**NASA TECHNICAL MEMORANDUM** NASA TM X-71545

NASA TM X-71545

(NASA-TM-X-71545) NORMALIZING PARAMETERS FOR THE CRITICAL FLOW RATE OF SIMPLE FLUIDS THROUGH NOZZLES (NASA) 11 p HC $3.00 CSCL 20D

N74-26793

Unclas
G3/12 40699

# NORMALIZING PARAMETERS FOR THE CRITICAL FLOW RATE OF SIMPLE FLUIDS THROUGH NOZZLES

by Robert C. Hendricks
Lewis Research Center
Cleveland, Ohio



TECHNICAL PAPER proposed for presentation at the Fifth International Cryogenic Engineering Conference sponsored by The International Institute of Refrigeration Kyoto, Japan, May 7-10, 1974

E-7899

# NORMALIZING PARAMETERS FOR THE CRITICAL FLOW RATE OF SIMPLE FLUIDS THROUGH NOZZLES

by Robert C. Hendricks

NASA Lewis Research Center
Cleveland, Ohio 44135 U.S.A.

## SUMMARY

Two-phase critical flow of simple fluids through a nozzle has been shown to nearly obey the principle of corresponding states. Quantum fluid departures from the principle were resolved as a function of temperature for parahydrogen and helium.

The critical flow rates were normalized using

$$G^* = \sqrt{\rho_c P_c / Z_c}$$

which nearly reduces the calculated equilibrium values of critical flow for classical fluids methane, oxygen, nitrogen and argon to a single isothermal curve. For the quantum fluids, p-hydrogen and helium, the normalizing parameter becomes dependent on the isotherm and quantum fluid of interest. The normalizing parameter

$$G^* [1 + \psi_Q]$$

where $\psi_Q = 0$ for classical fluids, nearly reduces the critical flow rates for all simple fluids to a single isothermal curve.

Experimental data for classical fluids nitrogen and methane for reduced pressures to 2.5 and reduced temperatures from 0.8 where taken and have been reported elsewhere. Experimental data for p-hydrogen, reduced pressures to 4.5 and reduced temperatures from 0.87, were taken and analyzed herein. These data support the procedures for normalizing two-phase critical flow of simple fluids, including quantum fluids, through nozzles.

## INTRODUCTION

The concept that there could be several thermodynamic processes which nearly obey the principle of corresponding states was advanced in Ref. 1 and extended to two phase choked flows through nozzles in Ref. 2. The critical flow rate for several simple fluids, c.f. methane, nitrogen, oxygen, and argon, were normalized using the parameter:

$$G^* = \sqrt{\frac{\rho_c P_c}{Z_c}} \tag{1}$$

where the values of $G^*$ for several fluids are given in table I. The calculated† normalized critical flow rate $(G/_{G^*})$ for the above four fluids were nearly collapsed to single isotherms. Experimental data for nitrogen and

†Isentropic, equilibrium expansion of a fluid through a nozzle was assumed; this does not advocate any particular theory but facilitates the establishment of normalization procedures.

methane verified the normalization procedure.

The question of how to apply this procedure to normalize the critical flow rates of quantum fluids para-hydrogen and helium is addressed in this paper.

## SYMBOL LIST

| | |
|---|---|
| f,g | functions defined by eqs. 5 and 6 |
| G | critical mass flow rate |
| $G^* = \sqrt{\dfrac{\rho_c P_c}{Z_c}}$ | normalizing parameter for G |
| P | pressure |
| m | molecular mass |
| T | temperature |
| Z | compressibility |
| $\Lambda$ | de Broglie wavelength |
| $\epsilon$ | intermolecular force constant |
| $\rho$ | density |
| $\psi_Q$ | empirical deviation function |
| $\sigma$ | collision diameter |
| $\dot{\omega}$ | mass flow rate |

Subscripts:

| | |
|---|---|
| c | thermodynamic critical condition |
| He | helium |
| o | stagnation condition |
| $pH_2$ | para-hydrogen |
| P.G. | perfect gas |
| R | reduced |
| 0,1 | reference conditions |

