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CONTRIBUTIONS OF SATELLITE-DETERMINED GRAVITY

RESULTS IN GEODESY

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ABSTRACT

Different forms of the theoretical gravity formula are summarized and methods of standardization of gravity anomalies obtained from satellite gravity and terrestrial gravity data are discussed in the context of three most commonly used reference figures, e.g., International Reference Ellipsoid, Reference Ellipsoid 1967, and Equilibrium Reference Ellipsoid. These methods are important in the comparison and combination of satellite gravity and gravimetric data as well as the integration of surface gravity data, collected with different objectives, in a single reference system. For ready reference, tables for such reductions are computed. Nature of the satellite gravity anomalies is examined to aid the geophysical and geodetic interpretation of these anomalies in terms of the tectonic features of the earth and the structure of the earth's crust and mantle. Computation of the Potsdam correction from satellite-determined geopotential is reviewed. The contribution of the satellite gravity results in decomposing the total observed gravity anomaly into components of geophysical interest is discussed. Recent work on the possible temporal variations in the geogravity field is briefly reviewed.

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INTRODUCTION

Analyses of the orbital data of the artificial earth satellites yield geopotential models which are customarily given in the form of spherical harmonic coefficients of geopotential. These coefficients are related to the more familiar gravity anomaly Δg and the geoidal undulation N as

$$\Delta g = \frac{GM}{a_e^2} \sum_{n=2} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^{n+2} (n-1) (\delta C_{nm} \cos m\lambda + \delta S_{nm} \sin m\lambda) P_{nm}(\sin \phi) \quad (1)$$

and

$$N = a_e \sum_{n=2} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^{n+1} (\delta C_{nm} \cos m\lambda + \delta S_{nm} \sin m\lambda) P_{nm}(\sin \phi) \quad (2)$$

where Δg is the gravity anomaly, GM the product of the gravitational constant and mass of the earth, a_e the equatorial radius of the earth, (r, ϕ, λ) the spherical coordinates of the computation point, N the geoidal undulation, P_{nm} $(\sin \phi)$ the associated Legendre's functions, and

$$\delta C_{nm} = \text{observed} \quad C_{nm} - \text{reference} \quad C_{nm}$$

$$\delta S_{nm} = \text{observed} \quad S_{nm} - \text{reference} \quad S_{nm} \quad (3)$$

The factor a_e in Equation (2) is a simplification of the factor $GM/a_e\gamma$ where γ is the average value of gravity over N and is generally approximated by the value of theoretical gravity at the appropriate point of the reference ellipsoid. The reference (C_{nm}, S_{nm}) refer to the geopotential coefficients which define the reference figure. A customary reference figure is an ellipsoid of revolution. For such an ellipsoid, because of the supposition of axial symmetry, all $C_{nm} = S_{nm} = 0$ for $m \neq 0$. Since such an ellipsoid also has equatorial symmetry, all the odd zonal harmonics also become zero. Hence, for such a reference surface, only the even-degree zonal harmonics need be considered. Of these, the harmonic coefficients of 6th degree or higher are of the order of 10^{-9} or smaller. In a second order theory, this corresponds to terms $\geq 0(f^3)$ which are usually neglected. Hence, in practice, such a reference ellipsoid is completely described by the 2nd and 4th degree zonal harmonics. Consequently, with the exception of δC_{20} and δC_{40}, all δC_{nm} and δS_{nm} in equation (3) equal the observed ones.

A combination of equation (2) and Stokes' formula can be used to compute detailed gravimetric geoid height at a point as outlined by Strange and Khan (1965) and Khan and Strange (1966).

