2up

X-921-74-161

PREPRINT

NASA TM X- 70677

DYNAMIC TECHNIQUES FOR STUDIES OF SECULAR VARIATIONS IN POSITION FROM RANGING TO SATELLITES

DAVID E. SMITH
RONALD KOLENKIEWICZ
RUSSELL W. AGREEN
PETER J. DUNN

(NASA-TM-X-70677) DYNAMIC TECHNIQUES FOR STUDIES OF SECULAR VARIATIONS IN POSITION FROM RANGING TO SATELLITES (NASA) 34 p HC $4.75 CSCL 22A

N74-27351

Unclas
G3/30 42161

MAY 1974

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-921-74-161

DYNAMIC TECHNIQUES FOR STUDIES OF SECULAR VARIATIONS IN POSITION FROM RANGING TO SATELLITES

David E. Smith
Ronald Kolenkiewicz
Russell W. Agreen
NASA, Goddard Space Flight Center
Greenbelt, Maryland

and

Peter J. Dunn
Wolf Research and Development Corporation
Riverdale, Maryland

May 1974

Presented at the IAG Symposium--The Earth's Gravitational Field and Secular Variations in Position, Sydney, Australia, November 1973

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

DYNAMIC TECHNIQUES FOR STUDIES OF SECULAR VARIATIONS IN POSITION FROM RANGING TO SATELLITES

David E. Smith
Ronald Kolenkiewicz
Russell W. Agreen
Peter J. Dunn

ABSTRACT

During the last few years NASA Goddard Space Flight Center has been applying satellite laser range measurements to problems in earth and ocean physics. Primary attention has been directed towards the measurement of the variation of latitude arising from polar motion and to the determination of the solid-earth and ocean tidal distortion of the earth's gravity field. These investigations have been successfully conducted using data obtained by a single laser station tracking a single satellite. It has further been demonstrated that these data can also be used to monitor the height or radial distance of the station and can contribute substantially to our knowledge of the gravity field.

Experiments involving two stations have also been conducted. Simultaneous range measurements to a satellite from two stations several hundred kilometers apart have been used to determine the relative location of one station with respect to the other. This technique is now being used in an experiment (SAFE) to measure the motion between points 900 km apart on opposite sides of the San Andreas fault system in California. A simulation of this experiment for a seven year observing period has indicated that it should be possible to determine the average relative motion of the two sides of the fault to an accuracy of about 5 millimeters per year.

At the present time all spacecraft equipped with laser retroreflectors are in relatively low orbits of 500 to 2000 km but in the next few years it is anticipated that geodynamic satellites at much greater altitude will be launched. At these greater altitudes of several thousand kilometers the perturbing effects of the earth's gravity field will be much smaller and should permit an improvement of at least an order of magnitude in the determination of the product of the earth's mass and gravitational constant.

In a few years it is anticipated that laser ranging measurements to high altitude satellites from a single site will permit the station to monitor its own latitude and height variations at the 5 to 10 cm level on a daily basis and determine the length of day to about 0.2 milliseconds. Multiple station experiments should enable crustal and tectonic motions to be measured to an accuracy of a few millimeters per year over a few years from which large scale strain fields could be derived. Observations of lower altitude satellites will yield information on the gravity field, the elastic response of the solid-earth to tidal forces and the amplitudes and phases of certain components of the ocean tides.

CONTENTS

PRECEDING PAGE BLANK NOT FILMED

ILLUSTRATIONS

TABLES

DYNAMIC TECHNIQUES FOR STUDIES OF SECULAR VARIATIONS IN POSITION FROM RANGING TO SATELLITES

1. INTRODUCTION

The successful development of high precision laser tracking systems over the last decade is now beginning to permit the investigation of geophysical parameters of considerable interest and importance. At Goddard Space Flight Center the accuracy of laser ranging has improved nearly two orders of magnitude since 1964 and toward the end of 1973 reached a level of about ten centimeters. With this quality it can reasonably be expected that laser ranging to satellites will be able to contribute significantly to the measurement of the motions of the earth, such as tectonic, polar, tidal and crustal, and to the determination of the earth's gravitational field in both space and time. However, until very recently the quality of the range measurements was of the order of fifty centimeters, a capability achieved in 1970, and it is upon data of this quality that all our experience so far has been based. In this paper we attempt to review the major aspects of the analysis of satellite laser range measurements conducted at Goddard Space Flight Center since 1970 for ultimate application to geodesy and geodynamics.

The investigations at GSFC have been restricted so far to those that can be achieved with one or two tracking systems. Until late 1973, Goddard Space Flight Center possessed one fixed laser system sited at its optical facility in Greenbelt, Maryland and one mobile tracking system that could be driven, or shipped, almost anywhere. Both these systems were one joule ruby lasers with a pulse rate of one-per-second.

A major factor that influenced the analyses was the limited knowledge of the perturbing forces affecting the motion of the satellite. In 1970, our long wavelength knowledge of the gravitational field was almost entirely derived from optical satellite tracking data accurate to about two seconds of arc (ten meters at about one thousand kilometers range)[1]. Thus, the laser observations were about an order of magnitude better than the data used to derive the field and any analysis might ultimately be limited by errors in the force field rather than the data. The position is hardly any different in 1973 than in 1970 and in none of the investigations conducted at GSFC in the last few years has any real evidence been uncovered which suggests that we have been limited by the quality of the data; even at the fifty centimeter level. Consequently, any modest achievements that can be established in the present environment should be well exceeded (even with the same data) when improved force models are available. It is therefore essential that research continue in the area of improving the earth's gravity field and in the modeling of other perturbing influences of satellite orbits for the full potential of the orbital dynamic technique to be fully realized.

Because of the difficulty of assessing the full value of the range data already available, predictions of future capabilities, such as detecting plate motion, are almost entirely dependent on simulations or error analyses; which are themselves based on assumed error models. In the absence of any other technique this approach has been used at GSFC for projecting future capabilities with future satellites. It is from these simulations and error analyses, as well as from our work over the last few years, that much of our optimism for the future is drawn.

In the following sections the major results obtained at GSFC over the last three years from the analysis of laser range data are described and discussed; followed by a brief description of some of our ideas and plans for the future.

2. LASER DATA

There are seven satellites presently in orbit carrying laser retroreflectors, Beacon Explorers B and C, and GEOS 1 and 2 launched by the United States and three satellites launched by France, D1-C, D1-D and PEOLE. At GSFC most of our tracking operations have been on the U.S. satellites, particularly Beacon Explorer C (BE-C), and it is the analysis of tracking data on this spacecraft that is described here.

The orbit of BE-C is nearly circular at an altitude of about 1000 km with an orbital inclination of 41 degrees. A typical pass of BE-C near a ground station lasts about 8 to 10 minutes and can be tracked by the laser station down to elevations of twenty degrees or less fairly routinely (weather permitting) and sometimes below ten degrees, thus, with a pulse rate of one-per-second, five hundred or more range measurements are obtained on a "good" pass of the satellite. Less than one hundred measurements are considered "poor" passes but this does not necessarily imply that the data or the pass is any less valuable than a "good" pass. The average number of measurements per pass over nearly eighteen months of tracking of BE-C by the fixed laser at GSFC was about two hundred and fifty.

The basic measurement of the laser system is the round-trip travel time of the laser pulse from the transmitter to the satellite and back down to the receiver.[2] This time interval is corrected for system delays (calibration constants), for variation in the pulse shape and height, converted to a range, and corrected for atmospheric effects and for spacecraft size and attitude. The whole system is calibrated by ranging to a calibration target at a known distance from the tracking system before and after every satellite pass. The tropospheric correction amounts to about 2.5 meters at zenith and is applied to each individual range measurement according to the elevation (E), pressure (P), temperature

(T), and station height above mean sea level (H), as shown below.

$$\delta\rho = \frac{2.238 + 0.0414PT^{-1} - 0.238H}{\sin E + 10^{-3} \cot E} \text{ meters} \tag{1}$$

The above formula is believed to provide the correction accurate to about 5 cm at 20 degrees elevation.

A correction for spacecraft attitude and size is also made before the data are analyzed. The laser pulse is reflected from the surface of the spacecraft and yet the orbital dynamics are concerned with the motion of the spacecraft center of mass. Thus, a range correction from the spacecraft surface to the center of mass is made to each measurement. For BE-C, which is magnetically stabilized, this correction is on average about 20 cm and varies slowly during a pass observed at GSFC between about 15 cm and 27 cm.

After all corrections have been applied the final range measurements are checked for internal consistency by fitting an orbit or a polynomial through each pass of data for each station. This procedure provides the rms noise level of the data and identifies any measurements that are obvious errors. Further, the noise level can be used as an initial check that the system appears to be working correctly. For example, a sudden increase in the noise level from 50 cm to 100 cm between two consecutive passes probably indicates a change in performance of the system that needs to be investigated.

Figure 1 shows the range residuals to an orbit fitted to each of four passes of data obtained by the fixed Goddard laser (GODLAS). These four passes were obtained on September 2, 1970. The rms fits of the data about these orbits varies between 48 cm and 53 cm. The noise levels associated with each pass only indicate the internal consistency of the data and say nothing about possible biases that could exist. It will be seen in later sections, that if these pass-to-pass biases exist, they are extremely constant or probably no larger than the noise level. The quality of the data shown in Figure 1 is typical of that collected during the Summer and Fall of 1970. During the latter part of 1971 the noise level of the data was generally higher at about 80 cm and is believed to have been the result of an unintentional increase in the length of the ruby pulse from about 20 to nearly 40 nanoseconds.

