &= \sum_{k=0}^{2n} \binom{2n}{k} (-1)^k \begin{cases} 0 & , \quad t \leq k \\ \dfrac{(t - k)^{2n}}{2n!} & , \quad t > k \end{cases} \\ &= \sum_{k=0}^{2n} \frac{(-1)^k}{k!\,(2n - k)!} \begin{cases} 0 & , \quad t \leq k \\ (t - k)^{2n} & , \quad t > k \end{cases} \end{aligned} \right\} \tag{A30}$$

To determine the probability of abort, evaluate equation (A30) at $t = \bar{x} - T_s$ where $\bar{x} = n$ is defined as the mean of $2n$ identical uniform distributions, that is, the probability that $Z < \bar{x} - T_s$ is $F_z(n - \sqrt{n})$. This evaluation yields

$$
\left.
\begin{aligned}
F_z(n - \sqrt{n}) &= \sum_{k=0}^{2n} \frac{(-1)^k}{k!(2n - k)!} \begin{cases} 0 & , \quad k \geq n - \sqrt{n} \\ (n - \sqrt{n} - k)^{2n} & , \quad k < n - \sqrt{n} \end{cases} \\
&= \sum_{k=0}^{k_{max}} \frac{(-1)^k}{k!(2n - k)!} (n - \sqrt{n} - k)^{2n}
\end{aligned}
\right\} \quad \text{(A31)}
$$

where k_{max} is the largest integer k so that $k < n - \sqrt{n}$ or $k_{max} = int[n - \sqrt{n}]$.

Table 3 gives, for various values of n, the values of k_{max}, $F_z(n - \sqrt{n})$,

TABLE 3.— PROBABILITY OF ABORT FOR VARIOUS n

n	k_{max}	$F_z(n - \sqrt{n})$	Z_g
1	0	0	$-\infty$
2	0	.00491	-2.58
4	1	.00615	-2.50
8	5	.00670	-2.47

and Z_g, the normalized Gaussian variable with the same area under the left-hand tail as $F_z(n - \sqrt{n})$.

Figure 22 is a plot of Z_g versus n. The previous calculations assume that all error sources are identical, that is, each has an identical effect on the landing time whereas, in fact, each has a different effect. To consider nonidentical sources, assume that n - 1 sources had identical effects and that the effect of the nth source was to vary from that of the others to zero. As it varies, the width of the probability curve decreases, and, by definition, the value of ΔT also decreases. The net effect should be that the area to the left of ΔT will change monotonically from the probability of abort due to n sources, to that due to n - 1 sources. From figure 22, note that this probability is very insensitive

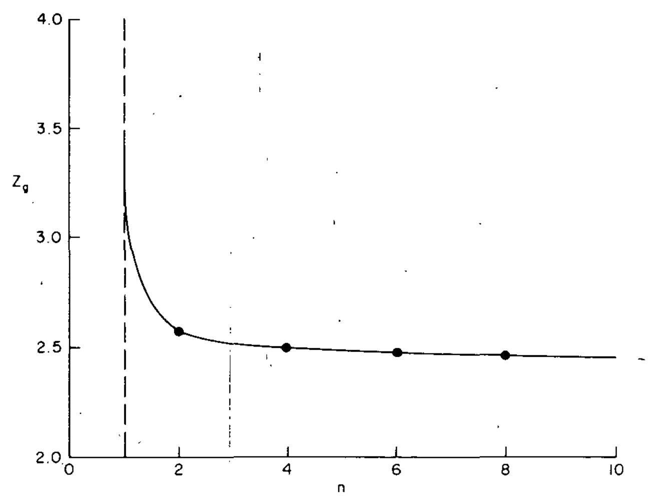


Figure 22.— Effect of number of error sources in probability of abort.

to n and, furthermore, the probability of abort decreases as n decreases. Therefore, the probability that the actual error will exceed the RSS of 1/2 the ranges (which implies $K = 1$) occurs with a probability of less than 0.006, as stated in the text. The Monte Carlo runs showed the probability of aborts from 0 to 0.006, which agrees with the above analysis.

APPENDIX B

PROGRAM LISTING AND DESCRIPTION

A listing of the complete program, consisting of a main program and two subroutines, HEAD1 and OUT, is presented. The main program first calls HEAD1, which reads input data, sets up the calculations, and prints heading information. The main program then does the major portions of the calculations and some output, and finally calls OUT for the remainder of the output. The programs have many comment cards and it is hoped that they are self-explanatory.

The computations for a single case are done from a data array called ACDTA, and the purpose of HEAD1 is to change the data in ACDTA as required. This allows a kind of DO LOOP action, under the control of HEAD1, so that a variety of conditions can be run simply. A listing of a sample input deck is discussed in detail to show how the required inputs are made.

INPUT DECK

The inputs are free form as controlled by the Ames INPUT routine.[1] This routine continues to read data cards and put the data into the named locations until a * is read. At that point, the program proceeds to the next operation, and subsequent data cards are read by a subsequent call to INPUT.

```
TITLE='RUN 109  1 VEL  VARIOUS M   MIXED STOL CTOL  EQUAL SPACING',   01
NAC=300,                                                              02
AC1=19.,6.,6.,60.,60.,60.,90.,2.,2.5,2.5,30.,'60',                    03
AC2=19.,6.,6.,60.,65.,65.,90.,2.,2.5,2.5,30.,'65',                    04
AC3=19.,6.,6.,60.,70.,70.,90.,2.,2.5,2.5,30.,'70',                    05
AC7=2.,2.,2.,30.,30.,30.,60.,2.,2.5,2.5,10.,'30',                     06
AC8=2.,2.,2.,30.,35.,35.,60.,2.,2.5,2.5,10.,'35',                     07
AC9=2.,2.,2.,30.,40.,40.,60.,2.,2.5,2.5,10.,'40',                     08
ACPROB='30',40.,'35',30.,'40',19.,                                    09
CHNGAC='50','65','70',                                                10
TVF=900.,                                                             11
SF=.1,                                                                12
NORUN=1   ,                                                           13
*                                                                     14
CHNGAC='30','35','40',                                                15
TVF=900.,                                                             16
SF=.1,                                                                17
M=2.,7.,                                                              18
*                                                                     19
ACPROB='30',120.,'35',90.,'40',57.,'60', 40.,'65',30.,'70',19.,       20
*                                                                     21
CHNGAC='60','65','70',                                                22
M=7.,                                                                 23
NORUN=1   ,                                                           24
*                                                                     25
CHNGAC='30','35','40',                                                26
M=2.,7.,                                                              27
*                                                                     28
ACPROB='30', 40.,'35',30.,'40',19.,'60', 40.,'65',30.,'70',19.,       29
*                                                                     30
CHNGAC='60','65','70',                                                31
M=19.,                                                                32
NORUN=1   ,                                                           33
*                                                                     34
CHNGAC='30','35','40',                                                35
M=2.,7.,                                                              36
*                                                                     37
```

Figure 23.— Input deck.

Figure 23 shows the sample input deck, where card 1 is a title card and card 2 defines the number of landings to be used in the statistical portion of the program. Cards 3 through 8 give information concerning the aircraft, one card for each aircraft type. Up to 12 aircraft types can be defined and loaded into AC1 through AC12, which are rows of the ACDTA table. The variables for each aircraft type are m, S_o, S_f, T_o, V_a, V_b, T_{vf}, $R\delta p$, $R\delta V_a$, $R\delta V_b$, $R\delta T_{vf}$, and type (in units of km, sec, and m/sec). Card 9 defines the proportion of the various aircraft types to be considered, and this proportion is used until a new proportion card is entered. In this case, only the three STOL vehicles are to be used.

