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THE ALBEDO OF PARTICLES IN REFLECTION NEBULAE

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ABSTRACT

The relation between the apparent angular extent of a reflection nebula and the apparent magnitude of its illuminating star derived by Hubble (1922) has been reconsidered under a less restrictive set of assumptions. A computational technique has been developed which permits the use of fits to the observed $m - \log a$ values to determine the albedo of particles composing reflection nebulae, providing only that a phase function and average optical thickness are assumed. Multiple scattering, anisotropic phase functions, and illumination by the general star field are considered, and the albedo of reflection nebular particles appears to be the same as that for interstellar particles in general. The possibility of continuous fluorescence contributions to the surface brightness is also considered.

I. INTRODUCTION

The scattering properties of interstellar particles are most frequently described in terms of the albedo, γ , and asymmetry parameter, g , of the phase function of Henyey and Greenstein (1941). Van de Hulst and de Jong (1969) analyzed Witt's (1968) observations of the diffuse galactic light (DGL) in order to determine γ and g . The existence of discrete clouds in the interstellar medium was taken into account by Mattila (1971) in his analysis of DGL observations. These investigations indicate that the interstellar particles have an albedo of $.3 < \gamma < .7$ and an asymmetry parameter of $g \geq .7$. The value of g may range from -1 to $+1$ as the scatterer changes from completely backward scattering to completely forward scattering. For isotropic scattering, $g=0$.

The starting point for this investigation is the observation that the values of γ and g of interstellar particles as determined from DGL observations appear to be in fundamental disagreement with the γ and g values used to interpret observations of surface brightness of reflection nebulae. Hubble (1922) obtained an excellent theoretical fit to reflection nebulae observations by assuming $g=0$ and $\gamma=1$ for the constituent particles. Henyey and Greenstein (1939) determine $\gamma \geq .8$ under the assumption of an isotropic phase function.

It is important to recognize at the outset that in seeking the cause of the discrepancy between the values of γ and g determined by DGL studies and reflection nebulae observations, an implicit assumption is being made. It is being assumed that reflection nebulae are composed of the same types of particles which comprise the general interstellar medium. While one certainly can imagine mechanisms which would invalidate this assumption, the simplest assumption should be maintained until observations require a more refined model.

II A GENERALIZED FORM OF HUBBLE'S RELATION

Assuming $\gamma=1$ and an isotropic phase function, Hubble (1922) derived a theoretical relationship between the apparent size of a reflection nebula and the apparent magnitude of its illuminating star. He finds

$$1) \quad m_{\star} = 10.6 - 5 \log a',$$

where m_{\star} is the apparent magnitude of the illuminating star and a' is the angular distance in arc minutes between this star and the most distant patch of nebulosity which can be detected under uniform exposure conditions. The intention of this present paper is to derive a relationship between the observable quantities a' and m_{\star} which can be interpreted in terms of γ and g ; it is intended that there be as few restrictive assumptions as possible.

Let us begin by listing the assumptions which will be made initially and then proceed to remove each of these assumptions. Initially, assume:

1. The nebula is illuminated by only one star.
2. Only single scattering occurs.

3. The intensity of light from the illuminating star decreases as r^{-2} , where r is the distance from the star to a point in the nebula.
4. The apparent size of the nebula is determined by the intensity of light reaching the parts most distant from the star, rather than by the absence of dust particles.
5. For any line of sight through the nebula, we may neglect the r^{-2} dependence of the stellar intensity.
6. For small regions, the nebula may be treated as a uniform plane-parallel structure.
7. There is no obscuring material between the observer and nebula.
8. The angles between the observer's line of sight and the front surface of the nebula are random.
9. Only energy entering the nebula as visible light can leave as visible light.