The laser systems also output the direction of the receiving telescope at the time of each return pulse but we have never used these data in any of our investigations because of their relatively low accuracy and high probability of biases. The angular field of the receiving telescope is about one minute of arc and the return pulse could come from anywhere in the field of view. Further, the telescope is computer driven to follow a predicted path across the sky and if the satellite is acquired in one segment of the field of the receiving telescope it may well remain in that "off-center" position throughout the pass. The angle measurements would then be biased by anything up to about 30 arcseconds. On a number of occasions the quality of the angle data has been estimated by comparing the position of the satellite computed from a large number range measurements with the angle data. This comparison suggested the rms noise of the angle data was about 30 arcseconds with biases of the order of 15 arcseconds.

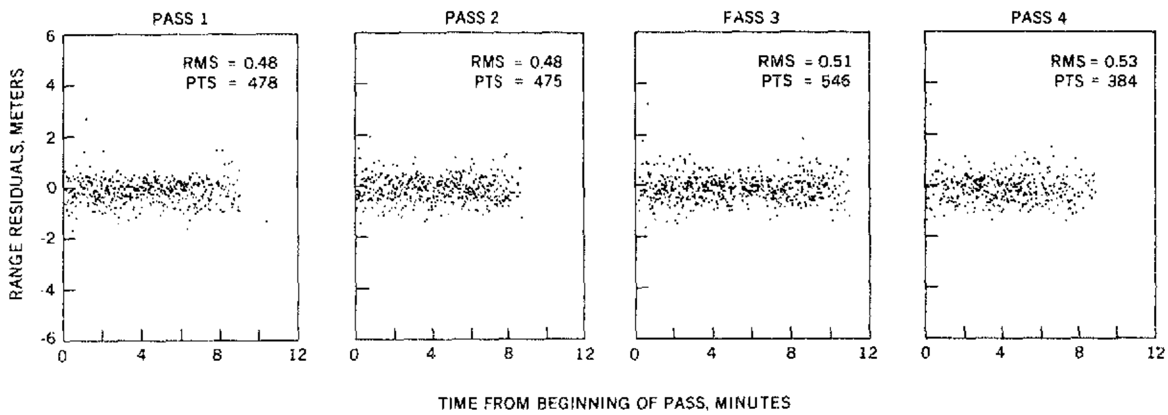

Figure 1. Laser Range Residuals to 4 Beacon Explorer C Passes on September 2, 1970

3. ORBIT DETERMINATION

The latitude of the GSFC Optical Site is approximately 39 N and because this is comparable to the orbital inclination of BE-C (41 degrees) the apparent motion of the satellite from the site is predominantly west to east. In addition, the site sees four consecutive passes of BE-C spanning about six hours each day (when the weather permits). In the analysis, an orbit has been determined for BE-C from laser data on every occasion that four consecutive passes were observed at the GSFC site (GODLAS). Thirty-six sets of orbital elements have been determined in a five-hundred day period between July 1970 and November 1971. In order to assess the quality of these BE-C orbits a simple analysis of the variations in the orbit parameters was conducted. For this test, twenty-eight sets of orbital elements obtained for the Summer and Fall of 1970 were used in a short arc minus long arc analysis.

The short minus long arc approach involves comparing orbit parameters obtained from short arcs (for example, four passes) with the same parameters obtained from a long arc (several weeks or months) and is basically equivalent to comparing observations (short arcs) with theory (long arcs). In situations, such as at GSFC, where numerical integration is used in the orbit determination programs, no "theory" exists and must be replaced with a numerical computation. This computation must provide the best orbit available and be equally representative of the true orbit at each of the observation points (short-arc orbits). The way this "theoretical" orbit can be produced is by fitting one single orbit to all the data covered by the short arcs. If this long-arc is sufficiently long it will average through perturbations that are not accounted for in the integration of the orbit and it then becomes equivalent to the theoretical orbit. The short arcs, on the other hand, are unable to average through any unmodeled perturbation that has a periodicity longer than the time span of the short arc. The short arc, therefore, absorbs the perturbations into the orbit parameters. Subtracting the orbit parameters of the long arc from the parameters of the short arcs reveals perturbations that are not computed in the long arc as well as errors, or deficiencies, in the modeling of forces that are included. Figure 2 shows the residuals in semi-major axis, argument of perigee, right-ascension of the node, eccentricity and mean anomaly obtained from a short minus long arc analysis of the BE-C data. The residuals in inclination are not shown in Figure 2 because they are described and discussed in considerable detail in later sections of this paper. It will be seen in these sections that the orbital inclination is the best determined parameter.

Figure 2 shows the variation in semi-major axis is bounded by about plus and minus three meters with the suggestion of a 70 to 80 day oscillation. This oscillation is probably more evident in the argument of perigee and mean anomaly, and to a lesser extent in the eccentricity. There is no evidence, however, of an oscillation in the nodal residuals which are dominated by a linear acceleration of about 1.5×10^{-5} degrees/day, equivalent to an error of about 3 parts in 10^6 in the precession of the node. In terms of satellite position, the residuals in mean anomaly and semi-major axis indicate a maximum difference between the short arc and long arc positions of less than 1 km. A fact to be noticed in Figure 2 is that the eccentricity and argument of perigee are negatively biased. The cause of these and the other trends and patterns in Figure 2 are at present unknown. The connection between these residuals and a possible error in GM, the product of the gravitational constant and the earth's mass, is being investigated. It should also be noted from Figure 2 that each of the elements appears to have high internal consistency suggesting that the patterns and trends clearly visible are caused by errors in the earth parameters (gravity, station position, etc.) rather than the data.

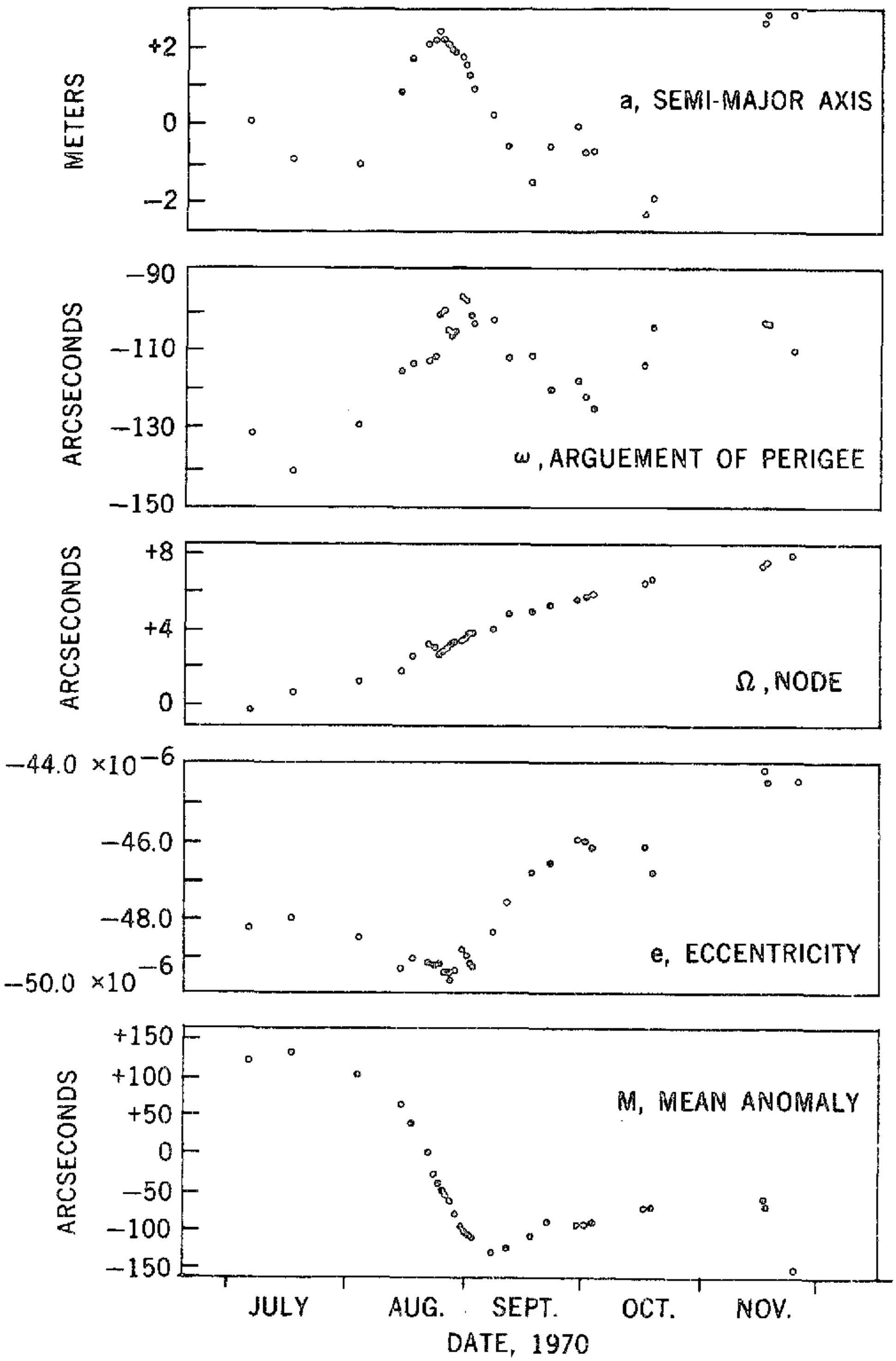

Figure 2. Beacon Explorer C Orbital Element Residuals

In general, the fit of the orbit to the laser data in a four-pass orbit never equals the quality of the data. Typically, the rms of fit of the laser data in a four-pass orbit is between 1 and 3 meters (in contrast to 40 to 60 cm on a single pass) and we have found that the fit is very dependent on the gravitational field being used in the orbit analysis. For the Goddard Earth Model 1 (GEM 1) the average rms of fit over 36 four-pass orbits containing nearly 36,000 laser range measurements is 1.32 meters. There appears to be no correlation between the number of measurements and the rms of fit. Further, we have found no evidence of poor data in the orbital arcs with the larger rms deviations.

