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ANALYSIS AND PLOTTING OF HBDY ELEMENTS
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PREFACE

This report describes changes made to the NASTRAN Thermal Analyzer for Variance Analysis and plotting of HBDY Elements. The theory and the User Information are given. Three demonstration problems were used to test the capability.

INTRODUCTION

The NASTRAN Thermal Analyzer has been extended to do variance analysis and plot the thermal boundary elements. The capability to compute the temperatures resulting from specified flux, boundary temperatures, radiation, etc., had been installed previously[1].

The objective of the variance analysis addition is to assess the sensitivity of temperature variances resulting from uncertainties inherent in input parameters for heat conduction analysis. The plotting capability provides the ability to check the geometry (location, size and orientation) of the boundary elements of a model in relation to the conduction elements.

VARIANCE ANALYSIS

Method

Variance analysis is the study of uncertainties of the computed results as a function of uncertainties of the input data. To study this problem using NASTRAN, a solution is made for both the expected values of all inputs, plus another solution for each uncertain variable. A variance analysis module subtracts the results to form derivatives, and then can determine the expected deviations of output quantities.

For heat transfer analysis, any real number in the Bulk Data Deck may be treated as an uncertain item. This can include conductivity, heat capacity, film coefficients, emissivities, applied heat, boundary temperatures, etc. The output quantities can include temperatures, heat of constraint at specified temperature boundaries, temperature gradients and fluxes within heat conduction elements, and radiation exchange between boundary elements.

Theory and Definitions, Variance Analysis

Variance analysis is a study of the statistical properties of random variables. Some terms will be defined using a notation similar to Feller[2]. Let U be a random variable, and E(U) be the expectation (or expected value) of the random variable. The *mean* value of a random variable U is given by

$$\mu = E(U) \tag{1}$$

A new random variable can be defined, $U-\mu$, which is the *deviation from the mean*. The expectation of the square of the deviation from the mean is called the *variance*.

$$\mathrm{Var}(U) = E((U-\mu)^2) \tag{2}$$

The positive square root of the variance is called the *standard deviation*.

$$\sigma = (\mathrm{Var}(U))^{\frac{1}{2}} \tag{3}$$

The standard deviation is a measure of the scatter of the values taken by the random variable, and can also be interpreted as the width of the probability distribution curve. Other measures of scatter are possible (which differ from the standard deviation by a constant factor). The term *deviation* will be used for any measure of scatter; for example, the range which contains ninety-nine percent of the data points. The deviation will be written Dev(U).

Variance analysis applied to structural analysis deals with the deviations of computed quantities as a result of deviations of data items. In the general case, there may be several data items which have uncertainty and for which a mean value and deviation are known. The question is to determine the mean and deviation of the output quantities. Let the data items be $D_1, D_2, \ldots, D_N$. Let the output quantity be U. From a standard structural analysis, the result is given by

$$U = U(D_1, D_2, \ldots, D_N) \tag{4}$$

If the data items are independent, or *uncorrelated*, then the deviation of the result is given by

$$\mathrm{Dev}(U) = \left[\sum_{j=1}^{N} \left(\frac{\partial U}{\partial D_j} \mathrm{Dev}(D_j) \right)^2 \right]^{\frac{1}{2}} \tag{5}$$

The basic requirement for Equation 5 to hold is that the deviations of the input quantities must be sufficiently small that the output quantities are linear functions of the inputs.

The derivatives will be computed from

$$\frac{\partial U}{\partial D_j} = \frac{\Delta U_j}{\delta} \tag{6}$$

where ΔU is the computed result with the jth quantity changed minus the basic result. δ is a user specified parameter. The user must select the size of the change of the input parameter sufficiently large that Equation 6 is evaluated accurately. Usually a one to ten percent change is about the right size. If the change is too small, accuracy will be lost in the subtraction needed to compute the numerator. If the change is too large, nonlinear behavior will be seen.

Plotting HBDY Elements

The boundary condition elements for the NASTRAN heat conduction analyzer can be plotted for undeformed structure plots. This has been integrated into the structural plotter. General user instructions are given in the NASTRAN User's Manual[3], Section 4. Some special instructions are given in the MSC/NASTRAN Application Manual[4], Sections 2.3 and 2.5. This section only describes changes from the documentation referenced above.