---

[1]This allows free-form input similar to NAMELIST, and a description of the subroutine can be obtained from the Ames Computer Library.

To reduce the number of input cards required to run multiple cases, the input procedure was arranged to set up a series of DO LOOPS. With this procedure, it is necessary to first indicate which of the aircraft types is to have variables changed by the DO LOOPS and then to indicate what variables are to be changed, and their new values. If more than one value is specified for a variable, the program will compute for all values specified, as in a DO LOOP. If more than one variable is to be changed, nested DO LOOPS are created. These DO LOOPS remain in force until a new indication of aircraft types is specified.

The particular set of input data shown in figure 23 computes landing rates for the following cases:

STOL only	$m_{STOL} = 2, 7$	
75 percent STOL	$m_{STOL} = 2, 7$	$m_{CTOL} = 19$
75 percent STOL	$m_{STOL} = 2, 7$	$m_{CTOL} = 7$
50 percent STOL	$m_{STOL} = 2, 7$	$m_{CTOL} = 7$
50 percent STOL	$m_{STOL} = 2, 7$	$m_{CTOL} = 19$

The remainder of the input cards provides the information to run these cases.

Card 10 indicates that, for the first case, some data for the three aircraft types indicated (CTOL) are to be changed and cards 11 and 12 indicate the required changes to T_{vf} and S_f. Card 13 indicates that no calculations are to be made at this point (since more changes will come in the next call to INPUT). Cards 15, 16, and 17 make the same changes for the STOL vehicles, while card 18 indicates that computations should be done for $m_{STOL} = 2$ and $m_{STOL} = 7$ and has thus set up a one-level DO LOOP that will be gone through twice. Up to four values of m could have been provided, and additional values for T_{vf} and S_f could also have been provided. If two values had been specified for both T_{vf} and S_f, then there would have been a three level DO LOOP, with two values for each level, resulting in eight sets of computations. Quantities that can be changed in this fashion and used to control DO LOOPS are m, S_o, S_f, T_o, V_a, V_b, T_{vf}, $R\delta p$, $R\delta V_a$, $R\delta V_b$, and $R\delta T_{vf}$. If all these variables are used, an eleven-level DO LOOP will exist, with up to four values for each level, resulting in 10^7 cases. Obviously, care must be exercised here.

Card 19 again indicates the end of a data read, and only two cases (STOL only, $m_{STOL} = 2, 7$) will be computed. The output from these two cases is shown in figures 24(a) and 24(b). Card 20 presents a new set of proportions of aircraft types and, since there is no reference to CHNGAC, the previously set-up DO LOOP will be used for the next cases (75 percent STOL, $m_{STOL} = 2, 7$, $m_{CTOL} = 19$). The output from the first of these cases is shown in figure 24(c).

Cards 22 and 23 indicate a change to the CTOL aircraft data, to set $m_{CTOL} = 7$, and cancel the initial DO LOOP setup. Card 24 indicates no computations. Cards 26 and 27 change variables for the STOL aircraft, and again set up the one-level, two-value, DO LOOP (75 percent STOL, m_{STOL} = 2, 7, m_{CTOL} = 7). Card 29 specifies 50 percent STOL, running the same DO LOOP (50 percent STOL, m_{STOL} = 2, 7, m_{CTOL} = 7). On card 37, the last case has been set up to run (50 percent STOL, m_{STOL} = 2, 7, m_{CTOL} = 19). The program will continue to read data cards until they are gone, and then it will stop. This particular input deck will compute data for 10 cases (as previously stated).

Two additional subroutines are called by the program. The first, RANDU(IX,NX,R), is a random number generator and (as used here) will return a random number, R, with a uniform probability distribution from 0 to 1. The variable IX is a starting point and NX is the starting point for the next random number. The second routine, VTITLE, prints a title line and page number at the top of each page of output. The title is read from the first data card.

DESCRIPTION OF OUTPUT

The first three pages of output from the sample input deck are shown in figures 24(a) through (c). The output includes not only the information discussed in the text, but also some material which is not discussed. This description defines all of the data in the output.

In figures 24(a)-(c), the first line is the case title and the second two lines are general heading information. This is followed by the "A/C DATA TABLE," which gives all the input quantities for each type of aircraft used for this particular case. In addition, the individual partial derivatives required in equation (10), along with the root sum square, are given in the last five columns.

The next two lines define the probability that the next aircraft is of a specified type. This is followed by a table defining the interarrival times and available takeoff times for all possible sequences of aircraft. The first line defines the type of the second aircraft, and the first column defines the type of the first aircraft. For each possible sequence, four entries are made in the table. The first, PROB, is the probability of that sequence, obtained by multiplying the probability of the individual types. The second item, T12, is the required interarrival time, as calculated by equation (12). Immediately following this is a letter, which defines the τ with the largest value, using the code indicated. The last item is the time available for takeoffs, which is the interarrival time minus the runway occupancy time of the first aircraft.

Some histogram information is presented next. The total number of aircraftin the histogram is specified. The histogram shows the distribution of interarrival time and available takeoff times, as calculated from the information immediately above. Also, average, maximum, and minimum times are shown.

```
RUN 109  1 VEL  VARIOUS M   MIXED STOL CTOL   EQUAL SPACING                           10/25/72  PAGE   1

  3 A/C TYPES USED       300 A/C TO LAND     DECELERATION=0.60M/SEC2     RANDOM START=   0
                 APPROX CONVERSION FACTORS            KM/2=MILES          M/S*2=KNOTS

                               A/C DATA TABLE
COMMON  MIN SEPARATION  RUNWAY  APPROACH VEL.  VEL CH      RANGE OF UNCERTAINTIES      A/C      2.4 S.D. OF TIME ERROR
 DIST.    O.M.     TD.   TIME   INIT.   FINAL   TIME   POSIT   IN VEL  FIN V  TIME-VB  A/C   TOTAL   ----PARTIALS DUE TO----
 M  KM   SO KM   SF KM  TO SEC  VA M/S VB M/S  TVF S   RDP KM  RDVA    RDVB   RDTVF         TDL SEC   DP     DVA   DVB   DTVF
 19.0    6.0      6.0   60.0   60.0    60.0   900.0    2.00    2.50    2.50    30.0   60    17.92   16.67  0.00  6.60  0.00
 19.0    6.0      6.0   60.0   65.0    65.0   900.0    2.00    2.50    2.50    30.0   65    16.38   15.38  0.00  5.62  0.00
 19.0    6.0      6.0   60.0   70.0    70.0   900.0    2.00    2.50    2.50    30.0   70    15.09   14.29  0.00  4.85  0.00
  2.0    6.0      6.0   30.0   30.0    30.0   900.0    2.00    2.50    2.50    10.0   30    33.45   33.33  0.00  2.78  0.00
  2.0    6.0      6.0   30.0   35.0    35.0   900.0    2.00    2.50    2.50    10.0   35    28.64   28.57  0.00  2.04  0.00
  2.0    6.0      6.0   30.0   40.0    40.0   900.0    2.00    2.50    2.50    10.0   40    25.05   25.00  0.00  1.56  0.00

A/C TYPE    30        35       40
PROBABILITY 0.449    0.337    0.213

A/C TYPE2   30              35                40
 A/C1    PROB  T12  TTO
 30      .202 200.F170.   .151 171.F141.   .096 150.F120.
 35      .151 210.I180.   .114 171.I141.   .072 150.F120.
 40      .096 217.I187.   .072 179.I149.   .046 150.I120.
NOTE--CRITICAL TIMES ARE  (I=TIN OR X1=MIN(M1,M2)), (M=TMED OR X2=DA2), (F=TFIN OR X1=0) , (L=TDEL OR RUNWAY CLEAR)

AFTER  302LANDINGS       AVG    MAX    MIN                          HISTOGRAM(DETERMINISTIC)
   NOMINAL                                     TIME= 0            60              120              180              240              288
LANDING INTERVALS,SEC   183.   217.   150.          0  0  0  0  0  0  0  0  0  0  0  0 65  0101  0 61 45 29  0  0  0  0  0  0
TIME AVAILABLE,T.O.     153.   187.   120.          0  0  0  0  0  0  0  0  0  0 65 79 22  0106 29  0  0  0  0  0  0  0  0  0

NO. OF ABORTS      ABORT RATIO            NO. OF NOM. LNDNGS  NOM. LNDNG RATE(/HR)       NO. OF ACT. LNDNGS    ACT.LNDNG RATE(/HR)
     0                0.000                      302                 19.6                      302                    19.6

TIME REQD FOR TO  NO. OF TAKEOFFS         NO. OF NOM. OPERNS  NOM. OPERN RATE(/HR)       NO.. OF ACT. OPERNS   ACT.OPERN RATE(/HR)
           36.          1089                    1391                 90.7                     1391                    90.7
           48.           759                    1061                 69.2                     1061                    69.2
           60.           631                     933                 60.8                      933                    60.8
           72.           458                     760                 49.5                      760                    49.5
           84.           436                     738                 48.1                      738                    48.0
           96.           301                     603                 39.3                      603                    39.3
          108.           301                     603                 39.3                      603                    39.3
          120.           301                     603                 39.3                      603                    39.3
```