An inspection of the residuals on a four-pass arc shows that they are far from random. Periodic trends are clearly evident in the residuals, of which a good example is shown in Figure 3a. This pattern reflects the errors or deficiencies in the gravitational field model (in this case GEM 1). The residual pattern for the Standard Earth II field is quite different from the GEM 1 although the pattern is still quasi-sinusoidal. The possibility that these patterns are a result of other errors, such as the position of the tracking station has been explored but this has been rejected because another tracking station only a few hundred kilometers away has the same pattern when the same gravitational field is used. It is not difficult to show that modifying the solar radiation pressure perturbation model has no effect on the residual patterns of the short four-pass orbits. It should further be mentioned that all four-pass orbits derived from data collected at the same station exhibit almost identical patterns.

It is interesting to note that if orbits are fitted to only two or three consecutive passes (instead of four) the rms fits of the observations about the orbits are significantly improved and the patterns less pronounced. Indeed, on two-pass orbits the patterns are no longer evident. Figures 3b and 3c show a three-pass and a two-pass orbit. The data in Figures 3b and c are subsets of that shown in Figure 3a.

The average range residual on a four pass orbit is usually a few centimeters but in our initial investigations we usually found this average to be a few decimeters and we postulated that it was caused by an error in the height of the station. Consequently, we made a determination of the station height from each four pass orbit in a simultaneous solution with the six orbit parameters. In all 36 cases the average residual became less than 10 cm and in most cases only a few centimeters. The values of the height recovered from each four-pass orbital arc are shown in Figure 4. They range over nearly 20 meters about a weighted mean of 9.29 meters with a rms deviation of 3.85 m. The mean height is only a few meters larger than the a priori value and was adopted as the "true" height of the station. The results shown in Figure 4 were obtained with the GEM 1 gravity field and from earlier experiments employing different gravity models we know that the distribution of the points is almost completely dependent on the gravity field used in the analysis. For example, one might reasonably suspect some of the outlying points in Figure 4 to be a result of poor data but this is not the case. The analysis of these data with other gravity fields has shown other points to be outliers and convinced the authors that Figure 4 is unable to shed any light on the general quality of the data but rather on the quality of the gravity field.

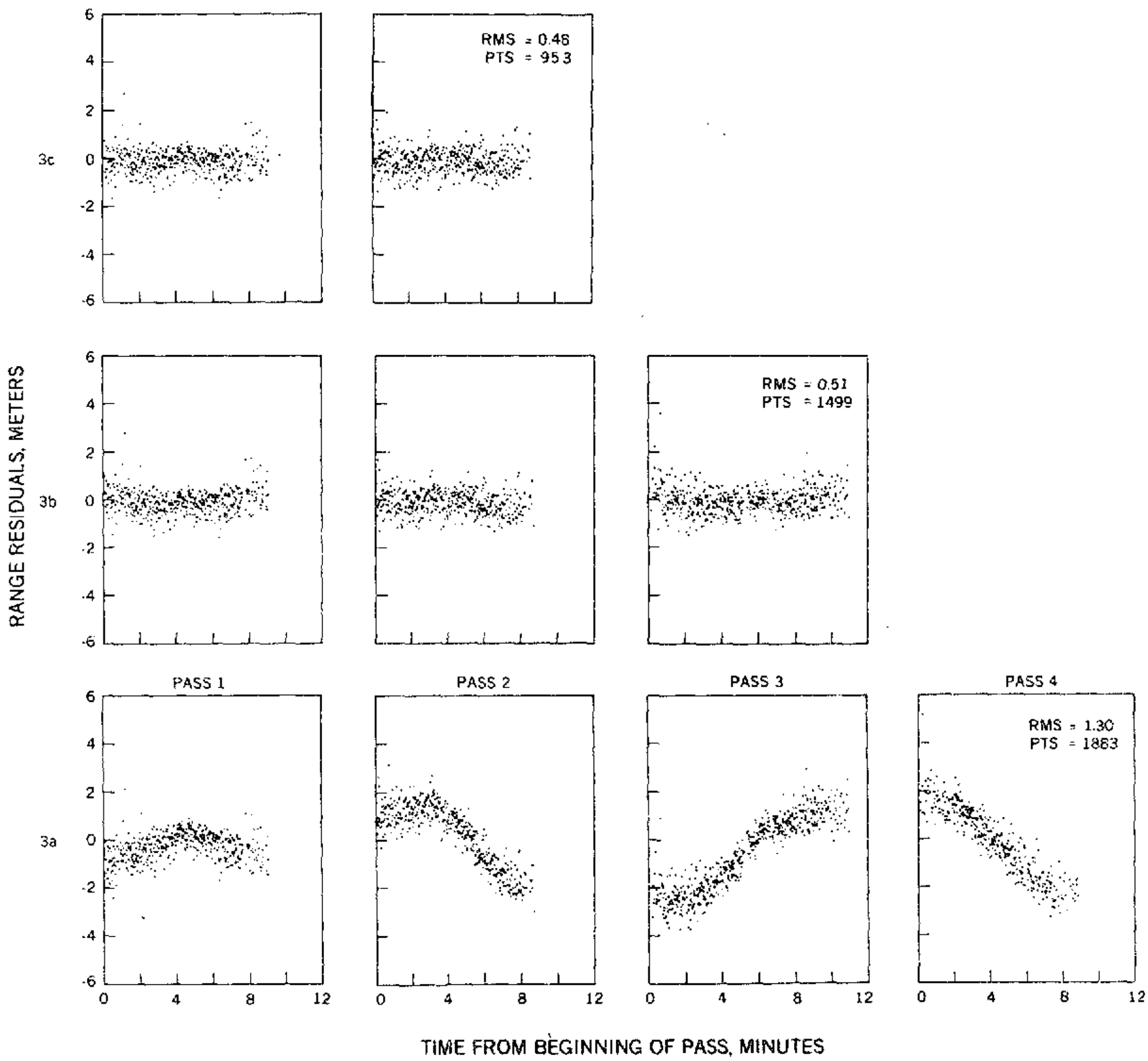

Figure 3. Laser Range Residuals to Beacon Explorer C.
3a, Residuals to a 4-pass Orbit; 3b, Residuals to a 3-pass Orbit;
3c, Residuals to a 2-pass Orbit

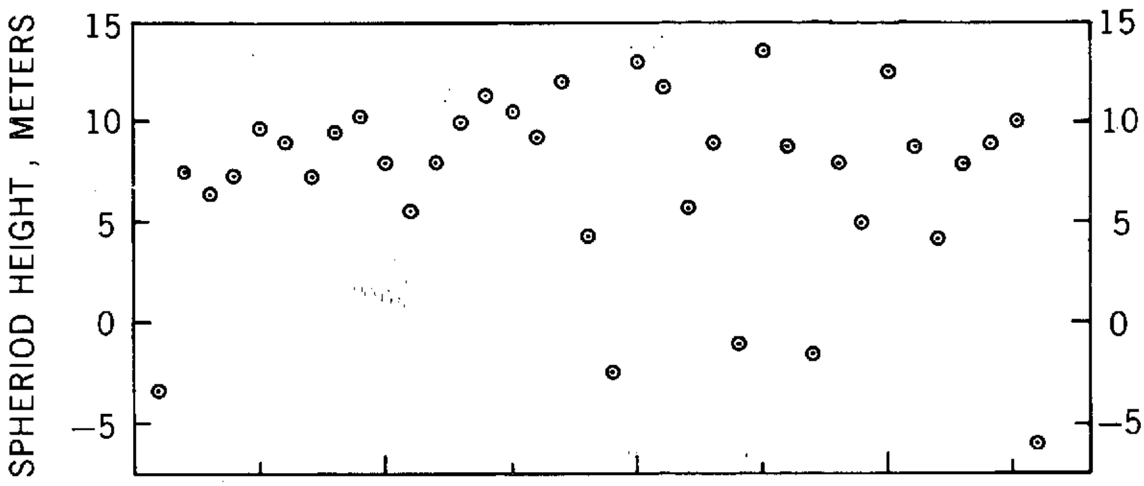

Figure 4. Height of Goddard laser obtained from thirty-six 4-pass orbital arcs of Beacon Explorer C. The rms deviation of a single height measurement is 3.85 meters.

In order to aid in the interpretation of Figure 4 a kind of spectral analysis of the 36 heights has been performed by fitting a large number of sine curves through the data and by determining the least squares amplitude and phase for different frequencies. The results of this analysis are shown in Figure 5 in which the square of the amplitude is plotted against frequency. A proper spectral analysis of the data could not be performed because of its sparcity and the irregular intervals between the measurements. If data had been obtained on all possible occasions the interval between the measurements would (on average) have been about 23 hours 39 minutes (determined by the precession of the orbit with respect to the sun). Thus the highest frequency that was removed was 1 cycle/day; the lowest was 0.01 cycles/day.

