Reports of the Department of Geodetic Science

Report No. 193

FREE GEOMETRIC ADJUSTMENT OF THE DOC/DOD COOPERATIVE WORLDWIDE GEODETIC SATELLITE (BC-4) NETWORK

by

James P. Reilly

and

M. Kumar, Ivan I. Mueller, N. Saxena

Prepared for the

National Aeronautics and Space Administration

Washington, D.C.

Grant No. NGR 36-008-093

OSURF Project No. 2514

The Ohio State University

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PREFACE AND ACKNOWLEDGEMENT

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D.C.. The contract is administered by the Office of University Affairs, NASA, Washington, D.C., 20546.

The authors wish to express their appreciation to the National Geodetic Survey for the observational data, and for other helpful information related to the analysis of the data. This report under a somewhat modified title was submitted to the Graduate School of The Ohio State University as a partial fulfillment of the requirements for the Ph.D. degree of Mr. Reilly. Acknowledgement is given to Professors Richard H. Rapp and U.A. Uotila for their helpful suggestions made as members of the dissertation reading committee.

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1. INTRODUCTION

The geometric adjustment of a network of ground stations, using optical observations to satellites, is hardly a new topic. Since the launch of the ANNA satellite in 1962, satellite observations have been used to solve a variety of previously unsolvable geodetic problems. One of the more basic problems was the determination of the Azimuth between two distant stations. The latest accomplishment is the establishment of a worldwide network of optical observing stations by the National Geodetic Survey. This geodetic network, shown in Figure 1.1, is composed of 49 observing stations, more or less evenly distributed throughout the world. Observations were made, using the BC-4 ballistic cameras, to the PAGEOS balloon satellite, beginning in June, 1966 and ending in November, 1970.

In January, 1971, the National Geodetic Survey was asked by NASA to transform the worldwide network data into what is referred to as Type I and Type II data for deposition in the Space Science Data Center at the Goddard Space Flight Center, Greenbelt, Maryland. The Type I data, called "partially reduced observations," consists of the adjusted x,y plate coordinates of each satellite image on the photographic plates. The Type II data, referred to as "fictitious satellite directions," consists of the Greenwich hour angles and declinations of generally seven fictitious images, calculated from a polynomial fit to the satellite

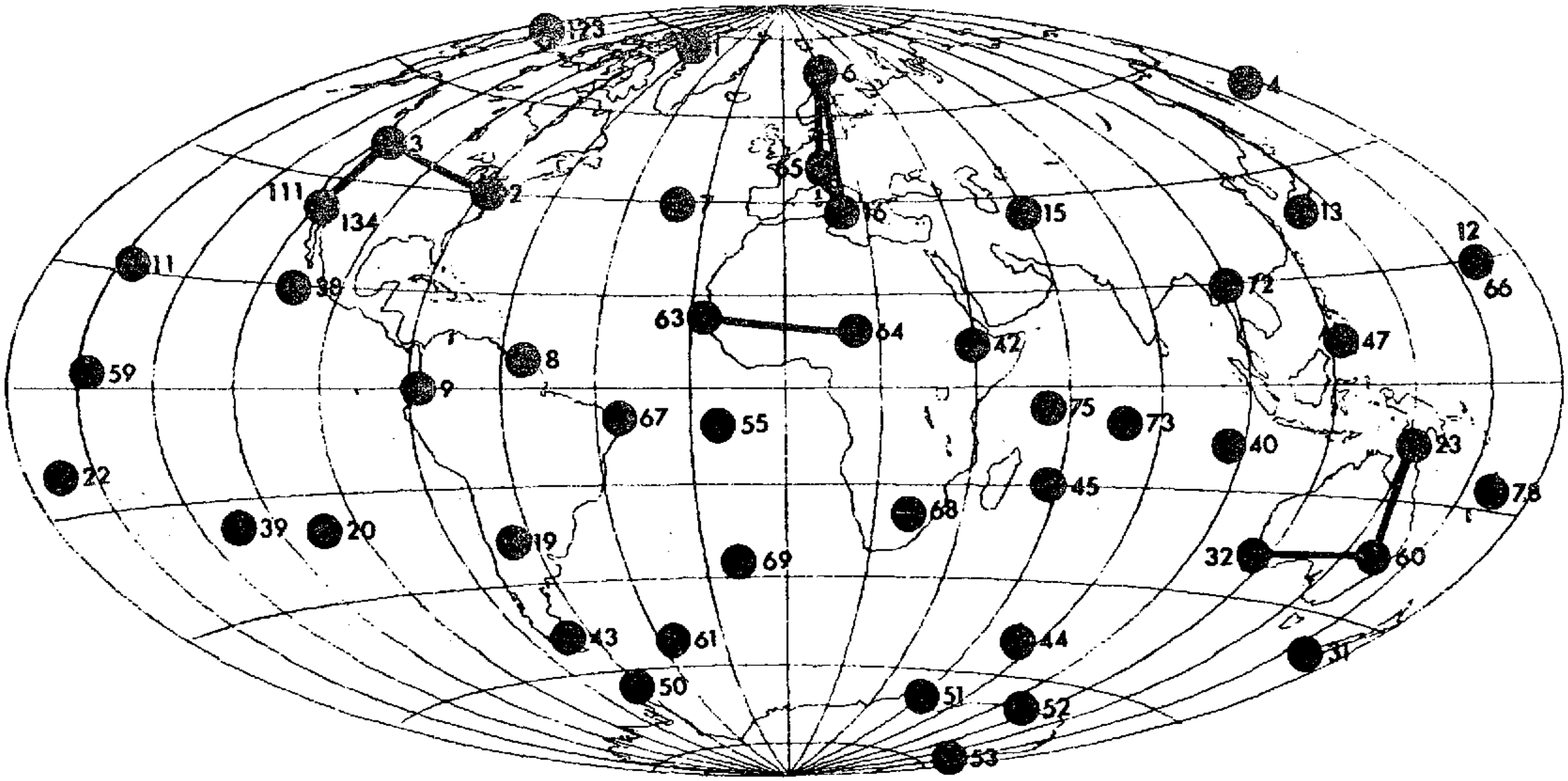

Figure 1.1. BC-4 Worldwide Geometric Satellite Network with the EDM Scalars

images, on each photographic plate. These fictitious images were reduced to simultaneouty to those from other ground stations observing the satellite during a particular event. Since all seven images on each plate were selected from a polynomial, a 14x14 variance-covariance matrix associated with the fictitious directions from each observing station is also provided as part of the data.

The purpose of this paper is to find the most practical and economical way to use these correlated observations for the accurate recovery of ground station positions, and then apply the result to the adjustment of the National Geodetic Survey worldwide network.

The paper is divided into four Chapters. Due to the variance-covariance matrix associated with the observations from each ground station, it was necessary to develop a mathematical model that would incorporate this matrix into the normal equations. The description of this model, and the mathematical development necessary to form reduced normal equations, is given in Chapter 2, following this introduction.

The description of the data, and the problems encountered in processing it, are discussed in Chapter 3. Also included in Chapter 3 are the results of preliminary adjustments performed using observations from a sub-network of the worldwide network only. This sub-network was used to analyze (1) the resulting differences using a generalized least squares solution and one utilizing observation equations, (2) the difference between a solution with correlated data and one where the correlations were ignored and (3) the degree of correlation that can be tolerated before a solution begins to weaken.

The results of the worldwide network adjustment are given in Chapter 4.

2. THEORETICAL CONSIDERATIONS

The mathematical model used as the basis for this work is the optical adjustment model developed in [Krakiwsky and Pope, 1967]. This program was developed for uncorrelated observations, with the observed directions being the right ascension and declination referenced to the true celestial coordinate system at the epoch of the observations. The modifications to this model, in order to use the Type II observations, are described in the following paragraphs.

2.1 General Development

The basic geometric figure used to describe the model is that of a single ground station observing one satellite position, illustrated in Figure 2.1. Here O is the origin of the average terrestrial coordinate system [Krakiwsky and Pope, 1967], G is the observing ground

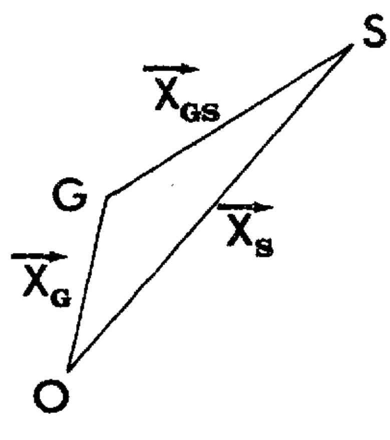

Figure 2.1. Single Station Observing One Satellite Position

station, and S is the satellite position. The mathematical model can be written as

$$F = \vec{X}_S - \vec{X}_G - \vec{X}_{GS} = 0. \qquad (2.1\text{-}1)$$

With the observed directions to the satellite in the true right ascension-declination coordinate system, the expanded version of Equation (2.1-1) is [Krakiwsky and Pope, 1967],

$$\begin{matrix} F_1 = \\ F_2 = \\ F_3 = \end{matrix} \begin{bmatrix} x_S - x_G \\ y_S - y_G \\ z_S - z_G \end{bmatrix} - \mathbf{S} \begin{bmatrix} r \cos\alpha \cos\delta \\ r \sin\alpha \cos\delta \\ r \sin\delta \end{bmatrix} \begin{matrix} = 0 \\ = 0 \\ = 0 \end{matrix} \qquad (2.1\text{-}2)$$

where

r = topocentric range to the satellite,

α = true topocentric right ascension of the satellite,

δ = true topocentric declination of the satellite,

and

$$\mathbf{S} = \begin{bmatrix} \cos(GAST) & \sin(GAST) & x \\ -\sin(GAST) & \cos(GAST) & -y \\ -x\cos(GAST) - y\sin(GAST) & -x\sin(GAST) + y\cos(GAST) & 1 \end{bmatrix},$$

GAST = Greenwich Apparent Sidereal Time,

x, y = the two components of polar motion.

If the observed quantities are the Greenwich hour angle (h) and declination (δ) in the average terrestrial coordinate system, such as the Type II data, the expanded version of Equation (2.1-1) is

$$\begin{aligned} F_1 &= x_S - x_G - r\cos h \cos\delta = 0 \\ F_2 &= y_S - y_G + r\sin h \cos\delta = 0 \qquad (2.1\text{-}3) \\ F_3 &= z_S - z_G - r\sin\delta = 0\,. \end{aligned}$$

The development necessary to form normal equations from uncorrelated observations, using the generalized least squares approach with the model defined by Equation (2.1-2), is described in detail in [Krakiwsky and Pope, 1967]. When the correlation between observations is introduced, the situation becomes much more complex. The Type II data was in the form of card images, on magnetic tapes, arranged as shown in Figure 2.2. In the model using uncorrelated observations, an event was defined as two or more ground stations simultaneously observing one satellite position. With the Type II data, an event is defined as two or more ground stations observing the seven simultaneous (fictitious) satellite positions. This is shown in Figure 2.3. The situation corresponding to Figure 2.1 is drawn in Figure 2.4. The resulting mathematical model is developed in the following paragraphs.

2.2 The Generalized Least Squares Development

The mathematical model needed for the Type II observations is that of Equation (2.1-3) expanded to include observations to all seven satellite positions, i.e.,

$$
\begin{aligned}
F_1 &= x_{S_1} - x_G - r_{S_1} \cos h_{S_1} \cos \delta_{S_1} = 0 \\
F_2 &= y_{S_1} - y_G + r_{S_1} \sin h_{S_1} \cos \delta_{S_1} = 0 \\
F_3 &= z_{S_1} - z_G - r_{S_1} \sin \delta_{S_1} = 0 \\
&\vdots \\
F_{19} &= x_{S_7} - x_G - r_{S_7} \cos h_{S_7} \cos \delta_{S_7} = 0 \\
F_{20} &= y_{S_7} - y_G + r_{S_7} \sin h_{S_7} \cos \delta_{S_7} = 0 \\
F_{21} &= z_{S_7} - z_G - r_{S_7} \sin \delta_{S_7} = 0.
\end{aligned}
\qquad (2.2\text{-}1)
$$

	Columns	Format	Contents
Card Format 1	2 - 6	I5	Event No.
	7	I1	The number of ground stations observing during the event.
	8 - 9	I2	Number of fictitious Satellite images (usually 7).
Card Format 2	2 - 6	I5	Station No. of the first observing ground station.
	7 - 30	6A4	Name of ground station.
	31 - 34	I4	Plate No.
	35 - 36	I2	Number of usable fictitious points.
Card Format 3	1 - 20 21 - 40 41 - 60 61 - 80	4E20.13	The upper-triangular part of the variance-covariance matrix. There will be N(2N + 1) terms in this matrix, where N is the number of fictitious satellite images on the Plate. Card Format 3 is repeated until all terms have been read.
Card Format 4	1 - 2	I2	Satellite image No.
	3 - 18	F16.9	Greenwich hour angle, in radians.
	19 - 34	F16.9	Declination, in radians.

Card Format 4 is repeated for each satellite image. Card Formats 2, 3 and 4 are repeated for each additional set of data in the event.

Figure 2.2. Card Formats for the Type II Data

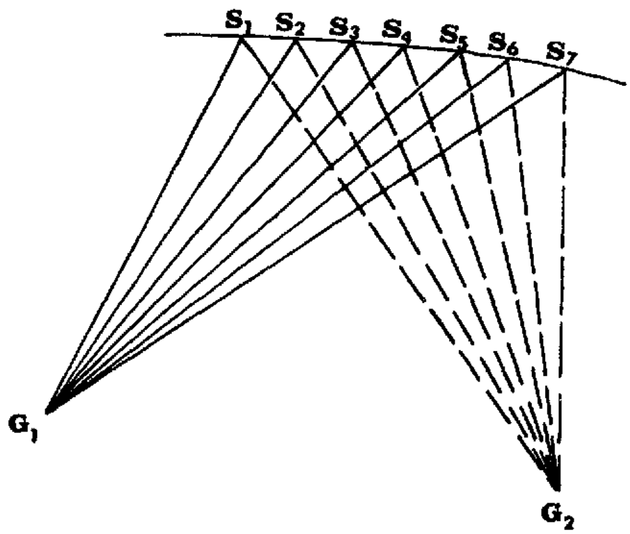

Figure 2.3. A Two-station Event using the Type II Data

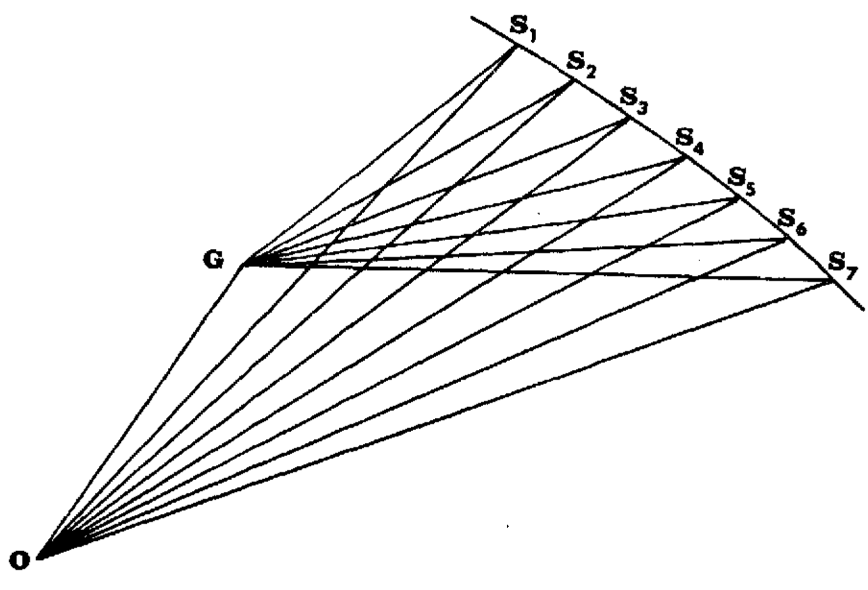

Figure 2.4. Single Station Observing Seven Satellite Positions

The generalized form of the linearized mathematical model is

$$A_1 X_G + A_2 X_S + BV + W = 0 \tag{2.2-2}$$

where

$$A_1 = \begin{bmatrix} \frac{\partial F_1}{\partial x_G} & \frac{\partial F_1}{\partial y_G} & \frac{\partial F_1}{\partial z_G} \\ \frac{\partial F_2}{\partial x_G} & \frac{\partial F_2}{\partial y_G} & \frac{\partial F_2}{\partial z_G} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\partial F_{21}}{\partial x_G} & \frac{\partial F_{21}}{\partial y_G} & \frac{\partial F_{21}}{\partial z_G} \end{bmatrix} = \begin{bmatrix} -I \\ -I \\ \cdot \\ \cdot \\ \cdot \\ -I \end{bmatrix}_{(21x3)},$$

$$A_2 = \begin{bmatrix} \frac{\partial F_1}{\partial x_{S_1}} & \frac{\partial F_1}{\partial y_{S_1}} & \frac{\partial F_1}{\partial z_{S_1}} & \cdots & \frac{\partial F_1}{\partial x_{S_7}} & \frac{\partial F_1}{\partial y_{S_7}} & \frac{\partial F_1}{\partial z_{S_7}} \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \frac{\partial F_{21}}{\partial x_{S_1}} & \frac{\partial F_{21}}{\partial y_{S_1}} & \frac{\partial F_{21}}{\partial z_{S_1}} & \cdots & \frac{\partial F_{21}}{\partial x_{S_7}} & \frac{\partial F_{21}}{\partial y_{S_7}} & \frac{\partial F_{21}}{\partial z_{S_7}} \end{bmatrix} = \begin{bmatrix} I \end{bmatrix}_{(21x21)}.$$

In the general case where the ranges r_{S_i} are also observed,

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial h_1} & \frac{\partial F_1}{\partial \delta_1} & \frac{\partial F_1}{\partial r_1} & \cdots & \frac{\partial F_1}{\partial h_7} & \frac{\partial F_1}{\partial \delta_7} & \frac{\partial F_1}{\partial r_7} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ \frac{\partial F_{21}}{\partial h_1} & \frac{\partial F_{21}}{\partial \delta_1} & \frac{\partial F_{21}}{\partial r_1} & \cdots & \frac{\partial F_{21}}{\partial h_7} & \frac{\partial F_{21}}{\partial \delta_7} & \frac{\partial F_{21}}{\partial r_7} \end{bmatrix}.$$

In practice the above matrix has the following form:

$$B = \begin{bmatrix} \boxed{3x3} & & & & & & \\ & \boxed{3x3} & & & & 0 & \\ & & \boxed{3x3} & & & & \\ & & & \boxed{3x3} & & & \\ & 0 & & & \boxed{3x3} & & \\ & & & & & \boxed{3x3} & \\ & & & & & & \boxed{3x3} \end{bmatrix}_{(21,21)} .$$

The vector V is the vector of observation residuals

$$V = \begin{bmatrix} h_1^a & - & h_1^o \\ \delta_1^a & - & \delta_1^o \\ r_1^a & - & r_1^o \\ & \vdots & \\ h_7^a & - & h_7^o \\ \delta_7^a & - & \delta_7^o \\ r_7^a & - & r_7^o \end{bmatrix}$$

where the superscript ^{o} means observed and the superscript ^{a} means adjusted. The approximate satellite positions are determined from the ground station coordinates, and the observations. The ranges can then be calculated. The vector W is the discrepancy vector determined by inserting the observations and the approximations to the parameters in the mathematical model, Equation (2.2-1). The vectors X_G and X_S are the corrections to the ground stations and satellite positions, respectively.

The reduced normal equations for a maximum of four co-observing stations, and the variation function, are formed as follows:

$$
\begin{aligned}
A_1X_1 + A_2X_S + B_1V_1 + W_1 &= 0 \\
A_1X_2 + A_2X_S + B_2V_2 + W_2 &= 0 \\
A_1X_3 + A_2X_S + B_3V_3 + W_3 &= 0 \\
A_1X_4 + A_2X_S + B_4V_4 + W_4 &= 0,
\end{aligned}
\qquad (2.2\text{-}3)
$$

and

$$
\begin{aligned}
\phi = {} & V_1'P_1V_1 + V_2'P_2V_2 + V_3'P_3V_3 + V_4'P_4V_4 - 2K_1'(A_1X_1 + A_2X_S + B_1V_1 + W_1) \\
& - 2K_2'(A_1X_2 + A_2X_S + B_2V_2 + W_2) - 2K_3'(A_1X_3 + A_2X_S + B_3V_3 + W_3) \\
& - 2K_4'(A_1X_4 + A_2X_S + B_4V_4 + W_4),
\end{aligned}
$$

where

V_{S_i}' = the transpose of the residual vector V_{S_i}

K_{S_i}' = the vector of Lagrangian Multipliers

P_{S_i} = the weight matrix of the observations.

Taking the partial derivatives of the variation function with respect to the V's, X_1, X_2, X_3, X_4 and X_S,

$$
\begin{aligned}
\frac{1}{2}\frac{\partial\phi}{\partial V_1} &= P_1V_1 - B_1'K_1 = 0 \longrightarrow V_1 = P_1^{-1}B_1'K_1 \\
\frac{1}{2}\frac{\partial\phi}{\partial V_2} &= P_2V_2 - B_2'K_2 = 0 \longrightarrow V_2 = P_2^{-1}B_2'K_2 \\
\frac{1}{2}\frac{\partial\phi}{\partial V_3} &= P_3V_3 - B_3'K_3 = 0 \longrightarrow V_3 = P_3^{-1}B_3'K_3 \\
\frac{1}{2}\frac{\partial\phi}{\partial V_4} &= P_4V_4 - B_4'K_4 = 0 \longrightarrow V_4 = P_4^{-1}B_4'K_4 \qquad (2.2\text{-}4) \\
\frac{1}{2}\frac{\partial\phi}{\partial X_1} &= -A_1'K_1 = 0
\end{aligned}
$$

$$\frac{1}{2}\frac{\partial\phi}{\partial X_2} = -A_1'K_2 = 0$$

$$\frac{1}{2}\frac{\partial\phi}{\partial X_3} = -A_1'K_3 = 0$$

$$\frac{1}{2}\frac{\partial\phi}{\partial X_4} = -A_1'K_4 = 0$$

$$\frac{1}{2}\frac{\partial\phi}{\partial X_S} = -A_2'K_1 - A_2'K_2 - A_2'K_3 - A_2'K_4 = -A_2'(K_1 + K_2 + K_3 + K_4) = 0.$$

At this point in the development there are 13 equations, namely (2.2-3) and (2.2-4). Four of these equations can be eliminated by substituting the first four equations of (2.2-4) into (2.2-3). This results in

$$\begin{aligned} A_1X_1 + A_2X_S + B_1P_1^{-1}B_1'K_1 + W_1 &= 0 \\ A_1X_2 + A_2X_S + B_2P_2^{-1}B_2'K_2 + W_2 &= 0 \\ A_1X_3 + A_2X_S + B_3P_3^{-1}B_3'K_3 + W_3 &= 0 \\ A_1X_4 + A_2X_S + B_4P_4^{-1}B_4'K_4 + W_4 &= 0\ . \end{aligned} \tag{2.2-5}$$

The development has now been reduced to the above four equations plus the last five in (2.2-4). Equations (2.2-5) can now be rearranged to solve for the Lagrangian multipliers (K's) as follows:

$$\begin{aligned} K_1 &= -(B_1P_1^{-1}B_1')^{-1}\ (A_1X_1 + A_2X_S + W_1) \\ K_2 &= -(B_2P_2^{-1}B_2')^{-1}\ (A_1X_2 + A_2X_S + W_2) \\ K_3 &= -(B_3P_3^{-1}B_3')^{-1}\ (A_1X_3 + A_2X_S + W_3) \\ K_4 &= -(B_4P_4^{-1}B_4')^{-1}\ (A_1X_4 + A_2X_S + W_4). \end{aligned} \tag{2.2-6}$$

Before proceeding further, it is necessary to explain how Equations (2.2-6) are actually solved. Looking specifically at the first equation in (2.2-6), the B_1 matrix is dimensioned 21x21 but the variance-covariance matrix associated with the observations is 14x14. When the B_1 matrix was developed, the range r_{S_i} was considered as an observed quantity. The P^{-1} matrix refers only to the actual observed quantities which are the Greenwich hour angle (h_i) and the declination (δ_i). Therefore, the P_i^{-1} matrices in Equation (2.2-6) have to be changed. The easiest way to explain this is to look only at that part of B_1 that corresponds to observations on the first satellite position:

$$B_1 = \begin{bmatrix} \frac{\partial F_1}{\partial h_1} & \frac{\partial F_1}{\partial \delta_1} & \frac{\partial F_1}{\partial r_1} \\ \frac{\partial F_2}{\partial h_1} & \frac{\partial F_2}{\partial \delta_1} & \frac{\partial F_2}{\partial r_1} \\ \frac{\partial F_3}{\partial h_1} & \frac{\partial F_3}{\partial \delta_1} & \frac{\partial F_3}{\partial r_1} \end{bmatrix} . \tag{2.2-7}$$

The matrix P_1 (not P_1^{-1}) would have to be

$$P_1 = \begin{bmatrix} \sigma_{h_1}^2 & \sigma_{h_1\delta_1} & \sigma_{h_1 r_1} \\ \sigma_{h_1\delta_1} & \sigma_{\delta_1}^2 & \sigma_{\delta_1 r_1} \\ \sigma_{h_1 r_1} & \sigma_{\delta_1 r_1} & \sigma_{r_1}^2 \end{bmatrix}^{-1} . \tag{2.2-8}$$

The range measurment is used in the algebraic derivation, but in the numerical computations are obtained to a sufficient accuracy from the observed quantities. Therefore, Equation (2.2-8) can be written as

$$P_1 = \begin{bmatrix} \sigma_{h_1}^2 & \sigma_{h_1\delta_1} & 0 \\ \sigma_{h_1\delta_1} & \sigma_{\delta_1}^2 & 0 \\ 0 & 0 & \infty \end{bmatrix}^{-1}$$

which is

$$P_1 = \begin{bmatrix} \begin{bmatrix} \sigma^2_{h_1} & \sigma_{h_1\delta_1} \\ \sigma_{h_1\delta_1} & \sigma^2_{\delta_1} \end{bmatrix}^{-1} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix} . \tag{2.2-9}$$

The expression $(B_1P_1^{-1}B_1')^{-1}$ can now be solved as follows:

$$(B_1P_1^{-1}B_1')^{-1} = (B_1')^{-1}P_1B_1^{-1} = (B_1^{-1})'P_1B_1^{-1} \tag{2.2-10}$$

where P_1 is defined by Equation (2.2-9).

The preceding description applies to the case of one satellite position. For the seven satellite positions the dimension of the P^{-1} matrix is 14x14, and the P_1 matrix must be dimensioned 21x21. An intermediate matrix $\overline{W}$ can be formed as follows:

$$\overline{W} = \begin{bmatrix} \sigma^2_{h_1} & \sigma_{h_1\delta_1} & \cdots & \sigma_{h_1\delta_7} \\ \cdot & \sigma^2_{\delta_1} & & \cdot \\ \cdot & & \ddots & \cdot \\ \cdot & & \sigma^2_{h_7} & \cdot \\ \sigma_{h_1\delta_7} & \cdots & & \sigma^2_{\delta_7} \end{bmatrix}^{-1} = \begin{bmatrix} \overline{W}_{1,1} & \overline{W}_{1,2} & \cdots & \overline{W}_{1,14} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \overline{W}_{14,1} & & \cdots & \overline{W}_{14,14} \end{bmatrix} . \tag{2.2-11}$$

Now the 21x21 version of Equation (2.2-9) will be

$$P_1 = \begin{bmatrix} \overline{W}_{1,1} & \overline{W}_{1,2} & 0 & \overline{W}_{1,3} & \overline{W}_{1,4} & 0 & \cdots & \overline{W}_{1,13} & \overline{W}_{1,14} & 0 \\ \overline{W}_{2,1} & \overline{W}_{2.2} & 0 & \overline{W}_{2,3} & \overline{W}_{2,4} & 0 & \cdots & \overline{W}_{2,13} & \overline{W}_{2.14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ & & & & & & & & & \end{bmatrix}$$

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \overline{W}_{13,1} & \overline{W}_{13,2} & 0 & \overline{W}_{13,3} & \overline{W}_{13,4} & 0 & \cdots & \overline{W}_{13,13} & \overline{W}_{13,14} & 0 \\ \overline{W}_{14,1} & \overline{W}_{14,2} & 0 & \overline{W}_{14,3} & \overline{W}_{14,4} & 0 & \cdots & \overline{W}_{14,13} & \overline{W}_{14,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} . \tag{2.2-12}$$

With P_1 defined by Equation (2.2-12), the matrix M^{-1} can be formed using the technique shown in Equation (2.2-10), namely

$$M_1^{-1} = (B_1 P_1^{-1} B_1')^{-1} = (B_1^{-1})' P_1 B_1^{-1}. \tag{2.2-13}$$

Using this to solve for the Lagrangian multipliers in Equations (2.2-6)

$$\begin{aligned} K_1 &= -M_1^{-1}(A_1X_1 + A_2X_S + W_1) \\ K_2 &= -M_2^{-1}(A_1X_2 + A_2X_S + W_2) \\ K_3 &= -M_3^{-1}(A_1X_3 + A_2X_S + W_3) \\ K_4 &= -M_4^{-1}(A_1X_4 + A_2X_S + W_4). \end{aligned} \tag{2.2-14}$$

Substitution of these into the last five equations in (2.2-4) gives

$$\begin{aligned} -A_1'K_1 &= A_1'M_1^{-1}A_1X_1 + A_1'M_1^{-1}A_2X_S + A_1'M_1^{-1}W_1 &= 0 \\ -A_1'K_2 &= A_1'M_2^{-1}A_1X_2 + A_1'M_2^{-1}A_2X_S + A_1'M_2^{-1}W_2 &= 0 \\ -A_1'K_3 &= A_1'M_3^{-1}A_1X_3 + A_1'M_3^{-1}A_2X_S + A_1'M_3^{-1}W_3 &= 0 \\ -A_1'K_4 &= A_1'M_4^{-1}A_1X_4 + A_1'M_4^{-1}A_2X_S + A_1'M_4^{-1}W_4 &= 0 \end{aligned} \tag{2.2-15}$$

and

$$\begin{aligned} -A_2'(K_1 + K_2 + K_3 + K_4) = {} & A_2'M_1^{-1}A_1X_1 + A_2'M_1^{-1}A_2X_S + A_2'M_1^{-1}W_1 \\ & + A_2'M_2^{-1}A_1X_2 + A_2'M_2^{-1}A_2X_S + A_2'M_2^{-1}W_2 \end{aligned}$$

$$+ A_2'M_3^{-1}A_1X_3 + A_2'M_3^{-1}A_2X_S + A_2'M_3^{-1}W_3$$

$$+ A_2'M_4^{-1}A_1X_4 + A_2'M_4^{-1}A_2X_S + A_2'M_4^{-1}W_4 = 0. \tag{2.2-16}$$

Writing the above in block notation results in the following equation:

$$\begin{bmatrix} A_1'M_1^{-1}A_1 & 0 & 0 & 0 & A_1'M_1^{-1}A_2 \\ 0 & A_1'M_2^{-1}A_1 & 0 & 0 & A_1'M_2^{-1}A_2 \\ 0 & 0 & A_1'M_3^{-1}A_1 & 0 & A_1'M_3^{-1}A_2 \\ 0 & 0 & 0 & A_1'M_4^{-1}A_1 & A_1'M_4^{-1}A_2 \\ A_2'M_1^{-1}A_1 & A_2'M_2^{-1}A_1 & A_2'M_3^{-1}A_1 & A_2'M_4^{-1}A_1 & \sum_{i=1}^{4}(A_i'M_i^{-1})A_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_S \end{bmatrix} + \begin{bmatrix} A_1'M_1^{-1}W_1 \\ A_1'M_2^{-1}W_2 \\ A_1'M_3^{-1}W_3 \\ A_1'M_4^{-1}W_4 \\ \sum_{i=1}^{4} A_2'M_i^{-1}W_i \end{bmatrix} = 0. \tag{2.2-17}$$

Equation (2.2-17) is referred to as the conventional normal equation for a four station event where the satellite position X_S is one of the parameters. If only three stations were involved, the fourth row and fourth column would be deleted from Equation (2.2-17) and the summation in the last row would be one to three. Since the satellite position is of no interest, it is eliminated from the solution. This is done by solving for X_S in terms of the other parameters and substituting this into the remaining equations:

$$X_S = - \left[\sum_{i=1}^{4}(A_i'M_i^{-1})A_2 \right]^{-1} \Big\{ A_2'M_1^{-1}A_1X_1 + A_2'M_2^{-1}A_1X_2 + A_2'M_3^{-1}A_1X_3 + A_2'M_4^{-1}A_1X_4 + \sum_{i=1}^{4} A_2'M_i^{-1}W_i \Big\}. \tag{2.2-18}$$

Insertion of Equation (2.2-18) into Equations (2.2-15) gives the reduced normal equation. Since the A_2 matrix is the identity matrix, the equations can be simplified to

$$A_1'M_1^{-1}A_1X_1 - A_1'M_1^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_1^{-1}A_1X_1 - A_1'M_1^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_2^{-1}A_1X_2$$

$$- A_1'M_1^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_3^{-1}A_1X_3 - A_1'M_1^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_4^{-1}A_1X_4$$

$$- A_1'M_1^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1} \sum_{i=1}^{4}\left(M_i^{-1}W_i\right) + A_1'M_1^{-1}W_1 = 0$$

$$A_1'M_2^{-1}A_1X_2 - A_1'M_2^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_1^{-1}A_1X_1 - A_1'M_2^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_2^{-1}A_1X_2$$

$$- A_1'M_2^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_3^{-1}A_1X_3 - A_1'M_2^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_4^{-1}A_1X_4$$

$$- A_1'M_2^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1} \sum_{i=1}^{4}\left(M_i^{-1}W_i\right) + A_1'M_2^{-1}W_2 = 0$$

$$A_1'M_3^{-1}A_1X_3 - A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_1^{-1}A_1X_1 - A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_2^{-1}A_1X_2$$

$$- A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_3^{-1}A_1X_3 - A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_4^{-1}A_1X_4 \qquad (2.2\text{-}19)$$

$$- A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1} \sum_{i=1}^{4}\left(M_i^{-1}W_i\right) + A_1'M_3^{-1}W_3 = 0$$

$$A_1'M_4^{-1}A_1X_4 - A_1'M_4^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_1^{-1}A_1X_1 - A_1'M_4^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_2^{-1}A_1X_2$$

$$- A_1'M_4^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_3^{-1}A_1X_3 - A_1'M_3^{-1}\left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}M_4^{-1}A_1X_4$$

$$- A_1' M_4^{-1} \left[\sum_{i=1}^{4} M_i^{-1} \right]^{-1} \sum_{i=1}^{4} \left(M_i^{-1} W_i \right) + A_1' M_4^{-1} W_4 = 0.$$

Combining terms in the above equations and writing in block notation, the result is:

$$NX + U = 0,$$

where

$$N = \begin{bmatrix} A_1' M_1^{-1} A_1 - A_1' M_1^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_1^{-1} A_1 ; & -A_1' M_1^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_2^{-1} A_1 ; & -A_1' M_1^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_3^{-1} A_1 ; & -A_1' M_1^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_4^{-1} A_1 \\ & A_1' M_2^{-1} A_1 - A_1' M_2^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_2^{-1} A_1 ; & -A_1' M_2^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_3^{-1} A_1 ; & -A_1' M_2^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_4^{-1} A_1 \\ & & A_1' M_3^{-1} A_1 - A_1' M_3^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_3^{-1} A_1 ; & -A_1' M_3^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_4^{-1} A_1 \\ \text{(same as above diagonal)} & & & A_1' M_4^{-1} A_1 - A_1' M_4^{-1} [\sum_{i=1}^{4} M_i^{-1}]^{-1} M_4^{-1} A_1 \end{bmatrix}$$

and

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

and

$$
U = \begin{bmatrix}
A_1'M_1^{-1}W_1 - A_1'M_1^{-1}[\sum_{i=1}^{4} M_i^{-1}]^{-1} \sum_{i=1}^{4} M_i^{-1}W_i \\
A_1'M_2^{-1}W_2 - A_1'M_2^{-1}[\sum_{i=1}^{4} M_i^{-1}]^{-1} \sum_{i=1}^{4} M_i^{-1}W_i \\
A_1'M_3^{-1}W_3 - A_1'M_3^{-1}[\sum_{i=1}^{4} M_i^{-1}]^{-1} \sum_{i=1}^{4} M_i^{-1}W_i \\
A_1'M_4^{-1}W_4 - A_1'M_4^{-1}[\sum_{i=1}^{4} M_i^{-1}]^{-1} \sum_{i=1}^{4} M_i^{-1}W_i
\end{bmatrix} .
$$

2.3 The Observation Equation Development

The basic generalized model given by Equation (2.1-3) can easily be rearranged so that the adjusted values of the observed quantities are a function of the parameters that result from the adjustment. The rearranged model is

$$
h = \tan^{-1}\left(\frac{y_S - y_G}{x_S - x_G}\right) \tag{2.3-1}
$$

$$
\delta = \tan^{-1}\left(\frac{z_S - z_G}{\{(x_S - x_G)^2 + (y_S - y_G)^2\}^{\frac{1}{2}}}\right) .
$$

The expanded version of this model, to include all seven satellite positions, is

$$
h_1 = \tan^{-1}\left(\frac{y_{S_1} - y_G}{x_{S_1} - x_G}\right)
$$

$$\delta_1 = \tan^{-1}\left(\frac{z_{S_1} - z_G}{\left\{(x_{S_1} - x_G)^2 + (y_{S_1} - y_G)^2\right\}^{\frac{1}{2}}}\right)$$

$$\begin{array}{l} \cdot \\ \cdot \\ \cdot \end{array} \qquad (2.3\text{-}2)$$

$$h_7 = \tan^{-1}\left(\frac{y_{S_7} - y_G}{x_{S_7} - x_G}\right)$$

$$\delta_7 = \tan^{-1}\left(\frac{z_{S_7} - z_G}{\left\{(x_{S_7} - x_G)^2 + (y_{S_7} - y_G)^2\right\}^{\frac{1}{2}}}\right) .$$

The observation equation form of the linearized mathematical model is

$$A_1X_G + A_2X_S + L - V = 0 \qquad (2.3\text{-}3)$$

where

$$A_1 = \begin{bmatrix} \frac{\partial h_1}{\partial x_G} & \frac{\partial h_1}{\partial y_G} & \frac{\partial h_1}{\partial z_G} \\ \frac{\partial \delta_1}{\partial x_G} & \frac{\partial \delta_1}{\partial y_G} & \frac{\partial \delta_1}{\partial z_G} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial h_7}{\partial x_G} & \frac{\partial h_7}{\partial y_G} & \frac{\partial h_7}{\partial z_G} \\ \frac{\partial \delta_7}{\partial x_G} & \frac{\partial \delta_7}{\partial y_G} & \frac{\partial \delta_7}{\partial z_G} \end{bmatrix} ,$$

$$A_2 = \begin{bmatrix} \frac{\partial h_1}{\partial x_{S_1}} & \frac{\partial h_1}{\partial y_{S_1}} & \frac{\partial h_1}{\partial z_{S_1}} & \cdots & \frac{\partial h_1}{\partial x_{S_7}} & \frac{\partial h_1}{\partial y_{S_7}} & \frac{\partial h_1}{\partial z_{S_7}} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \frac{\partial \delta_7}{\partial x_{S_1}} & \frac{\partial \delta_7}{\partial y_{S_1}} & \frac{\partial \delta_7}{\partial z_{S_1}} & \cdots & \frac{\partial \delta_7}{\partial x_{S_7}} & \frac{\partial \delta_7}{\partial y_{S_7}} & \frac{\partial \delta_7}{\partial z_{S_7}} \end{bmatrix},$$

$$L = \begin{bmatrix} h_1^c - h_1^o \\ \delta_1^c - \delta_1^o \\ \cdot \\ \cdot \\ \cdot \\ h_7^c - h_7^o \\ \delta_7^c - \delta_7^o \end{bmatrix}.$$

The conventional normal equations for a four-station event is

$$\begin{bmatrix} A_1'P_1A_1 & 0 & 0 & 0 & A_1'P_1A_2 \\ 0 & A_3'P_2A_3 & 0 & 0 & A_3'P_2A_4 \\ 0 & 0 & A_5'P_3A_5 & 0 & A_5'P_3A_6 \\ 0 & 0 & 0 & A_7'P_4A_7 & A_7'P_4A_8 \\ A_2'P_1A_1 & A_4'P_2A_3 & A_6'P_3A_5 & A_8'P_4A_7 & \mathbf{N} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_S \end{bmatrix} + \begin{bmatrix} A_1'P_1L_1 \\ A_3'P_2L_2 \\ A_5'P_3L_3 \\ A_7'P_4L_4 \\ \mathbf{U} \end{bmatrix} = 0 \qquad (2.3\text{-}4)$$

where

$$\mathbf{N} = A_2{}'P_1A_2 + A_4{}'P_2A_4 + A_6{}'P_3A_6 + A_8{}'P_4A_4$$

and

$$\mathbf{U} = A_2{}'P_1L_1 + A_4{}'P_2L_2 + A_6{}'P_3L_3 + A_8{}'P_4L_4.$$

As with the generalized case, the satellite position X_S is a nuisance parameter and should be eliminated from the equations. From Equation (2.3-4),

$$\begin{aligned} X_S = -\mathbf{N}^{-1}(&A_2{}'P_1A_1X_1 + A_4{}'\ P_2A_3X_2 + A_6{}'\ P_3A_5X_3 + A_8{}'P_4A_7X_4 \\ &+ A_2{}'P_1L_1 + A_4{}'P_2L_2 + A_6{}'P_3L_3 + A_8{}'P_4L_4). \end{aligned} \qquad (2.3\text{-}5)$$

Substituting Equation (2.3-5) into Equation (2.3-4) and writing in block notation,

$$\begin{bmatrix} N_1 - \bar{N}_1 \mathbf{N}^{-1} \bar{N}_1{}' & -\bar{N}_1 \mathbf{N}^{-1} \bar{N}_3{}' & -\bar{N}_1 \mathbf{N}^{-1} \bar{N}_5{}' & -\bar{N}_1 \mathbf{N}^{-1} \bar{N}_7{}' \\ & N_3 - \bar{N}_3 \mathbf{N}^{-1} \bar{N}_3{}' & -\bar{N}_3 \mathbf{N}^{-1} \bar{N}_5{}' & -\bar{N}_3 \mathbf{N}^{-1} \bar{N}_7{}' \\ & & N_5 - \bar{N}_5 \mathbf{N}^{-1} \bar{N}_5{}' & -\bar{N}_5 \mathbf{N}^{-1} \bar{N}_7{}' \\ \text{same as above diagonal} & & & N_7 - \bar{N}_7 \mathbf{N}^{-1} \bar{N}_7{}' \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} U_1 + \bar{N}_1\mathbf{N}^{-1}(U_2+U_4+U_6+U_8) \\ U_2 + \bar{N}_3\mathbf{N}^{-1}(U_2+U_4+U_6+U_8) \\ U_3 + \bar{N}_5\mathbf{N}^{-1}(U_2+U_4+U_6+U_8) \\ U_4 + \bar{N}_7\mathbf{N}^{-1}(U_2+U_4+U_6+U_8) \end{bmatrix} = 0. \qquad (2.3\text{-}6)$$

where

$N_1 = A_1{}'P_1A_1$	$N_3 = A_3{}'P_2A_3$	$N_5 = A_5{}'P_3A_5$	$N_7 = A_7{}'P_4A_8$
$N_2 = A_2{}'P_1A_2$	$N_4 = A_4{}'P_2A_4$	$N_6 = A_6{}'P_3A_6$	$N_8 = A_8{}'P_4A_8$
$\bar{N}_1 = A_1{}'P_1A_2$	$\bar{N}_3 = A_3{}'P_2A_4$	$\bar{N}_5 = A_5{}'P_3A_6$	$\bar{N}_7 = A_7{}'P_4A_8$
$\bar{N}_1{}' = A_2{}'P_1A_1$	$\bar{N}_3{}' = A_4{}'P_2A_3$	$\bar{N}_5{}' = A_6{}'P_3A_5$	$\bar{N}_7{}' = A_8{}'P_4A_7$
$U_1 = A_1{}'P_1L_1$	$U_3 = A_3{}'P_2L_2$	$U_5 = A_5{}'P_3L_3$	$U_7 = A_7{}'P_4L_4$
$U_2 = A_2{}'P_1L_1$	$U_4 = A_4{}'P_2L_2$	$U_6 = A_6{}'P_3L_3$	$U_8 = A_8{}'P_4L_4$

2.4 Formation of the Normal Equations

The normal equations described in Sections 2.2 and 2.3 were developed in order that the reduced normal equation N could be separated into 3x3 blocks, and the discrepancy vector U could be separated into 3x1 columns. The computer program to solve normal equations in this form had been developed several years ago. This adjustment program, called OSUGOP, can add different sets of normal equations, and apply constraints by using a sophisticated constraint package. The program is described in detail in [Reilly, Schwarz and Whiting, 1973].

