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Reports of the Department of Geodetic Science

Report No. 196

GEOMETRIC ADJUSTMENT OF THE SOUTH AMERICAN SATELLITE DENSIFICATION (PC-1000) NETWORK

by

Ivan I. Mueller and M. Kumar

Prepared for

National Aeronautics and Space Administration
Washington, D. C.

Contract No. NGR 36-008-093
OSURF Project No. 2514

The Ohio State University
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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

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1. INTRODUCTION

The basic purpose of this experiment is to compute reduced normal equations from the observational data of the South American Satellite Densification (PC-1000) Network obtained from the Defense Mapping Agency Aerospace Center, St. Louis. These reduced normal equations are to be combined with reduced normal equations of other satellite networks of National Geodetic Satellite Program [Mueller et al., 1973] so as to provide station coordinates from a single least square adjustment.

Details of this network, including instrumentation, are given in Huber [1971].

2. DATA

2.1 Terrestrial Data

Terrestrial data, which include base-lines, heights and survey coordinates of stations, provide the necessary relative position constraints between "collocated" stations of BC-4 World-net and the South-American Densification Net (Figure 1). Survey information regarding the observation stations is summarized in Table 2.1-1. Constraints used in this solution are given in Tables 2.1-2, 2.1-3, 2.1-4 and 2.1-5 [Mueller, et al. 1973]. Geoidal undulations (Table 2.1-5) are computed by using formula and constants as given in [Rapp, 1973].

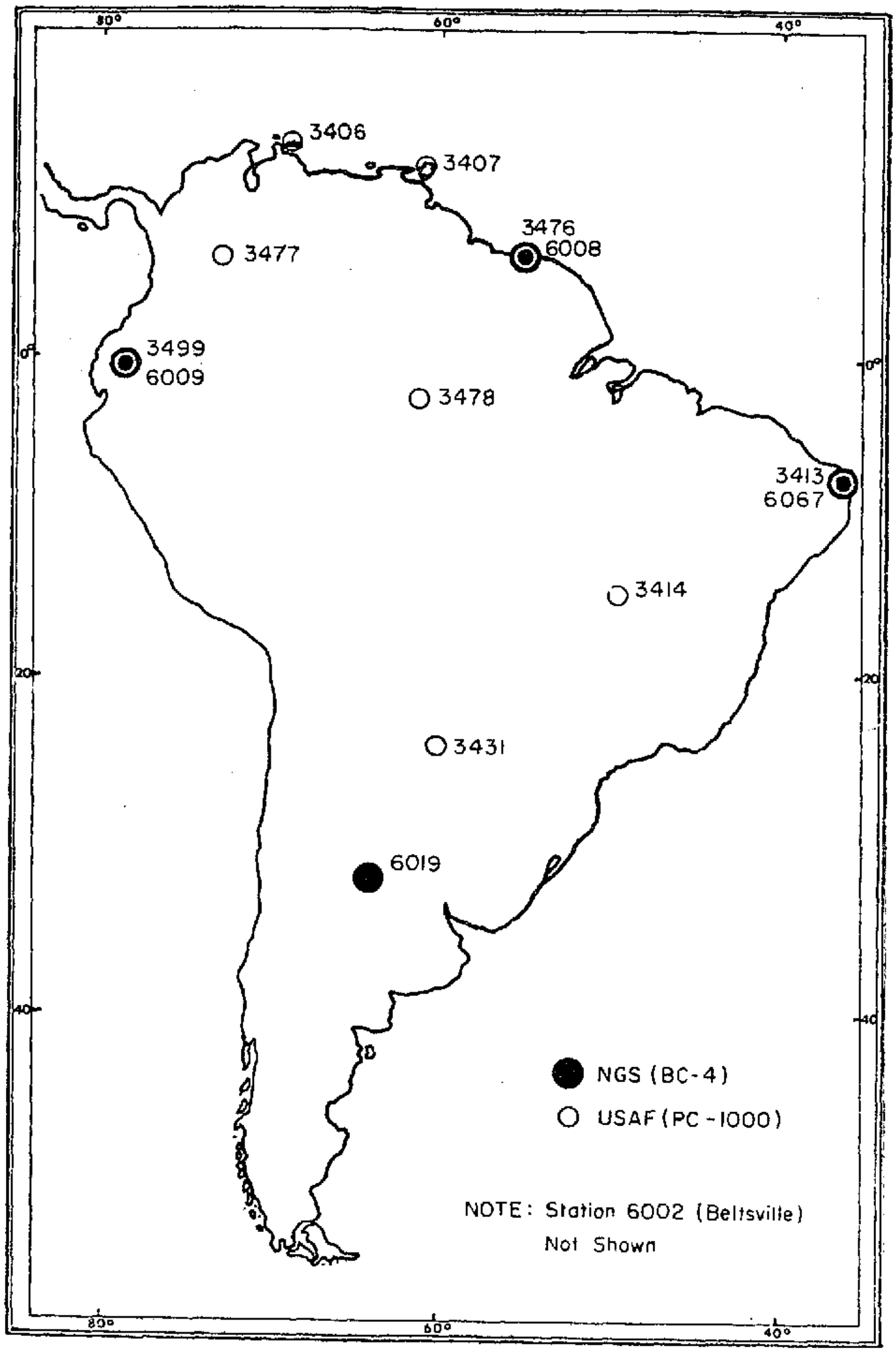


Fig. 1 South American densification net.