Figure 24(a) - Sample output.

```
RUN 109  1 VEL  VARIOUS M  MIXED STOL CTOL   EQUAL SPACING                              10/25/72  PAGE    2

   3 A/C TYPES USED        300 A/C TO LAND     DECELERATION=0.60M/SEC2      RANDOM START=   0
                   APPROX CONVERSION FACTORS            KM/2=MILES          M/S*2=KNOTS

                                A/C DATA TABLE
COMMON  MIN SEPARATION  RUNWAY  APPROACH VEL.  VEL CH       RANGE OF UNCERTAINTIES        A/C       2.4 S.D. OF TIME ERROR
 DIST.     O.M.     TD.   TIME    INIT.   FINAL   TIME   POSIT   IN VEL  FIN V  TIME-VB  A/C    TOTAL   ----PARTIALS DUE TO----
 M  KM  SO KM     SF KM  TO SEC  VA M/S VB M/S  TVF S   RDP KM   RDVA    RDVB   RDTVF          TDL SEC   DP     DVA   DVB   DTVF
 19.0    6.0       6.0    60.0   60.0    60.0   900.0    2.00    2.50    2.50    30.0    60    17.92   16.67  0.00  6.60  0.00
 19.0    6.0       6.0    60.0   65.0    65.0   900.0    2.00    2.50    2.50    30.0    65    16.38   15.38  0.00  5.62  0.00
 19.0    6.0       6.0    60.0   70.0    70.0   900.0    2.00    2.50    2.50    30.0    70    15.09   14.29  0.00  4.85  0.00
  7.0    6.0       6.0    30.0   30.0    30.0   900.0    2.00    2.50    2.50    10.0    30    34.72   33.33  0.00  9.72  0.00
  7.0    6.0       6.0    30.0   35.0    35.0   900.0    2.00    2.50    2.50    10.0    35    29.45   28.57  0.00  7.14  0.00
  7.0    6.0       6.0    30.0   40.0    40.0   900.0    2.00    2.50    2.50    10.0    40    25.59   25.00  0.00  5.47  0.00

A/C TYPE    30       35       40
PROBABILITY  0.449    0.337   0.213

A/C TYPE2   30                 35                  40
 A/C1   PROB   T12  TTO
 30     .202 200.F170.     .151 171.F141.     .096 150.F120.
 35     .151 233.I203.     .114 171.F141.     .072 150.F120.
 40     .096 258.I228.     .072 196.I166.     .046 150.I120.
NOTE--CRITICAL TIMES ARE   (I=TIN OR X1=MIN(M1,M2)), (M=TMED OR X2=DA2), (F=TFIN OR X1=0), (L=TDEL OR RUNWAY CLEAR)

AFTER  302LANDINGS        AVG     MAX     MIN                            HISTOGRAM(DETERMINISTIC)
    NOMINAL                                            TIME= 0             60              120              180              240           288
LANDING INTERVALS,SEC    192.    258.    150.              0  0  0  0  0  0  0  0  0  0  0  0 65  0 79  0 83  0  0 45  0 29  0  0  0
TIME AVAILABLE,T.O.      162.    228.    120.              0  0  0  0  0  0  0  0  0  0 65 79  0 22 61  0 45  0  0 29  0  0  0  0  0

NO. OF ABORTS       ABORT RATIO         NO. OF NOM. LNDNGS  NOM. LNDNG RATE(/HR)       NO. OF ACT. LNDNGS     ACT.LNDNG RATE(/HR)
      0                0.000                   302                 18.7                       302                  18.7

TIME REQD FOR TO  NO. OF TAKEOFFS       NO. OF NOM. OPERNS  NOM. OPERN RATE(/HR)       NO. OF ACT. OPERNS     ACT.OPERN RATE(/HR)
           36.          1163                  1465                 91.1                      1465                  91.1
           48.           833                  1135                 70.6                      1135                  70.6
           60.           676                   978                 60.8                       978                  60.8
           72.           487                   789                 49.1                       789                  49.1
           84.           436                   738                 45.9                       738                  45.9
           96.           375                   677                 42.1                       677                  42.1
          108.           330                   632                 39.3                       632                  39.3
          120.           301                   603                 37.5                       603                  37.5
     ACPROB='30',120.,'35',90.,'40',57.,'60', 40.,'65',30.,'70',19.,
     *
```

Figure 24(b).- Sample output - continued.

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RUN 109  1 VEL  VARIOUS M  MIXED STOL CTOL    EQUAL SPACING                                10/25/72  PAGE    3

   6 A/C TYPES USED        300 A/C TO LAND     DECELERATION=0.60M/SEC2      RANDOM START=   0
                   APPROX CONVERSION FACTORS          KM/2=MILES             M/S*2=KNOTS

                                 A/C DATA TABLE
COMMON  MIN SEPARATION  RUNWAY  APPROACH VEL.  VEL CH      RANGE OF UNCERTAINTIES       A/C      2.4 S.D. OF TIME ERROR
 DIST.     O.M.     TD.  TIME   INIT.   FINAL   TIME   POSIT   IN VEL  FIN V  TIME-VB  A/C   TOTAL   ----PARTIALS DUE TO----
 M KM   SO KM    SF KM  TO SEC  VA M/S  VB M/S  TVF S  RDP KM   RDVA   RDVB   RDTVF          TDL SEC    DP    DVA    DVB   DTVF
 19.0    6.0      6.0   60.0    60.0    60.0   900.0   2.00    2.50   2.50    30.0    60   17.92   16.67  0.00  6.60  0.00
 19.0    6.0      6.0   60.0    65.0    65.0   900.0   2.00    2.50   2.50    30.0    65   16.38   15.38  0.00  5.62  0.00
 19.0    6.0      6.0   60.0    70.0    70.0   900.0   2.00    2.50   2.50    30.0    70   15.09   14.29  0.00  4.85  0.00
  2.0    6.0      6.0   30.0    30.0    30.0   900.0   2.00    2.50   2.50    10.0    30   33.45   33.33  0.00  2.78  0.00
  2.0    6.0      6.0   30.0    35.0    35.0   900.0   2.00    2.50   2.50    10.0    35   28.64   28.57  0.00  2.04  0.00
  2.0    6.0      6.0   30.0    40.0    40.0   900.0   2.00    2.50   2.50    10.0    40   25.05   25.00  0.00  1.56  0.00