The logic used in developing the computer program to form reduced normal equations was as follows:

1. The observational data should be read directly from a magnetic tape.
2. The program should be such that the only card input would be the station coordinates and datum information.
3. The program output should be the reduced normal matrix, in 3x3 blocks, plus the discrepancy vector, both punched on data cards so that this deck of cards can be input into the existing adjustment program.

The computer program that was developed satisfied all three of the above requirements. The normal equations were formed using each of the seventeen magnetic tapes on which the Type II data was stored, separately, so that there would be seventeen sets of normal equations to add together for the adjustment.

The one difficulty with forming the normal equations as described above was the calculation of the residuals (v's) of the observations after adjustment, necessary to determine the V'PV. The technique used to compute this quantity is discussed below. A brief description of the computer program is also included.

2.4.1 Calculation of V'PV

As mentioned above, the determination of V'PV is essential in order to obtain the variance-covariance matrix of the parameters after the adjustment. When the reduced normal equations are formed using the methods discussed in this investigation, it is difficult to substitute the original observed quantities back into the linearized mathematical model, like Equations (2.2-2) or (2.3-3), to arrive at the residual vector V. Although it is not possible to get the individual residuals, v_i, the total V'PV can be calculated.

The complete derivation of V'PV, using the reduced normal equations, is given in [Krakiwsky and Pope, 1967, pgs 73-77]. The equation for V'PV, using the notation of this report, is

$$V'PV = \Sigma\,(W_i'M_i^{-1}W_i) - \Sigma\left[(M_i^{-1}W_i)'(\Sigma\,M_i^{-1})^{-1}(M_i^{-1}W_i)\right] - X'U \tag{2.4.1-1}$$

for the generalized least squares development, and for the method of observation Equations

$$V'PV = \Sigma\,(L_iP_i^{-1}L_i) + \Sigma\left[(A_{2_i}'P_i^{-1}L_i)(\Sigma\,A_{2_i}P_i^{-1}A_{2_i})^{-1}(A_{2_i}'P_i^{-1}L_i)\right] - X'U. \tag{2.4.1-2}$$

The first term in the above equations is the contribution from the ground stations. The second term is the contribution from the satellite positions. The third term requires the X vector, and this can be calculated only after the adjustment is completed. Therefore, the procedure used in this investigation was to separately sum the first and second terms during the formation of normal equations for each magnetic tape of data. This number was then punched onto a data card.[1] After all normal equations have been formed, this number is added to that of the other magnetic tapes and the adjustment is performed to obtain the X vector. V'PV is then calculated.

2.4.2 Computer Program to form Reduced Normal Equations

The complete description of the computer programs developed during this investigation would be a volume in itself. Therefore, this section will be limited to an explanation of the output of the programs, and the techniques used to analyze the results. A complete listing of the programs developed are given in the Appendix.

Basically, there were two programs developed. The first program formed reduced normal equations using the generalized least squares model described in Section 2.2. The second program also formed reduced normal equations, but here the method of observation equation described in Section 2.3 was used. The printed output from each of these programs is identical. Each program was written to read the observational data directly from the magnetic tapes, form the reduced normal equations in

---

[1]Also punched on the same card was the number of observations minus three times the number of satellite positions that were eliminated during the formation of the reduced normal equations. This number will be needed in order to calculate the degrees of freedom.

blocks, and then punched onto cards in a format such that the normal equations could be processed directly by the existing OSUGOP adjustment program. The additional printed information will be discussed here.

One of the requirements of this investigation was the comparison of results between correlated and uncorrelated observations. The choice of using either can be made by changing only one number on a control card used with the program. If full correlation is used, the program gives the option of computing and printing the matrix of correlation coefficients of the given variance-covariance matrix associated with each station in each event, this option also being exercised by changing only one number on the control card. If the matrix of correlation coefficients is printed, it will appear as in Figure 2.5.

An important part of the normal equation program output is the event adjustment. This is the adjustment for each satellite position prior to the formation of normal equations. The satellite position is computed from the given observations and the approximate coordinates of the camera stations. An example of the printed output, after an event adjustment, is shown in Figure 2.6. This shows the event to be number 0 of this particular run, and the event number given by the National Geodetic Survey as 6346. There are seven satellite positions in the event, and the computed coordinates of the satellite positions are given both as X, Y, Z, and ϕ, λ, h. The numbers 19, 20, and 43 on the left side of this figure are the station numbers of the three stations observing this event. On the same line as each station are the observations, which are the Greenwich hour angle and declination, expressed in radians. The last number on each line is similar to a

28912 7

EVENT NO. 2891 STATION NO. 6

1.000	0.116	0.214	-0.000	0.083	-0.030	0.174	0.014	0.028	0.015	0.004	-0.010	-0.076	-0.011
0.116	1.000	0.023	0.123	0.009	-0.057	0.044	0.144	0.036	0.036	0.006	-0.024	-0.019	-0.088
0.214	0.023	1.000	0.123	0.541	0.031	0.157	-0.075	0.292	-0.059	0.133	-0.012	0.097	0.031
-0.000	0.123	0.123	1.000	0.066	0.476	-0.015	-0.107	-0.018	-0.005	-0.017	-0.018	0.019	0.122
0.083	0.009	0.541	0.066	1.000	0.091	0.626	-0.064	0.266	-0.130	0.234	0.002	-0.041	0.030
-0.030	-0.057	0.031	0.476	0.091	1.000	-0.009	0.369	-0.092	-0.199	-0.025	0.045	0.003	-0.021
0.174	0.044	0.157	-0.015	0.626	-0.009	1.000	-0.004	0.547	-0.056	0.150	0.001	-0.045	0.020
0.014	0.144	-0.075	-0.107	-0.064	0.369	-0.004	1.000	-0.046	0.354	-0.048	-0.024	-0.009	-0.080
0.028	0.036	0.292	-0.018	0.266	-0.092	0.547	-0.046	1.000	0.070	0.384	0.042	0.228	0.038
0.015	0.036	-0.059	-0.005	-0.130	-0.199	-0.056	0.354	0.070	1.000	0.010	0.293	0.023	0.185
0.004	0.006	0.133	-0.017	0.234	-0.025	0.150	-0.048	0.384	0.010	1.000	0.134	-0.199	-0.028
-0.010	-0.024	-0.012	-0.018	0.002	0.045	0.001	-0.024	0.042	0.293	0.134	1.000	-0.040	-0.233
-0.076	-0.019	0.097	0.019	-0.041	0.003	-0.045	-0.009	0.228	0.023	-0.199	-0.040	1.000	0.133
-0.011	-0.088	0.031	0.122	0.030	-0.021	0.020	-0.080	0.038	0.185	-0.028	-0.233	0.133	1.000

EVENT NO. 2891 STATION NO. 16

1.000	0.103	0.449	0.060	0.153	0.010	0.198	-0.045	0.194	-0.095	0.038	-0.111	-0.006	-0.112
0.103	1.000	-0.023	0.481	-0.137	0.166	-0.176	0.186	-0.183	0.156	-0.176	0.014	-0.115	-0.002
0.449	-0.023	1.000	0.153	0.779	0.130	0.344	-0.044	0.178	-0.215	0.213	-0.276	0.053	-0.235
0.060	0.481	0.153	1.000	0.100	0.779	-0.067	0.202	-0.213	-0.087	-0.213	-0.011	-0.125	-0.003
0.153	-0.137	0.779	0.100	1.000	0.150	0.769	-0.012	0.397	-0.211	0.156	-0.283	0.134	-0.270
0.010	0.166	0.130	0.779	0.150	1.000	0.036	0.650	-0.127	0.097	-0.185	-0.079	-0.073	0.090
0.198	-0.176	0.344	-0.067	0.769	0.036	1.000	0.033	0.773	-0.073	0.294	-0.164	0.152	-0.233
-0.045	0.186	-0.044	0.202	-0.012	0.650	0.033	1.000	0.037	0.675	-0.010	0.197	0.001	0.174
0.194	-0.183	0.178	-0.213	0.397	-0.127	0.773	0.037	1.000	0.109	0.754	0.021	0.211	-0.151
-0.095	0.156	-0.215	-0.087	-0.211	0.097	-0.073	0.675	0.109	1.000	0.180	0.769	0.077	0.280
0.038	-0.176	0.213	-0.213	0.156	-0.185	0.294	-0.010	0.754	0.180	1.000	0.165	0.513	-0.025
-0.111	0.014	-0.276	-0.011	-0.283	-0.079	-0.164	0.197	0.021	0.769	0.165	1.000	0.132	0.570
-0.006	-0.115	0.053	-0.125	0.134	-0.073	0.152	0.001	0.211	0.077	0.513	0.132	1.000	0.130
-0.112	-0.002	-0.235	-0.003	-0.270	0.090	-0.233	0.174	-0.151	0.280	-0.025	0.570	0.130	1.000

Figure 2.5. Matrix of Correlation Coefficients Computed from the Variance-Covariance Matrix of the Observations

```
                    TEST DISTANCE =               200.00    SECONDS OF ARC

EVENT      0        6346
   19                          1.7160276           -0.4494284          0.8
   20                          0.7766793           -0.4440155          0.1
   43                          1.5084081           -0.0442105          0.8
 SATELLITE POSITION     1700356.201   -8881809.630   -5289410.721
 GEOD. COORD. OF SATELLITE     -30.425892    280.837718     4103737.7
                        RMS MISCLOSURE IN METERS=      15.3
   19                          1.7226323           -0.4946744          1.0
   20                          0.7683539           -0.4795793          0.2
   43                          1.5105150           -0.0813797          1.0
 SATELLITE POSITION     1684787.482   -8808501.308   -5480318.584
 GEOD. COORD. OF SATELLITE     -31.532332    280.828090     4137764.6
                        RMS MISCLOSURE IN METERS=      19.4
   19                          1.7296815           -0.5392995          1.2
   20                          0.7593536           -0.5145951          0.2
   43                          1.5126964           -0.1190654          1.1
 SATELLITE POSITION     1668956.258   -8731807.937   -5669136.666
 GEOD. COORD. OF SATELLITE     -32.631420    280.820732     4171708.8
                        RMS MISCLOSURE IN METERS=      21.4
   19                          1.7372069           -0.5832296          1.0
   20                          0.7496437           -0.5490213          0.3
   43                          1.5149562           -0.1572398          0.8
 SATELLITE POSITION     1652889.487   -8651785.291   -5855803.364
 GEOD. COORD. OF SATELLITE     -33.723202    280.815803     4205549.3
                        RMS MISCLOSURE IN METERS=      17.8
   19                          1.7452529           -0.6264005          1.2
   20                          0.7391878           -0.5828195          0.6
   43                          1.5172969           -0.1958751          0.7
 SATELLITE POSITION     1636605.664   -8568488.152   -6040267.776
 GEOD. COORD. OF SATELLITE     -34.807779    280.813414     4239267.2
                        RMS MISCLOSURE IN METERS=      20.2
   19                          1.7538638           -0.6687612          1.2
   20                          0.7279468           -0.6159533          0.4
   43                          1.5197225           -0.2349440          0.8
 SATELLITE POSITION     1620133.482   -8481982.864   -6222503.264
 GEOD. COORD. OF SATELLITE     -35.885311    280.813744     4272868.3
                        RMS MISCLOSURE IN METERS=      20.7
   19                          1.7630827           -0.7102609          0.3
   20                          0.7158776           -0.6483886          0.7
   43                          1.5222375           -0.2743969          0.9
 SATELLITE POSITION     1603504.618   -8392325.907   -6402427.427
 GEOD. COORD. OF SATELLITE     -36.955733    280.817013     4306317.4
                        RMS MISCLOSURE IN METERS=      17.9
NEW VPV  0.48539378489500+02
NEW VPV  0.61802340804 70+05
NEW VPV  0.48212119466900+02
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.61775628382 60+05
```

Figure 2.6. Printed Output after an Event Adjustment

residual, and used by the analyst as an indicator of the least squares fit. This is described in [Reilly, Schwarz and Whiting, 1973]. The most meaningful numbers in the event adjustment are the last lines labeled 'NEW VPV' and 'WPW CONTRIBUTION FROM SATELLITE POSITIONS.' In this particular example, there are three different values of 'NEW VPV,' one for each of the three observing stations. The sum of these three numbers is the first term in Equation (2.4.1-1). The number denoted 'WPW CONTRIBUTION FROM SATELLITE POSITIONS' is the second term in Equation (2.4.1-1). The sum of the 'NEW VPV' of each observing station, minus the 'WPW CONTRIBUTION FROM SATELLITE POSITIONS' must be a positive number. Each of the 'NEW VPV' numbers must also be positive numbers. If either of these are not positive the observational information is not useable. This will be discussed later with specific examples.

The remaining output from the formation of normal equations program is identical to that described in the OSUGOP report [Reilly, Schwarz and Whiting, 1973].

3. DATA

The Type II data used for this investigation was the result of a polynomial fit to the satellite images on the BC-4 camera plates. The order of the polynomial was six for almost every plate. In most cases this turned out to be a very good polynomial fit with very low correlation between the different coefficients. However, there were many polynomials where the correlation between the observations was so high that it was impossible to use the variance-covariance matrix of the observed quantities when using double precision arithmetic.

The Type II data is a second generation of the observed quantities not used by the National Geodetic Survey. The data used by the National Geodetic Survey are the x,y plate coordinates of the satellite images. The polynomial fit is not made to the satellite's path in space, but to the x and y coordinates of the satellite images on the camera plate where the x-axis is in the general direction of the satellite's trail [Bush, 1973].

Extensive studies were carried out by the National Geodetic Survey to determine what form of mathematical curve should be used to represent the satellite path. It was decided that a fifth-order polynomial was the optimum form. In forming this polynomial, all measured points were rotated to a coordinate system in which the x-axis is defined by the two end points of the satellite trail. The center image of the satellite trail became the zero-point of the time dimension. The mathematical model was the two polynomials

$$x = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
$$y = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \tag{3-1}$$

where t is the difference in time between the image time and the time of the center image. Normal equations were formed, and a solution was made for the coefficients of the polynomial. Using the computed coefficients, the polynomial was evaluated at each satellite image point by Equation (3-1), and the residuals in x and y were computed by

$$v_x = x^c - x$$
$$v_y = y^c - y$$

where x and y are the original rotated coordinates, and x^c and y^c are computed. The mean errors for a fifth-order polynomial are

$$S_x^c = \left(\frac{\Sigma v_x{}^2}{N - 6}\right)^{\frac{1}{2}}$$

$$S_y^c = \left(\frac{\Sigma v_y{}^2}{N - 6}\right)^{\frac{1}{2}}$$

$$S^c = \left(\frac{\Sigma v_x{}^2 + \Sigma v_y{}^2}{2(N - 6)}\right)^{\frac{1}{2}}$$

where N is the number of satellite images used to form the polynomials, and the superscript ^{c} means curve fit. These give a measure of the trails lack of adherence to a smooth curve, the x in the direction of motion, and the y perpendicular to it. As a residual check, a criterion was established that any residual greater than 20 μm must represent an erroneous point. If any such points were present, they were eliminated and the solution repeated.

The correlation present in the Type II data came about during the process of propagating from the x,y coordinates of the polynomials to the Greenwich hour angle and declination.

At the completion of the worldwide network observation program, 3672 successful plates were observed on 1702 successful events, composed of 1449 two-station, 238 three-station, and 15 four-station events [Schmid, 1972]. An event in this case is defined as two or more ground stations observing a satellite during the same time span. Approximately two-thirds of these events were transformed into the Type I and Type II data. The data sent to the Space Science Data Center consisted of 903 two-station, 216 three-station, and 15 four-station events. Observations were from 49 different ground stations.

The Type II data contained approximately 35,000 observations. Each event was recorded as a separate file on a 7-track magnetic tape, utilizing seventeen tapes in all. Table 3.1 is a listing of these magnetic tapes, showing the tape number given by National Geodetic Survey, the total number of events on each tape, and the number of two-, three-, and four-station events.

3.1 The Conditioning of the Variance-Covariance Matrix of the Observations

When an analyst uses the expression that a matrix is ill-conditioned, it may mean one thing to one person and something else to another. When stating that a matrix is ill-conditioned, it must be specified

TABLE 3.1

LISTING OF MAGNETIC TAPES CONTAINING THE TYPE II DATA

Tape No.	Number of Events on each Tape			
	Total No.	No. of 2 Station Events	No. of 3 Station Events	No. of 4 Station Events
7	87	73	12	2
8	90	76	13	1
15	90	70	17	3
16	90	70	20	-
20	90	74	14	2
24	90	62	25	3
28	90	68	20	2
32	89	71	17	1
34	30	19	11	-
36	29	22	7	-
39	60	40	20	-
41	30	28	2	-
44	60	55	5	-
46	30	26	4	-
49	60	47	13	-
52	60	50	9	1
55	59	52	7	-
Total	1134	903	216	15

in what operation the ill-conditioning is present. A matrix can be ill-conditioned when it is necessary to get an inverse, or to get its eigenvalues or eigenvectors, but it cannot be said that ill-conditioning with respect to one operation will also be ill-conditioned with respect to any other operation. An illustration is a matrix that has all of its eigenvalues close together, i.e., $\lambda_{max}/\lambda_{min} \approx 1$. A matrix such as this is well-conditioned for inverting matrices but ill-conditioned for determining eigenvalues.

As might be expected, there are varying degrees of ill-conditioning. A matrix can be referred to as well-conditioned, ill-conditioned, very ill-conditioned, or singular. Much has been written on this particular topic, and there are many different tests that can be used to determine the degree of ill-conditioning. One of the indicators is the ratio of the maximum and minimum eigenvalues. In the literature this is referred to by several different names, the two most common being the P-Number and the Spectral Norm [Ralston, 1965, pg 417]. Once a matrix has been classified as ill-conditioned, everything that this matrix is associated with also becomes poorly-determined. This can be illustrated using the mathematical development for the generalized least squares model in Chapter 2. Assume that four ground stations (1, 2, 3, 4) are involved in one event. The variance-covariance matrix of the observations associated with Station 1 is very highly correlated, and the variance-covariance matrices of the observations associated with the remaining three stations have very low correlation. Since P_1^{-1} is ill-conditioned, the matrix

$$M_1^{-1} = (B_1 P_1^{-1} B')^{-1}$$

will not be well-determined, even though the matrix B_1 is well-conditioned. Also, the expression $W'_1 M_1^{-1} W_1$ will result in a very large number due to the ill-conditioning of M_1^{-1}. In addition to this, the matrix

$$M^{-1} = \left[\sum_{i=1}^{4} M_i^{-1}\right]^{-1}$$

will not be well-determined. All of these expressions are necessary in the development of the reduced normal equations necessary to perform a station adjustment.

The above illustration is typical of many of the Type II events. An example of a very ill-conditioned variance-covariance matrix is one that would have a matrix of correlation coefficients as follows:

	h_1	∂_1	h_2	∂_2	h_3	∂_3	h_4	∂_4	h_5	∂_5	h_6	∂_6	h_7	∂_7
h_1	1.00	0.00	0.97	0.01	0.92	0.03	0.86	0.04	0.78	0.05	0.70	0.04	0.63	0.03
∂_1		1.00	-0.02	0.96	-0.03	0.85	-0.05	0.64	-0.06	0.35	-0.06	0.10	-0.06	-0.04
h_2			1.00	0.00	0.98	0.02	0.93	0.04	0.85	0.06	0.75	0.06	0.66	0.04
∂_2				1.00	-0.01	0.96	-0.02	0.79	-0.03	0.51	-0.04	0.23	-0.04	0.05
h_3					1.00	0.02	0.98	0.04	0.92	0.06	0.81	0.07	0.71	0.06
∂_3						1.00	0.01	0.93	0.00	0.71	-0.01	0.44	-0.01	0.25
h_4							1.00	0.04	0.97	0.07	0.90	0.08	0.80	0.07
∂_4								1.00	0.04	0.91	0.03	0.72	0.02	0.54
h_5									1.00	0.07	0.97	0.08	0.90	0.08
∂_5										1.00	0.07	0.94	0.06	0.82
h_6											1.00	0.08	0.97	0.08
∂_6												1.00	0.07	0.96
h_7													1.00	0.07
∂_7														1.00

As can be seen, the highest correlation is in the second diagonal row parallel to the main diagonal of the matrix. This means that h_1 and h_2 are highly correlated, as is h_2 and h_3, h_3 and h_4, δ_1 and δ_2, etc. As one moves further away from the diagonal the correlation decreases. In most cases the decrease is gradual. It should also be noted from this matrix of correlation coefficients that there is very little correlation between the Greenwich hour angles and the declinations. This is not always the case, but when correlation does exist it is low compared to the correlation between like observations.

3.1.1 The Use of Correlated Observations in an Adjustment

Due to the very high correlation of the variance-covariance matrix associated with some of the observations, it became necessary to determine which data was highly correlated, and determine a way to use the data. The indicator first used in this investigation to determine the conditioning was the P-Number, or Spectral Norm. This number is a good indicator of high correlation; the smaller the number, the better the conditioning.

It was found that the P-Number of the variance-covariance matrices varied from 1 to 10^{10}. The question then becomes "what is the largest value of the P-Number that can be accepted before the conditioning of the matrix is so poor that it can affect the results of an adjustment?"

At this point in the investigation it was noted that the magnitude of the P-Number was just one or two orders of magnitude larger than $W'M^{-1}W$ or $L'PL$. A few examples of this are shown here:

$W'M^{-1}W$, or $L'P^{-1}L$	P-Number
$.315 \times 10^8$	$.161 \times 10^{10}$
$.183 \times 10^6$	$.306 \times 10^8$
$.178 \times 10^4$	$.267 \times 10^6$
$.220 \times 10^4$	$.523 \times 10^5$
$.262 \times 10^6$	$.831 \times 10^8$
$.270 \times 10^4$	$.591 \times 10^6$

As was mentioned in Section 3.1, the examination of the matrix of correlation coefficients of any P^{-1} matrix shows the highest correlation to be between successive images. With a little practice, the conditioning of the matrix can be determined with a fair degree of accuracy by just looking at the matrix of correlation coefficients. To illustrate this, seven different P^{-1} matrices were selected where the value of $W'M^{-1}W$ varied from about 10 to 10^{10}. The matrix of correlation coefficients for each of these is given in Tables 3.2 through 3.8. Each of these tables will be discussed separately.

Table 3.2 illustrates a very well-conditioned matrix. The value of $W'M^{-1}W$ was 10. The second diagonal row parallel to the main diagonal has the highest correlation, but 0.58 is the largest correlation coefficient.

Table 3.3 is an example where the value of $W'M^{-1}W$ is approximately 4000. As can be seen, the correlation between successive like-observations varies from 0.72 to 0.91. There is essentially no correlation between the Greenwich hour angles and the declinations.

Table 3.4 is given here mainly to be compared with Table 3.3. In

TABLE 3.2

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #3538,
STATION 2, ON TAPE 24

1.00	-0.25	0.22	-0.10	0.24	-0.10	0.13	-0.12	0.11	-0.14	0.10	-0.09	0.06	-0.03
	1.00	-0.03	0.18	0.06	0.08	0.05	0.03	-0.02	0.09	-0.01	0.07	-0.00	0.03
		1.00	-0.31	0.50	-0.24	0.08	-0.09	0.13	-0.01	0.11	-0.01	0.14	-0.06
			1.00	-0.16	0.55	0.05	0.17	0.10	0.09	0.01	0.04	-0.07	0.12
				1.00	-0.34	0.43	-0.14	0.00	0.09	0.19	0.03	0.08	-0.02
					1.00	-0.04	0.58	0.14	0.06	-0.03	0.12	-0.11	0.06
						1.00	-0.19	0.45	-0.12	0.15	0.00	0.02	0.06
							1.00	-0.08	0.54	-0.08	0.18	-0.08	-0.03
								1.00	-0.31	0.43	-0.13	0.31	-0.00
									1.00	-0.16	0.47	-0.07	0.20
										1.00	-0.27	-0.04	0.02
											1.00	-0.02	-0.06
												1.00	-0.19
													1.00

$W'M^{-1}W = 10$

TABLE 3.3

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #4682,
STATION 23, ON TAPE 7

1.00	0.07	0.79	0.04	0.63	0.05	0.58	0.05	0.55	0.03	0.44	0.02	0.33	0.00
	1.00	0.02	0.72	0.01	0.54	0.01	0.47	0.01	0.41	0.01	0.32	0.01	0.31
		1.00	0.08	0.89	0.10	0.66	0.09	0.58	0.05	0.58	0.02	0.48	-0.00
			1.00	0.09	0.87	0.08	0.50	0.06	0.30	0.05	0.33	0.04	0.37
				1.00	0.13	0.88	0.12	0.73	0.07	0.60	0.02	0.55	-0.00
					1.00	0.12	0.80	0.10	0.49	0.07	0.35	0.05	0.43
						1.00	0.13	0.91	0.09	0.66	0.03	0.57	-0.01
							1.00	0.12	0.84	0.08	0.53	0.05	0.47
								1.00	0.09	0.88	0.04	0.66	-0.00
									1.00	0.07	0.85	0.03	0.57
										1.00	0.04	0.83	0.00
											1.00	0.02	0.78
												1.00	0.00
													1.00

$$W'M^{-1}W = 4000$$

TABLE 3.4

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #2661,
STATION 11, ON TAPE 7

1.00	0.24	0.66	0.16	0.33	0.05	0.15	-0.04	0.13	-0.10	0.16	-0.11	0.12	-0.11
	1.00	0.17	0.67	0.05	0.31	-0.05	0.05	-0.09	-0.01	-0.08	0.02	-0.06	0.03
		1.00	0.32	0.82	0.26	0.42	0.04	0.19	-0.14	0.17	-0.19	0.20	-0.17
			1.00	0.29	0.80	0.12	0.24	0.04	-0.11	-0.08	-0.10	-0.03	0.04
				1.00	0.35	0.83	0.18	0.51	-0.06	0.20	-0.18	0.10	-0.19
					1.00	0.29	0.72	0.13	0.23	-0.03	-0.07	-0.06	-0.06
						1.00	0.29	0.84	0.12	0.40	-0.06	0.10	-0.15
							1.00	0.29	0.77	0.14	0.27	0.00	-0.00
								1.00	0.29	0.77	0.14	0.38	-0.03
									1.00	0.32	0.77	0.19	0.37
										1.00	0.34	0.80	0.17
											1.00	0.34	0.80
												1.00	0.33
													1.00

$W'M^{-1}W = 70,000$

Table 3.4 the correlation between successive like-observations is less than in Table 3.3, but the value of $W'M^{-1}W$ is higher, namely 70,000. The difference is that in Table 3.4 the correlation between the Greenwich hour angles and declinations has increased. If every other image is eliminated and only images 1-3-5-7 are used, the value of $W'M^{-1}W$ is 20.

Table 3.5 is an example with a $W'M^{-1}W$ of 400,000. The second diagonal row shows the correlation to vary from 0.78 to 0.89, along with some correlation between the Greenwich hour angles and declinations. If every other image is eliminated, the resulting $W'M^{-1}W$ is 300.

Table 3.6 is an example with a $W'M^{-1}W$ of 2,500,000. The reason for such a large value is not at all obvious. The second diagonal row has lower correlation coefficients than Table 3.5, but the correlation between the Greenwich hour angles and declinations is higher. The correlation matrix for images 1-3-5-7 has a $W'M^{-1}W$ of 300.

Table 3.7 is one of the very bad matrices. The value of $W'M^{-1}W$ is approximately 14,000,000. The reason for this is obvious; the correlations in the second diagonal row are all greater than 0.9. The correlations in the fourth diagonal row, as well as the sixth diagonal row are higher than the other matrices examined. The correlation between the Greenwich hour angle and declination is essentially nonexistent. This lack of correlation between the Greenwich hour angle and declination caused the $W'M^{-1}W$ for the 1-3-5-7 image matrix to drop to 21.4.

Table 3.8 is an example of one of the worst matrices in the world network data. The lowest correlation in the second diagonal row is

TABLE 3.5

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #7765,
STATION 69, ON TAPE 7

1.00	-0.18	0.86	-0.14	0.54	-0.01	0.26	0.11	0.20	0.18	0.28	0.15	0.33	0.07
	1.00	-0.20	0.84	-0.16	0.41	-0.09	-0.02	-0.04	-0.15	-0.01	-0.05	0.01	0.08
		1.00	-0.19	0.85	-0.08	0.56	0.08	0.35	0.18	0.26	0.17	0.25	0.07
			1.00	-0.17	0.78	-0.11	0.31	-0.04	0.00	0.01	-0.07	0.03	0.01
				1.00	-0.10	0.89	0.03	0.65	0.12	0.30	0.14	0.11	0.06
					1.00	-0.09	0.81	-0.04	0.45	0.01	0.09	0.05	-0.07
						1.00	-0.02	0.88	0.04	0.47	0.06	0.14	0.01
							1.00	-0.03	0.84	-0.01	0.42	0.03	0.07
								1.00	-0.02	0.80	-0.03	0.45	-0.05
									1.00	-0.04	0.81	-0.01	0.44
										1.00	-0.09	0.85	-0.11
											1.00	-0.08	0.84
												1.00	-0.12
													1.00

$$W'M^{-1}W = 400{,}000$$

TABLE 3.6

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #7137,
STATION 19, ON TAPE 7

1.00	-0.04	0.77	0.07	0.32	0.09	-0.02	0.04	-0.08	0.05	0.00	0.11	-0,02	0.05
	1.00	-0.12	0.80	-0.07	0.41	0.04	0.16	0.04	0.21	-0.07	0.26	-0.06	0.14
		1.00	-0.05	0.76	0.09	0.27	0.11	-0.01	0.04	-0.15	0.00	-0.05	0.03
			1.00	-0.11	0.77	-0.04	0.41	0.06	0.24	0.03	0.22	-0.07	0.18
				1.00	0.01	0.76	0.15	0.14	0.11	-0.21	-0.04	-0.09	-0.02
					1.00	-0.07	0.84	-0.02	0.46	0.05	0.15	-0.04	0.15
						1.00	0.06	0.66	0.16	0.14	0.09	-0.04	0.00
							1.00	-0.05	0.77	-0.04	0.34	-0.05	0.14
								1.00	0.08	0.78	0.19	0.23	0.10
									1.00	-0.06	0.80	-0.09	0.32
										1.00	0.08	0.61	0.13
											1.00	-0.07	0.65
												1.00	0.04
													1.00

$W'M^{-1}W = 2,500,000$

TABLE 3.7

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #4887,
STATION 60, ON TAPE 24

1.00	-0.00	0.97	0.01	0.92	0.03	0.86	0.04	0.78	0.05	0.70	0.04	0.63	0.03
	1.00	-0.02	0.96	-0.03	0.85	-0.05	0.64	-0.06	0.35	-0.06	0.10	-0.06	-0.04
		1.00	0.00	0.98	0.02	0.93	0.04	0.85	0.06	0.75	0.06	0.66	0.04
			1.00	-0.01	0.96	-0.02	0.79	-0.03	0.51	-0.04	0.23	-0.04	0.05
				1.00	0.02	0.98	0.04	0.92	0.06	0.81	0.07	0.71	0.06
					1.00	0.01	0.93	0.00	0.71	-0.01	0.44	-0.01	0.25
						1.00	0.04	0.97	0.07	0.90	0.08	0.80	0.07
							1.00	0.04	0.91	0.03	0.72	0.02	0.54
								1.00	0.07	0.97	0.08	0.90	0.08
									1.00	0.07	0.94	0.06	0.82
										1.00	0.08	0.97	0.08
											1.00	0.07	0.96
												1.00	0.07
													1.00

$$W'M^{-1}W = 14{,}000{,}000$$

TABLE 3.8

MATRIX OF CORRELATION COEFFICIENTS FOR EVENT #4590,
STATION 12, ON TAPE 24

1.00	-0.02	0.98	-0.02	0.93	-0.03	0.86	-0.03	0.78	-0.04	0.68	-0.05	0.57	-0.05
	1.00	-0.02	0.97	-0.02	0.87	-0.03	0.71	-0.03	0.48	-0.04	0.22	-0.04	-0.03
		1.00	-0.02	0.98	-0.02	0.94	-0.03	0.87	-0.04	0.77	-0.04	0.65	-0.05
			1.00	-0.02	0.97	-0.03	0.86	-0.03	0.66	-0.04	0.40	-0.04	0.13
				1.00	-0.03	0.98	-0.03	0.94	-0.04	0.85	-0.04	0.74	-0.04
					1.00	-0.03	0.96	-0.03	0.82	-0.04	0.60	-0.04	0.33
						1.00	-0.03	0.98	-0.04	0.92	-0.04	0.82	-0.04
							1.00	-0.04	0.95	-0.04	0.79	-0.04	0.56
								1.00	-0.04	0.98	-0.04	0.91	-0.04
									1.00	-0.04	0.94	-0.04	0.79
										1.00	-0.04	0.98	-0.03
											1.00	-0.03	0.95
												1.00	-0.03
													1.00

$$W'M^{-1}W = 6,700,000,000$$

0.91 and the highest 0.98. However, there are six 0.98 values. The $W'M^{-1}W$ for this matrix is 6,700,000,000. As in Table 3.7, the correlation between the Greenwich hour angles and declinations is nonexistent. By eliminating every other image, the very high correlation in the second diagonal row is eliminated, with 100 as the $W'M^{-1}W$ for images 1-3-5-7.

The seven different matrices described above are typical examples of the actual data. To give an indication of how much data falls into each of the different categories of conditioning, two different subsets of the data were tested. This data was the observations on tape #44 and tape #52, a total of 120 events. For each event the value of $W'M^{-1}W$ for each station was abstracted and tabulated. These numbers were grouped together as follows:

Group No.	
0	$W'M^{-1}W < 1$
1	$1 < W'M^{-1}W < 10$
2	$10 < W'M^{-1}W < 100$
3	$100 < W'M^{-1}W < 10^3$
4	.
5	.
6	.
7	.
8	.
9	.
10	$10^9 < W'M^{-1}W < 10^{10}$
11	$10^{10} < W'M^{-1}W$

The results are shown in the form of a histogram in Figures 3.1 and 3.2. In Figure 3.1, 25 of the 116 plates has a $W'M^{-1}W$ greater than 10,000, which is 21.5%. In Figure 3.2 the amount is 24 out of the 129 plates for a total of 18.6%. If the number 10,000 were used as the cutoff point between acceptable and unacceptable data, approximately 20% of all plates would be rejected. If each of the rejected plates were part of a two-station event, these events would account for a total rejection factor of 40% of all data. If the rejection number were raised to 100,000 the percentage of plates rejected would be 11.2% and 10.8% for the two tapes.

Rather than select an arbitrary value of $W'M^{-1}W$ for a rejection criteria, another approach to the problem would be to use every other image (i.e., 1-3-5-7 or 2-4-6) in an event if the $W'M^{-1}W$ value in that event is extremely large. Using the same two data subsets, tape #44 and tape #52, the value of $W'M^{-1}W$ was computed for each station using, first, only observations to images 1-3-5-7, and secondly, observations to images 2-4-6. As before, each value of $W'M^{-1}W$ was abstracted and tabulated in the form of a histogram. These are shown in Figures 3.3 through 3.6. The value of $W'M^{-1}W$, in all cases, was less than 1×10^5. As was anticipated, the elimination of the polynomial endpoints by using images 2-4-6 did give better results, but not significantly better. The decrease in the $W'M^{-1}W$ by using images 2-4-6 is compensated by more observations and better geometry when using images 1-3-5-7.