NATURE OF THE SATELLITE-DETERMINED GRAVITY ANOMALIES

An important problem in the understanding and interpretation of satellite-determined gravity results and their combination with the surface gravity results is to define which of the well-known surface gravimetric anomaly types (e.g., free air, Bouguer, isostatic, etc.) most closely equals the satellite-determined gravity anomaly. The most obvious answer, perhaps, is the free air anomaly; but since the mean height of most of the satellites used in the gravity field determinations is several times greater than the depth of compensation, one must investigate how different the isostatic gravity anomaly is from the satellite-derived gravity anomaly. The solution of this problem becomes all the more important as one must assume one of the several possible isostatic compensation mechanisms in computing the isostatic reduction and thus, perhaps, run a greater risk of introducing, rather than removing, spurious factors in the gravity field.

The problem has been investigated by Jeffreys (1962) and Khan (1972a). The potential due to the isostatic reduction is found to be

$$U^{i}_{nm} = \frac{4\pi G}{2n+1} \sigma_{nm} S_{nm} \frac{R^{n+2}}{r^{n+1}} \left[1 - \left(\frac{R-d}{R}\right)^{n}\right] \tag{4}$$

where $\sigma_{nm}\ S_{nm}$ is a surface density layer at $r = R$ and d, the depth of compensation. The appropriate isostatic anomaly potential coefficients based on a recent geopotential model are given by Khan (1973).

REFERENCE FIGURES AND THEORETICAL GRAVITY

Reference Figures

There are three most commonly used reference systems: (1) International Reference Ellipsoid with equatorial radius $a_e = 6{,}378{,}388$ meters and flattening $f = 1/297.0$. Based on this is the International Gravity Formula. (2) Reference Ellipsoid 1967 with $a_e = 6{,}378{,}160$ meters and the second geopotential coefficient $C_2 = -1082.7 \times 10^{-6}$ corresponding to a flattening $f = 1/298.25$. These values are based on the satellite-determined data. Based on this system is the Gravity

Formula 1967. (3) Equilibrium Reference Ellipsoid with a flattening of 1/299.75 derived from the earth's polar or preferably, mean moment of inertia which in turn is determined from the ratio of the satellite-determined C_2 and a parameter H obtained from the precession of the moon. This is the hypothetical flattening which the earth would have if it were in complete fluid equilibrium.

Theoretical Gravity

Two types of formulas are available for describing gravity on the surface of a reference ellipsoid: (1) series expansion formulas which must be truncated to a finite number of terms, their accuracy depending upon the degree of such truncation; (2) closed formulas which are, of course, exact. It is found that, in practice, the series expansion formulas need be developed to second order terms only, i.e., to include quantities of the order of square of the flattening of the reference ellipsoid being considered. Cook (1957) has developed these formulas accurate to third order terms but the contribution of these additional terms is only of the order of 10^{-8} to 10^{-9}.

Series Expansion Formulas: The theoretical or normal gravity γ on the surface of an oblate spheroid, provided it is bounding equipotential, is

$$\gamma = \gamma_e [A_0 + A_2 P_2(\sin\phi) + A_4 P_4(\sin\phi)] \tag{5}$$

where

γ_e = normal gravity at the equator and is given by

$$\gamma_e = \frac{GM}{a_e^2}\left(1 - \frac{3}{2}C_2 + \frac{15}{8}C_4 - m_e\right) \tag{6}$$

$P_2(\sin\phi)$, $P_4(\sin\phi)$ = second and fourth degree Legendre's polynomials, respectively

$$A_0 = 1 + \frac{1}{3}\beta_1 + \frac{1}{5}\beta_2$$

$$A_2 = \frac{2}{3}\beta_1 + \frac{4}{7}\beta_2 \tag{7}$$

$$A_4 = \frac{8}{35}\beta_4$$

$$\beta_1 = 2m\left(1 + \frac{55}{56}m\right) + \frac{3}{2}C_2\left(1 + \frac{25}{7}m\right)$$

$$\beta_2 = -3C_2\left(m + \frac{3}{8}C_2\right) + \frac{9}{8}m^2 \tag{8}$$

In the above expressions, C_2 and C_4 are the second and fourth geopotential coefficients, respectively, and are related to the flattening f of the reference ellipsoid as

$$C_2 = -\frac{2}{3}f + \frac{1}{3}m + \frac{1}{3}f^2 - \frac{2}{21}mf$$

$$C_4 = \frac{4}{5}f^2 - \frac{4}{7}mf \tag{9}$$

with

$$m = \frac{\omega^2 a_e^2 b}{GM}$$

ω = rotational velocity

a, b = semi-major and semi-minor axes of the reference ellipsoid

and

GM = the product of the gravitational constant and mass of the earth.