Table 2.1-1

S U R V E Y   I N F O R M A T I O N   O F   O B S E R V A T I O N   S T A T I O N S

STATION NO	STATION NAME	DATUM CODE[1]	SURVEY COORDINATES[2] LATITUDE	LONGITUDE	ELL. H(M)	MSL[3] (M)	INSTR. HEIGHT[4] (M)	INSTR. TYPE	SOURCE CODE[5]
3406	CURACAO	41	12 5 26.843	291 9 45.803	-4.0	6.83	1.25	PC-1000	1
3407	TRINIDAD	41	10 44 35.844	298 23 25.652	237.0	254.80	1.25	PC-1000	1
3413	NATAL	41	- 5 54 56.253	324 49 57.605	63.0	36.90	*	PC-1000	1
3414	BRASILIA	41	-15 51 35.540	312 6 2.679	1059.0	1058.25	1.14	PC-1000	2
3431	ASUNCION	41	-25 18 56.192	302 25 15.376	162.0	149.74	1.65	PC-1000	2
3476	PARAMARIBO	41	5 26 54.645	304 47 44.226	8.6	18.27	1.25	PC-1000	1
3477	BOGOTA	41	4 49 2.379	285 55 35.482	2586.0	2557.90	1.25	PC-1000	2
3478	MANAUS	*	- 3 8 44.820	300 0 59.620	*	83.60	*	PC-1000	3
3499	QUITO	41	- 0 5 50.468	281 34 49.212	2706.4	2681.80	*	PC-1000	1
6002	BELTSVILLE	29	39 1 39.003	283 10 26.942	45.0	44.30	1.50	BC-4	1
6008	PARAMARIBO	41	5 26 55.325	304 47 42.832	8.7	18.38	1.49	BC-4	1
6009	QUITO	41	- 0 5 50.468	281 34 49.212	2706.7	2682.10	1.50	BC-4	1
6019	VILLA DOLORES	41	-31 56 33.954	294 53 41.342	621.0	608.18	1.50	BC-4	2
6067	NATAL	41	- 5 55 37.414	324 50 6.200	66.7	40.63	*	BC-4A	1

* INSUFFICIENT DATA

1 DATUM CODE :

29 -- NAD 1927
41 -- SAD 1969

2 GEODETIC COORDINATES OF THE INSTRUMENTAL REFERENCE POINT (OPTICAL/ELECTRONIC CENTER,ETC.) ON THE LOCAL GEODETIC DATUM

3 MEAN SEA LEVEL HEIGHT OF THE INSTRUMENTAL REFERENCE POINT

4 HEIGHT OF INSTRUMENTAL REFERENCE POINT ABOVE SURVEY MONUMENT

5 SOURCE CODE :

1 -- (CSC,1971)
2 -- (CSC,1972/73)
3 -- (HUBER,1971)

NOTE : ZERO IN THE LAST DIGIT MAY INDICATE THAT THE DIGIT IS UNKNOWN.

Table 2.1-2

Relative Position Constraints

Stations	Relative Coordinates (Meters)			Weights ($^1/\sigma^2$)
	Δ u	Δ v	Δ w	
3413-6067	-48.64	-289.13	1258.05	3.00
3476-6008	36.31	22.94	-20.80	3.00
3499-6009	0.00	0.00	0.00	100.00

Table 2.1-3

Station Position Constraints

Stations	Station Coordinates (Meters)			Weights		
	u	v	w	p_u	p_v	p_w
6002	1130 764.85	-4830 831.87	3994 704.05	0.2415	0.3425	0.2725
6008	3623 241.00	-5214 233.74	601 536.05	0.2212	0.2591	0.1163
6009	1280 834.24	-6250 955.94	-10 800.59	0.0776	0.0852	0.0593
6019	2280 627.09	-4914 543.17	3355 402.77	0.1779	0.1366	0.0741
6067	5186 397.12	-3653 933.25	-654 276.92	0.2315	0.2160	0.1464

Weights ($^1/\sigma^2$) based as per variances obtained in OSU WN14 Solution [Mueller et al., 1973]

Table 2.1-4

Chord Constraint

Station-Station	Chord Distance (Meters)	$\sigma \times 10^6$
6009-6067	4734 132.44	1.00

The chord length and the sigma were computed from the adjusted coordinates of the stations from OSU WN14 Solution [Mueller et al., 1973]

Table 2.1-5

Geoidal Undulations and Heights Used in the Constraints

STATION		NREF[1]	HCONSTR[2]	σHCONSTR[3]
NO.	NAME	(m)	(m)	(m)
3406	Curacao	-29.19	-41.02	4.0
3407	Trinidad	-38.57	194.88	4.0
3413	Natal	-12.03	-5.87	6.0
3414	Brasilia	-9.88	1021.23	6.0
3431	Asuncion	11.98	137.72	6.0
3476	Paramaribo	-28.31	-34.02	6.0
3477	Bogota	10.71	2551.44	6.0
3478	Manaus	-7.17	53.63	6.0
3499	Quito	16.73	2682.74	6.0
6002	Beltsville	-36.90	-6.73	2.5
6008	Paramaribo	-28.31	-33.91	4.0
6009	Quito	16.73	2683.04	6.0
6019	Villa Dolores	22.80	609.43	6.0
6067	Natal	-12.03	-2.14	6.0