3.1.2 Rearranging the Order of the Observed Quantities

It was shown in Section 3.1.1 that the rejection of observations on every other satellite image greatly improved the conditioning. This

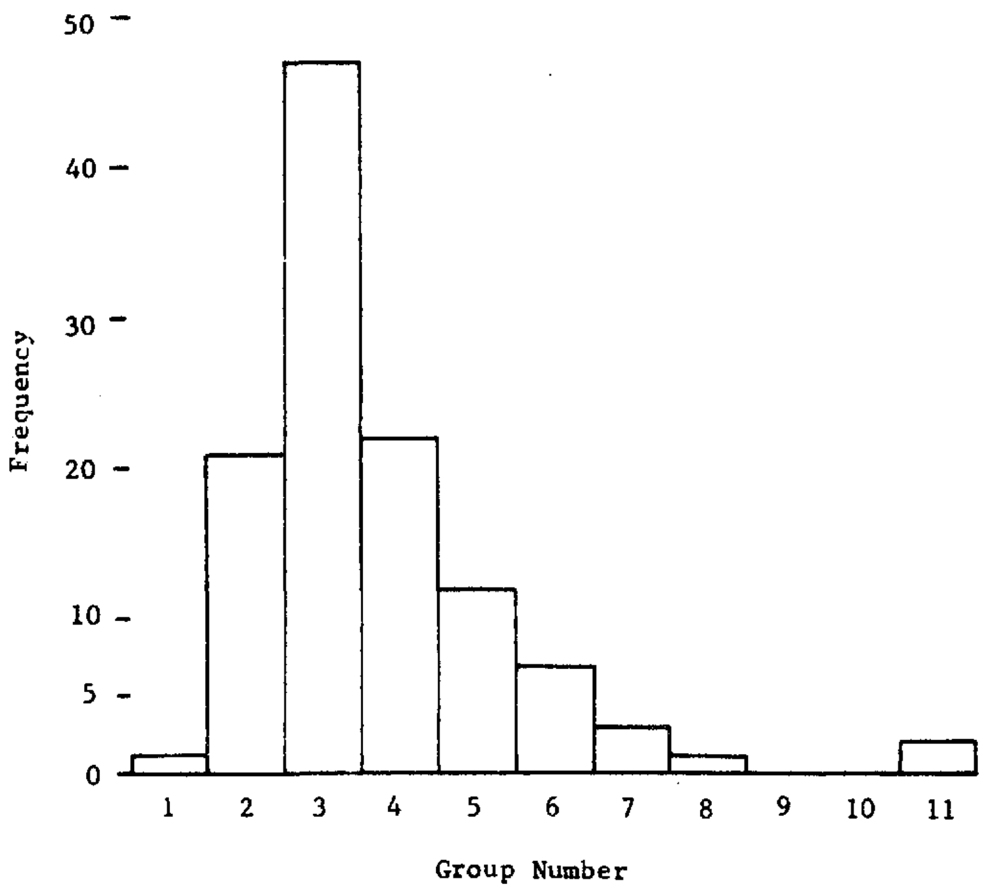

Figure 3.1. Histogram showing Group Number vs Frequency for Tape 44, Every Image

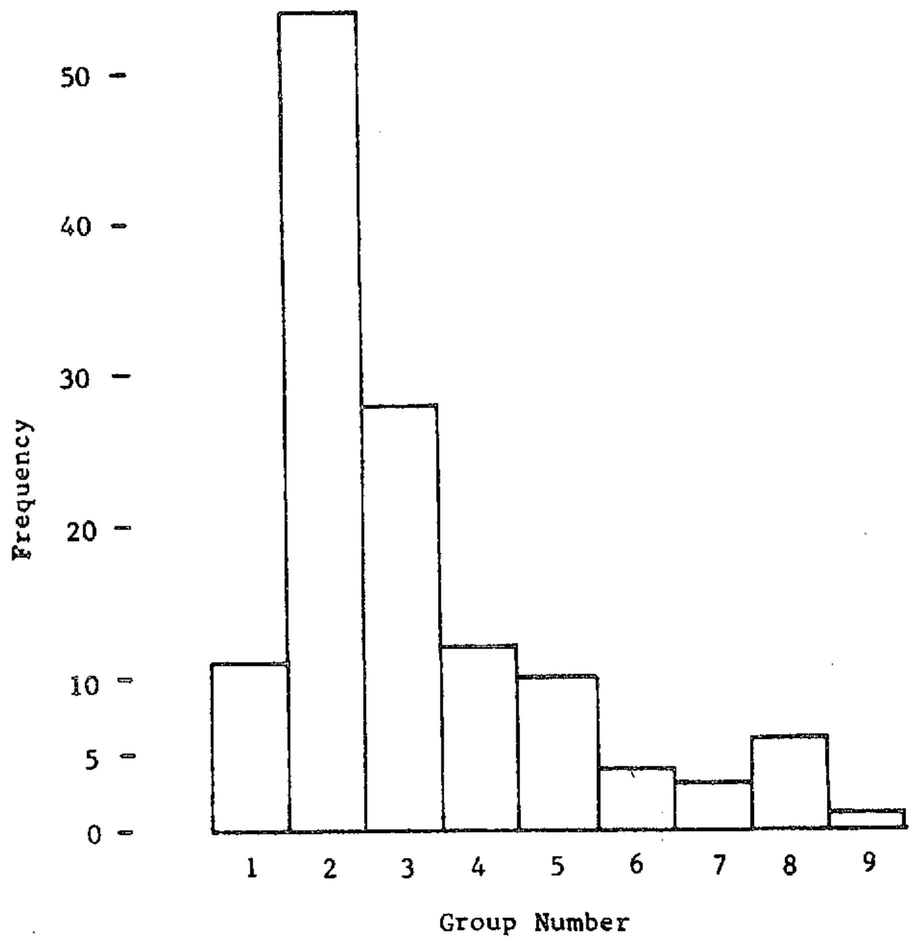

Figure 3.2. Histogram showing Group Number vs Frequency for Tape 52, Every Image

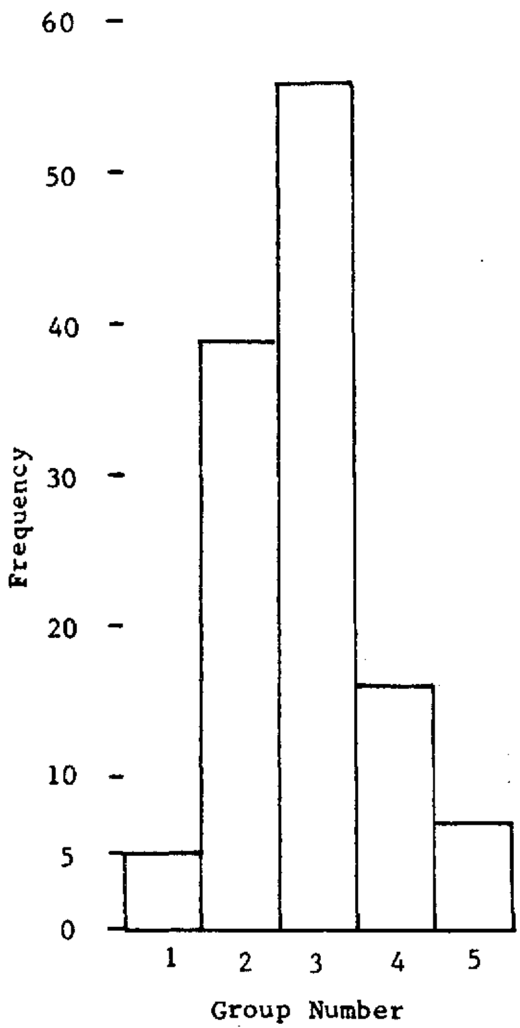

Figure 3.3. Histogram showing Group Number vs Frequency for Tape 44, Images 1-3-5-7

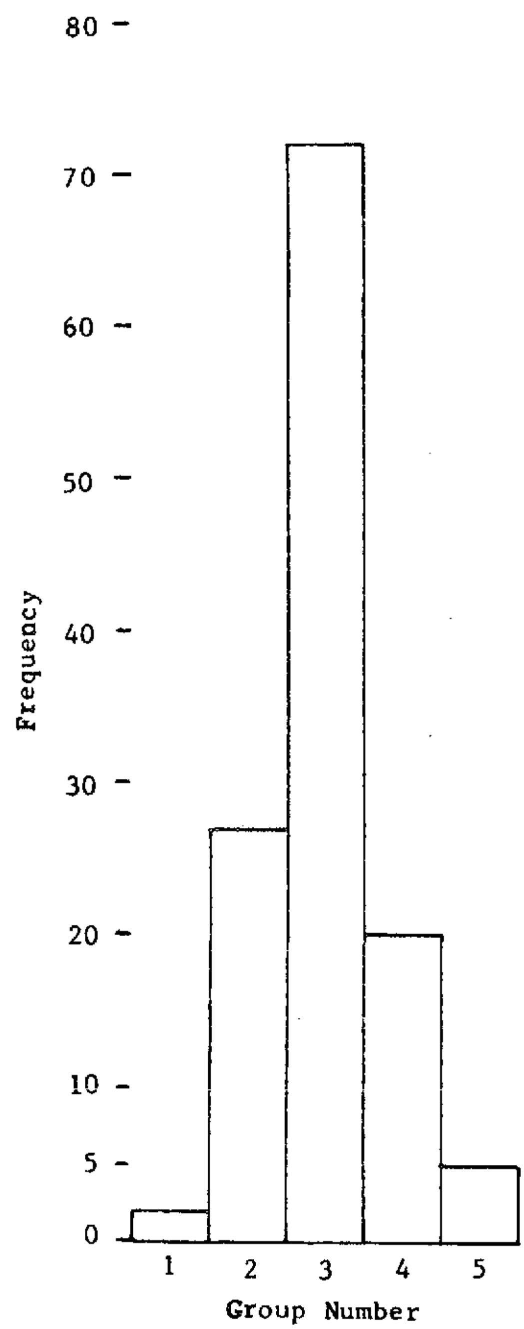

Figure 3.4. Histogram showing Group Number vs Frequency for Tape 52, Images 1-3-5-7

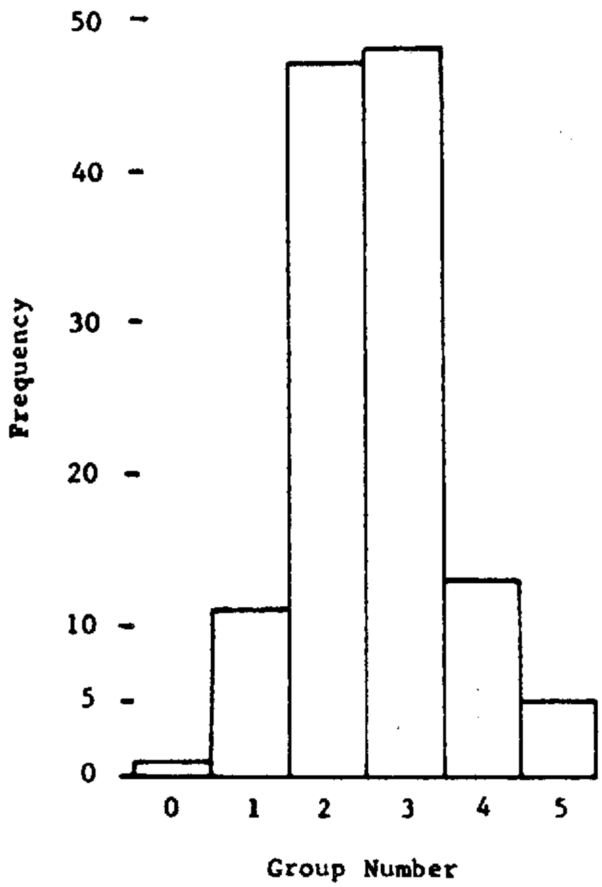

Figure 3.5. Histogram showing Group Number vs Frequency for Tape 44, Images 2-4-6

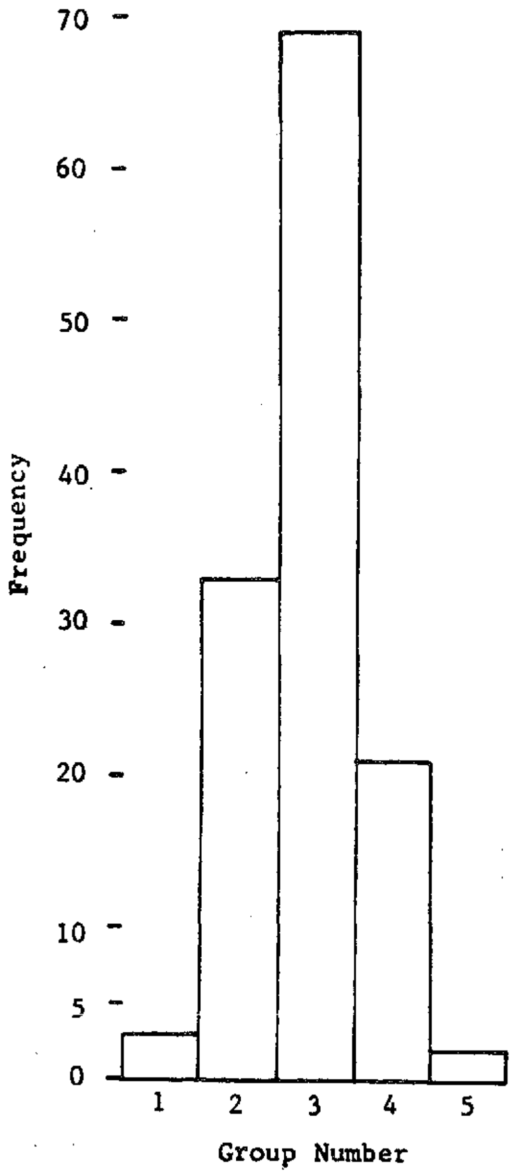

Figure 3.6. Histogram showing Group Number vs Frequency for Tape 52, Images 2-4-6

was done using satellite images 1-3-5-7 and satellite images 2-4-6. Although it is not a recommended practice, it is possible to get a solution using the above two sets of data simultaneously, i.e.,

$$P^{-1} = \begin{bmatrix} \boxed{\begin{array}{c} P^{-1} \text{ for Images} \\ \text{1-3-5-7} \end{array}} & 0 \\ 0 & \boxed{\begin{array}{c} P^{-1} \text{ for Images} \\ \text{2-4-6} \end{array}} \end{bmatrix}_{(14\text{x}14)} \qquad (3.1.2\text{-}1)$$

This is the case where the observations to all seven satellite images are used, but the correlation between successive images is neglected.

Suppose that instead of the Null matrices in the upper right and lower left portions of the P^{-1} matrix above, the actual values are inserted. In other words, the P^{-1} matrix will be 14x14 and full, the only change being the order of the observations. In experimenting with many different matrices and many different arrangements of observations, the results after the rearrangements were always the same as before. If the correlation between successive images was eliminated, as shown in Equation 3.1.2-1, the results would always be well-conditioned. However, once these off-diagonal elements were inserted, regardless of the order of the observations, the ill-conditioning would return.

3.1.3 The Diagonalization of a Matrix

There is a theorem in the matrix theory, called an existance theorem, which is the following [Hohn, 1966, pg 296]:

> If A is Hermitian, there exists a unitary matrix U such that U^*AU is a diagonal matrix whose diagonal elements are the eigenvalues of A^*: $U^*AU = D[\lambda_1, \lambda_2, \ldots, \lambda_n]$.

In this theorem, the superscript * means Hermitian. The matrix A is called Hermitian if $A=\bar{A}'$ where $\bar{A}'$ is the transpose of the complex conjugate. If the elements of A are real, $A'=A^*$ so that the property of being real and symmetric is a special case of the property of being Hermitian.

The matrix P^{-1}, being a matrix of real numbers and symmetric, is Hermitian. This theorem can be applied by defining D as a diagonal matrix where the diagonal elements are the eigenvalues of the P^{-1} matrix and the unitary matrix U is made up of the eigenvectors of the P^{-1} matrix. The U matrix is also symmetric. The theorem, in this case, is

$$UP^{-1}U = D. \tag{3.1.3-1}$$

This can be rearranged as follows:

$$P^{-1} = U^{-1}DU^{-1} \tag{3.1.3-2}$$

and

$$P = UD^{-1}U. \tag{3.1.3-3}$$

Equation (3.1.3-3) can be substituted in place of the weight matrix in the mathematical formulation described in Chapter 2.

Tests were performed using Equation (3.1.3-3) in place of the weight matrix, and the conclusions were that this method will improve the results when used with data that is very highly correlated. It is not a fool-proof method, however, since there are several matrices where it was very difficult to compute the eigenvalues.

3.2 Preliminary Numerical Results

After the completion of all the preliminary experiments described in Section 3.1, the next step was to perform an adjustment. Many different adjustments were necessary to answer the questions raised earlier in this investigation. To recapitulate, these questions were:

1. Is there a difference in the solution when using generalized least squares, as opposed to the method of observation equations?
2. How much correlation can be tolerated before the solution begins to weaken?
3. What is the difference in the results when the same data is used with and without correlation?
4. Is there any data that is unusable?

The formation of normal equations for the entire worldwide network is a very expensive as well as time consuming task. It was decided that it is not necessary to use the entire network to answer the questions listed above. The Type II data from the original seventeen tapes was organized into four separate geographic areas, with the data copied onto four other magnetic tapes. One of these areas is North and South America, shown in Figure 3.7. There are 237 events in this sub-network, which is considered sufficient to perform the necessary experiments.

3.2.1 The North and South America Sub-network

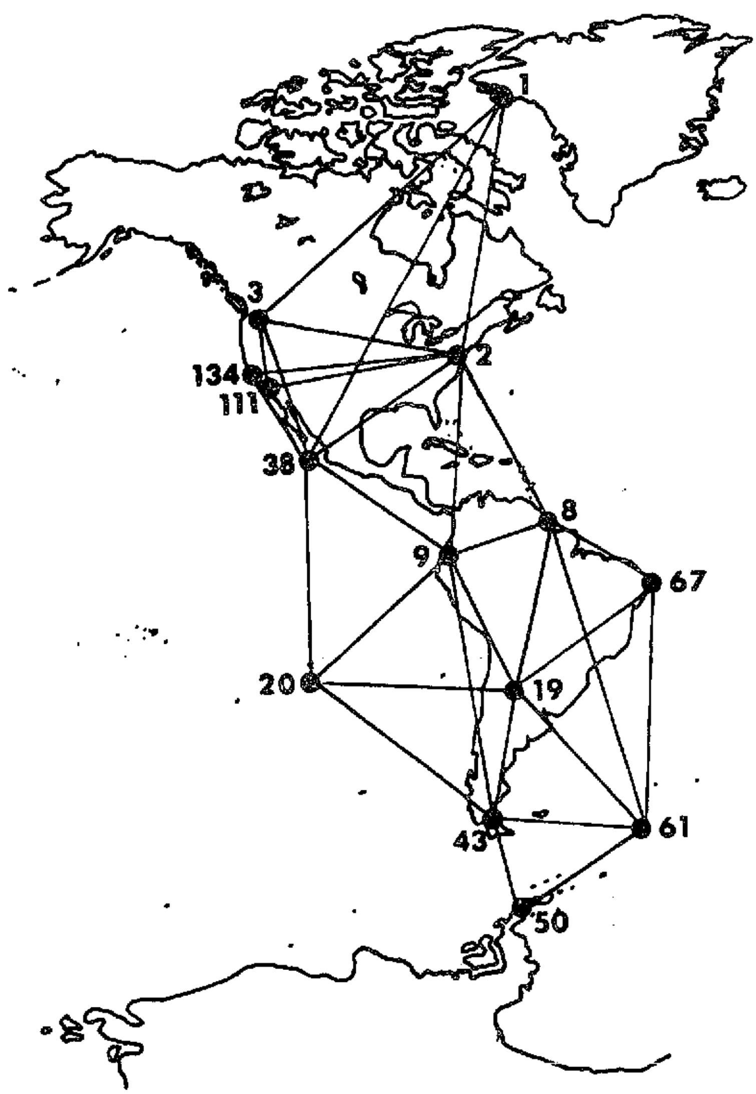

Figure 3.7. The North and South American Network

Of the 237 events in the North and South American network, 187 were well-conditioned (the value of $W'M^{-1}W$ was less than 10,000). The remaining 50 events were highly correlated. For the first series of experiments, all observations from the 187 good events and observations on images 1-3-5-7 from the 50 ill-conditioned events were used. This data was used to perform three different adjustments:

1. using the generalized least squares adjustment, without correlation, using double precision arithmetic,
2. using the generalized least squares adjustment, with correlation, using double precision arithmetic,
3. using the method of observation equation, with correlation, using double precision arithmetic.

The results of each of these three adjustments are given in Table 3.9. General information on the adjustments is contained in Table 3.10.

The first conclusion that can be drawn from the results is that the generalized least squares adjustment gave essentially the same results as the method of observation equations. What appeared to be an advantage in that the mathematical model was linear in the parameters, using the generalized method, was not really an advantage at all.

The second conclusion is that the technique of using only the better conditioned events plus images 1-3-5-7 from the highly correlated events gives essentially the same results as using all observations without correlation. This conclusion is not too surprising. Many analysts agree that correlation coefficients of 0.6 or less do not cause significant changes in a solution. Many of the differences in station coordinates are approaching the 1 σ level. The σ_o for the

TABLE 3.9

COORDINATES OF THE NORTH AND SOUTH AMERICAN NETWORK
STATIONS FROM THE THREE TEST ADJUSTMENTS

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates					
			Generalized, No Correlation	σ	Generalized, Full Correlation	σ	Obs. Eq., Full Correlation	σ
1	X	546551.3	- 0.5	2.8	1.4	3.2	2.6	3.0
	Y	-1389976.8	- 9.6	10.1	- 9.8	9.5	- 8.4	9.5
	Z	6180216.4	7.1	7.8	4.8	8.3	3.1	8.4
2	X	1130751.5	0.0	0.3	0.0	0.3	0.0	0.2
	Y	-4830822.5	0.0	0.3	0.0	0.3	0.0	0.2
	Z	3994698.9	0.0	0.3	0.0	0.3	0.0	0.2
3	X	-2127841.1	0.7	9.0	1.6	8.1	1.6	8.1
	Y	-3785839.5	- 8.1	3.6	- 3.7	3.9	- 3.1	3.7
	Z	4656032.3	0.8	3.2	- 1.8	4.0	- 2.2	4.0
8	X	3623218.3	-34.0	9.9	-28.5	11.1	-29.4	4.0
	Y	-5214222.7	3.9	4.8	4.0	6.0	5.2	6.2
	Z	601532.3	40.1	12.6	31.1	13.6	31.3	13.6
9	X	1280811.5	-20.4	5.3	-17.6	6.4	-17.6	6.4
	Y	-6250937.6	7.5	6.8	9.3	8.1	10.7	8.4
	Z	- 10814.6	41.6	13.9	34.2	14.7	33.1	14.6
19	X	2280596.7	-29.0	6.9	-24.3	8.2	-24.6	8.2
	Y	-4914539.4	3.9	5.2	3.9	6.7	4.1	6.7
	Z	-3355431.0	64.7	24.5	58.2	25.6	58.5	25.5

All units are in meters.

TABLE 3.9 (Cont'd)

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates					
			Generalized, No Correlation	σ	Generalized, Full Correlation	σ	Obs. Eq., Full Correlation	σ
20	X	-1888624.9	4.1	9.7	- 0.4	9.7	- 2.5	9.3
	Y	-5354875.8	4.6	6.2	- 0.3	8.0	0.3	7.9
	Z	-2895760.4	69.4	23.5	53.8	24.6	55.4	24.6
38	X	-2160990.2	1.3	9.2	3.1	8.4	1.8	8.4
	Y	-5642692.6	- 4.7	3.8	- 1.5	4.6	0.0	4.5
	Z	2035359.0	15.9	7.2	13.0	8.0	9.7	7.9
43	X	1371345.7	-18.4	5.5	-17.0	6.9	-16.6	6.8
	Y	-3614746.0	-10.0	6.6	- 7.1	7.7	- 6.8	7.6
	Z	-5055948.9	80.0	29.9	68.6	31.0	66.8	30.9
50	X	1192648.5	-17.8	6.3	-16.6	7.9	-15.7	7.8
	Y	-2451015.8	-16.4	10.3	-12.5	11.8	-13.2	11.7
	Z	-5747042.3	77.8	32.6	65.7	34.1	65.9	34.0
61	X	2999889.2	-34.8	9.1	-33.6	10.6	-34.0	10.6
	Y	-2219363.2	-23.9	10.8	-19.7	12.2	-18.7	12.1
	Z	-5155268.2	80.5	30.4	66.7	31.7	65.7	31.6
67	X	5186389.3	-52.6	15.7	-48.7	17.6	-47.4	17.7
	Y	-3653935.6	3.3	7.4	2.1	9.4	3.1	9.5
	Z	- 654306.7	49.8	16.6	44.9	17.9	46.6	18.0

All units are in meters.

TABLE 3.9 (Cont'd)

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates					
			Generalized, No Correlation	σ	Generalized, Full Correlation	σ	Obs. Eq., Full Correlation	σ
111	X	-2448865.4	2.0	10.0	3.3	9.1	2.1	9.1
	Y	-4667971.2	-10.1	3.3	- 8.1	4.4	- 5.7	4.4
	Z	3582743.9	5.0	3.8	2.9	5.1	- 0.1	5.1
134	X	-2448914.6	0.8	10.0	2.1	9.1	0.9	9.1
	Y	-4668062.8	- 2.6	3.3	- 0.6	4.4	1.7	4.4
	Z	3582433.7	19.9	3.8	17.8	5.1	14.8	5.1

All units are in meters.

TABLE 3.10

GENERAL INFORMATION ON THE ADJUSTMENTS

	Generalized, No Correlation	Generalized, Full Correlation	Observation Equ. Method Full Correlation
Number of observing stations	14	14	14
Station used as origin	Beltsville (2)	Beltsville (2)	Beltsville (2)
Chord constraint between Station 2 and Station 3 (3485363.23m)	1:364777	1:406009	1:406009
Number of Degrees of Freedom	1890	1890	1890
Quadratic sum of all the residuals (W'PW)	14203.5	11465.1	11472.7
Standard deviation of unit weight (σ_o)	2.7414	2.4630	2.4638

solution using uncorrelated observations is slightly higher than in the solution using correlated observations (see Table 3.10).

3.2.1.1 Adjustments using the Very Highly Correlated Observations

The next series of experiments was to use all 237 events in the North and South American Network, if possible, using full correlation. The adjustments were performed in sequence as follows:

1. A control adjustment using only the 187 well-conditioned events.
2. An adjustment using the data in 1, plus all events where $W'M^{-1}W$ was greater than 10,000 but less than 100,000.
3. Same as 2, plus all events where $W'M^{-1}W$ was greater than 100,000 but less than 1,000,000.
4. Same as 3, plus all events where $W'M^{-1}W$ was greater than 1,000,000 and less than 10,000,000.
5. The final adjustment used the same data as 4, plus all events where $W'M^{-1}W$ was greater than 10,000,000.

The results of each of the five adjustments are shown in Tables 3.11 and 3.12. Table 3.11 shows that the σ_0 decreased slightly when the observations had a $W'M^{-1}W$ less than 1,000,000. When the very highly correlated observations are added, the σ_0 increased. However, the weight coefficient matrix, N^{-1}, remains about the same for all adjustments. A very close examination of the weight coefficient matrices actually shows the numbers to decrease, although very slightly, with the addition of the highly correlated observations. The change in the statistics given in Table 3.12 is due almost entirely to the change of σ_0, which is due to the increase in $V'PV$.

TABLE 3.11

GENERAL INFORMATION ON THE NORTH AND SOUTH AMERICAN ADJUSTMENTS

	187 well-conditioned events	$W'M^{-1}W <$ 100,000	$W'M^{-1}W <$ 1,000,000	$W'M'^{-1}W <$ 10,000,000	$W'M^{-1}W <$ 100,000,000
Number of observing stations	14	14	14	14	14
Station used as origin	Beltsville (2)	Beltsville (2)	Beltsville (2)	Beltsville (2)	Beltsville (2)
Chord constraint between Station 2 and Station 3 (3485363.23m)	1:1000000	1:1000000	1:1000000	1:1000000	1:1000000
Number of events	187	209	223	231	233
Number of Degrees of Freedom	1610	1820	1974	2058	2072
Quadratic sum of all residuals (W'PW)	9614.2	10554.7	11722.4	13976.5	14470.4
Standard deviation of unit weight (σ_o)	2.4437	2.4082	2.4369	2.6060	2.6427

TABLE 3.12

COORDINATES OF THE NORTH AND SOUTH AMERICAN NETWORK STATIONS
AT THE DIFFERENT STAGES OF USING CORRELATED OBSERVATIONS

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates									
			187 well-conditioned events	σ	$W'M^{-1}W<$ $1x10^5$	σ	$W'M^{-1}W<$ $1x10^6$	σ	$W'M^{-1}W<$ $1x10^7$	σ	$W'M^{-1}W<$ $1x10^8$	σ
1	X	546551.3	1.5	3.1	1.4	3.1	1.5	3.1	1.5	3.3	1.4	3.4
	Y	-1389976.8	-10.5	9.5	- 9.3	9.4	- 9.6	9.4	- 9.8	10.1	- 9.8	10.2
	Z	6180216.4	4.6	8.3	4.5	8.2	5.7	8.2	5.4	8.7	5.4	8.8
2	X	1130751.5	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.3	0.0	0.3
	Y	-4830822.5	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.3	0.0	0.3
	Z	3994698.9	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.3	0.0	0.3
3	X	-2127841.1	1.5	8.0	2.1	7.9	2.0	8.0	1.8	8.5	1.8	8.7
	Y	-3785839.5	- 5.9	4.0	- 3.3	3.8	- 3.0	3.8	- 3.4	4.1	- 3.4	4.1
	Z	4656032.3	1.3	4.1	0.1	4.0	- 1.0	3.9	- 1.4	4.2	- 1.4	4.2
8	X	3623218.3	-28.3	11.6	-28.2	11.4	-28.5	11.1	-28.4	11.9	-28.5	12.1
	Y	-5214222.7	4.4	6.7	3.6	6.1	3.2	6.0	4.3	6.3	4.3	6.4
	Z	601532.3	32.7	14.3	32.2	14.0	35.4	13.5	33.1	14.4	32.9	14.6
9	X	1280811.5	-20.9	7.4	-19.2	7.2	-19.5	6.6	-18.5	7.0	-17.8	7.1
	Y	-6250937.6	10.1	8.6	8.7	8.2	10.2	8.0	9.4	8.6	9.5	8.7
	Z	- 10814.6	33.0	15.3	33.3	15.0	37.1	14.6	35.9	15.5	36.0	15.8

All units are in meters.

TABLE 3.12 (cont'd)

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates									
			187 well-conditioned events	σ	$W'M^{-1}W<$ $1x10^5$	σ	$W'M^{-1}W<$ $1x10^6$	σ	$W'M^{-1}W<$ $1x10^7$	σ	$W'M^{-1}W<$ $1x10^8$	σ
19	X	2280596.7	-31.2	9.1	-26.8	8.8	-26.5	8.4	-24.9	8.9	-24.5	9.0
	Y	-4914539.4	1.9	8.1	2.3	7.2	7.5	7.0	6.4	7.0	6.3	7.1
	Z	-3355431.0	52.1	27.1	60.5	26.1	56.3	25.4	57.0	27.1	58.6	27.5
20	X	-1888624.9	1.6	9.9	2.4	9.7	1.1	9.5	- 0.7	10.2	- 1.0	10.3
	Y	-5354875.8	4.6	9.3	9.1	8.8	2.1	8.0	- 0.8	8.3	- 1.0	8.5
	Z	-2895760.4	64.1	27.4	71.3	26.4	56.5	24.5	52.6	26.1	52.0	26.4
38	X	-2160990.2	1.4	8.4	1.7	8.3	3.6	8.3	3.0	8.9	3.0	9.0
	Y	-5642692.6	- 4.5	4.9	- 3.5	4.7	- 1.3	4.5	- 1.2	4.8	- 1.2	4.9
	Z	2035359.0	16.1	8.2	15.7	8.0	14.1	7.9	12.8	8.4	13.0	8.5
43	X	1371345.7	-14.0	8.0	-14.8	7.6	-18.2	7.2	-18.9	7.5	-17.5	7.5
	Y	-3614746.0	-20.2	9.8	-14.1	8.9	- 3.4	8.1	- 2.3	8.1	- 3.8	8.2
	Z	-5055948.9	71.0	32.7	74.9	31.6	64.8	30.9	64.6	32.9	67.8	33.3
50	X	1192648.5	-12.5	9.0	-14.2	8.6	-18.6	8.1	-19.1	8.5	-17.6	8.6
	Y	-2451015.8	-30.9	14.4	-24.9	13.1	- 7.5	12.2	- 6.7	12.5	- 9.1	12.6
	Z	-5747042.3	66.7	36.0	71.8	34.8	58.9	34.0	60.2	36.2	64.0	36.6

All units are in meters.

TABLE 3.12 (cont'd)

Station Number		Approximate Coordinates	Corrections to Approximate Coordinates									
			187 well-conditioned events	σ	W'M-1W< 1×10^5	σ	W'M-1W< 1×10^6	σ	W'M-1W< 1×10^7	σ	W'M-1W< 1×10^8	σ
61	X	2999889.2	-29.5	12.6	-32.7	11.5	-28.2	10.9	-32.3	11.3	-32.4	11.5
	Y	-2219363.2	-38.1	15.0	-32.3	13.6	-13.8	12.6	-13.4	12.9	-16.1	13.0
	Z	-5155268.2	62.9	33.5	69.1	32.4	60.6	31.6	61.9	33.6	65.1	34.1
67	X	5186389.3	-59.6	22.1	-57.0	18.7	-49.3	18.3	-46.0	18.7	-47.2	19.0
	Y	-3653935.6	7.1	11.0	1.5	10.0	8.0	9.6	4.4	9.9	3.5	10.1
	Z	- 654306.7	41.0	19.1	46.2	18.3	46.0	17.9	44.7	19.0	45.0	19.3
111	X	-2448865.4	1.7	9.1	2.5	9.0	4.0	9.1	2.9	9.7	2.9	10.0
	Y	-4667971.2	-10.4	5.0	-10.4	4.6	- 8.9	4.5	- 7.4	4.6	- 7.4	4.6
	Z	3582743.9	2.0	5.4	4.5	5.0	3.3	5.0	3.8	5.3	3.8	5.4
134	X	-2448914.6	0.5	9.1	1.3	9.0	2.8	9.1	1.7	9.7	1.7	9.8
	Y	-4668062.8	- 2.9	5.0	- 3.0	4.6	- 1.5	4.5	0.1	4.6	0.1	4.6
	Z	3582433.7	16.8	5.4	19.4	5.0	18.2	5.0	18.7	5.3	18.7	5.4

All units are in meters.

3.2.1.2 Formation of Normal Equations using 32 Digit Arithmetic

The normal equations used in all adjustments up to this point in time have been formed with the IBM 370/165 computer, using double precision arithmetic which is 14 digits. The last solution listed in Tables 3.11 and 3.12 had only 233 of the 237 events in the North and South American Network. The four missing events were rejected by the computer program because the contribution to V'PV was a negative number. These rejected events were then processed through the quadruple precision version of the generalized normal equation program using 32 digit arithmetic, the result being that these four events were now acceptable, since the reason for the original rejection was roundoff error. Figures 3.8 through 3.11 show the event adjustment using double precision for these four events, while the same events in Figures 3.12 through 3.15 use quadruple precision with changes in the numbers 'NEW VPV' and 'WPW CONTRIBUTION FROM SATELLITE POSITIONS.'

3.3 Conclusions on how to use the Data

The conclusions drawn from the experiments performed to this point in the investigation can be summarized in one sentence. 'Observations with high correlation should be processed using quadruple precision arithmetic.' The question is, "at what degree of correlation must the quadruple precision be used?" The results given in Tables 3.11 and 3.12 indicate that observations with a $W'M^{-1}W$ less than 10^6 can be processed using double precision arithmetic without loss of accuracy. However, there were some events, such as those shown in Figures 3.9 and 3.10, that were exceptions to this rule. From the standpoint of this

```
                    TEST DISTANCE =                200.00    SECONDS OF ARC

EVENT      0        7699
    9                               1.1931927           -0.4121166          0.1
   19                               1.5459385            0.4597032          0.1
 SATELLITE POSITION     2382909.821    -9029551.447    -1317543.931
 GEOD. COORD. OF SATELLITE      -8.066587     284.783388      3053434.2
                          RMS MISCLOSURE IN METERS=         1.9
    9                               1.1948048           -0.3744808          0.2
   19                               1.5441448            0.4790971          0.1
 SATELLITE POSITION     2391206.608    -9063693.121    -1199329.257
 GEOD. COORD. OF SATELLITE      -7.323809     284.779172      3072417.5
                          RMS MISCLOSURE IN METERS=         2.7
    9                               1.1963576           -0.3366853          0.1
   19                               1.5424798            0.4979706          0.0
 SATELLITE POSITION     2399045.133    -9096469.182    -1080928.684
 GEOD. COORD. OF SATELLITE      -6.584008     284.774411      3091526.5
                          RMS MISCLOSURE IN METERS=         1.0
    9                               1.1978497           -0.2987890          0.0
   19                               1.5409379            0.5163479          0.0
 SATELLITE POSITION     2406437.640    -9127884.327     -962353.458
 GEOD. COORD. OF SATELLITE      -5.847137     284.769170      3110760.8
                          RMS MISCLOSURE IN METERS=         0.4
    9                               1.1992852           -0.2608577          0.1
   19                               1.5395140            0.5342521          0.1
 SATELLITE POSITION     2413384.674    -9157936.872     -843624.170
 GEOD. COORD. OF SATELLITE      -5.113213     284.763463      3130112.2
                          RMS MISCLOSURE IN METERS=         1.5
    9                               1.2006697           -0.2229605          0.2
   19                               1.5382032            0.5517050          0.1
 SATELLITE POSITION     2419884.302    -9186618.502     -724768.828
 GEOD. COORD. OF SATELLITE      -4.382296     284.757288      3149565.3
                          RMS MISCLOSURE IN METERS=         2.6
    9                               1.2019921           -0.1851608          0.5
   19                               1.5370009            0.5687270          0.3
 SATELLITE POSITION     2425958.490    -9213913.264     -605808.777
 GEOD. COORD. OF SATELLITE      -3.654406     284.750800      3169102.5
                          RMS MISCLOSURE IN METERS=         8.0
NEW VPV  0.2458270415000D+01
NEW VPV  0.2439665970000D+07
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.2440833513210+07
THE ABOVE EVENT WAS REJECTED BECAUSE OF POOR CONDITIONING

THE P NUMBER FOR ONE OF THE STATIONS IN THE ABOVE EVENT IS  0.91284207D+09
```

Figure 3.8. Event 7699 using Double Precision Arithmetic

```
EVENT     0       7233
    2                          0.7950795          0.0080930          0.2
    8                          1.6818541          1.0526375          0.2
 SATELLITE POSITION    3407152.237   -7151727.759     4021010.666
 GEOD. COORD. OF SATELLITE      27.023159     295.473581      2510178.8
                        RMS MISCLOSURE IN METERS=         3.7
    2                          0.7870637          0.0554557          0.3
    8                          1.6978760          1.0834882          0.2
 SATELLITE POSITION    3383422.901   -7091004.761     4171845.611
 GEOD. COORD. OF SATELLITE      28.081836     295.507814      2522300.1
                        RMS MISCLOSURE IN METERS=         4.3
    2                          0.7784308          0.1040389          0.4
    8                          1.7154916          1.1129508          0.3
 SATELLITE POSITION    3358903.747   -7028132.473     4321455.239
 GEOD. COORD. OF SATELLITE      29.137396     295.544192      2534855.8
                        RMS MISCLOSURE IN METERS=         6.1
    2                          0.7691212          0.1537385          0.3
    8                          1.7348968          1.1410866          0.2
 SATELLITE POSITION    3333618.279   -6963114.350     4469784.812
 GEOD. COORD. OF SATELLITE      30.189707     295.582952      2547817.9
                        RMS MISCLOSURE IN METERS=         4.0
    2                          0.7590751          0.2044288          0.0
    8                          1.7563210          1.1679507          0.0
 SATELLITE POSITION    3307583.820   -6895967.947     4616791.888
 GEOD. COORD. OF SATELLITE      31.238681     295.624251      2561171.8
                        RMS MISCLOSURE IN METERS=         0.7
    2                          0.7482320          0.2559659          0.2
    8                          1.7800328          1.1935904          0.1
 SATELLITE POSITION    3280813.280   -6826727.786     4762457.934
 GEOD. COORD. OF SATELLITE      32.284318     295.668175      2574925.9
                        RMS MISCLOSURE IN METERS=         3.1
    2                          0.7365198          0.3081667          0.2
    8                          1.8063468          1.2180440          0.1
 SATELLITE POSITION    3253322.570   -6755413.386     4906713.043
 GEOD. COORD. OF SATELLITE      33.326379     295.714884      2589049.5
                        RMS MISCLOSURE IN METERS=         2.7
NEW VPV  0.270315597421D+01
NEW VPV  0.682665306170D+03
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.764305191101D+03
THE ABOVE EVENT WAS REJECTED BECAUSE OF POOR CONDITIONING

THE P NUMBER FOR ONE OF THE STATIONS IN THE ABOVE EVENT IS  0.91284207D+09
```

Figure 3.9. Event 7233 using Double Precision Arithmetic

```
EVENT    0        7743
   2                              1.4039851              -0.0830579             0.2
   9                              1.3783598               0.8101447             0.1
 SATELLITE POSITION    1936343.697   -9615287.151    3590788.424
 GEOD. COORD. OF SATELLITE      20.183363     281.386038      4069325.7
                            RMS MISCLOSURE IN METERS=        3.5
   2                              1.4046602              -0.0527146             0.0
   9                              1.3791300               0.8340770             0.0
 SATELLITE POSITION    1928944.462   -9590993.153    3740029.325
 GEOD. COORD. OF SATELLITE      20.999890     281.371645      4098153.7
                            RMS MISCLOSURE IN METERS=        0.5
   2                              1.4053380              -0.0221292             0.1
   9                              1.3798881               0.8573365             0.1
 SATELLITE POSITION    1921236.257   -9564698.259    3888474.001
 GEOD. COORD. OF SATELLITE      21.811776     281.357712      4126917.4
                            RMS MISCLOSURE IN METERS=        1.4
   2                              1.4060177               0.0086781             0.1
   9                              1.3806328               0.8799547             0.1
 SATELLITE POSITION    1913229.566   -9536416.262    4036096.114
 GEOD. COORD. OF SATELLITE      22.619072     281.344280      4155603.2
                            RMS MISCLOSURE IN METERS=        1.8
   2                              1.4066980               0.0396884             0.3
   9                              1.3813628               0.9019616             0.3
 SATELLITE POSITION    1904937.930   -9506171.842    4182881.262
 GEOD. COORD. OF SATELLITE      23.421865     281.331394      4184212.7
                            RMS MISCLOSURE IN METERS=        7.6
   2                              1.4073763               0.0708849             0.5
   9                              1.3820769               0.9233858             0.5
 SATELLITE POSITION    1896379.955   -9474004.128    4328834.671
 GEOD. COORD. OF SATELLITE      24.220300     281.319115      4212769.6
                            RMS MISCLOSURE IN METERS=       11.9
   2                              1.4080484               0.1022551             0.3
   9                              1.3827740               0.9442543             0.3
 SATELLITE POSITION    1887581.399   -9439970.972    4473991.082
 GEOD. COORD. OF SATELLITE      25.014609     281.307523      4241328.5
                            RMS MISCLOSURE IN METERS=        6.8
NEW VPV   0.120188504964D+03
NEW VPV   0.297724807739D+05
WPW CONTRIBUTION FROM SATELLITE POSITIONS   0.559696564716D+05
THE ABOVE EVENT WAS REJECTED BECAUSE OF POOR CONDITIONING

THE P NUMBER FOR ONE OF THE STATIONS IN THE ABOVE EVENT IS   0.91284207D+09
```

Figure 3.10. Event 7743 using Double Precision Arithmetic

```
EVENT      0       10301
   19                            0.6302922            0.5330065             0.1
   67                            1.6032289           -0.1963765             0.2
 SATELLITE POSITION     5079245.217   -6956326.333   -1311629.413
 GEOD. COORD. OF SATELLITE      -8.700431      306.135499      2334941.6
                          RMS MISCLOSURE IN METERS=        2.5
   19                            0.6284620            0.5190288             0.1
   67                            1.6061741           -0.2187057             0.1
 SATELLITE POSITION     5070022.973   -6941786.745   -1385523.471
 GEOD. COORD. OF SATELLITE      -9.200527      306.143003      2329466.5
                          RMS MISCLOSURE IN METERS=        1.3
   19                            0.6265574            0.5047741             0.3
   67                            1.6092495           -0.2410095             0.4
 SATELLITE POSITION     5060479.886   -6926776.051   -1459327.620
 GEOD. COORD. OF SATELLITE      -9.701255      306.150663      2324075.8
                          RMS MISCLOSURE IN METERS=        5.8
   19                            0.6245767            0.4902344             0.3
   67                            1.6124474           -0.2632745             0.3
 SATELLITE POSITION     5050641.242   -6911304.220   -1533034.225
 GEOD. COORD. OF SATELLITE      -10.202563     306.158579      2318792.3
                          RMS MISCLOSURE IN METERS=        5.2
   19                            0.6225175            0.4754010             0.0
   67                            1.6157720           -0.2854850             0.1
 SATELLITE POSITION     5040504.798   -6895370.474   -1606637.548
 GEOD. COORD. OF SATELLITE      -10.704436     306.166744      2313614.1
                          RMS MISCLOSURE IN METERS=        0.8
   19                            0.6203776            0.4602649             0.3
   67                            1.6192302           -0.3076245             0.3
 SATELLITE POSITION     5030062.733   -6878972.300   -1680132.231
 GEOD. COORD. OF SATELLITE      -11.206869     306.175130      2308534.8
                          RMS MISCLOSURE IN METERS=        5.5
   19                            0.6181543            0.4448168             0.3
   67                            1.6228268           -0.3296778             0.3
 SATELLITE POSITION     5019312.638   -6862108.690   -1753513.773
 GEOD. COORD. OF SATELLITE      -11.709857     306.183730      2303552.4
                          RMS MISCLOSURE IN METERS=        5.5
NEW VPV  0.1834255375210+07
NEW VPV  0.1724313677100+02
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.1834362249150+07
THE ABOVE EVENT WAS REJECTED BECAUSE OF POOR CONDITIONING

THE P NUMBER FOR ONE OF THE STATIONS IN THE ABOVE EVENT IS  0.3930803000+10
```

Figure 3.11. Event 10301 using Double Precision Arithmetic

```
                    TEST DISTANCE =                 200.00    SECONDS OF ARC

EVENT      0        7699
                                   1.1931927             -0.4121166          0.1
   9
  19                               1.5459385              0.4597032          0.1
 SATELLITE POSITION     2382909.821   -9029551.447   -1317543.931
 GEOD. COORD. OF SATELLITE      -8.066587     284.783388      3053434.2
                         RMS MISCLOSURE IN METERS=            1.9
   9                               1.1948048             -0.3744808          0.2
  19                               1.5441448              0.4790971          0.1
 SATELLITE POSITION     2391206.608   -9063693.121   -1199329.257
 GEOD. COORD. OF SATELLITE      -7.323809     284.779172      3072417.5
                         RMS MISCLOSURE IN METERS=            2.7
   9                               1.1963576             -0.3366853          0.1
  19                               1.5424798              0.4979706          0.0
 SATELLITE POSITION     2399045.133   -9096469.182   -1080928.684
 GEOD. COORD. OF SATELLITE      -6.584008     284.774411      3091526.5
                         RMS MISCLOSURE IN METERS=            1.0
   9                               1.1978497             -0.2987890          0.0
  19                               1.5409379              0.5163479          0.0
 SATELLITE POSITION     2406437.441   -9127884.327    -962353.458
 GEOD. COORD. OF SATELLITE      -5.847137     284.769170      3110760.8
                         RMS MISCLOSURE IN METERS=            0.4
   9                               1.1992852             -0.2608577          0.1
  19                               1.5395140              0.5342521          0.1
 SATELLITE POSITION     2413384.674   -9157936.872    -843624.170
 GEOD. COORD. OF SATELLITE      -5.113213     284.763463      3130112.2
                         RMS MISCLOSURE IN METERS=            1.5
   9                               1.2006697             -0.2229605          0.2
  19                               1.5382032              0.5517050          0.1
 SATELLITE POSITION     2419884.302   -9186618.501    -724768.828
 GEOD. COORD. OF SATELLITE      -4.382296     284.757288      3149565.3
                         RMS MISCLOSURE IN METERS=            2.6
   9                               1.2019921             -0.1851608          0.5
  19                               1.5370009              0.5687270          0.3
 SATELLITE POSITION     2425958.491   -9213913.264    -605808.777
 GEOD. COORD. OF SATELLITE      -3.654406     284.750800      3169102.5
                         RMS MISCLOSURE IN METERS=            8.0
NEW VPV  0.24722372724806108181610+01
NEW VPV  0.24408398622778196457230+07
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.24408335066660+07
```