A/C TYPE     30       35      40      60      65      70
PROBABILITY  0.337   0.253   0.160   0.112   0.084   0.053

A/C TYPE2    30              35              40              60              65              70
 A/C1   PROB  T12  TTO
 30     .114 200.F170.   .085 171.F141.   .054 150.F120.   .038 100.F 70.   .028  92.F 62.   .018  86.F 56.
 35     .085 210.I180.   .064 171.I141.   .040 150.F120.   .028 100.F 70.   .021  92.F 62.   .013  86.F 56.
 40     .054 217.I187.   .040 179.I149.   .026 150.I120.   .018 100.F 70.   .013  92.F 62.   .009  86.F 56.
 60     .038 233.I173.   .028 195.I135.   .018 167.I107.   .013 100.I 40.   .009  92.F 32.   .006  86.F 26.
 65     .028 236.I176.   .021 198.I138.   .013 169.I109.   .009 124.I 64.   .007  92.F 32.   .004  86.F 26.
 70     .018 238.I178.   .013 200.I140.   .009 171.I111.   .006 145.I 85.   .004 113.I 53.   .003  86.I 26.
NOTE--CRITICAL TIMES ARE  (I=TIN OR X1=MIN(M1,M2)), (M=TMED OR X2=DA2), (F=TFIN OR X1=0), (L=TDEL OR RUNWAY CLEAR)

AFTER  301LANDINGS        AVG     MAX     MIN                         HISTOGRAM(DETERMINISTIC)
   NOMINAL                                      TIME= 0             60            120            180            240            288
LANDING INTERVALS,SEC    166.    238.     86.        0  0  0  0  0  0  0 40 29  1  3  0 38  5 64  0 53 26 16 25  0  0  0  0  0
TIME AVAILABLE,T.O.      129.    187.     26.        0  0  9  4 13 47  0  2  5  7 36 64 12  0 85 16  0  0  0  0  0  0  0  0  0

NO. OF ABORTS     ABORT RATIO          NO. OF NOM. LNDNGS  NOM. LNDNG RATE(/HR)      NO. OF ACT. LNDNGS  ACT.LNDNG RATE(/HR)
      0              0.000                    301                 21.6                      301               21.6

TIME REQD FOR TO  NO. OF TAKEOFFS      NO. OF NOM. OPERNS  NOM. OPERN RATE(/HR)      NO. OF ACT. OPERNS  ACT.OPERN RATE(/HR)
          36.           867                  1168                 84.2                      1168              84.2
          48.           625                   926                 66.8                       926              66.8
          60.           503                   804                 57.9                       804              57.9
          72.           340                   641                 46.2                       641              46.2
          84.           328                   629                 45.3                       629              45.3
          96.           225                   526                 37.9                       526              37.9
         108.           220                   521                 37.5                       521              37.5
         120.           213                   514                 37.0                       514              37.0
```

Figure 24(c) - Sample output - concluded.

Next, data on the landing rate are presented. The Monte Carlo process was used to determine how many of the landings were required to abort because an aircraft was attempting to touch down before the runway was clear. The abort ratio (number of aborts/total number of landings) is printed, along with the nominal (or planned) number of landings and landing rate. The actual landings referred to in the last two columns are the nominal landings minus the aborts.

Finally, takeoff information is presented. The first line of this table (fig. 24(a)) is read as follows: if all aircraft use 36 sec for takeoff, then there will be sufficient time for 1089 aircraft to take off without interfering with the landing vehicles. Since there are 302 landings being considered (given on the previous line), there will be a total of 1391 operations, at a rate of 90.7 operations/hr. The last two columns consider the effect of aborts on the number of operations and the operation rate. Since there are no aborts (see previous line), the actual and nominal operations are the same.

```
C CALCULATE LANDING CAPACATIES FOR A SINGLE RUNWAY, MONTE CARLO J.S.WHITE
C    REVISED 4/25/72  SO THAT INITIAL TEST OCCURS AT X1=MIN(M1,M2)
      DIMENSION ACDTA(20,12),ITYPE(7)
      DIMENSION ISTTL(25),ISTTO(25),CP(8),T12M(12,12),TTOM(12,12),PROBM
     1    (12,12),ARROUT(28) ,ACPROB(2,7),CRIT(12,12),TT(4),ZZL(4)
      DATA  SIG/1.0/
      EQUIVALENCE  (TIN,TT(1)),(TMED,TT(2)),(TFIN,TT(3)),(TDFL,TT(4))
      DATA  ACDTA/240*0./,ZZL/'I   ','M   ','F   ','L   '/
      DATA  NPRNT,MXTYP/0,0/
      IX = 65539
    5 CALL HEAD1(ACDTA,CP,ITYPE,MXTYP,NAC,NRAN,SIG,ACC2,ACPROB,NSTOP)
      IF  (NSTOP)  5,6,5
C
C     INITIALIZE
C
    6 DO 9 I=1,25
      ISTTL(I)=0.
    9 ISTTO(I)=0.
      DO 8 I=1,144
    8 PROBM(I,1)='.
      TBR = 0.
      TOBR= 0.
      TOMX = 0.
      IABRT=0
      SUMT=0.
      LAC=1
      TMIN = 1.E6
      TMAX = 0.
      TOMN=1.E6
      IF (NRAN.EQ.0)  GO TO 11
      IX = 65539
C     INITIALIZE RANDOM NO GEN.
      DO 10 I = 1,NRAN
      CALL RANDU(IX,NTYP,R)
      IX = NTYP
   10 CONTINUE
C
C     CALCULATE  MATRICES OF  T12, TTO, AND PROB
C
   11 DO 80 IROW=1,MXTYP
      DO 80 ICOL=1,MXTYP
      JR1=ITYPE(IROW)
      JC2=ITYPE(ICOL)
C     TRANSFER TO LOCAL VARIABLES
      ZM2   =ACDTA(1 ,JC2)
```