Figure 1 shows the geometry assumed in deriving the generalized version of the Hubble relation. Starlight is incident at angle α_i , measured from the normal to the nebular surface. The star is located at distance r from the nebula, and the observer, whose distance from the nebula is R , detects light leaving the nebula in direction α_s , relative to the normal. The nebula is of physical thickness D and optical thickness τ_0 . We define the scattering angle, α , by

$$2) \quad \alpha = \alpha_i + \alpha_s.$$

Let us further define the quantities μ and μ_0 by

$$3) \quad \mu = \cos \alpha_s,$$

$$4) \quad \mu_0 = \cos \alpha_i.$$

Consider an arbitrary point, P , within the nebula, located a distance x behind the front surface of the nebula. If k is the extinction (either

scattering or absorption) per unit distance, the optical distance along the line of sight to the star from P to the front surface of the nebula is

$$\tau_1 = kx/\mu_0.$$

The intensity of starlight at P is

$$5) \quad I_P = \frac{L}{4\pi r^2} \cdot \exp\left[-\frac{kx}{|\mu_0|}\right],$$

where L is stellar luminosity and the absolute value sign arises to include cases for which the star is behind the nebula. If σ is the cross sectional area for extinction contained in a volume of cross section of A and length ds situated at P, the flux intercepted by grains at P will be

$$6) \quad I_P \sigma ds dA = I_P d\tau dA.$$

On interaction, the flux will be reduced to γ of its initial value by absorption and then reemitted into some new direction, θ , an angle measured from the original direction of motion of the light. The angular distribution of the scattered radiation is described by the phase function, $\mathcal{P}(\cos\theta)$.

It is normalized to γ so that

$$7) \quad \oint \mathcal{P}(\cos\theta) \frac{d\omega}{4\pi} = \gamma.$$

Since \mathcal{P} is normalized to γ and the scattering angles α and θ are equivalent, we may express the phase function in terms of α and explicitly indicate the γ dependence by the expression $p(\alpha, \gamma)$. The energy scattered into direction α is then

$$\frac{L}{4\pi r^2} \cdot \exp\left[-\frac{kx}{|\mu_0|}\right] \cdot p(\alpha, \gamma) d\tau dA.$$

On its trip to the front surface of the nebula, the intensity will be reduced by a factor of $\exp[-kx/\mu]$. The energy emerging from area dA contributed by the grains at P will then travel unhindered to the observer, giving rise to a flux of

$$8) \quad dI = \frac{L}{4\pi r^2} \cdot \exp\left[-\frac{\mu + \mu_0}{\mu \mu_0} \tau\right] p(\alpha, \gamma) \, d\tau \frac{dA}{4\pi R^2}.$$

The total flux reaching the observer will be the integral of this expression along the line of sight through the nebula. Since

$$d\tau = k \frac{dx}{\mu},$$

we may integrate equation 8) between the limits 0 and D. If we restrict ourselves to cases for which $\mu_0 \geq 0$ and make the replacement

$$\frac{dA}{R^2} = d\omega',$$

integration of equation 8) yields the nebular flux reaching the observer:

$$9) \quad I_n = p(\alpha, \gamma) \frac{L}{(4\pi r)^2} G(\alpha, \tau_0) \, d\omega',$$

where $\tau_0 = kD$

and

$$10) \quad G(\alpha, \tau_0) = \frac{\mu_0}{\mu + \mu_0} \{ 1 - \exp[-\tau_0 \frac{(\mu + \mu_0)}{\mu \mu_0}] \}.$$

For brevity, this has been written as $G(\alpha, \tau_0)$, although the actual angular dependence is on α_i and α_s .

The stellar flux reaching the observer is

$$11) \quad I_* = \frac{L}{4\pi R^2}.$$

Since R will be only poorly known, as will r, these terms may be eliminated from equations 9) and 11) as follows: denote by a the angular separation between the star and the most distant patch of nebulosity detected at Hubble's limiting surface brightness. Then inspection of Figure 1 and the use of the small angle approximation yields

$$12) \quad r \sin \alpha = aRc,$$

where c is a conversion constant from radians to seconds of arc. Dividing

equation 9) by equation 11) and eliminating r and R by use of equation 12), we may convert to the magnitude system, finding

$$13) \quad m_{\star} = m_n + 2.5 \log[p(\alpha, \gamma) G(\alpha, \tau_0) \sin^2 \alpha] - 5 \log a' - 11.64.$$

Here m_n is the limiting surface brightness which can be detected on the plate in magnitudes/" for some standard set of observing conditions and a' is the angular distance from the star to the most distant patch of nebulosity, measured in minutes of arc. The numerical constant arises from numerical constants in the equations and m_{\star} is the apparent magnitude of the illuminating star.