Figure 3.12. Event 7699 using Quadruple Precision Arithmetic

```
EVENT     1        7233
    2                         0.7950795            0.0080930           0.2
    8                         1.6818541            1.0526375           0.2
 SATELLITE POSITION    3407152.237   -7151727.759    4021010.666
 GEOD. COORD. OF SATELLITE    27.023159    295.473581     2510178.8
                       RMS MISCLOSURE IN METERS=        3.7
    2                         0.7870637            0.0554557           0.3
    8                         1.6978760            1.0834882           0.2
 SATELLITE POSITION    3383422.901   -7091004.761    4171845.611
 GEOD. COORD. OF SATELLITE    28.081836    295.507814     2522300.1
                       RMS MISCLOSURE IN METERS=        4.3
    2                         0.7784308            0.1040389           0.4
    8                         1.7154916            1.1129508           0.3
 SATELLITE POSITION    3358903.747   -7028132.473    4321455.239
 GEOD. COORD. OF SATELLITE    29.137396    295.544192     2534855.8
                       RMS MISCLOSURE IN METERS=        6.1
    2                         0.7691212            0.1537385           0.3
    8                         1.7348968            1.1410866           0.2
 SATELLITE POSITION    3333618.279   -6963114.350    4469784.812
 GEOD. COORD. OF SATELLITE    30.189707    295.582952     2547817.9
                       RMS MISCLOSURE IN METERS=        4.0
    2                         0.7590751            0.2044288           0.0
    8                         1.7563210            1.1679507           0.0
 SATELLITE POSITION    3307583.871   -6895967.947    4616791.886
 GEOD. COORD. OF SATELLITE    31.238681    295.624251     2561171.8
                       RMS MISCLOSURE IN METERS=        0.7
    2                         0.7482320            0.2559659           0.2
    8                         1.7800328            1.1935904           0.1
 SATELLITE POSITION    3280813.280   -6826727.785    4762457.934
 GEOD. COORD. OF SATELLITE    32.284318    295.668175     2574925.9
                       RMS MISCLOSURE IN METERS=        3.1
    2                         0.7365198            0.3081667           0.2
    8                         1.8063468            1.2180440           0.1
 SATELLITE POSITION    3253322.570   -6755413.385    4906713.043
 GEOD. COORD. OF SATELLITE    33.326379    295.714884     2589049.5
                       RMS MISCLOSURE IN METERS=        2.7
NEW VPV  0.26979143915430551355030+01
NEW VPV  0.76957128527540809529490+03
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.76430585079800+03
```

Figure 3.13. Event 7233 using Quadruple Precision Arithmetic

```
EVENT     2        7743
   2                             1.4039851            -0.0830579              0.2
   9                             1.3783598             0.8101447              0.1
 SATELLITE POSITION    1936343.698   -9615287.151    3590788.425
 GEOD. COORD. OF SATELLITE      20.183363     281.386038       4069325.7
                         RMS MISCLOSURE IN METERS=        3.5
   2                             1.4046602            -0.0527146              0.0
   9                             1.3791300             0.8340770              0.0
 SATELLITE POSITION    1928944.463   -9590993.153    3740029.325
 GEOD. COORD. OF SATELLITE      20.999890     281.371645       4098153.7
                         RMS MISCLOSURE IN METERS=        0.5
   2                             1.4053380            -0.0221292              0.1
   9                             1.3798881             0.8573365              0.1
 SATELLITE POSITION    1921236.257   -9564698.259    3888474.001
 GEOD. COORD. OF SATELLITE      21.811776     281.357712       4126917.4
                         RMS MISCLOSURE IN METERS=        1.4
   2                             1.4060177             0.0086781              0.1
   9                             1.3806328             0.8799547              0.1
 SATELLITE POSITION    1913229.567   -9536416.262    4036096.114
 GEOD. COORD. OF SATELLITE      22.619072     281.344280       4155603.2
                         RMS MISCLOSURE IN METERS=        1.8
   2                             1.4066980             0.0396884              0.3
   9                             1.3813628             0.9019616              0.3
 SATELLITE POSITION    1904937.931   -9506171.842    4182881.262
 GEOD. COORD. OF SATELLITE      23.421865     281.331395       4184212.7
                         RMS MISCLOSURE IN METERS=        7.6
   2                             1.4073763             0.0708849              0.5
   9                             1.3820769             0.9233858              0.5
 SATELLITE POSITION    1896379.955   -9474004.128    4328834.671
 GEOD. COORD. OF SATELLITE      24.220300     281.319115       4212769.6
                         RMS MISCLOSURE IN METERS=       11.9
   2                             1.4080484             0.1022551              0.3
   9                             1.3827740             0.9442543              0.3
 SATELLITE POSITION    1887581.399   -9439970.972    4473991.082
 GEOD. COORD. OF SATELLITE      25.014609     281.307523       4241328.5
                         RMS MISCLOSURE IN METERS=        6.8
NEW VPV  0.92856117162938089843230+01
NEW VPV  0.55964749642795168183120+05
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.5596964687070+05
```

Figure 3.14. Event 7743 using Quadruple Precision Arithmetic

```
EVENT     3      10301
   19                          0.6302927            0.5330065         0.1
   67                          1.6032289           -0.1963765         0.2
 SATELLITE POSITION    5079245.217   -6956326.333   -1311629.413
 GEOD. COORD. OF SATELLITE    -8.700431    306.135499     2334941.6
                      RMS MISCLOSURE IN METERS=          2.5
   19                          0.6284620            0.5190288         0.1
   67                          1.6061741           -0.2187057         0.1
 SATELLITE POSITION    5070022.974   -6941786.745   -1385523.471
 GEOD. COORD. OF SATELLITE    -9.200527    306.143003     2329466.5
                      RMS MISCLOSURE IN METERS=          1.3
   19                          0.6265574            0.5047741         0.3
   67                          1.6092495           -0.2410095         0.4
 SATELLITE POSITION    5060479.887   -6926776.051   -1459327.620
 GEOD. COORD. OF SATELLITE    -9.701255    306.150663     2324075.8
                      RMS MISCLOSURE IN METERS=          5.8
   19                          0.6245767            0.4902344         0.3
   67                          1.6124474           -0.2632745         0.3
 SATELLITE POSITION    5050641.243   -6911304.220   -1533034.225
 GEOD. COORD. OF SATELLITE   -10.202563    306.158579     2318792.3
                      RMS MISCLOSURE IN METERS=          5.2
   19                          0.6225175            0.4754010         0.0
   67                          1.6157720           -0.2854850         0.1
 SATELLITE POSITION    5040504.798   -6895370.474   -1606637.548
 GEOD. COORD. OF SATELLITE   -10.704436    306.166744     2313614.1
                      RMS MISCLOSURE IN METERS=          0.8
   19                          0.6203776            0.4602649         0.3
   67                          1.6192302           -0.3076245         0.3
 SATELLITE POSITION    5030062.733   -6878972.300   -1680132.231
 GEOD. COORD. OF SATELLITE   -11.206869    306.175130     2308534.8
                      RMS MISCLOSURE IN METERS=          5.5
   19                          0.6181543            0.4448168         0.3
   67                          1.6228268           -0.3296778         0.3
 SATELLITE POSITION    5019312.639   -6862108.690   -1753513.777
 GEOD. COORD. OF SATELLITE   -11.709857    306.183730     2303552.4
                      RMS MISCLOSURE IN METERS=          5.5
NEW VPV  0.18343666466209047737980+07
NEW VPV  0.16257748399400630567380+02
WPW CONTRIBUTION FROM SATELLITE POSITIONS  0.18343597986600+07
```

Figure 3.15. Event 10301 using Quadruple Precision Arithmetic

investigation it is very easy to decide which events should be processed in quadruple precision, since every Type II event had been processed in double precision and the troublesome data detected. In reality then, there are two different recommendations that can be given to anyone who is going to use highly correlated observations:

1. If all of the data can be processed using double precision arithmetic, the troublesome data can be detected. Then for the final formation of normal equations, the double precision arithmetic can be used to process all good events that have a $W'M^{-1}W$ less than 10^6, and quadruple precision arithmetic used to process all events with a higher $W'M^{-1}W$, plus any other troublesome data.
2. Same as 1, but use only every other image where $W'M^{-1}W > 10^6$.

4. THE WORLD NETWORK ADJUSTMENT

Once the proper method of handling the highly correlated observations had been determined, the final step in this investigation was to perform an adjustment of the entire 49 station network. For the purpose of gaining a better insight of the results, several adjustments were performed. The normal equations used for these adjustments were formed as suggested in Section 3.3. Earlier in the investigation, before the final data handling procedures were decided upon, the normal equations were formed by using every image if the value of $W'M^{-1}W$ was less than 10^4, and only images 1-3-5-7 if $W'M^{-1}W$ was larger. These normal equations used in the large worldwide network adjustment of the National Geodetic Satellite Program are described in [Mueller et al., 1973]. After it was determined that quadruple precision could be used to process the highly correlated observations, only the events with a $W'M^{-1}W$ greater than 10^4 were processed in this manner. This set of normal equations was then combined with the earlier set of every-image normal equations from events with a $W'M^{-1}W$ less than 10^4. By using this technique the cutoff point between double and quadruple precision was 10^4, and not 10^6 as suggested in Section 3.3.

Even though it was discovered that the data could be processed using quadruple precision, there was still the question of verifying the earlier procedure. For this reason, one of the adjustments included here was based on the 1-3-5-7 image data for the highly correlated observations as in [Mueller et al., 1973], and an additional adjustment was performed with the correlation between observations neglected.

4.1 The Adjustment

The network adjustments were performed using the approximate station coordinates listed in Table 4.1. These are geodetic coordinates referenced to an ellipsoid with the following dimensions:

$a = 6378155.0$ meters

$b = 6356769.7$ meters.

The parameters of the adjustment were the Cartesian coordinates of the 49 ground stations.

4.1.1 Constraints

The normal equations are singular as no constraints are applied during the process of forming normal equations. Since the orientation of the system is inherent in the optical observations, the minimum number of constraints needed is four (one for defining the scale, three for the definition of the origin of the coordinate system). In addition to these "minimum" constraints, any additional available geodetic information may be included in the solution in the form of constraints. In the case of the worldwide network there are three different types of geodetic information available:

1. Relative geodetic positions of nearby stations. On Wake Island, Stations 12 and 66, and in California, Stations 111 and 134, were co-located. The relative positions of these two pairs of stations were determined from the local survey. In all adjustments the relative positions of these stations were constrained to the known values listed in Table 4.2.
2. Scalars. Table 4.3 is a listing of eight baselines measured for the purpose of furnishing scale to this network.

TABLE 4.1

APPROXIMATE STATION COORDINATES

Station		Approximate Coordinates						
No.	Name	Latitude			Longitude (E)			Ell. Ht. (H)m
1	THULE	76°	30'	5.2500"	291°	27'	54.6990"	186.694
2	BELTSVILLE	39	1	39.4710	283	10	26.5940	-19.429
3	MOSES LAKE	47	11	6.6310	240	39	42.1940	326.364
4	SHEMYA	52	42	48.1210	174	7	27.7320	-38.810
6	TROMSO	69	39	45.2300	18	56	29.2500	81.216
7	TERCEIRA	38	45	36.1780	332	54	24.6100	73.057
8	PARAMARIBO	5	26	53.3500	304	47	39.7040	-58.913
9	QUITO	0	5	51.9460	281	34	46.4820	2661.306
11	MAUI	20	42	26.6550	203	44	37.4660	3087.464
12	WAKE ISLAND I	19	17	28.0290	166	36	40.8440	-65.870
13	KANOYA	31	23	42.6910	130	52	19.4960	32.220
15	MASHHAD	36	14	25.4760	59	37	45.1740	917.594
16	CATANIA	37	26	38.5920	15	2	45.2840	-10.989
19	VILLA DOLORES	-31	56	35.9850	294	53	37.3290	607.419
20	EASTER ISLAND	-27	10	36.4750	250	34	21.4760	210.046
22	TUTUILA	-14	19	55.0940	189	17	10.2930	-21.370
23	THURSDAY ISLAND	-10	35	3.4030	142	12	40.1730	32.500
31	INVERCARGILL	-46	24	58.8520	168	19	33.6360	-100.400
32	CAVERSHAM	-31	50	28.5010	115	58	34.5380	14.720
38	SOCORRO ISLAND	18	43	58.1350	249	2	40.5120	-31.052
39	PITCAIRN ISLAND	-25	4	6.7450	229	53	11.9690	299.418
40	COCOS ISLAND	-12	11	44.5170	96	50	5.0150	-99.590
42	ADDIS ABABA	8	46	12.7860	38	59	51.9550	1818.962
43	CERRO SOMBRERO	-52	46	53.3320	290	46	31.8480	79.731
44	HEARD ISLAND	-53	1	10.8490	73	23	36.2950	-15.510
45	MAURITIUS	-20	13	54.4340	57	25	32.7440	48.920
47	ZAMBOANGA	6	55	20.1450	122	4	11.3700	26.340
50	PALMER STATION	-64	46	26.5320	295	56	50.1240	9.733
51	MAWSON STATION	-67	36	6.3550	62	52	24.6430	-25.710
52	WILKES STATION	-66	16	45.8590	110	32	12.1800	-86.410
53	MCMURDO STATION	-77	50	41.2180	166	38	39.7330	-113.320
55	ASCENSION ISLAND	-7	58	16.1870	345	35	34.0190	40.923
59	CHRISTMAS ISLAND	2	0	19.0250	202	35	15.5260	25.562
60	CULGOORA	-30	18	34.7120	149	33	42.0430	109.480
61	SOUTH GEORGIA IS.	-54	17	2.1770	323	30	19.4660	-3.433
63	DAKAR	14	44	41.8850	342	30	59.7990	10.417
64	FORT LAMY	12	7	54.1410	15	2	7.2850	248.595
65	HOHENPEISSENBERG	47	48	3.6290	11	1	25.7350	916.461
66	WAKE ISLAND II	19	17	28.0290	166	36	40.8440	-65.870
67	NATAL	-5	55	39.6860	324	50	3.7920	1.641
68	JOHANNESBURG	-25	53	0.2920	27	42	23.7750	1489.425
69	TRISTAN DA CUNHA	-37	3	54.8070	347	41	4.6360	17.797
72	CHIANG MAI	18	46	10.6010	98	58	5.0640	170.500
73	DIEGO GARCIA	-7	21	7.1720	72	28	21.9230	-154.050
75	MAHE	-4	40	15.4600	55	28	48.8380	503.181
78	PORT VILA	-17	41	30.6000	168	18	25.4000	70.600
111	WRIGHTWOOD I	34	22	54.1420	242	19	4.9350	2239.267
123	POINT BARROW	71	18	48.0000	203	21	4.0000	18.100
134	WRIGHTWOOD II	34	22	43.9340	242	19	4.8940	2149.892

TABLE 4.2

RELATIVE POSITION CONSTRAINTS

Stations	Relative coordinates (meters)			Weights	Source of Information*
	Δ X	Δ Y	Δ Z		
12 - 66	1.93	42.34	- 25.67	100.00	NGS
111 - 134	53.73	90.04	305.32	100.00	NGS

*NGS National Geodetic Survey .

TABLE 4.3

BASELINES USED TO SCALE THE BC-4 WORLDWIDE NETWORK

Baseline Stations	Chord Distance (meters)	Estimated Standard Deviation		Source of Information*
		(m)	ppM	
2 - 3	3485363.232	3.5	1.00×10^{-6}	NGS
3 - 111	1425876.452	1.6	1.11×10^{-6}	NGS
6 - 65	2457765.810	3.5	1.43×10^{-6}	NGS,DGFI
16 - 65	1194793.601	1.4	1.18×10^{-6}	NGS,DGFI
6 - 16	3545871.454	3.5	1.00×10^{-6}	NGS,DGFI
63 - 64	3485550.755	4.1	1.18×10^{-6}	NGS
23 - 60	2300209.803	4.6	2.00×10^{-6}	NGS,DNP
32 - 60	3163623.866	6.3	2.00×10^{-6}	NGS,DNP

*NGS National Geodetic Survey.
DGFI Deutsche Geodätisches Forschungsinstitut.
DNP Division of National Mapping, Department of National Development, Australia.

3. The ellipsoidal (geodetic) heights, being the sum of the orthometric heights and the geoid undulations. In recent years, the geoid has been fairly well determined [Rapp, 1973], and the orthometric heights accurately measured through differential leveling. The height constraints used in these adjustments were calculated as described in [Mueller et al., 1973]. These are listed in Table 4.4. An additive term of -15 meters was applied, in most cases, to the geoid undulations resulting in a semi-major axis (of the level ellipsoid best fitting the geoid) of 6378140 meters. This value was decided upon a priori. In all adjustments, "inner" constraints were used to define the origin [Blaha, 1971] for reasons explained in [Mueller et al., 1973] and in Section 4.1.2.

4.1.2 The Selection of the Baselines

To get a feeling for the quality of the EDM baselines listed in Table 4.3, four preliminary adjustments were performed in which the four longest scalars were individually constrained to their measured lengths and their effect on the other (unconstrained) baselines investigated. The results are shown in Table 4.5 in the form of the differences "adjusted - measured" lengths (Δd). Only independent lines longer than 2000 km are shown, since the adjusted length of a short line, due to the geometry resulting from the high altitude of PAGEOS, is not reliable. From the table it is clear that holding the east-west Australian line (32-60) to its measured value generally results in

TABLE 4.4

HEIGHT CONSTRAINTS

No.	N_{Ref} (m)	MSL (m)	H_{Constr}* (m)
1	11.66	206.0	199.951
2	-36.90	44.300	5.444
3	-17.65	368.74	351.580
4	6.22	36.80	29.418
6	27.06	105.70	102.751
7	54.00	53.30	86.780
8	-28.31	18.380	- 12.091
9	16.73	2682.10	2707.282
11	1.75	3049.270	3056.381
12	13.75	3.50	7.244
13	34.27	65.90	72.214
15	-20.67	991.00	925.791
16	37.43	9.240	10.344
19	22.80	608.18	635.373
20	- 4.75	230.80	241.566
22	27.35	5.340	38.110
23	67.94	60.50	116.207
31	8.68	0.900	6.374
32	-30.51	26.30	- 30.292
38	-35.47	23.20	- 0.720
39	-16.68	339.40	338.356
40	-38.11	4.50	- 71.484
42	- 5.78	1886.460	1836.223
43	15.60	80.70	99.324
44	36.61	3.80	13.258
45	- 6.07	149.40	101.997
47	62.17	9.39	40.849
50	15.70	16.440	31.475
51	29.20	11.30	20.672
53	-56.10	19.00	- 43.009
55	16.26	70.940	65.505
59	16.07	2.750	27.763
60	27.33	211.080	226.551
61	11.28	4.20	8.904
63	27.20	26.30	30.600
64	10.35	295.40	268.163
65	44.23	943.20	953.516
66	13.74	5.30	9.434

TABLE 4.4 (cont'd)

No.	N_{Ref} (m)	MSL (m)	H_{Constr}* (m)
67	-12.03	40.630	17.645
68	24.65	1523.80	1513.482
69	25.52	24.800	32.742
72	-40.39	319.20	238.359
73	-73.64	3.90	- 113.795
75	-44.40	588.980	499.546
78	63.10	15.20	74.360
111	-33.18	2284.30	2257.368
123	- 1.40	8.30	- 7.364
134	-33.19	2198.400	2171.459

$^{*}H_{Constr} = N_{Ref} + MSL + \Delta N$

where

$\Delta N = \Delta a + \Delta x \cos\phi \cos\lambda + \Delta y \cos\phi \sin\lambda + \Delta z \sin\phi$

$\Delta a = 15.27m$

$\Delta x = 15.11m, \quad \Delta y = 26.82m, \quad \Delta z = 8.05m.$

(The set of constants were obtained through several iterations).

TABLE 4.5

ADJUSTED - GIVEN LENGTHS (m)

Solution	BC-D8	BC-D9	BC-D10	BC-D11
Line fixed / Line	2 - 3	63 - 64	32 - 60	6 - 16
2 - 3	0.0	- 8.6	33.8	12.4
6 - 16	-13.3	-20.9	22.1	0.0
63 - 64	6.1	0.0	40.5	19.1
23 - 60	- 9.5	-14.6	12.4	- 0.7
32 - 60	-29.5	-36.6	0.0	-17.5
$\sum \Delta d$ (m)	-46.2	-83.6	108.8	13.3
$\frac{\sum \Delta d}{\text{length}} \times 10^6$	- 2.89	- 5.23	6.81	0.83

unreasonable larger differences of opposite signs than in any other case.

To verify the suspicion that something is wrong with the given measured value of line 32-60, a free adjustment was performed in which both the origin and the scale constraints were "free". It is expected that the variance obtained from such an adjustment would primarily reflect the geometry of the situation. In other words, the variances of the various lengths would be due to the geometry of the network and free of the quality of the measured lengths ($\sigma_d^{msrd})^2$. If the estimated variance of the measured lengths are added to those obtained from the free adjustment ($\sigma_d^{free})^2$, an estimate is obtained for the maximum expected variance of the length differences $(\sigma_{\Delta d}^{est})^2$. If an actual length difference is found to be 2-3 times greater than this estimated standard deviation, the measured length becomes suspect. The result of such an analysis is shown in Table 4.6. From the table it is seen

TABLE 4.6

ADJUSTED - MEASURED LENGTHS (Δd) FROM A FREE ADJUSTMENT (SOLUTION BC-16)

Line	σ_d^{free} (m)	σ_d^{msrd*} (m)	$\sigma_{\Delta d}^{est}$ (m)	Δd(m)
2 - 3	4.2	3.5	5.5	- 5.0
3 - 111	3.7	1.6	4.0	9.8
6 - 65	4.0	3.5	5.3	7.7
16 - 65		1.4		
6 - 16	4.5	3.5	5.7	-17.2
63 - 64	4.4	4.1	6.0	2.4
23 - 60	4.4	4.6	6.4	-12.1
32 - 60	4.3	6.3	7.6	-33.1

*From Table 4.3.

again that line 32-60 is out of bound.

Another way of evaluating the effect of a scalar is through the semi-diameter of an ellipsoid best fitting the geoid resulting from a solution. In this method the undulations for each station are computed (N = Ellipsoidal Height - Mean Sea Level Height) and, after suitable transformations for shift in origin, are compared with some standard set of undulations (in this case with those in [Rapp, 1973]). The average difference of these two sets of undulations (ΔN) is equivalent (with opposite sign) to the difference between the semi-diameter of the reference ellipsoid (a = 6378155m) and that of the level ellipsoid of the same flattening to which the standard undulations refer. Thus,

a(level ellipsoid) = a(reference ellipsoid) - Δ N.

Three sets of such comparisons were performed: one with the baselines constrained with weights corresponding to the standard deviations listed in Table 4.3, one with all lines constrained to 1:3M, and one with all lines constrained to 1:30M. Within each set the adjustment was performed with all eight lines constrained and also without the line 32-60 (seven lines). The results are shown in Table 4.7. In addition to the semi-diameter of the best fitting level ellipsoid, the Table also contains the average standard deviations of a single coordinate ($\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$) as well as those of the heights (σ_H), and the ratios

adjusted - measured lengths/lengths : $\left(\sum \frac{\Delta d}{\text{length}}\right)$.

From the Table it is evident that though the varying type and number of constraints do not significantly change the quality of the coordinates, in the seven baseline solutions (BC-D2, BC-D8, BC-D10) the adjusted lengths agree much better with their measured values than in the eight baseline solutions (BC-D12, BC-D7, BC-D9). It is also seen that the inclusion of the single east-west Australian line increases the semi-diameter by the unreasonable amount of 6-9m (1.15ppM).

TABLE 4.7

THE SEMI-DIAMETER OF AN ELLIPSOID BEST FITTING THE GEOID, AS DETERMINED FROM SIX DIFFERENT ADJUSTMENTS

Solution	No. of lines constrained	Type of constraint	$\sum \frac{\Delta d}{\text{length}}$ x10^6	a (level ellipsoid) 6378000+(m)	σ (m)	σ_H (m)
BC-D12	8	as in Table 4.3	.81	124.1 ± 11.0	6.3	8.1
BC-D2	7		.19	118.4 ± 11.2	6.2	8.3
BC-D7	8	1:3M	.08	128.0 ± 10.8	6.1	7.7
BC-D8	7		.04	119.7 ± 11.2	6.2	7.9
BC-D9	8	1:30M	.02	127.0 ± 10.7	5.9	7.2
BC-D10	7		.01	118.0 ± 11.2	6.0	7.3

On the basis of the results in Tables 4.5 - 4.7, and also based on other calculations not reported here, the measured value of the Australian line 32-60 was rejected as a useful constraint.

The high standard deviations attached to the semi-diameters of the level ellipsoids in Table 4.7 also indicates the questionable value of only seven or eight baselines in scaling a global network regardless of their individual quality. The inclusion of height constraints in the solution is an attempt to obtain a better scale [Mueller et al., 1973, Section 5.1].

4.1.3 Results

The characteristics of some of the typical adjustments performed are given in Table 4.8. The Cartesian and geodetic coordinates resulting from the BC-D6 solution, which is considered the best, are listed

TABLE 4.8

GENERAL INFORMATION ON THE ADJUSTMENTS

Solution	BC-D1	BC-D2	BC-D3	BC-D4	BC-D6	BC-D11	BC-D13
No. of Observing Sta's	49	49	49	49	49	49	49
σ_o (a priori)	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Type of Observations	Correlated	Correlated	Correlated	Correlated	Correlated if $W'M^{-1}W < 10^4$, Images 1-3-5-7 for all others	Correlated	Uncorrelated
Origin Used	Inner	Inner	Inner	Inner	Inner	Inner	Inner
Scale Used	Inner	Baselines	Baselines and heights (additive term -15m)	Heights (additive term -15m)	Baselines and heights (additive term -15m)	Baselines and heights (w/o additive term of -15m)	Baselines and heights (additive term of -15m)
Degrees of Freedom	10207	10213	10261	10255	9403	10261	10928
V'PV	82523.5	82821.3	83597.6	83229.4	74165.4	83854.1	127274.4
σ_o(a postori)	2.84	2.85	2.85	2.85	2.81	2.86	3.41

in Table 4.9. Coordinates from other solutions are in the Appendix. Standard deviations of both types of coordinates are also given with the parameters of the error ellipsoid. The first page of the Table explains the format and the units used. The full variance-covariance matrix for solution BC-D6 cannot be presented here due to lack of space. However, the correlation coefficients ρ_{ij} between the coordinates of stations i and j (the off diagonal 3x3 matrices) are listed in Table 4.10 where $\rho_{ij} > 0.75$. The same type of table for the BC-D3 solution is given in Table 4.11. The 3x3 correlation coefficient matrices with any element greater than 0.99 are marked by asterisks. Comparison with Table 4.2 reveals that the two station pairs with the very high correlation have their relative positions constrained, therefore high correlations are expected. The correlation coefficients between the coordinates of a given station, i.e., the 3x3 matrices along the diagonal of the full correlation coefficient matrix, were less than 0.5 for all ground stations.

4.2 Comparison of Results

Several comparisons of the results were performed: a statistical comparison, a comparison with geometric information, and a comparison with dynamic solutions.

4.2.1 Statistical Comparison of Results

The statistical comparison was that described in [Fedorov, 1972, pgs 51-56]. This is a method of determining "which of the two different experiments is the best." The technique is to take the variance-covariance matrix of the parameters from two different adjustments,

TABLE 4.9

CARTESIAN AND GEODETIC COORDINATES (SOLUTION BC-D6)

Stn.No	x	σ_x	y	σ_y	z	σ_z
	ϕ	σ_ϕ	λ	σ_λ	H	σ_H
		a_1	A_1	r_1		
		a_2	A_2	r_2		
		a_3	A_3	r_3		

x, y, z Cartesian coordinates in meters (Orientation: x = the Greenwich meridian as defined by the B.I.H.; $y = \lambda = 90°$ (E); z = Conventional International Origin).

ϕ, λ Geodetic latitude and longitude in angular units (degrees, minutes and seconds of arc) computed from the Cartesian coordinates and referred to a rotational ellipsoid of a = 6378155.00m and b = 6356769.70m.

H Geodetic (ellipsoidal) height in meters referred to the same ellipsoid.

$\sigma_x, \sigma_y, \sigma_z$ Standard deviations of the Cartesian coordinates in meters.

$\sigma_\phi, \sigma_\lambda$ Standard deviations of the geodetic coordinates in seconds of arc.

σ_H Standard deviation of the geodetic height in meters.

a_1, A_1, r_1 Altitude (elevation angle), azimuth and magnitude of the semi-major axis of the error ellipsoid, respectively. Angles in degrees, magnitude in meters. Altitude is positive above the horizon. Azimuth is positive east reckoned from the north.

a_2, A_2, r_2 Same as above for the mean axis of the error ellipsoid.

a_3, A_3, r_3 Same as above for the minor axis of the error ellipsoid.

TABLE 4.9 (cont'd)

1	546563.84	3.09	-1389999.31	2.77	6180229.69	3.63
	76 30 4.55	0.10	291 27 55.18	0.50	205.62	4.00
		70.14	51.94	3.65		
		-17.93	78.33	3.36		
		-8.26	-14.36	2.39		
2	1130758.45	2.97	-4830847.71	2.52	3994704.06	3.09
	39 1 39.07	0.10	283 10 26.64	0.10	4.15	2.00
		7.64	-2.71	3.38		
		0.91	87.41	2.99		
		-82.31	4.15	2.09		
3	-2127839.92	2.62	-3785870.51	2.54	4656030.97	3.09
	47 11 5.97	0.10	240 39 42.97	0.10	343.38	2.00
		20.54	8.96	3.25		
		-0.29	98.85	2.58		
		-69.46	8.08	2.37		
4	-3851797.83	4.29	396404.77	5.10	5051322.61	6.05
	52 42 47.99	0.20	174 7 26.89	0.30	35.25	5.00
		19.89	32.26	6.65		
		57.16	-91.83	5.17		
		-24.98	-48.03	3.18		
6	2102925.09	2.60	721665.03	3.35	5958171.52	2.98
	69 39 45.17	0.10	18 56 26.83	0.30	101.46	3.00
		-6.79	134.09	3.55		
		73.20	67.32	2.90		
		15.30	-137.78	2.44		
7	4433646.05	2.96	-2268156.33	2.98	3971650.97	3.65
	38 45 36.19	0.10	332 54 24.15	0.10	94.91	3.00
		20.85	-18.88	3.78		
		11.79	75.68	3.06		
		65.76	-166.68	2.71		
8	3623240.75	4.13	-5214258.55	3.45	601534.13	6.30
	5 26 53.28	0.20	304 47 39.64	0.10	-16.73	3.00
		3.53	-15.82	6.45		
		10.06	74.81	4.11		
		-79.32	55.10	3.20		

TABLE 4.9 (cont'd)

9	1280823.16 - 0 5 51.92	4.71 0.20	-6250976.33 281 34 46.60	4.37 0.20	-10813.88 2701.59	6.21 4.00
		9.30 17.70 -69.86	-20.59 72.40 42.90	6.48 4.49 4.22		
11	-5466029.16 20 42 26.40	4.12 0.10	-2404429.92 203 44 38.49	3.87 0.10	2242209.08 3077.39	4.82 4.00
		20.18 -59.46 -21.91	32.22 83.69 -49.28	5.17 4.13 3.38		
12	-5858559.06 19 17 27.89	3.59 0.10	1394494.54 166 36 40.35	4.25 0.10	2093793.15 -1.23	4.75 4.00
		26.19 22.56 -54.21	5.05 -96.74 -41.93	4.77 4.27 3.54		
13	-3565891.08 31 23 42.44	4.25 0.20	4120693.82 130 52 17.98	5.60 0.20	3303412.97 72.03	5.52 4.00
		-4.44 42.10 -47.56	-134.10 -48.13 -39.23	6.72 4.56 3.74		
15	2604344.13 36 14 25.92	2.70 0.10	4444149.31 59 37 44.40	3.15 0.10	3750308.08 924.91	3.31 3.00
		2.80 31.00 58.84	-28.62 63.06 -123.27	3.56 3.10 2.44		
16	4896386.96 37 26 38.91	2.30 0.10	1316166.75 15 2 44.41	2.89 0.10	3856659.21 8.14	2.75 2.00
		0.59 23.32 66.67	139.14 48.88 -129.50	3.06 2.65 2.19		
19	2280611.66 -31 56 35.52	3.86 0.10	-4914565.71 294 53 37.42	3.72 0.10	-3355432.85 634.01	4.49 4.00
		-11.89 -14.59 71.01	18.43 111.57 70.73	4.70 3.88 3.42		

TABLE 4.9 (cont'd)

20	-1888625.12	5.74	-5354908.80	4.68	-2895771.87	5.84
	-27 10 36.34	0.20	250 34 21.87	0.20	243.01	6.00
		-44.90	-10.45	5.99		
		-29.36	113.64	5.87		
		-30.75	-136.81	4.30		
22	-6099963.34	3.93	-997362.81	4.11	-1568592.08	5.34
	-14 19 54.56	0.20	189 17 9.14	0.10	38.25	4.00
		-12.32	22.65	5.55		
		-67.42	144.33	3.93		
		-18.62	-71.57	3.82		
23	-4955380.48	3.30	3842227.47	3.01	-1163857.66	4.05
	-10 35 3.40	0.10	142 12 40.54	0.10	105.06	3.00
		7.57	22.04	4.28		
		-7.64	-66.94	3.32		
		79.21	-112.20	2.65		
31	-4313822.27	3.48	891323.89	3.84	-4597269.95	3.82
	-46 24 58.07	0.10	168 19 32.89	0.20	-0.55	3.00
		-7.65	-114.35	4.11		
		-38.94	-18.12	3.83		
		-50.02	146.43	3.14		
32	-2375426.16	3.42	4875525.40	3.17	-3345427.38	3.85
	-31 50 25.99	0.10	115 58 33.64	0.10	-11.74	3.00
		-25.40	-1.63	3.86		
		1.62	-90.86	3.51		
		64.54	2.54	3.06		
38	-2160989.14	2.93	-5642722.80	3.11	2035359.96	4.42
	18 43 57.87	0.10	249 2 40.91	0.10	-4.39	3.00
		5.13	-4.70	4.50		
		-29.92	-91.74	3.13		
		-59.55	76.51	2.79		
39	-3724776.32	6.46	-4421242.58	5.79	-2686101.58	5.80
	-25 4 6.73	0.20	229 53 12.39	0.20	333.17	6.00
		-38.14	85.14	6.59		
		-44.77	-56.04	6.32		
		20.49	12.20	5.04		

TABLE 4.9 (cont'd)

40	-741987.44	4.72	6190769.42	3.82	-1338543.07	4.25
	-12 11 43.99	0.10	96 50 4.26	0.20	-71.86	4.00
		-3.87	-70.61	4.82		
		-33.96	22.00	4.25		
		55.76	13.69	3.70		
42	4900740.47	3.41	3968235.05	3.28	966327.34	3.91
	8 46 12.53	0.10	38 59 52.21	0.10	1831.42	3.00
		1.99	0.76	3.92		
		14.40	-89.75	3.55		
		75.46	98.46	3.11		
43	1371359.19	4.33	-3614769.69	4.27	-5055956.83	5.03
	-52 46 52.79	0.20	290 46 32.07	0.20	102.35	4.00
		-18.57	12.98	5.58		
		-3.68	104.22	4.15		
		-71.04	-155.00	3.74		
44	1098883.63	6.97	3684590.18	6.39	-5071884.22	7.91
	-53 1 10.44	0.30	73 23 36.37	0.40	21.10	6.00
		-24.14	19.82	8.53		
		-16.25	-77.69	7.22		
		60.30	-18.42	5.20		
45	3223422.80	3.50	5045320.34	3.44	-2191806.21	4.19
	-20 13 53.57	0.10	57 25 32.71	0.10	97.37	3.00
		-19.36	6.71	4.19		
		-10.26	-86.94	3.68		
		67.89	-23.40	3.23		
47	-3361970.34	4.03	5365784.20	4.14	763610.75	5.70
	6 55 20.21	0.20	122 4 10.18	0.10	51.32	4.00
		12.76	-9.10	5.75		
		67.05	-131.43	4.08		
		-18.74	-94.69	4.02		
50	1192663.94	5.52	-2451027.47	6.32	-5747058.42	6.21
	-64 46 26.25	0.30	295 56 50.79	0.40	31.64	5.00
		-14.50	6.16	8.25		
		1.28	95.82	4.74		
		-75.44	-179.11	4.30		

TABLE 4.9 (cont'd)

51	1111322.72	4.91	2169248.74	3.64	-5874342.42	4.50
	-67 36 5.87	0.10	62 52 24.92	0.40	22.51	4.00
		-18.71	-49.50	5.15		
		-48.31	62.85	4.28		
		35.63	26.45	3.58		
52	-902618.85	4.43	2409507.49	3.88	-5816559.31	5.66
	-66 16 45.48	0.10	110 32 10.72	0.30	-2.56	6.00
		-77.15	10.98	5.72		
		-10.30	-131.77	4.55		
		7.61	-43.16	3.64		
53	-1310861.25	4.62	311248.58	4.48	-6213282.87	4.35
	-77 50 40.92	0.20	166 38 35.27	0.70	-43.76	4.00
		21.66	157.28	4.97		
		-14.62	-118.66	4.38		
		-63.44	119.88	4.05		
55	6118337.62	3.74	-1571755.41	3.79	-878618.02	5.03
	- 7 58 15.71	0.20	345 35 33.65	0.10	61.27	4.00
		-10.27	6.49	5.05		
		30.27	90.41	4.20		
		57.68	-66.86	3.25		
59	-5885340.45	3.78	-2448374.79	3.91	221663.01	5.18
	2 0 18.14	0.20	202 35 16.50	0.10	29.46	4.00
		13.53	12.50	5.30		
		-21.92	96.94	3.88		
		-63.86	-48.13	3.65		
60	-4751643.60	3.32	2792039.78	3.20	-3200168.80	3.59
	-30 18 34.49	0.10	149 33 42.26	0.10	222.73	3.00
		1.85	-144.71	3.86		
		-16.40	-55.26	3.41		
		73.49	-48.45	2.76		
61	2999903.86	4.22	-2219381.41	5.82	-5155273.81	5.47
	-54 17 1.69	0.20	323 30 19.14	0.30	14.33	5.00
		11.30	125.67	6.20		
		-38.18	44.71	5.33		
		49.58	22.10	3.84		

TABLE 4.9 (cont'd)

63	5884470.82	2.81	-1853501.10	2.89	1612845.73	4.26
	14 44 42.10	0.10	342 30 59.48	0.10	31.74	3.00
		10.96	7.69	4.29		
		22.57	102.31	3.01		
		64.64	-106.42	2.64		
64	6023384.69	3.06	1617924.69	2.86	1331723.00	3.75
	12 7 54.57	0.10	15 2 6.62	0.10	268.14	3.00
		-2.06	-3.74	3.80		
		87.73	-28.74	3.04		
		0.96	86.22	2.83		
65	4213562.18	2.27	820825.27	2.92	4702776.01	2.59
	47 48 4.39	0.10	11 1 24.51	0.10	951.16	2.00
		-6.17	137.19	3.15		
		9.54	48.23	2.61		
		78.61	-165.24	1.91		
66	-5858561.01	3.59	1394452.19	4.26	2093818.81	4.76
	19 17 28.76	0.10	166 36 41.78	0.10	-0.22	4.00
		26.04	5.32	4.78		
		22.64	-96.44	4.28		
		-54.28	-41.89	3.54		
67	5186402.25	4.23	-3653954.36	4.53	-654298.51	5.17
	- 5 55 39.35	0.20	324 50 3.54	0.20	22.05	4.00
		-4.55	42.02	6.04		
		37.04	128.57	3.87		
		-52.59	137.99	3.69		
68	5084823.85	3.76	2670326.47	3.22	-2768097.01	4.80
	-25 52 59.76	0.10	27 42 23.35	0.10	1505.47	4.00
		-24.83	-1.85	4.80		
		49.46	-59.09	3.76		
		29.74	72.82	3.22		
69	4978421.73	6.79	-1086880.87	7.05	-3823193.56	8.14
	-37 3 54.41	0.30	347 41 4.26	0.30	35.49	6.00
		-9.96	11.44	8.60		
		16.74	98.41	7.27		
		70.37	-69.07	5.94		

TABLE 4.9 (cont'd)

72	-941704.81	6.14	5967431.33	4.30	2039298.76	4.54
	18 46 10.55	0.10	98 58 3.88	0.20	231.28	5.00
		-1.22	-75.34	6.22		
		60.57	12.49	4.79		
		29.40	-164.65	3.90		
73	1905126.73	3.68	6032265.39	3.92	-810735.14	4.42
	- 7 21 6.86	0.10	72 28 21.71	0.10	-110.23	4.00
		-12.79	-14.39	4.46		
		39.76	64.72	4.02		
		47.40	-90.09	3.53		
75	3602810.47	4.12	5238225.36	3.94	-515950.02	4.34
	- 4 40 14.81	0.10	55 28 48.40	0.10	500.54	4.00
		-22.87	-20.91	4.47		
		38.06	-91.62	4.12		
		43.22	45.74	3.79		
78	-5952297.68	9.77	1231893.65	8.36	-1925980.18	13.39
	-17 41 31.77	0.50	168 18 25.52	0.30	74.31	7.00
		16.26	-9.82	15.90		
		-14.63	-95.45	7.69		
		-67.85	34.42	5.71		
111	-2448861.73	3.04	-4667995.39	2.75	3582750.92	3.19
	34 22 53.97	0.10	242 19 5.50	0.10	2259.50	2.00
		8.79	0.24	3.37		
		5.56	91.10	3.27		
		-79.58	33.06	2.22		
123	-1881805.73	5.30	-812443.30	4.94	6019581.22	4.98
	71 18 47.57	0.20	203 21 5.75	0.60	-2.49	5.00
		2.04	57.43	5.94		
		48.69	-34.89	5.12		
		-41.24	-30.78	3.97		
134	-2448915.48	3.04	-4668085.46	2.75	3582445.59	3.20
	34 22 43.88	0.10	242 19 5.28	0.10	2173.52	2.00
		8.55	-1.87	3.38		
		5.87	89.01	3.27		
		-79.61	33.10	2.22		

TABLE 4.10

STATION TO STATION CORRELATION COEFFICIENTS $\rho_{ij} > 0.75$
(SOLUTION BC-D6)

STA.NO. 11	WITH STA.NO.	59	* STA.NO. 12	WITH STA.NO.	66
0.627	-0.118	-0.092	0.994	-0.100	-0.162
-0.055	0.834	-0.145	-0.100	0.995	-0.029
-0.052	-0.299	0.317	-0.162	-0.029	0.996
STA.NO. 16	WITH STA.NO.	65	STA.NO. 19	WITH STA.NO.	43
0.732	0.023	-0.274	0.775	0.129	0.034
0.015	0.856	-0.192	0.178	0.607	0.089
-0.289	-0.184	0.697	0.066	0.111	0.563
STA.NO. 23	WITH STA.NO.	60	STA.NO. 31	WITH STA.NO.	60
0.836	0.251	-0.132	0.848	0.259	-0.088
0.182	0.788	-0.017	0.342	0.557	-0.171
-0.226	-0.173	0.641	-0.021	-0.023	0.608
STA.NO. 38	WITH STA.NO.	111	STA.NO. 38	WITH STA.NO.	134
0.823	-0.113	0.148	0.829	-0.112	0.147
-0.042	0.434	-0.040	-0.040	0.436	-0.041
0.163	-0.097	0.383	0.165	-0.099	0.384
STA.NO. 50	WITH STA.NO.	61	* STA.NO. 111	WITH STA.NO.	134
0.275	-0.371	0.097	0.991	-0.208	0.089
0.108	0.798	-0.059	-0.207	0.989	0.136
-0.077	-0.377	0.275	0.090	0.135	0.992

* $\rho_{ij} > 0.95$

TABLE 4.11

STATION TO STATION CORRELATION COEFFICIENTS $\rho_{ij} > 0.75$
(SOLUTION BC-D3)

STA.NO. 11	WITH STA.NO.	59	* STA.NO. 12	WITH STA.NO.	66
0.607	-0.207	-0.033	0.997	-0.038	-0.043
-0.132	0.846	-0.180	-0.038	0.998	-0.088
0.013	-0.312	0.346	-0.043	-0.089	0.998
STA.NO. 16	WITH STA.NO.	65	STA.NO. 19	WITH STA.NO.	43
0.748	0.034	-0.397	0.764	0.148	0.091
0.036	0.865	-0.236	0.219	0.565	0.012
-0.473	-0.239	0.720	0.075	0.062	0.492
STA.NO. 23	WITH STA.NO.	60	STA.NO. 31	WITH STA.NO.	60
0.816	0.309	-0.217	0.835	0.289	-0.179
0.266	0.775	0.034	0.363	0.520	-0.184
-0.277	-0.097	0.696	-0.128	-0.019	0.572
STA.NO. 50	WITH STA.NO.	61	* STA.NO. 111	WITH STA.NO.	134
0.220	-0.376	0.136	0.997	-0.240	0.172
0.158	0.825	-0.122	-0.241	0.995	0.237
-0.074	-0.433	0.279	0.173	0.237	0.996

* $\rho_{ij} > 0.95$

subtract one from the other, and then test to see if the resulting matrix is positive-definite. It is possible to use this test to make comparisons between all possible combinations of the different adjustments listed in Table 4.8. However, the only comparison made was that between the critical solutions BC-D3 and BC-D6. The constraints used in these two solutions were identical, but BC-D3 used all observations whereas BC-D6 used only observations to images 1-3-5-7 for the highly correlated events. The test indicated that BC-D6 is preferred to BC-D3. If this test is valid, it means that the addition of the highly correlated observations actually caused the adjustment to deteriorate.