```
      ZK2    =ACDTA(14,JC2)
      SF2O2  =ACDTA(15,JC2)
      DA2    =ACDTA(16,JC2)
      VA2    =ACDTA(5 ,JC2)
      DAVA2  =ACDTA(18,JC2)
      VB2    =ACDTA(6 ,JC2)
      SO2O2  =ACDTA(17,JC2)
      TDL2   =ACDTA(20,JC2)
      ZM1    =ACDTA(1 ,JR1)
      ZK1    =ACDTA(13,JR1)
      SF1O2  =ACDTA(15,JR1)
      DA1    =ACDTA(16,JR1)
      VA1    =ACDTA(5 ,JR1)
      DAVA1  =ACDTA(18,JR1)
      VB1    =ACDTA(6 ,JR1)
      TDL1   =ACDTA(20,JR1)
      TO1    =ACDTA(4 ,JR1)
C
C     COMPUTE T12    START WITH TFIN
      SF12=SF1O2+SF2O2
      IF  (SF12.LT.ZM2*ZK2) GO TO 22
      SS=SF1O2+SO2O2
      GO TO 25
   22 SS=SF12/ZK2
   25 IF (SS.LT.DA2) GO TO 27
      VALP2=VA2
      TFNDA2=DAVA2
      GO TO 30
   27 VALP2=VB2
      TFNDA2=0.
   30 TFIN=SS/VALP2-TFNDA2
C
C     END OF  TFIN COMPUTATION.
C     CHECK ON M2 VS M1 FOR TIN AND TMED
      IF (ZM2.LT.ZM1) GO TO 40
C     CASE III FOR TIN
      ZMDM=ZM2
      IF  (ZM1.LT.DA1) GO TO 32
      VBET1=VA1
      TINDA1=DAVA1
      GO TO 35
   32 VBET1=VB1
      TINDA1=0.
   35 IF (SF12+ZK1*ZM1.LT.ZK2*ZM2) GO TO 37
      SSI=SF1O2+SO2O2
```

```
      ZMK1=ZK1*ZM1
      GO TO 50
   37 SSI=SF12/ZK2
      ZMK1=ZK1*ZM1/ZK2
      GO TO 50
C
C     START OF CASE 2 FOR TIN
   40 ZMDM=ZM1
      IF(ZM2.LT.DA1) GO TO 42
      VBET1=VA1
      TINDA1=DAVA1
      GO TO 45
   42 VBET1=VB1
      TINDA1=0.
   45 SSI=SF102+SO202
      ZMK1=ZK1*ZM2
C
C     CONTINUE BOTH CASE 2 AND 3
   50 IF (SSI+ZMK1.LT.DA2) GO TO 52
      VBET2=VA2
      TINDA2=DAVA2
      GO TO 55
   52 VBET2=VB2
      TINDA2=0.
   55 TIN=(SSI+ZMK1)/VBET2-ZMK1/(VBET1*ZK1)+ TINDA1-TINDA2
C     END OF TIN COMPUTATION
C     START  CASE IV  TMED
      IF (ZM2.GT.DA2)  GO TO 57
      SSM=(SF102+SO202)/ZK1
      ZDK2=DA2/ZK1
      GO TO 60
   57 SSM=SF12/ZK1
      ZDK2=DA2*ZK2/ZK1
   60  IF (SSM+DA1.GT.ZDK2)  GO TO 62
      IF (SSM+ZMDM.LT.ZDK2) GO TO 69
      TMDA1=DAVA1
      VGAM1=VA1
      GO TO 68
   62 IF (SSM.GT.ZDK2)  GO TO 69
      IF (ZMDM.GT.DA1)  GO TO 64
      IF (SSM+ZMDM.LT.ZDK2)GO TO 69
      GO TO 66
   64 IF (ZM2.LT.ZM1) GO TO 66
      IF (SSM/VB1+ZM2/VA1.LT.ZDK2/VB1+DAVA1) GO TO 69
   66 TMDA1=0.
```

```
      VGAM1=VB1
   68 TMED=SSM/VGAM1+DA2/VB2-ZDK2/VGAM1+TMDA1
      GO TO 70
   69 TMED=0.
C
C     ALL TESTS FOR T12 COMPLETED
C     COMPUTE SPACING FOR RUNWAY CLEARANCE
   70 TDEL=T01+SQRT (TDL1+TDL2)
C     COMPUTE MAX OF THE VARIOUS T'S
      JK=1
      DO 72 JJ=2,4
C     TT CONTAINS TIN,TMED,TFIN,TDEL
      IF   (TT(JJ).GT.TT(JK))   JK=JJ
   72 CONTINUE
C     LOAD  MATRICES
      T12=TT(JK)
      T12M(JC2,JR1)=T12
C     CRIT DEFINES WHICH IS THE MAX T
      CRIT(JC2,JR1)=ZZL(JK)
      TTOM(JC2,JR1)=T12-T01
   80 PROBM(JC2,JR1)=ACPROB(2,ICOL)*ACPROB(2,IROW)
C
C     END OF  MATRIX CALCULATIONS
C
C     COMPUTE HISTOGRAMS
C
      DO 100 IROW =1,MXTYP
      DO 100 ICOL =1,MXTYP
      JR1 = ITYPE(IROW)
      JC2 = ITYPE(ICOL)
      T12 = T12M(JC2,JR1)
      TTO = TTOM(JC2,JR1)
      PR = PROBM(JC2,JR1)
      IF(T12.GT.TMAX) TMAX = T12
      IF(T12.LT.TMIN) TMIN = T12
      IF(TTO.GT.TOMX) TOMX = TTO
      IF(TTO.LT.TOMN) TOMN = TTO
      ZNAC=NAC
C
C     COMPUTE MEAN VALUES OF T12 AND TTO
      TBR=TBR+T12*PR
      TOBR=TOBR+TTO*PR
C
C     COUNT INTO LANDING HISTOGRAM
      N = T12/12. +1.
```

```
      IF(N.GT.25) N=25
      KAC=PR*ZNAC+.5
      ISTTL(N) = ISTTL(N)+KAC
C
C     COUNT INTO TAKEOFF HISTOGRAM
      N = TTO /12.0+1.
      IF(N.GT.25) N = 25
      ISTTO(N) = ISTTO(N) +KAC
C
C     LAC=TOTAL NO OF A/C IN HISTOGRAM
  100 LAC=LAC+KAC
      ZLAC=LAC
C
C     SUMT= TOTAL TIME FOR ALL A/C TO LAND
      SUMT=TBR*(ZLAC-1.)
C
C     PRINT TABLES OF T12,TTO, AND PROB
C
      DO 110 I=1,MXTYP
      JR1=ITYPE(I)
  110 ARROUT(I)=ACDTA(12,JR1)
      WRITE (6,1110)(ARROUT(I),I=1,MXTYP)
 1110 FORMAT  (1HO,'A/C_TYPE2',3X,A4,6(13X,A4))
      WRITE (6,1111)
 1111 FORMAT          (2X,'A/C1',3X,'PROB',2X,'T12',2X,'TTO')
      MX4=MXTYP*4
      DO 130  I=1,MXTYP
      JR1=ITYPE(I)
      DO 120  J=1,MXTYP
      J3=4*J
      JC2=ITYPE(J)
C
C     LOAD ARROUT WITH NONZERO VALUES FROM THE MATRICES
      ARROUT(J3-3)=PROBM(JC2,JR1)
      ARROUT(J3-2)='12M (JC2,JR1)
      ARROUT(J3-1)=CRIT(JC2,JR1)
  120 ARROUT(J3)   ='TOM (JC2,JR1)
  130 WRITE (6,1130)  ACDTA(12,JR1),(ARROUT(J),J=1,MX4)
 1130 FORMAT  (2X,A4,7(3X,F4.3,F5.0,A1,F4.0))
      WRITE (6,1140)
 1140 FORMAT (' NOTE--CRITICAL TIMES ARE  (I=TIN OR X1=MIN(M1,M2)), ',
     X     '(M=TMED OR X2',
     1'=DA2), (F=TFIN OR X1=0), (L=TDEL OR RUNWAY CLEAR)')
C
C     START OF STATISTICS LOOP
```