## ANALYSIS

Gunn et.al. [3], found that quantum deviations in the reduced Joule-Thomson curve could be expressed in terms of the de Broglie wavelength

$$\Lambda = \frac{h}{\sigma\sqrt{m\epsilon}} \tag{2}$$

The inversion curve was then expressed as the sum of the classical corresponding states part $f_0\ (T_R)$ and a quantum deviation $f_1\ (T_R, \Lambda^2)$

$$Pr = f_0\ (T_R) + f_1\ (T_R, \Lambda^2) \tag{3}$$

In this report the physical thermodynamic critical constants, table I, will be used. This represents a departure from Ref. 3 where the constants were modified for quantum effects.

The isentropic equilibrium calculations for para-hydrogen and helium were performed using the techniques of Ref. 4 and the thermophysical properties program GASP Ref. 5 for para-hydrogen and other fluids and HELP (internal NASA-Lewis Research Center program) for helium.

In Fig. 1, the reduced critical flow curves for the classical fluids methane, nitrogen, oxygen and argon and the quantum fluids para-hydrogen and helium for $T_R = 1.0$ are given. Also presented are $T_R = 0.8$ and $T_R = 1.2$ isotherms for nitrogen, para-hydrogen and helium. For the $T_R = 1.0$ isotherm, while the critical flow rate G varied nearly a factor of 10, the reduced curves for the above simple fluids deviate $\pm$ 7% at low pressure to $\pm$ 1% at $P_R = 4$. This success implies that the normalization procedures of Ref. 2 can be extended to quantum fluids. Departures along other isotherms are pronounced and dealt with subsequently.

Reduced critical flow rate plots for para-hydrogen and helium for selected isotherms were constructed, Figs. 2 and 3, respectively. Fig. 4 represents the reduced critical flow rate plot for classical fluids (e.g. nitrogen and oxygen).

Fig. 1 and overlays of Figs. 2 to 4 reveals considerable differences between para-hydrogen, helium, and the classical fluids except near $T_R = 1$. The problem now is how to resolve these differences. Assume that the reducing parameter for critical flow rate may be expressed as the sum of the classical reducing parameter and the quantum deviation:

$$G^* \; (1 + \psi_Q) \tag{4}$$

It was found that while Eq. (4) is a function of both temperature and pressure, the pressure dependence appeared to be minimal see Fig. 1. Selecting a reduced pressure of 3 ($P_R = 3$) as best representing the range of interest, one forms the following functions:

$$f(T_R) = \frac{(G/G^*)\,pH_2}{(G/G^*)\text{classical}} - 1 \tag{5}$$

$$g(T_R) = \frac{(G/G^*)\text{He}}{(G/G^*)\text{classical}} - 1 \tag{6}$$

where classical refers to nitrogen, oxygen etc. Using Figs. 2 and 4, $f(T_R)$ was found and plotted as Fig. 5. Similarly using Figs. 3 and 4, $g(T_R)$ was found (see fig. 5). If the functions $f(T_R)$ and $g(T_R)$ are valid, i.e. pressure independent, one should now be able to reduce the critical flow rate of say para-hydrogen and nitrogen to a set of single isotherms. To verify the functions $f(T_R)$ and $g(T_R)$ the critical flow rates for para-hydrogen and helium were calculated using

$$G_R = \frac{G_{pH_2}}{[1 + f(T_R)]G^*} \quad \text{and} \quad G_R = \frac{G_{He}}{[1 + g(T_R)]G^*} \qquad (7)$$

and the results are given as Fig. 6 along with nitrogen. The agreement between calculated isentropic equilibrium flow for classical fluids (dashed lines) and para-hydrogen (the circles) is quite good, with the exception of the region quite close to the saturation boundary for $T_R < 1$. Similar results were found for $g(T_R)$, eq. (6) (crosses on Fig. 6).

## COMPARISON WITH DATA

A limited number of tests were run with para-hydrogen in the blow down facility described in Ref. 4. The data and associated calculated parameters are given as table II.