1. From [Rapp, 1973]
2. HCONSTR = MSL+NREF+ΔN, where ΔN is a correction term for the differences of position and size of the ellipsoids used [Mueller et al., 1973]
3. Used in Computing the Weights of the Height Constraints

2.2 Satellite Observational Data

Data of South America Densification Net was obtained from the Defense Mapping Agency/Aerospace Center, St. Louis, Missouri. This data, which is a punched card-deck, is used without any modification. No major blunders are detected. (See Table 2.2-1)

3. THEORETICAL BACKGROUND

3.1 Normal Equations for Optical Observations

A set of reduced normal equations of the form

$$\overline{N} X + \overline{U} = 0$$

is obtained from the optical observations. The symmetric coefficient matrix is composed of 3 x 3 blocks of the form [Mueller, 1968]

$$\underset{3\times3}{\overline{N}_{kk}} = \sum_j M_{kj}^{-1} - \sum_j M_{kj}^{-1} \left(\sum_i M_{ij}^{-1}\right)^{-1} M_{kj}^{-1} + P_k$$

$$\underset{3\times3}{\overline{N}_{kl}} = -\sum_j M_{kj}^{-1} \left(\sum_i M_{ij}^{-1}\right)^{-1} M_{lj}^{-1}$$

where

$$M_{ij} = B_{ij} \; P_{ij}^{-1} \; B_{ij}'$$

$$B_{ij} = SR_3 \; (-\alpha) \; R_2 \; (-90^\circ + \delta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\delta & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

S = Polar motion matrix

and

k, l	Denotes particular ground stations
j	Denotes particular simultaneous event
i	Denotes any ground station participating in an event
$\sum_i$	Is the summation over all ground stations involved in event j
$\sum_j$	Is the summation over all events observed by ground station k and/or l
$\underset{3\times3}{P_k}$	Weight matrix associated with any particular ground station
$\underset{3\times3}{P_{ij}}$	Weight matrix of any observed direction

Table 2.2-1

Summary of Simultaneous Observations by Line (SA Network)

Line Station-Station	No. of Pairs	Line Station-Station	No. of Pairs
6002-6008	23	3406-3478	14
6002-3406	14	3406-3499	4
6002-3407	11	3407-3431	16
6002-3476	7	3407-3476	19
6002-3477	7	3407-3477	23
6008-6009	10	3407-3478	9
6008-6019	36	3413-3414	29
6008-6067	14	3413-3431	2
6008-3406	26	3414-3431	22
6008-3477	3	3476-3477	15
6008-3478	6	3477-3478	2
6009-6019	7	3477-3499	5
6009-3406	14		
6009-3407	6		
6009-3476	6		
6009-3477	5		
6009-3499	9		
6019-6067	35		
6019-3406	19		
6019-3407	38		
6019-3431	4		
6019-3476	19		
6019-3477	6		
6067-3407	3		
3406-3407	9		
3406-3413	25		
3406-3414	41		
3406-3431	53		
3406-3476	20		
3406-3477	13		

Finally, the vector of constant terms is expressed by

$$\overline{U}_k = \sum_j M_{kj}^{-1} [X_k^{\circ} - (\sum_i M_{ij}^{-1})^{-1} \sum_i M_{ij}^{-1} X_i^{\circ}]$$

where as usual, the superscript ($^{\circ}$) denotes initial approximate values.

3.2 "Constraints" Contributions to the Normal Equations

3.21 General

Since the coefficient matrix of normal equations is singular, a unique least squares solution is not possible. A minimal set of constraints to the normal equations provides a unique solution [Blaha, 1971].

Two alternative definitions exist for the term "constraints". The absolute constraints represent certain conditions which have to be fulfilled exactly and with no uncertainties while the relative constraints (or weighted constraints) have the same characteristics as the observations.

In general the contribution of the functional constraint equations

$$G(X, L_c) = 0$$

to the normal equations can be found by bordering the normal equations matrix

$$\begin{vmatrix} \overline{N} & C' \\ C & -P_c^{-1} \end{vmatrix} \begin{vmatrix} X \\ -K_c \end{vmatrix} + \begin{vmatrix} \overline{U} \\ W^c \end{vmatrix} = 0, \text{ where } C = \frac{\partial G}{\partial X}.$$

After elimination of K_c, it is easy to find

$$[\overline{N} + C'P_c C]X + \overline{U} + C'P_c W^c = 0$$

or

$$[\overline{N} + N^c]X + \overline{U} + U^c = 0 \tag{1}$$

where N^c and U^c are the contributions to the coefficient matrix and constant vector of the normal equations due to the application of constraints. The quantities $\overline{N}$ and $\overline{U}$ correspond to the original normal equations without constraints (Section 3.1).

After the constraints are added the normal equations will take the usual form

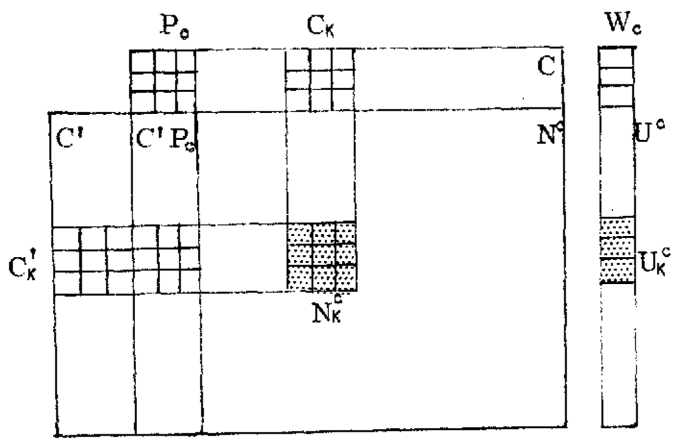
$$NX + U = 0$$

and we are in the position to obtain the contribution from a new set of constraints.

