2

CR 137527

(NASA-CR-137527) ANALYTICAL STUDY OF STRESSES RECORDED IN THE DH 2011 ROTOR BLADES Final Report (Giravions Dorand Co.) 9~~2~~3 p HC $7.75 CSCL 01C

N74-27501

Unclas
G3/02 42835

GIRAVIONS DORAND

93

NASA CR-137527

Analytical Study of Stresses Recorded in the DH 2011 Rotor Blades

Document DH 2011-C E1

July 1973

Contract N° NAS2 - 3673
Modification N° 19

Prepared by :

Marcel KRETZ	Jean-Noël AUBRUN	Marc LARCHE
Chief Engineer	Research Engineer	Research Engineer

The work reported in this document has been made possible through support and sponsorship of the United States Government through its National Aeronautics and Space Administration, Ames Research Center, in joint participation with U. S. Army Air Mobility Laboratory.

GIRAVIONS DORAND

5, Rue Jean Macé - 92153 SURESNES
FRANCE

Tel. : 772-18-20

/

ABSTRACT

An analytical study of stresses in the blades recorded during the March 1971 tests of the DH 2011 jet flap rotor has been performed and presented. The main objective of the study was to compare the experimental results with analyticaly determined stresses. The comparison extended over 15 specific flight cases has been only partially successful. In fact computed 3P and 4P stress components showed only a poor correlation with the test data obtained. It is believed that the simplified model of aeroelastic effects used is mainly responsible for this lack of agreement with test results.

ANALYTICAL STUDY OF STRESSES RECORDED IN THE DH 2011 ROTOR BLADES

Final Report

TABLE OF CONTENTS

1. INTRODUCTION

The March 1971 Wind tunnel tests of the DH 2011 jet flap rotor have showed high correlation between stresses and multicyclic control effects. By a suitable choice of these multicyclic effects, reductions of 50% in stresses and vibrations have been obtained. Though these results correspond to what could have been expected, no reliable analytical methods exist to predict and explain the experimental phenomena. The purpose of the present study is to attempt with existing analytical means the determination of stresses and their comparison with recorded data.

The existing means consist of the computer program (Ref. 1), known as the Evans - Mc Cloud Program, EMCP, and an analytical method for determination of stresses and vibrations on the rotor blades from known aerodynamic load distributions (Ref. 2). The Giravions Dorand Stress Program, GDSP, is based on transfer functions and transfer matrices suitable for investigation of complex rotary wing problems.

The stresses will be analyzed at selected flight configurations, mainly at advance ratios of 0.4, which have been used in the stress and vibration reduction study (Ref. 3).

2. SYMBOLS

Symbols of Ref. 1 are used throughout this document and will not be restated.

T	period
c_n	complex Fourier coefficient of n-th order
x	t/T non dimensional time
N	number of blade elements
m_n	tip mass of n-th blade element
m'_n	central mass of n-th blade element
k_n	equivalent spring effect between n-th and n + 1st blade element
ℓ_n	length of n-th blade element
θ_n	angle between n-th and n + 1st blade element

F_n	shear at the end of the n-th blade element
σ	bending stress
f_n	aerodynamic force applied on m_n
z_n	ordinate at the end of the n th blade element
γ_n	angle between the n th blade element and the reference line
EI	local rigidity of the blade
$\frac{dm}{dr}$	mass distribution of the blade (blade density)
h_i	blade longeron thickness
R	rotor radius
Ω	rotation speed (rd/s).

Upperscripts ($\dot{x}$) and ($\ddot{x}$) indicates first and second order derivatives $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$.

3. BASIC TEST DATA

The basic stress data has been processed to obtain their analytical content. The following method has been adopted : the stress signals have been originally recorded on photographic strips of CEC recorders (fig. 1). The period then has been divided into 50 to 60 intervals defined by X , the corresponding ordinate Y was noted. Traces n°'s 19 and 20 corresponding respectively to the stress at 45% radius (4.2)and 70% radius (4.3), have thus been treated for 73 test points.

The Fourier analysis of the stress signals has been obtained at the Ames Research Center by a standard computer routine.

In fact, only 15 of these analyses are of use for this study, since we have only 15 simulations. The simulations are defined such that the rotor is in the same aerodynamic configuration as in certain windtunnel tests.

In particular, the cyclic and multicyclic pitch law was defined by Fourier analysis of the flap deflection signal for these same tests. The deflection, produced by a CIMATRAN transducer, is given by trace n° 24 of the C.E.C. recordings.

These signals were first analyzed by Giravions Dorand, using a mechanical harmonic analyzer and were then reanalyzed more accurately by NASA, using a computer.

We therefore possess 15 simulations giving the radial distribution of forces on the blade as a function of the azimuthal position of the blade. These simulations, numbered SIM 11 through SIM 25, correspond respectively to run-points 9-3, 9-4, 9-5, 9-6, 12-10, 12-11, 12-12, 12-13, 14-10, 14-11, 14-12, 14-13, 16-8, 16-9 and 16-10, for which we know the Fourier coefficients of stress at 0.45R and 0.7R.

4. CALCULATION PROCEDURE

The program described in the present note has been motivated by the need to correlate the theoretical results obtained from the Evans - Mc Cloud simulation program (Ref. 1) with the experimental data gathered during the March 1971 wind tunnel tests of the DORAND jet flap rotor. In the E. M. C. program, the aerodynamic forces are computed at several stations on the blade and for several blade azimuthal angles ψ . On other hand, blade stresses have been measured at 0.45 and 0.7R, as well as the vibratory forces on the rotor hub, as a function of ψ (Ref. 2 and 3).

The purpose of the Giravions Dorand Stress Program is to compute the blade stresses and vibratory forces from the aerodynamic forces. The method is based upon the computation of transfer functions obtained from the dynamic and elastic characteristics of the blade (Ref. 4 and 5), which are then used to compute the Fourier coefficients of the stresses from the Fourier coefficients of the forces.

The G. D. Stress Program is composed of three independent programs.

1. BFHA (Blade forces harmonic analysis)

   Computes the aerodynamic forces Fourier coefficients from the values of these forces given for discrete values of ψ .

2. BSTM (Blade stresses transfer matrices)

   Computes transfer matrices parameters from blade characteristics.

3. STRESS

   Computes the Fourier coefficients of blade deflections and stresses and of the vertical vibratory force on the rotor hub, using the output cards of BFHA and BSTM.

In this chapter of the present note the mathematical methods involved in these programs will be described. The following chapter will be devoted to the description and use of the FORTRAN IV programs.

4.1 The fast Fourier analysis (Program FHARM)

4.1.1 The complex Fourier serie

A periodic function $F(t)$, of period T, may be expanded in a complex Fourier serie

$$F(t) = \sum_{n=-\infty}^{+\infty} c_n \, e^{2\pi j n t / T} \qquad (1)$$

The complex coefficients c_n are given by :

$$c_n = \int_0^1 f(x) \, e^{-2\pi j n x} \, dx \qquad (2)$$

where $f(x) = F(Tx)$

and $x = t/T$

These coefficients are related to the coefficients of the cosine/sine expansion by the relations :

$$\begin{cases} A_0 = C_0 \\ A_n = 2 \times \text{Real part } (C_n) \\ B_n = -2 \times \text{Imag. part } (c_n) \end{cases} \qquad (3)$$

The computation of the Fourier coefficients amounts therefore to that of the integrand in (2), which may be approximated, in the case of digital computation, by the sum :

$$S_n = \sum_{p=1}^{N'} f_p \, e^{-2\pi j n x_p} \left(x_{p+1} - x_p \right) \qquad (4)$$

The value of F is assumed to be known for N' points of the interval $(0, T)$, so that

$$f_p = f(x_p) = F(t_p)$$

$$x_1 = 0$$

$$x_{N+1} = 1$$

4.1.2 Principles of fast analysis

A direct computation of the sum in (4) is not desirable because :

1) it replaces the function f by a series of steps
2) the exponential FORTRAN IV function has to be called up for each p and each n, which is a time consuming process.

The method of the fast analysis will consist in replacing the actual function f by linearly interpolated segments between two known values f_p and f_{p+1}, and to compute the exact solution for the Fourier coefficients of this interpolated function $\hat{f}$.

Also, a recursive algorithm is used which deduces the value of the exponential function needed at a particular step from its value at the previous step by simple multiplication, using the fact that :

$$e^{(k+1)z} = (e^{z}) \times e^{kz}$$

The FORTRAN IV exponential function is therefore called up only once, at the beginning of the program, and subsequent calls are replaced by a multiplication, which is some 10 times faster.

4.1.3 Equation of the fast analysis

The function f is presumed known at equidistant points x_p. If D is the interval between two points :

$$D = x_{p+1} - x_p = 1/N' \qquad (5)$$

$$x_p = (p-1)\, D \qquad (6)$$

Equation (2) may be written as :

$$C_n = \sum_{p=1}^{N'} \int_{p=1}^{N'} f(x)\, e^{-2\pi j n x}\, dx \qquad (7)$$

Between x_p and x_{p-1}, the function f is approximated by the linear function :

$$\hat{f}(x) = f_p + \frac{f_{p+1} - f_p}{x_{p+1} - x_p}\,(x - x_p) \qquad (8)$$

Replacing f by $\hat{f}$ in Eq. 7, the integrand can be evaluated analytically and C_n is found to be equal to :

$$C_n = \frac{1}{2\pi j n}\left[A_n \sum_{p=1}^{N'} f_p E_n^{p-1} + B_n \sum_{p=1}^{N'} f_{p+1} E_n^{p-1}\right] \quad (9)$$

where

$$\left\{\begin{array}{l} E_n = e^{-2\pi j n D} \\ A_n = 1 - (1 - e^{2\pi j n D}) / 2\pi j n D \\ B_n = -e^{-2\pi j n D} + (1 - e^{2\pi j n D}) / 2\pi j n D \end{array}\right. \quad (10)$$

Since $N'D = 1$, it follows that

$$E_n^{N'} = 1$$

$$f_{N'+1} = f_1$$

Therefore

$$\sum_{p=1}^{N'} f_p E_n^{p-1} = f_1 + \sum_{p=1}^{N'} f_{p+1} E_{N'}^{p} - f_{N'+1} E_n^{N'} = \sum_{p=1}^{N'} f_{p+1} E_n^{p}$$

Hence(9) may be written as :

$$C_n = \frac{1}{2\pi j n}\left(A_n + B_n / E_n\right) \sum_{p=1}^{p=N'} f_p E_n^{p-1}$$

or, using definitions (10) :

$$2C_n = \frac{1 - \text{Real Part}(E_n)}{\pi^2 n^2 D} \sum_{p=1}^{p=N'} f_p E_n^{p-1} \quad (11)$$

This is the formula which is used in the program for computating the Fourier coefficients.

The quantities E_n are obtained recursively by :

$$E_n = E_1 E_{n-1} \quad (12)$$

The sum over p, by the recursive formula :

$$S_1 = 0$$
$$S_{p+1} = E_n S_p + f_{N'-p+1} \tag{13}$$

giving :

$$\sum_{p=1}^{p=N'} f_p E_n^{p-1} = S_{N'+1} \tag{14}$$

Finally, for $n = o$, a separate computation is performed :

$$C_o = D \sum_{p=1}^{p=N'} f_p \tag{15}$$

4.2 The method of transfer

4.2.1 Principle of the method

In this method, the blade is replaced by an equivalent system of N rigid elements linked to each other by an elastic junction. Assuming small deflections and negligible structural coupling, the three deflection modes (flap bending, in-plane bending and torsional) are treated separately.

For the bending modes, deflections occur in a plane and each rigid element is constructed with three masses, m_1, m_2, m_3, chosen in order to reproduce the same total mass, center of gravity and inertia of the actual blade portion this element represents. The elasticity of the junction is modeled by a stiffness coefficient k relating the local torque to the angle between two adjacent elements

The equivalent blade is obtained by linking these elements and adding the masses at the junction

This system is defined by the (N + 1) coordinates of the element tips. The coordinate z_o defines the position of the blade root and is either taken equal to zero for fixed hub conditions, or has to be defined by the hub equations.

For computation purpose, it is convenient to consider that the equivalent blade is constructed with two-mass elements (m'_i , central mass and m_i ,tip mass) as sketched in fig. 4.

4.2.2 Definition of the blade elements

Let ℓ_r be the blade linear density ($\ell_r = \frac{\partial M}{\partial r}$). An element of length ℓ , representing a portion of the blade between R_1 and R_2 ($\ell = R_2 - R_1$), is such that :

$$\begin{cases} m_1 + m_2 + m_3 = \int_{R_1}^{R_2} \ell_r \, dr = I_1 & \text{(total mass equation)} \\ (\frac{1}{2} m_1 + m_2)\ell = \int_{R_1}^{R_2} \ell_r \, r \, dr = I_2 & \text{(center of mass equation)} \\ (\frac{1}{4} m_2 + m_3)\ell^2 = \int_{R_1}^{R_2} \ell_r \, r^2 dr = I_3 & \text{(inertia equation)} \end{cases} \quad (1)$$

The values of I_1 , I_2 and I_3 are obtained from the blade mass distribution.Using the solution of system (1) gives :

$$\begin{cases} m_1 = I_1 - 3 I_2 / \ell + 2 I_3 / \ell^2 \\ m_2 = 4 I_2 / \ell - 4 I_3 / \ell^2 \\ m_3 = - I_2 / \ell + 2 I_3 / \ell^2 \end{cases} \quad (2)$$

Central and tip masses of the i th element are finally given by :

$$\begin{cases} m_i = m_3 (i) + m_1 (i+1) \\ m'_i = m_2 (i) \end{cases} \quad (3)$$

4.2.3 Computation of the stiffness coefficient

The stiffness coefficient k_i to be associated with the hinge between element ($i-1$) and i is evaluated by equating the deflection energy for the real and equivalent blade.

For the real blade, if r'_{i-1} and r'_i are the radial distances of the centers of elements ($i-1$) and i respectively, this energy is

$$W = \frac{1}{2} \int_{r'_{i-1}}^{r'_i} \frac{M}{EI} \, dr \quad (4)$$

where M is the bending moment, I the section inertia and E, the Young modulus.

For the equivalent blade :

$$W_e = \frac{1}{2} k_i \theta_i^2 \qquad (5)$$

where θ_i is the angle between elements $(i-1)$ and i. It is assumed that the bending moment M is constant and equal to $k_i \theta_i$.

Combining (4) and (5) then leads to :

$$k_i = 1 / \int_{r'_{i-1}}^{r'_i} \frac{dr}{EI} \qquad (6)$$

Knowing the values of m_i, m'_i (equation 3) and k_i (equation 6) completely defines the mathematical model equivalent to the real blade.

The calculation of integrals I_1, I_2 and I_3 and of equation 6 is performed simply by the trapezoidal method, knowing the mass per meter and the local section inertia at a certain number of points.

A sufficiently large number of points will be taken to ensure good accuracy for the blade data. For example, 19 data points are required to adequately define the section inertia and mass curves in the case of the DH 2011 blade (see figures 2 and 3).

4.2.4 Dynamic equation of the equivalent blade

The motion equation of the blade subject to forces f_i and rotating at constant speed Ω is obtained by means of Lagrange equations. See figure 4 for the notations and sign convention used.