III. CORRECTIONS FOR ILLUMINATION BY THE GENERAL STELLAR FIELD

The generalized Hubble relation between m_{\star} and a' which has just been derived is based on the nine assumptions listed earlier. Since the starting point of this investigation was the fact that the albedo (or equivalently, the surface brightness) of reflection nebulae appears to be higher than expected on the basis of DGL studies, we must examine the assumptions in order to guarantee that we have not overlooked any contributions to the surface brightness in formulating the equation.

Clearly both the assumptions of single scattering and of illumination of the nebula by only one source are suspect. Both of these assumptions may be removed, at least approximately, by specializing the geometry to the case of a plane parallel structure in order to derive correction terms. The assumption is being made here that the correction terms devised for a plane parallel nebula will not differ radically from the correction terms for an irregular geometry.

The procedure for deriving the correction term for illumination of the nebula by the general stellar field (hereafter abbreviated GSF) is to compute the surface brightness to be expected from a plane parallel nebula

illuminated by an isotropic radiation field. The surface brightness of the illuminating sphere is equal to the average integrated surface brightness of the sky as seen by the nebula. It is then possible to determine the GSF component of the surface brightness of the reflection nebula as a function of γ , τ_0 , and phase function.

Chandrasekhar (1950) has obtained solutions in closed form for plane parallel atmospheres for both semi-infinite atmospheres and atmospheres of finite optical thickness. Of the phase functions considered by Chandrasekhar, only two will be of interest to us. They are

$$14) \quad p(\cos\theta) = \gamma$$

and

$$15) \quad p(\cos\theta) = \gamma(1 + x\cos\theta), \quad -1 \leq x \leq 1.$$

Equation 14) describes an isotropic scatterer and Equation 15) describes a scatterer whose asymmetry parameter is given by

$$g = x/3.$$

Clearly the largest value which g may attain for equation 15) is .33. Although this is only about half of the value of g which one finds for general interstellar particles, it is the largest value for which closed form solutions could be found. The numerical values required for computations for the case of semi-infinite atmosphere were taken from Chandrasekhar (1950). The corresponding functions for finite optical thickness were taken from Sobouti (1963) and from Carlstedt and Mullikin (1969). The intensity of light reflected in any direction was calculated as a function of γ for $\tau_0 = .6, 1, 2, 3$, and ∞ with $g=0$ and for $\tau_0=\infty$ for $g=.33$. The GSF correction coefficients, which have the physical interpretation of being the fraction of incident GSF intensity which is diffusely reflected by the nebula, are listed in Table I. The angle of observation assumed in this

case was 0° , but it was found that changing α_s to 45° resulted in a change in surface brightness of less than 10% in all cases.

Two points are worthy of note: first, for finite τ_0 , the unavailability of the required tabulated functions necessitated using the results of the isotropic phase function calculations for the phase function of equation 15). To some extent this can be qualitatively justified by noting that in the finite case the nebula is illuminated from both sides and the extent to which light is forward (or backward) scattered by one surface is about the same extent to which light is forward (or backward) scattered by the other side. The second point is that neither the assumed illuminating geometry (a sphere) nor the plane parallel nebular structure is realistic. If one maintains the same average sky intensity, but changes the illuminating geometry from a uniform sphere to a sphere with a brighter band, the GSF intensity increases. Quantitatively, half of the total intensity was taken as coming from band of angular extent 30° and the remainder of the sphere contributed the remaining half of the incident light. This change increased the GSF correction by 50%. Unpublished calculations by Witt and Stephens employed the Monte Carlo technique for spherical nebulae and find the GSF correction to be four times the values given in Table 1. Calculations were carried out using the values given in Table 1 and repeated after multiplying each value in Table 1 by a factor of four. The larger values yield GSF contributions which are in good agreement with Mattila's (1970b) observations of surface brightnesses of dark nebulae.