```
C
      DO 210 IAC = 1,LAC
C     DETERMINE TYPE A/C
      CALL RANDU(IX,NTYP,R)
      IX = NTYP
      DO 150 I =1,MXTYP
      IF(R.LT.CP(I   ))GO TO 160
  150 CONTINUE
      WRITE(6,1025) R,CP
 1025 FORMAT(1X,'A/C TYPE NOT FOUND EXIT CALLED R='F6.4,2X,'CP=',10F6.4)
      CALL EXIT
C
C     GENERATE RANDOM VARIABLES AND LOCAL VARIABLES
  160 JTYPA = ITYPE(I)
      CALL RANDU(IX,NTYP,R1)
      IX = NTYP
      CALL RANDU(IX,NTYP,R2)
      IX = NTYP
      R1=(R1+R2)/2.0
      DELP2 = (R1-.5) * ACDTA(8,JTYPA)
      CALL RANDU(IX,NTYP,R1)
      IX=NTYP
      CALL RANDU(IX,NTYP,R2)
      IX=NTYP
      R1 = (R1+R2)/2.0
      DELVA2 = (R1-.5) * ACDTA(9,JTYPA)
      CALL RANDU(IX,NTYP,R1)
      IX=NTYP
      CALL RANDU(IX,NTYP,R2)
      IX=NTYP
      R1=(R1+R2)/2.0
      DELVB2=(R1 -.5)* ACDTA(10,JTYPA)
      CALL RANDU(IX,NTYP,R1)
      IX=NTYP
      CALL RANDU(IX,NTYP,R2)
      IX=NTYP
      R1=(R1+R2)/2.0
      DELTVF=(R1 -.5)* ACDTA(11,JTYPA)
      JC2=JTYPA
      VA2 = ACDTA( 5,JTYPA)
      VB2 = ACDTA( 6,JTYPA)
      DA2 = ACDTA(16,JTYPA)
      ZM2 = ACDTA( 1,JTYPA)
      TVF2= ACDTA( 7,JTYPA)
      TO2 = ACDTA( 4,JTYPA)
```

```
  170 IF (ZM2.GT.DA2) GO TO 180
      T2ACT=(ZM2+DELP2)/(VB2+DELVB2)
      GO TO 190
C     DAPDDA=DA + DELTA(DA)
  180 DAPDDA=(TVF2+DELTVF)*(VB2+DELVB2)+(VA2+DELVA2-VB2-DELVB2)/ACC2*
     1          (VB2+DELVB2)
      T2ACT=(ZM2+DELP2-DAPDDA)/(VA2+DELVA2)+DAPDDA/(VB2+DELVB2)
  190 DELT2=ACDTA(19,JTYPA)-T2ACT
      IF (IAC.EQ.1)  GO TO 200
      T12A=T12M(JC2,JR1)-DELT1+DELT2
C
C     COUNT NO OF LANDINGS ABORTED
      IF (T12A.LT.T01)    IABRT=IABRT+1
  200 DELT1=DELT2
      JR1=JC2
      T01=T02
  210 CONTINUE
C
C     PRINT REST OF RESULTS
      CALL OUT(TBR,TMAX,TMIN,ISTTL,TOBR,TOMX ,TOMN,ISTTO,LAC,
     X IABRT,SUMT)
      GO TO 5
      END
```

```
C SUBROUTINE HEAD1  READS AND PRINTS CASE INFORMATION
      SUBROUTINE HEAD1(ACDTA,CP,ITYPE,MXTYP,NAC,NRAN,SIG,ACC2,ACPROB,
     1     NSTOP)
      DIMENSION   ACDTA(20,12),ACPROB(2,7),ITYPE(7),TITL(20),DTA(4,11).
     1      ICOL(11),CHNGAC(12),ACPRO(2,7),IROW(12),ICNT(11),II(11),
     2     CP(8),DATA(4,11)
      DATA  ACC/.6/
      DATA BLANK,KRUN/' ',0/
      DATA KD,KR/0,0/,JTL/0/
C     CONVERT METERS TO KILOMETERS FOR INPUT AND OUTPUT
      DO 11  I=1,12
      ACDTA(1,I)=ACDTA(1,I)/1000.
      ACDTA(2,I)='CDTA(2,I)/1000.
      ACDTA(3,I)=ACDTA(3,I)/1000.
   11 ACDTA(8,I)='CDTA(8,I)/1000.
C
C     IF KRUN IS NOT ZERO, THEN WE ARE IN MIDDLE OF DO LOOP
C            DONT READ NEW DATA
C            GO TO DATA CHANGE POINT
      IF  (KRUN.NE.0)  GO TO 600
C     INITIALIZE VARIABLES
   13 DO 12 I=1,7
   12 ACPRO(2,I)='1.
      DO 14 I=1,11
      II(I)=1
      DO 14 J=1,4
   14 DTA(J,I)=-1.
      DO  15  I=1,12
   15 CHNGAC(I)='.
      NORUN=0
      NRAN=0
      NSTOP=0
C
C     READ DATA INPUT AS NEEDED
      CALL   INPUT (5HTITLE,TITL,3HNAC,NAC,4HNRAN,NRAN,
     1     3HAC1,ACDTA(1,1),3HAC2,ACDTA(1,2),3HAC3,ACDTA(1,3),3HAC4,
     2     ACDTA(1,4),3HAC5,ACDTA(1,5),3HAC6,ACDTA(1,6),3HAC7,ACDTA(1,7),
     3     3HAC8,ACDTA(1,8),3HAC9,ACDTA(1,9),4HAC10,ACDTA(1,10),4HAC11,
     4     ACDTA(1,11),4HAC12,ACDTA(1,12),6HACPROB,ACPRO,1HM,DTA(1,1),
     6     2HSO,DTA(1,2),2HSF,DTA(1,3),2HTO,DTA(1,4),2HVA,DTA(1,5),
     7     3HTVF,DTA(1,7),3HRDP,DTA(1,8),6HCHNGAC,CHNGAC,3HSIG,SIG,2HVB.
     8     DTA(1,6),4HRDVA,DTA(1,9),4HRDVP,DTA(1,10),5HRDTVF,DTA(1,11),
     9     3HACC,ACC,5HNORUN,NORUN)
      ACC2=2.*ACC
C
```

```
C       SET ACPROB ARRAY FROM INPUT FOR 7 AC TYPES
C
        DO 170 I=1,7
C
C       IS THIS A/C TYPE USED
        IF  (ACPRO(2,I).LT.0.)    GO TO 180
C
C       STORE PROPORTIONS AND LABEL IN ACPROB
        ACPROB(2,I)=ACPRO(2,I)
        ACPROB(1,I)=ACPRO(1,I)
C       IS THERE DATA FOR THIS TYPE
        ACPRO (2,I)= -1.
        DO 150  J=1,12
        IF  (ACPROB(1,I).EQ.ACDTA(12,J))  GO TO 160
  150   CONTINUE
        WRITE (6,1150)  ACPROB(1,I)
 1150   FORMAT  (1X,'A/C',A4,' IN ACPROB HAS NO DATA')
        NSTOP=1
        GO TO 170
  160   ITYPE(I)=J
  170   CONTINUE
C
C       MXTYP IS NO OF TYPES OF A/C AVAILABLE
        MXTYP=7
        GO TO 185
  180   IF (I.GT.1)  MYTYP=I-1
        IF  (MXTYP.EQ.0)      NSTOP=1
C       DETERMINE A/C PROBABILITIES AND CUM. PROB.
  185   SPROB = 0.0
        DO 190 J ='1,MYTYP
        SPROB = SPROB + ACPROB(2,J)
  190   CONTINUE
        DO 191 J = 1,MXTYP
  191   ACPROB(2,J) ='ACPROB(2,J)/SPROB
        MN = MXTYP
        CP(1) =ACPROB(2,1)
        DO 192 I = 2,MN
  192   CP(I  ) = CP(I-1)+ ACPROB(2,I)
C
C       SET UP DTA ARRAY FOR DO-LOOPS
C       DTA CONTAINS VARIABLES TO BE CHANGED IN DO LOOPS
   18   IKD=0
        DO 20 I=1,11
        IF (DTA(1,I).LT.0.) GO TO 20
        IF  (IKD.EQ.0)  KD=0
```