The rotational velocity ω is a fixed quantity. Consequently, the selection of C_2 (or the flattening f), a_e and either GM or γ_e, defines the reference system completely.

A more familiar form of the theoretical gravity formula, or what is also known as the standard gravity formula, is

$$\gamma = \gamma_e [1 + \alpha_2 \sin^2 \phi + \alpha_4 \sin^2 2\phi] \tag{10}$$

which is obtained from equation (5) by the substitution

$$\begin{aligned} P_2(\sin \phi) &= \frac{1}{2} (3 \sin^2 \phi - 1) \\ P_4(\sin \quad) &= \frac{1}{8} (35 \sin^4 \phi - 30 \sin^2 \phi + 3). \end{aligned} \tag{11}$$

The coefficients α_2 and α_4 in this case are

$$\alpha_2 = \beta_1 + \beta_2$$

$$\alpha_4 = -\frac{1}{4}\beta_2 \tag{12}$$

It is again seen from equations (6), (8), (10) and (12) that a knowledge of ω, GM, a_e and f, or ω, γ_e, a_e and f, defines the standard gravity completely.

Closed formulas: The normal gravity γ is given on the surface of an ellipsoid by the exact closed formula (Heiskenan and Moritz, 1967):

$$\gamma = \frac{a_e \gamma_e \cos^2 \phi + b \gamma_p \sin^2 \phi}{[a^2 \cos^2 \phi + b^2 \sin^2 \phi]^{1/2}} \tag{13}$$

where

$$b = a_e(1 - f)$$

$$\left.\begin{aligned}
&\text{equatorial gravity} \quad \gamma_e = \frac{GM}{a_e b}\left(1 - m - \frac{m}{6} e' \frac{q_0'}{q_0}\right) \\
&\text{polar gravity} \quad \gamma_p = \frac{GM}{a_e^2}\left(1 + \frac{m}{3} e' \frac{q_0'}{q_0}\right) \\
&e' = \frac{\sqrt{a_e^2 - b^2}}{b}
\end{aligned}\right] \tag{14}$$

$$
\left.\begin{aligned}
q_0' &= 3\left(1+\frac{b^2}{E^2}\right)\left(1-\frac{b}{E}\tan^{-1}\frac{E}{b}\right)-1 \\
q_0 &= \frac{1}{2}\left[\left(1+3\frac{b^2}{E^2}\right)\tan^{-1}\frac{E}{b}-3\frac{b}{E}\right] \\
\text{and} \quad E &= \sqrt{a^2-b^2}
\end{aligned}\right] \tag{15}
$$

Numerical Gravity Formulas: With the adopted geometric parameters of the International Reference Ellipsoid and γ_e = 978,049.0 milligals, ω = 0.72,921, 151.10^{-4} sec^{-1}, equations (8), (10) and (12) yield the International Gravity Formula, i.e.,

$$
\gamma = 978\,049.0[1 + 0.0\,052\,883 \sin^2\phi - 0.0\,000\,059 \sin^2 2\phi] \text{ mgals} \tag{16}
$$

The value of γ_e in this formula is an adopted quantity, obtained from the gravimetric measurements and is tied to the Potsdam system. GM in this case is derived from adopted γ_e and a_e.