The potential energy U is given by :

$$U = \frac{1}{2} \sum_{i=1}^{N} k_i \theta_i^2 \quad \text{where } \theta_i = \gamma_i - \gamma_{i-1}$$

The kinetic energy T is given by :

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \dot{z}_i^2 + \frac{1}{2} \sum_{i=1}^{N} m'_i \left(\frac{\dot{z}_{i-1} + \dot{z}_i}{2} \right)^2 +$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \left(m_i r_i^2 \Omega^2 + m'_i r'^2_i \Omega^2 \right)$$

Since the angles are small :

$$\sin \gamma_i = \gamma_i = \frac{z_i - z_{i-1}}{\ell_i}$$

enabling θ_i to written in the form :

$$\theta_i = \frac{z_i}{\ell_i} - z_{i-1} \left(\frac{1}{\ell_i} + \frac{1}{\ell_{i+1}} \right) - \frac{z_{i-2}}{\ell_{i-1}} \qquad (1)$$

It is also possible to write :

$$r_i = \sum_{j=1}^{i} \ell_j \cos \gamma_j = \sum_{j=1}^{i} \ell_j \left(1 - \frac{1}{2} \gamma_j^2 \right) \qquad (2)$$

$$r'_i = r_i - \frac{\ell_i}{2} \cos \gamma_i = r_i - \frac{\ell_i}{2} \left(1 - \frac{1}{2} \gamma_i^2 \right) \qquad (3)$$

The partial derivatives of the Lagrange equations can be expressed as follows, using equations 1, 2 and 3 :

$$\frac{\partial u}{\partial z_i} = \frac{k_i \theta_i}{\ell_i} - k_{i+1} \theta_{i+1} \left(\frac{1}{\ell_{i+1}} + \frac{1}{\ell_i} \right) + \frac{k_{i+2} \theta_{i+2}}{\ell_{i+1}} \qquad (4)$$

$$- \frac{\partial T}{\partial z_i} = \Omega^2 \gamma_i \left(m_i r_i + m'_i \frac{r'_i}{2} \right) + \Omega^2 \frac{\gamma_{i+1}}{2} m'_{i+1} r'_{i+1}$$

$$- \left(\gamma_{i+1} - \gamma_i \right) \Omega^2 \sum_{j=i+1}^{N} \frac{(m_j + m'_j r'_j)}{m_j r_j + m'_j r'_j} \qquad (5)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{z}_i} \right) = m_i \ddot{z}_i + \frac{1}{4} m'_i \ddot{z}_i + \frac{1}{4} m'_{i+1} \ddot{z}_i + \frac{1}{4} m'_i \ddot{z}_{i-1} + \frac{1}{4} m'_{i+1} \ddot{z}_{i+1} \qquad (6)$$

The writting of these expressions may be simplified by letting :

$$\begin{cases} u_i = \dfrac{1}{\ell_{i-1}} + \dfrac{1}{\ell_i} \\ a_i = m_i r_i + \dfrac{m'_i r'_i}{2} \end{cases}$$

$$\begin{cases} b_i = \frac{1}{2} m'_{i+1} \; r'_{i+1} \\ c_i = \sum_{j=i+1}^{N} m_j r_j + m'_j r'_j \\ m''_i = m'_i + m'_{i+1} \end{cases}$$

The final equation may therefore be written in the form :

$$f_i = \frac{k_i \theta_i}{\ell_i} - k_{i+1} \theta_{i+1} \; u_{i+1} + \frac{k_{i+2} \; \theta_{i+2}}{\ell_{i+1}} + m_i \ddot{z}_i + \frac{1}{4} m'_i \ddot{z}_{i-1} \tag{7}$$

$$+ \frac{1}{4} m''_i \ddot{z}_i + \frac{1}{4} m'_{i+1} \ddot{z}_{i+1} + \gamma_i a_i \Omega^2 + \gamma_{i+1} b_i \Omega^2 - (\gamma_{i+1} - \gamma_i) c_i \Omega^2$$

where

$$\gamma_i = (z_i - z_{i-1}) \; / \; \ell_i$$

$$\theta_i = \frac{z_i}{\ell_i} - z_{i-1} \; u_i + \frac{z_{i-2}}{\ell_{i-1}}$$

Equation 7 may be expressed in the following matrix form :

$$F = EM . \ddot{Z} + EK . Z + \Omega^2 EC . Z \tag{8}$$

where

$$F = \begin{vmatrix} f_1 \\ \vdots \\ f_i \\ \vdots \\ f_N \end{vmatrix} \qquad Z = \begin{vmatrix} z_1 \\ \vdots \\ z_i \\ \vdots \\ z_N \end{vmatrix} \qquad \ddot{Z} = \begin{vmatrix} \ddot{z}_1 \\ \vdots \\ \ddot{z}_i \\ \vdots \\ \ddot{z}_N \end{vmatrix}$$

F being the column vector of the forces and Z that of the displacements.

EM, EK and EC are square matrices ($N \times N$) which represent successively the effects of mass, elasticity and centrifugal forces.

The elements of these matrices may be calculated from the following formulas (M_i^j being the term in the i th line and j th column).

$$\text{Matrix } EM \begin{cases} EM_i^{i-1} = \frac{1}{4} m'_i \\ EM_i^i = m_i + \frac{1}{4}(m'_i + m'_{i+1}) \\ EM_i^{i+1} = \frac{1}{4} m'_{i+1} \end{cases} \qquad (9)$$

$$\text{Matrix } EK \begin{cases} EK_i^{i-2} = \frac{k_i}{\ell_i \ell_{i-1}} \\ EK_i^{i-1} = -\frac{k_i u_i + k_{i+1} u_{i+1}}{\ell_i} \\ EK_i^i = \frac{k_i}{\ell_i^2} + k_{i+1} u_{i+1}^2 + \frac{k_{i+2}}{\ell_{i+1}^2} \\ EK_i^{i+1} = \frac{k_{i+1} u_{i+1} + k_{i+2} u_{i+2}}{\ell_{i+1}} \\ EK_i^{i+2} = \frac{k_{i+2}}{\ell_{i+1} \ell_{i+2}} \end{cases} \qquad (10)$$

$$\text{Matrix } EC \begin{cases} EC_i^{i-1} = -\frac{a_i + c_i}{\ell_i} \\ EC_i^i = \frac{a_i}{\ell_i} - \frac{b_i}{\ell_{i+1}} + c_i u_{i+1} \\ EC_i^{i+1} = \frac{b_i - c_i}{\ell_{i+1}} \end{cases} \qquad (11)$$

<u>Remark</u> : The non-defined terms of these matrices are zero.

It may also be seen that these three matrices are symmetrical (since $c_i - b_i = a_{i+1} + c_{i+1}$) which simplifies calculation.

Equations 9, 10 and 11 thus enable the blade motion equation to be expressed in linear differential matrix form, which relates the flapping displacements of the N points selected on the blade (at the junctions of the equivalent blade elements) to the aerodynamic forces applied to these points.

$$F = EM.\ddot{Z} + [EK + \Omega^2 EC].Z$$

This equation is easily resolved with the Fourier coefficients of the displacement forces, since this amounts to a matrix inversion.

Thus we can write :

$$F_n = \left[-n^2 \Omega^2 EM + EK + \Omega^2 EC \right] Z_n$$

where F_n is a vector containing the n-order Fourier components (sine or cosine) of the forces on the various points,

Z_n: same definition for the displacements

The n-th harmonic is thus given by :

$$Z_n = \left[-n^2 \Omega^2 EM + EK + \Omega^2 EC \right]^{-1} F_n$$

Remark : This equation should be applied twice for each harmonic, once for the sine and once for the cosine.

4.2.5 Calculation of Flapping natural Vibration Modes

Taking the basic matrix equation :

$$F = EM . \ddot{Z} + \left[EK + \Omega^2 EC \right] Z$$

it is seen that the resonant frequencies can be easily calculated by writing that for these frequencies this equation cannot be inverted.

Let ω_R be a resonant pulse , giving :

$$F = \left[-\omega_R^2 \; EM + EK + \Omega^2 EC \right] Z$$

The conditions of resonance impose :

determinant $\left[-\omega_R^2 \; EM , EK + \Omega^2 EC \right] = 0$

This equation is true if ω_R^2 is a eigenvalue of

$$EM^{-1} \left[EK + \Omega^2 EC \right]$$

The various natural frequencies are obtained by calculating the eigen-values of $EM^{-1}[EK + \Omega^2 EC]$.

It is apparent that the number of values obtained is finite and equal to the number N of elements of the equivalent blade, whilst in reality it is infinite. This raises a problem of accuracy and therefore of convergence of the method.

We have, for example, evaluated the rate of convergence by calculating the natural frequencies of the first three modes of vibration of a uniform bar, for different numbers of elements constituting the equivalent bar. Two cases of calculation were considered : zero speed of rotation and a rotation of 33.3 rd /s.

The results are shown in figures 5 and 6.

Since we theoretically know the required values for a uniform bar, we note that :

- the method provides good accuracy for the first 3 modes with a small number (approximately 6) of elements.
- convergence is more rapid for zero rotational speed than for $\Omega = 33.3$ rd /s.

The application of this method to the DH 2011 blade enabled us to determine the first three modes (flapping and the first 2 bending modes) and to compare them with the values calculated previously. The calculation was made for a 6-element equivalent blade. See figure 7.

4.2.6 Calculation of blade bending stresses

The blade motion equation enables us to know the Fourier coefficients of blade distortion.

Knowing the cross-section thichness of the blade spar is therefore sufficient for determing local stress in this spar.

See figure 8 for the following calculation.

If h_i is the half-width of the spar, we can express the stress σ_i as :

$$\sigma_i = E \frac{\Delta e_i}{e_i}$$

For small angles, however, $\Delta \ell_i = h\theta_i$

whence $\sigma_i = E\theta_i \cdot \dfrac{h_i}{\ell_i}$

Since $\theta_i = \gamma_i - \gamma_{i-1} = \dfrac{z_i - z_{i-1}}{\ell_i} - \dfrac{z_{i-1} - z_{i-2}}{\ell_{i-1}}$

then finally :

$$\sigma_i = E h_i \left(\frac{z_i}{\ell_i^2} - \frac{z_{i-1}}{\ell_i} \left(\frac{1}{\ell_i} + \frac{1}{\ell_{i-1}} \right) + \frac{z_{i-2}}{\ell_i \ell_{i-1}} \right)$$

This equation can be written in matrix form :

$$\sigma = ES.Z$$

where σ is a column vector containing the stresses at the N points on the blade.

This equation can also be written using Fourier coefficients :

$$\sigma_n = ES \, . \, Z_n$$

giving finally the Fourier coefficients of the stresses.

Matrix ES is defined as follows :

$$\begin{cases} ES_i^{i-2} = E \, . \, h_i \, . \, \dfrac{1}{\ell_i \ell_{i-1}} \\ ES_i^{i-1} = - \, E \, . \, h_i \, . \, \dfrac{u_i}{\ell_i} \\ ES^i = E \, . \, h_i \, . \, \dfrac{1}{\ell_i^2} \end{cases}$$

The stresses can therefore be easily calculated by simple matrix products.

4.2.7 Calculation of hub vibrations

Vertical hub vibrations are determined by calculating the blade hinge force. In order to obtain the total force, both blades must be considered, which results in doubling the even-order Fourier coefficients and eliminating the odd-order coefficients. Odd-order vibration harmonics cannot occur in a two-bladed rotor.

The force applied by one blade is written :

$$F = \theta_1 F_c - \frac{k_2 \theta_2}{\ell_1} \qquad (1)$$

where F_c is the centrifugal force.

Equation 1 can also be written in the form :

$$\boxed{F = \frac{\gamma_1}{\ell_1}\left(F_c + \frac{k_2}{\ell_1} + \frac{k_2}{\ell_2}\right) - \frac{k_2 \gamma_2}{\ell_1 \ell_2}}$$

Since the Fourier coefficients of γ_3 and γ_2 are known, this equation gives the coefficients of the force applied to the hub by the blade.

The constant term corresponds to lift.

5. DESCRIPTION AND USE OF THE FORTRAN IV PROGRAMS

As already stated, the computational program consists in fact of three independent programs :

BFHA	:	Blade forces harmonic analysis
BSTM	:	Blade stresses tranfer matrices
STRESS	:	Computation of blade deflections and stresses.

The following is a brief description of each of these programs, for which the listings are given in Appendix 1.

5.1 BFHA : Blade forces harmonic analysis

The distribution of forces along the blade is given at a certain number of points on the blade (17 in the present case) and for different azimuthal positions (24 in the present case, i. e. every 15°). The purpose of the program is to provide the Fourier coefficients of these forces (at the same radial points as the data).

PROGRAM INPUTS : Read operations are performed in the following order :

- commentary cards : the number of cards is not limited (format 20 A 4).
- ✱ INPUT DATA : this card (punched starting with column 1) indicates that commentary is complete and that computing data will follow.

- the next card contains in order :
  - . simulation identification n° in F 6.2 format (column 1),
  - . number of azimuthal positions where the forces are given, in I3 format (column 26),
  - . number of radial points in I3 format (column 37),
  - . number of harmonics required in I3 format (column 47),
  - . unit (Newtons per meter in present case). This enables a factor to be given for changing from the forces as given by the simulations to forces in Newtons/meter. NW/M is contained in column 68.
- the next cards define the positions of the radial points (values in fractions of radius) measured from the blade root. Format F 10.4 is used. Seven values are given per card (columns 11, 21, 31, 41, 51, 61 and 71).
- Set of cards corresponding to one simulation (from EMC program).Put the last 4 cards of this set of cards ahead.
- Card ✱ END (column 1) indicates there are no more cases to be treated.

  If this card is not present, the program starts reading new data (same presentation as before : commentary cards, etc.).

PROGRAM OUTPUTS :

- line-printer output containing input data, and the Fourier coefficients harmonic by harmonic for all the radial points defined by the input data.
- punched-card output containing the same data and directly usable for computing stresses.

  This set of cards is entitled PUNCH 1.

5.2 BSTM : Blade stresses transfer matrices

The purpose of this program is to compute the transfer matrices EM, EC, EK and ES defined in § 4.1.2 and the natural vibration modes.

Since there are a greater number of inputs and outputs for this program, the following description refers to Appendix 1, which gives the listings.

PROGRAM INPUTS

1) - Two cards containing : blade identification, rotor radius, distance between the axis of rotation and the blade root, unit of length,

material modulus of elasticity and its unit, and the number of data points of the blade characteristics (FORMAT 501).

- the next cards contain the radial positions of the blade characteristic data points (FORMAT 502).
- the next cards contain the blade longeron thickness (FORMAT 502).
- card containing the units of mass and inertia (FORMAT 500).
- option 1 card :
  Two possibilities : COMPUTED BLADE ELEMENT PARAMETERS
  or GIVEN BLADE ELEMENT PARAMETERS
- option 2 card :
  Three possibilities : CONSTANT VALUE OF ELEMENT LENGTHS
  GIVEN VALUES OF ELEMENT LENGTHS
  OPTIMIZED VALUES OF ELEMENT LENGTHS

= if option 1 is "COMPUTED", the next cards contain the mass per meter (density) and the cards contain the blade section inertia (FORMAT 502).

2) - card NBE containing the number of equivalent blade elements (FORMAT 503)

= if option 2 is "GIVEN", the blade element lengths are read (FORMAT 502)

= if option 1 is "GIVEN", the spring constants, and the masses assumed to be concentrated at the center and extremity of each element are read (FORMAT 504)

- option 3
  Two possibilities : PRINT BLADE ELEMENT PARAMETERS
  NO PRINT
- option 4
  Three possibilities : PUNCH AND PRINT TRANSFER MATRICES
  PRINT ONLY
  NO PRINT NO PUNCH
- option 5
  Two possibilities : CONTINUE
  EIGENMODES

= if option 5 is "EIGENMODES", the next card gives the speed of rotation in radians/second (FORMAT 505), and option 5 is reread. This loop repeats until a "CONTINUE" card is encountered.

- option 6
  Three possibilities : NEW BLADE CONFIGURATION
  CHANGE VALUE OF NBE
  END OF COMPUTATION

= if option 6 is "NEW BLADE CONFIGURATION", the sequence returns to 1.

= if option 6 is "CHANGE VALUE OF NBE", the sequence returns to 2.

NOTE : Cards or groups of cards marked in the margin by "-" are necessary, whilst those marked by"=" depend on the contents of certain options.

PROGRAM OUTPUTS

The line-printer output is shown in figures 9 through 12.

- the first page lists the mechanical characteristics of the blade :spar tickness,mass distribution and section inertia, followed by the total mass average section inertia and the 1st resonant frequency of a uniform blade having the same average characteristics.
- the second page lists the characteristics of the equivalent blade, i. e. the masses constituting each element and the spring constants linking them. The four matrices EC, EK, EM and ES are then printed out.
- the third and fourth pages gives two examples of natural mode computations (EIGENMODE option) for rotational speeds of zero and 318 rpm.

The punched-card output contains matrices EC, EK, EM and ES, in addition to certain characteristics such as radius, the number of elements, etc.

This set of cards may be directly used for the third program and is entitled PUNCH 2.

It should be noted a punched-card output is obtained only if the PUNCH option has been requested.

5.3 STRESS : Stress and vibration computation

The purpose of this program is to compute the rotor stresses and vibration due to the applied forces (output of program BFHA), using the matrices defining the equivalent blade (output of program BSTM).

PROGRAM INPUTS

The input cards are stacked in the following order :

- PUNCH 2 (cards produced by BSTM)
- a punch option card :

PUNCH in column 1	:	punched-card output of the blade distortion Fourier coefficients
PUNCH in column 40	:	punched-card output of the blade distortion as a function of blade azimuth (ψ)
unpunched card	:	neither option selected

- PUNCH 1 (cards produced by BFHA)

Three cases are possible following this computation :

1) There are no more computations and computation therefore stops : add the following input cards :

   1 unpunched card
   1 ✱ END card (column 1)
   2 unpunched cards

2) It is required to enter another computation which uses the same matrix data for the equivalent blade, only the system of applied forces being changed.

   In this case add the following input cards :

   1 punching option card
   the new stack of PUNCH 1 cards produced by BFHA followed by the same procedure as before.

3) It is required to enter another computation with changing : the matrix data for the equivalent blade and the system of applied forces.

   In this case add following input card

   1 unpunched card
   1 ✱ END card
   the new stack of PUNCH 2 cards produced by BSTM.

   Followed by the same procedure as before.

PROGRAM OUTPUTS

The following data are printed out in succession :

- EC, EK, EM and ES matrices
- Fourier coefficients of the forces and coefficients C_T and M_T (force and moment) for the rotor
- Fourier coefficients of the blade distortion

- blade distortion as a function of ψ.
- Fourier coefficients of blade stresses
- * stresses as a function of ψ.
- * Fourier coefficients of vibration (vertical force)
- * Fourier coefficients of stresses at 0.45 R
- * Fourier coefficients of stresses at 0.7 R
- * vibration as a function of ψ
- * stresses at 0.45 R as a function of ψ.
- various parameters of rotor operation with its cyclic control in particular (including multicyclic)

Results marked * are punched out onto carts independently of the PUNCH option.