```
      JKD=1
      DATA(1,I)=DTA(1,I)
      KD=KD+1
C     ICOL GIVES COLUMNS OF ACDTA TO BE CHANGED
      ICOL(KD)=I
      DO  16 J=2,4
      IF  (DTA(J,I).LT.0)   GO TO 17
C
C     DATA CONTAINS ALL VALUES TO BE USED FOR EACH VARIABLE IN ACDTA
      DATA(J,I)=DTA(J,I)
   16 CONTINUE
C
C     ICNT DEFINES LENGTH OF DO LOOPS
      ICNT(KD)=4
      GO TO 20
   17 ICNT(KD)=J-1
   20 CONTINUE
C
C     IF KD=0  SKIP DO LOOPS,    USE ACDTA AS IS
      IF (KD.EQ.0) GO TO 200
C     DETERMINE WHICH A/C HAVE CHANGED DATA USING CHNGAC
      DO 40 I=1,12
      IF (CHNGAC(I).EQ.0.)  GO TO 50
      IF(I.EQ.1) KR=0
      DO 30 J=1,12
C
C     FIND A/C TYPE TO BE CHANGED
      IF (CHNGAC(I).EQ.ACDTA(12,J))  GO TO 35
   30 CONTINUE
      NSTOP=-1
      WRITE  (6,1030) CHNGAC(I)
 1030 FORMAT (1X,'A/C',A4,' IN CHNGAC HAS NO DATA')
      GO TO 39
C
C     IROW GIVES ROWS (A/C TYPES) OF ACDTA TO BE CHANGED
   35 KR=KR+1
      IROW(KR)=J
   39 CHNGAC(I)=0.
   40 CONTINUE
      GO TO 100
   50 IF (KR.NE.0) GO TO 100
      WRITE (6,2100)
 2100 FORMAT(' DTA DECK PRESENT, BUT NO CHNGAC CARD')
      NSTOP=-1
  100 KRUN=1
```

```
C
C      PUT PROPER DATA IN ACDTA, USING IROW,ICOL
  105 DO 110 K=1,KD
      ICOL=ICOL(K)
C
C      II IS THE DO LOOP COUNTER FOR ALL LOOPS
C            KD IS NO. OF DO LOOPS
      IDT=II(K)
      DO 110 L=1,KR
      LROW=IROW(L)
      ACDTA(LCOL,LROW)= DATA(IDT,LCOL)
  110 CONTINUE
C
C      WHEN SEVERAL ACDTA ITEMS ARE TO BE CHANGED
C             IT MAY BE DESIRABLE TO NOT COMPUTE ON THIS DATA
      IF (NOPUN.EQ.1)    GO TO 430
C
C      WRITE HEADING INFO
C
  200 CALL VTITLE (TITL,20,JTL)
      JTL=1
      SIG2=2.4*SIG
  300 WRITE (6,1300) MXTYP,NAC,ACC,NRAN,SIG2
 1300 FORMAT (1H0,I4,' A/C TYPES USED',5X,I5,' A/C TO LAND',5X,
     1     'DECELERATION=',F4.2,'M/SEC2'      ,5X,'RANDOM START=',I5/
     2        20X,'APPROX CONVERSION FACTORS',10X,'KM/2=MILES',10X,
     3        'M/S*2=KNOTS'//1H0,30X,'A/C DATA TABLE'/
     4    T3,'COMMON',T11,'MIN SEPARATION',T27,'RUNWAY',T36,
     5    'APPROACH VEL.',T51,'VEL CH',T63,'RANGE OF UNCERTAINTIES',
     6    T91,'A/C',T102,F3.1,' S.D. OF TIME ERROR'/
     7    T4,'DIST.',T13,'O.M.',T22,'TD.',T28,'TIME',T36,'INIT.',
     8    T44,'FINAL',T53,'TIME',T60,'POSIT',T67,'IN VEL',T75,'FIN V',
     9    T82,'TIME-VB',T91,'A/C',T99,'TOTAL',T107,'---PARTIALS DUE TO',
     X    '------'/
     A    T4,'M  KM',T12,'SO KM',T20,'SF KM',  T27,'TO SEC',
     B    T36,'VA M/S',T43,'VB M/S',T51,'TVF S'   T59,'RDP KM',
     C    T68,'RDVA',T76,'RDVB',T83,'RDTVF',T98,'TOL SEC',T107,'DP',
     E    T114,'DVA',T121,'DVB',T128,'DTVF')
      DO 400 I=1,12
      IF (ACDTA(6,I).GT.80.0.OR.ACDTA(6,I).LT.25.)  GO TO 400
C      COMPUTE THE REST OF ACDTA TABLE
C      NOTE THAT ACDTA(1,I),(2,I),(3,I),(8,I),ARE IN KM,   THE REST-METERS
C
C      SFO2
      ACDTA(15,I)=ACDTA(3,I)/2.E-3
```

```
C       S002
        ACDTA(17,I)=ACDTA(2,I)/2.E-3
C       ZK1
        ACDTA(13,I)=1.+(ACDTA(2,I)-ACDTA(3,I))/(2.*ACDTA(1,I))
C       ZK2
        ACDTA(14,I)=2.-ACDTA(13,I)
C       DA
        ACDTA(16,I)=(ACDTA(7,I)+(ACDTA(5,I)-ACDTA(6,I))/ACC2)*ACDTA(6,I)
C       DAVA
        ACDTA(18,I)=ACDTA(16,I)*(1.-ACDTA(5,I)/ACDTA(6,I))/ACDTA(5,I)
        IF  (ACDTA(1,I)*1.E3.GT.ACDTA(16,I))   GO TO 320
C       TNOM   FOR    M LT DA
        ACDTA (19,I)='CDTA(1,I)*1.E3/ACDTA(6,I)
C       TDL    FOR    M LT DA
        TDL1= ACDTA(8,I)/(2.*ACDTA(6,I))*1.E3
        TDL2=0.
        TDL3= ACDTA(1,I)*ACDTA(10,I)/(2.*ACDTA(6,I)**2)*1.E3
        TDL4=0.
        GO TO 325
C       TNOM   FOR   M GE DA
  320   ACDTA(19,I)=ACDTA(1,I)*1.E3/ACDTA(5,I)   -ACDTA(18,I)
C       TDL    FOR   M GE DA     COMPUTED IN 4 STEPS
        TDL1=ACDTA(8,I)*1.E3/(2.*ACDTA(5,I))
        TDL2=((ACDTA(5,I)**2-ACDTA(6,I)**2)/ ACC2
       1     +ACDTA(7,I)*ACDTA(6,I)-ACDTA(1,I)*1.E3)*
       2       ACDTA(9,I)/(2.*ACDTA(5,I)**2)
        TDL3=((ACDTA(6,I)-ACDTA(5,I))/ACC -ACDTA(7,I))*ACDTA(10,I)/
       1      (2.*ACDTA(5,I))
        TDL4=(ACDTA(5,I)-ACDTA(6,I))*ACDTA(11,I)/(2.*ACDTA(5,I))
  325   ACDTA(20,I)=(TDL1**2+TDL2**2+TDL3**2+TDL4**2)*SIG*SIG
        ACD=SQRT(ACDTA(20,I))
C
C       NOW WRITE A/C DATA FOR THIS CONDITION
C
        WRITE  (6,1400)(ACDTA(J,I),J=1,12),ACD,TDL1,TDL2,TDL3,TDL4
 1400   FORMAT (F7.1,6F8.1,3F8.2,F8.1,3X,A4,F9.2,4F7.2)
        IF (ACDTA(5,I).GE.ACDTA(6,I))   GO TO 400
        WRITE (6,1401)
 1401   FORMAT (1X,'DATA ERROR, VA MUST BE GREATER THAN VB')
        NSTOP=-1
  400   CONTINUE
C       WRITE A/C PROBABILITIES
        WRITE  (6,1410)(ACPPOB(1,J),J=1,MYTYP)
 1410   FORMAT (1HO,'A/C TYPE',1X,8(4X,A4))
        WRITE (6,1420) (ACPPOB(2,J),J=1,MYTYP)
```

