A NOTE ON PARALLEL AND PIPELINE COMPUTATION OF FAST UNITARY TRANSFORMS

by

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ABSTRACT

This correspondence discusses the parallel and pipeline organization of fast unitary transforms algorithms such as the Fast Fourier Transform and points out the efficiency of a combined parallel-pipeline processor of a transform such as the Haar transform in which (2^n-1) hardware "butterflies" generate a transform of order 2^n every computation cycle.

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8

Algorithms for all fast unitary transforms, such as the Fast Fourier transform (FFT), fast Walsh-Hadamard transform (FWT) and other fast unitary transform [1], require n stages of computation for transforms of order 2^n. Each stage of computation can be in turn decomposed into at most 2^{n-1} "butterflies" [2], each performing a rotation by a matrix of order 2. Some or all of the butterflies at one stage of computation can operate in parallel (see [3], [4] for FFT) and fast unitary transforms have thus a greater potential in applications with the development of low cost parallel circuitry. For example, we show in Fig. 1a the FFT Cooley-Tukey algorithm of order 4 with 2 butterflies in each of its 2 stages of computation. If τ seconds is the time required to perform a butterfly operation, each stage can be performed in τ seconds with the highest possible degree of parallelism which uses 2^{n-1} butterflies. Thus, a transform of order 2^n can be performed in $n\tau$ seconds as compared to $n2^{n-1}\tau$ seconds with sequential computation (which requires only one butterfly).

If a number of successive transforms have to be computed, it is possible to increase further the throughput rate with several transformers working simultaneously, each operating on a different input vector and each possibly at a different stage of computation (see [5] for FFT): this is generally referred to as a pipeline organization. Parallel and pipeline organizations can be combined conveniently with $n2^{n-1}$(at most) butterflies working in parallel and one transform of order 2^n is obtained every τ seconds on the average. Fig. 1b shows a possible organization of the FFT Cooley-Tukey algorithm of order 4. All stages of this pipeline algorithm are identical: the 2 first butterflies perform the first stage of Fig. 1a and the 2 last butterflies perform the second stage. The input vector is entered in the first 4 cells and its FFT transform obtained in the same cells after 2 cycles. This algorithm can be wired-in and will give the transform coefficients in any order but it requires a large amount of hardware and requires the access at its input of two sets of $n2^n$ storage cells.[1]

Some transforms, however, do not require 2^{n-1} butterflies at each stage of computation and then a pipeline algorithm can be implemented with much less hardware. We consider now in particular a pipeline algorithm for the Fast Haar Transform (FHT). Although less known, the FHT is closely related to the FWT [6], has a fast algorithm [7], is certainly a transform of interest for signal encoding [8], [9] and other applications [10]. A pipeline-parallel algorithm for the FHT requires only (2^n-1) butterflies and still produces a transform of order 2^n at every cycle. We show in Fig. 2a the Haar matrix of order 8 and in Fig. 2b a possible organization of the FHT of the same order. The number of butterflies decreases for successive stages and this is the property which can be exploited in a pipeline processor. In Fig. 3, we show a stage of a possible organization of the pipeline FHT of order 8.

Many other transforms can have similar pipeline algorithms with reduced amount of hardware: the Modified generalized discrete transforms [11], the WFH transforms [1], the Slant Haar transforms [12] and other generalized Slant transforms [13]. In all cases, the pipeline-parallel algorithm needed to perform a transform of order 2^n in one cycle is the total number of butterflies appearing in the flow diagram of the algorithm. By contrast, parallel processing requires the maximum number of butterflies needed at any stage.

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FOOTNOTE

[1]The computation can be also performed "in place" with $n2^n$ storage cells only followed by cyclic shifts by 2^n cells.