There are several types of HBDY elements, as follows:

Type	Number of Primary Grid Points	Normals Available
PØINT	1	yes
LINE	2	yes
AREA3	3	yes
AREA4	4	yes
REV	2	no
ELCYL	2	no

The secondary grid points are used for "ambient" conditions, and are ignored by the plotter. Type PØINT must have a nonzero associated area (see AF on the associated PHBDY data card) and a defined normal direction (see V1, V2, V3 on the CHBDY data card) to be plotted. It is plotted as a hexagon with approximately the correct area. Type LINE must have a nonzero width (see AF on the associated PHBDY data card) and a normal defined in order to be able to plot. The others plot via their associated geometry defined by primary grid points.

Input Data Card 1VARY

Description: Defines functional relationships for bulk data changes. Used for variance analysis.

Format and Example:

1	2	3	4	5	6	7	8	9	10
1VARY	MNEM	FIELD	K	A	B	ID1	ID2	ID3	
1VARY	MAT4	3	1	.01	1.0	7	THRU	9	

Field	Contents
MNEM	Name of bulk data cards to be modified (BCD).
FIELD	Location of data items to be changed (integer > 0).
K	Parameter index (integer > 0).
A, B	Coefficients (real).
ID1 thru ID3	Identifiers of data cards to be modified (integer) or ID2 may be "THRU." See Remark 2.

Remarks:

1. If $ID > 0$, the identifiers ID1 thru ID3 refer to field 2 of the card type referenced by MNEM. If field 2 is not unique, then the following method is used. If $ID < 0$, then ID is the sorted parent card number within the MNEM to change. The first card of a MNEM is numbered 1, and continuation cards are not counted. For a given MNEM, positive and negative ID's can not be mixed.

2. Several 1VARY cards may reference the same MNEM and FIELD, but they must have unique values of K. If, for example, three cards appeared with fields 2, 3, 7, 8, and 9 as above and fields 4, 5, and 6 values of

K	A	B
1	.01	1.0
2	.02	0.
3	.04	1.0

then the function defined would be $D = [D_o(1+.01S_1) + .02S_2](1+.04S_3)$. Here, D refers to the item in field 3 of the MAT4 data cards whose ID's are 7, 8, and 9. D_o is the value found on the card. The S's are variance parameters. In the general case, a sequence is defined by

$$D_k = \begin{cases} D_{k-1}(1+a_kS_k)^{b_k} & b_k \neq 0 \\ D_{k-1} + a_kS_k & b_k = 0 \end{cases},$$

and then D = last value of D_k in sequence.

Figure 1. 1VARY Bulk Data Card

For each case, it is required that the data item be expressed in terms of a parameter, S_k, whose mean is zero and deviation is unknown.

$$k = .167(1+S_1) \quad , \quad Dev(S_1) = .025 \tag{9}$$

$$T_b = 350.+S_2 \quad , \quad Dev(S_2) = 20. \tag{10}$$

The above two examples are expressed in terms of percentage and absolute deviations. The 1VARY Bulk Data card (see Figure 1) defines the functional relationship between data items and parameters.

To illustrate the format of the data card, let us assume that the basic value of k is in field 3 of a MAT4 data card of ID - 201; and that the basic value of T_b is given in fields 4, 6 and 8 of a TEMP Bulk Data card that is the first TEMP card in the deck. The functions are given by

1VARY	MNEM	FIELD	K	A	B	ID1	ID2	ID3
1VARY	MAT4	3	1	1.0	1.0	201		
1VARY	TEMP	4	2	1.0		-1		
1VARY	TEMP	6	2	1.0		-1		
1VARY	TEMP	8	2	1.0		-1		

Examination of the data card description shows that any data item can depend on more than one parameter. Another important feature is that many items may depend upon a single parameter, such as for the temperatures at three points in the above example. If (for example) the three temperatures were independent, then separate values of k should be assigned for the temperatures, since they depend upon independent parameters.