Comparing the normalized flow rate $(G/G^*)pH_2$ in the data section to that in the Calculated Parameters Section, reasonably good agreement can be found. A few selected data points from table II can be compared to the calculated isotherms of Fig. 2, see Fig. 7. As is the case with nitrogen and methane, the calculated isotherms lie above the data - up to 10% for $T_R = 0.8$, to being close at $T_R \approx 1.$; to 5-10% below for $T_R > 1$; and good agreement prevails in the gaseous regime.

In Ref. 2 critical flow rate data for nitrogen and methane are compared at selected isotherms. The equilibrium calculations serve as convenient reference lines and their use does not advocate any particular model; the comparison is between the data. Unfortunately the para-hydrogen data are not along equivalent isotherms, consequently no direct comparison with the data of Ref. 2 is made. The indirect comparison between calculations for classical fluids and experimental data, similar results for para-hydrogen and the agreement illustrated in Fig. 6, establish Eq. (7) as a technique to normalize the critical flow rate for simple fluids.

## CONCLUSIONS

The normalization parameters for two phase flow of simple fluids through a nozzle has been investigated. Quantum fluid departures from the classical fluids were resolved as functions of temperature.

The generalized normalization parameter for simple fluids is:

$$G_R = \frac{G}{[1 + \psi_Q]G^*}$$

where

$$\psi_Q = \begin{cases} 0 & \text{for classical fluids} \\ f(T_R) & \text{for para-hydrogen} \\ g(T_R) & \text{for helium} \end{cases}$$

$$G^* = \sqrt{\frac{\rho_c P_c}{Z_c}}$$ $\rho_c$, $P_c$, $Z_c$ are the physical critical constants

Along the isotherm $T_R = 1$, $\psi_Q$ is quite small and $G_R \approx G/G^*$. While the critical flow rate $G$ varied nearly a factor of 10, the reduced curves for simple fluids deviate $\pm$ 7% at low pressure to $\pm$ 1% at $P_R = 4$.

In general, the agreement between calculated characteristics and experimental characteristics for para-hydrogen is quite good. The prediction of the critical flow rate for nitrogen or oxygen and the prediction of critical flow rate for para-hydrogen also showed good agreement.

## REFERENCES


1. Hendricks, R. C., Peller, I. C., and Baron, A. K., NASA Tech. Note D-6807 (1972).
2. Hendricks, R. C. and Simoneau, R. J., NASA Tech. Memo X-68193 (1973).
3. Gunn, R. D., Chueh, P. L., and Prausnitz, J. M., Cryogenics, 6, 324-329, (1966).
4. Hendricks, R. C., Simoneau, R. J., and Ehlers, R. C., NASA Tech. Memo X-68107 (1972).
5. Hendricks, R. C., Baron, A., Peller, I., and Pew, K. J., Proceedings of the 13th International Congress on Refrigeration, Washington DC, (1971).

E-7899

TABLE I. - CRITICAL CONSTANTS USED IN THE REDUCING PARAMETERS

| FLUID | $P_c$ $MN/m^2$ | $T_c$ K | $\rho_c$ gm/cc | $Z_c = \frac{P_c}{\rho_c R T_c}$ $Z_c$ | $G^*$ $gm/cm^2 sec$ |
|---|---|---|---|---|---|
| Nitrogen | 3.417 | 126.3 | .3105 | .2937 | 6010.4 |
| Oxygen | 5.083 | 154.78 | .4325 | .2922 | 8673.9 |
| Methane | 4.627 | 190.77 | .162 | .2889 | 5093.7 |
| Argon | 4.865 | 150.7 | .531 | .2921 | 9404.2 |
| Para-hydrogen | 1.2925 | 32.976 | .03143 | .3023 | 1158 |
| Helium | .22746 | 5.2014 | .06964 | .3023 | 724 |