Figure 5 is essentially noisy throughout the whole frequency range. One peak around 0.032 cycles/day (31.2 days) might be significant but those near 0.31 (3.2 days) are almost certainly noise. The 31 day period is difficult to explain. It is close to a month and therefore one is tempted to ask it if is indirectly caused by the moon. This is unlikely, however, because most lunar effects have periods determined primarily by the motion of the moon with respect to the orbit, which for BE-C are around 11 and 85 days.

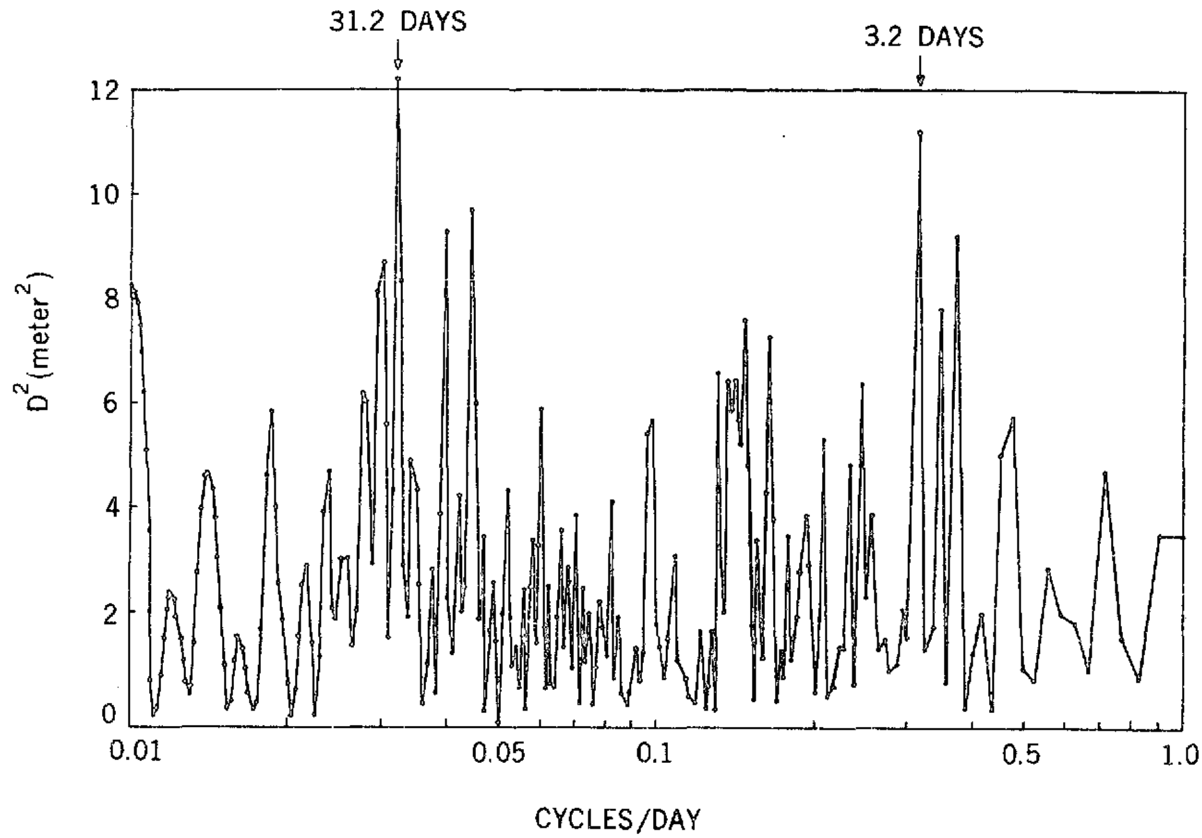

Figure 5. Frequency Analysis of the Height Determinations of the Goddard Laser

4. SOLID EARTH AND OCEAN TIDES

One of the major aims of this analysis was the detection of the perturbations of the orbit by the solid-earth and ocean-tides. Since the latitude of the Goddard tracking station (GODLAS) was within a few degrees of the apex of the Beacon Explorer C orbit most of the observations were obtained when the satellite had a predominant motion from west to east. Thus, during a four pass observing period the observations were clustered around the position of maximum latitude attained by the satellite, and hence the orbital inclination was a well determined parameter. The variations in orbital inclination obtained from the 36 four pass orbits have been analyzed using the "short-arc minus long-arc" technique in order to isolate the tidal perturbations.

The largest perturbations of the orbital inclination result from the non-sphericity of the earth; which cause both short and long period effects. The largest short period term is caused by the earth's oblateness given by [3]

$$\delta i = -\frac{3}{2} J_2 \left(\frac{R}{p}\right)^2 \sin 2i \left[\left\{\frac{1}{2}\sin^2 u + \frac{1}{3} e \cos\omega\right\} + \left\{\frac{1}{3} e \sin^2 u \cos\nu - \frac{1}{3} e \cos u \cos\omega\right\}\right]$$

$$+ \text{ terms of order } J_2 \text{ squared} \qquad (2)$$

where δi is the perturbation in inclination, J_2 is the second degree zonal harmonic coefficient, R is the earth's equatorial radius, p is the semi-latus rectum, i is the orbital inclination, u is the argument of latitude, e is the eccentricity and ω is the argument of perigee. Since $J_2 \sim 1.1 \times 10^{-3}$, the amplitude of δi is about 60 arcseconds, or about 2 km projected onto the earth's surface. An important aspect of the perturbation given by equation 2 is that the major term $\left(\frac{1}{2}\sin^2 u\right)$ is only a function of the projected motion of the satellite on the earth's surface and consequently can be ignored if the orbit inclination is determined for the same argument of latitude (u). For example, suppose the orbit of the satellite is determined at the position of maximum northerly latitude $\left(u = \frac{\pi}{2}\right)$ then the terms inside the brackets in equation 2 reduce to

$$\frac{1}{2} + \frac{2}{3} e \sin\left\{\frac{1}{2}(\omega - \nu)\right\}$$

which has an amplitude of about 120 meters for BE-C. Thus, the short period perturbations of the inclination by the gravity field can be kept to a minimum by determining the orbit at the maximum latitude position.

The J_2^2 terms and other geopotential coefficients cause short period perturbations (in the inclination) of about ten meters. Slightly longer period terms, the so-called m-daily terms of low order (m), can cause perturbations equivalent to a few tens of meters in the orbital inclination with periods of 24 hours, 12 hours, 8 hours etc. The next largest period is that of the primary resonance which is about 5.5 days for Beacon Explorer C and associated with terms of order 13. The argument of the primary thirteenth order resonance is

$$\omega + M + 13\,(\Omega - \theta)$$

where M is the mean anomaly, Ω is the right ascension of the node, and θ is the sidereal time. After approximately five and a half days the ground track of the satellite repeats itself, that is, when the argument of the primary resonance has moved through 2π. The amplitude of this resonance (for BE-C) is about 0.35 arc-seconds in the inclination (11 meters).

There are also long-period perturbations in the inclination that are associated with the odd zonal harmonics in the gravity field. The principal terms take the form[3]

$$\delta i = (AJ_3 + BJ_5 + CJ_7 \ldots) \sin \omega \tag{3}$$

where A, B, C are functions of the orbit parameters and J_3, J_5, J_7 are the odd zonal harmonics. The perigee rotation period, and therefore the period of the perturbation, for BE-C is 70 days and the amplitude of the perturbation is about 2 arcseconds which is equivalent to about 60 meters projected onto the earth's surface. Other long period perturbations are caused by the gravitational attraction of the sun and moon, and contain several periods ranging from 10 days to 85 days. In addition, there are very long period terms with periods up to about 19 years, but all these luni-solar perturbations can be computed with adequate accuracy from a knowledge of the positions of the sun and moon.

The major unknown perturbations of long period are those associated with the solid-earth and ocean-tides and are effectively indirect luni-solar perturbations. The sun and moon raise ocean and body tides that involve sufficient mass to perturb the motion of the satellite. These tidal disturbances of the gravity field can be largely represented by a second degree spherical harmonic with axial symmetry in the approximate direction of the tide rising body. With this representation the major terms in the tidal perturbation of inclination can be written[4,5,6]

$$\begin{aligned}\delta i = \frac{3}{4} m_D \frac{n_D^2}{n} \left(\frac{R}{a}\right)^5 \frac{k_2}{(1-e^2)^2} \cos^2\left(\frac{i_D}{2}\right) \Bigg\{ &\frac{\sin i \cos^2\left(\frac{i_D}{2}\right)}{2(\mathring{L}-\mathring{\Omega})} \cos 2(L-\Omega) \\ &- \frac{2 \cos i \tan\left(\frac{i_D}{2}\right) \cos i_D}{(\mathring{\Omega}-\mathring{\Omega}_D)} \cos(\Omega-\Omega_D) \\ &- \frac{\cos i \sin i_D}{(2\mathring{L}-\mathring{\Omega})} \cos(2L-\Omega) \Bigg\}\end{aligned} \tag{4}$$

where m_D is the ratio of the mass of the tide raising body to the mass of the earth, n_D the mean motion of the disturbing body, n the mean motion of the satellite and a the semi-major axis. k_2 is Love's number of second degree, i_D is the inclination of the orbit of the disturbing body to the earth's equator, L is the longitude of the disturbing body, and Ω and Ω_D are the right ascension of the ascending nodes of the orbits of the satellite and disturbing body. The dot quantities refer to time derivatives. The Love number is defined as the ratio of the tidal potential to the tide raising potential.