The satellite determined 'Geodetic Reference System 1967' has the adopted parameters

$$
\left.\begin{aligned}
a_e &= 6,378,160.0 \text{ meter} \\
C_2 &= -1082.7\times 10^{-6} \\
GM &= 3.98,603\times 10^{20} \text{ cm}^3 \text{ sec}^{-2}
\end{aligned}\right] \text{(Reference Ellipsoid 1967)}
$$

The corresponding flattening f = 1/298.25 from equation (9) and γ_e = 978.0318 $\times 10^3$ milligals from equation (6). The resulting 'Gravity Formula 1967' is

$$\gamma = 978.0318\ [1 + 0.0\ 053\ 024\ \sin^2\phi - 0.0\ 000\ 058\ \sin^2 2\phi] \times 10^3\ \mathrm{mgals} \tag{17}$$

Here γ_e is independent of the Potsdam Gravity System. The value of C_4 used in equation (6) is not the satellite-derived value for the actual earth but the theoretical value expected from the value of C_2 (equation 9).

STANDARDIZATION OF THE GRAVITY ANOMALIES

It frequently happens that the gravity anomalies computed on two different reference systems need be integrated or compared or, the gravity data obtained from two independent techniques of measurements need be compared and combined. For example, the terrestrial gravity data are usually tied to the Potsdam Gravity System and the anomalies are usually computed on the basis of International Gravity Formula or equivalently, referred to the International Reference Ellipsoid. The satellite gravity results are independent of the Potsdam System and are usually referred to the 'Reference Ellipsoid 1967' (Geodetic Reference System 1967) or equivalently, to the 'Gravity Formula 1967'. For the geophysical analysis of the crust and mantle from gravity anomalies on the other hand, the best reference figure is regarded to be the Equilibrium Reference Ellipsoid (Khan and O'Keefe, in press). Hence, it is important to consider the conversion from one reference to the other and to compute tables to serve as ready reference for such conversions.

Let

$$\left.\begin{aligned} \Delta g^I &= g_p^O - \gamma_p^I \\ \\ \Delta g^S &= g^O - \gamma^S = (g_p^O - d) - \gamma^S \end{aligned}\right] \tag{18}$$

Δg is the gravity anomaly, g^o the observed gravity and γ the theoretical gravity. The superscripts S and I refer to the Gravity Formula 1967 and International Gravity Formula, respectively and subscript P to the Potsdam System. It is clear that g_p^O and γ^I are related to the Potsdam System while g^O and γ^S are independent of it. Also, $g_p^O = g^O + d$ where d is the datum shift correction i.e., the quantity by which the Potsdam Gravity System is in error.

Equation (18) gives

$$\Delta g^S = \Delta g^I - d + \gamma_P^I - \gamma^S \tag{19}$$

or

$$\Delta g^I = \Delta g^S + d + \gamma^S - \gamma_P^I$$

From equation (10), γ_P^I and γ^S are

$$\gamma_P^I = \gamma_e^I [1 + \alpha_2^I \sin^2 \phi + \alpha_4^I \sin^2 2\phi] \tag{20}$$

and

$$\gamma^S = \gamma_e^S [1 + \alpha_2^S \sin^2 \phi + \alpha_4^S \sin^2 2\phi]$$

Let

$$\gamma_e^I = \gamma_e^S + \delta\gamma_e$$

$$\alpha_2^I = \alpha_2^S + \delta\alpha_2 \tag{21}$$

$$\alpha_4^I = \alpha_4^S + \delta\alpha_4$$

Then

$$\gamma_P^I - \gamma^S = \delta\gamma_e [1 + \alpha_2^S \sin^2\phi + \alpha_4^S \sin^2 2\phi] + \gamma_e^I [\delta\alpha_2 \sin^2 +$$
$$\delta\alpha_4 \sin^2 2\phi] = \delta\gamma + \delta e_2 \tag{22}$$

where the first three terms are denoted by $\delta\gamma$ and the last two by δe_2. Since γ_e^I is tied to the Potsdam system while γ_e^S is independent of it, the quantity $\delta\gamma$ contains the datum shift correction d, i.e.,

$$\delta\gamma = \delta e_1 + d$$

Hence

$$\left.\begin{aligned} \Delta g^S &= \Delta g^I + \delta e_1 + \delta e_2 = \delta g^I + \delta e \\ \text{or} \quad \Delta g^I &= \Delta g^S - \delta e \end{aligned}\right] \tag{23}$$

The correction δe_1 accounts for the difference in the equatorial gravity values of the two formulas under consideration, corrected for the Potsdam datum shift. This correction obviously arises from changes in the value of GM and equatorial radius a_e of the reference ellipsoid. The quantity δe_2 stems primarily from changes in the flattening of the reference ellipsoid.