TABLE II. - HYDROGEN CHOKED FLOW DATA

$P_c = 1.2928\ MN/m^2$; $T_c = 32.976$ K; $\rho_c = .03143$ gm/cc; $\sqrt{\frac{P_c \rho_c}{Z_c}} \to 1158$

| DATA | | | | | | | | | | | CALCULATED PARAMETERS | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Run | $P_o$ $MN/m^2$ | $P_R$ | $T_o$ K | $T_R$ | $P_t/P_o$ | G $gm/cm^2 sec$ | $\left.\frac{G}{G^*}\right)_{pH_2}$ | $\dot{w}$ lbm/sec | $P_t$ $MN/m^2$ | $\frac{P_t/P_o}{(P_t/P_o)P.G.}$ | $P_t/P_o$ | $(P_t/P_o)_R$ | G $gm/cm^2 sec$ | $\left.\frac{G}{G^*}\right)_{pH_2}$ | $1 + \psi_Q(T_R) = 1 + f(T_R)$ |
| 1200 | 3.3 | 2.552 | 27.25 | .826 | .116 | 1871 | 1.616 | .278 | .384 | .22 | .1034 | .1960 | 1926 | 1.662 | 0.957 |
| 1201 | 2.84 | 2.196 | 32.23 | .977 | .261 | 1418 | 1.224 | .21 | .74 | .495 | .2768 | .5248 | 1482 | 1.2784 | 1.01 |
| 1202 | 2.00 | 1.547 | 70.0 | 2.122 | .508 | 285.8 | .247 | .042 | 1.02 | .963 | .4776 | .9056 | 279 | .241 | 1.12 |
| 1203 | 4.202 | 3.25 | 284.5 | 8.63 | .517 | 260. | .224 | .0387 | 2.17 | .980 | .5317 | 1.008 | 262 | .226 | |
| 1204 | 5.216 | 4.034 | 285.8 | 8.67 | .511 | 321.8 | .278 | .0478 | 2.67 | .969 | .53 | 1.006 | 325 | .280 | |
| 1205 | 2.583 | 1.998 | 273.5 | 8.29 | .532 | 166.6 | .144 | .0247 | 1.375 | 1.009 | .5332 | 1.011 | 165 | .142 | |
| 1206 | 3.505 | 2.711 | 29.45 | .893 | .1898 | 1853. | 1.6 | .275 | .665 | .360 | .1379 | .262 | 1894 | 1.634 | 0.974 |
| 1207 | 2.779 | 2.15 | 30.32 | .919 | .2278 | 1566 | 1.352 | .2327 | .633 | .432 | .2170 | .411 | 1577 | 1.361 | 0.983 |
| 120[illegible] | 2.488 | 1.924 | 37.37 | 1.133 | .274 | 953.8 | .824 | .141 | .682 | .520 | | | | | 1.127 |
| 1209 | 2.034 | 1.573 | 65.31 | 1.98 | .4664 | 307.6 | .268 | .0457 | .949 | .884 | .4754 | .901 | 298 | .257 | 1.127 |
| 1210 | 4.614 | 3.569 | 28.65 | .869 | .1003 | 2207 | 1.906 | .328 | .463 | .190 | .0821 | .1557 | 2277 | 1.965 | 0.967 |
| 1211 | 4.123 | 3.189 | 29.1 | .882 | .1122 | 2050 | 1.770 | .305 | .463 | .212 | .1043 | .1978 | 2124 | 1.833 | 0.971 |
| 1212 | 4.216 | 3.261 | 28.75 | .872 | .1065 | 2088 | 1.803 | .31 | .449 | .202 | .0951 | .1803 | 2155 | 1.859 | 0.968 |
| 1213 | 3.461 | 2.677 | 29.46 | .893 | .1429 | 1835 | 1.58 | .273 | .494 | .271 | .1406 | .2665 | 1878 | 1.620 | 0.974 |
| 1214 | 2.137 | 1.653 | 31.04 | .941 | .2884 | 1266 | 1.093 | .188 | .616 | .547 | .3455 | .6551 | 1240 | 1.070 | 0.991 |
| 1215 | 1.571 | 1.215 | 32.25 | .978 | .4762 | 896.7 | .774 | .133 | .748 | .903 | .6303 | 1.195 | 756 | .653 | 1.01 |
| 1216 | 5.797 | 4.484 | 28.57 | .866 | .0766 | 2506 | 2.164 | .375 | .444 | .145 | .0576 | .1093 | 2599 | 2.242 | 0.966 |
| 1217 | 4.114 | 3.182 | 30.32 | .319 | .1309 | 1989 | 1.718 | .296 | .538 | .748 | .1253 | .2375 | 2049 | 1.768 | 0.981 |