CAPTIONS

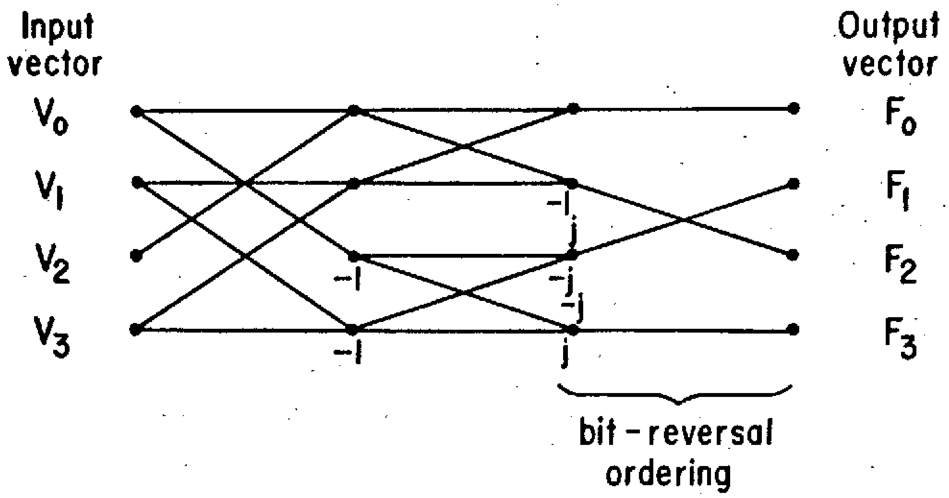
Fig. 1a : FFT Cooley-Tukey Algorithm of order 4

Fig. 1b : Pipeline FFT Cooley-Tukey Algorithm of order 4

Fig. 2a : Haar matrix of order 8

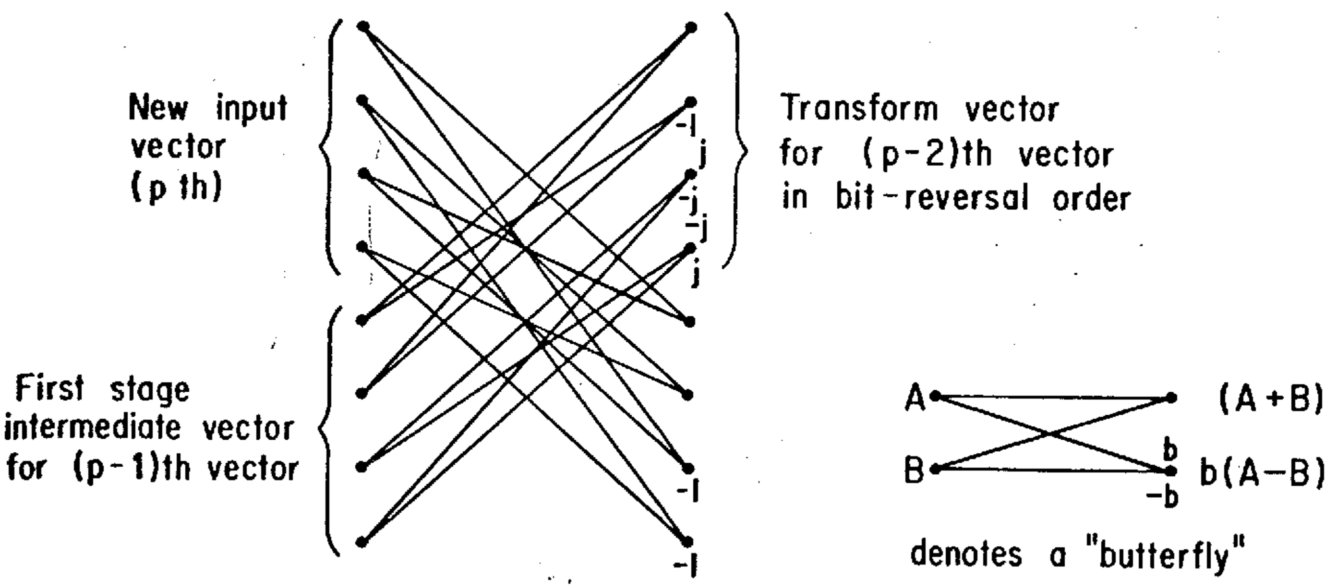
Fig. 2b : Fast Haar Transform of order 8

Fig. 3 : Pipeline Fast Haar Transform of order 8.