Remark

Certain results obtained on cards, such as the 0.45 R and 0.7 R stress Fourier coefficients and vibration, have the same format as that used for the input data of the multicyclic analysis program (see reference 4).

5.4 Conclusions concerning presentation of results

This report does not cover all results obtained on the line printer, but tables have been prepared comparing real cases and simulations for the stress Fourier coefficients.

In addition curves, obtained by means of a curve plotting program and comparing stress curves for real and simulated cases, are presented.

6. SUMMARY OF RESULTS

The following presents the results of the stress calculations at 0.45R for the 15 simulations at μ = 0.4 performed with the Evans -Mc Cloud program.

These 15 cases were computed with an equivalent blade consisting of 6 rigid elements of equal lenght. It has been estimated from the natural frequency calculation that 6 elements provide sufficient accuracy for computing blade stresses.

The Fourier coefficients of the computed stresses are given in Table I and should be compared with the measured stresses given in Table II. In order to facilitate comparison, measured and computed stresses have been plotted against ψ on the same curve for each of the 15 cases.

Examination of the results leads to a first remark concerning the radial distribution of the blade stresses. The stresses measured during the runs are maximum at 0.45R. For example, for the 15 runs analyzed, the mean stress σ_o is approximately 7.75 hectobars at 0.45R and 5.0 hectobars at 0.7R. The computed stresses, however, show a maximum at 0.7R. For example, the mean stress σ_o is approximately 6.0 hectobars at 0.7R and 4.8 hectobars at 0.45R. (See Figure 13, where these distributions have been plotted for Run-PT 9.03).

Since total blade lift is the same in either case (measured and simulated), there are two main reasons for this anomaly :

- the distributions of aerodynamic loading are certainly different for the real and simulated cases,
- the mechanical characteristics of the blade, i.e. section density and inertia are not accurately known.

This remark led to plotting the measured and computed stresses as ratios of their mean values in order to be able to compare the two curves.

Comparison of computed and measured results does not allow positive conclusions to be drawn. It is possible, however, to state that the simulated cases contain very small 3rd and 4th harmonic components, whilst on the contrary the real cases contain high values.

Correlation for the 1st and 2nd harmonics is much better, as shown by Tables I and II, in which the relative amplitudes and signs of the coefficients are very similar. For example, it is interesting to note the variations of the 1st and 2nd harmonic coefficients for runs 14-10 to 14-13, i.e. decreased 1st harmonic and increased 2nd harmonic. The overall correlation remains nevertheless rather poor, as shown in Figures 14 to 28, because of the absence of higher harmonics.

When this was noted, simulation with a smaller airfoil stalling angle was requested (12° instead of 16°) in order to attempt to introduce larger lift discontinuities, which would therefore contain greater values of higher order harmonics of aerodynamic force. This change did not produce the expected result, the 3rd and 4th harmonic contents remaining far too small. This is explained that the stalled sector for the blade at $\mu = 0.4$ occupies only a very small part of the disk. The value of stalling angle was therefore returned to 16°.

7. CONCLUSION

The purpose of the work described in this report was to create a program for computing helicopter blade stresses from a knowledge of the local forces applied to the rotor disk.

Local forces can be computed by means of the Evans - Mc Cloud simulation program for the DH 2011 rotor.

During the DH 2011 rotor runs in the 40' x 80' Ames windtunnel in March 1971 the blade bending stresses were measured for various aerodynamic configurations of the rotor (in particular, multicyclic control). Fifteen of these configurations were selected and simulated with the Evans - Mc Cloud program.

GD's stress program enabled blade bending stresses to be computed for these 15 simulations, the purpose being to examine the correlation between measured and computed values.

Comparison showed fair agreement for the 1st and 2nd harmonics of stress, but very poor correlation for higher order harmonics (the computed values containing only very small amplitudes).

Consequently, although the Evans - Mc Cloud program produces good results for rotor performance, it cannot be used for analyzing dynamic phenomena, such as vibration or stresses.

In order to obtain a better description of blade dynamic loads, several effects need to be included in the simulation program :

- unsteady aerodynamics
- blade flexibility
- wake effects
- Mach number effects.

It is, however, difficult to state which of these effects has the greatest influence on the 3P and 4P harmonic contents of local aerodynamic loads.

Notwithstanding the inclusion of these effects in the simulation program would certainly improve the accuracy of aerodynamic force computation.

8. <u>REFERENCES</u>

1 "Analytical Investigation of a Helicopter Rotor driven and controlled by a Jet Flap"
William T. EVANS and John L. Mc CLOUD III
Doc. NASA NT D 3028 — Sept. 1965

2 Computer Program for Stress Determination of a Helicopter Blade
Doc. DH 2011-D NT 79 — Dec. 1971

3. March 1971 Wind Tunnel Tests of the Dorand DH 2011 Jet Flap Rotor
Doc. DH 2011-D E5 NASA CR 114693 and CR 114694, June 1973

APPENDIX 1

This appendix contains the listings of the FORTRAN IV programs used for the stress computation.

The three main programs are :

1) BFHA : Blade forces harmonic analysis ; which requires the subroutine FHARM3.

2) BSTM : Blade stresses transfer matrices ; which requires the subroutines, ATEIG, B2C, CLEAR, EIGMOD, HSBG, IPOL1, LICOM, MINV, MOMENT, PROD1 and STRAM.

3) STRESS : Stress computation.

   This program is composed in fact, of a short MAIN program and of a subroutine named STRESS. The other subroutines required are B2C, FSUM3, IPOL1, LICOM, MINV, MOMENT and PROD.