Table 4.12 shows the average standard deviation of the coordinates

TABLE 4.12

AVERAGE STANDARD DEVIATIONS
(METERS)

Solution	σ	σ_H
BC-D1	5.6	6.1
BC-D2	6.4	8.3
BC-D3	4.6	3.9
BC-D4	4.6	4.0
BC-D6	4.3	3.9
BC-D11	4.6	3.9
BC-D13	3.5	3.3

and heights for the solutions. The average standard deviation was computed as follows:

$$\sigma = \frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{3} .$$

The results of solution BC-D13, the adjustment using uncorrelated observations, are significantly lower than any of the other adjustments. It can be seen that the addition of weighted heights decreases the standard deviation significantly, whereas, the omission of the geometric scalers has almost no effect. Also, the addition of highly correlated observations (BC-D3) makes the results somewhat worse (compare with BC-D6).

4.2.2 Comparisons with Geometric Information

The geometric information available for a comparison of the results was the eight measured baselines and the station heights. To repeat the differences in the solutions: BC-D3 and BC-D6 had constraints on chords and heights (with additive term), BC-D2 had chord constraints and no heights, BC-D4 had height constraints (with additive term) but no chords, and BC-D11 was identical to BC-D3 but the height constraints were without the additive term. The BC-D13 solution had the same constraints as BC-D3 and BC-D6 but the correlation between observations was neglected.

4.2.2.1 Chord Comparisons

Table 4.13 shows the differences between the adjusted and measured chord lengths for each of the solutions (see Table 4.3 for the measured chord lengths).

The results indicate that the sum of the differences in solution BC-D2 (scalars constrained) is even greater than those in BC-D4 (height

TABLE 4.13

CHORD LENGTH COMPARISONS

Line	Adjusted Minus Given Length											
	BC-D2		BC-D3		BC-D4		BC-D6		BC-D11		BC-D13	
	m	ppM	m	ppM	m	ppM	m	ppM	m	ppM	m	ppM
2-3*	0.5±2.9	0.14	5.9±2.6	1.69	15.5±4.1	4.44	5.3±2.6	1.53	9.4±3.2	2.69	6.2±2.4	1.77
3-111	0.9±1.5	0.65	1.5±1.5	1.04	13.0±4.1	9.09	2.0±1.4	1.40	1.8±1.5	1.28	2.8±2.7	2.00
6-16*	- 0.1±2.4	0.03	2.8±2.2	0.78	6.4±4.2	1.81	1.6±2.2	0.44	5.0±2.2	1.42	2.4±3.9	0.67
6-65	3.4±2.3	1.37	6.0±2.1	2.45	20.2±3.9	8.20	5.4±2.1	2.22	8.2±2.1	3.32	6.4±3.6	2.61
16-65	- 2.2±1.3	1.88	- 1.7±1.3	1.44	-14.2±3.7	11.90	- 2.2±1.2	1.84	- 1.4±1.3	1.19	- 2.4±4.2	2.04
63-64*	3.7±3.3	1.05	7.7±2.9	2.21	14.5±4.1	4.15	8.6±2.9	2.47	11.5±2.9	3.32	10.5±3.4	0.30
23-60*	- 6.5±3.4	2.83	- 0.9±3.1	0.40	- 1.7±4.1	0.73	0.2±3.1	0.08	2.1±3.1	0.93	- 0.3±3.7	0.15
32-60*	-25.7±4.8	8.13	-12.2±3.9	3.86	-12.0±4.0	3.80	-14.9±3.9	4.72	- 5.6±3.9	1.86	-17.2±2.4	5.44
Sum**	-28.1	1.76	3.3	0.21	22.7	1.42	0.8	0.05	22.4	1.41	1.6	0.1
Average**		2.43		1.79		2.99		1.85		2.05		1.67

**Computed from independent long line only (marked by *).

constraints only). This probably is due, again, to the erroneous measured value of line 32-60. Leaving this line out of the comparison, the sum in BC-D2 decreases to -2.4m (.19ppM), while in BC-D4 increases to 34.7m (2.71ppM), which makes more sense, and serves as another indication of the problematic nature of line 32-60. The superiority of solution BC-D6 over BC-D3 is again evidenced, though not very significantly. The same can be said for the uncorrelated solution BC-D13 over BC-D6. The results of BC-D11 indicate that if the small differences between the adjusted and measured lengths are to be maintained, the inclusion of the additive term is justified.

In solution BC-D4 the large differences found at the short lines (3-111, 6-65 or 16-65) are due to the fact that with a high satellite like PAGEOS their lengths can not be determined accurately. For the same reason, the inclusion of their measured lengths as constraints to determine accurately the positions at Stations 111 and 65 is a must, though their effect on the global scale is negligable.

4.2.2.2 Comparison of the Geometric Solutions

Another method of making a comparison of results is through coordinate transformations and residual analysis. This is a least squares adjustment where the observations are the coordinate difference of the adjusted station coordinates from two different solutions. The parameters are the three translations, three rotation angles, and the scale difference. The method of computing these parameters is described in [Kumar, 1972]. For this investigation, solution BC-D6 is referred to as the standard solution and a transformation was performed between this and the other solutions. The results are shown in Tables 4.14 thru 4.18

where the parameters DX, DY and DZ are the three translations (in meters), DELTA is the scale difference (in parts per million), and OMEGA, PSI and EPSILON are the three rotation angles in seconds of arc about the Z, Y and X axes respectively. The positive rotations are counter-clockwise when viewed from the positive end of the respective axes towards the origin. The units in the variance-covariance matrix for the elements corresponding to the rotations are radians, squared.

Table 4.14 contains the results of the transformation between BC-D6 and BC-D2. The effect of the missing heights in BC-D2 is very noticable in the difference in scale. It is also noticable in some of the residuals (e.g., Stations 44, 123) where the improvement coming from the heights is in evidence.

Table 4.15 contains the results of the transformation between BC-D6 and BC-D3. The systematic differences between the two solutions appear to be insignificant. An examination of the residual differences, however, shows some approaching the 2σ value (e.g., Station 123). The addition of the highly correlated observations in solution BC-D3 seems to have caused a possibly significant change in some of the station coordinates.

The results given in Table 4.16 between BC-D6 and BC-D4 show the scale difference to be insignificant even though there were no baselines constrained in BC-D4. The residuals are small and within the noise level.

Table 4.17 contains the results of the transformation between solutions BC-D6 and BC-D11. The elimination of the additive form causes a noticable difference in scale. The residuals are smaller than those in the transformation between BC-D6 and BC-D4.

TABLE 4.14

TRANSFORMATION: BC-D6 - BC-D2

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-0.75	0.04	-0.15	-3.42	-0.08	-0.02	0.08

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 0.52

0.449D+00	0.674D-04	-0.203D-03	-0.113D-09	0.943D-10	0.677D-09	0.102D-10
0.674D-04	0.420D+00	-0.184D-03	-0.256D-10	0.445D-09	-0.436D-10	-0.518D-09
-0.203D-03	-0.184D-03	0.639D+00	-0.159D-09	0.338D-10	-0.329D-09	0.349D-09
-0.113D-09	-0.256D-10	-0.159D-09	0.323D-15	-0.149D-18	-0.265D-18	0.660D-18
0.943D-10	0.445D-09	0.336D-10	-0.149D-18	0.670D-15	-0.579D-16	0.273D-16
0.677D-09	-0.436D-10	-0.329D-09	-0.265D-18	-0.579D-16	0.853D-15	0.581D-17
0.102D-10	-0.518D-09	0.349D-09	0.660D-18	0.273D-16	0.581D-17	0.852D-15

COEFFICIENTS OF CORRELATION

0.100D+01	0.155D-03	-0.379D-03	-0.941D-02	0.544D-02	0.346D-01	0.521D-03
0.155D-03	0.100D+01	-0.355D-03	-0.220D-02	0.265D-01	-0.230D-02	-0.274D-01
-0.379D-03	-0.355D-03	0.100D+01	-0.111D-01	0.164D-02	-0.141D-01	0.150D-01
-0.941D-02	-0.220D-02	-0.111D-01	0.100D+01	-0.321D-03	-0.505D-03	0.126D-02
0.544D-02	0.265D-01	0.164D-02	-0.321D-03	0.100D+01	-0.766D-01	0.361D-01
0.346D-01	-0.230D-02	-0.141D-01	-0.505D-03	-0.766D-01	0.100D+01	0.681D-02
0.521D-03	-0.274D-01	0.150D-01	0.126D-02	0.361D-01	0.681D-02	0.100D+01

TABLE 4.14 (cont'd)

RESIDUALS V

	V1(8C4-D6)				V2(8C4-D2)			V1 - V2		
1	-0.5	1.3	1.3	1	0.6	-1.9	-4.2	-1.1	3.2	5.5
2	0.4	-0.1	-0.7	2	-0.6	0.5	2.7	1.0	-0.7	-3.4
3	-2.0	0.4	0.0	3	3.0	-1.2	-0.0	-5.0	1.6	0.1
4	-3.1	3.6	-0.1	4	5.8	-5.6	0.4	-9.0	9.3	-0.6
6	-0.8	0.2	0.7	6	1.9	-0.3	-2.6	-2.7	0.5	3.2
7	1.9	-1.7	1.8	7	-5.7	2.6	-5.2	7.6	-4.3	7.0
8	-2.6	1.4	-0.7	8	5.4	-5.5	0.8	-8.0	7.0	-1.4
9	-2.3	1.4	0.1	9	2.8	-5.4	-0.1	-5.1	6.8	0.2
11	-1.6	-3.4	1.9	11	4.5	5.0	-3.3	-6.1	-8.4	5.2
12	1.5	-4.7	0.3	12	-6.3	7.0	-0.6	7.8	-11.7	1.0
13	3.6	-0.3	-4.3	13	-7.1	0.7	11.3	10.7	-1.0	-15.6
15	-0.4	1.2	-1.0	15	0.8	-2.9	2.9	-1.2	4.1	-3.9
16	0.9	1.7	-0.6	16	-4.0	-2.1	2.3	4.9	3.9	-2.9
19	-1.6	1.1	-0.9	19	2.3	-3.3	2.0	-3.9	4.5	-3.0
20	-2.7	0.3	0.7	20	3.7	-1.0	-1.1	-6.4	1.3	1.8
22	0.1	-2.5	3.8	22	-0.4	2.9	-4.9	0.6	-5.3	8.7
23	2.2	-0.9	0.3	23	-7.9	2.5	-0.4	10.1	-3.4	0.7
31	2.2	1.4	0.4	31	-6.1	-1.6	-1.2	8.3	2.9	1.6
32	2.6	-0.2	-2.7	32	-4.5	0.5	5.5	7.2	-0.7	-8.2
38	-2.5	-0.1	-1.2	38	4.3	0.3	2.1	-6.7	-0.3	-3.2
39	0.7	0.4	2.1	39	-1.3	-0.9	-3.9	2.1	1.3	6.0
40	2.4	0.9	-0.1	40	-2.6	-2.8	0.1	5.2	3.6	-0.2
42	-0.3	0.3	0.3	42	0.9	-0.5	-0.4	-1.3	0.8	0.8
43	-1.3	1.8	-0.7	43	1.5	-3.4	2.0	-2.8	5.2	-2.7
44	-1.6	0.7	-3.4	44	1.7	-0.9	8.1	-3.3	1.6	-11.4
45	-0.6	0.6	2.0	45	1.2	-1.7	-3.1	-1.9	2.3	5.1
47	2.4	-0.8	0.9	47	-4.5	2.2	-1.1	6.9	-3.0	2.0
50	-1.1	1.4	-0.6	50	1.2	-1.7	1.8	-2.3	3.1	-2.4
51	-1.6	0.2	0.2	51	1.8	-0.3	-0.9	-3.5	0.5	1.1
52	-0.7	-0.2	0.7	52	0.8	0.3	-1.7	-1.5	-0.6	2.4
53	-0.1	1.5	0.6	53	0.1	-1.6	-2.4	-0.1	3.1	3.0
55	0.9	-0.3	-2.4	55	-3.0	0.4	2.8	3.9	-0.7	-5.1
59	-0.4	-2.3	2.4	59	1.4	3.5	-3.1	-1.7	-5.8	5.5
60	2.3	-1.0	-0.3	60	-7.6	2.0	0.5	9.8	-2.9	-0.8
61	-0.8	1.0	-1.5	61	1.2	-1.2	4.2	-1.9	2.2	-5.8
63	0.9	-1.2	0.2	63	-4.3	1.5	-0.3	5.2	-2.7	0.5
64	0.6	1.6	-0.6	64	-2.4	-2.2	0.8	3.0	4.1	-1.4
65	0.5	1.4	0.1	65	-2.2	-1.7	-0.3	2.7	3.1	0.4
66	1.5	-4.7	0.3	66	-6.3	7.0	-0.6	7.8	-11.7	1.0
67	0.6	-1.2	-2.1	67	-1.4	1.8	2.3	2.0	-3.0	-4.4
68	-1.3	0.5	2.2	68	3.9	-0.7	-3.7	-5.2	1.2	6.0
69	0.6	1.0	-3.2	69	-1.1	-1.1	5.6	1.7	2.1	-8.8
72	-2.3	-3.0	-2.2	72	3.2	10.7	4.1	-5.5	-13.7	-6.3
73	0.1	0.8	0.7	73	-0.1	-2.5	-0.8	0.1	3.3	1.5
75	-1.1	0.4	0.6	75	2.0	-0.9	-0.7	-3.1	1.3	1.3
78	2.0	0.9	-0.0	78	-9.9	-1.2	0.1	11.9	2.1	-0.1
111	-1.4	-0.8	-0.2	111	2.6	3.0	0.6	-4.1	-3.8	-0.8
123	1.2	1.5	1.7	123	-8.2	-7.9	-23.5	9.4	9.4	25.2
134	-1.4	-0.8	-0.2	134	2.6	3.0	0.6	-4.0	-3.7	-0.7

TABLE 4.15

TRANSFORMATION: BC-D6 - BC-D3

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
0.22	-0.19	0.02	-0.01	-0.09	-0.02	0.08

VARIANCE - COVARIANCE MATRIX

$\sigma_0^2 =$ 0.17

0.935D-01	-0.112D-04	-0.803D-04	-0.111D-10	-0.181D-10	0.188D-09	0.274D-11
-0.112D-04	0.932D-01	-0.110D-04	0.242D-11	0.628D-10	-0.785D-11	-0.133D-09
-0.803D-04	-0.110D-04	0.128D+00	-0.173D-10	0.692D-11	-0.926D-10	0.169D-10
-0.111D-10	0.242D-11	-0.173D-10	0.188D-16	0.502D-20	0.111D-19	0.629D-19
-0.181D-10	0.628D-10	0.692D-11	0.502D-20	0.154D-15	-0.137D-16	0.569D-17
0.188D-09	-0.785D-11	-0.926D-10	0.111D-19	-0.132D-16	0.192D-15	0.306D-17
0.274D-11	-0.133D-09	0.169D-10	0.629D-19	0.569D-17	0.306D-17	0.194D-15

COEFFICIENTS OF CORRELATION

0.100D+01	-0.120D-03	-0.733D-03	-0.835D-02	-0.476D-02	0.442D-01	0.645D-03
-0.120D-03	0.100D+01	-0.101D-03	0.183D-02	0.166D-01	-0.185D-02	-0.312D-01
-0.733D-03	-0.101D-03	0.100D+01	-0.111D-01	0.156D-02	-0.187D-01	0.338D-02
-0.835D-02	0.183D-02	-0.111D-01	0.100D+01	0.932D-04	0.185D-03	0.104D-02
-0.476D-02	0.166D-01	0.156D-02	0.932D-04	0.100D+01	-0.767D-01	0.329D-01
0.442D-01	-0.185D-02	-0.187D-01	0.185D-03	-0.767D-01	0.100D+01	0.158D-01
0.645D-03	-0.312D-01	0.338D-02	0.104D-02	0.329D-01	0.158D-01	0.100D+01

TABLE 4.15 (cont'd)

RESIDUALS V

	V1(BC4-D6)				V2(BC4-D3)			V1 - V2		
1	0.3	1.4	0.2	1	-0.4	-1.5	-0.2	0.7	2.9	0.4
2	-0.5	-0.5	-0.4	2	0.5	0.5	0.5	-1.0	-0.9	-0.9
3	-0.6	-0.3	0.2	3	0.6	0.3	-0.3	-1.2	-0.6	0.5
4	-3.7	4.2	-2.8	4	4.8	-5.9	4.4	-8.5	10.0	-7.3
6	0.1	0.2	-0.8	6	-0.1	-0.2	0.8	0.2	0.3	-1.5
7	0.5	0.4	0.2	7	-0.5	-0.4	-0.2	1.0	0.7	0.3
8	-1.2	-0.6	0.7	8	1.2	0.6	-0.7	-2.3	-1.3	1.4
9	-1.3	-0.7	0.8	9	1.3	0.8	-0.9	-2.6	-1.5	1.7
11	-0.7	-2.0	-0.4	11	0.7	2.1	0.5	-1.4	-4.1	-0.9
12	-0.3	-3.0	0.8	12	0.3	3.7	-0.9	-0.6	-6.6	1.7
13	3.0	1.7	-3.1	13	-3.4	-2.3	5.0	6.4	4.1	-8.0
15	-0.3	0.4	-0.3	15	0.3	-0.4	0.3	-0.6	0.8	-0.6
16	0.9	0.4	-0.6	16	-0.9	-0.5	0.6	1.8	0.9	-1.2
19	-1.1	0.8	-0.9	19	1.2	-0.8	0.9	-2.3	1.6	-1.9
20	-1.6	-0.1	-0.1	20	1.7	0.1	0.1	-3.3	-0.2	-0.2
22	-0.3	-1.7	2.8	22	0.3	1.8	-2.9	-0.6	-3.6	5.7
23	0.6	-0.7	-0.7	23	-0.6	0.7	0.7	1.1	-1.4	-1.4
31	0.5	0.7	-0.1	31	-0.5	-0.7	0.1	1.1	1.5	-0.2
32	2.7	-0.7	-2.1	32	-2.6	0.7	2.2	5.5	-1.5	-4.3
38	-1.9	-0.7	-1.2	38	2.4	0.8	1.4	-4.3	-1.5	-2.5
39	-0.7	-0.4	0.3	39	0.7	0.4	-0.3	-1.5	-0.9	0.6
40	1.9	1.9	0.5	40	-2.0	-1.9	-0.5	3.9	3.7	1.0
42	0.3	-0.1	0.9	42	-0.3	0.1	-0.9	0.6	-0.2	1.8
43	-0.7	1.3	-0.5	43	0.7	-1.3	0.5	-1.4	2.5	-1.0
44	-1.2	-0.6	-0.4	44	1.3	0.6	0.4	-2.5	-1.2	-0.8
45	0.1	1.0	2.1	45	-0.1	-1.0	-2.4	0.2	2.0	4.5
47	2.3	-0.4	2.6	47	-2.5	0.4	-2.9	4.8	-0.8	5.5
50	-0.8	0.6	-0.4	50	0.8	-0.6	0.4	-1.6	1.2	-0.8
51	-1.3	-0.5	0.1	51	1.3	0.5	-0.1	-2.5	-1.1	0.1
52	-1.2	-0.5	0.1	52	1.2	0.5	-0.1	-2.4	-1.0	0.2
53	-0.7	0.3	-0.0	53	0.7	-0.3	0.0	-1.4	0.6	-0.1
55	-0.0	0.7	-0.0	55	0.0	-0.7	0.0	-0.1	1.4	-0.1
59	-0.2	-1.1	0.4	59	0.3	1.2	-0.5	-0.5	-2.4	0.9
60	0.8	-0.4	-0.2	60	-0.8	0.4	0.2	1.6	-0.8	-0.4
61	-0.3	0.3	-0.6	61	0.3	-0.3	0.6	-0.7	0.5	-1.2
63	-0.2	0.4	0.6	63	0.2	-0.4	-0.6	-0.5	0.7	1.2
64	-0.2	-0.1	0.4	64	0.2	0.1	-0.4	-0.4	-0.2	0.9
65	0.7	0.9	-0.1	65	-0.8	-0.9	0.1	1.5	1.8	-0.2
66	-0.3	-3.0	0.8	66	0.3	3.7	-0.9	-0.6	-6.6	1.7
67	-0.3	0.6	-0.0	67	0.3	-0.6	0.0	-0.7	1.3	-0.0
68	0.2	0.3	1.3	68	-0.2	-0.3	-1.3	0.4	0.5	2.6
69	0.1	1.7	0.0	69	-0.1	-1.6	-0.0	0.1	3.3	0.0
72	-3.2	-2.8	0.2	72	4.1	3.4	-0.3	-7.2	-6.2	0.5
73	0.5	1.2	1.0	73	-0.5	-1.2	-1.0	0.9	2.4	2.0
75	-0.6	0.2	1.0	75	0.6	-0.2	-1.0	-1.2	0.4	2.0
78	1.1	0.6	-1.1	78	-1.1	-0.7	1.1	2.3	1.3	-2.3
111	0.4	0.0	0.0	111	-0.6	-0.0	-0.0	1.0	0.0	0.1
123	1.2	1.4	0.3	123	-7.8	-7.1	-0.7	9.0	8.4	1.0
134	0.4	0.0	0.0	134	-0.6	-0.0	-0.0	1.0	0.0	0.1

TABLE 4.16

TRANSFORMATION: BC-D6 - BC-D4

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
0.33	-0.02	-0.46	0.09	-0.08	-0.01	0.06

VARIANCE - COVARIANCE MATRIX

$\sigma_0^2 =$ 0.39

0.223D+00	-0.278D-04	-0.137D-03	-0.499D-10	-0.525D-10	0.390D-09	0.406D-11
-0.278D-04	0.222D+00	-0.424D-04	0.801D-11	0.134D-09	-0.159D-10	-0.286D-09
-0.137D-03	-0.424D-04	0.311D+00	-0.709D-10	0.158D-10	-0.205D-09	0.664D-10
-0.499D-10	0.801D-11	-0.709D-10	0.912D-16	0.221D-19	0.515D-19	0.305D-18
-0.525D-10	0.134D-09	0.158D-10	0.221D-19	0.363D-15	-0.306D-16	0.133D-16
0.390D-09	-0.159D-10	-0.205D-09	0.515D-19	-0.306D-16	0.455D-15	0.610D-17
0.406D-11	-0.286D-09	0.664D-10	0.305D-18	0.133D-16	0.610D-17	0.457D-15

COEFFICIENTS OF CORRELATION

0.100D+01	-0.125D-03	-0.521D-03	-0.111D-01	-0.584D-02	0.388D-01	0.403D-03
-0.125D-03	0.100D+01	-0.162D-03	0.178D-02	0.150D-01	-0.158D-02	-0.284D-01
-0.521D-03	-0.162D-03	0.100D+01	-0.133D-01	0.149D-02	-0.172D-01	0.557D-02
-0.111D-01	0.178D-02	-0.133D-01	0.100D+01	0.121D-03	0.253D-03	0.150D-02
-0.584D-02	0.150D-01	0.149D-02	0.121D-03	0.100D+01	-0.752D-01	0.327D-01
0.388D-01	-0.158D-02	-0.172D-01	0.253D-03	-0.752D-01	0.100D+01	0.134D-01
0.403D-03	-0.284D-01	0.557D-02	0.150D-02	0.327D-01	0.134D-01	0.100D+01

TABLE 4.16 (cont'd)

RESIDUALS V

	V1(BC4-D6)				V2(BC4-D4)			V1 - V2		
1	-0.0	2.5	0.5	1	0.0	-3.0	-0.6	-0.1	5.5	1.1
2	1.4	-0.9	-1.4	2	-2.1	1.0	1.6	3.5	-1.9	-3.0
3	-2.0	0.9	1.2	3	2.5	-1.0	-1.4	-4.5	2.0	2.6
4	-4.1	4.6	-2.7	4	5.4	-6.6	4.2	-9.5	11.2	-6.9
6	-1.0	1.0	0.6	6	1.4	-1.1	-0.7	-2.4	2.1	1.3
7	1.3	-0.1	-0.2	7	-1.4	0.1	0.3	2.7	-0.2	-0.5
8	-1.0	-0.4	0.8	8	1.0	0.4	-0.8	-1.9	-0.9	1.6
9	-0.8	-0.7	0.9	9	0.8	0.7	-1.0	-1.7	-1.4	1.9
11	-1.4	-1.8	-1.1	11	1.5	1.9	1.3	-2.8	-3.8	-2.4
12	-0.3	-3.2	0.6	12	0.3	4.0	-0.8	-0.7	-7.2	1.4
13	3.1	1.4	-2.6	13	-3.6	-1.9	4.2	6.7	3.3	-6.7
15	0.1	1.5	-0.5	15	-0.1	-1.7	0.5	0.2	3.2	-1.0
16	1.0	1.2	0.0	16	-1.1	-1.3	-0.0	2.1	2.5	0.0
19	-0.9	0.6	0.3	19	0.9	-0.6	-0.3	-1.7	1.2	0.5
20	-2.2	-0.2	1.0	20	2.2	0.2	-1.1	-4.4	-0.5	2.1
22	-0.3	-2.0	3.6	22	0.3	2.1	-3.7	-0.6	-4.1	7.3
23	0.7	-1.2	-0.3	23	-0.7	1.2	0.4	1.5	-2.4	-0.7
31	0.7	0.4	0.9	31	-0.7	-0.4	-1.0	1.4	0.7	1.9
32	2.9	-1.2	-1.3	32	-3.0	1.3	1.4	5.8	-2.5	-2.7
38	-3.0	-0.5	-2.4	38	4.0	0.6	2.8	-7.0	-1.1	-5.2
39	-0.9	-0.4	1.5	39	1.0	0.4	-1.5	-1.9	-0.8	3.0
40	2.4	1.6	1.4	40	-2.5	-1.6	-1.4	4.9	3.1	2.8
42	0.1	0.4	1.3	42	-0.1	-0.5	-1.3	0.2	0.9	2.6
43	-0.3	0.8	0.8	43	0.3	-0.8	-0.8	-0.6	1.7	1.7
44	-1.2	-0.9	0.6	44	1.2	0.8	-0.6	-2.5	-1.7	1.1
45	0.1	1.0	3.2	45	-0.1	-1.0	-3.6	0.3	2.0	6.8
47	2.5	-0.8	3.0	47	-2.7	0.8	-3.3	5.2	-1.6	6.3
50	-0.3	-0.3	1.0	50	0.3	0.3	-1.0	-0.7	-0.6	2.0
51	-1.3	-0.8	1.1	51	1.3	0.8	-1.1	-2.6	-1.5	2.2
52	-1.2	-0.9	1.2	52	1.2	0.9	-1.1	-2.3	-1.8	2.3
53	-0.7	-0.1	1.0	53	0.7	0.1	-1.0	-1.4	-0.2	2.0
55	-0.1	-0.4	-0.5	55	0.1	0.5	0.5	-0.2	-0.9	-1.0
59	-0.5	-1.1	0.8	59	0.5	1.2	-0.9	-1.0	-2.3	1.8
60	1.0	-0.7	0.7	60	-1.0	0.8	-0.7	1.9	-1.5	1.3
61	-0.3	-0.6	0.7	61	0.3	0.6	-0.7	-0.6	-1.3	1.3
63	-0.3	-0.9	0.6	63	0.3	1.2	-0.7	-0.6	-2.2	1.3
64	0.1	1.4	0.4	64	-0.1	-1.8	-0.5	0.2	3.2	0.9
65	2.0	3.4	-3.8	65	-4.3	-4.0	6.2	7.3	7.4	-10.0
66	-0.3	-3.2	0.6	66	0.3	4.0	-0.8	-0.7	-7.2	1.4
67	-0.6	-0.3	0.1	67	0.6	0.3	-0.1	-1.2	-0.6	0.2
68	-0.0	0.7	2.6	68	0.0	-0.7	-2.8	-0.1	1.4	5.4
69	-0.1	0.1	0.9	69	0.1	-0.1	-0.9	-0.2	0.3	1.8
72	-2.4	-3.0	0.5	72	3.1	3.7	-0.6	-5.5	-6.7	1.1
73	0.8	1.0	1.8	73	-0.8	-1.0	-1.9	1.6	2.0	3.8
75	-0.7	0.2	1.8	75	0.7	-0.2	-1.9	-1.3	0.4	3.7
78	1.1	0.4	0.1	78	-1.1	-0.4	-0.1	2.2	0.9	0.2
111	-1.3	-1.5	-2.8	111	1.9	2.2	4.7	-3.2	-3.7	-7.4
123	0.9	1.6	0.3	123	-5.8	-9.3	-0.6	6.7	11.1	0.9
134	-1.3	-1.5	-2.8	134	1.9	2.2	4.7	-3.2	-3.7	-7.4

TABLE 4.17

TRANSFORMATION: BC-D6 - BC-D11

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
0.36	-0.40	0.36	2.28	-0.09	-0.01	0.09

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 0.19

0.106D+00	-0.129D-04	-0.895D-04	-0.142D-10	-0.205D-10	0.212D-09	0.310D-11
-0.129D-04	0.106D+00	-0.128D-04	0.309D-11	0.711D-10	-0.889D-11	-0.150D-09
-0.895D-04	-0.128D-04	0.145D+00	-0.220D-10	0.784D-11	-0.105D-09	0.191D-10
-0.142D-10	0.309D-11	-0.220D-10	0.240D-16	0.642D-20	0.142D-19	0.804D-19
-0.205D-10	0.711D-10	0.784D-11	0.642D-20	0.175D-15	-0.150D-16	0.644D-17
0.212D-09	-0.889D-11	-0.105D-09	0.142D-19	-0.150D-16	0.218D-15	0.346D-17
0.310D-11	-0.150D-09	0.191D-10	0.804D-19	0.644D-17	0.346D-17	0.219D-15

COEFFICIENTS OF CORRELATION

0.100D+01	-0.122D-03	-0.721D-03	-0.887D-02	-0.476D-02	0.442D-01	0.644D-03
-0.122D-03	0.100D+01	-0.104D-03	0.194D-02	0.166D-01	-0.185D-02	-0.312D-01
-0.721D-03	-0.104D-03	0.100D+01	-0.118D-01	0.156D-02	-0.187D-01	0.338D-02
-0.887D-02	0.194D-02	-0.118D-01	0.100D+01	0.991D-04	0.196D-03	0.111D-02
-0.476D-02	0.166D-01	0.156D-02	0.991D-04	0.100D+01	-0.767D-01	0.329D-01
0.442D-01	-0.185D-02	-0.187D-01	0.196D-03	-0.767D-01	0.100D+01	0.158D-01
0.644D-03	-0.312D-01	0.338D-02	0.111D-02	0.329D-01	0.158D-01	0.100D+01

TABLE 4.17 (cont'd)

RESIDUALS V

	V1(BC4-D6)				V2(BC4-D11)			V1 - V2		
1	0.8	1.0	-0.3	1	-0.8	-1.1	0.4	1.6	2.1	-0.7
2	-1.5	-0.2	-0.0	2	1.6	0.2	0.0	-3.1	-0.4	-0.1
3	0.3	-0.8	-0.1	3	-0.4	0.8	0.1	0.7	-1.6	-0.1
4	-3.5	4.1	-3.0	4	4.5	-5.8	4.6	-8.1	10.0	-7.6
6	1.3	0.3	-1.9	6	-1.4	-0.4	2.0	2.6	0.7	-3.9
7	0.4	1.0	0.9	7	-0.4	-1.1	-0.9	0.7	2.1	1.8
8	-1.4	-0.8	1.0	8	1.4	0.8	-1.0	-2.8	-1.5	1.9
9	-1.7	-0.8	1.0	9	1.8	0.8	-1.0	-3.5	-1.5	2.0
11	-0.7	-1.9	-0.5	11	0.7	2.0	0.6	-1.4	-3.8	-1.1
12	-0.5	-2.9	0.3	12	0.6	3.6	-0.3	-1.1	-6.6	0.6
13	2.7	1.7	-3.2	13	-3.0	-2.3	5.2	5.7	4.1	-8.4
15	-0.0	-0.5	0.6	15	0.0	0.5	-0.6	-0.1	-1.0	1.1
16	0.3	-0.3	0.5	16	-0.3	0.3	-0.5	0.6	-0.6	1.0
19	-1.5	0.7	-1.4	19	1.5	-0.7	1.4	-3.0	1.5	-2.8
20	-1.4	-0.1	-0.7	20	1.5	0.1	0.7	-2.9	-0.1	-1.5
22	-0.5	-1.4	2.3	22	0.5	1.5	-2.4	-1.0	-2.9	4.7
23	0.4	-0.7	-1.9	23	-0.4	0.7	1.9	0.7	-1.4	-3.7
31	0.3	1.0	-0.4	31	-0.3	-1.0	0.4	0.6	2.1	-0.8
32	2.2	-0.7	-2.4	32	-2.3	0.7	2.5	4.5	-1.4	-4.9
38	-1.3	-0.7	-0.6	38	1.6	0.9	0.9	-2.9	-1.6	-1.7
39	-0.7	-0.4	-0.4	39	0.8	0.4	0.4	-1.5	-0.8	-0.8
40	1.4	2.0	0.2	40	-1.5	-2.0	-0.2	2.9	3.9	0.4
42	0.5	-0.6	1.2	42	-0.5	0.6	-1.2	1.0	-1.3	2.5
43	-1.1	1.4	-1.1	43	1.1	-1.4	1.1	-2.2	2.7	-2.2
44	-1.5	-0.7	-1.2	44	1.6	0.6	1.1	-3.1	-1.3	-2.3
45	0.3	0.9	1.8	45	-0.3	-0.9	-1.9	0.5	1.7	3.7
47	2.0	-0.3	2.2	47	-2.1	0.3	-2.4	4.1	-0.6	4.5
50	-1.3	1.0	-1.2	50	1.3	-1.0	1.2	-2.6	2.0	-2.3
51	-1.5	-0.5	-0.5	51	1.5	0.5	0.5	-3.1	-1.0	-1.1
52	-1.6	-0.3	-0.5	52	1.6	0.3	0.5	-3.2	-0.7	-1.0
53	-1.0	0.5	-0.6	53	1.0	-0.5	0.6	-2.0	1.1	-1.1
55	-0.2	1.3	1.0	55	0.2	-1.4	-1.1	-0.3	2.7	2.1
59	-0.4	-1.0	0.1	59	0.4	1.0	-0.1	-0.9	-2.0	0.1
60	0.5	0.1	-0.4	60	-0.5	-0.1	0.4	1.1	0.1	-0.7
61	-0.6	0.6	-1.3	61	0.6	-0.6	1.3	-1.1	1.3	-2.5
63	-0.5	1.3	1.6	63	0.5	-1.3	-1.6	-1.0	2.6	3.2
64	-0.6	-1.3	1.3	64	0.6	1.3	-1.3	-1.3	-2.5	2.5
65	0.7	0.6	-0.0	65	-0.7	-0.6	0.0	1.4	1.2	-0.1
66	-0.5	-2.9	0.3	66	0.6	3.6	-0.3	-1.1	-6.6	0.6
67	-0.3	1.2	0.6	67	0.3	-1.2	-0.6	-0.6	2.4	1.2
68	0.4	-0.1	0.9	68	-0.4	0.1	-0.9	0.8	-0.2	1.8
69	0.2	2.6	-0.2	69	-0.2	-2.5	0.2	0.4	5.1	-0.4
72	-3.7	-2.7	0.2	72	4.7	3.3	-0.3	-8.4	-5.9	0.5
73	0.4	1.3	0.8	73	-0.5	-1.3	-0.8	0.9	2.5	1.6
75	-0.3	0.1	0.8	75	0.3	-0.1	-0.9	-0.5	0.3	1.7
78	1.1	0.8	-2.4	78	-1.1	-0.8	2.4	2.3	1.6	-4.7
111	1.2	0.3	0.9	111	-1.7	-0.3	-1.0	2.9	0.7	1.9
123	1.3	1.2	0.3	123	-8.6	-6.5	-0.7	9.9	7.7	1.0
134	1.2	0.3	0.9	134	-1.7	-0.4	-1.0	2.9	0.7	1.9

TABLE 4.18

TRANSFORMATION: BC-D6 - BC-D13

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
0.17	0.05	-0.50	0.01	0.02	0.05	-0.02

VARIANCE - COVARIANCE MATRIX

$\sigma_0^2 =$ 0.15

0.659D-01	-0.584D-05	-0.468D-04	-0.626D-11	-0.106D-10	0.133D-09	0.199D-11
-0.584D-05	0.653D-01	-0.146D-04	0.115D-11	0.352D-10	-0.478D-11	-0.988D-10
-0.468D-04	-0.146D-04	0.888D-01	-0.118D-10	0.414D-11	-0.516D-10	0.197D-10
-0.626D-11	0.115D-11	-0.118D-10	0.128D-16	0.565D-21	0.368D-20	0.387D-19
-0.106D-10	0.352D-10	0.414D-11	0.565D-21	0.106D-15	-0.923D-17	0.297D-17
0.133D-09	-0.478D-11	-0.516D-10	0.368D-20	-0.923D-17	0.130D-15	0.201D-17
0.199D-11	-0.988D-10	0.197D-10	0.387D-19	0.297D-17	0.201D-17	0.132D-15

COEFFICIENTS OF CORRELATION

0.100D+01	-0.893D-04	-0.613D-03	-0.685D-02	-0.403D-02	0.454D-01	0.678D-03
-0.893D-04	0.100D+01	-0.192D-03	0.126D-02	0.134D-01	-0.164D-02	-0.337D-01
-0.613D-03	-0.192D-03	0.100D+01	-0.110D-01	0.135D-02	-0.152D-01	0.577D-02
-0.685D-02	0.126D-02	-0.110D-01	0.100D+01	0.154D-04	0.901D-04	0.943D-03
-0.403D-02	0.134D-01	0.135D-02	0.154D-04	0.100D+01	-0.784D-01	0.251D-01
0.454D-01	-0.164D-02	-0.152D-01	0.901D-04	-0.784D-01	0.100D+01	0.153D-01
0.678D-03	-0.337D-01	0.577D-02	0.943D-03	0.251D-01	0.153D-01	0.100D+01

TABLE 4.18 (cont'd)

RESIDUALS V

	V1(BC4-06)				V2(BC4-013)			V1 - V2		
1	-0.7	0.8	0.2	1	0.5	-0.6	-0.1	-1.2	1.3	0.3
2	0.0	0.2	-0.4	2	-0.0	-0.1	0.3	0.1	0.3	-0.7
3	-0.8	-0.5	-0.9	3	0.5	0.4	0.6	-1.3	-0.9	-1.5
4	-0.2	0.4	3.6	4	0.1	-0.3	-2.5	-0.3	0.7	6.0
6	-0.3	1.7	-0.5	6	0.2	-1.2	0.4	-0.6	2.9	-0.9
7	0.2	0.3	0.6	7	-0.2	-0.2	-0.4	0.4	0.5	1.0
8	-2.1	0.7	1.6	8	1.4	-0.5	-0.9	-3.5	1.1	2.5
9	0.2	-0.0	1.7	9	-0.1	0.0	-1.0	0.3	-0.1	2.7
11	-1.5	-3.2	3.2	11	1.0	2.0	-2.0	-2.5	-5.3	5.2
12	0.7	-2.1	0.3	12	-0.5	1.3	-0.2	1.2	-3.4	0.5
13	1.7	-0.8	-0.3	13	-1.1	0.5	0.2	2.8	-1.3	-0.5
15	0.9	0.2	-0.0	15	-0.6	-0.1	0.0	1.5	0.3	-0.0
16	0.8	1.6	0.2	16	-0.6	-1.0	-0.2	1.4	2.6	0.4
19	-0.7	0.1	-1.7	19	0.5	-0.1	1.0	-1.2	0.2	-2.6
20	0.8	1.3	4.0	20	-0.6	-1.0	-2.3	1.3	2.3	6.4
22	-0.3	-1.5	0.8	22	0.2	0.9	-0.4	-0.6	-2.4	1.2
23	2.2	0.9	-1.1	23	-1.4	-0.6	0.6	3.6	1.5	-1.7
31	1.3	1.5	-0.2	31	-0.8	-0.9	0.1	2.0	2.5	-0.4
32	-1.1	1.9	-4.1	32	0.6	-1.2	2.3	-1.7	3.1	-6.5
38	-1.2	-0.4	-0.1	38	0.8	0.3	0.1	-2.0	-0.7	-0.1
39	-3.2	0.3	2.8	39	2.5	-0.2	-1.7	-5.7	0.5	4.5
40	-1.1	-0.1	0.8	40	0.6	0.0	-0.4	-1.7	-0.1	1.2
42	0.6	-0.5	-0.5	42	-0.4	0.3	0.2	0.9	-0.8	-0.7
43	-0.8	-0.0	-0.1	43	0.5	0.0	0.1	-1.2	-0.0	-0.2
44	0.0	-3.2	-2.9	44	-0.0	1.9	2.0	0.0	-5.1	-4.9
45	0.2	-1.2	1.2	45	-0.1	0.8	-0.7	0.3	-2.0	1.8
47	0.6	-0.1	-0.6	47	-0.4	0.0	0.3	1.0	-0.1	-1.0
50	-1.0	1.4	-0.9	50	0.7	-0.9	0.7	-1.7	2.4	-1.7
51	0.1	-0.3	0.4	51	-0.1	0.2	-0.3	0.1	-0.5	0.8
52	-1.5	0.2	0.8	52	0.9	-0.1	-0.6	-2.5	0.3	1.4
53	-0.2	0.9	0.8	53	0.2	-0.6	-0.6	-0.4	1.5	1.5
55	0.2	0.4	-1.2	55	-0.1	-0.3	0.7	0.3	0.7	-1.8
59	0.1	-1.7	1.2	59	-0.0	1.0	-0.7	0.1	-2.7	1.8
60	1.6	1.4	-1.0	60	-1.0	-0.8	0.5	2.6	2.2	-1.5
61	1.4	1.5	0.9	61	-0.8	-0.9	-0.6	2.2	2.4	1.5
63	0.3	-0.1	0.4	63	-0.3	0.1	-0.2	0.6	-0.2	0.6
64	-0.8	1.0	0.0	64	0.5	-0.6	-0.0	-1.3	1.7	0.0
65	0.1	1.0	-0.9	65	-0.1	-0.6	0.7	0.1	1.6	-1.6
66	0.7	-2.1	0.3	66	-0.5	1.3	-0.2	1.2	-3.4	0.5
67	1.8	1.1	-0.3	67	-1.1	-0.7	0.1	3.0	1.7	-0.4
68	-1.5	-0.9	1.8	68	1.0	0.6	-1.0	-2.5	-1.5	2.8
69	-1.0	-1.8	1.3	69	0.6	1.0	-0.8	-1.7	-2.7	2.1
72	-0.1	-1.8	-0.9	72	0.1	1.2	0.6	-0.2	-3.0	-1.5
73	0.7	-0.3	2.0	73	-0.5	0.2	-1.1	1.2	-0.6	3.1
75	-0.9	-1.2	1.0	75	0.6	0.8	-0.6	-1.5	-2.0	1.6
78	-0.9	2.4	3.1	78	0.8	-1.6	-2.0	-1.7	4.0	5.1
111	-0.4	-1.2	-1.0	111	0.3	0.9	0.7	-0.7	-2.1	-1.8
123	0.2	0.8	1.7	123	-0.1	-0.5	-1.3	0.3	1.2	3.0
134	-0.4	-1.2	-1.1	134	0.3	0.9	0.7	-0.7	-2.1	-1.8

The results given in Table 4.18 show the differences between BC-D6 and BC-D13 solutions to be insignificant with the residuals smaller than in any previous transformations. These results are in agreement with those of the North and South American sub-network described in Section 3.2.1. The conclusion there was that the use of only the better conditioned events, plus images 1-3-5-7 from the highly correlated events gave results similar to that of using all observations without correlation. The only noticable change between solutions BC-D6 and BC-D13 is in the σ_o (a posteri), shown in Table 4.8, and the station coordinates σ's shown in that table of adjusted station coordinates in the Appendix. As with the North and South American sub-network, the a posteri σ_o increased to 3.4 (the a posteri σ_o was in the range of 2.81 to 2.86 for all other solutions) and the standard deviation of all station coordinate components decreased.