Input
vector
V_0
V_1
V_2
V_3
Output
vector
F_0
F_1
F_2
F_3
-1
-1
-1
j
-j
-j
j
bit-reversal
ordering

(a)

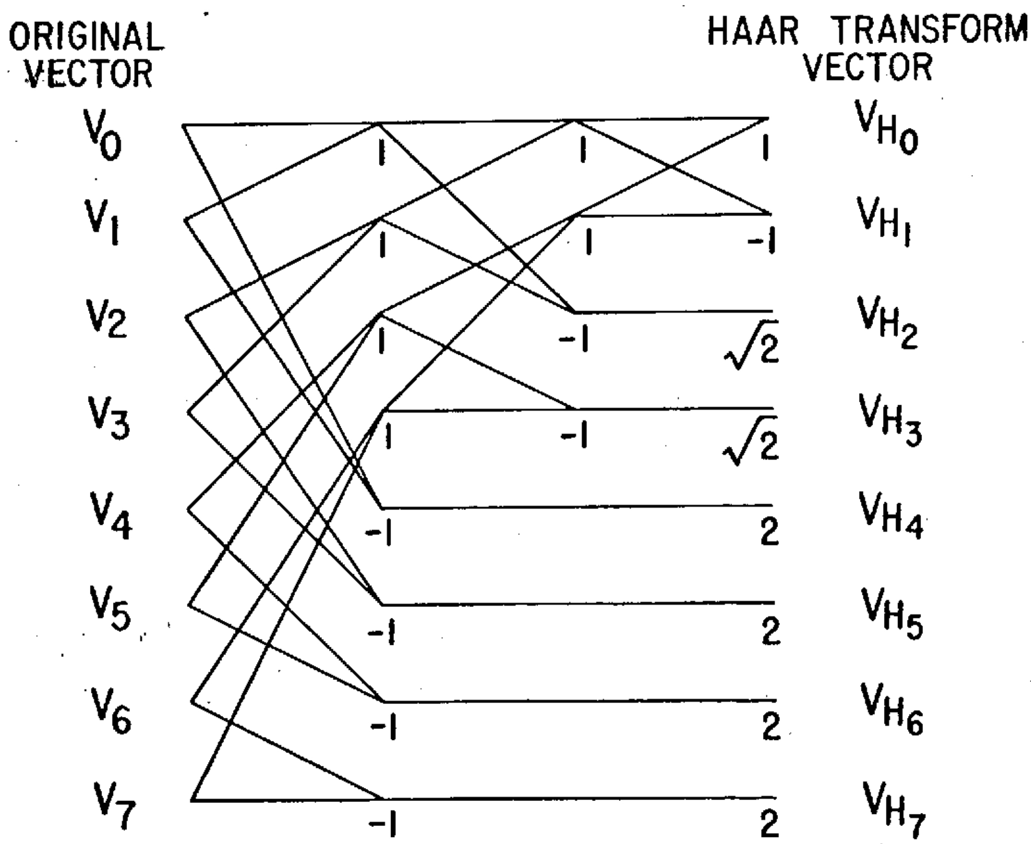


New input
vector
(p th)
First stage
intermediate vector
for (p-1)th vector
Transform vector
for (p-2)th vector
in bit-reversal order
-1
j
-j
-j
j
-1
-1
A
B
(A+B)
b
b(A−B)
−b
denotes a "butterfly"