The listings of the subroutines are also joined to this appendix.

```
C       BLADE FORCES HARMONIC ANALYSIS
C
      DIMENSION H(2),TYPE(2),UF(2),T(20),R(20),DELTA(9),COM(7)
      COMMON FO(20),FN(12,20,2),F(24,20,2)
      DATA H,TYPE,DATIN,END/'A','B','FZ','FX','*INP','*END'/
  500 FORMAT(20A4)
  501 FORMAT(F6.2,4X,'RPM=',F6.1,'NPTS=',I3,2X,'NPOST=',I3,1X,
     1'NHARM=',I3,1X,'FACTOR=',E10.3,2A4/('  X/R =    ',7F10.4))
  502 FORMAT(I3,6E11.4)
  503 FORMAT(7A4,12X,F5.2,19X,F5.2/10X,9F7.2//
     1  3X,F6.2,6X,F7.2,8X,F6.3,6X,F10.7)
  600 FORMAT('1SET NUMBER',I3/)
  601 FORMAT(10X,20A4)
  602 FORMAT(//' RUN IDENTIFICATION',2X,F6.2,5X,'RPM=',F6.1//
     11X,I3,'POINTS PER STATION',I10,' STATIONS AT X/R ='//(1X,18F7.4))
  603 FORMAT(1X,A2,I3,17F7.4)
  612 FORMAT(/' FORCES VALUES MUST BE MULTIPLIED BY',1PE11.3,
     1' TO OBTAIN',1X,2A4//' AMES DATA CARDS INPUT ...'/)
  604 FORMAT('1FOURIER COEFFICIENTS FOR ',A2,I10,' HARMONICS ANALYSED'/)
  605 FORMAT(1X,A2,1X,A1,I2,4X,1P10E11.3/(11X,10E11.3))
  700 FORMAT(60X,'OPTION CARD ********')
  701 FORMAT(F6.2,4X,' ALPHAS=',F6.2,' CJR=',F10.7,' MU=',F6.3,' SL=',
     1  F5.2,' THETA=',F5.2/F6.2,'DELT',9F7.3)
  705 FORMAT(A2,1X,I1,I3,3X,1P7E10.3/(10X,7E10.3))
      NSET=1
 6600 WRITE(6,600)NSET
      WRITE(7,700)
 5500 READ(5,500)T
      WRITE(6,601)T
      WRITE(7,500)T
      IF(T(1).EQ.END) STOP
      IF(T(1).NE.DATIN)      GO TO 5500
      READ(5,501) RUN,RPM,NPTS,NPOST,NHARM,FACTOR,UF,(R(I),I=1,NPOST)
      READ(5,503) COM,SL,THETA,DELTA,ALFAS,OMR,AMU,CJR
      WRITE(6,601) COM
      RAD = 19.68
      OMEGA = OMR/RAD
      RPM = 9.549*OMEGA
      FACTOR = 0.9625*OMEGA**2*(RAD*0.3048)**3
      WRITE(7,501) RUN,RPM,NPTS,NPOST,NHARM,FACTOR,UF,(R(I),I=1,NPOST)
      WRITE(6,602) RUN,RPM,NPTS,NPOST,(R(I),I=1,NPOST)
      WRITE(6,612) FACTOR,UF
      NC=(NPOST-1)/6+1
 1001 DO1 K=1,NPTS
      I1=1
      I2=6
 1002 DO2 J=1,NC
 1003 DO3 NTYPE=1,2
    3 READ(5,502) KPSI,(F(K,I,NTYPE),I=I1,I2)
      I1=I2+1
      I2=I2+6
      IF(I2.GT.NPOST) I2=NPOST
    2 CONTINUE
    1 CONTINUE
      WRITE(6,603)((TYPE(NTYPE), K,(F(K,I,NTYPE),I=1,NPOST),K=1,NPTS) ,
     1 NTYPE = 1,2)
C       START THE HARMONIC ANALYSIS
      NTYPE = 1
C       STATEMENT 1004 SUPPRESSED FOR ANALYSING FZ ONLY (NTYPE = 1)
C1004 DO 4 NTYPE=1,2
```

```
      L=1
      N=0
      CALL FHARM3(NPTS,NHARM,NPOST,NTYPE)
      WRITE(6,604) TYPE(NTYPE),NHARM
      WRITE(6,605) TYPE(NTYPE),H(L),N,(FO(K),K=1,NPOST)
      WRITE(7,705) TYPE(NTYPE),L,N,(FO(K),K=1,NPOST)
 2004 DO 4 N=1,NHARM
 3004 DO 4 L=1,2
      WRITE(7,705) TYPE(NTYPE),L,N,(FN(N,K,L),K=1,NPOST)
    4 WRITE(6,605) TYPE(NTYPE),H(L),N,(FN(N,K,L),K=1,NPOST)
      WRITE(7,701) RUN,ALFAS,CJR,AMU,SL,THETA,RUN,DELTA
      NSET=NSET+1
      GO TO 6600
      END
```

```
C      BLADE STRESSES TRANFER MATRICES PROGRAM                                B
       COMMON/BLADE/BEL(20),BER(20),BECM(20),BETM(20),BEK(20),
      1 A(20),B(20)
       COMMON/TRANS/OMEGA,ES(50),EM(60),EC(50),EK(100)
       COMMON/WORK/W1(20),W2(20),W3(20),W4(400),W5(400)
       DIMENSION TEXT(19),CODE(7),BD(30),BI(30),HE(30),RS(30),
      1 UL(3),UM(3),UI(3),UE(3),BLID(4)
       DATA CODE/'GIVE','OPTI','PRIN','EIGE','NEW ','PUNC','CHAN'/
  500 FORMAT(20A4)
  501 FORMAT('1BLADE ',4A4,7X,'R=',1PE10.3,1X,'RO=',E10.3,1X,3A4/1X,
     1'E=',E10.3,1X,3A4,4X,'NUMBER OF BLADE DATA POINTS',I3)
  502 FORMAT(10X,1P7E10.3)
  503 FORMAT('1NUMBER OF BLADE ELEMENTS =',I3)
  504 FORMAT(10X,3E10.3)
  505 FORMAT('OMEGA=',F5.1)
  506 FORMAT(A4,1X,I1)
  600 FORMAT(/' *PROGRAM OPTION* ',20A4)
  601 FORMAT(/' RADIUS',1PE12.3,1X,3A4/' TOTAL MASS',E12.3,1X,3A4/
     1' AVERAGED SECTIONAL INERTIA',E15.3,1X,3A4/
     2' UNIFORM BLADE FREQUENCY',E12.3,' RAD/SEC')
  602 FORMAT( ' ELEMENT',7X,'LENGTH',3X,'RAD. DIST. SPRING CTE. CENTRAL
     1MASS  TIP MASS'//(4X,I3,4X,1P5E12.3))
  603 FORMAT(/' BLADE STRUCTURAL DATA'/' ELEMENT',7X,'RAD.DIST',4X,'THIC
     1KNESS',2X,'MASS(KG/M)',2X,'SECT.INERT'//(4X,I3,4X,1P4E12.3))
C
C
    1 NOPT = 1
C
C             READING OF THE FOLLOWING DATAS=
C        BLID=BLADE IDENTIFICATION
C             BRAD=BLADE RADIUS,RO=DISTANCE FROM THE ROTOR AXIS TO THE/
C               BEGINNING OF THE BLADE.
C             UL=LENGTH UNITS,E=ELASTICITY MODULE,UE=MODULE UNITS.
C             NR=NUMBER OF BLADE DATA POINTS.
C
       READ(5,501) (BLID(I),I=1,4),BRAD,RO,UL,E,UE,NR
       WRITE(6,501)(BLID(I),I=1,4),BRAD,RO,UL,E,UE,NR
       E=(1.E+07)*E
C
C             READING OF RS AND HE.
C              HE=THICKNESS OF THE BLADE LONGERON,
C                   RS =RADIAL DISTANCES WHERE BLADE CHARACTERISTICS
C                       ARE MEASURED.
C
       READ(5,502) (RS(I),I=1,NR)
       READ(5,502)  (HE(I),I=1,NR)
C
C             READING OF MASS UNITS (UM) AND INERTIA UNITS (UI).
C
       READ(5,500) UM,UI
C
C             READING OF OPTION 1=
C        TWO CASES                     COMPUTED BLADE ELEMENTS PARAMETERS
C                                      GIVEN    BLADE ELEMENTS PARAMETERS
C
       READ(5,500)  OPT1,TEXT
       WRITE(6,600) OPT1,TEXT
C                                                                      92
C             READING OF OPTION 2=
C        THREE CASES                   CONSTANT VALUE OF ELEMENT LENGTHS
```

```
C                                              GIVEN     VALUE OF ELEMENT LENGTHS
C                                              OPTIMIZED VALUE OF ELEMENT LENGTHS
C
      READ(5,500)  OPT2,TEXT
      WRITE(6,600) OPT2,TEXT
C
C             RESET TO ZERO (S.P. CLEAR )
C
      CALL CLEAR(1)
C
C             IF OPTION 1 NOT EQUAL 'GIVEN' READ BLADE DENSITY (BD) AND
C             SECTION INERTIA (BI).
C
      IF(OPT1.EQ.CODE(1))     GO TO 2
      READ(5,502)  (BD(I),I=1,NR)
      READ(5,502)  (BI(I),I=1,NR)
      WRITE(6,603) (I,RS(I),HE(I),BD(I),BI(I),I=1, NR)
C
C             NBE=EQUIVALENT BLADE NUMBER OF ELEMENTS.
C
C
C
    2 READ(5,503)    NBE
C
C             RESET TO ZERO.
C
      CALL CLEAR(2)
C
      IF(OPT2.EQ.CODE(1))     GO TO 5502
C
C             IF OPTION 2 EQUAL'GIVEN'READ THE LENGTHS OF BLADE ELEMENTS.
C
C
      IF(OPT2.EQ.CODE(2))     GO TO 30
C             IF OPTION 2 NOT EQUAL'GIVEN' OR 'OPTI',COMPUTE THE LENGTH
C             OF THE BLADE ELEMENTS.
C
      BELS = (BRAD-RO)/FLOAT(NBE)
 1003 DO 3 I=1,NBE
    3 BEL(I) = BELS
      GO TO 4
 5502 READ(5,502) (BEL(I),I=1,NBE)
C
C             IF OPTION 1 NOT EQUAL 'GIVEN' COMPUTE THE MASS,GRAVITY
C             CENTER AND INERTIA OF EACH BLADE ELEMENTS.
C
    4 IF(OPT1.EQ.CODE(1))     GO TO 20
C          COMPUTE INVERSE VALUES OF SECTION INERTIA AND STORE IN W2
 1005 DO 5 I=1,NR
    5 W2(I)=1./BI(I)
C
C       COMPUTE BLADE ELEMENT TOTAL MASS, CG AND INERTIA (SM,SM1,SM2)
C          ASSUME FREE HINGE BLADE (BEK(1)=0.)
C
      BEK(1)=0.
C
      X1 = RO
      K1 = 1
      XM1=RO+BEL(1)/2.
      KM1=1
C
```

```
C
C              LOOP FROM 1006 TO 6 COMPUTES BLADE ELEMENTS PARAMETERS=
C                   BECM=CENTRAL MASS
C                   BETM=TIP MASS
C                   BEK =SPRING CONSTANT.
C
C
 1006 DO 6 I=1,NBE
      BER(I) = X1 + BEL(I)
      X2 = BER(I)
      CALL MOMENT(W1,2,RS,X1,X2,X1,BD,K1,NR,KO)
      SM = W1(1)
      SM1 = W1(2)/BEL(I)
      SM2 = W1(3)/BEL(I)**2
C
C
C       COMPUTE THE EQUIVALENT MASSES (EM1,EM2,EM3)
C
      EM1 = SM - 3.*SM1 + 2.*SM2
      EM2 = 4.*(SM1-SM2)
      EM3 = 2.*SM2 - SM1
C
C
C       COMPUTE BLADE ELEMENTS CENTRAL AND TIP MASSES( BECM,BETM)
C
      BECM(I) = EM2
      BETM(I) = EM3
      IF(I.EQ.1)        GO TO 7
      BETM(I-1) = BETM(I-1) + EM1
C
C       COMPUTE BLADE ELEMENT SPRING CONSTANT BEK
C
      XM2=BER(I-1)+BEL(I)/2.
      CALL MOMENT(W1,0,RS,XM1,XM2,XM1,W2,KM1,NR,KO)
      BEK(I)=E/W1(1)
      XM1=XM2
      GO TO 6
    7 BETMO = EM3
    6 X1 = X2
      GO TO 40
C
C
C                   IF OPTION EQUAL 'GIVEN',
C       READ GIVEN VALUES OF BLADE ELEMENTS PARAMETERS
C
   20 BRAD=RO
 1021 DO 21 J=1,NBE
      READ(5,504)   BEK(J),BECM(J),BETM(J)
      BER(J) = BRAD + BEL(J)
   21 BRAD = BER(J)
C
C
C       COMPUTE AVERAGED PARAMETERS
C
      GO TO 40
   30 CONTINUE
C
C       CALL VARBEL(NR,NBE)  IN CASE OF OPTIMIZATION OF THE LENGTHS
C
   40 IF(NOPT.EQ.0)      GO TO 41
      BMASS = 0.
```

```
      BRAD = BER(NBE)
      BIAV = 0.
 1050 DO 50 I=1,NBE
      BMASS = BMASS + BECM(I) + BETM(I)
      IF(I.EQ.1) GO TO 50
      BIAV=BIAV+BEK(I)*(BEL(I-1)+BEL(I))/2.
   50 CONTINUE
      BIAV=BIAV/(FLOAT(NBE-1)*E)
      BMASS = BMASS + BETMO
C
C
C       COMPUTE THE EQUIVALENT UNIFORM BLADE EIGENFREQUENCY
C
       OMUB = 15.4*SQRT(E*BIAV/(BMASS*(BRAD-RO)**3))
      WRITE(6,601) BRAD,UL,BMASS,UM,BIAV,UI,OMUB
C
C
C             READ THE OPTION 3.
C             TWO POSSIBILITIES          PRINT BLADE ELEMENTSS PARAMETERS.
C                                        NO PRINT.
C
   41 WRITE(6,503)  NBE
      READ(5,500) OPT,TEXT
      IF(OPT.NE.CODE(3))     GO TO 200
      WRITE(6,600) OPT,TEXT
      WRITE(6,602) (I,BEL(I),BER(I),BEK(I),BECM(I),BETM(I),I=1,NBE)
C
C             READING OF THE OPTION 4
C             THREE CASES      PUNCH THE TRANSFER MATRICES
C                              PRINT ONLY
C                              NO PRINT OR PUNCH
C
  200 READ(5,500) OPT,TEXT
      WRITE(6,600) OPT,TEXT
      IS = 0
      IF(OPT.EQ.CODE(3))  IS=6
      IF(OPT.EQ.CODE(6)) IS=7
C
C
C      COMPUTATION OF THE BLADE STRESSES TRANSFER MATRICES  EM,EK,EC
C
      CALL STRAM(NBE,NR,IS,RO,BLID,HE,RS,EM,ES,EC,EK)
C
C
C
C             READ THE OPTION 5          CONTINUE
C                                        EIGENMODES
C
  207 READ(5,500) OPT,TEXT
      WRITE(6,600) OPT,TEXT
C
C             IF OPTION 5 EQUAL 'EIGEN' , READ THE VALUE OF OMEGA ,
C             COMPUTE EIGEN MODES OF THE BLADE,AND READ OPTION 5 AGAIN.
C
      IF(OPT.NE.CODE(4))     GO TO 210
C
      READ(5,505) OMEGA
      CALL EIGMOD(EM,EC,EK,OMEGA,OMUB,NBE)
      GO TO 207
C
C             READ OPTION 6
```

```
C
  210 READ(5,500) OPT,TEXT
      WRITE(6,600) OPT,TEXT
C
C             THREE CASES      NEW BLADE COMPUTATION
C                              CHANGE THE VALUE OF NBE
C                              END THE COMPUTATIONS
C
      IF(OPT.EQ.CODE(5))       GO TO 1
      NOPT = 0
      IF(OPT.EQ.CODE(7))       GO TO 2
      STOP
      END
```

```
      COMMON CF,BB,RO
      DIMENSION BER(15),EC(15,3),EK(15,5),EM(15,3),ES(15,3),T(20),R(20),
     1D2(2),ID1(2),IW1(15),IW2(15),SIGMA(15,21),F2R1(36,3),FC(3,21),
     2W1(20),W2(20),W3(20),W4(20),W5(225),W6(225),W7(225),F2R(8,15),
     3BEL(15),ID2(2),BLID(4)
 5700 READ(5,700) BLID,NBE,CF,BB,RO,(BER(I),I=1,NBE)
  700 FORMAT('1BLADE IDENTIFICATION ',4A4,2X,'NBE=',I3,3X,'CF=',
     11PE10.3,' BB=',E10.3/(10X,7E10.3))
      WRITE(6,700) BLID,NBE,CF,BB,RO,(BER(I),I=1,NBE)
      IF(NBE.EQ.0)     STOP
      NBE2=NBE**2
      CALL  STRESS(NBE,NBE2,BER,EC,EK,EM,ES,BEL,IW1,IW2,SIGMA,F2R,
     1W1,W2,W3,W4,W5,W6,W7)
      GO TO 5700
      END
```

```
      SUBROUTINE STRESS(NBE,NBE2,BER,EC,EK,EM,ES,BEL,IW1,IW2,SIGMA,F2R,
     1W1,W2,W3,W4,W5,W6,W7)
      COMMON CF,BB,RO
      DIMENSION BER(NBE),EC(NBE,3),EK(NBE,5),EM(NBE,3),ES(NBE,3),
     1T(20),R(20),D2(2),ID1(2),IW1(NBE),IW2(NBE),SIGMA(NBE,21),
     2F2R1(36,3),FC(3,21),W1( 1 ),W2( 1 ),W3( 1 ),W4( 1 ),W5(NBE2),
     3W6(NBE2),W7(NBE2),F2R(8,NBE),BEL(NBE),ID2(2)
      DATA DATIN,END,ID2,YES   /'*INP','*END','A','B','PUNC'/
  599 FORMAT('    X/R =   ',10F7.4/(10X,10F7.4))
  600 FORMAT(1X,'Z % ' ,A1,I2,2X,10F7.3/(10X,10F7.3))
  601 FORMAT(' PSI=',F5.0,'DEGREES'/(10X,10F7.3))
  602 FORMAT('1')
  603 FORMAT(//' DIRECT INTEGRATION OF FORCES'/' 1000*CT=',F7.3,
     1 '     1000*MT=',F7.3)
  604 FORMAT('1  BLADE DEFLECTION (100*Z/R) RADIAL DISTRIBUTION FOR DIF
     1ERENT AZIMUTHAL ANGLES')
  605 FORMAT(1X,20A4)
  606 FORMAT(1X,2A4,1X,1P7E10.3/(10X,7E10.3))
  607 FORMAT('1  BLADE DEFLECTION (100*Z/R) FOURIER COEFFICIENTS')
  610 FORMAT('1  STRESS (10000*SIGMA/E) FOURIER COEFFICIENTS')
  611 FORMAT('1  STRESS (10000*SIGMA/E) RADIAL DISTRIBUTION FOR DIFFERE
     1T AZIMUTHAL ANGLES')
  620 FORMAT(1X,'SIG ' ,A1,I2,2X,10F7.3/(10X,10F7.3))
  701 FORMAT(' EC',7X,1P3E10.3)
  702 FORMAT(' EK',7X,1P5E10.3)
  703 FORMAT(' EM',7X,1P3E10.3)
  704 FORMAT(' ES',7X,1P3E10.3)
  705 FORMAT(20A4)
  706 FORMAT(2A4,2X,1P7E10.3/(10X,7E10.3))
  708 FORMAT(F6.2,4X,'RPM=',F6.1,'NPTS=',I3,2X,'NPOST=',I3,1X,'NHARM=',
     1I3,1X,'FACTOR=',E10.3,2A4/(' STATION',2X,7F10.4))
  750 FORMAT('1 HUB AXIAL FORCE COEFF. (1000*CFZ) HARMONIC ANALYSIS'
     1/F6.2,'CFZ3',10F7.3/(10X,10F7.3))
  751 FORMAT(' 0.45R STRESS FOURIER COEFFICIENTS(10000*SIGMA/E)'
     1/F6.2,'SIG ',10F7.3/(10X,10F7.3))
  752 FORMAT(' 0.70R STRESS FOURIER COEFFICIENTS(10000*SIGMA/E)'
     1/F6.2,'SIG7',10F7.3/(10X,10F7.3))
  753 FORMAT(' FORCE COEFF.(1000*CFZ) VERSUS BLADE AZIMUTHAL ANGLE BY I
     1CREMENT OF 10 DEG.'/(' CFZ(PSI) ',10F7.3))
  754 FORMAT(' 0.45R SIGMA VERSUS AZIMUTHAL ANGLE BY INCREMENT OF 10 DE
     1.'/(' SIG0.45R ',10F7.3))
  800 FORMAT(80('.')/' RUN NUMBER=',F6.2,5X,'STRESS PROGRAM OUTPUT CARD
     1')
      READ(5,701) ((EC(I,J),J=1,3),I=1,NBE)
      WRITE(6,701) ((EC(I,J),J=1,3),I=1,NBE)
      READ(5,702) ((EK(I,J),J=1,5),I=1,NBE)
      WRITE(6,702) ((EK(I,J),J=1,5),I=1,NBE)
      READ(5,703) ((EM(I,J),J=1,3),I=1,NBE)
      WRITE(6,703) ((EM(I,J),J=1,3),I=1,NBE)
      READ(5,704) ((ES(I,J),J=1,3),I=1,NBE)
      WRITE(6,704) ((ES(I,J),J=1,3),I=1,NBE)
      NO4=NBE+1
      NO7=NBE+2
      BRAD = BER(NBE)
      RO4=0.45*BER(NBE)
      RO7=0.7*BER(NBE)
      BEL(1)=BER(1)-RO
      DO 6 I=2,NBE
    6 BEL(I)=BER(I)-BER(I-1)
    1 WRITE(6,602)
```

```
      READ(5,705) OPT1,(T(I),I=1,9),OPT2,(T(I),I=10,18)
      WRITE(6,605)OPT1,(T(I),I=1,9),OPT2,(T(I),I=10,18)
 1000 READ(5,705) T
      WRITE(6,605) T
      IF(T(1).EQ.END) RETURN
      IF(T(1).NE.DATIN) GO TO 1000
      READ(5,708) RUN, RPM ,NPTS,NPOST,NHARM,FACTOR,D2,(R(I),I=2,NPOST),
     1RADIUS
      WRITE(6,708) RUN, RPM ,NPTS,NPOST,NHARM,FACTOR,D2,(R(I),I=2,NPOST)
     1,RADIUS
      OMEGA = RPM*3.14159/30.
      OM2=OMEGA**2
C              FACTOR = 0.9625*OMEGA**2*BER(NBE)**3   (0.9625 = PI*RHO/4)
C              MULTIPLY FACTOR TO OBTAIN Z/R IN %
      FACTOR = 100.*FACTOR
      QR = 0.9625*OM2*BRAD**3/250.
      WRITE(7,800) RUN
      N1HARM=NHARM+1
      R(1)=RO
      NPOST1=NPOST+1
      R(NPOST1)=RADIUS
      DO 180 I=2,NPOST1
  180 R(I)=R(I)*BER(NBE)
      NHARM1=2*NHARM+1
      CALL B2C(W7,EK,NBE,5,3)
      CALL B2C(W5,EC,NBE,3,2)
      CALL LICOM(W6,W7,1.,W5,OM2,NBE,NBE)
      CALL B2C(W7,EM,NBE,3,2)
      W1(1)=0.
      NR=-1
C
 1002 DO 2 NHA=1,NHARM1
      READ(5,706)  ID1,(W1(I),I=2,NPOST1)
      WRITE(6,606) ID1,(W1(I),I=2,NPOST1)
      IF(NHA.NE.1)       GO TO 220
      K1 = 1
      CALL MOMENT(D2,1,R,RO,BRAD,0.,W1,K1,NPOST1,KO)
      CT = 0.02*FACTOR*D2(1)/(QR*BRAD)
      CMT =0.01*FACTOR*D2(2)/(QR*BRAD**2)
  220 K1=1
      DO 3 I=1,NBE
      Y2=BER(I)
      Y1=Y2-BEL(I)
      CALL MOMENT(D2,1,R,Y1,Y2,Y1,W1,K1,NPOST1,KO)
      W4(I)=D2(2)/BEL(I)
      W3(I)=D2(1)-W4(I)
      IF(I.EQ.1) GO TO 3
      W2(I-1)=(W3(I)+W4(I-1))*FACTOR
    3 CONTINUE
      W2(NBE)=W4(NBE)*FACTOR
      NOH=NHA/2
      IF(NOH.EQ.NR) GO TO 10
      OM2N2=-NOH*OM2
      CALL LICOM(W5,W6,1.,W7,OM2N2,NBE,NBE)
C              ELIMINATE FLAPPING ANGLE UNDETERMINATION AT ROTOR FREQUENCY
      IF(NOH.NE.1)       GO TO 210
      NBE2S = NBE2-NBE
      DO 200 I=1,NBE
      W5(I*NBE) = 0.
  200 W5(I+NBE2S) = 0.
      W5(NBE2) = 1.
```

```
      CALL MINV(W5,NBE,DET,IW1,IW2)
      W5(NBE2) = 0.
      GO TO 10
  210 CALL MINV(W5,NBE,DET,IW1,IW2)
   10 CONTINUE
      CALL PROD(W5,NBE,NBE,W2,1,W4)
      DO 11 I=1,NBE
   11 SIGMA(I,NHA)=W4(I)/BER(NBE)
    2 NR=NOH
C
      WRITE(6,603) CT,CMT
      DO 77 I=1,NBE
   77 W1(I)=BER(I)/BER(NBE)
      N=1
      NHA=0
      WRITE(6,607)
      WRITE(6,599) (W1(I),I=1,NBE)
      IF(OPT1.NE.YES) GO TO 78
      WRITE(7,607)
      WRITE(7,599) (W1(I),I=1,NBE)
   78 CONTINUE
      DO 50 K=1,N1HARM
      K1=K-1
      DO 51 J=1,N
      NHA=NHA+1
      IF(OPT1.NE.YES) GO TO 51
      WRITE(7,600) ID2(J),K1,(SIGMA(I,NHA),I=1,NBE)
   51 WRITE(6,600) ID2(J),K1,(SIGMA(I,NHA),I=1,NBE)
   50 N=2
      CALL FSUM3(8,NHARM1,F2R,SIGMA,NBE)
      WRITE(6,604)
      WRITE(6,599) (W1(I),I=1,NBE)
      IF(OPT2.NE.YES) GO TO 49
      WRITE(7,604)
      WRITE(7,599) (W1(I),I=1,NBE)
   49 CONTINUE
      PSI=0.
      DO 52 I=1,8
      WRITE(6,601) PSI,(F2R(I,J),J=1,NBE)
      IF(OPT2.NE.YES) GO TO 52
      WRITE(7,601) PSI,(F2R(I,J),J=1,NBE)
   52 PSI=PSI+45.
      CZ1=(BB+CF*OM2)/BEL(1)+BB/BEL(2)
      CZ2=-BB/BEL(2)
      QR = 100.*QR
      DO 60 NHA=1,NHARM1
      N1=NHA/2
   60 FC(1,NHA)=(CZ1*SIGMA(1,NHA)+CZ2*SIGMA(2,NHA))*(1+(-1)**N1)/QR
      CALL B2C(W5,ES,NBE,3,2)
      I04=1
      I07=1
      DO 15 NHA=1,NHARM1
C             STORE Z/R IN W4
      DO 16 I=1,NBE
   16 W4(I)=SIGMA(I,NHA)/100.
C             COMPUTE THE STRESSES IN W2
      CALL PROD(W5,NBE,NBE,W4,1,W2)
      I1=I04
      CALL IPOL1(W2,BER,VAL,R04,NBE,I1,I04)
      FC(2,NHA)=VAL
      I1=I07
```

```
      CALL IPOL1(W2,BER,VAL,R07,NBE,I1,I07)
      FC(3,NHA)=VAL
      DO17 I=1,NBE
   17 SIGMA(I,NHA)=W2(I)
   15 CONTINUE
      N=1
      NHA=0
      X04=0.45
      X07=0.7
      WRITE(6,610)
      WRITE(6,599) (W1(I),I=1,NBE),X04,X07
      DO 70K=1,N1HARM
      K1=K-1
      DO 71 J=1,N
      NHA=NHA+1
   71 WRITE(6,620) ID2(J),K1,(SIGMA(I,NHA),I=1,NBE),FC(2,NHA),FC(3,NHA)
   70 N=2
      CALL FSUM3(8,NHARM1,F2R,SIGMA,NBE)
      WRITE(6,611)
      WRITE(7,611)
      WRITE(6,599) (W1(I),I=1,NBE)
      WRITE(7,599) (W1(I),I=1,NBE)
      PSI=0.
      DO 73 I=1,8
      DO 74 II=6,7
   74 WRITE(II,601) PSI,(F2R(I,J),J=1,NBE)
   73 PSI=PSI+45.
      DO 100 II=6,7
      WRITE(II,750) RUN,(FC(1,NHA),NHA=1,NHARM1)
      WRITE(II,751) RUN,(FC(2,NHA),NHA=1,NHARM1)
  100 WRITE(II,752) RUN,(FC(3,NHA),NHA=1,NHARM1)
      CALL FSUM3(36,NHARM1,F2R1,FC,3)
      DO 101 II=6,7
      WRITE(II,753) (F2R1(I,1),I=1,36)
  101 WRITE(II,754) (F2R1(I,2),I=1,36)
      DO 102 I=1,2
      READ(5,705) T
      WRITE(6,705) T
  102 WRITE(7,705) T
      GO TO 1
      END
```

```
C        SUBROUTINE ATEIG
C
C        PURPOSE
C           COMPUTE THE EIGENVALUES OF A REAL ALMOST TRIANGULAR MATRIX
C
C        USAGE
C           CALL ATEIG(M,A,RR,RI,IANA,IA)
C
C        DESCRIPTION OF THE PARAMETERS
C           M      ORDER OF THE MATRIX
C           A      THE INPUT MATRIX, M BY M
C           RR     VECTOR CONTAINING THE REAL PARTS OF THE EIGENVALUES
C                  ON RETURN
C           RI     VECTOR CONTAINING THE IMAGINARY PARTS OF THE EIGEN-
C                  VALUES ON RETURN
C           IANA   VECTOR WHOSE DIMENSION MUST BE GREATER THAN OR EQUAL
C                  TO M, CONTAINING ON RETURN INDICATIONS ABOUT THE WAY
C                  THE EIGENVALUES APPEARED (SEE MATH. DESCRIPTION)
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY A
C                  IN THE CALLING PROGRAM WHEN THE MATRIX IS IN DOUBLE
C                  SUBSCRIPTED DATA STORAGE MODE.
C                  IA=M WHEN THE MATRIX IS IN SSP VECTOR STORAGE MODE.
C
C        REMARKS
C           THE ORIGINAL MATRIX IS DESTROYED
C           THE DIMENSION OF RR AND RI MUST BE GREATER OR EQUAL TO M
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           QR DOUBLE ITERATION
C
C        REFERENCES
C           J.G.F. FRANCIS - THE QR TRANSFORMATION---THE COMPUTER
C           JOURNAL, VOL. 4, NO. 3, OCTOBER 1961, VOL. 4, NO. 4, JANUAR