In satellite-derived gravity anomalies, it is more convenient to work with spherical harmonics and hence to incorporate the above corrections in terms of the coefficients of these harmonics. In this case the correction δe to the order of accuracy required here, is given as

$$- \delta e = \frac{GM}{a_e^2} [\delta C_2 P_2(\sin \phi) + 3 \delta C_4 P_4(\sin \phi)] \tag{24}$$

where allowance has been made for the fact that the point to which the correction refers to on the spheroid is shifted to the geoid because of the disturbing masses, i.e., as the correction is to be applied to the gravity anomalies, the Bruns' term has been taken into consideration. In equation (24), δC_2 and δC_4 are the differences between the second and fourth spherical harmonic coefficients of the satellite-determined ellipsoid and the International Reference Ellipsoid, respectively. Since none of the parameters in equation (24) is derived from the terrestrial gravity data (which are tied to the Potsdam System), the correction is independent of the Potsdam System and is equal to δe to which the datum shift correction d has already been applied (with due regard to the sign, of course).

For formulas relating to the Equilibrium Reference Ellipsoid, the above development must be followed closely with appropriate change of superscript.

The customary way to allow for the correction term due to equation (24) while computing the satellite-determined Δg with respect to the preferred reference ellipsoid has already been described in equations (1) and (3).

The correction factors for the interconversions of the various reference ellipsoids are computed using equation (24), or equation (22) with allowance for the correction d, or equation (13) for the closed formulas, are listed in Table 1 for each 1° latitude. The number of decimal places should not be regarded as indicating the degree of accuracy and the numbers should be rounded to first place after decimal in applying the correction. For any fractional degrees, the values can be safely interpolated linearly.

Correction to the Potsdam Absolute Gravity Value

As noted before, the Potsdam System affects equation (22) through γ_e^I (equation 20) which is derived from terrestrial gravity data tied to the Potsdam System. From equations (22) and (23)

$$(\gamma_P^I - \gamma^S) = d + \delta e$$

None of the parameters used in equation (24) is tied to the Potsdam System. Hence, the correction given by this equation is independent of this system. The difference of the corrections obtained from equations (22) and (24), therefore, should give an estimate of the value by which the Potsdam gravity value is in error, i.e.,

$$d = (\gamma_P^I - \gamma^S) - \delta e \tag{25}$$

The datum shift correction d is computed from equation (25) via equations (22) and (26), using the standard parameters of the International Gravity Formula and the 'Geodetic Reference System 1967.' The results of this analysis are published in Khan (1972b).

REMOVAL OF THE REGIONAL GRAVITY EFFECT

In the process of geophysical interpretation of a gravity anomaly it is often necessary to split the anomaly in the part which stems from sources of local extent and the part which is contributed by the regional sources. Several methods have been devised to estimate the regional gravity effect (the areal extent of the local and regional sources being defined arbitrarily in a given problem) but perhaps the most commonly used is the graphical method in which the regional part of the gravity anomaly is usually estimated by the general trend of the gravity anomaly curve. Such a method has its obvious limitations. The satellite determinations of the long wavelength components of the gravity field provide us with a relatively more objective and reliable method of estimating the regional effect. Let Δg_t be the total anomaly and Δg_e and Δg_r its local and regional components, respectively, i.e.,

$$\Delta g_e = \Delta g_t - \Delta g_r \tag{26}$$

Where

$$\Delta g_r = \frac{GM}{a_e^2} \sum_{n=2}^{N} \sum_{m}^{n} \left(\frac{a_e}{r}\right)^{n+2} (n-1) (\delta C_{nm} \cos m\lambda + \delta S_{nm} \sin m\lambda) P_{nm}(\sin \phi)$$

N is the number of harmonic degree to which the regional effect is desired to be computed and is arbitrarily determined by the requirements of the specific problem. The other symbols in the above equation are the same as in equations (1), (2) and (3).