Input Data Card 1VARY

Description: Defines functional relationships for bulk data changes. Used for variance analysis.

Format and Example:

1	2	3	4	5	6	7	8	9	10
1VARY	MNEM	FIELD	K	A	B	ID1	ID2	ID3	
1VARY	MAT4	3	1	.01	1.0	7	THRU	9	

Field	Contents
MNEM	Name of bulk data cards to be modified (BCD).
FIELD	Location of data items to be changed (integer > 0).
K	Parameter index (integer > 0).
A, B	Coefficients (real).
ID1 thru ID3	Identifiers of data cards to be modified (integer) or ID2 may be "THRU." See Remark 2.

Remarks:

1. If ID > 0, the identifiers ID1 thru ID3 refer to field 2 of the card type referenced by MNEM. If field 2 is not unique, then the following method is used. If ID < 0, then ID is the sorted parent card number within the MNEM to change. The first card of a MNEM is numbered 1, and continuation cards are not counted. For a given MNEM, positive and negative ID's can not be mixed.

2. Several 1VARY cards may reference the same MNEM and FIELD, but they must have unique values of K. If, for example, three cards appeared with fields 2, 3, 7, 8, and 9 as above and fields 4, 5, and 6 values of

K	A	B
1	.01	1.0
2	.02	0.
3	.04	1.0

then the function defined would be $D = [D_0(1+.01S_1) + .02S_2](1+.04S_4)$. Here, D refers to the item in field 3 of the MAT4 data cards whose ID's are 7, 8, and 9. D_0 is the value found on the card. The S's are variance parameters. In the general case, a sequence is defined by

$$D_k = \begin{cases} D_{k-1}(1+a_k S_k)^{b_k} & b_k \neq 0 \\ D_{k-1} + a_k S_k & b_k = 0 \end{cases},$$

and then D = last value of D_k in sequence.

Figure 1. 1VARY Bulk Data Card

the data cards would be

1PARM	JRUN	K1	SK1	K2	SK2	K3	SK3
1PARM	1	1	.01				
1PARM	2	2	.01				

This data card allows several parameters to be varied for any JRUN. Thus, one could simultaneously increase the conductivity by twenty percent and the boundary temperature by one hundred degrees by using the 1PARM data card. For variance analysis, the parameters are usually varied one at a time.

The deviations of the parameters are supplied on a VARIAN Bulk Data card (Figure 3), and a scale factor, δ, is supplied on a PARAM Bulk Data card with the name DELTA. Let U be any output quantity and D_i be the data items. These are related by Equation 5. NASTRAN computes

$$\mathrm{Dev}(U) = \left[\sum_{j=1}^{N} \left(\frac{\partial U}{\partial S_k} \mathrm{Dev}(S_k) \right)^2 \right]^{\frac{1}{2}} \tag{13}$$

The value for $\mathrm{Dev}(S_k)$ is determined as follows:

$$\mathrm{Dev}(S_k) = \frac{\delta}{S_k} \cdot \frac{\mathrm{Dev}(D_k)}{\left(\frac{\partial D_k}{\partial S_k} \right)} \tag{14}$$

Frequently, one can choose all S_k (on 1PARM data cards) to be equal to δ, making the first factor 1. The values for the two special cases are

Input Data Card 1PARM

Description: Defines parameter values for modifying bulk data. Is used with 1VARY data cards.

Format and Example:

1	2	3	4	5	6	7	8	9	10
1PARM	JRUN	K1	SK1	K2	SK2	K3	SK3	X	
1PARM	3	1	.97	3	1.0			X	

Field	Contents
JRUN	The run index for which the parameters are to be used (integer > 0).
K1, K2, K3	Index for nonzero parameters (integer).
SK1, SK2, SK3	The values of the parameters (real).

Remarks: 1. The default value is given by

$$S_k = \begin{cases} 1.0 & k = \text{JRUN} \\ 0.0 & k \neq \text{JRUN} \end{cases} .$$

These values are used if no 1PARM cards are found for any value JRUN

2. If there are more than three nonzero S_k's for any value of JRUN, additional 1PARM data cards may be introduced with the same value for JRUN.