Constraints can be applied between two station k and l or to a single station. The contribution of these constraints to the matrix $\overline{N}$ (3 x 3) blocks) and $\overline{U}$ (3 x 1 blocks) can be schematically expressed in two different ways.

Assuming that the matrix P_c is always diagonal it is possible to express:

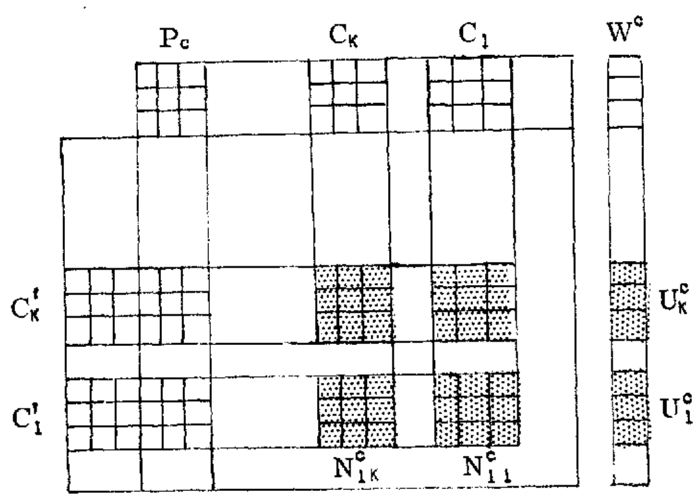
(a) Contribution to the normals due to the constraint applied to station k



$$N_K^c = C_K^t \; P_c \; C_K \qquad \text{(2a)}$$

$$U_K^c = C_K^t \; P_c \; W^c$$

(b) Contribution to the normals due to the constraint between stations k and l



$$N_{KK}^c = C_K^t \; P_c \; C_K \; ; \; N_{Kl}^c = C_K^t \; P_c \; C_l$$

$$U_K^c = C_K^t \; P_c \; W^c ; \; U_l^c = C_l^t \; P_c \; W^c$$

(2b)

These blocks obtained as indicated above for the corresponding case will be the only ones computed and added to the original normal equations as expressed by formula (1).

3.22 Relative Position Constraints

Relative position constraints are used in order to combine the normal equations obtained from various satellite nets and to constrain "double" stations or closely situated stations of the same net. The expression for the combination of normals can be written as follows:

$$[\overline{N} + N^R]\ X + \overline{U} + U^R = 0$$

where N^R and U^R' computed from (2a), (2b), are the contribution to the original normal equations $(\overline{N}X + \overline{U} = 0)$.

If the relative position $(\Delta u^\circ\ ,\ \Delta v^\circ\ ,\ \Delta w^\circ\ )$ of two stations is known, along with the standard deviation of these relative positions, the constraints can be formed. In this case the functional constraint equations are

$$u_k^\circ - u_l^\circ = \Delta u^\circ$$

$$v_k^\circ - v_l^\circ = \Delta v^\circ$$

$$w_k^\circ - w_l^\circ = \Delta w^\circ$$

Therefore

$$\underset{3\times3}{C_k^R} = \underset{3\times3}{I}\ ;\ \underset{3\times3}{C_l^R} = \underset{3\times3}{-I}$$

and

$$\underset{3\times1}{U_k^R} = 0\ ;\ \underset{3\times1}{U_l^R} = 0 \quad \text{because } W^R = G^R\ (X^\circ, L_c^\circ) = 0$$

where

$$P_R = \begin{vmatrix} \frac{1}{\sigma^2_{\Delta u^\circ}} & 0 & 0 \\ 0 & \frac{1}{\sigma^2_{\Delta v^\circ}} & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{\Delta w^\circ}} \end{vmatrix}$$

and

$$\underset{3\times3}{N^R_{KK}} = I\ P_R\ I = \underset{3\times3}{P_R}$$

$$\underset{3\times3}{N^R_{11}} = I\ P_R\ I = \underset{3\times3}{P_R}$$

$$\underset{3\times3}{N^R_{K1}} = \underset{3\times3}{N^R_{1K}} = I\ P_R\ (-I) = \underset{3\times3}{-P_R}$$

Thus, the diagonal elements of P_R are added to each element of the diagonal of the blocks $_{KK}$ and $_{11}$ of the matrix of the combined normals $\overline{N}$, and subtracted from the diagonal elements of the blocks $_{K1}$ and $_{1K}$ of $\overline{N}$. There is no contribution to the vector $\overline{U}$.