Once δg_e is isolated and explained in terms of known or possible local and perhaps shallow-seated sources, Δg_r can then be computed more accurately and assigned to broad and perhaps relatively more deep seated anomalous sources.

The removal of the regional effect in equation (26) is, in fact, equivalent to an arbitrary extension of the definition of normal potential, i.e., instead of adopting a simple ellipsoid of revolution as the reference figure, one chooses to select a more complicated geoidal surface as the reference figure which must be defined to a higher harmonic degree. The implication of this equivalence in terms of equation (26) is that Δg_e can be directly written as:

$$\Delta g_e = \frac{GM}{a_e^2} \sum_{n=N} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^{n+2} (n-1) (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin \phi) \qquad (27)$$

with C_{nm} and S_{nm} replacing δC_{nm} and δS_{nm} of equation (1) and N defined as above. Note that equation (27) is just another statement of equation (26).

The techniques provided by equation (26) or (27) can be very useful in the investigation of interrelationships of gravity with elevation, heat flow, seismic data, tectonic activity zones or various geological provinces, etc.

SECULAR VARIABILITY OF THE GEOGRAVITY FIELD

It has long been speculated that earth's gravity field shows secular variations. Various methods have been applied to investigate such variations. Some involve the graphical techniques (Tuman, 1966) while others involve the use of geophysical evidence such as seismic reflections from the core-mantle boundary, stationary and displaced correlations of the earth's gravity and magnetic fields (Vogel, 1960, 1968; Egyed, 1964; Hide, 1970; Khan, 1970, 1971) as well as the tidal records (Khan, 1971). Evidence to date is inconclusive at best. Khan (in prep.) has suggested that such variations can be traced by comparing the satellite-determined models of the gravity field as a function of time. If a_t, b_t be the geopotential harmonic coefficients of the gravity field at the present time t and a_0, b_0 the coefficients of the gravity field in the past at $t = 0$, then the comparison of the two sets of coefficients will yield the drift angle between the two versions of the field over time t. If the drift angles computed for different harmonic degrees are randomly distributed, there is no reason to believe that the gravity field has any secular or long period variations. If, however, these drift angles show a preference for a single-cluster distribution, the assumption of secular or long-period variation in the field is supported. The available geopotential models have been analyzed with the help of this method and screened through the significance tests adopted from the theory of circular distributions. The investigations to date (Khan, in prep.) indicate no evidence of any systematic secular or long-period variations in the earth's gravity field. Such investigations are possible, of course, because of the global coverage, and perhaps the accuracy, furnished by the satellite-determined gravity results.

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Table 1. Corrections in Milligals for the Interconversion of the International, 1967 and Equilibrium Reference Ellipsoids

Latitude (Degrees)	(1)	(2)	(3)
90.00	9.10	-9.00	-18.11
89.00	9.10	-9.00	-18.10
88.00	9.09	-.899	-18.07
87.00	9.07	-.897	-18.03
86.00	9.04	-8.94	-17.97
85.00	9.00	-8.90	-17.90
84.00	8.95	-8.86	-17.81
83.00	8.90	-8.80	-17.71
82.00	8.84	-8.74	-17.59
81.00	8.77	-8.68	-17.45
80.00	8.70	-8.60	-.17.30
79.00	8.61	-8.52	-17.13
78.00	8.52	-8.43	-16.94
77.00	8.42	-8.33	-16.75
76.00	8.31	-8.22	-16.53
75.00	8.20	-8.11	-16.31
74.00	8.08	-7.99	-16.06
73.00	7.95	-7.86	-15.81
72.00	7.81	-7.73	-15.54

Table 1. (continued)