Figure 2. 1PARM Bulk Data Card

Input Data Card: VARIAN

Description: Defines the values of the deviation of the parameters, for use in variance analysis.

Format and Examples:

1	2	3	4	5	6	7	8	9	10
VARIAN	DEL S1	DEL S2	DEL S3	DEL S4					ABC
VARIAN	1.0	0.	1.3-2						789

+BC									
+89									

etc.

Field	Contents
DEL S1, DEL S2, etc.	The deviation of the parameters used in variance analysis. (Real, a blank field ends the card)

Remarks: 1. The deviation of any output quantity Ø will be computed by

$$\text{DEVIATION}(\emptyset) = \left[\Sigma\left(\frac{\partial\emptyset}{\partial S_k}\,\Delta S_k\right)^2\right]^{1/2} ,$$

where $\partial\emptyset/\partial S_k$ are partial derivatives and ΔS_k are values supplied on this data card.

2. Deviations will be computed only if JRUN (the loop index) is equal to the number of entries on this card. Only one logical card may be in the deck.

Figure 3. VARIAN Bulk Data Card

The value for $Dev(S_k)$ is determined as follows:

$$Dev(S_k) = \frac{\delta}{S_k} \cdot \frac{Dev(D_k)}{\left(\frac{\partial D_k}{\partial S_k}\right)} \tag{14}$$

Frequently, one can choose all S_k (on 1VARY data cards) to be equal to δ, making the first factor 1. The values for the two special cases are

$$Dev(S_k) = \frac{\delta}{S_k} \begin{cases} \frac{Dev(D_k)}{E(D_k)} & \text{percentage form} \\ Dev(D_k) & \text{absolute form} \end{cases} \tag{15}$$

For the above example of data (see Equations 9 and 10), the Bulk Data cards used are

```
PARAM   DELTA   .01
VARIAN  .025    20.
```

All of the above Bulk Data cards must be in the Bulk Data Deck on restart, either as data supplied to the checkpoint run or as input data to the restart. The user is responsible to assure that data variations are large enough that numerical differences are not negligible, and yet small enough that the output varies linearly.

A special parameter MESH can be given the value YES to cause printout of VIEW data. This uses a special VIEW Bulk Data card defined by GSFC.

The Case Control Deck

Standard output requests are made. The values of the output for the changed parameters will be output. Wherever possible (i.e., when output was requested for the original data *and* the changed data) the derivatives will be output. The word "DERIVATIVES" and the value of the index JRUN are automatically printed to identify derivatives. On the final execution, derivations are computed which are labeled "VARIANCES."

Standard plot requests can be made which will plot the HBDY elements. A new PLØT parameter "NØRMALS" is used to request vectors normal to the HBDY elements. The mneumonic HBDY will be recognized in set requests, and HB is used to label elements in plots.

The Executive Control Deck

A special ALTER is required to include the VARIAN and PLTHBDY modules. These ALTERS are supplied for the three heat rigid formats. The user should request DIAG 40 in order to get a message about data cards affected by 1VARY and 1PARM data cards. For JRUN = 0, request CHKPNT YES.

The NASTRAN Card

The NASTRAN card can be used to control looping. The initial and final values of JRUN are specified as SYSTEM(61) and SYSTEM(62). Using looping (called the automatic mode), the only data items that can be changed are those specified by 1VARY and 1PARM. Single executions can be made with JRUN specified by SYSTEM(61) where any data item can be changed. For the initial checkpoint run, JRUN = 0, so no SYSTEM(61) need be supplied (i.e., the default is 0).

The Job Submission

It is suggested that the MERGE3 program be used with NASTRAN. This will allow the following:

1. $VIEW data cards may be in the input data deck. MERGE3 will change these to VIEW since NASTRAN ignores all DATA cards with $ in column 1.
2. Rigid Format ALTERS can be merged into the deck.
3. Checkpoint Dictionaries can be merged into the deck.

Details of how to use MERGE3 are given in the MSC/NASTRAN Application Manual.