A comparison of residuals for Stations 65 and 111 (in Tables 4.14 through 4.18 shows these to be smaller in solutions that have the chords constrained. The fact that chords 3-111 and 16-65 improve the positions of these stations is not surprising. As mentioned earlier, these lines are too short to be determined accurately from observations on PAGEOS without some additional constraints.

4.2.2.3 Height Comparisons

The idea of the height comparison is to compare undulations computed from the geometric solutions with some reference undulations calculated from an independent method. The reference set selected was that of [Rapp, 1973] based on combined satellite and surface gravity observations. Tables 4.19 and 4.20 contain the results of the comparisons. In

TABLE 4.19

HEIGHT RESIDUALS (SOLUTION BC-D6)

XO= 15.1 YO= 26.8 ZO= 8.1 CONSTANT= -15.3

STN. NO.	NOSUGC	N REF	NOSUGC-N REF	RESIDUALS
1	2.91	11.66	-8.75	6.00
2	-52.69	-36.90	-15.79	-1.05
3	-40.38	-17.65	-22.73	-7.98
4	-2.58	6.22	-8.80	5.94
6	11.30	27.06	-15.76	-1.02
7	47.62	54.00	-6.38	8.36
8	-47.69	-28.31	-19.38	-4.64
9	-3.76	16.73	-20.49	-5.75
11	7.93	1.75	6.18	20.92
12	-10.08	13.75	-23.83	-9.09
13	19.20	34.27	-15.07	-0.33
15	-36.50	-20.67	-15.83	-1.09
16	20.91	37.43	-16.52	-1.78
19	6.32	22.80	-16.48	-1.74
20	-18.44	-4.75	-13.69	1.06
22	12.28	27.35	-15.07	-0.33
23	47.50	67.94	-20.44	-5.69
31	-13.75	8.68	-22.43	-7.68
32	-27.42	-30.51	3.09	17.83
38	-53.84	-35.47	-18.37	-3.62
39	-37.04	-16.68	-20.36	-5.61
40	-53.79	-38.11	-15.68	-0.93
42	-25.52	-5.78	-19.74	-5.00
43	3.32	15.60	-12.28	2.46
44	28.93	36.61	-7.68	7.06
45	-25.97	-6.07	-19.90	-5.16
47	57.49	62.17	-4.68	10.07
50	0.46	15.70	-15.24	-0.50
51	15.48	29.20	-13.72	1.03
53	-72.43	-56.10	-16.33	-1.58
55	-2.91	16.26	-19.17	-4.42
59	2.76	16.07	-13.31	1.43
60	8.07	27.33	-19.26	-4.52
61	1.37	11.28	-9.91	4.83
63	13.64	27.20	-13.56	1.18
64	-4.50	10.35	-14.85	-0.10
65	27.34	44.23	-16.89	-2.15
66	-10.87	13.74	-24.61	-9.87
67	-22.49	-12.03	-10.46	4.29
68	1.41	24.65	-23.24	-8.50

TABLE 4.19 (cont'd)

XO= 15.1 YO= 26.8 ZO= 8.1 CONSTANT= -15.3

STN. NO.	NOSUGC	N REF	NOSUGC-N REF	RESIDUALS
69	13.05	25.52	-12.47	2.28
72	-62.48	-40.39	-22.09	-7.34
73	-85.28	-73.64	-11.64	3.10
75	-58.53	-44.40	-14.13	0.61
78	47.75	63.10	-15.35	-0.61
111	-45.64	-33.18	-12.46	2.28
123	-11.01	-1.40	-9.61	5.13
134	-45.73	-33.19	-12.54	2.21

AVERAGE	SIGMA
-0.1474D+02	0.6229D+01

SEMI-MAJOR AXIS

6378140.26

TABLE 4.20

HEIGHT RESIDUALS (SOLUTION BC-D2)

XO= 12.7   YO= 28.0   ZO= 5.2   CONSTANT= -37.9

STN. NO.	NOSUGC	N REF	NOSUGC-N REF	RESIDUALS
1	-17.43	11.66	-29.09	7.48
2	-79.02	-36.90	-42.12	-5.54
3	-63.00	-17.65	-45.35	-8.78
4	-19.01	6.22	-25.23	11.34
6	-11.69	27.06	-38.75	-2.18
7	32.79	54.00	-21.21	15.36
8	-82.61	-28.31	-54.30	-17.73
9	-34.80	16.73	-51.53	-14.95
11	-2.20	1.75	-3.95	32.63
12	-38.87	13.75	-52.62	-16.05
13	-16.14	34.27	-50.41	-13.84
15	-60.18	-20.67	-39.51	-2.94
16	-1.75	37.43	-39.18	-2.61
19	-18.86	22.80	-41.66	-5.09
20	-38.70	-4.75	-33.95	2.62
22	-7.52	27.35	-34.87	1.70
23	19.65	67.94	-48.29	-11.72
31	-37.10	8.68	-45.78	-9.21
32	-44.06	-30.51	-13.55	23.02
38	-74.80	-35.47	-39.33	-2.76
39	-60.93	-16.68	-44.25	-7.68
40	-70.13	-38.11	-32.02	4.55
42	-49.51	-5.78	-43.73	-7.16
43	-18.40	15.60	-34.00	2.57
44	19.61	36.61	-17.00	19.57
45	-47.91	-6.07	-41.84	-5.26
47	32.34	62.17	-29.83	6.75
50	-18.70	15.70	-34.40	2.17
51	-4.69	29.20	-33.89	2.68
53	-92.91	-56.10	-36.81	-0.24
55	-22.59	16.26	-38.85	-2.28
59	-12.41	16.07	-28.48	8.09
60	-17.22	27.33	-44.55	-7.97
61	-16.50	11.28	-27.78	8.79
63	-6.16	27.20	-33.36	3.21
64	-25.72	10.35	-36.07	0.50
65	4.18	44.23	-40.05	-3.48
66	-39.67	13.74	-53.41	-16.84
67	-43.15	-12.03	-31.12	5.45
68	-26.90	24.65	-51.55	-14.98

TABLE 4.20 (cont'd)

XO= 12.7 YO= 28.0 ZO= 5.2 CONSTANT= -37.9

STN. NO.	NOSUGC	N REF	NOSUGC-N REF	RESIDUALS
69	-2.96	25.52	-28.48	8.09
72	-97.41	-40.39	-57.02	-20.45
73	-103.22	-73.64	-29.58	6.99
75	-81.33	-44.40	-36.93	-0.36
78	19.70	63.10	-43.40	-6.83
111	-64.65	-33.18	-31.47	5.10
123	-14.68	-1.40	-13.28	23.29
134	-64.76	-33.19	-31.57	5.00

AVERAGE	SIGMA
-0.36570+02	0.11250+02

SEMI-MAJOR AXIS

6378118.43

the tables, NOSUGC denotes the quantity H-MSL-dH, where H is the ellipsoidal height from the geometric solution, MSL is the mean sea level height, and dH is the change in height due to the fact that the reference ellipsoid is not geocentric, while the reference undulations (N REF) are referred to the geocentric level ellipsoid. The coordinates of the geocenter with respect to the center of the reference ellipsoid (x_o, y_o, z_o) were estimated from a least squares fit using the following model [Heiskanen and Moritz, 1967, p207]:

$$N\ REF - (H\text{-}MSL) = Ax_o + By_o + Cz_o + \Delta a$$

where Δa is the difference between the semi-diameter of the reference ellipsoid and that of the level ellipsoid of the same flattening, and the coefficients A, B, C are functions of the station locations. Since the semi-diameter of the reference ellipsoid is known ($a = 6378155$m), a by-product of the fit is the semi-diameter of the level ellipsoid best fitting the geoid. These quantities for the different solutions are listed in Table 4.21.

In the height comparison tables, in case of a uniform global station distribution, the average value of NOSUGC - N REF should be equal to the additive terms from the best fit or the difference between the semi-diameters of the reference ellipsoid and that of the level ellipsoid. In the case of solution BC-D6, as seen on the last page of Table 4.19, this value is -14.7m. The root mean square value of the residuals is $\pm$ 6.3m. Thus, the dimension of the level ellipsoid is 6378140.3 $\pm$ 6.3m, the same as in Table 4.21. In the case of the BC-D2 solution, the results given in Table 4.20 show the semi-major axis to be reduced to

TABLE 4.21

SEMI-DIAMETER OF THE LEVEL ELLIPSOID BEST FITTING THE GEOID AS DETERMINED FROM THE DIFFERENT SOLUTIONS

Solutions	X_o (m)	Y_o (m)	Z_o (m)	a (level ellipsoid) 6378000 + (m)
BC-D1	12.7	29.0	6.0	108.0 ± 11.3
BC-D2	12.7	28.0	5.2	118.4 ± 11.2
BC-D3	14.4	27.6	8.0	140.1 ± 6.5
BC-D4	14.3	27.7	7.5	140.6 ± 6.6
BC-D6	15.1	26.8	8.0	140.3 ± 6.3
BC-D11	14.5	27.6	8.3	154.8 ± 6.5
BC-D13	15.4	26.8	8.1	140.3 ± 7.7

6378118.4m. The root mean square value of the residuals, however, is ±11.2 meters due to the lack of height constraints. Comparison of the residuals in BC-D6 show marked improvements over those in BC-D2, as expected.

Other solutions produced semi-diameters as listed in Table 4.21. The residuals from solutions BC-D3, BC-D4, BC-D11 and BC-D13 are similar to those in BC-D6, and are not reproduced here.

4.2.3 Comparisons with Dynamic Solutions

In addition to the geometric comparisons described above, comparisons were made between solution BC-D6 and three different dynamic solutions having common stations. These were solution NWL-9D of the Naval Weapons Laboratory [American Geophysical Union, 1974], SAO-III of the

Smithsonian Astrophysical Observatory [American Geophysical Union, 1974], and solution GEM-6 of the National Aeronautics and Space Administration [American Geophysical Union, 1974]. The comparison with solution NWL-9D was possible because ground surveys tie 22 Doppler stations with BC-4 cameras. The Smithsonian Astrophysical Observatory had in their adjustment 48 of the 49 BC-4 stations (seven of these BC-4 cameras were connected directly by ground surveys). The GEM-6 adjustment included 47 of the 49 BC-4 stations (sixteen connected directly by ground surveys). The results of these transformations are given in Tables 4.22 through 4.24. Separate transformations were performed to compare BC-D6 with SAO-III and GEM-6 solutions using only the stations connected directly by ground survey. These results are given in Tables 4.25 and 4.26.

The shifts are within the noise level of those determined through the best fit procedure listed in Table 4.21. There the geocenter seems to be very near to the geometric center defined by the center of the best fitting level ellipsoid.

The scale differences are also negligably small. This is through design by defining the scale of a = 6378140m (through the a priori additive term of -15m).

The ω rotations are sizable as seen from the illustration in Figure 4.1. The largest difference occurs between the BC-D6 and the NWL-9D, where $\omega = 0\overset{''}{.}63$, which is about 20m on the equator. The differences for the SAO-III solutions are smaller, but still significant. The GEM-6 solution has an ω difference of less than 1m.

TABLE 4.22

TRANSFORMATION: BC-D6 - NWL-9D

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
19.82	20.92	6.10	0.37	0.63	-0.17	-0.27

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 2.12

0.173D+01	-0.291D-02	0.604D-02	-0.829D-08	-0.302D-09	0.103D-07	0.602D-09
-0.291D-02	0.176D+01	-0.345D-02	0.222D-08	0.353D-08	-0.758D-09	-0.111D-07
0.604D-02	-0.345D-02	0.218D+01	-0.178D-07	0.275D-09	-0.472D-08	-0.126D-09
-0.829D-08	0.222D-08	-0.178D-07	0.105D-13	-0.821D-17	0.408D-17	0.294D-16
-0.302D-09	0.353D-08	0.275D-09	-0.821D-17	0.617D-14	-0.314D-15	0.687D-15
0.103D-07	-0.758D-09	-0.472D-08	0.408D-17	-0.314D-15	0.609D-14	0.355D-15
0.602D-09	-0.111D-07	-0.126D-09	0.294D-16	0.687D-15	0.355D-15	0.782D-14

COEFFICIENTS OF CORRELATION

0.100D+01	-0.167D-02	0.311D-02	-0.614D-01	-0.293D-02	0.100D+00	0.518D-02
-0.167D-02	0.100D+01	-0.176D-02	0.163D-01	0.339D-01	-0.732D-02	-0.946D-01
0.311D-02	-0.176D-02	0.100D+01	-0.117D+00	0.237D-02	-0.410D-01	-0.967D-03
-0.614D-01	0.163D-01	-0.117D+00	0.100D+01	-0.102D-02	0.510D-03	0.324D-02
-0.293D-02	0.339D-01	0.237D-02	-0.102D-02	0.100D+01	-0.513D-01	0.989D-01
0.100D+00	-0.732D-02	-0.410D-01	0.510D-03	-0.513D-01	0.100D+01	0.514D-01
0.518D-02	-0.946D-01	-0.967D-03	0.324D-02	0.989D-01	0.514D-01	0.100D+01

TABLE 4.22 (cont'd)

RESIDUALS V

	V1(BC4-D6)				V2(NWL-9D)			V1 - V2		
1	1.4	-0.8	-5.8	18	-0.8	0.5	2.3	2.2	-1.3	-8.1
2	-0.6	4.5	-0.3	742	0.4	-3.7	0.2	-1.0	8.2	-0.4
3	2.6	-0.7	-1.2	738	-2.0	0.5	0.7	4.5	-1.2	-1.9
4	5.9	-12.1	-3.4	739	-1.7	2.5	0.5	7.5	-14.6	-3.9
6	0.3	-7.7	2.2	818	-0.2	3.6	-1.3	0.4	-11.3	3.5
8	0.5	5.8	3.8	815	-0.2	-2.5	-0.5	0.7	8.3	4.3
11	2.3	-9.5	-3.6	811	-0.7	1.9	0.8	3.0	-7.4	-4.5
12	-3.5	2.8	3.4	708	10.2	-5.8	-5.7	-13.8	8.6	9.1
15	0.9	1.8	-2.0	817	-0.6	-1.0	1.0	1.5	2.8	-3.0
16	1.0	0.8	0.1	812	-1.0	-0.5	-0.0	2.0	1.3	0.1
22	-5.1	-4.6	-5.9	117	1.7	1.4	1.1	-6.9	-6.1	-6.9
23	-6.1	8.1	-0.3	744	2.9	-4.7	0.1	-9.0	12.8	-0.3
31	-4.4	3.2	-0.1	809	1.9	-1.1	0.0	-6.3	4.3	-0.2
38	-0.3	0.8	0.1	831	0.9	-1.9	-0.1	-1.2	2.7	0.2
43	6.6	-3.3	27.3	847	-1.8	0.9	-5.7	8.4	-4.2	32.9
53	2.3	-1.3	-1.7	19	-0.6	0.3	0.5	2.8	-1.6	-2.2
55	-1.5	-5.5	10.1	722	0.6	2.0	-2.1	-2.1	-7.5	12.2
60	-2.2	0.6	-0.0	805	7.5	-2.3	0.0	-9.6	3.0	-0.0
64	-1.9	1.3	-2.1	822	1.1	-0.8	0.8	-3.0	2.1	-2.8
65	-1.6	-4.6	-1.0	830	1.6	2.9	0.8	-3.3	-7.5	-1.8
68	3.6	-0.7	1.9	115	-1.3	0.4	-0.4	4.9	-1.1	2.4
75	6.8	2.9	-7.8	717	-2.1	-1.0	2.2	8.9	3.9	-10.0

TABLE 4.23

TRANSFORMATION: BC-D6 - SAO-III

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
24.12	27.44	-2.00	0.35	0.60	0.10	-0.23

VARIANCE - COVARIANCE MATRIX

$\sigma_0^2 =$ 0.81

0.101D+01	0.793D-04	0.384D-04	-0.991D-10	0.139D-08	0.119D-08	0.179D-10
0.793D-04	0.964D+00	0.504D-03	0.861D-09	0.258D-09	-0.638D-10	-0.140D-08
0.384D-04	0.504D-03	0.112D+01	-0.700D-09	0.174D-10	-0.722D-10	-0.175D-08
-0.991D-10	0.861D-09	-0.700D-09	0.102D-14	0.847D-18	-0.118D-17	0.228D-17
0.139D-08	0.258D-09	0.174D-10	0.847D-18	0.195D-14	-0.225D-15	-0.193D-16
0.119D-08	-0.638D-10	-0.722D-10	-0.118D-17	-0.225D-15	0.228D-14	0.555D-16
0.179D-10	-0.140D-08	-0.175D-08	0.228D-17	-0.193D-16	0.555D-16	0.229D-14

COFFFICIENTS OF CORRELATION

0.100D+01	0.802D-04	0.359D-04	-0.308D-02	0.312D-01	0.247D-01	0.375D-03
0.802D-04	0.100D+01	0.484D-03	0.274D-01	0.594D-02	-0.136D-02	-0.301D-01
0.359D-04	0.484D-03	0.100D+01	-0.207D-01	0.371D-03	-0.143D-02	-0.349D-01
-0.308D-02	0.274D-01	-0.207D-01	0.100D+01	0.599D-03	-0.771D-03	0.151D-02
0.312D-01	0.594D-02	0.371D-03	0.599D-03	0.100D+01	-0.107D+00	-0.920D-02
0.247D-01	-0.136D-02	-0.143D-02	-0.771D-03	-0.107D+00	0.100D+01	0.245D-01
0.379D-03	-0.301D-01	-0.349D-01	0.151D-02	-0.920D-02	0.245D-01	0.100D+01

TABLE 4.23 (cont'd)

RESIDUALS V

	V1(BC4-D6)			V2(SAO-III)				V1 - V2		
1	0.4	0.7	0.6	6001	-2.9	-6.7	-3.2	3.2	7.4	3.8
2	-1.8	-3.4	1.0	6002	1.1	2.9	-0.6	-2.9	-6.4	1.5
3	-0.0	-0.4	0.6	6003	0.2	2.0	-2.2	-0.2	-2.3	2.8
4	0.4	-0.4	1.9	6004	-4.7	3.6	-11.5	5.1	-4.1	13.4
6	-0.2	-0.3	0.2	6006	2.7	2.8	-2.6	-2.8	-3.1	2.8
7	-1.0	0.3	-1.0	6007	11.5	-3.4	7.3	-12.5	3.7	-8.3
8	-0.4	1.1	-2.2	6008	2.3	-9.2	5.3	-2.6	10.3	-7.4
9	1.3	1.4	-0.9	6009	-7.7	-9.6	3.0	9.0	11.0	-3.8
11	0.7	-2.0	9.0	6011	-0.4	1.2	-3.4	1.1	-3.1	12.4
12	0.9	-0.3	-0.2	6012	-8.0	2.0	1.0	9.0	-2.3	-1.2
13	3.8	1.1	2.0	6013	-9.9	-1.6	-3.1	13.7	2.7	5.2
15	-0.1	-0.6	0.5	6015	1.1	3.9	-3.1	-1.2	-4.5	3.7
16	-0.2	-0.2	0.4	6016	3.2	1.9	-4.2	-3.4	-2.1	4.6
19	8.0	2.5	-0.1	6019	-5.4	-1.8	0.0	13.4	4.3	-0.1
20	1.3	1.0	1.3	6020	-9.2	-10.7	-8.8	10.6	11.7	10.2
22	-0.1	0.1	-0.1	6022	0.7	-0.7	0.2	-0.9	0.6	-0.3
23	-0.2	-0.1	-0.4	6023	0.9	0.6	1.2	-1.1	-0.7	-1.6
31	-0.7	-0.1	-1.7	6031	2.8	0.4	5.9	-3.5	-0.5	-7.6
32	3.2	0.6	4.0	6032	-18.3	-3.8	-18.1	21.5	4.4	22.0
38	-0.1	0.0	0.4	6038	0.6	-0.0	-0.8	-0.7	0.0	1.2
39	1.1	0.8	0.6	6039	-7.6	-7.0	-5.2	8.8	7.8	5.8
40	2.3	0.8	-2.5	6040	-10.5	-5.5	14.5	12.8	6.3	-17.1
42	-2.7	-2.6	0.3	6042	4.3	4.5	-0.4	-7.0	-7.1	0.8
43	2.2	0.9	-0.9	6043	-11.4	-4.5	3.3	13.6	5.3	-4.2
44	1.2	2.9	0.4	6044	-7.8	-22.2	-2.1	9.0	25.0	2.6
45	-0.5	-0.5	-1.7	6045	2.0	2.0	5.1	-2.5	-2.5	-6.9
47	2.1	0.8	2.0	6047	-12.7	-4.8	-6.2	14.8	5.7	8.2
50	1.2	1.8	-1.4	6050	-9.2	-10.0	8.2	10.5	11.8	-9.6
51	1.5	0.1	-1.2	6051	-7.0	-0.6	6.6	8.5	0.7	-7.8
52	2.6	0.8	-1.6	6052	-14.8	-5.6	5.6	17.4	6.4	-7.2
53	2.4	0.1	-1.7	6053	-11.9	-0.7	9.2	14.3	0.9	-10.9
55	-1.9	0.4	1.0	6055	10.1	-2.1	-2.9	-12.0	2.5	3.9
59	-0.1	-0.2	0.5	6059	0.6	0.8	-1.3	-0.8	-1.0	1.9
60	-2.8	2.1	-5.3	6060	2.5	-2.0	4.0	-5.3	4.2	-9.3
61	1.5	1.0	0.0	6061	-11.6	-4.0	-0.2	13.1	5.0	0.2
63	-1.0	0.0	0.0	6063	9.4	-0.1	-0.2	-10.4	0.1	0.2
64	-0.8	-0.3	0.2	6064	4.7	2.4	-1.0	-5.5	-2.7	1.3
65	-0.2	-0.3	-0.7	6065	4.8	3.2	11.2	-5.0	-3.5	-11.9
67	-1.8	7.6	4.7	6067	1.5	-5.4	-2.6	-3.3	13.0	7.3
68	-5.3	-1.4	-14.3	6068	2.0	0.8	3.4	-7.3	-2.2	-17.8
69	-0.4	0.1	2.2	6069	3.5	-0.7	-13.4	-3.9	0.8	15.6
72	1.1	-0.1	0.5	6072	-3.2	0.7	-3.0	4.3	-0.8	3.5
73	-0.2	-1.0	-0.7	6073	1.3	5.7	3.2	-1.5	-6.7	-4.0
75	-0.5	-1.0	-0.6	6075	2.4	4.7	2.6	-2.9	-5.7	-3.2
78	-6.9	1.0	15.6	6078	22.6	-4.4	-27.3	-29.4	5.4	43.0
111	-0.5	-0.3	1.6	6111	0.5	0.3	-1.4	-1.1	-0.6	3.0
123	0.5	-0.4	1.0	6123	-3.0	3.0	-7.6	3.4	-3.4	8.7
134	-0.5	-0.3	1.5	6134	0.6	0.4	-1.4	-1.1	-0.7	2.9

TABLE 4.24

TRANSFORMATION: BC-D6 - GEM-6

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
26.77	22.47	9.45	0.13	0.02	0.11	0.01

VARIANCE - COVARIANCE MATRIX

$\sigma_0^2 = 0.61$

0.118D+01	0.404D-03	-0.507D-04	-0.666D-09	0.111D-08	0.417D-09	0.187D-10
0.404D-03	0.113D+01	0.255D-03	0.549D-09	0.160D-08	-0.195D-09	-0.587D-09
-0.507D-04	0.255D-03	0.128D+01	-0.292D-09	0.137D-09	-0.149D-08	-0.143D-08
-0.666D-09	0.549D-09	-0.292D-09	0.104D-14	0.630D-18	-0.967D-18	0.182D-17
0.111D-08	0.160D-08	0.137D-09	0.630D-18	0.233D-14	-0.254D-15	0.233D-17
0.417D-09	-0.195D-09	-0.149D-08	-0.967D-18	-0.254D-15	0.265D-14	0.913D-16
0.187D-10	-0.587D-09	-0.143D-08	0.182D-17	0.233D-17	0.913D-16	0.268D-14

COEFFICIENTS OF CORRELATION

0.100D+01	0.350D-03	-0.413D-04	-0.190D-01	0.211D-01	0.745D-02	0.332D-03
0.350D-03	0.100D+01	0.212D-03	0.160D-01	0.311D-01	-0.356D-02	-0.107D-01
-0.413D-04	0.212D-03	0.100D+01	-0.799D-02	0.251D-02	-0.256D-01	-0.244D-01
-0.190D-01	0.160D-01	-0.799D-02	0.100D+01	0.404D-03	-0.582D-03	0.109D-02
0.211D-01	0.311D-01	0.251D-02	0.404D-03	0.100D+01	-0.102D+00	0.932D-03
0.745D-02	-0.356D-02	-0.256D-01	-0.582D-03	-0.102D+00	0.100D+01	0.343D-01
0.332D-03	-0.107D-01	-0.244D-01	0.109D-02	0.932D-03	0.343D-01	0.100D+01

TABLE 4.24 (cont'd)

RESIDUALS V

	V1(RC4-D6)			V2(GEM-6)				V1 - V2		
2	-3.5	-0.2	1.1	6002	3.5	0.3	-1.0	-7.0	-0.5	2.1
3	0.2	-0.7	-0.4	6003	-2.9	8.2	3.1	3.2	-8.8	-3.4
4	0.0	-0.8	0.9	6004	-1.2	14.6	-11.4	1.2	-15.4	12.3
6	-0.0	0.6	-0.6	6006	1.1	-13.0	15.7	-1.1	13.6	-16.3
7	-0.6	0.9	-0.4	6007	8.7	-11.6	3.4	-9.4	12.5	-3.8
8	-0.6	0.5	0.8	6008	8.2	-10.0	-4.6	-8.8	10.6	5.4
9	0.2	0.1	0.7	6009	-2.5	-1.9	-5.2	2.6	2.0	5.9
11	0.6	-4.6	6.1	6011	-0.5	4.9	-4.2	1.1	-9.5	10.3
12	0.3	-0.3	1.4	6012	-3.0	2.2	-9.0	3.7	-2.4	10.4
13	0.3	-0.1	0.1	6013	-4.6	0.6	-1.1	4.9	-0.7	1.2
15	0.2	-1.0	-1.2	6015	-1.8	8.1	8.8	2.0	-9.0	-9.9
16	0.0	0.8	-0.7	6016	-0.7	-8.0	7.1	0.8	8.8	-7.7
19	3.3	0.2	0.2	6019	-3.5	-0.2	-0.1	6.8	0.4	0.3
20	0.3	0.4	0.8	6020	-5.1	-7.8	-11.8	5.5	8.2	12.7
22	-0.8	-0.8	0.9	6022	6.2	5.5	-3.9	-7.0	-6.2	4.8
23	-0.8	0.4	-0.5	6023	7.1	-4.4	2.9	-7.8	4.8	-3.4
31	-1.2	0.5	-0.5	6031	7.9	-2.5	2.7	-9.1	3.0	-3.2
32	2.7	1.2	4.8	6032	-11.4	-6.0	-15.9	14.1	7.2	20.7
38	-0.3	-0.2	1.0	6038	3.5	2.2	-4.9	-3.8	-2.4	5.8
39	-0.0	0.1	0.4	6039	0.3	-1.9	-8.1	-0.3	2.0	8.5
40	3.7	0.5	-1.8	6040	-20.1	-4.5	12.3	23.8	5.1	-14.2
42	0.1	-1.6	-3.1	6042	-0.3	3.8	5.1	0.4	-5.4	-8.3
43	0.3	0.2	2.4	6043	-1.9	-1.0	-11.6	2.2	1.2	14.0
44	2.9	-0.5	-0.2	6044	-15.2	3.2	0.8	18.1	-3.8	-1.0
45	1.0	-0.2	-0.5	6045	-10.3	1.7	3.5	11.3	-1.9	-4.0
47	0.3	0.2	-0.7	6047	-3.5	-2.2	1.2	3.8	2.4	-1.4
50	0.2	0.2	-0.3	6050	-1.3	-1.1	2.0	1.5	1.3	-2.4
51	3.3	0.5	0.6	6051	-16.5	-4.3	-3.8	19.8	4.8	4.4
52	1.0	0.6	0.4	6052	-5.9	-4.5	-1.5	6.9	5.1	1.9
53	1.1	-0.8	-0.2	6053	-5.1	4.0	1.0	6.2	-4.8	-1.1
55	-1.5	1.3	0.4	6055	8.5	-7.6	-1.1	-9.9	8.9	1.5
59	-0.1	-0.7	1.4	6059	1.1	5.2	-6.2	-1.3	-5.9	7.5
60	-2.5	1.3	-1.3	6060	11.0	-6.0	4.8	-13.4	7.3	-6.1
61	1.4	1.5	3.1	6061	-11.6	-6.2	-14.9	13.0	7.7	18.0
63	-0.6	0.9	-0.8	6063	5.9	-9.0	3.4	-6.5	10.0	-4.1
64	-1.1	0.4	-0.7	6064	6.0	-2.2	2.5	-7.1	2.6	-3.3
65	-0.2	0.6	-0.5	6065	4.4	-10.9	11.4	-4.5	11.5	-12.0
67	-2.1	2.2	0.3	6067	11.7	-10.5	-1.3	-13.8	12.7	1.6
68	1.5	1.7	-3.2	6068	-1.0	-1.4	1.3	2.5	3.1	-4.4
69	-1.1	1.1	3.7	6069	8.3	-7.9	-20.4	-9.3	9.0	24.1
72	2.6	-0.4	-0.4	6072	-13.4	4.0	3.4	15.9	-4.4	-3.8
73	1.3	-0.3	-0.5	6073	-13.4	2.6	3.7	14.7	-2.9	-4.2
75	0.2	-0.4	-1.0	6075	-2.1	3.4	7.4	2.4	-3.7	-8.4
78	-0.0	0.1	-0.2	6078	0.3	-2.6	1.4	-0.3	2.8	-1.6
111	-0.7	-1.6	-0.1	6111	2.6	7.6	0.5	-3.3	-9.2	-0.6
123	1.0	-1.1	-1.0	6123	-24.0	30.3	28.6	25.0	-31.4	-29.7
134	-0.7	-1.6	-0.1	6134	2.6	7.6	0.4	-3.3	-9.3	-0.5

TABLE 4.25

TRANSFORMATION: BC-D6 - SAO-III (7 STATIONS)

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
23.50	26.51	-1.32	0.19	0.26	-0.07	-0.10

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 1.85

0.720D+01	-0.239D-02	0.629D-01	-0.723D-07	0.697D-07	0.133D-07	-0.298D-07
-0.239D-02	0.676D+01	0.385D-01	0.815D-07	0.706D-07	-0.117D-07	-0.476D-07
0.629D-01	0.385D-01	0.884D+01	-0.313D-07	0.438D-07	-0.957D-07	-0.161D-06
-0.723D-07	0.815D-07	-0.313D-07	0.690D-13	0.602D-15	-0.110D-14	0.497D-15
0.697D-07	0.706D-07	0.438D-07	0.602D-15	0.588D-13	-0.644D-14	-0.280D-13
0.133D-07	-0.117D-07	-0.957D-07	-0.110D-14	-0.644D-14	0.808D-13	0.201D-13
-0.298D-07	-0.476D-07	-0.161D-06	0.497D-15	-0.280D-13	0.201D-13	0.104D-12

COEFFICIENTS OF CORRELATION

0.100D+01	-0.343D-03	0.789D-02	-0.102D+00	0.107D+00	0.174D-01	-0.344D-01
-0.343D-03	0.100D+01	0.498D-02	0.119D+00	0.112D+00	-0.159D-01	-0.567D-01
0.789D-02	0.498D-02	0.100D+01	-0.401D-01	0.608D-01	-0.113D+00	-0.168D+00
-0.102D+00	0.119D+00	-0.401D-01	0.100D+01	0.945D-02	-0.147D-01	0.586D-02
0.107D+00	0.112D+00	0.608D-01	0.945D-02	0.100D+01	-0.934D-01	-0.357D+00
0.174D-01	-0.159D-01	-0.113D+00	-0.147D-01	-0.934D-01	0.100D+01	0.219D+00
-0.344D-01	-0.567D-01	-0.168D+00	0.586D-02	-0.357D+00	0.219D+00	0.100D+01

TABLE 4.25 (cont'd)

RESIDUALS V

	V1(BC4-D6)			V2(SAO-111)				V1 - V2		
11	-1.9	-0.3	5.4	6011	1.0	0.2	-2.0	-2.9	-0.5	7.4
13	3.8	2.1	2.2	6013	-10.0	-3.2	-3.4	13.8	5.3	5.5
19	8.1	2.6	-2.1	6019	-5.4	-1.9	1.0	13.5	4.5	-3.1
42	-1.2	-3.3	2.6	6042	1.9	5.5	-3.2	-3.1	-8.8	5.8
67	-1.8	5.9	4.9	6067	1.4	-4.1	-2.7	-3.2	10.0	7.6
68	-1.3	-1.4	-11.6	6068	0.5	0.7	2.8	-1.8	-2.2	-14.4
111	-3.2	-0.6	-0.5	6111	3.3	0.8	0.5	-6.5	-1.4	-1.0

TABLE 4.26

TRANSFORMATION: BC-D6 - GEM-6 (16 STATIONS)

SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
26.25	22.04	9.72	0.30	0.03	0.08	-0.05

VARIANCE - COVARIANCE MATRIX

σ_0^2 = 1.02

0.319D+01	0.127D-02	0.221D-02	-0.134D-07	0.100D-07	0.407D-08	-0.320D-09
0.127D-02	0.299D+01	0.563D-02	0.944D-08	0.173D-07	-0.396D-08	-0.722D-08
0.221D-02	0.563D-02	0.355D+01	-0.535D-08	0.367D-08	-0.202D-07	-0.164D-07
-0.134D-07	0.944D-08	-0.535D-08	0.120D-13	0.270D-16	-0.477D-16	0.691D-16
0.100D-07	0.173D-07	0.367D-08	0.270D-16	0.146D-13	-0.289D-14	-0.103D-14
0.407D-08	-0.396D-08	-0.202D-07	-0.477D-16	-0.289D-14	0.193D-13	0.165D-14
-0.320D-09	-0.722D-08	-0.164D-07	0.691D-16	-0.103D-14	0.165D-14	0.179D-13

COEFFICIENTS OF CORRELATION

0.100D+01	0.412D-03	0.656D-03	-0.682D-01	0.464D-01	0.164D-01	-0.134D-02
0.412D-03	0.100D+01	0.173D-02	0.498D-01	0.830D-01	-0.165D-01	-0.312D-01
0.656D-03	0.173D-02	0.100D+01	-0.259D-01	0.161D-01	-0.773D-01	-0.650D-01
-0.682D-01	0.498D-01	-0.259D-01	0.100D+01	0.204D-02	-0.313D-02	0.471D-02
0.464D-01	0.830D-01	0.161D-01	0.204D-02	0.100D+01	-0.172D+00	-0.636D-01
0.164D-01	-0.165D-01	-0.773D-01	-0.313D-02	-0.172D+00	0.100D+01	0.888D-01
-0.134D-02	-0.312D-01	-0.650D-01	0.471D-02	-0.636D-01	0.888D-01	0.100D+01

TABLE 4.26 (cont'd)

RESIDUALS V

	V1(BC4-D6)				V2(GEM-6)			V1 - V2		
2	-3.3	0.5	1.3	6002	3.4	-0.7	-1.2	-6.7	1.2	2.6
3	0.3	-0.5	-0.4	6003	-3.3	6.7	3.1	3.6	-7.2	-3.5
9	0.2	0.2	0.8	6009	-3.1	-2.9	-6.2	3.3	3.1	7.1
11	1.0	-4.2	5.9	6011	-1.0	4.5	-4.0	2.0	-8.6	9.9
15	0.1	-0.9	-1.3	6015	-1.6	7.2	9.7	1.8	-8.1	-11.0
19	3.7	0.4	0.9	6019	-4.0	-0.4	-0.7	7.7	0.8	1.6
32	2.9	1.1	4.6	6032	-12.1	-5.4	-15.1	14.9	6.5	19.6
40	3.8	0.5	-2.0	6040	-20.5	-4.2	13.5	24.2	4.8	-15.5
42	0.1	-1.4	-3.3	6042	-0.2	3.4	5.5	0.3	-4.8	-8.8
53	1.3	-0.9	-0.2	6053	-6.2	4.6	0.8	7.5	-5.5	-0.9
55	-1.4	1.5	0.5	6055	8.3	-8.2	-1.7	-9.8	9.7	2.3
60	-2.3	1.2	-1.5	6060	10.0	-5.5	5.5	-12.3	6.7	-7.0
64	-1.2	0.5	-0.7	6064	6.1	-3.0	2.5	-7.3	3.5	-3.3
67	-2.0	2.3	0.6	6067	11.5	-11.3	-2.1	-13.5	13.6	2.7
68	1.7	1.6	-3.2	6068	-1.1	-1.4	1.2	2.7	3.1	-4.4
111	-0.5	-1.3	-0.1	6111	2.1	6.4	0.3	-2.7	-7.7	-0.4

The rotations about the x and y axes are seen in Figure 4.2. The largest differences occur with respect to the NWL-9D solution, but the differences for SAO-III are also significant. As with the ω rotation, the GEM-6 solution was in better agreement. It should be noted that the SAO-III and GEM-6 transformations which included only the stations connected by survey agreed very well.