| 1218 | 2.752 | 2.127 | 32.34 | .981 | .2692 | 1425 | 1.231 | .212 | .74 | .510 | .2937 | .5569 | 1435 | 1.238 | 0.987 |
| 1219 | 3.602 | 2.786 | 30.56 | .928 | .1581 | 1833 | 1.583 | .272 | .57 | .3 | .1567 | .2971 | 1889 | 1.613 | 0.983 |
| 1220 | 2.805 | 2.17 | 31.32 | .95 | .2326 | 1517 | 1.310 | .225 | .652 | .441 | .2431 | .4705 | 1528 | 1.318 | 0.995 |
| 1221 | 2.117 | 1.638 | 32.21 | .977 | .312 | 1185 | 1.023 | .176 | .6806 | .592 | .4145 | .7859 | 1132 | 0.977 | 1.01 |
| 1222 | 5.724 | | 28.07 | .851 | .0754 | 2502 | 2.161 | .372 | .431 | .143 | .0539 | .1023 | 2601 | 2.245 | 0.963 |
| 1223 | 4.154 | 3.213 | 29.79 | .903 | .1222 | 2020 | 1.744 | .3 | .508 | .232 | .1143 | .2168 | 2087 | 1.801 | 0.977 |
| 1224 | 2.709 | 2.096 | 31.28 | .948 | .2398 | 1495 | 1.291 | .222 | .65 | .455 | .2589 | .4909 | 1490 | 1.286 | 0.996 |
| 1199 | 2.832 | 2.19 | 27.81 | .843 | .1475 | 1696 | 1.465 | .252 | .418 | .280 | .1409 | .2671 | 1731 | 1.493 | 0.961 |
| 1198 | 2.090 | 1.617 | 28.95 | .878 | .2433 | 1398 | 1.208 | .208 | .508 | .461 | .2553 | .4840 | 1355 | 1.170 | 0.969 |
| 1197 | 1.192 | .922 | 30.72 | .932 | .3939 | 892 | .77 | .1326 | .47 | .747 | .7121 | 1.350 | 605 | .522 | 0.987 |
| 1196 | 3.871 | 2.994 | 40.34 | 1.223 | .3126 | 1339 | 1.156 | .199 | 1.21 | .593 | | | | | 1.257 |
| 1195 | 2.972 | 2.299 | 33.4[illegible] | 1.016 | .2746 | 1467 | 1.267 | .2174 | .816 | .521 | .2969 | .5630 | 1459 | 1.259 | 1.035 |
| 1194 | 2.017 | 1.56 | 269.[illegible] | 8.18 | .546 | 135 | .116 | .0201 | 1.101 | 1.035 | .5338 | 1.012 | 129 | .112 | |
| 1193 | 4.701 | 3.636 | 281.2 | 8.53 | .516 | 292 | .252 | .0434 | 2.427 | .978 | .5304 | 1.006 | 295 | .255 | |
| 1192 | 3.692 | 2.856 | [illegible]8.8 | 8.45 | .5216 | 231 | .199 | .0344 | 1.93 | .989 | .5319 | 1.008 | 233 | .201 | |
| 1191 | 3.756 | 2.906 | 277.3 | 8.41 | .524 | 238 | .206 | .0354 | 1.97 | .994 | .5319 | 1.008 | 238 | .205 | |
| 1190 | 2.631 | 2.035 | 276.6 | 8.39 | .536 | 167 | .144 | .0248 | 1.41 | 1.016 | .5330 | 1.011 | 167 | .144 | |
| 1189 | 2.561 | 1.981 | 268. | 8.13 | .545 | 198 | .171 | .0294 | 1.4 | 1.033 | .5335 | 1.011 | 165 | .142 | |
| 1188 | 4.314 | 3.337 | 273 | 8.28 | .524 | 326 | .282 | .0484 | 2.26 | .994 | .5304 | 1.006 | 275 | .237 | |
| 1187 | 2.976 | 2.302 | 270.3 | 8.2 | .536 | 227 | .196 | .0337 | 1.6 | 1.016 | .533 | 1.011 | 191 | .164 | |
| 1186 | 3.024 | 2.339 | 272.1 | 8.25 | .531 | 228 | .199 | .0339 | 1.6 | 1.007 | .5326 | 1.010 | 193 | .167 | |
| 1185 | 4.611 | 3.567 | 288. | 8.73 | .515 | 341 | .294 | .0507 | 2.38 | .976 | .5304 | 1.006 | 286 | .247 | |
| 1184 | 3.47 | 2.684 | 288. | 8.73 | .524 | 256 | .221 | .0381 | 1.82 | .994 | .5320 | 1.009 | 216 | .186 | |
| 1183 | 3.45 | 2.669 | 98. | 8.73 | .524 | 251 | .217 | .0373 | 1.8 | .994 | .5316 | 1.008 | 214 | .185 | |