During the first run (JRUN = 0), the user must request tapes NPTP and INPT. Punch output of a checkpoint dictionary will occur. For later runs, the restart tape (now called ØPTP for read only), the tape INPT (for read and write), and the checkpoint dictionary must be supplied.

DEMONSTRATION PROBLEMS

Three demonstration problems are used to illustrate variance analysis and plotting of HBDY elements. The problems were chosen to incorporate many features of heat conduction problems. The actual runs have not been incorporated in this report.

Variance Analysis, Nonlinear, Steady

The model is shown in Figure 4. The variance values are given in Figure 5. Two computer runs are required.

Run 1. This is a CHKPNT run using the basic values. An ALTER for Heat Sol 3 is used, which saves solutions on a user tape INPT for variance calculations.

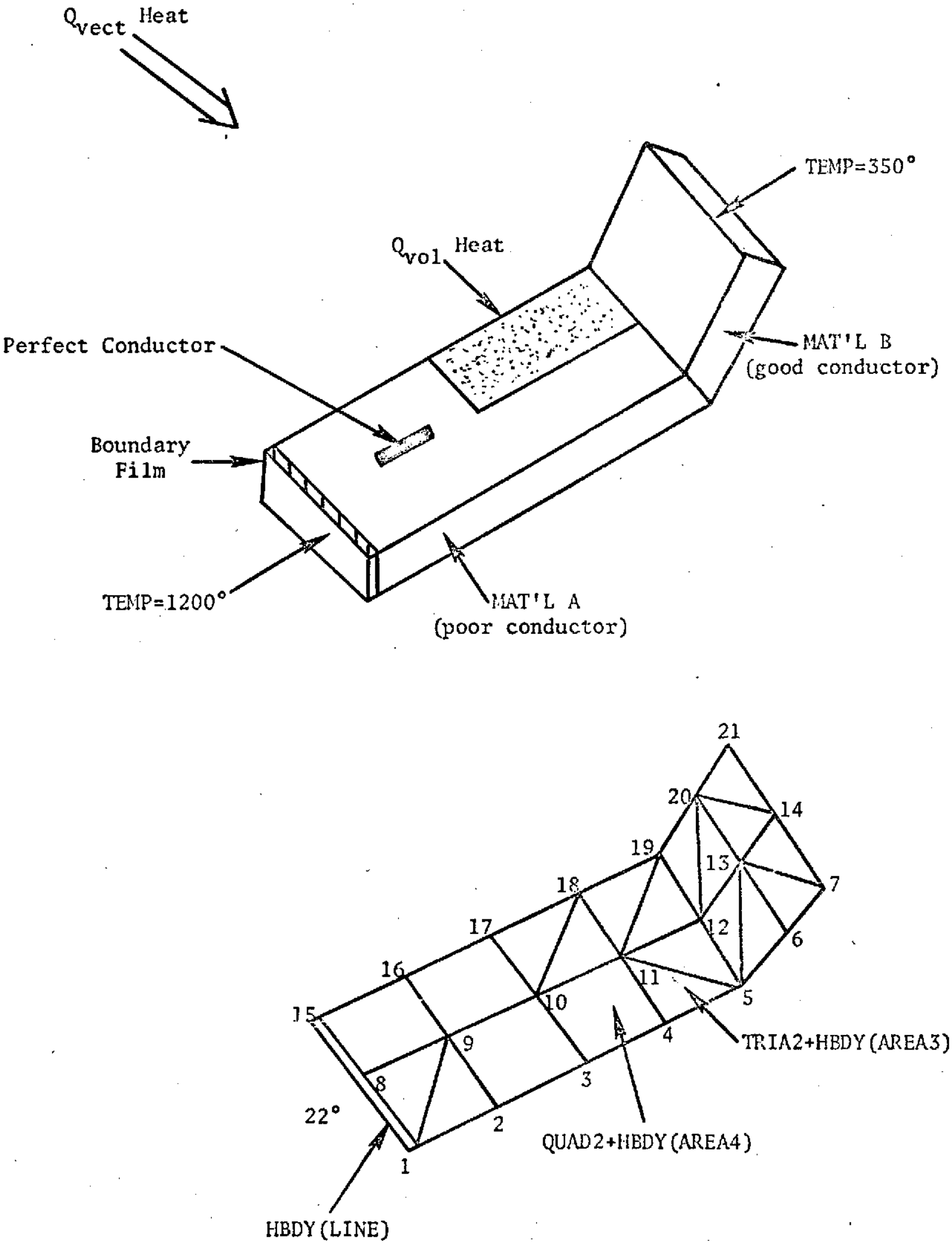


Figure 4. Variance Demonstration Problem

A bent plate of a uniform thickness (δ) is composed of two materials A and B. The system is subjected to thermal boundary conditions consisting of a directional energy flux (Q_{vect}), an internal energy source (Q_{vol}), a prescribed temperature boundary (T_b), and a convective coupling with a fluid of constant temperature (T_f). Diffuse-gray radiations among surface elements and to an environment of temperature T_∞ take place on the upper side of the plate while the underside is insulated perfectly. All dimensions, thermophysical properties, values of various parameters including the radiation exchange factors (A_iF_{ij}) of the finite-element model are given. Determine partial derivatives of the grid point temperatures with respect to each uncertain parameter and the resulting temperature intervals of each temperature based on