The amplitudes of these perturbations (of BE-C) amount to a few tenths of an arcsecond for each of the terms in equation 4 for both the sun and moon. Their periods are about 10, 12 and 85 days for the moon and 35 days, 58 days and 85 days for the sun. For Beacon Explorer C the expected total perturbation has a peak-to-peak variation of about 2.1 arcseconds, or about 70 meters projected onto the earth's surface.

The initial step in isolating the tidal perturbations was to determine the maximum latitude reached by the satellite on each pass of the 36 four-pass orbits. This was accomplished with the aid of an ephemeris which was generated for each orbital arc from a least squares fit to the data. The second step was to derive the maximum latitude of each pass from the long-arc orbit described in Section 3. Finally, the long-arc maximum latitudes were subtracted from the short-arc maximum latitudes and the "max-lat" residuals showed very clearly the effects of the tides on the orbit. From these residuals an estimation of the Love number k_2 (amplitude) and the phase of the tides were made. Initially this estimate was only made from the first five months of data[7] and subsequently extended through the following year[8]. The observed max-lat residuals are shown in Figure 6 together with the best-fit theoretical curve for $k_2 = 0.245$ and ϕ (phase) = 3.2 degrees. The precision with which the data fit the curve indicate a standard deviation of 0.005 for k_2 and 0.5 degrees for ϕ.

The actual procedure of deriving the values of k_2 and ϕ extended over several months[9]. In the initial analysis of the tides the differences between the laser data and the theory shown in Figure 6 contained an oscillation of amplitude about 2 meters and period 5.5 days. This pattern is characteristic of an error in the resonance terms (order 13 for BE-C) in the gravity field and consequently an adjustment to two coefficients was made. An approximately 19% decrease in C_{19}^{13} and approximately 18% increase in S_{19}^{13}, with respect to the GEM 1 values, removed the sinusoidal residual pattern[8]. At this stage the analysis was restarted from the beginning with this new gravity field which we referred to as GEM 1*. Having re-analyzed the orbits and redetermined the station height (with GEM 1*) we found almost no change from our original analysis with GEM 1 with the exception of removing the patterns in the residuals in inclination (maximum latitude) in the tide analysis. It is with the GEM 1* field that the laser points in Figure 6 have been computed and from which the Love number and phase already given were derived.

Because the determination of k_2 and ϕ was obtained from a separate analysis of the residuals in inclination rather than from a large least squares adjustment from the raw laser data it was necessary to undertake a second re-analysis of all the observational data with the tides modeled (using the recovered values of k_2 and ϕ) in order to check that the tidal solution had converged. Only insignificantly small changes in the recovered heights for the Goddard laser were obtained and there was no change from the original best fit values for k_2 and ϕ.

The most important single result evident from Figure 6 is that the amplitude of the perturbation is significantly smaller than expected. From seismic data we know that k_2 should be around 0.30 and therefore that there must be some additional perturbations of the BE-C orbit that are tending to compensate for the solid-earth tides. Since the agreement between the theoretical curve and the data is so good in Figure 6 it must be concluded that the functional form of the perturbation must be very similar to that of the solid-earth tide, that is, a second degree spherical harmonic. Thus, it has been suggested that the additional perturbation is caused by the global ocean tides[10]. The ocean tides, however, contain many components which can be represented in spherical harmonic form but it is only those of second order, and principally second and fourth degree, that actually perturb the spacecraft to any significant extent[11]. We may therefore argue that this tidal analysis has identified a perturbation of BE-C equivalent to $(k_2)_{ocean} = 0.245 - 0.30 = -0.055$ from the ocean tides.

The solution for the phase of the tidal perturbations ($\phi = 3.2°$) is more difficult to interpret. The phase of the solid-earth tide is believed to be small, of the order of a degree or less, and the difference with the result here suggests a larger phase lag for the ocean tides. This is not inconsistent with observations of the tides except that each tidal component is believed to have a different phase and thus a single observation of the phase lag of the tidal gravitational field (of even degree and order two) must be a composite value and hence difficult to interpret. We must await further observations of the ocean tidal perturbations of other satellites before the contributions and phase lags from each tidal component can be identified.

5. VARIATION OF LATITUDE

In the analysis previously described it was necessary to include polar motion which has the effect of changing the latitude, and to a small extent the longitude, of the laser tracking station. The magnitude of the variation in latitude[12] amounts to about 0.3 arcseconds in a year (10 meters on the earth's surface) and is monitored routinely by the Bureau International d'le Heure (BIH). In our computations the 5-day smoothed mean position of the pole published by BIH has been used. The accuracy of the BIH data is probably about 0.02 arcseconds but

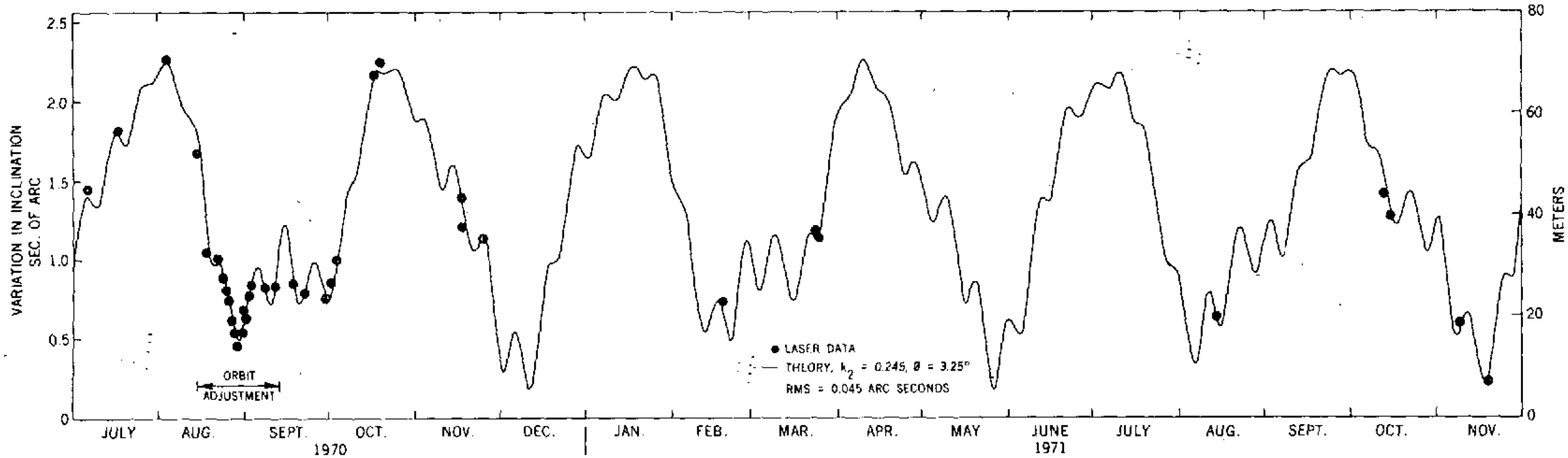

Figure 6. Tidal Perturbations of the Orbital Inclination of Beacon Explorer C

because these values are averaged over 5 days it is possible for real departures of several hundredths of an arcsecond to exist with respect to a continuous smooth curve.

During the seventeen months of laser data the latitude of the Goddard station varied by approximately 15 meters due to the combined effects of the annual and Chandlerian motions of the pole. In order to determine our sensitivity to these motions we suppressed the modeling of polar motion in the 36 short orbital arcs so that the variation of latitude of the tracking station would be forced into the orbital inclination and hence into the values of maximum latitude. Subtracting the long-arc maximum latitudes from the short-arc maximum latitudes revealed, as expected, the variation in latitudes of the tracking site[8,13] shown in Figure 7. In analyzing this variation it was essential to include the tidal effects and use the improved resonant gravity terms described in the previous section. The BIH smoothed variation of latitude is also shown in Figure 7 and the rms fit of the laser data about the BIH curve is 1.38 meters (0.045 arcseconds). At the present time it is probably correct to assume that the differences between the laser and BIH results of Figure 7 are primarily due to errors in the laser values rather than the BIH and that the scatter reflects errors in the gravitational field model used in the orbit computations. As our knowledge of the gravity field improves over the next few years the scatter in the latitude variation (from the laser) of Figure 7 should decrease correspondingly.