In order to incorporate the GSF correction into the calculation, note that the term m_n in equation 13) refers to the limiting magnitude/ \square " which will just blacken a photographic plate under a set of standard

observing conditions. We now recognize that the intensity corresponding to this surface brightness limit, I_{lim} , is the sum of two terms:

$$16) \quad I_{lim} = I_s + I_{GSF},$$

where I_s is the nebular surface brightness due to the illuminating star. If we denote the GSF correction factors of Table 1 by $\Delta(\gamma, g\tau_0)$ and the average surface brightness of the sky seen by the nebula by I_0 ,

$$I_{lim} = I_s + I_0 \cdot \Delta(\gamma, g\tau_0).$$

Physically, for a given I_{lim} , a nebula's apparent size is now increased since the two sources of surface brightness are additive. Rearranging this last equation, dividing by I_{lim} , and converting to the magnitude system, we find

$$17) \quad -D = m_{lim} - m_s = 2.5 \log \left(\frac{I_{lim}}{I_{lim} - I_0 \cdot \Delta} \right).$$

D is thus a positive term which is to be added to the right side of equation 13) to include the effect of GSF illumination.

IV. THE MULTIPLE SCATTERING CORRECTION

The second assumption underlying equation 13) is that of single scattering. The basis of the multiple scattering correction technique is the empirical observation that the difference between the exact and the single scattering solutions to the radiative transfer problem is independent of α to a good approximation. That is, for a set values of γ, g , and τ_0 , it is approximately true that

$$18) \quad I(\gamma, g, \mu, \mu_0, \tau_0) = I^{(1)}(\gamma, g, \mu, \mu_0, \tau_0) + C(\gamma, g, \tau_0),$$

where the multiple scattering correction coefficient, C , must be determined empirically from a plot of the complete scattered intensity, I , and the

single scattered intensity, $I^{(1)}$, as a function of μ for a given μ_0 . Typical curves are shown in Figure 2. That the validity of this approximation does not depend on the particular geometry assumed can be seen from the work of Mattila (1970a), who solves the radiative transfer problem for a spherical nebula using the Monte Carlo technique. He presents plots of a single and multiple scattering intensities similar to those in Figure 2 and his plots show that equation 18) holds true for a spherical geometry. Empirically it was found that C is very insensitive to μ_0 .

The physical reason that C has little dependence on μ and μ_0 is easy to understand: C is a measure of the intensity of the radiation which has undergone more than one scattering and thus, the original directionality which the incident light had has been destroyed by several scatterings.

Numerical values of C were determined by measuring the distance between the exact and single scattering curves and expressing the results as a fraction of the incident intensity. This method is quite simple unless $\gamma > .9$, in which case the approximation of equation 18) sometimes fails. In such instances, an estimate of the average distance between the two curves was made. Table 2 lists the values of C . Note that there is a slight difference between the front and rear surface terms.

Denoting the values of Table 2 by $C(\gamma, g, \tau_0)$, we can see how to modify equation 13) to account for multiple scattering. The addition of this scattered intensity will modify equation 9), making the correct expression for the nebular surface brightness

$$19) \quad I_n = \left\{ \frac{LpG}{4\pi r^2} + \frac{LC}{4\pi r^2} \right\} \frac{d\omega}{4\pi} .$$

Exactly the same steps which take one from equation 9) to equation 13) can be taken starting with equation 19), leading to the result

$$20) \quad m_{\star} = m_{lim} + D - 5 \log[a'] + 2.5 \log[\sin^2 \alpha \{ p(\alpha, \gamma) G(\alpha, \tau_0) + C(\gamma, g, \tau_0) \}] \\ - 11.64 + \left[\frac{|\mu_0| - \mu_0}{2\mu_0} \right] \cdot 2.5 \log[\exp(-\tau_0/\mu_0)]$$

Here C is multiplied by $\sin \alpha$ to correct for projection effects due to inclination of the line between the star and nebula to the line of sight. Note that the GSF correction term D , as given in equation 17], has also been incorporated into this equation. The term m_n , representing only the nebular surface brightness, has been replaced by $m_{lim} + D$ to account for the fact that the plate limit can be reached by a combination of illumination from both the illuminating star and the GSF. The last term of equation 20) is an artifice so designed that it is zero for $\mu_0 > 0$ (star in front of nebula) and for $\mu_0 < 0$ (star behind nebula), this term corrects for the fact that the star's apparent magnitude is increased by the extinction within the nebula.