(1) the specified variances of parameters with entries directly in absolute values or in percentage as given in following data list;

(2) the uncertainty effect on temperatures of each parameter(s) to be evaluated for perturbations ($\Delta S/S$) of 5% for every parameter excluding F_{ij}.

Data:

	A	B
k watt/cm-k	.167 (±2.5%)	3.933 (±0.04)
ρc_p watt-sec/cm^3-k	3.966 (±0.15)	3.449
$\epsilon_{IR} = \alpha_{IR}$	.9 (±3%)	.8 (±3%)
α_{solar}	.95	.75

$h = 2.5 \times 10^{-3}$ watt/cm^2-k (±15%) $T_b = 350$ °K (±20)

$Q_{vect} = 0.13$ watt/cm^2 (±5%) $T_f = 1200$ °K (±30)

$Q_{vol} = 0.52$ watt/cm^3 $T_{abs} = 273.00$ °K

$A_i F_{ij}$ = given in the listing of RADMTX (±7% to those elements only that i = 5, 6, 15, 16 and j = 7, 8, 17, 18)

$\sigma = 5.6696 \times 10^{-12}$ watt/cm^2-K^4

Figure 5. Variance Values for Demonstration Problems 1 and 3

Run 2. This is a restart, demonstrating two loops through variance analysis. For additional loops, it would be necessary to change the "NASTRAN" card and the VARIAN Bulk Data card. The ALTER for Heat Sol 3 is used, along with special Bulk Data cards for variance analysis. For this example, the special cards are the input data deck; however, they could have been in the data deck of the original run. A user message is supplied (request DIAG 40 in the Executive Control Deck) which tells which items have been changed. This message appears after the sorted data echo. Restart bits are set which then determine the modules which must be executed to form the changed solution. Output requests produce values for the new solution, the derivatives of the solution, and the deviations.

Plotting HBDY Elements

A demonstration problem illustrates several options available for plotting. Rigid Format 1 - Heat (Statics) was used. The combination of parameters used is shown in the table below.