In order to better understand the causes of the scatter of residuals in Figure 7 an attempt was made to determine any major periodicities within the residuals to the BIH curve. The same method as used in analyzing the station heights of Section 3 (Fig. 5) was employed. A set of sinusoidal oscillations was fitted through the residuals with frequencies between 1 cycle/day and 0.01 cycles/day. The square of the amplitudes (D) of the recovered oscillations are shown against frequency in Figure 8. As might be expected the variation of D^2 with frequency is largely noise but there may be one or two peaks that are significant. There is a peak near 0.165 cycles/day (6.0 day period) which is one of the resonant periods for the odd degree thirteenth order geopotential terms, and another unidentified peak at about 0.035 cycles/day (28.2 day period). A comparison of Figures 5 and 8 is important in their interpretation since the laser data are the source of both the height and latitude measurements. The only difference in technique used to generate these figures is that Figure 8 is an analysis of residuals obtained from differencing short and long orbit arcs while Figure 5 is based purely on a short arc analysis. Hence errors in the short-arc orbit computation will appear in both figures while errors in the long arc only affect Figure 8. There is probably only one significant peak in the height analysis of Figure 5, at about 0.032 cycles/day (31.2 day period). This frequency is reasonably close to the peak at 0.035 cycles/day seen in Figure 8 but a frequency this low in the orbit

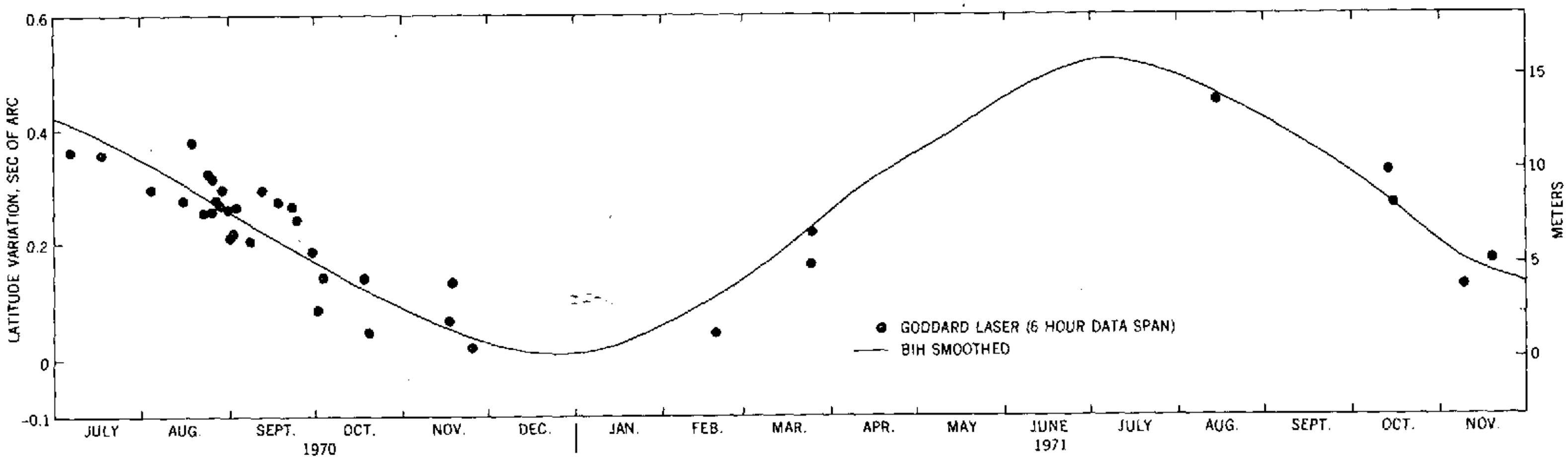

Figure 7. Variation of Latitude of the Goddard Laser

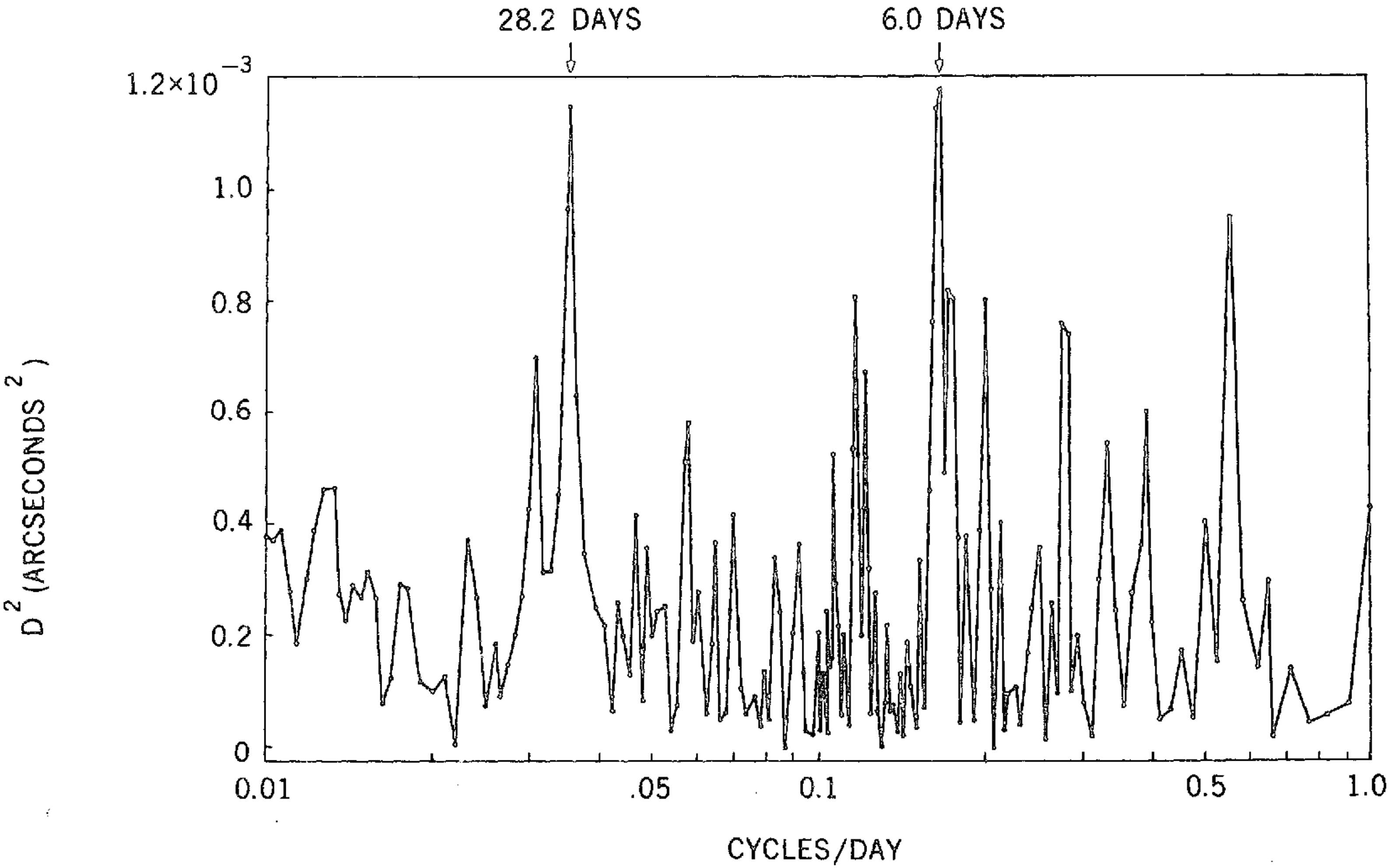

Figure 8. Frequency Analysis of the Variation in Latitude of the Goddard Laser

perturbations is difficult to explain. Further, the amplitudes of these oscillations (if they exist) are so large, 3.5 meters in height and 0.034 arcseconds (1 meter) in latitude, that their origin cannot be geophysical. However, it is possible that the orbit is resonant with terms of very high degree and order, say 27, and this possibility is being investigated.

6. TWO STATION EXPERIMENTS

The results described in the previous sections were obtained with a single laser tracking system and the fundamental objective of that work was to try and develop techniques that will permit a laser tracking system to monitor its own latitude and height variation at a geophysically useful level. In this section we describe some of our experiments using two laser systems in which the fundamental objective is the measurement of phenomena such as tectonic and fault motion by the precise determination of the position of one of the tracking systems with respect to the other.

During August and September 1970 a second GSFC laser tracking system was operating from a site near Seneca Lake, New York [14]. This station was 408 km due north of the laser system at the GSFC Optical Site with which the polar motion and tidal data were obtained. The original purpose of establishing the Seneca station was to determine polar motion in a joint experiment with the Goddard laser. However, it was subsequently found that the second station was not essential for polar motion studies but that with two stations tracking the same satellite on the same passes it was possible to determine the distance between the lasers very precisely.

There were five occasions when the Seneca station observed four consecutive passes of BE-C simultaneously with the Goddard Station. From these data, during August and September 1970, the position of the Seneca station has been determined. In these calculations the position of the Goddard station has been assumed known with the following coordinates:

Goddard: Lat. 39° 01' 13.88" N

Long. 283° 10' 18.50" E

Height 9.29 meters

The values of the latitude and longitude were adopted from early work at GSFC on tracking station positions and are not crucial in the analysis. The height value is a dynamic average of the values in Figure 4, derived by simultaneously adjusting the orbital parameters on each of the 36 orbital arcs together with the station height (a single least squares value for all 36 arcs). The numerical value of the height is dependent on the latitude adopted for the station (as above), the product of the gravitational constant (G) and the mass of the earth (M), the gravitational field (GEM 1*, see Section 4) and the mean equatorial radius (R_E) of the earth. Throughout these analyses we have used the following values:

$$GM = 3.986013 \times 10^{20} \ cm^3/sec^2$$

$$R_E = 6378155 \text{ meters.}$$

The results of our determination of the Seneca position are shown in Table 1. The position of Seneca has been determined from each four-pass orbital arc and Table 1 shows that the range of values is 0.055 arcseconds in latitude (1.7 meters), 0.116 arcseconds in longitude (2.7 meters at the latitude of Seneca) and 4.32 meters in height. Because Seneca is due north of Goddard the baseline values have the same range as the latitude values. The baseline length according to survey is 408,698.77 meters which is about three meters less than the value obtained from the coordinates given in Table 1; the accuracy of the survey is about ±2 meters.