V. THE COMPUTATIONAL TECHNIQUE FOR DETERMINING THE ALBEDO

With equation 20) as a starting point, we can now devise a computational technique for determining the albedo corresponding to any assumed phase function for the particles composing reflection nebulae. The only restrictions in practice are that it must be possible to determine the correction terms C and D .

The observational data which are employed in determining γ are observations of m_{\star} and $\log a'$. As shown by Hubble (1922) and by Dorschner and Görtler (1966), the points on a plot of m_{\star} as a function of $\log a'$ can be well fit by a relation of the form

$$21) \quad m_{\star} = H_0 - 5 \log a'$$

Note that H_0 , the intercept on the m_{\star} axis on this plot, is the single

number which results from the observational data. Subtracting equation 21) from equation 20) yields

$$22) \quad q = m_{im} + D(I_0, \Delta) - H_0 - 11.64 + 2.5 \log [\sin^2 \alpha \{ p(\alpha, \gamma) G(\alpha, \tau_0) + C(\gamma, g, \tau_0) \} + \frac{\mu_0 - |\mu_0|}{2\mu_0}] \cdot \log [\exp(-\tau_0/\mu_0)] > .$$

The condition that the theoretical and observational equations agree is satisfied when $q=0$. One new feature has appeared in equation 22)--the brackets around the last two terms. These brackets indicate averaging over all values of α and τ_0 , a necessary step before equations 20) and 21) may be subtracted. This can be readily seen when it is recalled that equation 20) was derived for a specific nebula, under the assumption that α and τ_0 have some unique value, while equation 21) is a result obtained from observations of many different nebulae with various α and τ_0 values.

The proper technique of averaging over angles is not trivial. Neither Hubble (1922) nor Zanstra (1927) has considered this correction in detail. Since the nebular surface brightness does not depend on α in a simple fashion, it was felt that the best approach to the angular averaging required in equation 22) was a numerical computation. The method which was adopted was to start by assuming a phase function (from the limited choice of two) and an average optical thickness of τ_0 , of .6, 1, 2, or ∞ . A value of γ was then also assumed. These assumptions then determine the correction coefficients (C and D of equation 22). All angles of incidence and scattering were assumed equally probable and the argument of the logarithm was numerically averaged over all angles of incidence and scattering taking angular increments of 10° . The value of q could then be calculated. The assumed value of γ was then changed and the process was repeated and

tables of γ and q were computed. The value of γ corresponding to $q=0$ was then determined by interpolating the table.

VI. RESULTS OF THE CALCULATIONS

The formalism which has been developed allows one to calculate the albedo of reflection nebular particles if one assumes a phase function and average τ_0 value, given the two observational values m_{lim} and H_0 . The observational data were taken from the original work of Hubble (1922) and from the more extensive work of Dorschner and Gürtler (1966), who determined $\log a'$ values from the Palomar Sky Survey prints in both the red and blue bandpasses.

Table 3 summarizes the results of the calculations. Here I_0 is the average GSF intensity in units of 10th magnitude stars per square degree, m_{lim} is the plate limit in m/\square'' , and H_0 is the m -axis intercept given by each author. The uncertainties in γ values of Table 3 were determined by varying each of the input parameters by its maximum uncertainty and in such a direction as to make γ as large (or small) as possible. Uncertainties in C were as high as $\pm 30\%$ in some cases, corresponding to an uncertainty of ± 0.05 in γ . The largest uncertainty in γ arises from the GSF correction.