C           1962.  J. H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -
C           CLARENDON PRESS, OXFORD, 1965.
C
C     ..................................................................
C
      SUBROUTINE ATEIG(M,A,RR,RI,IANA,IA)
      DIMENSION A(1),RR(1),RI(1),PRR(2),PRI(2),IANA(1)
      INTEGER P,P1,Q
C
      E7=1.0E-8
      E6=1.0E-6
      E10=1.0E-10
      DELTA=0.5
      MAXIT=30
C
C        INITIALIZATION
C
      N=M
   20 N1=N-1
      IN=N1*IA
      NN=IN+N
      IF(N1) 30,1300,30
   30 NP=N+1
C
```

```
C         ITERATION COUNTER
C
      IT=0
C
C         ROOTS OF THE 2ND ORDER MAIN SUBMATRIX AT THE PREVIOUS
C         ITERATION
C
      DO 40 I=1,2
      PRR(I)=0.0
   40 PRI(I)=0.0
C
C         LAST TWO SUBDIAGONAL ELEMENTS AT THE PREVIOUS ITERATION
C
      PAN=0.0
      PAN1=0.0
C
C         ORIGIN SHIFT
C
      R=0.0
      S=0.0
C
C         ROOTS OF THE LOWER MAIN 2 BY 2 SUBMATRIX
C
      N2=N1-1
      IN1=IN-IA
      NN1=IN1+N
      N1N=IN+N1
      N1N1=IN1+N1
   60 T=A(N1N1)-A(NN)
      U=T*T
      V=4.0*A(N1N)*A(NN1)
      IF(ABS(V)-U*E7) 100,100,65
   65 T=U+V
      IF(ABS(T)-AMAX1(U,ABS(V))*E6) 67,67,68
   67 T=0.0
   68 U=(A(N1N1)+A(NN))/2.0
      V=SQRT(ABS(T))/2.0
      IF(T)140,70,70
   70 IF(U) 80,75,75
   75 RR(N1)=U+V
      RR(N)=U-V
      GO TO 130
   80 RR(N1)=U-V
      RR(N)=U+V
      GO TO 130
  100 IF(T)120,110,110
  110 RR(N1)=A(N1N1)
      RR(N)=A(NN)
      GO TO 130
  120 RR(N1)=A(NN)
      RR(N)=A(N1N1)
  130 RI(N)=0.0
      RI(N1)=0.0
      GO TO 160
  140 RR(N1)=U
      RR(N)=U
      RI(N1)=V
      RI(N)=-V
  160 IF(N2)1280,1280,180
C
C         TESTS OF CONVERGENCE
```

```
C
  180 N1N2=N1N1-IA
      RMOD=RR(N1)*RR(N1)+RI(N1)*RI(N1)
      EPS=E10*SQRT(RMOD)
      IF(ABS(A(N1N2))-EPS)1280,1280,240
  240 IF(ABS(A(NN1))-E10*ABS(A(NN))) 1300,1300,250
  250 IF(ABS(PAN1-A(N1N2))-ABS(A(N1N2))*E6) 1240,1240,260
  260 IF(ABS(PAN-A(NN1))-ABS(A(NN1))*E6)1240,1240,300
  300 IF(IT-MAXIT) 320,1240,1240
C
C        COMPUTE THE SHIFT
C
  320 J=1
      DO 360 I=1,2
      K=NP-I
      IF(ABS(RR(K)-PRR(I))+ABS(RI(K)-PRI(I))-DELTA*(ABS(RR(K))
     1     +ABS(RI(K)))) 340,360,360
  340 J=J+I
  360 CONTINUE
      GO TO (440,460,460,480),J
  440 R=0.0
      S=0.0
      GO TO 500
  460 J=N+2-J
      R=RR(J)*RR(J)
      S=RR(J)+RR(J)
      GO TO 500
  480 R=RR(N)*RR(N1)-RI(N)*RI(N1)
      S=RR(N)+RR(N1)
C
C        SAVE THE LAST TWO SUBDIAGONAL TERMS AND THE ROOTS OF THE
C        SUBMATRIX BEFORE ITERATION
C
  500 PAN=A(NN1)
      PAN1=A(N1N2)
      DO 520 I=1,2
      K=NP-I
      PRR(I)=RR(K)
  520 PRI(I)=RI(K)
C
C        SEARCH FOR A PARTITION OF THE MATRIX, DEFINED BY P AND Q
C
      P=N2
      IPI=N1N2
      DO 580 J=2,N2
      IPI=IPI-IA-1
      IF(ABS(A(IPI))-EPS) 600,600,530
  530 IPIP=IPI+IA
      IPIP2=IPIP+IA
      D=A(IPIP)*(A(IPIP)-S)+A(IPIP2)*A(IPIP+1)+R
      IF(D)540,560,540
  540 IF(ABS(A(IPI)*A(IPIP+1))*(ABS(A(IPIP)+A(IPIP2+1)-S)+ABS(A(IPIP2+2
     1 ))) -ABS(D)*EPS) 620,520,560
  560 P=N1-J
  580 CONTINUE
  600 Q=P
      GO TO 680
  620 P1=P-1
      Q=P
      DO 660 I=1,P1
      IPI=IPI-IA-1
```

```
      IF(ABS(A(IPI))-EPS)680,680,660
  660 Q=Q-1
C
C         QR DOUBLE ITERATION
C
  680 II=(P-1)*IA+P
      DO 1220 I=P,N1
      II1=II-IA
      IIP=II+IA
      IF(I-P)720,700,720
  700 IPI=II+1
      IPIP=IIP+1
C
C         INITIALIZATION OF THE TRANSFORMATION
C
      G1=A(II)*(A(II)-S)+A(IIP)*A(IPI)+R
      G2=A(IPI)*(A(IPIP)+A(II)-S)
      G3=A(IPI)*A(IPIP+1)
      A(IPI+1)=0.0
      GO TO 780
  720 G1=A(II1)
      G2=A(II1+1)
      IF(I-N2)740,740,760
  740 G3=A(II1+2)
      GO TO 780
  760 G3=0.0
  780 CAP=SQRT(G1*G1+G2*G2+G3*G3)
      IF(CAP)800,860,800
  800 IF(G1)820,840,840
  820 CAP=-CAP
  840 T=G1+CAP
      PSI1=G2/T
      PSI2=G3/T
      ALPHA=2.0/(1.0+PSI1*PSI1+PSI2*PSI2)
      GO TO 880
  860 ALPHA=2.0
      PSI1=0.0
      PSI2=0.0
  880 IF(I-Q)900,960,900
  900 IF(I-P)920,940,920
  920 A(II1)=-CAP
      GO TO 960
  940 A(II1)=-A(II1)
C
C         ROW OPERATION
C
  960 IJ=II
      DO 1040 J=I,N
      T=PSI1*A(IJ+1)
      IF(I-N1)980,1000,1000
  980 IP2J=IJ+2
      T=T+PSI2*A(IP2J)
 1000 ETA=ALPHA*(T+A(IJ))
      A(IJ)=A(IJ)-ETA
      A(IJ+1)=A(IJ+1)-PSI1*ETA
      IF(I-N1)1020,1040,1040
 1020 A(IP2J)=A(IP2J)-PSI2*ETA
 1040 IJ=IJ+IA
C
C         COLUMN OPERATION
C
```

```
      IF(I-N1)1080,1060,1060
 1060 K=N
      GO TO 1100
 1080 K=I+2
 1100 IP=IIP-I
      DO 1180 J=Q,K
      JIP=IP+J
      JI=JIP-IA
      T=PSI1*A(JIP)
      IF(I-N1)1120,1140,1140
 1120 JIP2=JIP+IA
      T=T+PSI2*A(JIP2)
 1140 ETA=ALPHA*(T+A(JI))
      A(JI)=A(JI)-ETA
      A(JIP)=A(JIP)-ETA*PSI1
      IF(I-N1)1160,1180,1180
 1160 A(JIP2)=A(JIP2)-ETA*PSI2
 1180 CONTINUE
      IF(I-N2)1200,1220,1220
 1200 JI=II+3
      JIP=JI+IA
      JIP2=JIP+IA
      ETA=ALPHA*PSI2*A(JIP2)
      A(JI)=-ETA
      A(JIP)=-ETA*PSI1
      A(JIP2)=A(JIP2)-ETA*PSI2
 1220 II=IIP+1
      IT=IT+1
      GO TO 60
C
C        END OF ITERATION
C
 1240 IF(ABS(A(NN1))-ABS(A(N1N2))) 1300,1280,1280
C
C        TWO EIGENVALUES HAVE BEEN FOUND
C
 1280 IANA(N)=0
      IANA(N1)=2
      N=N2
      IF(N2)1400,1400,20
C
C        ONE EIGENVALUE HAS BEEN FOUND
C
 1300 RR(N)=A(NN)
      RI(N)=0.0
      IANA(N)=1
      IF(N1)1400,1400,1320
 1320 N=N1
      GO TO 20
 1400 RETURN
      END
```

```
      SUBROUTINE B2C(A,B,N,M,M1)
C        CE S.P. PERMET DE PASSER D'UNE MATRICE BANDE A UNE
C        MATRICE COLONNE.
C        N=NOMBRE DE LIGNES DE LA MATRICE BANDE.
C        M=NOMBRE DE DIAGONALES DANS LA BANDE.
C        M1=RANG DE LA DIAGONALE PRINCIPALE.
C        A=MATRICE COLONNE DE DIM.=N**2 (SORTIE)
C        B=MATRICE BANDE DE DIM.=N*M (ENTREE)
      DIMENSION A(1),B(N,M)
      N2=N**2
      DO 1 I=1,N2
    1 A(I)=0.
      K1=1
      DO 2 I=1,N
      J1=MAX0(M1-I+1,1)
      IF(I.GT.M1) K1=K1+1
      K=K1
      DO 2 J=J1,M
      IK=(K-1)*N+I
      IF(IK.GT.N2) GO TO 2
      A(IK)=B(I,J)
    2 K=K+1
      RETURN
      END
```

```
9 ( JUNE 70 )                                    OS/360  FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=56,SIZE=0000K,
                      SOURCE,EBCDIC,NOLIST,DECK,LOAD,MAP,NOEDIT,NOID,NOXREF
              SUBROUTINE CLEAR(N)
      C          INITIALISATION DES CB A 0 SI N=1 OU DES CT A 0 SI N=2.
              COMMON/BLADE/CB(140)
              COMMON/TRANS/CT(281)
              GO TO (1001,1002),N
         1001 DO 1 I=1,140
            1 CB(I)=0.
              RETURN
         1002 DO 2 I=1,281
            2 CT(I)=0.
              RETURN
              END
```

```
      SUBROUTINE EIGMOD(EM,EC,EK,OMEGA,OMUB,NBE)
      COMMON/WORK/W1(20),W2(20),W3(20),W4(400),W5(400)
      DIMENSION EM(1),EC(1),EK(1),IW1(20),IW2(20)
      DOUBLE PRECISION DEFMOD(2),DM
      DATA DEFMOD/'FLAPPING','BENDING '/
  600 FORMAT('1BLADE EIGENMODES AT OMEGA =',F7.2,' FOR',I3,' ELEMENTS'//
     1' ROTOR FREQUENCY',F7.2,' CPS', 8X,'ROTOR RPM',F7.1//' **RESONANCE
     2 FREQUENCY OF UNIFORM BLADE (FUB)',F7.2,' CPS'///
     33X,'MODE',9X,'EIGENVALUES',12X,'EIGENFREQUENCIES',3X,'F/FUB'/
     4 17X,'(RAD/S)**2'/14X,'REAL',8X,'IMAG'/)
  601 FORMAT(1X,A8,I2,1P2E12.3,8X,0PF7.2,8X,F6.2)
      FUB = OMUB/6.2832
      FROT = OMEGA/6.2832
      RPM = 60.*FROT
      CALL B2C(W4,EK,NBE,5,3)
      CALL B2C(W5,EC,NBE,3,2)
      X2=+OMEGA**2
      CALL LICOM(W4,W4,+1.,W5,X2,NBE,NBE)
      CALL B2C(W5,EM,NBE,3,2)
      CALL MINV(W5,NBE,DELT,IW1,IW2)
      CALL PROD1(W5,W4,W1,NBE,NBE)
      WRITE(6,600) OMEGA,NBE,FROT,RPM,FUB
      CALL HSBG(NBE,W4,NBE)
      CALL ATEIG(NBE,W4,W1,W2,IW1,NBE)
      DM = DEFMOD(1)
 1001 DO 1 I=1,NBE
      MODE = I-1
      K = NBE - MODE
      FREQ = SQRT(ABS(W1(K)))/6.2832
      FREQR = FREQ/FUB
      WRITE(6,601) DM,MODE,W1(K),W2(K),FREQ,FREQR
    1 DM = DEFMOD(2)
      RETURN
      END
```

```
      SUBROUTINE FHARM3(NPTS,NHARM,NPOST,NTYPE)
      COMMON FO(20),FN(12,20,2),F(24,20,2)
      COMPLEX EJD,EJND,CN
      N1 = NPTS + 1
      PI=3.141592
      PIDX=PI**2/NPTS
      EJD = CMPLX(0.,-2.*PI/FLOAT(NPTS))
      EJD = CEXP(EJD)
 1010 DO10 I=1,NPOST
      FO(I)=0.
 1001 DO1 K=1,NPTS
    1 FO(I)=FO(I)+F(K,I,NTYPE)
   10 FO(I)=FO(I)/NPTS
      EJND=(1.,0.)
 1002 DO2 N=1,NHARM
      EJND=EJND*EJD
      COR = (1. - REAL(EJND))/(PIDX*FLOAT(N**2))
 1011 DO11 I=1,NPOST
      CN=(0.,0.)
 1003 DO3 K=1,NPTS
    3 CN=CN*EJND+F(N1-K,I,NTYPE)
      CN = COR*CN
      FN(N,I,1)=REAL(CN)
   11 FN(N,I,2)=-AIMAG(CN)
    2 CONTINUE
      RETURN
      END
```

```
C     ...................................................................
C
C        SUBROUTINE HSBG
C
C        PURPOSE
C           TO REDUCE A REAL MATRIX INTO UPPER ALMOST TRIANGULAR FORM
C
C        USAGE
C           CALL HSBG(N,A,IA)
C
C        DESCRIPTION OF THE PARAMETERS
C           N      ORDER OF THE MATRIX
C           A      THE INPUT MATRIX, N BY N
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY
C                  A IN THE CALLING PROGRAM WHEN THE MATRIX IS IN
C                  DOUBLE SUBSCRIPTED DATA STORAGE MODE.  IA=N WHEN
C                  THE MATRIX IS IN SSP VECTOR STORAGE MODE.
C
C        REMARKS
C           THE HESSENBERG FORM REPLACES THE ORIGINAL MATRIX IN THE
C           ARRAY A.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SIMILARITY TRANSFORMATIONS USING ELEMENTARY ELIMINATION
C           MATRICES, WITH PARTIAL PIVOTING.
C
C        REFERENCES
C           J.H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -
C           CLARENDON PRESS, OXFORD, 1965.
C
C     ...................................................................
C
      SUBROUTINE HSBG(N,A,IA)
      DIMENSION A(1)
      DOUBLE PRECISION S
      L=N
      NIA=L*IA
      LIA=NIA-IA
C
C        L IS THE ROW INDEX OF THE ELIMINATION
C
   20 IF(L-3) 360,40,40
   40 LIA=LIA-IA
      L1=L-1
      L2=L1-1
C
C        SEARCH FOR THE PIVOTAL ELEMENT IN THE LTH ROW
C
      ISUB=LIA+L
      IPIV=ISUB-IA
      PIV=ABS(A(IPIV))
      IF(L-3) 90,90,50
   50 M=IPIV-IA
      DO 80 I=L,M,IA
      T=ABS(A(I))
      IF(T-PIV) 80,80,60
   60 IPIV=I
```

```
      PIV=T
   80 CONTINUE
   90 IF(PIV) 100,320,100
  100 IF(PIV-ABS(A(ISUB))) 180,180,120
C
C        INTERCHANGE THE COLUMNS
C
  120 M=IPIV-L
      DO 140 I=1,L
      J=M+I
      T=A(J)
      K=LIA+I
      A(J)=A(K)
  140 A(K)=T
C
C        INTERCHANGE THE ROWS
C
      M=L2-M/IA
      DO 160 I=L1,NIA,IA
      T=A(I)
      J=I-M
      A(I)=A(J)
  160 A(J)=T
C
C        TERMS OF THE ELEMENTARY TRANSFORMATION
C
  180 DO 200 I=L,LIA,IA
  200 A(I)=A(I)/A(ISUB)
C
C        RIGHT TRANSFORMATION
C
      J=-IA
      DO 240 I=1,L2
      J=J+IA
      LJ=L+J
      DO 220 K=1,L1
      KJ=K+J
      KL=K+LIA
  220 A(KJ)=A(KJ)-A(LJ)*A(KL)
  240 CONTINUE
C
C        LEFT TRANSFORMATION
C
      K=-IA
      DO 300 I=1,N
      K=K+IA
      LK=K+L1
      S=A(LK)
      LJ=L-IA
      DO 280 J=1,L2
      JK=K+J
      LJ=LJ+IA
  280 S=S+A(LJ)*A(JK)*1.0D0
  300 A(LK)=S
C
C        SET THE LOWER PART OF THE MATRIX TO ZERO
C
      DO 310 I=L,LIA,IA
  310 A(I)=0.0
  320 L=L1
      GO TO 20
```

```
360 RETURN
    END
```

```
      SUBROUTINE IPOL1(Y,X,VAL,ARG,N,I1,I2)
C         SOUS PROG. D'INTERPOLATION LINEAIRE D'UNE FONCTIONY DONNEE
C         PAR Y(I) ET X(I).ARG=ABSCISSE D'INTERPOL. ET VAL LA VALEUR
C         CORRESPONDANTE.
C         N=NOMBRE DE POINTS DEFINISSANT LA FONCTION.
C         I1=VALEUR DE DEPART DE LA VARIATION D'INDICE.
C         I2=INDICE DE LA PREMIERE VALEUR SUP. OU EGALE A ARG.
      DIMENSION X(1),Y(1)
      DO 1 I=I1,N
      I2=I
      IF(X(I).GE.ARG) GO TO 2
    1 CONTINUE
      GO TO 3
    2 IF(I2.EQ.1) GO TO 3
      VAL=(Y(I2)*(ARG-X(I2-1))+Y(I2-1)*(X(I2)-ARG))/(X(I2)-X(I2-1))
      RETURN
    3 VAL=Y(I2)
      RETURN
      END
```

```
      SUBROUTINE LICOM(X,X1,C1,X2,C2,N,M)
C        CE S.P. EFFECTUE LA COMBINAISON LINEAIRE DE DEUX MATRICES
C        COLONNES X1 ET X2 DE DIM.=N*M,AFFECTEES RESPECTIVEMENT
C        DES COEFF. C1 ET C2.LE RESULTAT EST DONNE EN X.
      DIMENSION X1(1),X2(1),X(1)
      MN=N*M
      DO 1 I=1,MN
    1 X(I)=C1*X1(I)+C2*X2(I)
      RETURN
      END
```

```
C
C       ...............................................................
C
C          SUBROUTINE MINV
C
C          PURPOSE
C             INVERT A MATRIX
C
C          USAGE
C             CALL MINV(A,N,D,L,M)
C
C          DESCRIPTION OF PARAMETERS
C             A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
C                 RESULTANT INVERSE.
C             N - ORDER OF MATRIX A
C             D - RESULTANT DETERMINANT
C             L - WORK VECTOR OF LENGTH N
C             M - WORK VECTOR OF LENGTH N
C
C          REMARKS
C             MATRIX A MUST BE A GENERAL MATRIX
C
C          SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C             NONE
C
C          METHOD
C             THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
C             IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
C             THE MATRIX IS SINGULAR.
C
C       ...............................................................
C
      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
C
C          ............................................................
C
C          IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C          C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C          STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,D,BIGA,HOLD
C
C          THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C          APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C          ROUTINE.
C
C          THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C          CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT
C          10 MUST BE CHANGED TO DABS.
C
C          ............................................................
C
C          SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
```

```
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
C
C        INTERCHANGE ROWS
C
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
C
C        INTERCHANGE COLUMNS
C
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
C
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C
C        REDUCE MATRIX
C
      DO 65 I=1,N
      IK=NK+I
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=A(IK)*A(KJ)+A(IJ)
   65 CONTINUE
```

```
C
C          DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C
C          PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C          REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
   80 CONTINUE
C
C          FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
      END
```

```
      SUBROUTINE MOMENT(FXN,N,X,X1,X2,XOR,F,K1,K2,K0)
C TO COMPUTE THE MOMENT UP TO THE NTH ORDER OF A MONOTONICALLY TABULATED
C FUNCTION F,IN THE INTERVAL (X1,X2).LE.(X(K1),X(K2)) WITH RESPECT TO PO
      DIMENSION F(1),FXN(1),X(1)                                 -IONT XOR
      NM=N+1
 1001 DO 1 K=K1,K2
C SEARCH FOR FIRST VALUE OF X LESS THAN X1
      IF(X(K).GT.X1) GO TO 2
      I1=K
      I2=I1
C SEARCH FOR FIRST VALUE OF X LESS THAN X2
    2 IF(X(K).LT.X2) GO TO 1
      I2=K-1
      GO TO 1003
    1 CONTINUE
 1003 DO 3 M=1,NM
    3 FXN(M)=0.
 1004 DO 4 I=I1,I2
      J=I+1
      XI=X(I)
      XJ=X(J)
      IF(I.EQ.I1) XI=X1
      IF(I.EQ.I2) XJ=X2
      A=F(I)
      C=(F(J)-A)/(X(J)-X(I))
      Z1=XI-XOR
      Z2=XJ-XOR
      A=A+C*(XOR-X(I))
 2004 DO 4 M=1,NM
      Z12=Z1*(XI-XOR)
      Z22=Z2*(XJ-XOR)
      FXN(M)=FXN(M)+A*(Z2-Z1)/M+C*(Z22-Z12)/(M+1)
      Z1=Z12
    4 Z2=Z22
      K0=I1
      K1=I2
      RETURN
      END
```

```
      SUBROUTINE PROD(A,L,N,B,M,AB)
      DIMENSION A(1),B(1),AB(1)
      DO 1 I=1,M
      IM = (I-1)*N
      DO 2 J=1,L
      JI = J + (I-1)*L
      AB(JI) = 0.
      DO 2 K=1,N
      KI=K+IM
      JK=J+(K-1)*L
    2 AB(JI) = AB(JI) + A(JK)*B(KI)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PROD1(A,B,X,N,M)
C     COMPUTE THE PRODUCT A.B OF TWO N*N AND N*M MATRICES
C     STORED COLUMNWISE AND STORE IT IN B
      DIMENSION A(1),B(1),X(1)
      DO 1 I=1,M
      IM=(I-1)*M
      DO 2 J=1,N
      X(J)=0.
      DO 2 K=1,N
      KI=K+IM
      JK=J+(K-1)*N
    2 X(J)=X(J)+A(JK)*B(KI)
      DO 1 K=1,N
      KI=K+IM
    1 B(KI)=X(K)
      RETURN
      END
```

```
      SUBROUTINE STRAM(NBE,NR,IS,RO,BLID,HE,RS,EM,ES,EC,EK)
      COMMON/WORK/W1(20),W2(20),W3(20), U(400), C(400)
      COMMON/BLADE/BEL(20),BER(20),BECM(20),BETM(20),BEK(20),
     1 A(20),B(20)
      DIMENSION ES(NBE,3),EM(NBE,3),EC(NBE,3),EK(NBE,5),HE(1),RS(1),
     1 BLID(4)
  600 FORMAT(/1X,4A4,I3,' ELEMENTS',3X,'CF=',1PE10.3,3X,'BB=',E10.3)
  700 FORMAT('1BLADE IDENTIFICATION ',4A4,2X,'NBE=',I3,3X,'CF=',1PE10.3,
     1' BB=',E10.3/(' BER',6X,7E10.3))
  701 FORMAT(' EC',7X,1P3E10.3)
  702 FORMAT(' EK',7X,1P5E10.3)
  703 FORMAT(' EM',7X,1P3E10.3)
  704 FORMAT(' ES',7X,1P3E10.3)
      NBE1 = NBE-1
      NBE2 = NBE-2
      EM(1,1) = BECM(1)/4.
      BEL(NBE+1) = 1.
      U(1) = 1./BEL(1)
C
C RADIAL DISTANCES FROM THE BLADE ELEMENT CENTERS ARE STORED IN W1
C
      W1(1) = BEL(1)/2. + RO
      IF(NBE.EQ.1)          GO TO 1203
 1202 DO 202 I=2,NBE
      W1(I) = BER(I-1) + BEL(I)/2.
  202 U(I) = 1./BEL(I) + 1./BEL(I-1)
C
C       COMPUTE A,B AND C. (STORE THE MI*RI IN W2 AND W3)
C
      W3(NBE+1)=0.
 1203 DO 203 I=1,NBE
      W2(I) = BETM(I)*BER(I)
      W3(I) = BECM(I)*W1(I)
      A(I) = W2(I) + W3(I)/2.
  203 W1(I) = BEK(I)*U(I)
C
C       W1 CONTAINS NOW THE PRODUCTS BEK*U
C
      W1(NBE+1) = 0.
      C(NBE) = 0.
      IF(NBE.EQ.1)          GO TO 1206
 1204 DO 204 I=2,NBE
      K=NBE-I+2
  204 C(K-1) = C(K) + W2(K) + W3(K)
C
C              COMPUTE          EM =          MASS MATRIX
C                        EC =                 CENTRIFICAL MATRIX
C                        EK =                 RIGIDITY MATRIX.
C
 1205 DO 205 I = 2,NBE
      ES(I-1,1) = 1./(BEL(I)*BEL(I-1))
      ES(I-1,2) = -U(I)/BEL(I)
      ES(I-1,3) = 1./BEL(I)**2
      EK(I,1) = BEK(I)*ES(I-1,1)
      EK(I,2) = -(W1(I)+W1(I+1))/BEL(I)
      EC(I,1) = -(A(I)+C(I))/BEL(I)
  205 EM(I,1) = BECM(I)/4.
C            FREE HINGE
      ES(1,1) = 0.
C
```

```
      DO 226 I=1,NBE1
      EC(I,3) = EC(I+1,1)
      EM(I,3) = EM(I+1,1)
  226 EK(I,4) = EK(I+1,2)
      IF(NBE.LT.3)       GO TO 208
      DO 207 I=1,NBE2
  207 EK(I,5) = EK(I+2,1)
  208 CONTINUE
 1206 DO 206 I=1,NBE
      B(I)=W3(I+1)/2.
      EK(I,3) = BEK(I)/BEL(I)**2 + W1(I+1)*U(I+1) + BEK(I+2)*ES(I,3)
      EC(I,2) = C(I)*U(I+1) + A(I)/BEL(I) - B(I)/BEL(I+1)
  206 EM(I,2) = EM(I,1) + EM(I,3) + BETM(I)
C       COMPUTE TOTAL CENTRIFUGAL EFFECT
C
      CF = C(1) + BECM(1)*(BER(1)-BEL(1)/2.) + BETM(1)*BER(1)
C
      I1 = 1
      ARG = RO
      R104 =  5000*BER(NBE)
C            COMPUTE THE ELASTICITY MATRIX  ES.
C            10000*(SIGMA/E) = ES*(Z/BRAD)
C
 1222 DO 222 I=1,NBE
      CALL IPOL1(HE,RS,VAL,ARG,NR,I1,I2)
      VAL = VAL*R104
      ES(I,1) = ES(I,1)*VAL
      ES(I,2) = ES(I,2)*VAL
      ES(I,3) = ES(I,3)*VAL
      ARG = BER(I)
  222 I1 = I2
C        COMPUTE AXIAL FORCE COEFFICIENT DUE TO BLADE BENDING=BB
      BB=BEK(2)/BEL(1)
C
      IF(IS.EQ.0)        RETURN
      WRITE(6,600) BLID,NBE,CF,BB
C
C       PUNCH THE RESULTS
 2223 DO 223 K=6,IS
      IF(K.EQ.6)      GO TO 7701
      WRITE(K,700) BLID,NBE,CF,BB,RO,(BER(I),I=1,NBE)
 7701 WRITE(K,701) ((EC(I,J),J=1,3),I=1,NBE)
      WRITE(K,702) ((EK(I,J),J=1,5),I=1,NBE)
      WRITE(K,703) ((EM(I,J),J=1,3),I=1,NBE)
  223 WRITE(K,704) ((ES(I,J),J=1,3),I=1,NBE)
      RETURN
      END
```