The major sources of error in the satellite solution for the Goddard-Seneca baseline are estimated to be:

gravity	2.5 meters
GM (1 part in 10^6)	0.2 meters
range biases (1 meter)	0.3 meters
refraction (5%)	0.1 meters
Goddard heigth (10 meters)	1.6 meters

where the gravity error model[15] is taken as one quarter of the difference between the Smithsonian Astrophysical Observatory's Standard Earth 1 field[16] and the Applied Physics Laboratory's 3.5 Model[17]. There is of course considerable uncertainty in the above estimates for the accuracy of the parameters used in the analysis but a total accuracy of 2 to 3 meters appears reasonable. Gravity errors dominate the solution but because of the technique that has been employed all the orbital arcs (4 passes) have very similar geometric distribution with respect to the tracking stations. Hence, the errors on one 4-pass orbit are very similar to the errors on any other 4-pass orbit so that the repeatability of the baseline measurement can be expected to be considerably better than the accuracy. This situation is supported by Table 1 in which the standard deviation of an individual baseline measurement is probably better than 1 meter even though our best estimate of the accuracy is, perhaps, 3 meters.

Table 1

Results of the Determination of the Position of the Seneca Laser and the Baseline between Goddard and Seneca

Time of First Pass Y - M - D	Range Meas.		R. M. S. Fit to Orbit		Adjusted Seneca Position			Baseline Deviation From Mean
	Goddard	Seneca	Goddard (cm.)	Seneca (cm.)	Lat. 42° 42'	Long. 283° 10'	Height	
70-08-22	1792	580	153	135	4.900"	17.278"	191.48 m.	+96 cm. (max.)
70-09-01	2002	817	105	106	4.845	17.334	189.16	-80 cm. (min.)
70-09-02	1883	1522	128	134	4.874	17.282	190.95	+14 cm.
70-09-07	1351	1399	147	145	4.848	17.253	189.81	-69 cm.
70-09-11	778	917	82	202	4.879	17.218	193.48	+38 cm.

MEAN BASELINE	408,701.92 m.	INDIVIDUAL BASELINE SIGMA	74 cm.
BASELINE SPREAD	176 cm.	MEAN BASELINE SIGMA	33 cm.

The internal accuracies (noise standard deviation) of the Goddard-Seneca experiment are of particular interest because they represent the ultimate capability of the technique if our knowledge of everything affecting the motion of the spacecraft were known perfectly. Based on laser range measurements of 1 meter noise (no biases) the standard deviation of latitude would be about 15 cm, longitude about 8 cm, and height 9 cm. Thus, the technique has the capability of reaching the one centimeter level in all coordinates with laser systems of the 5-10 cm noise level, which are projected to be available in 1974.

With the introduction of radar altimeters, satellite-to-satellite tracking techniques and more accurate laser data in greater quantities, significant improvements in the gravity field, GM and station coordinates can be projected such that 10 cm precision relative positioning should be a realizable objective from a single four-pass orbital arc. In 1972 a plan was formulated for applying this technique to the measurement of motion along the San Andreas Fault in California which is the boundary between the tectonic plates of the Pacific and North America. In its simplest concept, one laser tracking station would be established on the western side of the fault, on the Pacific plate, near San Diego and a second station on the eastern side of the fault, the North American plate, near Quincy in northern California (see Figure 9). The distance between the two stations is nearly 900 km and the angle between the baseline and the fault is about 15° so that the change in baseline length over several years will be very similar to the gross fault motion across the plate boundary. Present estimates of the fault motion are between three and five centimeters/year.

A simulation of this experiment[18], the San Andreas Fault Experiment (SAFE), has been completed and is summarized in Table 2. The experiment has been simulated to last for eight years, using the BE-C satellite and stations at San Diego and Quincy. Table 2 shows the effects of various errors on the baseline between the two stations when each year of measurements is composed of sixteen simultaneous short-arcs of three consecutive passes. (Three passes were used because the simulation suggested that three passes gave a greater precision than four passes). The error sources in the simulation were 1 part in 10^6 for GM, $\frac{1}{4}$ (SAO Standard Earth 1 - APL 3.5) for gravity, 10 cm range biases, 10% error in solar radiation and air drag models, $\frac{1}{4}$ of the nominal values of the 19th degree, 13th order resonant terms in the geopotential, and 5 meters in each coordinate at San Diego.

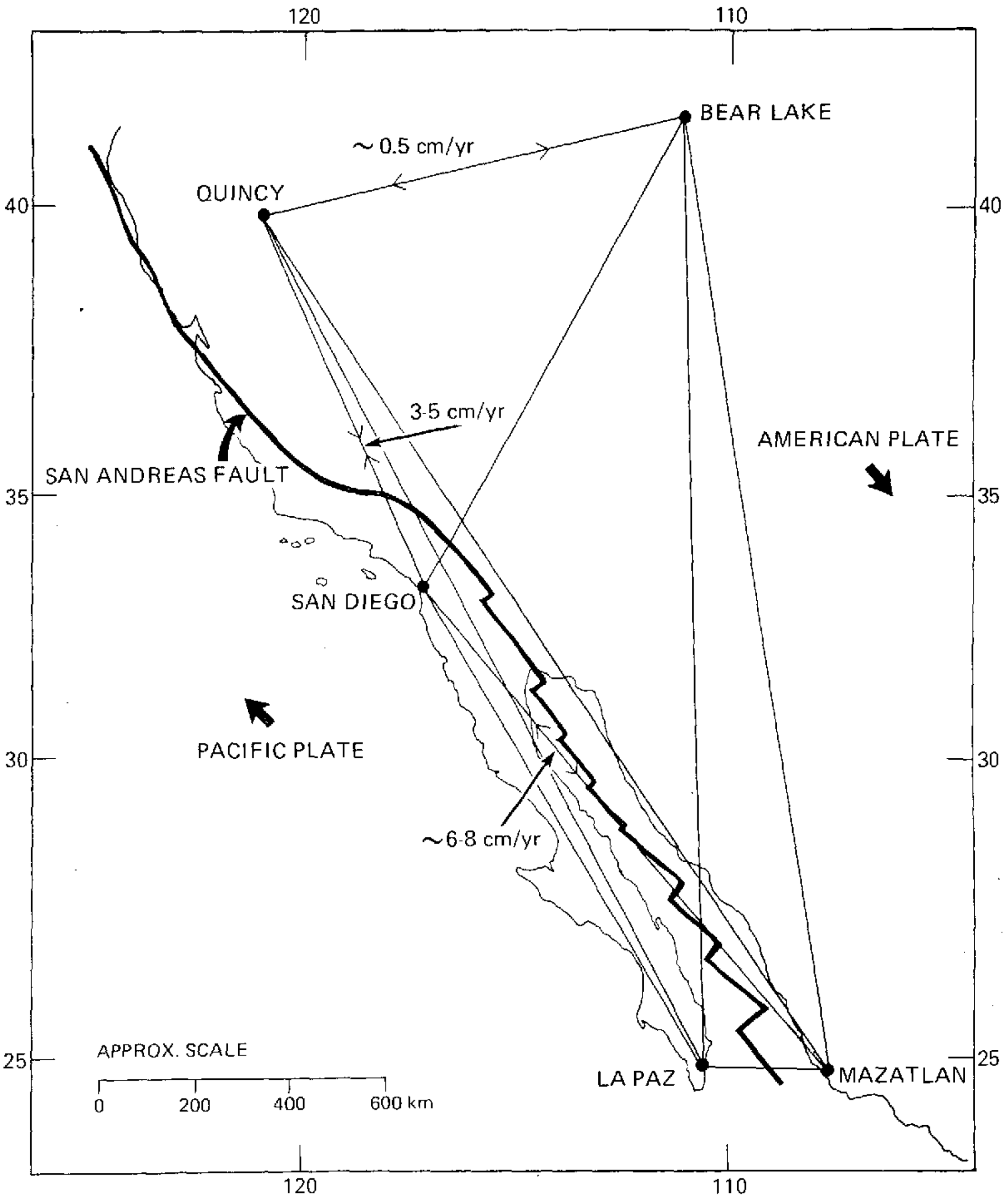

Figure 9. Proposed Site Locations for the San Andreas Fault Experiment (SAFE)

Table 2

A simulation of the San Andreas Fault Experiment. Effects of model errors on the determination of the baseline between San Diego and Quincy. All measurements are in centimeters.

Year	GM	Gravity Model	Quincy Bias	San Diego Bias	Solar Radiation	Atmospheric Drag	C 19,13	S 19,13	San Diego		
									Longitude	Latitude	Height
1970	102	-62	-2	7	-1	-1	1	0	0	-14	-110
1971	95	-68	-2	8	-2	-1	0	0	0	0	-99
1972	109	-53	-3	7	0	-1	1	0	0	-2	-113
1973	102	-66	-2	7	0	-1	0	0	0	-9	-107
1974	102	-57	-2	7	0	-1	1	0	0	-8	-108
1975	98	-61	-2	7	1	-1	0	0	0	-4	-103
1976	102	-75	-3	7	1	-1	1	0	0	-9	-107
1977	99	-64	-3	7	0	-1	0	0	0	-5	-104
Average	101	-63	-2.4	7.1	-.1	-1	.5	0	0	-6.4	-106
RMS	3.8	6.3	.5	.3	.9	0	.5	0	0	4.2	4.1

From Table 2 we obtain the root-sum-squares as ±9.5 cm for the baseline precision based on the a priori magnitudes of the error sources already given. These a priori estimates are believed to reflect our knowledge in 1972-73 and can be expected to improve considerably during the present decade. If we postulate that there will be the following improvements in our knowledge by 1980 (over 1972-73 values)

GM	by a factor	20	(to 5 parts in 10^8)
Gravity	by a factor	7	
San Diego Position	by a factor	20	(to 25 cm)
Laser Systems	by a factor	5	(to 2 cm)

the precision of the baseline measurement will be 1 to 2 cm with an accuracy of about 10 to 15 cm. This will permit the determination of the change in baseline (plate motion) to better than 0.5 cm/year over a seven year period.