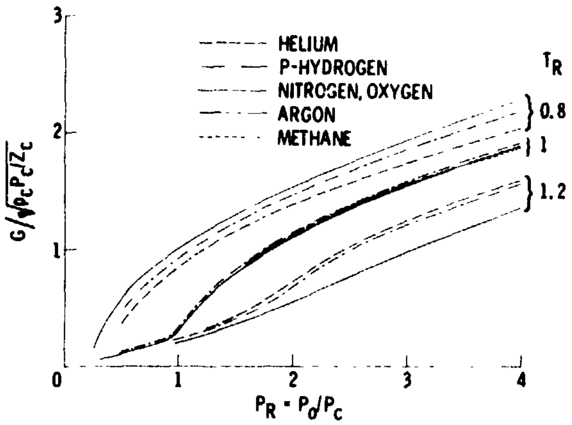


Figure 1. - Reduced critical flow for several fluids along selected isotherms.

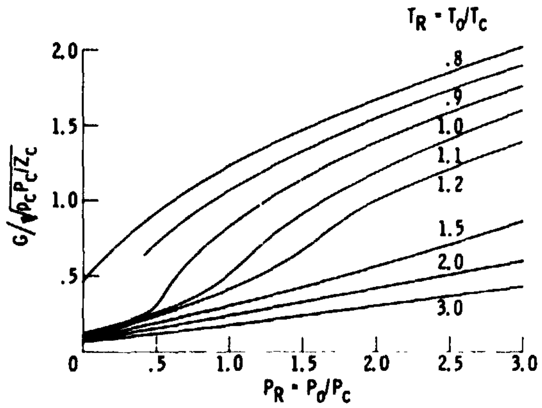


Figure 2. - Critical flow rate of para-hydrogen computed by isentropic equilibrium expansion.

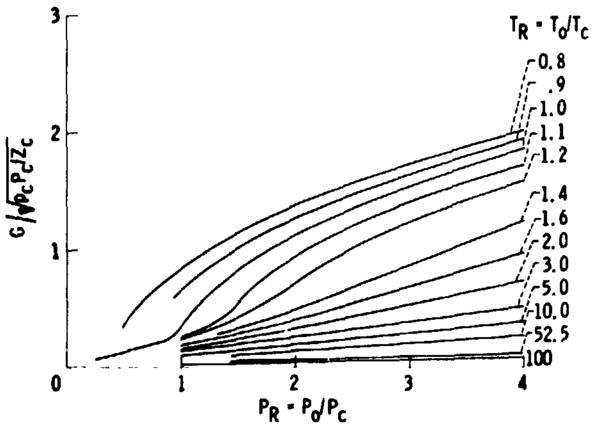


Figure 3. - Critical flow rate for helium computed by isentropic equilibrium expansion.