3.23 Length (Chord) Constraints

Chord constraints are introduced when scalar information is available between ground stations (e.g., distances determined through high precision geodimeter traversing). The functional constraint equation in this case is

$$G^c\ (X,\ L_c) = 0$$

or

$$[(u_K - u_1)^2 + (v_K - v_1)^2 + (w_K - w_1)^2]^{\frac{1}{2}} = L_{K1} \tag{3}$$

$$\underset{1\times3}{C^c_K} = \left[\frac{u^\circ_K - u^\circ_1}{L^\circ_{K1}},\ \frac{v^\circ_K - v^\circ_1}{L^\circ_{K1}},\ \frac{w^\circ_K - w^\circ_1}{L^\circ_{K1}}\right]$$

and

$$\underset{1\times3}{C^c_1} = -\left[\frac{u^\circ_K - u^\circ_1}{L^\circ_{K1}},\ \frac{v^\circ_K - v^\circ_1}{L^\circ_{K1}},\ \frac{w^\circ_K - w^\circ_1}{L^\circ_{K1}}\right]$$

and

$$P_c^{-1} = \frac{\sigma^2_{K1}}{\sigma^2_0} = \frac{\text{variance of the chord}}{\text{a priori variance of unit weight}}$$

Then the contribution to the normals are obtained by applying (2a) and (2b) as under:

$$\underset{3\times3}{N^c_{\kappa\kappa}} = (C^c_\kappa)' \, P_c \, C^c_\kappa$$

$$\underset{3\times3}{N^c_{\iota\iota}} = (C^c_\iota)' \, P_c \, C^c_\iota$$

$$\underset{3\times3}{N^c_{\kappa\iota}} = (C^c_\kappa)' \, P_c \, C^c_\iota$$

$$\underset{3\times3}{U^c_\kappa} = (C^c_\kappa)' \, P_c \, W^c$$

$$\underset{3\times3}{U^c_\iota} = (C^c_\iota)' \, P_c \, W^c$$

The first three expressions in the above are added respectively to the blocks $N_{\kappa\kappa}$, $N_{\iota\iota}$ and $N_{\kappa\iota}$ of $\overline{N}$; the last two expressions are added respectively to the constant vectors U_κ and U_ι of $\overline{U}$.

3.24 Station Position Constraints

Station position constraint is used for the purpose of defining the origin of the coordinate system. If the station coordinates $(u^\circ_\kappa, v^\circ_\kappa, w^\circ_\kappa)$ of station κ are to be constrained and if the computed (known) variances of its known coordinates are $\sigma^2_{u^\circ_\kappa}$, $\sigma^2_{v^\circ_\kappa}$, $\sigma^2_{u^\circ_\kappa}$, then the equations given in section 3.22 are valid by merely deleting the terms with index ι, i.e., $\Delta u^\circ = u^\circ_k$, $\Delta v^\circ = v_k{}^\circ$, $\Delta w^\circ_k = w^\circ_k$. Hence

$$\underset{3\times3}{N^s_{\kappa\kappa}} = I \, P_s \, I = \underset{3\times3}{P_s}$$

where

$$P_s = \begin{bmatrix} \frac{1}{\sigma^2_{u^\circ_\kappa}} & 0 & 0 \\ 0 & \frac{1}{\sigma^2_{v^\circ_\kappa}} & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{w^\circ_\kappa}} \end{bmatrix}$$

3.25 Height Constraints

If the geodetic (ellipsoidal) heights H_κ of the station κ is to be constrained, then

$$\underset{3\times 3}{N^H_{\kappa\kappa}} = (C^H_\kappa)' \; P_H \; C^H_\kappa$$

where

$$\underset{1\times 3}{C^H_\kappa} = [\cos\varphi^\circ_\kappa \; \cos\lambda^\circ_\kappa \, , \; \cos\varphi^\circ_\kappa \; \sin\lambda^\circ_\kappa \, , \; \sin\varphi^\circ_\kappa]$$

and

$$P_H = \frac{1}{\sigma^2_{H_\kappa}} \; .$$

Here φ°_κ and λ°_κ are the approximate geodetic coordinates and $\sigma^2_{H_\kappa}$ is the variance of the height for station κ.

The constant vector U^H_κ can be computed from

$$U^H_\kappa = (C^H_\kappa)' \; P_H \; W^H$$

where

$$W^H = H_\kappa - H^\circ_\kappa, \; H^\circ_\kappa \text{ being the approximate height.}$$

3.26 Inner Constraints (Free Adjustment)

Even though the selection of a coordinate system is arbitrary in the case of a minimum constraint adjustment, e.g., in the case of ranging, the selection of the six coordinates (at more than two stations) to be constrained is very critical, since one set of constraints would give a different solution than another set. The "best" solution is arrived at in a coordinate system defined through the use of a set of constraint equations called "inner" constraints [Rinner et al., 1967]. In this sense, "best" means resulting in the smallest covariance matrix for the unknowns. Covariance matrices may be compared by means of their traces, and the inner constraint equations are characterized by the property that the trace of the covariance matrix obtained with their use is a minimum among those obtained by adjusting a given set of observations augmented by a minimal set of constraint equations. This property also implies that the mean square uncertainty of the unknowns is smaller when the inner adjustment equations are used. The resulting adjustment is called a "free" one. The functional inner constraints equations can be written as

$$C^I X = 0$$

where X is the set of corrections of the approximate coordinates of the unknown points and in the most general application when the "best" origin, orientation and scale are sought

$$C^I = \begin{bmatrix} C_1^I \\ \\ C_2^I \\ \\ C_3^I \end{bmatrix} = \left[\begin{array}{ccc|ccc|c} & \underset{3\times 3}{I} & & & \underset{3\times 3}{I} & & \cdots \\ \hline 0 & w_1^o & -v_1^o & 0 & w_2^o & -v_2^o & \\ -w_1^o & 0 & u_1^o & -w_2^o & 0 & u_2^o & \cdots \\ v_1^o & -u_1^o & 0 & v_2^o & -u_2^o & 0 & \\ \hline u_1^o & v_1^o & w_1^o & u_2^o & v_2^o & w_2^o & \cdots \end{array}\right]$$

The symbols (u_i^o, v_i^o, w_i^o) denote the approximate coordinates of the ith unknown point where both the ground points and the satellite positions are considered.