The full plans for SAFE include the establishment of a third laser site near Bear Lake, Utah and proposals for two sites in Mexico, on either side of the Gulf of California. These additional sites will enable the gross plate motion and crustal deformation to be measured along a 2000 km stretch of the western United States and Mexico. Motions along the fault in the Gulf of California are estimated to be much larger than those in southern California, with estimates generally in the 6 to 8 cm/year range. The site in Utah will enable the spreading rate across northern California and Nevada to be estimated, although this motion is probably only of the order of 0.5 cm/year and consequently very difficult to measure.

The SAFE experiment is planned to begin in the Spring of 1974 but a preliminary tracking experiment was conducted in September 1972 between the San Diego and Quincy sites[19]. In this test experiment both sites successfully tracked four consecutive passes of BE-C on four simultaneous occasions from which estimates of the position of Quincy with respect to San Diego have been obtained. During this period the laser systems were performing at about the 60 to 80 cm noise level, implying biases of this order could be in the data. Essentially the same techniques were used in this analysis as had been used in the Goddard-Seneca experiment. From all the four-pass orbital arcs obtained at San Diego the height of the station was derived. The latitude, longitude and recovered height of the station used in the subsequent analysis were:

San Diego:	Lat.	32° 36' 02.53" N
	Long.	243° 09' 32.87"E
	Height	989.5 meters.

Based on the above coordinates the results shown in Table 3 were obtained with the GEM 1* gravity field. Unfortunately, there are only four determinations and any statistics concerning the solutions must, therefore, be treated with caution. However, the four solutions do agree at least as well as might be expected from our knowledge of the data and the models. Further, there is a very obvious correlation among the recovered latitudes and longitudes in Table 3. In addition, this linear variation is approximately perpendicular to the direction of the San Diego-Quincy baseline showing that there is a tendency for the Quincy solutions to lie on an arc centered on the primary station (San Diego) thereby giving a better determination of the baseline length than of the individual coordinates of the second station (Quincy).

7. FUTURE SATELLITES AND INVESTIGATIONS

In the course of the investigations described in the previous sections it became increasingly obvious that the full potential of the techniques could not be achieved without significant improvements in many geophysical parameters, including GM and the gravity field. The major problem area has probably been the gravity field so far, partly because our experience has been with relatively low altitude spacecraft like BE-C. However, this may change if the planned high altitude satellites such as TIMATION III and LAGEOS are launched. TIMATION, expected to be at an altitude of about 14,000 km, will be much less affected by a lack of knowledge of the higher harmonic coefficients of the gravity field and therefore, in principle, a much better spacecraft to use for satellite geodynamic research.

In order to assess the impact of TIMATION III on an experiment, such as SAFE, a series of simulations of the determination of the San Diego-Quincy baseline have been completed. In summary, the major error sources are now only GM and the position of San Diego. Further, results of the same accuracy and precision can be obtained from a few days of TIMATION tracking compared to about two months of BE-C tracking and because the gravity perturbations are very small, high altitude spacecraft offer the possibility of determining GM. Simulations of a small world-wide network of laser stations indicate that an improvement in GM by about two orders of magnitude (to 1 part in 10^8) should be realizable in the next five years[20]. Thus the TIMATION and LAGEOS type spacecraft are expected to have a major impact on this type of investigation.

Projecting the type of techniques used on Beacon Explorer C into the future with more advanced spacecraft and better orbits, and further realizing that there will be substantial improvements in our knowledge of the gravity field through the application of newer tracking techniques, such as satellite-to-satellite tracking and altimetry, we anticipate that geophysically useful measurements of fault motion, tectonic plate motions and polar motions will be obtained within the next five years. The accuracies that will be achievable from about a week

Table 3

Results of the Determination of the Position of the Quincy Laser and the Baseline between San Diego and Quincy

Time of First Pass Y-M-D	Range Meas.		R. M. S. Fit to Orbit		Adjusted Quincy Position			Baseline Deviation from Mean
	Quincy	Sandie	Quincy (cm.)	Sandie (cm.)	Lat. 39° 58'	Long. 239° 03'	Height	
72-09-17	623	1047	299	357	24.447"	37.787"	1062.76 m.	+35 cm. (max.)
72-09-18	327	920	205	250	24.411	37.732	1061.42	-19 cm.
72-09-30	649	930	190	153	24.402	37.686	1062.24	+15 cm.
72-10-03	439	578	127	98	24.373	37.659	1064.24	-30 cm. (min.)

MEAN BASELINE	896,275.17 m.	INDIVIDUAL BASELINE SIGMA	30 cm.
BASELINE SPREAD	65 cm.	MEAN BASELINE SIGMA	15 cm.

of data will probably be of the order of 5 to 10 cm in distance measuring for distances up to about 10,000 km, about 5 to 10 cm in the variation of latitude and 10 to 20 cm in the variation in height of a station from six to twelve hours of tracking. In addition, it should also be possible to determine the variations in the length of day (LOD) to a few tenths of a millisecond on a daily basis, although other techniques, such as very long baseline interferometry, may be better suited to this measurement. In the laser technique for LOD studies the essential requirement is a suitable satellite in a highly inclined orbit to the equator so that a station, or stations, can measure the time and longitude of the satellite crossing a given latitude. Daily measurements of this type permit the rotation of the earth with respect to the plane of the orbit of the satellite to be derived.

REFERENCES

1. Gaposchkin, E. M. and Lambeck, K., "1969 Smithsonian Standard Earth II," Smithsonian Astrophysical Observatory, Spec. Rep. 315, 1970.

2. Johnson, T. C., H. H. Plotkin and P. L. Spadin, "A laser satellite ranging system, 1, Experiment description," IEEE J. Quantum Electron., QE-3(11), 435-439, 1967.

3. Merson, R. H., "The motion of a satellite in an axi-symmetric gravitational field," Geophys. J., Roy. Astron. Soc., Vol. 4, p. 17-52, 1961.

4. Kozai, Y., "Effects of the tidal deformation of the earth on the motion of close earth satellites," Publ. Astron. Soc. Japan, Vol. 17, p. 395-402, 1965.

5. Musen, P. and Felsentreger, T., "On the determination of the long period tidal perturbations in the elements of artificial earth satellites," Celestial Mechanics, Vol. 7, p. 256-279, 1973.

6. Fisher, D. and T. L. Felsentreger, "Effects of the solar and lunar tides on the motion of an artificial earth satellite," Goddard Space Flight Center, Rpt. X-547-66-560, 1966.

7. Smith, D. E., R. Kolenkiewicz and P. J. Dunn, "A Determination of the earth tidal amplitude and phase from the orbital perturbations of the Beacon Explorer C spacecraft," Nature, Vol. 244, p. 498, 24 August 1973.

8. Kolenkiewicz, R., D. E. Smith and P. J. Dunn, "Polar motion and earth tides from Beacon Explorer C," Presented at the First International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Athens, Greece, May 1973.

9. Dunn, P. J., D. E. Smith and R. Kolenkiewicz, "Techniques for the analyses of geodynamic effects using laser data," Presented at the First International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Athens, Greece, May 1973.

10. Lambeck, K., and A. Cazenave, "Fluid Tidal Effects on Satellite Orbits and other Temporal Variations in the Geopotential," Bulletin No. 7, Groupe Recherches de Geodesie Spatiale, January 1973.

11. Musen, P., "A semi-analytical method of computation of oceanic tidal perturbations in the motion of artificial satellites," Goddard Space Flight Center, Rpt. X-590-73-190, July 1973.

12. Munk, W. H. and G. J. F. MacDonald, "The Rotation of the Earth," Cambridge University Press, London, 1960.

13. Smith, D. E., R. Kolenkiewicz, P. J. Dunn, H. H. Plotkin and T. S. Johnson, "Polar Motion from Laser Tracking of Artificial Satellites," Science, Vol. 178, pp. 405-6, 27 October 1972.

14. Smith, D. E., R. Kolenkiewicz and P. J. Dunn, "Geodetic Studies by Laser Ranging to Satellites," Geophysical Monograph Series Vol. 15, pp. 187-196, The Use of Artificial Satellites for Geodesy, Edited by Henriksen, Mancini and Chovitz, American Geophysical Union, 1972.

15. Martin, C. F. and N. A. Roy, "An error model for the SAO 1969 standard earth," Geophysical Monograph Series Vol. 15, pp. 161-167, The Use of Artificial Satellites for Geodesy, Edited by Henriksen, Mancini and Chovitz, American Geophysical Union, 1972.

16. Lundquist, C. A. and G. Veis (Eds.), "Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth, Vol. 1," Smithsonian Astrophysical Observatory, Cambridge, Mass., 1966.

17. Guier, W. H. and R. R. Newton, "The earth's gravitational field deduced from the Doppler tracking of five satellites," J. Geophys. Res., 70(18), 4613-4626, 1965.

18. Agreen, R. W. and D. E. Smith, "A simulation of the San Andreas Fault Experiment," Goddard Space Flight Center, Rpt. X-592-73-216, May 1973.

19. Smith, D. E. and F. O. Vonbun, "The San Andreas Fault Experiment," Presented at the 24th Congress of the International Astronautical Federation, Baku, USSR, 7-13 October 1973.

20. Carpenter, L. (Edit.), "Preliminary Study of the Application of the TIMATION III Satellite to Earth Physics," Goddard Space Flight Center Rpt. X-553-72-50, March 1972.