E-7899

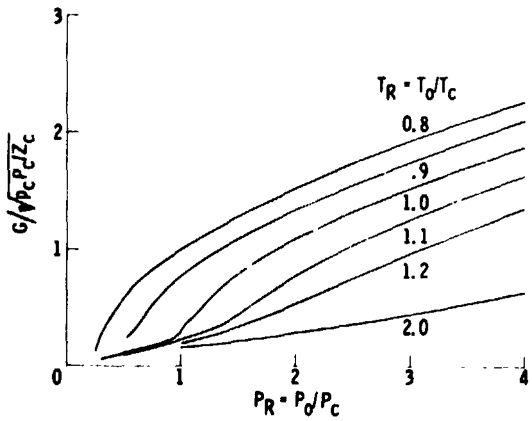


Figure 4. - Critical flow rate for nitrogen and oxygen computed by isentropic equilibrium expansion.

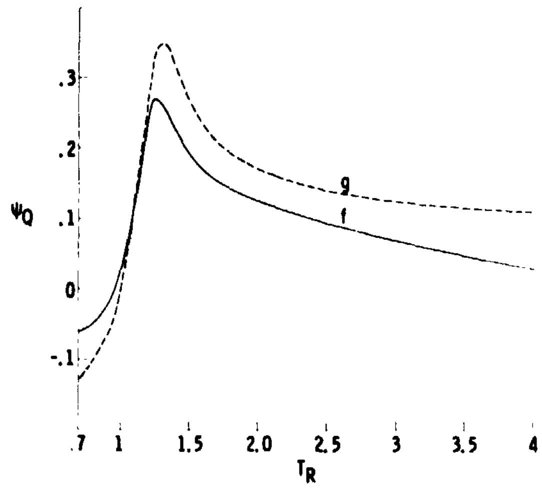


Figure 5. - Quantum deviation functions for critical flow of para-hydrogen and helium through a nozzle based on isentropic equilibrium expansion.

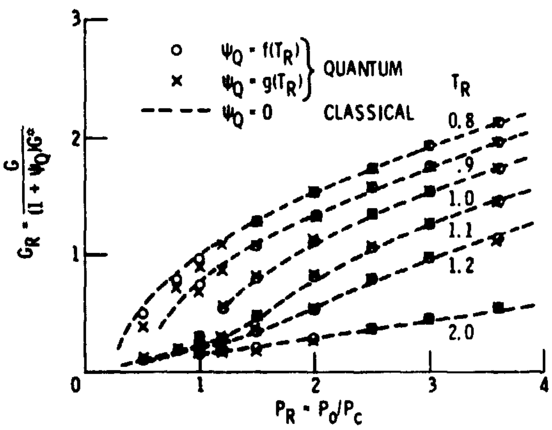


Figure 6. - Calculated classical (nitrogen-oxygen) para-hydrogen and helium critical flow rate along selected isotherms.

E-7899

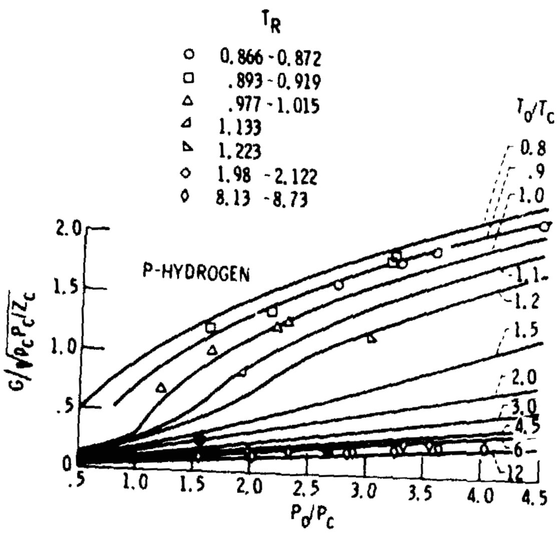


Figure 7. - Comparison of critical flow rate data with calculated equilibrium values for para-hydrogen.