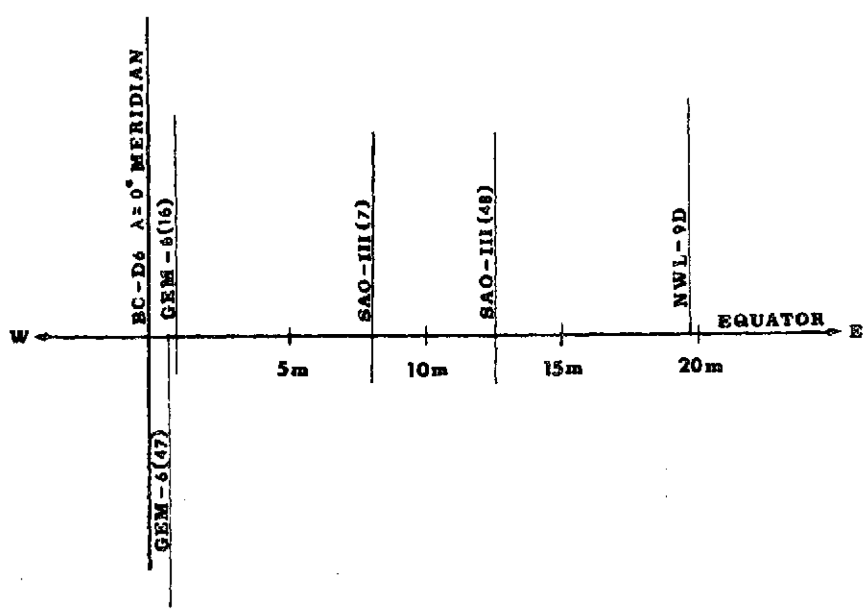

Figure 4.1. Dynamic Zero Meridians Relative to the BC-D6 Zero Meridian

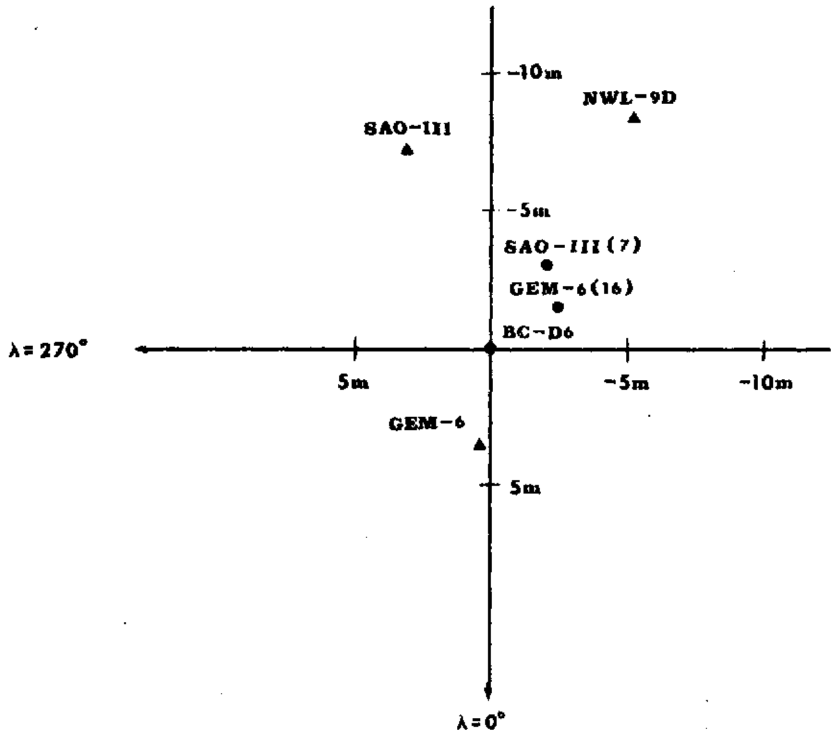

Figure 4.2. Dynamic Pole Positions Relative to the BC-D6 Pole

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APPENDIX A

TABLE A.1

COORDINATES OF THE BC-4 WORLDWIDE NETWORK STATIONS FROM THE GEOMETRIC ADJUSTMENTS

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordinates BC-D1	σ	BC-D2	σ	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
1	Thule	X	546551.3	8.0	3.7	9.6	3.5	14.5	3.2	13.8	3.5	12.6	3.1	16.8	3.2	9.8	2.7
		Y	-1389976.8	- 9.1	3.2	-11.8	3.4	-17.1	2.9	-15.2	3.0	-22.5	2.8	-21.0	2.9	-21.7	2.4
		Z	6180216.4	-14.2	5.0	- 1.6	6.5	14.3	3.9	15.1	3.8	13.3	3.6	27.7	3.8	13.3	3.3
2	Beltsville	X	1130751.5	4.2	3.9	5.4	3.3	8.5	3.1	13.1	3.7	7.0	3.0	9.2	3.1	5.8	2.6
		Y	-4830822.5	- 0.5	3.6	- 7.5	5.0	-24.3	2.6	-26.1	2.6	-25.2	2.5	-34.8	2.6	-25.4	2.1
		Z	3994698.9	-17.3	4.8	- 9.9	5.9	6.2	3.2	3.6	3.3	5.2	3.1	16.7	3.2	3.8	2.6
3	Moses Lake	X	-2127841.1	6.8	3.4	4.3	3.2	2.1	2.8	- 1.5	3.0	1.2	2.6	- 0.6	2.8	- 1.5	2.2
		Y	-3785839.5	- 8.9	3.1	-15.5	4.6	-30.9	2.6	-29.0	2.6	-31.0	2.5	-40.6	2.6	-32.2	2.1
		Z	4656032.3	-22.3	4.9	-15.5	5.8	1.0	3.3	2.7	3.4	- 1.3	3.1	11.4	3.3	- 4.1	2.6
4	Shemya	X	-3851751.5	-37.3	5.3	-43.0	5.9	-54.4	4.9	-55.9	4.9	-46.3	4.3	-62.7	4.9	-47.8	3.5
		Y	396384.1	28.9	6.3	29.4	6.2	31.0	6.0	31.8	6.1	20.7	5.1	31.8	6.1	21.4	4.2
		Z	5051266.2	29.6	10.2	38.6	11.1	49.3	7.6	49.8	7.6	56.4	6.1	60.8	7.6	61.0	5.0
6	Tromso	X	2102908.3	2.9	3.3	6.2	3.9	17.3	2.7	15.0	3.1	16.8	2.6	24.6	2.7	14.9	2.2
		Y	721686.9	-19.2	3.8	-20.4	3.7	-18.3	3.4	-17.1	3.6	-21.8	3.4	-16.3	3.5	-19.5	2.8
		Z	5958153.2	-10.6	5.5	0.9	5.9	16.4	3.0	19.6	3.4	18.3	3.0	28.0	3.0	17.6	2.8
7	Terceira	X	4433636.1	- 2.0	3.8	2.8	5.2	12.4	3.0	14.7	3.1	9.9	3.0	22.4	3.0	9.4	2.5
		Y	-2268138.8	- 7.7	3.6	-10.8	3.6	-13.6	3.1	-15.1	3.3	-17.6	3.0	-17.4	3.1	-17.8	2.5
		Z	3971637.0	3.7	4.9	8.2	6.2	14.9	3.7	13.9	3.8	14.0	3.6	25.9	3.7	15.5	2.9
8	Paramaribo	X	3623218.4	- 2.2	5.2	3.5	6.0	22.6	4.1	23.4	4.2	22.4	4.1	30.5	4.2	18.6	3.3
		Y	-5214222.7	- 2.3	5.1	- 9.6	6.8	-35.6	3.5	-35.8	3.5	-35.8	3.5	-47.8	3.5	-35.1	3.0
		Z	601532.3	- 0.6	6.7	0.2	6.8	5.1	6.3	4.4	6.4	1.8	6.3	7.6	6.4	4.4	4.8
9	Quito	X	1280811.5	1.7	5.1	3.9	5.2	11.9	4.8	13.0	4.8	11.6	4.7	14.1	4.8	11.6	3.8
		Y	-6250937.6	- 1.2	6.5	-10.3	8.5	-39.9	4.4	-40.5	4.4	-38.7	4.4	-54.4	4.4	-38.9	3.9
		Z	- 10814.6	3.1	6.6	3.3	6.6	4.9	6.3	3.9	6.3	0.7	6.2	5.7	6.3	2.7	4.8

All units are in meters.

TABLE A.1 (cont'd)

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordinates													
				BC-D1	σ	BC-D2	σ	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
11	Maui	X	-5466047.1	37.6	5.2	30.5	6.9	17.9	4.3	15.8	4.3	18.0	4.1	5.6	4.3	14.8	3.4
		Y	-2404405.6	-21.3	4.3	-25.8	4.7	-30.1	4.0	-30.0	4.0	-24.4	3.9	-35.4	4.0	-29.3	3.1
		Z	2242220.0	-18.0	5.9	-12.3	6.3	-10.4	5.3	-12.5	5.3	-11.0	4.8	- 5.1	5.3	- 7.9	3.8
12	Wake Island I	X	-5858501.6	-21.9	5.3	-31.0	7.4	-58.3	3.8	-59.0	3.8	-57.4	3.6	-72.0	3.8	-56.6	3.1
		Y	1394466.1	7.8	5.1	10.7	5.2	20.0	4.7	19.6	4.8	28.5	4.3	23.1	4.7	25.4	3.3
		Z	2093775.9	5.2	6.1	10.6	6.4	18.8	5.2	18.4	5.2	17.2	4.7	22.8	5.2	15.8	3.7
13	Kanoya	X	-3565896.5	31.6	5.1	26.0	6.0	10.6	4.5	10.5	4.5	5.4	4.2	1.8	4.5	7.9	3.4
		Y	4120638.9	32.7	7.4	40.0	8.2	58.6	6.5	58.2	6.5	54.9	5.6	67.9	6.5	53.7	4.4
		Z	3303398.7	-19.3	8.4	-14.3	8.9	4.9	7.0	6.5	7.0	14.3	5.5	12.2	7.0	12.7	4.4
15	Mashhad	X	2604328.6	0.5	3.2	3.3	3.9	13.6	2.8	14.9	2.9	15.5	2.7	20.2	2.8	16.7	2.2
		Y	4444161.0	-25.6	4.1	-20.0	4.8	- 8.4	3.3	- 5.7	3.4	-11.7	3.1	- 0.1	3.3	-11.7	2.5
		Z	3750292.7	- 7.4	4.3	- 3.3	5.6	12.8	3.4	12.9	3.5	15.4	3.3	23.3	3.4	16.0	2.6
16	Catania	X	4896372.5	- 7.5	3.3	1.8	4.7	16.2	2.3	17.2	2.5	14.5	2.3	26.3	2.3	15.3	1.9
		Y	1316185.2	-17.5	3.3	-15.5	3.2	-14.1	2.9	-12.8	3.0	-18.5	2.9	-12.6	2.9	-16.5	2.3
		Z	3856639.7	- 0.3	4.0	2.7	5.5	17.4	2.8	18.8	3.1	19.5	2.7	28.7	2.8	20.8	2.3
19	Villa Dolores	X	2280596.7	0.5	4.2	4.3	4.7	14.7	3.9	15.6	3.9	14.9	3.9	19.4	3.9	14.4	3.1
		Y	-4914539.4	1.2	4.6	- 5.8	6.4	-25.4	3.7	-25.9	3.7	-26.3	3.7	-37.0	3.7	-26.1	3.0
		Z	-3355431.0	14.1	5.9	8.2	6.6	- 1.9	4.5	- 1.0	4.5	- 1.9	4.5	- 9.9	4.5	- 4.9	3.5
20	Easter Island	X	-1888624.9	3.3	6.6	1.0	6.7	- 1.3	5.9	- 2.6	5.9	- 0.2	5.7	- 4.9	5.9	1.6	5.0
		Y	-5354875.8	- 7.4	6.2	-15.5	7.8	-35.4	4.7	-35.8	4.7	-33.0	4.7	-47.9	4.7	-30.3	4.0
		Z	-2895760.4	6.8	7.1	2.3	7.6	- 9.3	5.9	- 8.5	5.9	-11.4	5.8	-16.6	5.9	- 6.6	4.4
22	Tutuila	X	-6099896.7	-36.7	5.2	-45.9	7.5	-66.7	4.1	-67.4	4.1	-66.6	3.9	-80.9	4.1	-66.8	3.2
		Y	- 997387.0	21.8	4.4	19.3	4.5	17.3	4.2	17.1	4.2	24.2	4.1	15.3	4.2	22.5	3.2
		Z	-1568593.2	18.3	5.9	15.6	6.0	7.8	5.5	8.5	5.5	1.2	5.3	3.5	5.5	0.2	3.9

All units are in meters.

TABLE A.1 (cont'd)

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordinates BC-D1	σ	BC-D2	σ	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
23	Thursday Island	X	-4955317.3	-29.6	4.5	-38.6	6.2	-63.6	3.3	-63.7	3.3	-63.2	3.3	-75.2	3.3	-58.8	2.7
		Y	3842192.5	9.2	3.7	16.2	4.9	30.8	3.0	30.7	3.1	34.9	3.0	39.3	3.1	37.1	2.4
		Z	-1163844.4	-11.1	4.8	-10.3	4.8	-15.9	4.0	-15.4	4.4	-13.3	4.0	-20.7	4.0	-16.5	3.0
31	Invercargill	X	-4313741.0	-52.0	4.3	-59.6	5.8	-80.8	3.5	-80.9	3.5	-81.3	3.5	-90.9	3.5	-77.9	2.8
		Y	891290.8	29.3	4.1	29.5	4.1	30.7	3.8	30.9	3.9	33.1	3.8	32.9	3.8	36.4	3.0
		Z	-4597214.3	-29.1	4.8	-39.0	6.2	-56.0	3.8	-54.9	3.9	-55.7	3.8	-66.8	3.8	-57.7	3.0
32	Caversham	X	-2375439.4	29.0	3.9	25.8	4.5	16.7	3.5	16.9	3.5	13.3	3.4	10.4	3.5	13.0	2.6
		Y	4875498.6	- 1.4	3.8	7.4	5.7	22.9	3.2	23.0	3.2	26.8	3.2	33.7	3.2	30.6	2.5
		Z	-3345507.0	88.0	4.7	80.5	5.5	73.5	3.9	74.7	4.0	79.6	3.8	65.5	4.0	72.4	2.9
38	Socorro Island	X	-2160990.2	5.4	3.8	3.1	3.8	- 0.5	3.3	- 3.6	3.4	1.0	2.9	- 3.9	3.3	- 1.9	2.4
		Y	-5642692.6	- 2.4	4.4	-11.4	6.4	-32.0	3.5	-32.4	3.5	-30.2	3.1	-45.2	3.5	-30.9	2.6
		Z	2035359.0	-11.8	5.6	- 6.9	5.9	0.9	4.7	- 2.6	4.8	0.9	4.4	6.9	4.7	- 0.7	3.5
39	Pitcairn Island	X	-3724765.6	10.1	8.2	4.8	8.8	-10.3	6.6	-11.3	6.6	-10.7	6.5	-18.7	6.6	-16.0	5.6
		Y	-4421211.4	- 9.9	7.6	-17.4	8.7	-35.0	5.9	-34.9	5.9	-31.2	5.8	-45.3	5.9	-30.2	4.9
		Z	-2686087.6	7.5	7.4	2.9	7.8	-11.2	5.8	-10.2	5.8	-13.9	5.8	-18.2	5.8	-11.3	4.5
40	Cocos Island	X	- 742006.4	24.7	5.0	23.5	5.1	20.3	4.9	21.4	4.9	18.9	4.7	17.8	4.9	18.3	3.5
		Y	6190736.4	5.0	4.4	15.0	6.9	35.7	3.8	36.3	3.8	33.0	3.8	49.8	3.8	33.3	3.0
		Z	-1338553.1	15.5	4.7	11.6	4.9	8.5	4.3	10.4	4.3	10.0	4.3	5.0	4.3	11.0	3.1
42	Addis Ababa	X	4900734.9	-22.1	4.0	-14.4	5.8	4.8	3.4	5.1	3.4	5.5	3.4	16.4	3.4	6.8	2.7
		Y	3968220.4	- 0.9	3.8	4.3	4.7	16.7	3.3	18.3	3.4	14.6	3.3	24.6	3.3	13.5	2.6
		Z	966333.1	-10.1	4.3	-10.3	4.4	- 5.9	4.0	- 5.0	4.0	- 5.8	3.9	- 2.9	4.0	- 5.3	2.9
43	Cerro Sombrero	X	1371345.7	4.0	4.6	6.5	4.8	13.5	4.4	14.4	4.4	13.5	4.3	15.9	4.4	13.5	3.4
		Y	-3614746.0	- 2.8	4.8	- 7.9	5.8	-22.9	4.3	-23.5	4.3	-23.7	4.3	-31.3	4.3	-23.4	3.4
		Z	-5055948.9	16.0	7.3	7.6	8.6	- 7.6	5.1	- 6.5	5.1	- 7.9	5.0	-19.8	5.1	- 8.7	4.1
44	Heard Island	X	1098875.8	- 4.2	7.2	- 1.5	7.3	3.6	7.0	4.0	7.0	7.8	7.0	5.6	7.1	9.7	5.6
		Y	3684559.0	12.8	6.4	18.7	7.1	28.2	6.2	28.9	6.2	31.2	6.4	36.1	6.2	26.6	4.9
		Z	-5071862.6	- 7.8	11.2	-17.8	12.2	-24.1	7.9	-22.8	7.9	-21.6	7.9	-36.9	7.9	-26.5	6.6

All units are in meters.

TABLE A.1 (cont'd)

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordiantes BC-D1	σ	BC-D2	σ	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
45	Mauritius	X	3223392.5	10.3	3.8	14.9	4.8	28.5	3.5	29.2	3.6	30.3	3.5	36.3	3.6	31.9	2.8
		Y	5045274.9	23.8	4.0	31.1	5.6	47.8	3.5	48.8	3.5	45.5	3.4	58.8	3.5	43.6	2.8
		Z	-2191814.4	23.6	4.9	18.2	5.2	10.4	4.4	12.5	4.4	8.2	4.2	4.7	4.4	10.7	3.2
47	Zamboanga	X	-3361988.3	39.1	4.6	33.5	5.5	20.7	4.2	20.9	4.2	17.9	4.0	12.4	4.2	19.3	3.1
		Y	5365744.0	8.8	5.3	18.2	7.0	38.2	4.2	38.2	4.3	40.2	4.1	50.4	4.3	40.5	3.2
		Z	763605.8	0.0	6.5	2.1	6.4	8.6	6.0	9.5	6.0	5.0	5.7	9.5	6.0	3.1	4.1
50	Palmer Station	X	1192648.5	7.1	5.8	9.1	5.9	14.6	5.6	15.8	5.6	15.4	5.5	16.6	5.6	15.2	4.4
		Y	-2451015.8	1.2	6.6	- 2.2	7.0	-12.5	6.4	-13.9	6.4	-11.6	6.3	-17.7	6.4	- 8.9	5.2
		Z	-5747042.3	11.6	10.0	1.7	11.3	-16.0	6.2	-14.7	6.2	-16.1	6.2	-30.2	6.3	-18.3	5.5
51	Mawson Station	X	1111310.9	0.0	5.1	2.8	5.2	8.1	4.9	8.4	4.9	11.8	4.9	10.3	4.9	13.9	4.0
		Y	2169218.4	18.6	3.8	21.5	4.2	27.1	3.6	27.8	3.6	30.3	3.6	31.7	3.6	30.4	3.1
		Z	-5874303.6	- 7.2	6.6	-19.1	8.6	-39.7	4.5	-38.6	4.5	-38.8	4.5	-54.0	4.5	-38.2	4.0
52	Wilkes Station	X	- 902620.3	1.9	4.7	1.2	4.7	- 2.2	4.5	- 2.1	4.5	1.5	4.4	- 4.9	4.5	0.9	3.5
		Y	2409459.5	33.3	4.1	36.5	4.6	44.0	3.9	44.4	3.9	48.0	3.9	49.5	3.9	48.9	3.1
		Z	-5816487.2	-39.4	6.7	-51.2	8.7	-72.9	5.6	-71.7	5.6	-72.1	5.7	-87.0	5.6	-71.3	4.7
53	McMurdo Station	X	-1310845.0	-11.5	4.8	-13.0	4.9	-18.1	4.5	-18.1	4.5	-16.2	4.6	-21.5	4.5	-14.8	3.7
		Y	311214.8	33.6	4.7	32.8	4.7	31.1	4.5	31.3	4.5	33.8	4.5	31.9	4.5	36.0	3.5
		Z	-6213216.8	-29.9	6.6	-42.4	8.8	-66.2	4.3	-65.4	4.3	-66.1	4.3	-81.1	4.3	-65.6	3.8
55	Ascension Island	X	6118319.0	- 7.0	4.7	1.8	6.7	19.4	3.8	20.1	3.8	18.6	3.7	33.3	3.8	19.3	3.1
		Y	-1571738.8	- 8.7	4.2	-10.1	4.1	-13.2	3.8	-15.5	4.0	-16.6	3.8	-15.7	3.8	-16.3	3.1
		Z	- 878629.6	11.5	5.7	9.7	5.4	11.7	5.0	10.1	5.2	11.6	5.0	12.3	5.1	10.7	3.8
59	Christmas Island	X	-5885347.6	33.8	5.0	25.4	7.1	7.8	3.9	6.6	3.9	7.1	3.8	- 5.8	3.9	7.1	3.1
		Y	-2448345.0	-24.8	4.3	-29.4	4.8	-34.7	4.0	-34.7	4.0	-29.8	3.9	-40.2	4.0	-32.0	3.1
		Z	221690.0	-21.8	5.8	-21.2	5.8	-24.6	5.4	-24.6	5.4	-27.0	5.2	-24.5	5.4	-27.5	3.9
60	Culgoora	X	-4751553.4	-58.0	4.3	-66.3	6.1	-89.9	3.3	-90.0	3.3	-90.2	3.3	-101.1	3.3	-86.5	2.7
		Y	2791993.4	27.8	3.7	30.9	4.6	42.1	3.2	42.4	3.4	46.4	3.2	49.0	3.2	49.4	2.5
		Z	-3200117.7	-34.6	4.4	-42.3	5.1	-52.4	3.6	-51.2	3.7	-51.	3.6	-59.7	3.6	-54.2	2.7

All units are in meters.

TABLE A.1 (cont'd)

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordinates: BC-D1	σ	BC-D2	σ	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
61	S. Georgia Island	X	2999889.2	- 2.5	4.6	2.5	5.3	14.7	4.2	15.3	4.2	14.6	4.2	21.3	4.2	18.2	3.3
		Y	-2219363.2	- 6.6	6.1	- 9.5	6.4	-18.8	5.9	-20.1	5.9	-18.3	5.8	-23.5	5.9	-15.7	4.6
		Z	-5155268.2	15.3	7.9	6.7	9.2	- 6.2	5.5	- 5.0	5.5	- 5.6	5.5	-18.8	5.5	- 4.1	4.7
63	Dakar	X	5884455.6	- 8.1	4.2	0.6	6.1	15.8	2.8	16.5	2.9	15.2	2.8	28.9	2.9	15.5	2.4
		Y	-1853486.3	- 7.0	3.6	- 8.3	3.3	-11.1	2.9	-14.4	3.3	-14.8	2.9	-13.6	2.9	-15.6	2.4
		Z	1612833.8	7.4	4.9	7.3	5.2	13.4	4.3	13.2	4.4	11.9	4.3	19.6	4.3	13.4	3.2
64	Fort Lamy	X	6023363.7	- 7.0	3.9	2.4	5.9	20.3	3.0	21.7	3.1	21.0	3.1	33.3	3.0	19.8	2.5
		Y	1617939.9	-14.6	3.5	-13.8	3.1	-12.6	2.9	- 9.1	3.2	-15.2	2.9	-11.3	2.9	-14.1	2.3
		Z	1331706.1	9.6	4.2	9.9	4.5	16.7	3.8	16.6	3.9	16.9	3.8	21.7	3.8	18.1	2.8
65	Hohenpeissenberg	X	4213551.5	- 3.0	3.3	- 1.7	4.6	12.4	2.3	18.8	2.7	10.6	2.3	22.1	2.3	9.9	1.9
		Y	820849.1	-15.8	3.4	-20.0	3.2	-18.6	3.0	-13.3	3.1	-23.9	2.9	-17.3	3.0	-23.0	2.4
		Z	4702734.5	2.5	5.1	25.4	5.6	40.7	2.6	31.1	3.3	41.5	2.6	51.9	2.6	40.6	2.2
66	Wake Island II	X	-5858501.6	-23.8	5.3	-33.0	7.4	-60.3	3.8	-60.9	3.8	-59.4	3.6	-74.0	3.9	-58.5	3.1
		Y	1394466.1	-34.6	5.1	-31.6	5.2	-22.3	4.7	-22.7	4.8	-13.9	4 3	-19.3	4.7	-17.0	3.4
		Z	2093775.9	30.9	6.1	36.2	6.4	44.4	5.2	44.1	5.2	42.9	4.8	48.4	5.2	41.5	3.7
67	Natal	X	5186389.3	- 9.7	5.1	- 2.0	6.7	14.0	4.2	14.1	4.2	12.9	4.2	26.1	4.2	16.0	3.3
		Y	-3653935.6	- 3.7	5.3	- 7.7	5.7	-15.8	4.6	-18.0	4.7	-18.8	4.5	-23.2	4.6	-17.4	3.5
		Z	- 654306.7	8.9	5.6	7.1	5.5	9.2	5.2	8.5	5.2	8.2	5.2	9.6	5.2	8.3	3.8
68	Johannesburg	X	5084799.2	- 7.7	4.6	0.5	6.5	24.0	3.8	24.3	3.8	24.7	3.8	36.1	3.8	23.4	3.1
		Y	2670327.0	-10.6	3.6	- 7.5	3.9	0.9	3.2	2.5	3.3	- 0.5	3.2	6.0	3.2	- 2.1	2.6
		Z	-2768104.8	28.2	5.7	21.6	6.2	8.9	4.9	11.2	4.9	7.8	4.8	2.0	4.9	11.6	3.7
69	Tristan Da Cunha	X	4978402.8	- 4.2	8.5	3.4	9.6	19.5	6.8	19.9	6.8	18.9	6.8	31.3	6.8	18.5	5.3
		Y	-1086867.2	- 6.9	7.3	- 7.6	7.2	-10.0	7.0	-12.7	7.1	-13.7	7.0	-11.0	7.0	-16.4	5.2
		Z	-3823192.5	9.1	10.4	3.2	10.7	- 0.9	8.1	- 0.2	8.1	- 1.0	8.1	- 9.7	8.1	1.7	6.3
72	Chiang Mai	X	- 941730.1	21.9	7.1	20.0	7.2	15.8	6.9	17.6	6.9	25.2	6.1	12.6	6.9	25.2	4.6
		Y	5967368.6	19.7	6.2	29.4	8.2	56.9	4.8	57.1	4.8	62.7	4.3	70.6	4.6	59.8	3.5
		Z	2039280.6	- 0.7	6.0	2.3	6.3	16.3	5.1	17.4	5.1	18.1	4.5	21.0	5.1	16.4	3.5

All units are in meters.

TABLE A.1 (cont'd)

Station No.	Station Name		Approximate Coordinates	Corrections to Approximate Coordinates BC-D1	c	BC-D2	c	BC-D3	σ	BC-D4	σ	BC-D6	σ	BC-D11	σ	BC-D13	σ
73	Diego Garcia	X	1905107.0	8.1	4.0	10.4	4.3	18.2	3.8	19.3	3.8	19.7	3.7	22.6	3.8	21.8	2.9
		Y	6032224.7	15.1	4.7	24.0	6.8	43.4	3.9	44.0	3.9	40.6	3.9	57.0	3.9	40.2	3.1
		Z	- 810739.1	8.4	4.9	5.4	4.9	3.3	4.5	5.2	4.5	3.9	4.4	1.2	4.5	7.5	3.3
75	Mahe	X	3602799.9	-12.3	4.5	- 7.3	5.6	7.4	4.1	7.9	4.1	10.6	4.1	16.4	4.1	10.0	3.3
		Y	5238233.9	-31.2	4.7	-23.7	6.3	- 6.9	4.0	- 6.1	4.0	- 8.5	3.9	4.7	4.0	-10.6	3.1
		Z	- 515970.0	23.1	4.8	20.5	4.8	19.6	4.4	21.4	4.4	20.0	4.3	18.3	4.4	22.4	3.2
78	Port Vila	X	-5952304.2	47.5	21.0	37.2	22.0	8.3	9.8	7.7	9.8	6.6	9.8	- 5.1	9.8	5.5	9.0
		Y	1231898.6	-11.0	9.9	-10.0	10.0	- 7.1	8.4	- 6.9	8.5	- 4.9	8.4	- 4.3	8.5	- 0.2	6.7
		Z	-1925944.7	-27.2	18.3	-29.6	18.3	-37.8	13.4	-36.0	13.4	-35.4	13.4	-44.3	13.4	-32.3	10.8
111	Wrightwood I	X	-2448865.4	11.1	4.2	9.0	4.1	7.1	3.6	2.5	3.7	3.7	3.0	3.6	3.6	1.7	2.5
		Y	-4667971.2	- 7.0	4.4	-11.6	5.4	-23.9	2.9	-28.5	3.3	-24.2	2.7	-34.0	2.9	-26.4	2.3
		Z	3582743.9	-14.5	5.5	- 3.9	6.2	9.2	3.4	1.1	4.1	7.0	3.2	19.8	3.4	3.8	2.7
123	Point Barrow	X	-1881801.8	12.6	14.1	11.7	14.0	6.0	13.5	3.5	13.5	- 3.9	5.3	2.8	13.5	- 5.1	4.1
		Y	- 812422.7	- 4.5	11.5	- 6.6	11.4	-10.7	11.3	- 8.7	11.3	-20.6	4.9	-13.3	11.3	-19.7	4.0
		Z	6019606.9	-33.0	17.8	-20.6	18.6	-24.2	7.2	-24.1	7.2	-25.7	5.0	-10.1	7.3	-23.7	4.4
134	Wrightwood II	X	-2448914.6	6.6	4.2	4.5	4.1	2.6	3.6	- 2.0	3.7	- 0.9	3.0	- 0.9	3.6	- 2.8	2.5
		Y	-4668062.8	- 5.5	4.4	-10.1	5.4	-22.5	2.9	-27.0	3.3	-22.7	2.8	-32.6	2.9	-24.9	2.3
		Z	3582433.7	- 9.6	5.5	0.9	6.2	14.1	3.4	6.0	4.1	11.9	3.2	24.6	3.4	8.5	2.7

All units are in meters.

APPENDIX B

FORMING NORMAL EQUATIONS USING CORRELATED OBSERVATIONS

The computer programs for the formation of normal equations using uncorrelated observations is described in detail in the OSUGOP Report [Reilly, Schwarz and Whiting, 1972]. The purpose of the computer programs described in this Appendix is to form reduced normal equations using the correlated satellite observations in the NGS/DOD (BC-4) worldwide network.

The programs described herein are not incorporated into the OSUGOP program. They are separate programs which result in the reduced normal equations punched on data cards. These cards are then input into the OSUGOP program for the adjustments. Even though these programs are physically separate from OSUGOP, many subroutines from OSUGOP have been incorporated. The program logic, up to the point of reading the observations, is the same as described in [Reilly, Schwarz and Whiting, 1972, pgs 7-9]. A separate subroutine had to be developed to read the Type II data from the magnetic tapes, sort and merge the observations, compute the matrix of correlation coefficients if needed, use either full correlation or no correlation, and store this data on two disks in such a way that the data could be processed. This is the subroutine READIN. The subroutine ASD360, from OSUGOP, was modified to read the data from one of the disks for the purpose of computing the approximate satellite positions, and from the second disk to form the A, B, W, M^{-1} and all other matrices described in Chapter 2 that are necessary in the development. The subroutine DEDIT, from OSUGOP, was modified to compute the approximate satellite positions from the observations and the approximate satellite positions. The subroutine FORMRN, also from OSUGOP, had to be modified to use larger sub-blocks in order to form the reduced normal equations. The names of these subroutines, even though modified, have not been changed in these programs. All other subroutines except for the driver, MAIN, are the same as described in the OSUGOP Report.

The reduced normal equations were formed by two different techniques, the generalized least squares and the method of observation equations. The only change necessary in going from one method to the other is one subroutine. If the observation equation method is used, subroutine ASD360 is replaced by a modified version to handle the observation equation mathematics. It is not necessary to have both subroutines in the same program, since they both perform the same task. The analyst should decide which technique to use. If it is desired to use images 1-3-5-7, subroutines READIN and ASD360 must be modified.

Input to the Program

As with OSUGOP, the input is made up of card packets. The deck setup is shown in Figure A.1. The entire program is input, either as a source program or object deck. The description of the title packet, datum packet and station coordinate packet are described in [Reilly, Schwarz and Whiting, 1972, pgs 5-6]. The problem codes card is used only to define PCODE(12) and PCODE(13), which are the following:

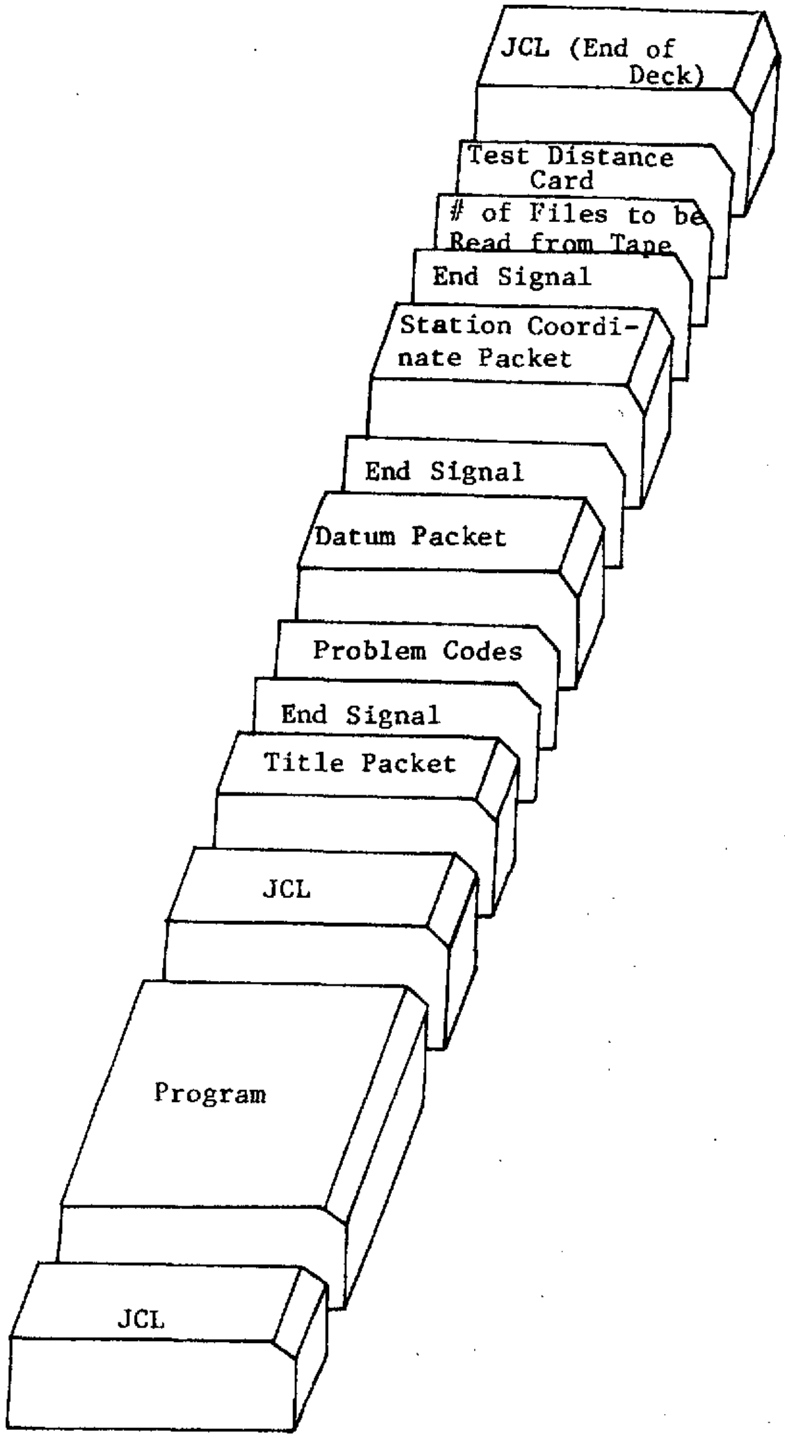

Figure A.1. Deck Setup for Formation of Normal Equations from the Type II Data

PCODE(12) = 3 means read only the diagonal elements of the variance-covariance matrix.

PCODE(13) = 1 means compute and print correlation coefficient matrix for each station.

There are two additional cards required in the data stream to complete the input deck. These are a card telling how many files (events) are to be read from the magnetic tape, and a second card with the test distance. The card formats are as follows:

File Card

Columns	Format	Contents
1-2	I2	The number of files to be read from the magnetic tape.

Test Distance Card

Columns	Format	Contents
1-10	F10.0	Rejection criteria, in seconds of arc, to be applied to each observation during the event adjustment.