```
 1420 FORMAT (1X,'PROBABILITY',8F8.3)
C     CONVERT KILOMETERS TO METERS FOR COMPUTATION
  430 DO 450  I=1,12
      ACDTA(1,I)='CDTA(1,I)*1000.
      ACDTA(2,I)=ACDTA(2,I)*1000.
      ACDTA(3,I)='CDTA(3,I)*1000.
  450 ACDTA(8,I)=ACDTA(8,I)*1000.
      IF (NSTOP) 500,520,510
  500 WRITE  (6,1500)
 1500 FORMAT (' ERROR IN CHNGAC, THIS CASE CANCELLED')
      GO TO 515
  510 WRITE (6,1510)
 1510 FORMAT  (1X,'ERROR IN PROPORTIONS, THIS CASE CANCELLED')
  515 CONTINUE
  520 IF (NORUN.EQ.1)  NSTOP=1
      RETURN
C     DO LOOPS FOR CYCLING THRU DATA
  600 DO 620 I=1,KD
      II(I)=II(I)+1
      IF  (II(I).LE.ICNT(I)) GO TO 105
      II(I)=1
  620 CONTINUE
      KRUN=0
      GO TO  13
      END
```

```
C       SUBROUTINE OUT    PRINTS HISTOGRAMS, LANDING AND TAKEOFF RATES
        SUBROUTINE OUT(TBR,TMAX,TMIN,ISTTL,TOBR,TOMX,TOMN,ISTTO,IAC,IABRT,
       1 SUMT)
        DIMENSION ISTTL(25),ISTTO(25)
  200   FORMAT ('0AFTER',I5,'LANDINGS',6X,
      A                 'AVG',4X,'MAX',4X,'MIN',30X,'HISTOGRAM'
      B          '(DETERMINISTIC)'/5X,'NOMINAL',
      1T49,'TIME=  0',12X,' 60',12X,'120',12X,'180',12X,'240',9X,'288')
  201   FORMAT(' LANDING INTERVALS,SEC',3F7.0,10X,25I3)
  202   FORMAT(' TIME AVAILABLE,T.O.',2X,3F7.0,10X,25I3)
C
C       PRINT MAX, MEAN, MIN, TIMES AND HISTOGRAMS
C
        WRITE(6,200)IAC
        WRITE(6,201) TBR,TMAX,TMIN, (ISTTL(I),I=1,25)
        WRITE(6,202) TOBR,TOMX,TOMN,(ISTTO(J),J=1,25)
C
C       COMPUTE LANDING RATES W & W/O ABORTS AND PRINT
        ZLR=3600./TBR
        ZAC=IAC
        ZAB=IABRT
        ZLRA=3600.*(ZAC-ZAB-1)/SUMT
        ABTRAT=ZAB/ZAC
        IACA=IAC-IABRT
        WRITE  (6,205)  IABRT,ABTRAT,IAC,ZLR,IACA,ZLRA
  205   FORMAT  (1H0,'NO. OF ABORTS',T20,'ABORT RATIO',
      X             T40,'NO. OF NOM. LNDNGS',T60,'NOM. LNDNG RATE(/HR)',
      1    T90,'NO. OF ACT. LNDNGS',T110,'ACT.LNDNG RATE(/HR)',
      Y     /5X,I5,T25,F5.3,
      2    T46,I4,T66,F5.1,T96,I4,T116,F5.1)
C
C       HEADING FOR TAKEOFF TABLE
C
        WRITE(6,210)
  210   FORMAT(1H0,'TIME REQD FOR TO',T20,'NO. OF TAKEOFFS',T40,'NO. OF NO
       AM. OPERNS',T60,'NOM. OPERN RATE(/HR)',
       1                        T90,'NO. OF ACT. OPERNS',T110,'ACT.OPERN RAT
       IE(/HR)')
C
C       CALCULATE TAKEOFFS AND TOTAL OPERATIONS RATE
C
        DO 300  I=4,11
        ZTTO=(I-1)*12.
        ISUM=0.
        DO 250 J=I,25
```

```
  250 ISUM=ISUM+(J-1)/(I-1)*ISTTO(J)
      IF  (ISUM.EQ.0)    GO TO 400

      IOP=ISUM+IAC
      ZNOPR=3600.*(IOP-1)/SUMT
      IOPA=IOP-IABRT

      ZAOPR=3600.*(IOPA-1)/SUMT
C
C     PRINT TAKEOFF DATA
C
  300 WRITE(6,1300)ZTTO,ISUM,IOP,ZNOPR,IOPA,ZAOPR
 1300 FORMAT(10X,F4.0,T26,I4,T46,I4,T66,F5.1,T96,I4,T116,F5.1)
      RETURN
  400 WRITE (6,1400) ZTTO
 1400 FORMAT (10X,F4.0,' OR GREATER      NO SLOTS AVAILABLE FOR TAKEOFF')
      RETURN
      END
```

REFERENCES

1. Blumstein, Alfred: The Landing Capacity of a Runway. Operations Res., vol. 70, no. 6, Dec. 1959, pp. 752–763.

2. Simpson, Robert W.: Analytical Methods of Research into Terminal Area Air Traffic Operations. J. Aircraft, vol. 2, no. 3, May-June 1965, pp. 185–193.

3. Odoni, Amedeo R.: An Analytical Investigation of Air-Traffic in the Vicinity of Terminal Areas. Ph.D. Thesis, Massachusetts Institute of Technology, Sept. 1969.

4. Anon: Analysis of a Capacity Concept for Runway and Final-Approach Path Aerospace. NBS Rep. 10111, SRDS-RD-69–47, Nov. 1969.

5. Rinker, Robert E.: Determination of STOL Air Terminal Traffic Capacity Through Use of Computer Simulation. M.S. Thesis, Naval Postgraduate School, Monterey, Calif.

6. Anon: The National Aviation System Plan – 1972–1981 FAA. March 1971, AD–756580.

7. Erzberger, Heinz; and Pecsvaradi, Thomas: 4-D Guidance System Design with Application to STOL Air Traffic Control.

NASA-Langley, 1974 A-5044

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