The value of γ which one determines for particles of reflection nebulae is seen to depend on the average optical thickness, τ_0 , of such nebulae. An upper limit to the average optical thickness is probably about 3, a value determined by Lynds (1965) for a high latitude dark nebula and by Mattila (1970b) for the Southern Coalsack. While there are undoubtedly many nebulae much thicker than this, it must be remembered that τ_0 is the average thickness. The lower limit to the optical thickness is

set by selection effects and is about $\tau_0 = .6$. The average τ_0 value is probably between 1 and 3.

Inspection of Table 3 indicates that if one accepts the restriction that $1 \leq \tau_0 \leq 3$, the albedo determined for the particles comprising reflection nebulae is seen to agree to within observational uncertainties with the value of albedo determined for interstellar particles in general. The physical reason that the value which is determined for γ decreases with increasing τ_0 is that as nebulae become more optically thick, they reflect more of the incident light, and thus requiring lower reflectivity of the particles.

VII. SELECTION EFFECTS AND THE BASIC ASSUMPTIONS

The results of the previous section indicate that the albedo of reflection nebular particles seems to be the same as the value for interstellar particles as determined by DGL studies to within observational uncertainty. It will now be shown that the alteration of most of the basic assumptions will raise the value of γ above that shown in Table 3.

In what follows it is useful to note that any effect which decreases $\log a'$ will result in the determination of a value of γ which is lower than the true value.

Assumption 1: This assumption that the nebula is illuminated by only one star has been removed by the inclusion of the term $D(I_0, \Delta)$ in equation 22).

Assumption 2: The correction for multiple scattering has been made by including the term $C(\gamma, g, \tau_0)$ in equation 22).

Assumption 3: The intensity of light reaching the front surface of the nebula could be less than the value used in deriving the theoretical relationship due to the presence of matter between the star and the surface of the nebula. Although it is difficult to estimate the magnitude of this effect,

its direction is clear--intervening matter makes nebulae appear smaller, causing the γ value determined from equation 22) to be too small. Thus the larger albedo values derived by Hubble (1922) and Henyey and Greenstein (1939) are not due to intervening matter.

Assumption 4: Nebulae having their apparent angular dimensions limited by a lack of scattering dust will necessarily appear smaller than they would if their extent were determined by photon limitations. Since equation 22) was derived on the assumption of all nebulae being photon limited, the existence of dust limited nebulae reduces the apparent sizes of some nebulae, thus again causing the value of γ derived from equation 22) to be too small. The magnitude of this effect is also difficult to estimate, but dust limitations are not responsible for the large albedo determined for reflection nebulae by earlier workers.

Assumption 5: Including the r^{-2} dependence of stellar flux within the nebula will reduce the flux inside the nebula, resulting in lower surface brightness values, smaller a' values, and consequently a value of γ which is lower than the actual value.

Assumption 6: Inspection of the steps leading to equation 22) reveals that the assumption of a uniform plane parallel nebular geometry never enters directly into the calculation. The entire argument leading to equation 13) is independent of geometry, except for the term $G(\alpha, \tau_0)$ of equation 8) and 10), which involve only optical distances and directions. The plane parallel assumption enters most importantly into the calculation in the determination of the correction terms C and D. While the importance of departure of the assumed geometry from a plane parallel structure is difficult to estimate, the similarities between the plane parallel results computed here and the results of Mattila (1970a) (see Section IV) do not indicate strong dependence of results on the specific geometry.

Assumption 7: The possibility that there is obscuring matter between the nebula and the observer must be considered under two different circumstances. The first is the case for which the extinction is constant over the field of view. Cederblad (1946) has shown that this leaves the form of the Hubble relation unaltered. In this case, the value of γ determined from equation 22) will be unaltered also. This will not hold true if the extinction changes across the face of the nebula.