0.45R COMPUTED STRESSES FOURIER COEFFICIENTS

σ in hectobars

$$\sigma = \sigma_0 + \sigma_1 \cos\Psi + \sigma_1' \sin\Psi + \sigma_2 \cos 2\Psi + \sigma_2' \sin 2\Psi + \dots$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Simulation number	Corresponding RUN.PTS		σ_0	σ_1	σ_1'	σ_2	σ_2'	σ_3	σ_3'	σ_4	σ_4		
11	9.03		4,8	0.91	0,09	0.29	0.25	0.22	-0.21	-0.006	0.116		
12	9.04		5.12	0.9	0.39	-0.04	0.14	0.1	-0.28	-0.03	-0.004		
13	9.05		4.8	0.86	0.28	-0.38	-0.05	+0.06	-0.35	-0.04	-0.11		
14	9.06		4.45	0.66	-0.14	-0.68	0.21	0.7	-0.32	-0.2	0 03		
15	12.10		4.6	0.76	0.57	0.06	0 09	0.07	-0.25	-0.01	0.02		
16	12.11		4.6	1.04	0.08	0.36	-0.2	0.1	-0.35	0.1	0.2		
17	12.12		4.8	0.82	-0.24	0.3	-0.56	-0.1	-0.45	0.38	0.03		
18	12.13		4.8	0.75	-0.36	0.30	-0.81	-0.15	-0.5	0.46	-0.02		
19	14.10		5.02	0.74	0.65	-0.04	0	0.11	-0.2	0.04	0.006		
20	14.11		5.06	0.66	0.12	-0.09	-0.37	0.25	-0.4	0.13	-0.14		
21	14.12		5.02	0.33	0.13	-0.46	-0.82	+0.37	-0.72	0 24	-0.29		
22	14.13		5.12	0.21	0.11	-0.33	-1.34	0.72	-0.68	0.37	-0 21		
23	16.08		4.8	0.37	0.82	-0.12	0.08	0.04	-0.27	0.04	-0.02		
24	16.09		4.71	0.24	0.76	0.29	0.11	-0.08	-0.14	-0.07	-0.12		
25	16.10		4.62	0.16	0.63	0.72	0.05	-0.2	0.04	-0.17	-0.24		