It is also possible to design a set of constraints that will result in the "best" solution for only a subset of the points. In the adjustment reported here we were only interested in the gound station unknowns implying that the trace of only that portion of the covariance matrix corresponding to the ground station unknowns should be minimized, while the variances of the satellite position unknowns should not be included in the minimum sum. The constraint equations that will produce such a solution have the same form as those producing the "best" solution for all the points; however, 3 x 3 blocks of zeros are inserted into those positions of I which correspond to unknowns whose variances are not to be included in the minimum sum.

The inner adjustment constraint equations can be given a geometrical interpretation that appeals to intuition. Let X_i^o denote the set of approximate coordinates of the ith unknown point, dX_i denote the corrections to these coordinates, and X_i denote the adjusted coordinates, i.e.,

$$X_i = X_i^\circ + dX_i$$

The first set of constraint equations, $C_1^I X = 0$, is then equivalent to the set of conditions

$$\sum_i dX_i = 0$$

The geometrical interpretation of these conditions is that the center of gravity of all the points will not change after adjustment, i.e.,

$$\sum_i X_i^\circ \times dX_i = 0$$

If the center of the system remains fixed, then the cross products $X_i^\circ \times dX_i$ reflect rotations of the points around the fixed center. These constraint equations insure that the sums of the rotations around all three coordinate axes are zero. The corresponding geometrical interpretation is that the mean orientation of the system of points will not change after adjustment either.

Thus, the respective equations $C_1^I X = 0$ and $C_2^I X = 0$ effectively specify the origin and the orientation of the adjustment coordinate system. A third "inner adjustment" equation $C_3^I X = 0$ specifies the scale of the system. However, this scale equation is only used when the observations themselves do not determine the scale.

A more complete description of the inner adjustment is described in [Blaha, 1971].

In summary, if the normal equations with the contribution of all the constraints (except inner constraints) are represented by

$$[\bar{N} + N^R + N^C + N^S + N^H]X + \bar{U} + U^R + U^C + U^S + U^H = 0 \tag{5}$$

or

$$NX + U = 0$$

then the inner adjustment can be obtained by bordering the coefficient matrix N of the normal equations as

$$\begin{bmatrix} N & (C^I)' \\ C^I & 0 \end{bmatrix} \begin{bmatrix} X \\ -K \end{bmatrix} = \begin{bmatrix} -U \\ 0 \end{bmatrix} \tag{6}$$

Upon the addition of any kind of constraint to the normal equations, it becomes necessary to consider also its contribution to Σ V'PV. The degrees of freedom change as well. In order to compute the proper variance of unit weight the latter must be taken into consideration.

4. THE SOLUTION

Using the specified constraint of Section 3.2, SA-10 solution is computed. A general information of this solution is given in Table 4-1. The solution is obtained by using the general OSUGOP program (Reilly et al., 1972].

The station coordinates of SA-10 solution are given in Table 4-2 with their standard deviations.

Table 4-1

General Information on the SA-10 Geometric Adjustment

No. of observing stations	14
σ_o (a priori)	1.0
No. of degrees of freedom	649
Σ V'PV	721.76
$\hat{\sigma}_o$ (a posteriori)	2.50
Number of constraints used:	
Relative position constraints	3
Length (Chord) constraint	1
Station position constraints	5
Height constraints	14
Inner constraint defines the origin of the coordinate system.	

Table 4-2

Cartesian and Geodetic Coordinates
(Solution SA 10)

Sta. No.	u	σ_u	v	σ_v	w	σ_w
	φ	σ_φ	λ	σ_λ	H	σ_H
		a_a a_b a_c	A_a A_b A_c	r_a r_b r_c		

u, v, w — Cartesian coordinates in meters (Orientation: u = the Greenwich meridian as defined by the B.I.H.; v = λ = 90° (E); w = Conventional International Origin).

φ, λ — Geodetic latitude and longitude in angular units (degrees, minutes and seconds of arc) computed from the Cartesian coordinates and referred to a rotational ellipsoid of a = 6378155.00 m and b = 6356769.70 m.

H — Geodetic (ellipsoidal) height in meters referred to the same ellipsoid.

σ_u, σ_v, σ_w — Standard deviations of the Cartesian coordinates in meters.

σ_φ, σ_λ — Standard deviations of the geodetic coordinates in seconds of arc.

σ_H — Standard deviations of the geodetic height in meters.

a_a, A_a, r_a — Altitude (elevation angle), azimuth and magnitude of the major semi axis of the error ellipsoid, respectively. Angles in degrees, magnitude in meters. Altitude is positive above the horizon. Azimuth is positive east reckoned from the north.

a_b, A_b, r_b — Same as above for the mean axis of the error ellipsoid.

a_c, A_c, r_c — Same as above for the minor axis of the error ellipsoid.