The Type II data processed by these programs is on magnetic tapes, and the subroutine READIN will read all the data. For each file (or event) to be read from a tape, there must be a Data Definition (DD) statement in the Job Control Language (JCL). This means that if 90 events are to be read from a tape, there must be 90 DD statements. The program was designed to process the data from only one tape. After the data has been processed the set of normal equations for that particular tape will be punched onto cards. Experience has shown that when a data tape is being processed, the station coordinate packet should include only the coordinates of stations that have observations on that particular tape. Otherwise, blocks of zeros will appear with the punched output, with the number 1000 (not 999) at the end of each row, and these must be removed before the OSUGOP program performs an adjustment.

```
C
C              PROBLEM  CODE  DEFINITIONS
C
C     COLUMN    MEANING
C        1. OVERALL PROBLEM CODE
C           PCODE(1)=1  MEANS  OPTICAL  PROGRAM,GEOMETRIC MODE, OSU FORMAT
C                    2  MEANS  RANGE,GEOMETRIC MODE
C                    3  MEANS  SOLUTION ONLY RUN
C                    4 MEANS ORBITAL MODE,OPTICAL OBSERVATIONS
C                    5  MEANS ORBITAL MODE,RANGE OBSERVATIONS
C                    6  MEANS ORBITAL MODE,MIXED OBSERVATIONS.
C           PCODE(1)=7  MEANS  OPTICAL  PROGRAM,GEOMETRIC MODE, GEOS FORMAT
C        2. PERFORM SOLUTION?
C           PCODE(2)=1  MEANS YES
C                    0  MEANS NO
C           PCODE(1)=3 IMPLIES PCODE(2)=1
C        3. MAXIMUM NUMBER OF ITERATIONS?
C           PCODE(1) MUST EQUAL 1, 2, OR 7
C           PCODE(2) MUST EQUAL 1,
C           PCODE(5) MUST EQUAL 1,  FOR ONE OR MORE COMPLETE ITERATIONS
C        5. FORM NORMALS?
C
C              PROCESSING CODES
C                                       1 MEANS YES,  0 MEANS NO
C        6. SIMULATE GUIDE MATRIX?
C        7. PRINT NORMALS?
C        8. PERFORM  SUMMARY BY OBSERVED LINES?
C        9. PUNCH NORMALS IN ASD FORMAT?
C       10. SUMMARIZE RESULTS
C            PCODE(10)=0 DO NOT PRINT SUMMARY
C                     =1 PRINT THE DX'S AND STANDARD DEVIATIONS
C                     =2 PRINTS THE X,Y,Z'S AND STANDARD DEVIATIONS
C                     =3 PRINTS THE LATITUDE,LONGITUDE AND HEIGHT
C                     =4 PRINTS BOTH X,Y,Z & LAT.,LONG. & H
C       11. PRINT SATELLITE POSITION FOR EACH EVENT?
C            0 MEANS NO
C            1 MEANS PRINT XYZ AND GEODETIC COORDINATES
C            2 MEANS PRINT XYZ ONLY
C            3 MEANS PRINT GEODETIC COORDINATES ONLY
C       12. THIS PARAMETER DESCRIBES WHERE THE STANDARD DEVIATIONS OF THE
C            INDIVIDUAL OBSERVATIONS (USED TO FORM THE WEIGHTS) ARE TO BE FOUND
C            PCODE(12)=0 MEANS TO READ THE OBSERVATIONAL STANDARD DEVIATION
C            FROM THE CARD CONTAINING THE OBSERVATION.
C            PCODE(12)=1 MEANS TO ASSOCIATE A SINGLE STANDARD DEVIATION WITH
C            ALL OBSERVATIONS FROM A GIVEN STATION.** THE STANDARD DEVIATIONS
C            TO BE ASSOCIATID WITH EACH STATION ARE GIVEN IN COLUMNS 73-79 OF
C            THE CARD CONTAINING THE INPUT COORDINATES OF THE STATION.
C            PCODE(12)=2 MEANS TO ASSOCIATE A SINGLE STANDARD DEVIATION WITH
C            ALL OBSERVATIONS.** THIS NUMBER IS FOUND IN COLS. 21-30 OF THE
C            CARD CONTAINING THE TEST DISTANCE (OPTICAL) OR TEST VARIANCE
C            (RANGE).
C            ** IN THE CASE OF OPTICAL OBSERVARTIONS, THIS NUMBER IS INTERPRETE
C            AS THE STANDARD DEVIAION OF THE DECLINATION AND OF THE RIGHT
C            ASCENSION TIMES THE COSINE OF THE DECLINATION, AND THE
C            COVARIANCE IS SET TO ZERO.
C           PCODE(12)=3 MEANS TO READ ONLY THE DIAGONAL ELEMENTS OF THE
C           VARIANCE-COVARIANCE MATRIX (CPGS CORRELATED DATA ONLY)
C       13. COMPUTE AND PRINT CORRELATION MATRIX FOR EACH STATION
```

```
C           ( C&GS CORRELATED DATA ONLY).
C        CODES WHICH APPLY TO ORBITAL MODE PROCESSING ONLY
C         14. TREAT COORDINATES OF CENTER OF MASS AS UNKNOWNS? (ORBITAL MODE ONLY)
C         15. PUNCH UPDATED ORBIT ELEMENTS?  (ORBITAL MODE ONLY)
C
C
C          SOLUTION CODES
C
C         16. WRITE NORMALS AND INVERSE DURING SOLUTION PROCESSING?
C                0 MEANS PRINT NOTHING
C                1 MEANS PRINT PIVOT ELEMENTS
C                2 MEANS ALSO PRINT NORMALS AND INVERSE
C                3 MEANS ALSO PRINT REARRANGED NROMALS AND INVERSE
C         17. PUNCH ADJUSTED STATION XYZ AND VARIANCES FOR INPUT TO BADEKAS'
C                DATUM TRANSFORMATION PROGRAM?
C         18. PUNCH ADJUSTED STATION POSITIONS?
C         19. COMPUTE EIGENVECTORS OF VARIANCE-COVARIANCE MATRIX
C         20. COMPUTE CORRELATION COEFFICIENTS
C
      COMMON/NSTA/NSTA
      INTEGER*2 ENDSIG/1HE/,CONTIN
      INTEGER*2 PCODE(20)
      COMMON/PCODES/PCODE
      REAL*8 TITLE(10)
    3 CONTINUE
      WRITE(6,6001)
 6001 FORMAT(1H1,20(/))
    4 READ(5,5001) TITLE,CONTIN
 5001 FORMAT(9A8,A7,A1)
      IF(CONTIN.EQ.ENDSIG) GO TO 5
      WRITE(6,6012) TITLE
 6012 FORMAT(30X,9A8,A7)
      GO TO 4
    5 CONTINUE
C
      READ(5,5050) PCODE
 5050 FORMAT(80I1)
      WRITE(6,6050) PCODE
 6050 FORMAT(////10X,'PROBLEM CODES',10X,20I1)
      CALL STAIN
      CALL READIN
      CALL ASD360
      CALL FORMRN
      STOP
      END
```

```
      SUBROUTINE STAIN
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/PCODES/PCODE
      INTEGER ENDSIG/1HE/,CONTIN
      COMMON/NSTA/NSTA
      COMMON/STAORD/KORDER(150)
      INTEGER STANAM,IDS*2
      INTEGER*2 PLUS/1H+/
      INTEGER*2 ISGNP,IPHID,IPHIM,LONGD,LONGM,ISGNL
      COMMON/STALOC/STAUVW(3,150),DATPRM(2,15),DATNAM(4,15),
     1STANAM(5,150),IDS(150)
      COMMON/STAPLH/STAPLH(2,150)
      COMMON/OBSD/OBSD(150),OVOBSD
      MAXSTA=150
      WRITE(6,6000)
 6000 FORMAT(1H1)
 6001 FORMAT(1H1,20(/))
      WRITE(6,6002)
 6002 FORMAT(/////4X,29HDATUMS INVOLVED IN ADJUSTMENT,//)
C   INPUT DATUMS
   10 READ(5,5002) IDD,AE,BE,CONTIN
 5002 FORMAT(I2,2F12.3,53X,A1)
      IF(CONTIN.EQ.ENDSIG) GO TO 30
      DATPRM(1,IDD)=AE
      DATPRM(2,IDD)=BE
      READ(5,5003)(DATNAM(I,IDD),I=1,4)
 5003 FORMAT(4A8)
      WRITE(6,6003) IDD,(DATNAM(I,IDD),I=1,4),(DATPRM(I,IDD),I=1,2)
 6003 FORMAT(6HODATUM,I3,3X,4A8,3HA= ,F10.2,12H METERS  B= ,F10.2,
     17H METERS)
      GO TO 10
C
   30 CONTINUE
C          STATION INPUT
      WRITE(6,6005)
 6005 FORMAT(1H1///40X,29HINPUT COORDINATES OF STATIONS)
      KSTA=0
   35 KSTA=KSTA+1
      READ(5,5005)IDD,IDTS,(STANAM(I,KSTA),I=1,5),ISGNP,IPHID,IPHIM,PHIS
     1,LONGD,LONGM,FLONGS,H,CONTIN
 5005 FORMAT(I4,I2,4A4,A2,A1,2(2I3,F8.4), F10.2,16X,A1)
      IF(CONTIN.EQ.ENDSIG) GO TO 50
      PHI=ANRADD(ISGNP,IPHID,IPHIM,PHIS)
      ISGNL=PLUS
      FLONG=ANRADD(ISGNL,LONGD,LONGM,FLONGS)
      KORDER(KSTA)=IDD
      IDS(KSTA)=IDTS
      STAPLH(1,KSTA)=PHI
      STAPLH(2,KSTA)=FLONG
      CALL UVWD(DATPRM(1,IDTS),DATPRM(2,IDTS),PHI,FLONG,H,STAUVW(1,KSTA)
     1,STAUVW(2,KSTA),STAUVW(3,KSTA))
      WRITE(6,6006)IDD,(STANAM(I,KSTA),I=1,5),IDTS,(DATNAM(I,IDTS),I=1,4
     1),ISGNP,IPHID,IPHIM,PHIS,ISGNL,LONGD,LONGM,FLONGS,H
 6006 FORMAT(1H0,I4,8X,4A4,A2,10X,5HDATUM,I4,4X,4A8/10X,20HGEODETIC COOR
     1DINATES,2(6X,A1,2I3,F8.4),F12.4)
      WRITE (6,6007) (STAUVW(I,KSTA),I=1,3)
 6007 FORMAT(10X,21HCARTESIAN COORDINATES,3F16.3)
      GO TO 35
   50 CONTINUE
      NSTA=KSTA-1
      NSTAUN=3*NSTA
      RETURN
      END
```

```
      SUBROUTINE READIN
C     THIS SUBROUTINE IS USED TO READ THE DATA FROM THE ORIGINAL TAPE
C     AND PUT THE DATA ON UNITS 3 AND 4 FOR PROCESSING
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION SIGAE(14,14), GHA(7), RDEC(7), ALFS(28), DEC(28),
     1KSTATE(50),ID(4),COR(14,14)
      DIMENSION TEMP1(28),TEMP2(28)
      INTEGER*4 CONTIN/1H /,ENDSIG/1HE/
      INTEGER*2 IGHH,IGHM,IDCH,IDCM
      COMMON/NSTA/NSTA
      INTEGER*2 PCODE(20)
      COMMON/PCODES/PCODE
C
C
      PI  = 3.1415926535897973D0
      SPR  =(180.D0 *3600.D0)/PI
      KT=0
      REWIND 3
      REWIND 4
      READ(5,5015) KOUNT
      READ(5,5020) TO
      WRITE(3) TO
 5020 FORMAT(F10.0)
  125 KT=KT+1
      N=1
    1 READ(8,5000 ,ERR=50,END=100) NEVENT,NSTE,NOPTS
 5000 FORMAT(1X,I5,I1,I2)
      WRITE(6,7000) NEVENT,NSTE,NOPTS
 7000 FORMAT(1X,I5,I1,I2)
      IS=1
      NN=NSTE*7
    2 READ(8,5005) IST,NUPTS
 7005 FORMAT(1X,I6,28X,I2)
 7006 FORMAT(I6,28X,I2)
      ID(N)=IST
      KSTA=KSTAID(IST)
      IF(KSTA.EQ.0) GO TO 80
      KSTATE(N)=KSTA
 5005 FORMAT(I6,28X,I2)
      NO=2*NUPTS
      READ(8,5010)((SIGAE(I,J),J=I,NO),I=I,NO)
 5010 FORMAT(4E20.13)
 7010 FORMAT(1H ,4E20.13)
 7011 FORMAT(4E20.13)
      IF(PCODE(13).EQ.0) GO TO 200
      DO 49 I=1,14
   49 COR(I,I)=1.0
      DO 51 I=1,13
      K=I+1
      DO 51 J=K,14
   51 COR(I,J)=SIGAE(I,J)/DSQRT(SIGAE(I,I)*SIGAE(J,J))
      WRITE(6,6396) NEVENT,IST
      DO 53 I=1,14
      DO 53 J=1,14
   53 COR(J,I)=COR(I,J)
      DO 52 I=1,14
   52 WRITE(6,6397)(COR(I,J),J=1,14)
 6396 FORMAT(1H ,'EVENT NO.',I5,5X,'STATION NO.',I5)
```

```
6397 FORMAT(1H ,14F9.3)
 200 DO 5 I=1,NUPTS
     READ(8,5015) IUPTS,GHA(IUPTS),RDEC(IUPTS)
5015 FORMAT(I2,2E16.9)
7015 FORMAT(1H ,I2,2F16.9)
   5 CONTINUE
     K=1
     DO  6 I=1,13,2
     SIGAE(I,I)=SIGAE(I,I)*DCOS(RDEC(K))**2
   6 K=K+1
     IF(PCODE(12).EQ.3) GO TO 300
     K=1
     DO 8 I=1,13,2
     M=I+1
     DO 7 J=M,14,2
   7 SIGAE(I,J)=SIGAE(I,J)*DCOS(RDEC(K))
   8 K=K+1
     K=1
     L=2
     DO 10 I=1,13,2
     M=I+2
     IF(M.GT.14) GO TO 10
     DO 9 J=M,13,2
     SIGAE(I,J)=SIGAE(I,J)*DCOS(RDEC(K))*DCOS(RDEC(L))
   9 L=L+1
     K=K+1
  10 L=K+1
     K=2
     L=2
     DO 12 I=2,12,2
     M=I+1
     DO 11 J=M,13,2
     SIGAE(I,J)=SIGAE(I,J)*DCOS(RDEC(K))
  11 K=K+1
     L=L+1
  12 K=L
 520 DO 4 I=1,14
     DO 4 J=I,14
     SIGAE(J,I)=SIGAE(I,J)
   4 CONTINUE
     GO TO 530
 300 DO 527 I=1,14
     DO 527 J=1,14
 527 COR(I,J)=SIGAE(I,J)
     DO 528 I=1,14
     DO 528 J=1,14
 528 SIGAE(I,J)=0.0D0
     DO 529 I=1,14
 529 SIGAE(I,I)=COR(I,I)
 530 CONTINUE
C
     J=1
     DO 20 I=N,NN,NSTE
     ALFS(I)=GHA(J)
     DEC(I)=RDEC(J)
     J=J+1
  20 CONTINUE
     WRITE(3) NEVENT,NSTE,IST,NUPTS,SIGAE,(GHA(IS),RDEC(IS),IS=1,NUPTS)
```

```
     1,CONTIN
C      WRITE(6,6847) NEVENT,IST,NSTE
 6847 FORMAT(1H ,3I5)
 6910 FORMAT(1H ,4D22.14)
      IF(N.EQ.NSTE) GO TO 60
      N=N+1
      GO TO 2
   60 WRITE(4) NEVENT,NSTE,NN,(ALFS(IS),DEC(IS),IS=1,NN),(KSTATE(L),L=1,
     *NSTE),(ID(L),L=1,NSTE),CONTIN
      GO TO 1
   80 WRITE(6,6398) IST
 6398 FORMAT(1HO,'STATION',I5,1X,'NOT FOUND IN INPUT LIST')
      STOP
   50 WRITE(6,75)
   75 FORMAT('ERROR FOUND WHILE READING TAPE')
  100 CONTINUE
      IF(KT.LT.KOUNT) GO TO 125
      BACKSPACE 3
      WRITE(3) NEVENT,NSTE,IST,NUPTS,SIGAE,(GHA(IS),RDEC(IS),IS=1,NUPTS)
     1,ENDSIG
C      RETURN
      END
```

```
      SUBROUTINE ASD360
C   S/360 VERSION OF ASD PROGRAM FOR OPTICAL SATELLITE DIRECTIONS
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER*2 PCODE(20)
      COMMON/PCODES/PCODE
      INTEGER*4 ENDSIG/1HE/,CONTIN,DELCOD(2)/1H ,1H*/,ECODE
      COMMON/NSTA/NSTA
      COMMON/DEDITC/ALFS( 4),DEC( 4),          S(3),D( 4),SDC(3, 4),EVSUM,
     1      STAXYZ(3,50),GOI,
     2TD,KSTATE(50),IPASS(50),NSTE,NSUSED,ECODE
      INTEGER STANAM,IDS*2
      COMMON/STALOC/STAUVW(3,150),DATPRM(2,15),DATNAM(4,15),
     1STANAM(5,150),IDS(150)
      COMMON/STAORD/KORDER(150)
      DIMENSION SSDC(21,4),AW(21),A(21,21),AT(21,21),DDN(21,21),W(21,21)
     1,WW(14,14),AL(28),OC(28),NOBSTA(150),A1(21,3),A1T(3,21),ID(4),
     2        DDK(21),TMP(14,14),          O1(21,21),AM(21,21),
     3BT(21,21),TEMP1(21,21),TEMP2(21,21),BN(3,21,4),TEMP3(21),
     4DN(3,3,50),DK(3,50),XX(3),               AK1(21),TA(21)
      DIMENSION VPV(4)
      COMMON/WPW/WPW,XPU,IDEGF,NFSTA
      REAL*4 VPVSTA(150)
      MAXSTE=50
      PI  = 3.14159265358979730D0
      PI2  = 2.D0*PI
      RPD  = 180.D0/PI
      SPR  =(180.D0 *3600.D0)/PI
      WPWSP=0.0
C
      REWIND 2
      REWIND 3
      REWIND 4
      READ(3) TD
      WRITE(6,6004) TD
 6004 FORMAT(//20X,'TEST DISTANCE =',F20.2,'    SECONDS OF ARC')
C
      KEVENT=0
      EPR=0.0
      DO 70 KSTA=1,NSTA
      NOBSTA(KSTA)=0
      VPVSTA(KSTA)=0.0
      DO 70 I=1,3
      DK(I,KSTA)=0.
      DO 70 J=1,3
      DN(I,J,KSTA)=0.
   70 CONTINUE
      DO 80 I=1,21
      DO 80 J=1,3
      A1(I,J)=0.0
      A1T(J,I)=0.0
   80 CONTINUE
      J=1
  313 DO 314 I=J,21,3
      A1T(J,I)=-1.0
  314 A1(I,J)=-1.0
      J=J+1
      IF(J.LT.4) GO TO 313
  317 CONTINUE
```

```
      DO 81 I=1,21
      DO 81 J=1,21
      A(I,J)=0.0
   81 CONTINUE
      DO 82 I=1,21
      DO 82 J=1,21
      W(I,J)=0.0
   82 CONTINUE
C     START DATA INPUT
  210 CONTINUE
      READ(4) IEVENT,NSTE,NN,(AL(IS),DC(IS),IS=1,NN),(KSTATE(L),L=1,
     1NSTE),(ID(IS),IS=1,NSTE),CONTIN
 6325 FORMAT(1H ,2D21.14)
      DO 272 IS=1,NSTE
      KSTA=KSTATE(IS)
      DO 272 M=1,3
  272 STAXYZ(M,IS)=STAUVW(M,KSTA)
      WRITE(6,6008) KEVENT,IEVENT                                       JPR
 6008 FORMAT(/ 1X,'EVENT',I6,5X,I6)                                     JPR
      LL=0
      JJ=0
      KEVENT=KEVENT+1
      DO 301 K=1,7
      DO 275 I=1,NSTE
      L=I+LL
      ALFS(I)=AL(L)
      DEC(I)=DC(L)
  275 CONTINUE
      CALL DEDIT
      DO 280 IS=1,NSTE
  280 WRITE(6,6010) ID(IS),ALFS(IS),DEC(IS),D(IS),DELCOD(IPASS(IS))
 6010 FORMAT(I7,20X,F15.7,5X,F15.7,5X,F10.1,2X,A1)
      DO 305 I=1,3
      J=I+JJ
      DO 305 IS=1,NSTE
      SSDC(J,IS)=SDC(I,IS)
  305 CONTINUE
      LL=LL+NSTE
      JJ=JJ+3
      IF(FCODE.GT.1) GO TO 630
      IF(PCODE(11)) 610,630,610
  610 IF(PCODE(11)-3) 611,612,611
  611 WRITE(6,6024) S
 6024 FORMAT('  SATELLITE POSITION',3F15.3)
      IF(PCODE(11)-2) 612,630,612
  612 IDTS=IDS(KSTATE(1))
      CALL UVWTG2(S,DATPRM(1,IDTS),PHI,FLAM,H)
      PHI=PHI*RPD
      FLAM=FLAM*RPD
      WRITE(6,6023) PHI,FLAM,H
 6023 FORMAT('  GEOD. COORD. OF SATELLITE',2F14.6,F14.1)
  630 CONTINUE
      WRITE(6,6012) GQI
 6012 FORMAT(10X,'GQI=',F10.5)
      IF(FCODE.GT.1) GO TO 290
      IF(NSUSED.EQ.0) GO TO 290
      RMSMC=DSQRT(EVSUM/DFLOAT(NSUSED))
      WRITE(6,6011) RMSMC
```

```
 6011 FORMAT(1H+,27X,'RMS MISCLOSURE IN METERS=',F10.1)
      GO TO 300
  290 WRITE(6,6015) ECODE
 6015 FORMAT(1H+,27X,'ENTIRE EVENT DELETED, KODE=',I4)
      GO TO 302
C
C     SET UP OBSERVATION EQUATIONS FOR THIS EVENT AND COMPUTE CONTRIBUTOONS
C        TO THE NORMAL EQUATIONS
  300 CONTINUE
  301 CONTINUE
 6903 FORMAT(1H ,6D20.13)
      DO 310 I=1,21
      AW(I)=0.0
      DDK(I)=0.
      DO 310 J=1,21
      DDN(I,J)=0.0
  310 CONTINUE
  302 CONTINUE
C
      JS=0
      DO 390 IS=1,NSTE
      READ(3) NEVENT,NSTB,IST,NUPTS,WW,(AL( I),  DC( I), I=1,NUPTS),
     *CONTIN
      IF(IPASS(IS).GT.1) GO TO 390
      CALL VERSOL(WW,TMP,14,14)
      K=1
      L=2
      N=1
    3 M=1
      DO 10 I=K,L
      DO 5 J=1,21,3
      W(I,J)=TMP(N,M)
      W(I,J+1)=TMP(N,M+1)
      W(I,J+2)=0.
      M=M+2
    5 CONTINUE
      N=N+1
      M=1
   10 CONTINUE
      K=K+3
      L=K+1
      IF(K.GT.21) GO TO 15
      GO TO 3
   15 CONTINUE
      DO 303 I=1,NUPTS
  303 AL(I)=PI2-AL(I)
 6900 FORMAT(1H ,7D15.8)
      JS=JS+1
C     JS IS THE COUNTER FOR NON DELETED STATIONS IN THE EVENT.
      I=1
      J=1
      L=1
  311 M=L+1
      N=L+2
      RSQCSD=SSOC(L,IS)**2+SSOC(M,IS)**2
      RSQ=RSQCSD+SSOC(N,IS)**2
      RCD=DSQRT(RSQCSD)
      RANGE=DSQRT(RSQ)
```

```
      SG=DSIN(AL(I))
      CG=DCOS(AL(I))
      SD=DSIN(DC(I))
      CD=DCOS(DC(I))
      A(L,L)=SD*CG*RANGE
      A(L,M)=SG*CD*RANGE
      A(L,N)=-CD*CG*RANGE
      A(M,L)=SD*SG*RANGE
      A(M,M)=-CG*CD*RANGE
      A(M,N)=-CD*SG*RANGE
      A( N,L)=-CD*RANGE
      A( N,M)=0.
      A( N,N)=-SD*RANGE
      AW(L)=       SSDC(L,IS)-RANGE*DCOS( AL(I))*DCOS(DC(I))
      AW(M)=       SSDC(M,IS)-RANGE*DSIN(AL(I))*DCOS(DC(I))
      AW(N)=       SSDC(N,IS)-RANGE*DSIN(DC(I))
      I=I+1
      J=J+2
      L=L+3
      IF(J.EQ.15) GO TO 312
      GO TO 311
  312 CONTINUE
      KSTA=KSTATE(IS)
C    ELIMINATE  DELETED STATIONS FROM THE LIST OF STATIONS INVOLVED IN
C        THE EVENT.
      KSTATE(JS)=KSTATE(IS)
C
      CALL VERSOL(A,BT,21,21)
      DO 940 I=1,21
      DO 940 J=1,21
  940 A(I,J)=BT(I,J)
      DO 821 I=1,21
      DO 821 J=1,21
  821 AT(J,I)=A(I,J)
      CALL DGMPRD(AT, W,TEMP1,21,21,21)
      CALL DGMPRD(TEMP1,A,TEMP2,21,21,21)
      CALL DGMPRD(A1T,TEMP2,BN(1,1,JS),3,21,21)
      CALL DGMPRD(TEMP2,AW,TEMP3,21,21,1)
      DO 915 I=1,3
      DO 915 J=1,21
  915 BN(I,J,JS)=-BN(I,J,JS)
      DO 916 I=1,21
      DDK(I)=DDK(I)+TEMP3(I)
 6910 FORMAT(1H ,4D22.14)
      DO 916 J=1,21
  916 DDN(I,J)=DDN(I,J)+TEMP2(I,J)
      DO 330 I=1,3
      DO 325 J=1,3
      TERM=0.0
      DO 320 II=1,21
      DO 320 JJ=1,21
  320 TERM=TERM+A1(II,I)*TEMP2(II,JJ)*A1(JJ,J)
      DN(I,J,KSTA)=DN(I,J,KSTA)+TERM
  325 CONTINUE
      TERM=0.0
      DO 328 II=1,21
      DO 328 JJ=1,21
  328 TERM=TERM+A1(II,I)*TEMP2(II,JJ)*AW(JJ)
```

```
      DK(I,KSTA)=DK(I,KSTA)-TERM
  330 CONTINUE
      CALL DGMPRD(SSDC(1,IS),TEMP2,AK1,1,21,21)
      CALL DGMPRD(AK1,SSDC(1,IS),VPVTO,1,21,1)
      WRITE(6,938) VPVTO
  938 FORMAT(1H ,'NEW VPV',D20.12)
      KNO=KORDER(KSTA)                                                  JPR
C     WRITE(7,7043) NEVENT,KNO,VPVTO                                    JPR
 7043 FORMAT(5X,2I5,D15.7)                                              JPR
      VPV(IS)=VPVTO
      VPVSTA(KSTA)=VPVSTA(KSTA)+VPVTO
      NOBSTA(KSTA)=NOBSTA(KSTA)+14
  390 CONTINUE
      IF(ECODE.LT.2) GO TO 670
      KEVENT=KEVENT-1
      GO TO 602
C
C         FORM REDUCED NORMAL EQUATIONS.
C
C INVERT DDN
  670 CALL VERSOL(DDN,BT,21,21)
      CALL DGMPRD(DDK,BT,TA,1,21,21)
      CALL DGMPRD(TA,DDK,TB,1,21,1)
      WRITE(6,939) TB
  939 FORMAT(1H ,'WPW CONTRIBUTION FROM SATELLITE POSITIONS',D20.12)
      VPVS=0.
      DO 943 I=1,NSTE
      VPVS=VPVS+VPV(I)
      IF(VPV(I).LT.100000.) GO TO 943
      CALL DEIGN1(DDN,RR,21,1)
      PNO=DDN(1,1)/DDN(21,21)
      IF(PNO.GT.0.) GO TO 943
      KSTA=KSTATE(I)
      KNO=KORDER(KSTA)
      WRITE(6,6985) KNO,PNO
 6985 FORMAT(1H0,'THE P NUMBER FOR STATION NO.',I5,1X,'IS',D16.8)
      GO TO 944
  943 CONTINUE
      TEST=VPVS-TB
      IF(TEST.GT.0.) GO TO 942
  944 DO 941 I=1,NSTE
      KSTA=KSTATE(I)
      VPVSTA(KSTA)=VPVSTA(KSTA)-VPV(I)
  941 NOBSTA(KSTA)=NOBSTA(KSTA)-14
      ECODE=2
      KEVENT=KEVENT-1
      WRITE(6,6983)
 6983 FORMAT(1H ,'THE ABOVE EVENT WAS REJECTED BECAUSE OF POOR CONDITION
     1ING')
      WRITE(6,6984) PNO
 6984 FORMAT(1H0,'THE P NUMBER FOR ONE OF THE STATIONS IN THE ABOVE EVEN
     1T IS',D16.8)
      GO TO 602
  942 CONTINUE
      WPWSP=WPWSP+TB
      NSUSED=JS
      WRITE(2) NSUSED,BT,DDK,(((BN(I,J,JS),I=1,3),J=1,21),KSTATE(JS),
     1JS=1,NSUSED),CONTIN
```

```
  602 CONTINUE
C
C     TEST FOR END OF INPUT
      IF(CONTIN.EQ.ENDSIG) GO TO 700
      GO TO 210
C
C
C
  700 CONTINUE
C
CHECK TO SEE IF END SIGNAL HAS BEEN WRITTEN ON DATA SET FT02
      IF(FCODE.EQ.1) GO TO 710
      BACKSPACE 2
C   READ AND REWRITE LAST RECORD FROM LAST GOOD EVENT
       READ(2) NSUSED,BT,DDK,(((BN(I,J,JS),I=1,3),J=1,21),KSTATE(JS),
     1JS=1,NSUSED)
      BACKSPACE 2
      WRITE(2) NSUSED,BT,DDK,(((BN(I,J,JS),I=1,3),J=1,21),KSTATE(JS),
     1JS=1,NSUSED),CONTIN
  710 CONTINUE
      WRITE(2)        ((( DN(I,J,KSTA),I=1,3),DK(J,KSTA),J=1,3),
     XKSTA=1,NSTA)
C     WRITE(6,6018)(KORDER(KSTA),((DN(I,J,KSTA),J=1,3),I=1,3),
C    1KSTA=1,NSTA)
 6018 FORMAT((I5/3(3D18.7/)))
      WPW=0.0
      NOBS=0
      WRITE(6,6019)
 6019 FORMAT(1H1,8(/),10X,'ANALYSIS OF MISCLOSURES BY STATION'//
     1T10,'STATION',T20,'NUMBER OF OBSERVATIONS',T50,'RMS MISCLOSURE')
      DO 750 KSTA=1,NSTA
      NOBS=NOBS+NOBSTA(KSTA)
      WPW=WPW+VPVSTA(KSTA)
      RMSMC=0.0
      IF(NOBSTA(KSTA).GT.0) RMSMC=DSQRT(VPVSTA(KSTA)/DFLOAT(NOBSTA(KSTA)
     1))
      WRITE(6,6020) KORDER(KSTA),NOBSTA(KSTA),RMSMC
 6020 FORMAT(T10,I7,T35,I7,T50,F14.2)
  750 CONTINUE
      IDEGF=NOBS-21*KEVENT
      RMSMC=DSQRT(WPW/DFLOAT(IDEGF))
      WRITE(6,6021) NOBS,KEVENT,IDEGF,WPW,RMSMC
 6021 FORMAT(////10X,'TOTAL NUMBER OF GOOD OBSERVATIONS',T60,I8//
     110X,'TOTAL NUMBER OF GOOD EVENTS',T60,I8,//
     210X,'CORRESPONDING DEGREES OF FREEDOM',T60,I8//
     310X,'TOTAL SUM OF SQUARES OF MISCLOSURES',T60,F11.2//
     410X,'CORRESPONDING STANDARD DEVIATION OF UNIT WEIGHT',T60,F11.2)
      WPW=WPW-WPWSP
      WRITE(6,6022) WPW
 6022 FORMAT(1H0,9X,'WPW INCLUDING CONTRIBUTION FROM SATELLITE POSITION'
     1/15X,'(I.E., VPV+UX)',T60,F11.2)
      RETURN
      END
```

```
      SUBROUTINE UVWD(A,B,PHI,LAMDA,H,U,V,W)
      DOUBLE PRECISION PHI,LAMDA,N,E2,FAC,U,V,W,SP
      REAL*8 A,B,H
      E2=1.0-(B/A)**2
      SP=DSIN(PHI)
      N=A/DSQRT(1.0-E2*SP*SP)
      FAC=(N+H)*DCOS(PHI)
      U=FAC*DCOS(LAMDA)
      V=FAC*DSIN(LAMDA)
      W=(N*(1.0-E2)+H)*SP
      RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION DPDOT(X,Y,N)
      DOUBLE PRECISION X(N),Y(N)
      DPDOT=0.0
      DO 10 I=1,N
   10 DPDOT=DPDOT+X(I)*Y(I)
      RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION ANRADD(ISGN,IDEG,MIN,SEC)
      INTEGER*2 MINUS/1H-/, PLUS/1H+/,AMPSAN/1H&/,ISGN,IDEG,MIN
      DOUBLE PRECISION SEC
      IF(IDEG.GE.0) GO TO 10
      ISGN=MINUS
      IDEG=-IDEG
   10 CONTINUE
      ANRADD=(DFLOAT((IDEG*60+MIN)*60)+SEC)/206264.8062500
      IF(ISGN.EQ.MINUS)ANRADD=-ANRADD
      IF(ISGN.EQ.AMPSAN) ISGN=PLUS
      RETURN
      END
```

```
      SUBROUTINE UVWTG2(UVW,DATUM,PHI,LAM,H)
C     CONVERT RECTANGULAR TO GEODETIC COORDINATES
C     ALIAS FOR UVWTG
      IMPLICIT REAL*8(A-Z)
      DIMENSION UVW(3),DATUM(2)
      LAM=DATAN2(UVW(2),UVW(1))
      IF(LAM.LT.0.0) LAM=LAM+6.2831853071795800
      OME2=(DATUM(2)/DATUM(1))**2
      E2=1.0-OME2
      P=DSQRT(UVW(1)**2+UVW(2)**2)
      WP=UVW(3)/P
      TP1=WP/OME2
      PHI1=DATAN(TP1)
    5 TTP=TP1*TP1
      SECP=DSQRT(1.0+TTP)
      N=DATUM(1)*SECP/DSQRT(1.0+OME2*TTP)
      H=P*SECP-N
      TP2=WP/(1.0-E2*N/(N+H))
      PHI=DATAN(TP2)
      IF(DABS(PHI-PHI1).LT.1.D-12) RETURN
      PHI1=PHI
      TP1=TP2
      GO TO 5
      END
```

```
      SUBROUTINE DEDIT
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/DEDITC/ALFS( 4),DEC( 4),         S(3),D( 4),SDC(3, 4),  SUM,
     1     STAXYZ(3,50),GQI,
     2TD,KSTATE(50),IPASS(50),NSTE,NSUSED,KODE
C  EDIT DATA BASED ON PRELIMINARY STATION POSITIONS AND DELETE BAD
C     OBSERVATIONS AND BAD EVENTS,BASED ON THE DISTANCE CRITERION TO
C  THIS SUBROUTINE IS DIMENSION FOR A MAXIMUM OF MAXSTE=50 STATIONS
C     PARTICIPATING IN ANY ONE EVENT. ALL AFFECTED ARRAYS ARE IN
C     COMMON BLOCK /DEDITC/.
C
C  THE NUMBER OF STATIONS PARTICIPATING IN THE EVENT IS NSTE.
C  THE NUMBER OF STATIONS NOT DELETED IS NSUSED.
C
      COMMON/STALOC/STAUVW(3,150)
      DIMENSION Q(3,3),RHS(3),QI(3,3),VI(3),U(3,4)
C
      PI=3.141592653589793D0
      TPI=2.*PI
      MAXSTE=50
C  INITIALIZE
      KODE=1
      DO 110 IS=1,NSTE
  110 IPASS(IS)=1
C          IPASS=1 MEANS THIS DIRECTION OK
C          IPASS=2 MEANS THIS DIRECTION DELETED FROM EVENT
C
C  FORM UNIT VECTORS FOR ALL DIRECTIONS IN THIS EVENT
      DO 125 IS=1,NSTE
      STS=TPI-ALFS(IS)
      CA=DCOS(STS)
      SA=DSIN(STS)
      CD=DCOS(DEC(IS))
      SD=DSIN(DEC(IS))
      U(1,IS)=CA*CD
      U(2,IS)=SA*CD
      U(3,IS)=SD
  125 CONTINUE
C
C  INITIALIZE ARRAYS FOR THIS ITERATION
  130 CONTINUE
      NSUSED=0
      DO 140 I=1,3
      RHS(I)=0.0
      S(I)=0.0
      DO 140 J=1,3
      Q(I,J)=0.0
  140 CONTINUE
C
C  ACCUMULATE EQUATIONS
      DO 190 IS=1,NSTE
      IF(IPASS(IS).EQ.2) GO TO 190
      NSUSED=NSUSED+1
      DO 170 I=1,3
      DO 169 J=1,3
  169 QI(I,J)=U(I,IS)*U(J,IS)
  170 QI(I,I)=QI(I,I)-1.0
      DO 175 I=1,3
```

```
      DO 175 J=1,3
      Q(I,J)=Q(I,J)+QI(I,J)
      RHS(I)=RHS(I)+QI(I,J)*STAXYZ(J,IS)
  175 CONTINUE
  190 CONTINUE
C
C TEST FOR DELETION OF WHOLE EVENT
      IF(NSUSED.LT.2) GO TO 420
C
C  INVERT AND SOLVE
C     THE SATELLITE POSITION S IS SELECTED IN SUCH A WAY THAT THE SUM OF
C     THE SQUARES OF THE DISTANCES FROM S OF THE NON-DELETED RAYS IS MINIMIZED.
      DET=1.0
      CALLDMINV(Q,3,DET,QI(1,1),QI(1,2))
      GQI=DABS(DET/DFLOAT(NSUSED))
      IF(GQI.LT.1.0D-4) GO TO 430
      CALL DGMPRD(Q,RHS,S,3,3,1)
C
C  COMPUTE DISTANCE FROM S FOR EACH RAY
      ISMAX=0
      DMAX=0.0
      SUM=0.0
      DO 280 IS=1,NSTE
      DO 270 I=1,3
      DO 269 J=1,3
  269 QI(I,J)=U(I,IS)*U(J,IS)
      QI(I,I)=QI(I,I)-1.0
      VI(I)=S(I)-STAXYZ(I,IS)
  270 CONTINUE
      DDI=DPDOT(VI,U(1,IS),3)
      DDI=DABS(DDI)
      DI=0.0
      DO 275 I=1,3
      DI=DI+(VI(I)-DDI*U(I,IS))**2
      SDC(I,IS)=VI(I)
  275 CONTINUE
      D(IS)=DSQRT(DI)/DDI*206264.80625
      IF(IPASS(IS).EQ.2) GO TO 280
      SUM=SUM+DI
C  TEST D AGAINST TD AND DELETE IF NECESSARY
      IF(D(IS).LT.DMAX) GO TO 280
      DMAX=D(IS)
      ISMAX=IS
  280 CONTINUE
      IF(DMAX.LT.TD) RETURN
      IPASS(ISMAX)=2
C
C   GO BACK AND MAKE ANOTHER PASS THROUGH THE DATA
      GO TO 130
  400 CONTINUE
C  DELETE WHOLE EVENT
      DO 410 IS=1,NSTE
  410 IPASS(IS)=2
      NSUSED=0
      RETURN
  420 CONTINUE
C  DELETE FOR INSUFFICIENT GOOD OBSERVATIONS
      KODE=2
      GO TO 400
C   DELETE FOR INSUFFICIENT GEOMETRICAL SEPARATION BETWEEN OBSERVATIONS
  430 CONTINUE
      KODE=3
      GO TO 400
      END
```

```
      INTEGER FUNCTION KSTAID(ID)
      COMMON/STAORD/KORDER(150)
      COMMON/NSTA/NSTA
      KSTAID=0
C   SEARCH TABLE OF STATION IDENTIFIERS FOR THE INTERNAL NUMBER OF THIS STATION
      DO 10 I=1,NSTA
      IF(KORDER(I).NE.ID) GO TO 10
      KSTAID=I
      RETURN
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DMSTR(A,R,N,MSA,MSR)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),R(1)
      DO 20 I=1,N
      DO 20 J=1,N
      IF(MSR) 5,10,5
    5 IF(I-J) 10,10,20
   10 CALL LOC(I,J,IR,N,N,MSR)
      IF(IR) 20,20,15
   15 R(IR)=0.0
      CALL LOC(I,J,IA,N,N,MSA)
      IF(IA) 20,20,18
   18 R(IR)=A(IA)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FORMRN
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NSTA/NSTA
      INTEGER*2 PCODE(20)
      COMMON/PCODES/PCODE
      COMMON/WPW/WPW,XPU,IDEGF,IFSTA
      DIMENSION DDN(21,21),DDK(21),L1(3),L2(3),BNDDNI(3,21),TN(3,3),
     1TK(3),CN(3,21,4),DN(21,3),DDL(3)
      INTEGER*2 L,LSOLVE
      INTEGER CONTIN,ENDSIG/1HE/
      COMMON/STAORD/KORDER(150)
      COMMON/NORMEQ/LSOLVE
      DIMENSION REDN(3,3,1275),U(3,50),L(1275)
      DIMENSION BN(3,3,50),LG(50)
C  FORM REDUCED NORMAL ECUATIONS FOR UP TO 50 STATIONS
      DIMENSION KSTATE(50)
      LOC(K)=(K*(K+1))/2
      MAXSTA=50
      IF(NSTA.GT.MAXSTA) GO TO 901
C
C   THE REDUCED NORMAL EQUATIONS ARE STORED AS 3 X 3 BLCCKS IN THE ARRAY REDN.
C   ONLY THE UPPER TRIANGULAR PART OF THE REDUCED NORMAL EQUATIONS IS STORED.
C     THE BLOCKS OF THE REDUCED NORMAL EQUATIONS ARE NUMBERED
C       ACCORDING TO THE FOLLOWING SCHEME:
C
C        1     2     4     7    11
C              3     5     8    12
C                    6     9    13
C                         10    14
C                               15             ET CETERA
C
C  L(1275) IS THE GUIDE MATRIX
C  L=1  SIGNIFIES A NON ZERO BLOCK
C  L=0  SIGNIFIES A ZERO BLOCK
      IB=LOC(NSTA)
      DO 100 JB=1,IB
      DO 99 I=1,3
      DO 99 J=1,3
   99 REDN(I,J,JB)=0.0
  100 L(JB)=0
C
      BACKSPACE 2
      READ(2)       (((BN(I,J,KSTA),I=1,3),U(J,KSTA)),J=1,3),
     XKSTA=1,NSTA)
      REWIND 2
C
C  STASH  DIAGONAL BLOCKS
      DO 110 KSTA=1,NSTA
      IB    =LOC(KSTA)
      DO 108 I=1,3
      DO 108 J=1,3
  108 REDN(I,J,IB)=BN(I,J,KSTA)
  110 CONTINUE
C
      FDEGF=IDEGF
      IF(PCODE(9).EQ.1) WRITE(7,7010) FDEGF,WPW
 7010 FORMAT(16X,2F16.6)
C   READ BLOCKS FROM EACH EVENT AND REDUCE NORMAL EQUATIONS
```

```
C
  150 READ(2) NSTE,DDN,DDK,(((CN(I,J,IS),I=1,3),J=1,21),
     1KSTATE(IS),IS=1,NSTE),CONTIN
C
      DO 180 IS=1,NSTE
      ISTA=KSTATE(IS)
      IB=ISTA
      CALL DGMPRD(CN(1,1,IS),DDN,BNDDNI,3,21,21)
      CALL DGMPRD(BNDDNI,DDK,TK,3,21,1)
      DO 155 I=1,3
  155 U(I,ISTA)=U(I,ISTA)-TK(I)
      DO 180 JS=1,NSTE
      JSTA=KSTATE(JS)
      JB=JSTA
C   SKIP IF (ISTA.GT.JSTA), SINCE ONLY THE UPPER TRIANGULAR PART OF THE
C        REDUCED NORMAL EQUATIONS IS BEING COMPUTED AND SAVED.
      IF(ISTA.GT.JSTA) GO TO 180
C   (IB,JB) GIVES THE ROW AND COLUMN NUMBER OF THE BLOCK IN THE REDUCED
C        NORMAL EQUATIONS CURRENTLY BEING PROCESSED.
C
C  SET INDICATOR
      NB=LOC(JB-1)
      NB=IB+NB
      L(NB)=L(NB)+7
C  PERFORM REDUCTION
      DO 156 I=1,3
      DO 156 J=1,21
  156 DN(J,I)=CN(I,J,JS)
      CALL DGMPRD(BNDDNI,DN,TN,3,21,3)
 6910 FORMAT(1H ,3D20.12)
      DO 130 I=1,3
      DO 130 J=1,3
  130 REDN(I,J,NB)=REDN(I,J,NB)-TN(I,J)
  180 CONTINUE
C   IF END OF DATA, GO OUT OF LOOP
      IF(CONTIN.EQ.ENDSIG) GO TO 400
C   IF NOT, RETURN TO PROCESS ANOTHER EVENT
      GO TO 150
C
C   ENTER HERE WHEN ALL EVENTS HAVE BEEN PROCESSED.
  400 CONTINUE
C
C  SIMULATE KRAKIWSKI'S GUIDE MATRIX
      IF(PCODE(6).NE.1) GO TO 441
C
      WRITE(6,6001)
 6001 FORMAT(1H1,10(/),20X,'GUIDE MATRIX')
      DO 440 ISTA=1,NSTA
      IB=0
      LG(1)=1000
      DO 435 JSTA=ISTA,NSTA
      JB=LOC(JSTA-1)+ISTA
      IF(L(JB).EQ.0) GO TO 435
      IB=IB+1
      LG(IB)=KORDER(JSTA)
  435 CONTINUE
C
      IB=IB+1
```

```
      IF(IB.GT.1) LG(IB)=999
  439 WRITE(6,6002) KORDER(ISTA),(LG(I),I=1,IB)
 6002 FORMAT(20X,I5,5X,18I5,200(/30X,18I5))
  440 CONTINUE
  441 CONTINUE
C
C  PRINT NORMALS IN ASD FORMAT,AND PUNCH IF DESIRED.
      WRITE(6,6003)
 6003 FORMAT(1H1//'              NORMAL EQUATIONS (SEE GUIDE MATRIX)'//)
      DO 450 ISTA=1,NSTA
      DO 442 I=1,3
  442 DDL(I)=-U(I,ISTA)
      IB=0
      JB=LOC(ISTA)
      IF(L(JB).GT.0) IB=1
C    PUNCH NORMALS
      IF(PCODE(9).NE.1) GO TO 443
      WRITE(7,7001) KORDER(ISTA)
 7001 FORMAT(14I5)
      WRITE(7,7006) DDL
 7006 FORMAT(3(D16.9,5X))
      WRITE(7,7008) ((REDN(I,J,JB),J=1,3),I=1,3)
 7008 FORMAT(3D16.9/3D16.9/3D16.9)
C
  443 CONTINUE
C    PRINT DIAGONAL BLOCK
      IF(PCODE(7).NE.1) GO TO 444
      WRITE(6,6004) KORDER(ISTA)
 6004 FORMAT(//I5)
      WRITE(6,6006) DDL
 6006 FORMAT(/3(F16.10,5X))
      WRITE(6,6008) ((REDN(I,J,JB),J=1,3),I=1,3)
 6008 FORMAT(3F16.10)
  444 CONTINUE
C    PRINT OFF-DIAGONAL BLOCKS
      KSTA=ISTA+1
      IF(ISTA.EQ.NSTA) GO TO 448
      DO 445 JSTA=KSTA,NSTA
      JB=LOC(JSTA-1)+ISTA
      IF(L(JB).EQ.0) GO TO 445
      IB=IB+1
      IF(PCODE(9).NE.1) GO TO 7445
      WRITE(7,7001) KORDER(JSTA)
      WRITE(7,7008) ((REDN(I,J,JB),J=1,3),I=1,3)
 7445 CONTINUE
      IF(PCODE(7).NE.1) GO TO 445
      WRITE(6,6004) KORDER(JSTA)
      WRITE(6,6008) ((REDN(I,J,JB),J=1,3),I=1,3)
  445 CONTINUE
  448 I=1000
      IF(IB.GT.0) I=999
      IF(PCODE(7).EQ.1) WRITE(6,6004) I
      IF(PCODE(9).EQ.1) WRITE(7,7001) I
  450 CONTINUE
      IF(PCODE(8).NE.1) GO TO 478
      WRITE(6,6010)
 6010 FORMAT(10(/),20X,'OBSERVATIONS ON EACH LINE')
      IB=NSTA-1
```

```
      DO 475 ISTA=1,IB
      KSTA=ISTA+1
      DO 475 JSTA=KSTA,NSTA
      WRITE(6,6011) KORDER(ISTA),KORDER(JSTA),L(LOC(JSTA-1)+ISTA)
 6011 FORMAT(8I10)
  475 CONTINUE
  478 CONTINUE
      RETURN
  901 CONTINUE
      WRITE(6,9001) MAXSTA,NSTA
 9001 FORMAT('    FORMRN IS PRESENTLY DIMENSIONED TO HANDLE ONLY',I5,
     1'    UNKNOWN STATIONS.'/20X,' THIS PROBLEM HAS',I5,'  UNKNOWN STATI
     2IONS.'/10X,'EXECUTION IS TERMINATED BY PROGRAM.')
      STOP
      END
```

```
      SUBROUTINE VERSOL (ORGMAT,VERMAT, I, M)
      IMPLICIT REAL*8(A-H,O-Z)
COMMENT I IS THE NUMBER OF ROWS IN ORGMAT, AND M IS I PLUS THE NUMBER OF
C      OF UNKNOWN COLUMNS. THE ORIGINAL VALUES OF ORGMAT ARE RETAINED.
      DIMENSION ORGMAT (I,M), VERMAT (I,M)
      DIMENSION P(72)
      N=I-1
      MI=M-1
      DO 1 J=1,I
      DO 1 K=1,I
    1 VERMAT(J,K)=ORGMAT(J,K)
      DO 5 K=1,I
      DO 2 J=1,MI
    2 P (J) = VERMAT (1,J+1)/VERMAT (1,1)
      P(M)=1.0D0/VERMAT(1,1)
      DO 4 L=1,N
      DO 3 J=1,MI
    3 VERMAT (L,J) = VERMAT (L+1,J+1) - VERMAT (L+1,1) * P(J)
    4 VERMAT (L,M) = - VERMAT (L+1,1) * P(M)
      DO 5 J=1,M
    5 VERMAT (I,J) = P(J)
      RETURN
      END
```