GIRAVIONS DORAND

TABLE 1

Page 62

Doc' DH.2011C.E1

0.45 R MEASURED STRESSES FOURIER COEFFICIENTS

σ in hectobars

$$\sigma = \sigma_0 + \sigma_1 \cos\Psi + \sigma_1' \sin\Psi + \sigma_2 \cos 2\Psi + \sigma_2' \sin 2\Psi + \dots$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	RUN.PT		σ_0	σ_1	σ_1'	σ_2	σ_2'	σ_3	σ_3'	σ_4	σ_4'		
	9.03		7.93	1.64	0.025	0.173	0.497	2.38	-0 567	1.27	0.641		
	9.04		8.25	1.46	0.389	-0.225	0.224	2.40	-0.775	-1.79	0.385		
	9.05		7.97	1.37	0.238	-0.547	0.0866	2.24	-1.06	-1.36	0.468		
	9.06		7.59	1.28	-0.198	-0 722	-0.1125	2.33	-1.42	-1.06	0.595		
	12.10		7.77	1.34	0.670	-0.0219	0.196	2.68	-0.084	-1.8	0 255		
	12.11		7.65	1.56	0.213	0.396	-0.116	2.92	-0.323	-3 05	-0.660		
	12.12		7.51	1.41	-0.419	0.426	-0.429	1.87	-2.82	-2.12	0,928		
	12.13		7.54	1.26	-0.536	0,350	-0.775	1.24	-3.89	-2.37	1.4		
	14.10		8.11	1.215	0.888	-0.333	0.0897	2.97	-0 226	-2.17	-0.165		
	14.11		7.83	1.26	0.008	-0.168	-0.289	2.69	-2.22	-1.19	2.18		
	14.12		7.75	1.	-0.27	-0.414	-0.631	3.04	-4.92	-0.856	3.05		
	14.13		7.74	0.798	-0.006	-0.801	-1.01	3.75	-5.91	-1.05	3.0		
	16.08		7.84	1.28	0.558	-0.711	0.435	1.06	-1.71	-0 586	2.15		
	16.09		7.83	0.911	0.924	0.0515	0.260	0.923	-0.006	-0797	0.541		
	16.10		7.73	0 852	0.841	0.556	0.115	0.718	-0.777	-0.833	0.823		

GIRAVIONS DORAND

TABLE II'

Doc' DH.2011.C.E1

Page 63

FIG.1 _ STRESS RECORDINGS

Referance and sign conventions

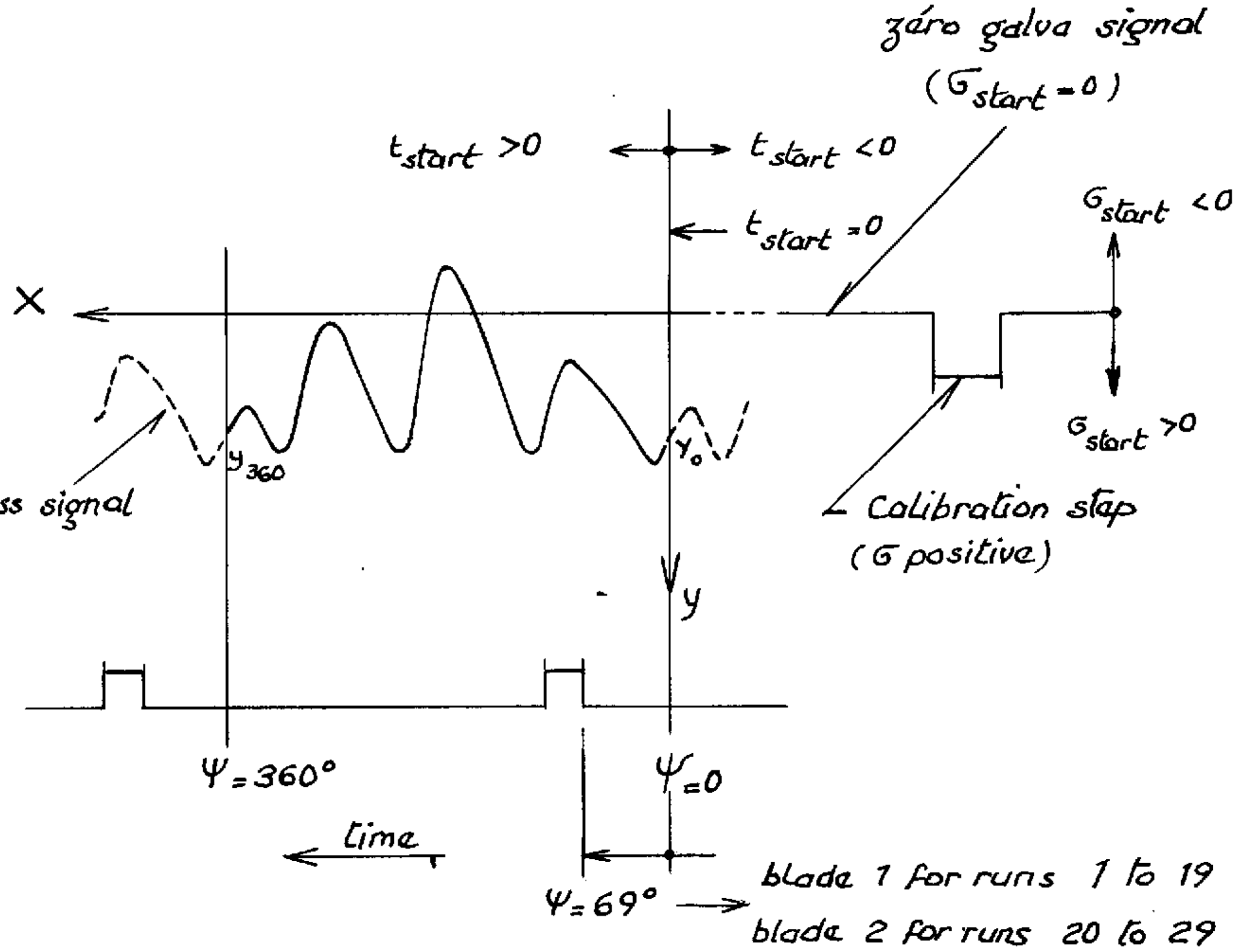
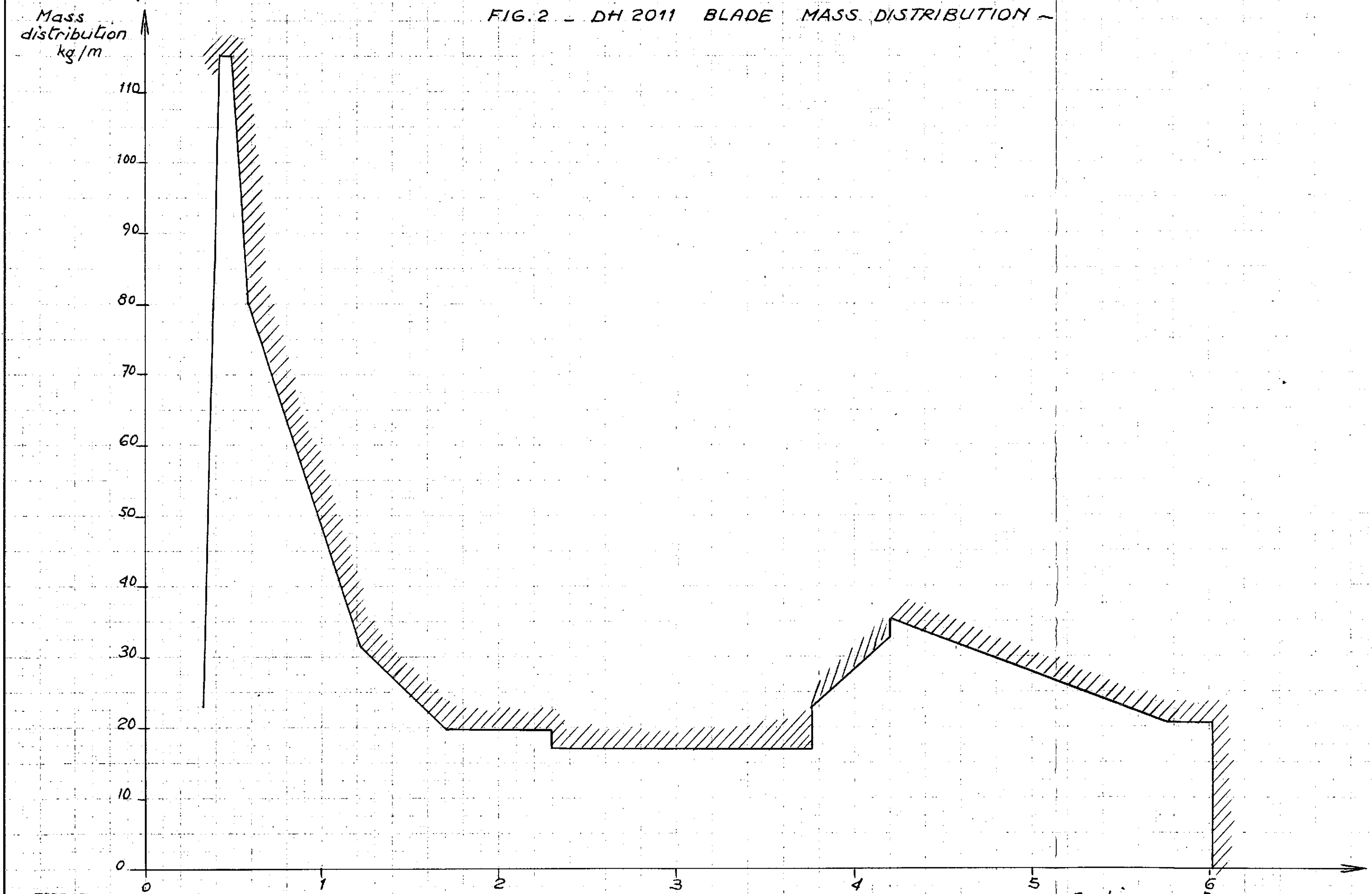


FIG. 2 _ DH 2011 BLADE MASS DISTRIBUTION _



FOLDOUT FRAME 1

FOLDOUT FRAME 2

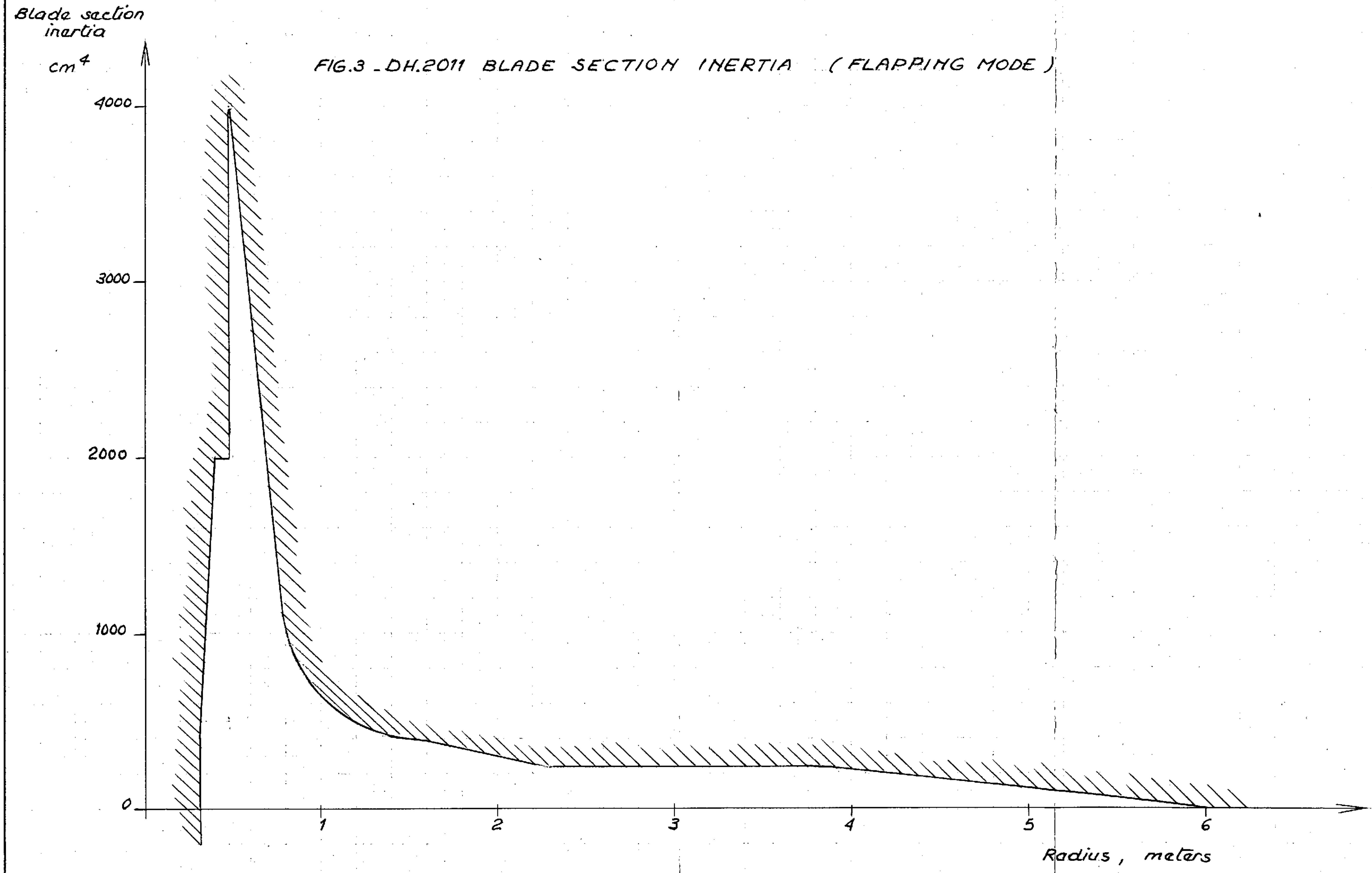
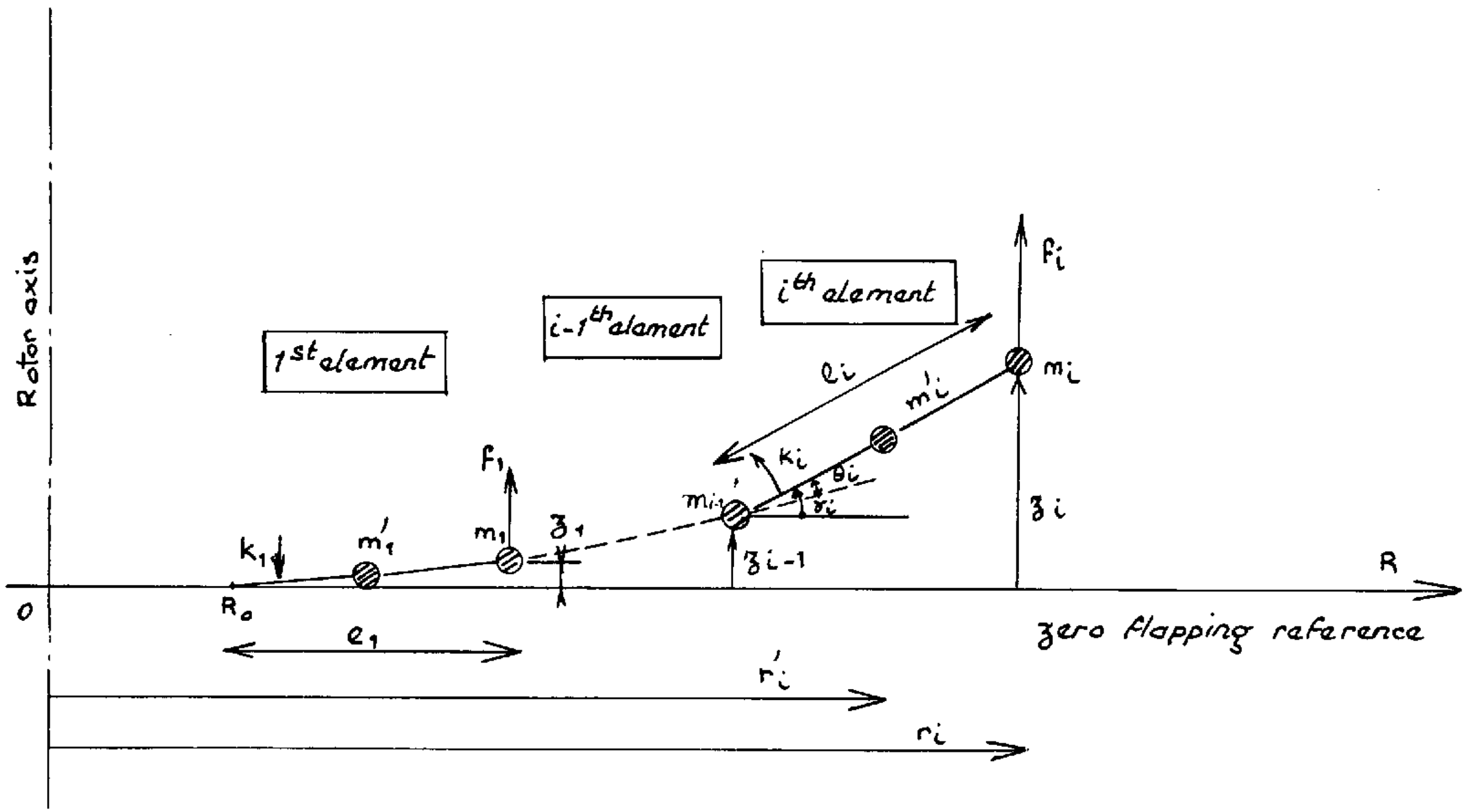