In constructing $m - \log a'$ diagram there may be a selection effect which operates to make nebulae appear too large. To visualize the origin of this effect, consider a star situated in front of a plane parallel nebula. With no intervening extinction, one would see a star in the center of a nebula whose surface brightness fades evenly into the night sky background. However, the introduction of rapidly varying extinction over this scene results in some random value of extinction for the star and a distribution of extinction for the "edges" observed for the nebula. An observer will select the patch of nebulosity experiencing the lowest extinction of $\log a'$. This leads to an overestimate of $\log a'$ for every case except that for which the extinction suffered by the star is less than that experienced by every point at the detectable "edge" of the nebula. This effect would cause an overestimate of γ . If the difference between the extinction suffered by the star and by the edge of the nebula is small (less than .1 magnitude), the change in H_0 will not be enough to alter γ by more than about 0.05, even if the effect occurs in every nebula observed. If the effect is very large, and an observer recognizes the presence of foreground extinction, the nebula can be omitted and the results will be unaffected by this effect. However, the probability of the presence of an intervening cloud across whose face the extinction changes by more than .1

magnitude and whose position overlaps a reflection nebula is difficult to estimate realistically. A rather hard upper limit to the magnitude of this effect can be obtained by adopting the standard cloud picture of the interstellar medium as discussed by Münch (1952) or Spitzer (1968). We assume the interstellar medium to be composed of uniform clouds characterized by an extinction of .2 magnitude and a density such that there are typically about 8 clouds in a 1 kpc. line of sight. For a rough upper limit to the magnitude of this effect, suppose that in every reflection nebula, the star is behind two more standard clouds (or .4 magnitude) than some point on the nebula's edge. This increases the value of H_0 by .4, corresponding to a change in γ of at most .03. Thus, although intervening matter between the nebula and the observer might raise the value determined for γ , the magnitude of the effect is small.

Assumption 8: The assumption that the distribution of angles α_i and α_s is random appears to be sound since the sun does not occupy a special point in the Galaxy. However, there is a possible selection effect which would cause observers to preferentially detect nebulae whose angles α_s are near 90° . To estimate the possible importance of this effect, the calculations of equation 22) were repeated, but with all contributions to the angular average were weighted double if α_s were larger than 45° . The effect was found to have a very small influence on γ .

Examination of the assumptions to this point indicates that the values given in Table 3 are lower limits in the sense that the alteration of any assumption except the seventh or eighth would increase the value obtained for γ . However, in those cases for which the magnitude of effects can be estimated, the change in γ is seen to quite small and there appears to be no reason to believe that the value determined for the

albedo of reflection nebular particles by this technique is unreliable.

VIII. THE POSSIBILITY OF FLUORESCENCE

The possibility that the ninth assumption is invalid and that some radiation leaving the nebula as visible light entered in another form can be discarded on the basis of the previous paper by Rush and Witt (1974). However, it is of interest to be able to determine how sensitive the value which is obtained for the albedo is to any possible fluorescence. This would permit the rejection of any model of interstellar particles which implied the existence of fluorescence whose intensity would significantly alter the albedo determined for the particle.

With the formalism developed above, it is a simple matter to include the effect of fluorescence. For simplicity, assume the fluorescent light is emitted isotropically. Let us then define a fluorescence parameter, Φ , as the ratio of the component of surface brightness due to fluorescence to the component due to reflection. Inspection of equation 22) shows that the reflected surface brightness is given by the terms

$$\sin^2 \alpha \{ p(\alpha, \gamma) \cdot G(\alpha, \tau_0) + C(\gamma, g, \tau_0) \} .$$

Since the term in brackets is the reflected intensity, multiplying this by a factor of

$$1 + \Phi$$

will add in the additional contribution to the surface brightness due to fluorescence. With this modification, a value was assumed for Φ and the calculations were repeated with $\Phi = .1$ and $\Phi = .3$. The results are shown as the last two sets of entries in Table 3.

IX. CONCLUSIONS

A reformulation of the Hubble relation for reflection nebulae has

resulted in a computational technique for calculation γ from observations of m and $\log a'$ if one assumes a phase function and τ_0 value. Multiple scattering and illumination of the nebula by the general stellar field are taken into account.