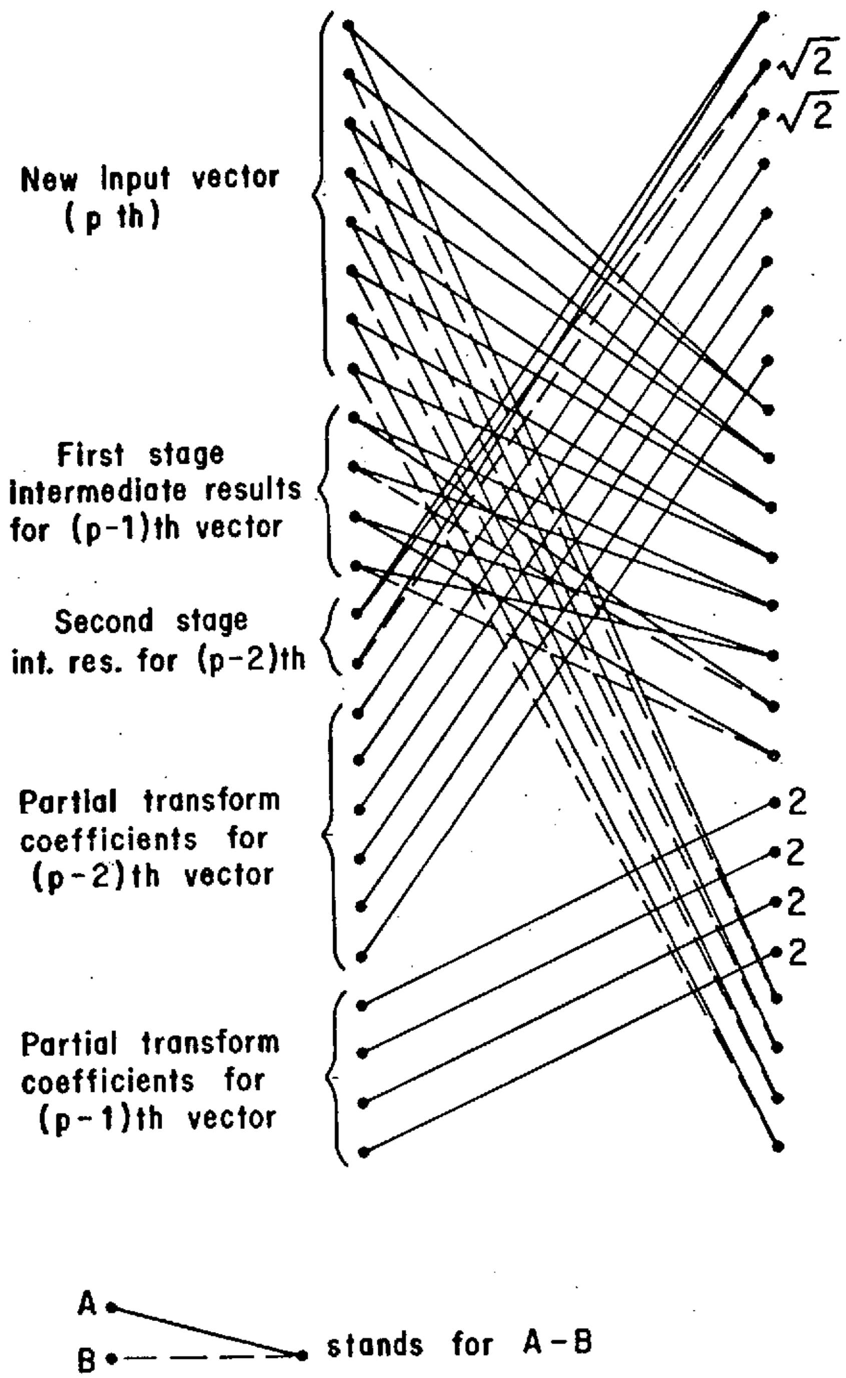
(b)

$$[H_8] = \frac{1}{\sqrt{8}} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{vmatrix}$$

(a)



(b)



New input vector
(p th)
First stage
intermediate results
for (p-1)th vector
Second stage
int. res. for (p-2)th
Partial transform
coefficients for
(p-2)th vector
Partial transform
coefficients for
(p-1)th vector
√2
√2
2
2
2
2
A
B
stands for A-B