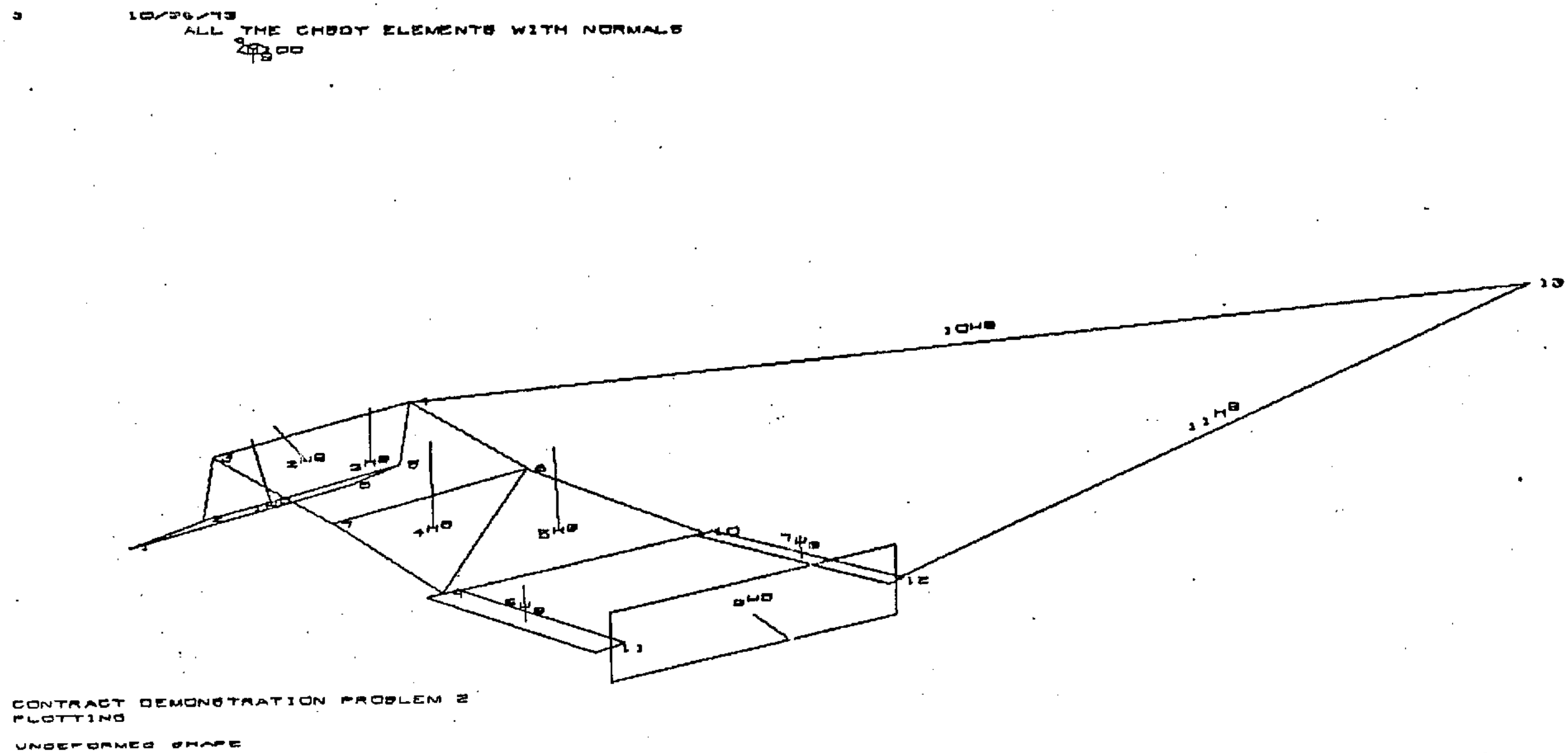
Plot	Set	Axes	View	Normals	Element Label	Grid Label	PTITLE	Symbol
1	1	X Y Z	Default		x	x	x	1
2	2	X Y Z	Default			x	x	
3	3	X Y Z	Default	x	x	x	x	
4	4	X Y Z	Default			x	x	
5	2	X MY MZ	Default				x	
6	2	MZ X MY	0.,0.,0.		x	x	x	
7	2	Y X MZ	0.,0.,0.		x	x	x	

A sample plot is reproduced in Figure 6.

Variance Analysis, Transient (DEMØ3)

The model is exactly the same as was used for DEMØ1. Initial conditions were chosen to be the estimates of DEMØ1, with steady loads. The asymptotic values should agree with the DEMØ1 results. Runs 3 and 4 show the check-point and restart phases. No deviation data was produced.

A special modeling technique was used to enforce transient boundary conditions. The generally accepted method (not used here) is with a good conductor to "ground" plus a large heat. The illustrated method uses extra points plus DMIG. The "load" applied to the extra point is the enforced temperature. The "temperature" or solution of the extra point is the heat required to maintain the specified temperature.



Sample Plot of HBDY Elements

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