It has been argued that alteration of most of the assumptions made in the derivation of this formulation would result in an underestimate of the value of γ obtained by this method. The alteration of assumptions #7 and #8 would result in a very small (.05-.07) overestimate of γ . Thus, the values of γ obtained from reflection nebulae appear to be in substantial agreement with those values obtained for interstellar particles by DGL workers.

The calculations described here indicate that the problem of the large extent of nebula B10 cannot be resolved by additional surface brightness contributions from the galaxy or fluorescence. In order to reconcile the size of this nebula with the Hubble relation, the dust would have to be a 100% efficient converter of ultraviolet radiation into visible and the star would have to be more than a hundred times more luminous in the ultraviolet than in the visible. The possibility suggested by Struve (1961) that B10 contains a group of newly formed stars at its center appears to be a more fruitful hypothesis.

I would like to thank Dr. Adolf Witt for suggesting this problem and for his guidance and criticism during its solution.

TABLE I GENERAL STELLAR FIELD CORRECTION COEFFICIENTS

τ_0	$\gamma=1$	$\gamma=.9$	$\gamma=.8$	$\gamma=.7$	$\gamma=.6$	$\gamma=.3$
.6	.12	.10	.08	.06	.05	.02
1.0	.17	.13	.10	.08	.06	.02
2.0	.23	.16	.11	.08	.06	.02
$\infty, g=0$.50	.30	.14	.11	.08	.03
$\infty, g=.33$.31	.20	.11	.08	.05	.01

Table II The Multiple Scattering Correction Coefficients, $C(\gamma, g, \tau_0)$

	τ_0	g	$\gamma=1$	$\gamma=.9$	$\gamma=.8$	$\gamma=.7$	$\gamma=.6$	$\gamma=.5$	$\gamma=.4$	$\gamma=.3$
Front	.6	0	.20	.16	.12	.09	.07	.05	.04	.03
	1.0	0	.27	.21	.15	.10	.07	.05	.03	.02
	2.0	0	.41	.29	.17	.11	.06	.04	.02	.01
	∞	0	.82	.57	.17	.11	.06	.04	.02	.01
	∞	.33	.46	.32	.17	.11	.06	.04	.02	.01
Rear	.6	0	.19	.14	.10	.08	.06	.04	.02	.02
	1.0	0	.21	.15	.10	.07	.05	.03	.02	.01
	2.0	0	.21	.12	.04	.03	.03	.02	.01	.00

Table III Summary of Results of Model Calculations

Observer	$I_0 \frac{S_{10}}{\square}$	$m_{lim} \frac{m}{\square \pi}$	H_0	τ_0	$\gamma(g=0)$	$\gamma(g=1/3)$
Hubble (1922)	66	23.25	11.1	.6 1.0 2.0 3.0	.89-1.00 .76-.90 .56-.63 .34-.38	.94-1.00 .81-1.00 .64-.75 .48-.51
Dorschner & Gürtler (1966) Blue	66	23.6	12.0	.6 1.0 2.0 3.0	.93-1.00 .81-1.00 .67-.82 .45-.55	.97-1.00 .83-1.00 .72-.84 .60-.67
Dorschner & Gürtler (1966) Red	116	22.4	10.8	.6 1.0 2.0 3.0	>1 .92-1 .74-.83 .50-.57	>1 .96-1 .80-.88 .64-.69
Dorschner & Gürtler (1966) Blue + 10% fluorescence	66	23.6	12.0	.6 1.0 2.0 3.0	.91-1.00 .79-1.00 .64-.79 .43-.51	.95-1.00 .83-1.00 .70-.84 .55-.65
Dorschner & Gürtler (1966) Blue + 30% fluorescence	66	23.6	12.0	.6 1.0 2.0 3.0	.86-1.00 .75-.98 .60-.72 .39-.45	.91-1.00 .80-1.00 .67-.80 .53-.60

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FIGURE CAPTIONS

Figure 1 - The symbols used in the derivation of the generalized Hubble Relation.

Figure 2 - The relation between the exact (solid line) and single scattered (broken line) solutions to the radiative transfer problem are seen to be constant, independent of angle. The assumed values of parameters are shown on each plot. Ordinate units are incident intensity.