Table 4.2 (Con't)

3406	2251789.08	5.57	-5816902.96	3.94	1327210.38	7.37
	12 5 26.57	0.24	291 9 43.15	0.20	-46.08	3.53
		8.88	-32.18	8.16		
		-5.04	57.03	4.83		
		-79.77	-62.24	3.33		
3407	2979880.94	7.18	-5513533.18	4.49	1181138.43	6.20
	10 44 35.20	0.20	298 23 23.08	0.25	185.59	3.71
		-3.40	118.77	8.31		
		20.28	30.03	5.44		
		-69.41	19.66	3.37		
3413	5186345.51	2.00	-3654218.56	2.10	-653018.68	2.71
	- 5 54 57.55	0.09	324 49 55.45	0.07	-4.76	1.97
		-3.79	3.66	2.71		
		-8.55	94.23	2.14		
		80.64	69.95	1.96		
3414	4114977.26	7.50	-4554124.02	5.71	-1732149.84	7.17
	-15 51 37.37	0.24	312 6 0.26	0.27	1002.05	4.88
		2.88	50.36	9.10		
		-11.13	-39.07	5.84		
		-78.50	126.06	4.82		
3431	3093028.20	7.17	-4870063.97	6.08	-2710839.99	10.51
	-25 18 58.25	0.36	302 25 12.20	0.26	130.54	4.94
		-5.20	11.62	11.19		
		-1.83	101.79	7.07		
		-84.48	-148.88	4.85		
3476	3623275.84	2.14	-5214208.28	1.87	601517.56	2.84
	5 26 52.81	0.09	304 47 41.51	0.07	-39.47	1.81
		3.15	-6.32	2.85		
		10.83	84.28	2.20		
		-78.72	67.69	1.79		

Table 4.2 (Con't)

3477	1744632.76	9.75	-6114278.13	6.57	532213.19	9.76
	4 49 0.44	0.32	285 55 31.56	0.34	2542.42	5.37
		-2.76	47.57	13.26		
		-43.15	140.16	5.79		
		46.72	134.64	4.90		
3478	3185743.08	11.19	-5514590.21	9.39	-347640.88	20.43
	- 3 8 43.73	0.67	300 0 53.09	0.43	36.97	5.75
		-0.19	-30.23	23.24		
		-21.96	59.85	7.91		
		68.04	59.31	5.31		
3499	1280825.47	2.99	-6250950.21	2.60	-10793.24	4.01
	- 0 5 51.25	0.13	281 34 46.84	0.10	2676.43	2.61
		2.51	14.32	4.07		
		10.43	104.78	2.90		
		-79.26	90.96	2.60		
6002	1130763.54	2.10	-4830831.02	1.63	3994704.49	1.86
	39 1 39.39	0.06	283 10 27.00	0.09	-7.30	1.54
		1.79	105.75	2.11		
		11.74	15.38	1.93		
		-78.12	24.27	1.52		
6008	3623239.59	2.07	-5214231.39	1.79	601538.31	2.79
	5 26 53.49	0.09	304 47 40.11	0.07	-39.20	1.74
		3.25	-5.65	2.80		
		11.07	84.98	2.12		
		-78.46	68.21	1.71		
6009	1280825.47	2.99	-6250950.22	2.59	-10793.24	4.01
	- 0 5 51.25	0.13	281 34 46.84	0.10	2676.43	2.61
		2.51	14.32	4.07		
		10.48	104.78	2.90		
		-79.22	91.01	2.59		
6019	2280626.58	2.43	-4914540.71	2.64	-3355401.90	3.65
	-31 56 34.95	0.11	294 53 38.34	0.10	603.72	2.77
		-24.10	-0.19	3.67		
		-46.19	117.61	2.63		
		33.99	72.26	2.41		

Table 4.2 (Con't)

6067	5186394.24	1.92	-3653929.59	2.03	-654276.72	2.64
	- 5 55 38.71	0.09	324 50 4.04	0.07	-0.90	1.90
		-3.90	3.53	2.65		
		-11.21	94.31	2.06		
		78.11	74.64	1.89		

In the tables the rotations ω, ψ, and ϵ are about the w, v, and u axes, respectively. The scale factor Δ is in units of ppm. The unit in the variance-covariance matrix, for the elements corresponding to the rotations, is radian squared.

5. COMPARISON WITH OTHER SOLUTIONS

Table 5.1-1 summarizes the transformation parameters between SA 10 and South American 1969 Datum, and Table 5.1-2 between SA 10 and WN14 Solution [Mueller et al., 1973].

Table 5.1-1

SA10 -TO- SAD-69
****

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DU METERS	DV METERS	DW METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
59.65	16.52	36.33	5.80	-0.79	0.12	0.21

VARIANCE - COVARIANCE MATRIX

σ_o^2 = 1.13

0.857D+02	0.584D+01	0.106D+02	-0.447D-05	0.967D-05	-0.277D-05	-0.126D-05
0.584D+01	0.749D+02	0.186D+01	0.738D-05	0.564D-05	-0.358D-06	0.130D-05
0.106D+02	0.186D+01	0.923D+02	0.137D-05	0.161D-05	-0.644D-05	-0.979D-05
-0.447D-05	0.738D-05	0.137D-05	0.145D-11	0.721D-14	0.319D-14	0.110D-13
0.967D-05	0.564D-05	0.161D-05	0.721D-14	0.187D-11	-0.268D-12	-0.152D-12
-0.277D-05	-0.358D-06	-0.644D-05	0.319D-14	-0.268D-12	0.136D-11	0.440D-12
-0.126D-05	0.130D-05	-0.979D-05	0.110D-13	-0.152D-12	0.440D-12	0.168D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.730D-01	0.119D+00	-0.401D+00	0.764D+00	-0.257D+00	-0.105D+00
0.730D-01	0.100D+01	0.224D-01	0.708D+00	0.476D+00	-0.355D-01	0.116D+00
0.119D+00	0.224D-01	0.100D+01	0.118D+00	0.122D+00	-0.576D+00	-0.788D+00
-0.401D+00	0.708D+00	0.118D+00	0.100D+01	0.438D-02	0.227D-02	0.705D-02
0.764D+00	0.476D+00	0.122D+00	0.438D-02	0.100D+01	-0.168D+00	-0.856D-01
-0.257D+00	-0.355D-01	-0.576D+00	0.227D-02	-0.168D+00	0.100D+01	0.292D+00
-0.105D+00	0.116D+00	-0.788D+00	0.705D-02	-0.856D-01	0.292D+00	0.100D+01