Latitude (Degrees)	(1)	(2)	(3)
71.00	7.67	-7.58	-15.26
70.00	7.52	-7.44	-14.96
69.00	7.37	-7.28	-14.65
68.00	7.21	-7.12	-14.33
67.00	7.04	-6.96	-14.00
66.00	6.87	-6.79	-13.65
65.00	6.69	-6.61	-13.30
64.00	6.50	-6.43	-12.93
63.00	6.31	-6.24	-12.56
62.00	6.12	-6.05	-12.17
61.00	5.92	-5.85	-11.77
60.00	5.72	-5.65	-11.37
59.00	5.51	-5.45	-10.96
58.00	5.30	-5.24	-10.54
57.00	5.08	-5.02	-10.11
56.00	4.87	-4.81	- 9.67
55.00	4.64	-4.59	- 9.23
54.00	4.42	-4.37	- 8.79
53.00	4.19	-4.14	- 8.33
52.00	3.96	-3.91	- 7.87

Table 1. (continued)

Latitude (Degrees)	(1)	(2)	(3)
51.00	3.73	-3.68	- 7.41
50.00	3.50	-3.45	- 6.95
49.00	3.26	-3.22	- 6.48
48.00	3.02	-2.99	- 6.01
47.00	2.79	-2.75	- 5.54
46.00	2.55	-2.51	- 5.06
45.00	2.31	-2.28	- 4.59
44.00	2.07	-2.04	- 4.11
43.00	1.83	-1.81	- 3.64
42.00	1.59	-1.57	- 3.16
41.00	1.35	-1.33	- 2.69
40.00	1.12	-1.10	- 2.22
39.00	0.88	-0.87	- 1.75
38.00	0.65	-0.64	- 1.29
37.00	0.42	-0.41	- 0.82
36.00	0.19	-0.18	- 0.37
35.00	-0.04	0.04	0.08
34.00	-0.27	0.26	0.53
33.00	-0.49	0.48	0.97
32.00	-0.70	0.70	1.40

Table 1. (continued)

Latitude (Degrees)	(1)	(2)	(3)
31.00	-0.92	0.91	1.83
30.00	-1.13	1.12	2.25
29.00	-1.34	1.32	2.66
28.00	-1.54	1.52	3.06
27.00	-1.74	1.72	3.45
26.00	-1.93	1.91	3.84
25.00	-2.12	2.09	4.21
24.00	-2.30	2.27	4.57
23.00	-2.48	2.45	4.92
22.00	-2.65	2.62	5.26
21.00	-2.81	2.78	5.59
20.00	-2.97	2.94	5.90
19.00	-3.12	3.09	6.21
18.00	-3.27	3.23	6.50
17.00	-3.41	3.37	6.77
16.00	-3.54	3.50	7.03
15.00	-3.66	3.62	7.28
14.00	-3.78	3.74	7.52
13.00	-.389	3.84	7.73
12.00	-3.99	3.95	7.94

Table 1. (concluded)

Latitude (Degrees)	(1)	(2)	(3)
11.00	-4.09	4.04	8.13
10.00	-4.18	4.12	8.30
9.00	-4.25	4.20	8.46
8.00	-4.32	4.27	8.60
7.00	-4.39	4.33	8.72
6.00	-4.44	4.39	8.83
5.00	-4.49	4.43	8.92
4.00	-4.53	4.47	9.00
3.00	-4.56	4.50	9.06
2.00	-4.58	4.52	9.10
1.00	-4.59	4.53	9.12
0.0	-4.59	4.54	9.13

Column (1): For change from the "Reference Ellipsoid 1967" to the International Reference Ellipsoid, add the correction to the gravity anomaly. For reverse operation subtract the correction.

Column (2): For change from the "Reference Ellipsoid 1967" to the Equilibrium Reference Ellipsoid, add the correction to the gravity anomaly. For reverse operation subtract the correction.

Column (3): For change from the International Reference Ellipsoid to the Equilibrium Reference Ellipsoid, add the correction to the gravity anomaly. For reverse operation subtract the correction.

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