Table 5.1-1 (Con't)

RESIDUALS V

	V1(SA10)				V2(SAD-69)			V1 - V2		
3414	0.4	-2.8	1.3	3414	-0.7	8.5	-2.6	1.2	-11.3	3.9
3431	3.0	-0.1	7.8	3431	-5.9	0.3	-7.1	9.0	-0.4	14.9
3477	17.7	0.9	8.3	3477	-18.7	-2.1	-8.7	36.4	3.1	16.9
6008	-0.2	0.3	1.0	6008	3.8	-8.2	-12.5	-4.0	8.4	13.5
6009	-0.9	-0.3	-2.6	6009	10.0	4.9	16.4	-10.9	-5.2	-19.0
6019	-0.4	0.5	-0.9	6019	7.4	-7.3	7.1	-7.8	7.8	-8.1
6067	-0.2	-0.2	-0.5	6067	4.1	3.9	7.3	-4.3	-4.0	-7.8

Table 5.1-2

SA10 -TO- WN14
****

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DU METERS	DV METERS	DW METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
1.63	-7.78	-4.92	-0.44	-0.13	0.07	0.10

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 0.92

0.365D+01	0.521D+00	0.880D+00	-0.171D-06	0.403D-06	-0.339D-07	-0.134D-06
0.521D+00	0.315D+01	0.462D+00	0.244D-06	0.286D-06	-0.304D-07	-0.120D-06
0.880D+00	0.462D+00	0.363D+01	-0.444D-07	0.162D-06	-0.168D-06	-0.277D-06
-0.171D-06	0.244D-06	-0.444D-07	0.505D-13	-0.155D-15	0.390D-15	0.351D-15
0.403D-06	0.286D-06	0.162D-06	-0.155D-15	0.856D-13	-0.884D-14	-0.284D-13
-0.339D-07	-0.304D-07	-0.168D-06	0.390D-15	-0.884D-14	0.431D-13	0.740D-14
-0.134D-06	-0.120D-06	-0.277D-06	0.351D-15	-0.284D-13	0.740D-14	0.535D-13

COEFFICIENTS OF CORRELATION

0.100D+01	0.154D+00	0.242D+00	-0.398D+00	0.720D+00	-0.855D-01	-0.302D+00
0.154D+00	0.100D+01	0.137D+00	0.612D+00	0.551D+00	-0.825D-01	-0.292D+00
0.242D+00	0.137D+00	0.100D+01	-0.104D+00	0.291D+00	-0.424D+00	-0.629D+00
-0.398D+00	0.612D+00	-0.104D+00	0.100D+01	-0.236D-02	0.835D-02	0.675D-02
0.720D+00	0.551D+00	0.291D+00	-0.236D-02	0.100D+01	-0.145D+00	-0.420D+00
-0.855D-01	-0.825D-01	-0.424D+00	0.835D-02	-0.145D+00	0.100D+01	0.154D+00
-0.302D+00	-0.292D+00	-0.629D+00	0.675D-02	-0.420D+00	0.154D+00	0.100D+01

Table 5.1-2 (Con't)

RESIDUALS V

	V1(SA10)				V2(WN14)			V1 - V2		
3406	6.6	-5.3	-14.5	3406	-1.2	1.5	3.0	7.8	-6.7	-17.6
3407	4.9	3.5	-4.3	3407	-2.1	-1.9	3.1	7.0	5.4	-7.4
3413	0.1	-0.2	1.0	3413	-0.1	0.3	-0.9	0.2	-0.5	1.9
3414	-1.4	-6.6	-1.2	3414	1.5	7.6	1.2	-2.9	-14.2	-2.4
3431	6.3	-5.8	9.1	3431	-7.1	6.7	-9.7	13.3	-12.5	18.8
3476	-0.8	0.3	-0.3	3476	0.8	-0.4	0.3	-1.7	0.7	-0.6
3477	6.6	-2.3	-1.5	3477	-7.2	2.4	1.4	13.8	-4.7	-2.9
3478	8.0	2.4	-15.4	3478	-22.4	-5.8	45.4	30.4	8.2	-60.7
3499	2.0	-0.5	-2.8	3499	-2.9	0.9	2.9	4.9	-1.4	-5.7
6002	-0.5	0.7	1.1	6002	0.5	-0.8	-1.1	-1.0	1.5	2.2
6008	-0.8	0.4	-0.3	6008	0.9	-0.5	0.3	-1.7	0.8	-0.6
6019	-1.8	1.6	0.5	6019	1.7	-1.7	-0.5	-3.5	3.4	1.1
6067	0.1	-0.2	0.9	6067	-0.1	0.2	-0.9	0.1	-0.3	1.9

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