Blade section inertia
cm^4
FIG.3 _ DH.2011 BLADE SECTION INERTIA (FLAPPING MODE)
4000
3000
2000
1000
0
1
2
3
4
5
6
Radius , meters

FIG. 4 – EQUIVALENT BLADE NOTATIONS



R_o = Radial location of the blade flapping hinge

FIG. 5 CONVERGENCE STUDY OF THE METHOD OF TRANSFERS UNIFORM BEAM TEST AT $\Omega = 0$

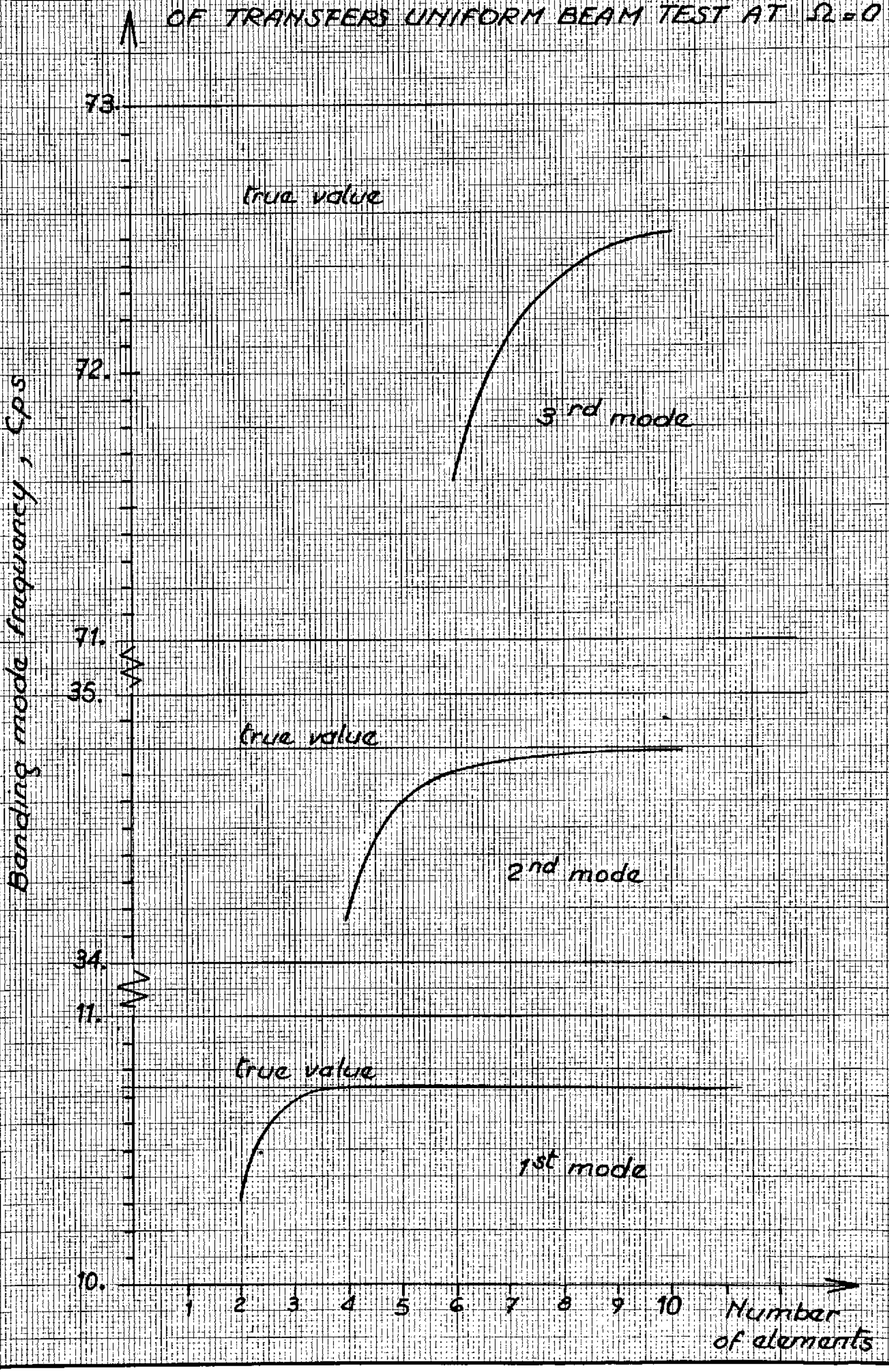


FIG. 6 CONVERGENCE STUDY OF THE METHOD OF TRANSFERS UNIFORM BEAM TEST AT $\Omega = 33.30$

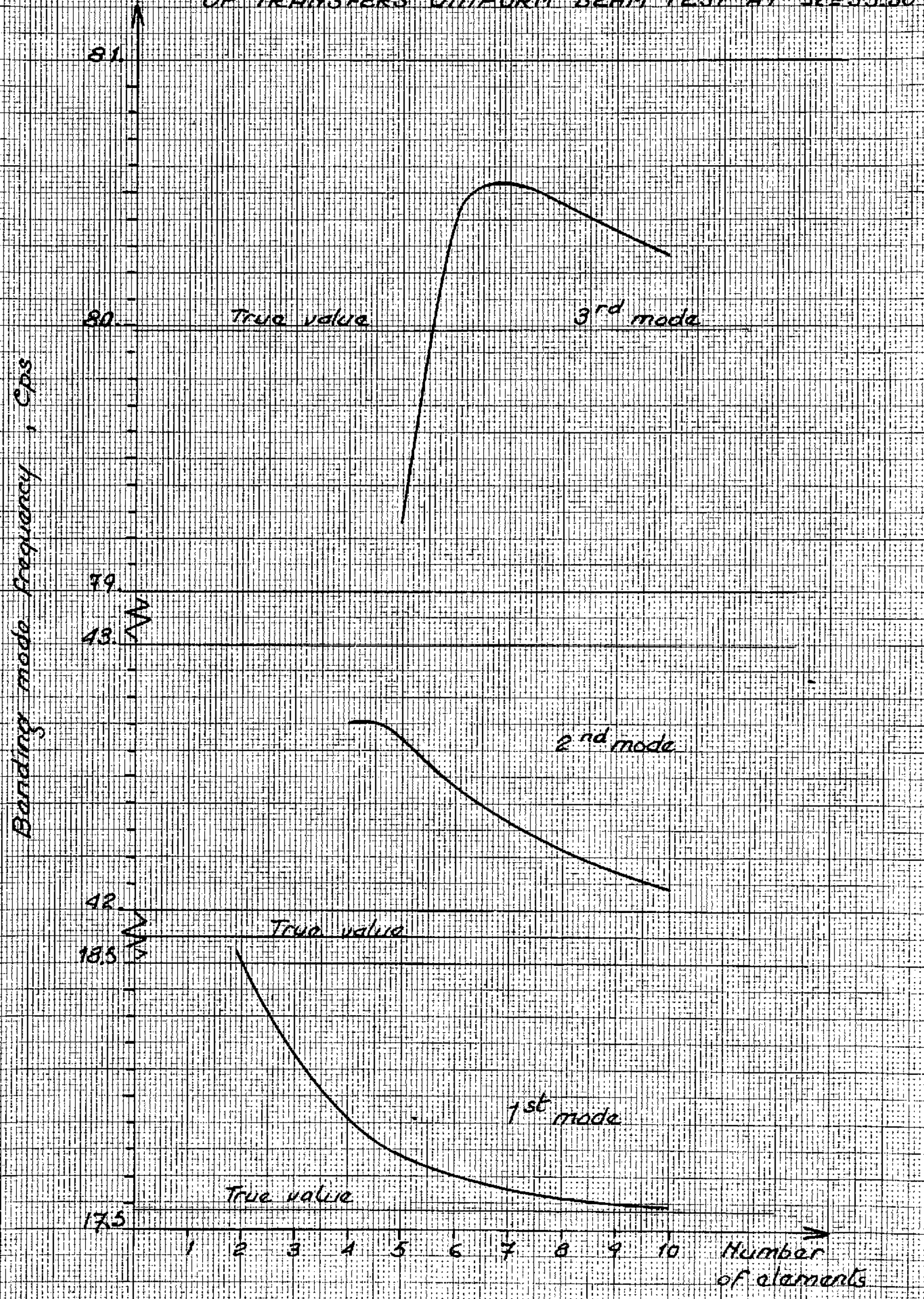


FIG. 7 - DH. 2011 D BLADE MODE FREQUENCIES AS A FUNCTION OF ROTOR R.P.M.

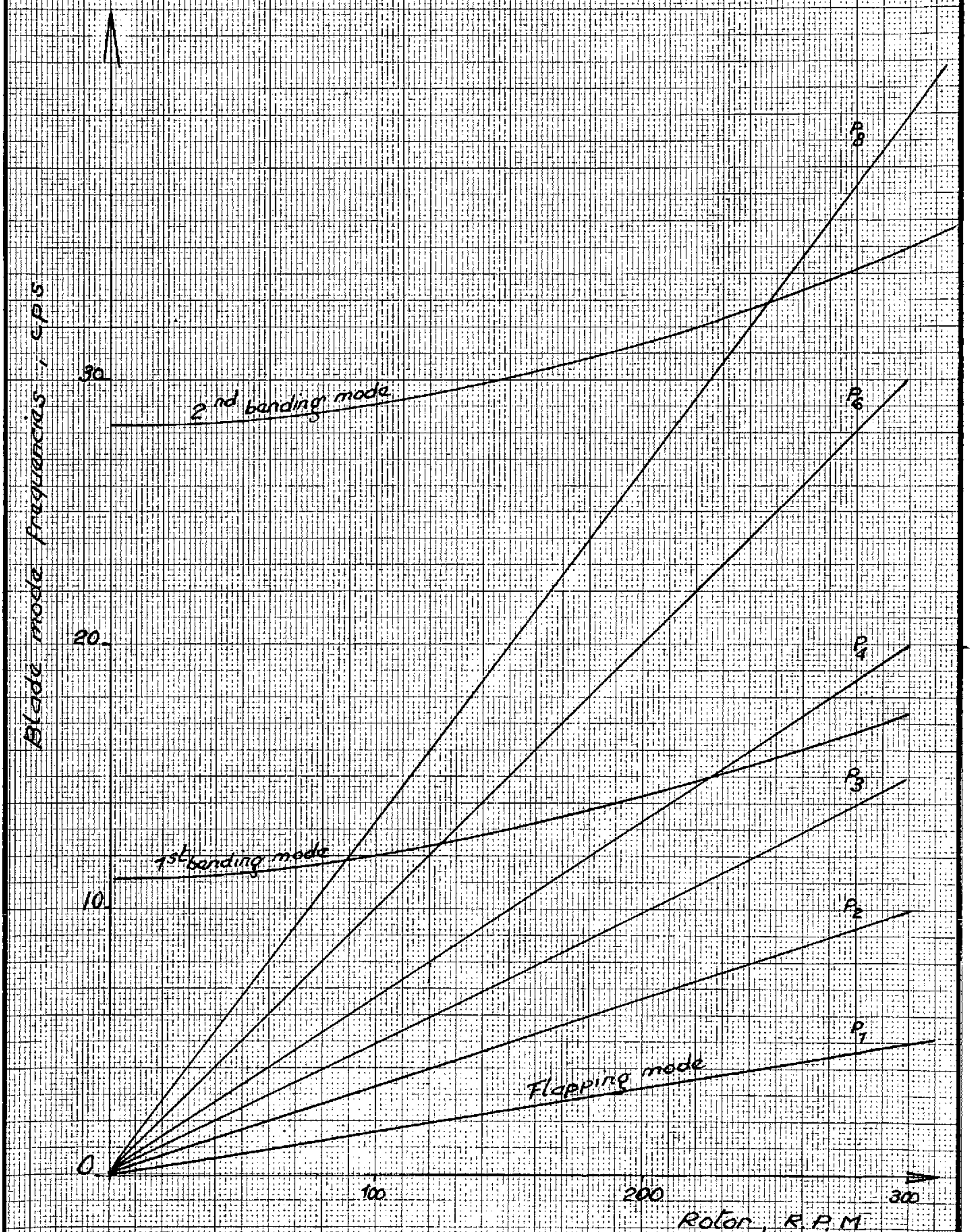
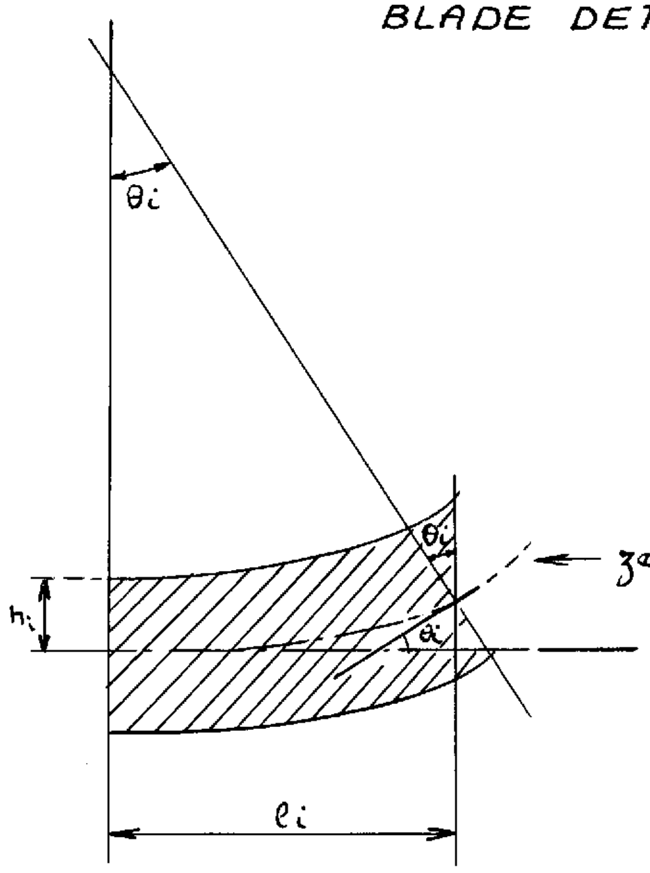
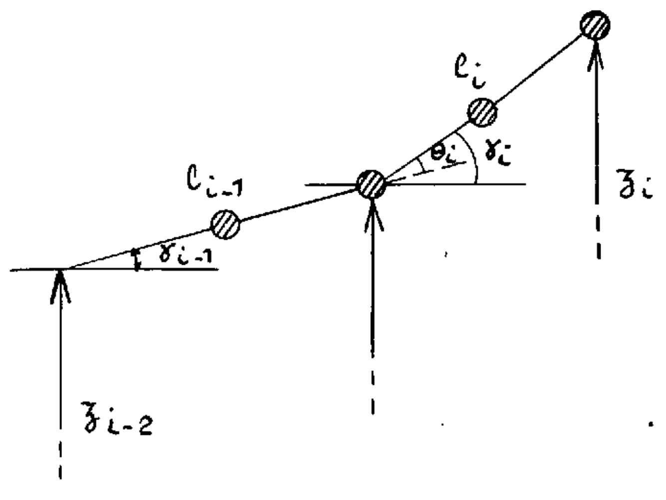


FIG. 8 _ STRESS COMPUTATION FROM BLADE DEFLECTION





```
BLADE UNIFORM BLADE          R= 6.000E 00 RO= 3.150E-01 METRES
E= 2.000E 04 HECTOBARS       NUMBER OF BLADE DATA POINTS  7

*PROGRAM OPTION* COMPUTED VALUES OF BLADE ELEMENTS PARAMETERS

*PROGRAM OPTION* CONSTANT VALUES OF ELEMENTS LENGTHS

BLADE STRUCTURAL DATA
ELEMENT      RAD.DIST    THICKNESS   MASS(KG/M)  SECT.INERT

   1       3.150E-01   6.500E-02   5.000E 01   5.000E-06
   2       1.000E 00   6.500E-02   5.000E 01   5.000E-06
   3       1.500E 00   6.500E-02   5.000E 01   5.000E-06
   4       3.000E 00   6.500E-02   5.000E 01   5.000E-06
   5       4.000E 00   6.500E-02   5.000E 01   5.000E-06
   6       5.000E 00   6.500E-02   5.000E 01   5.000E-06
   7       6.000E 00   6.500E-02   5.000E 01   5.000E-06

RADIUS   6.000E 00 METRES
TOTAL MASS   2.842E 02 KILOGRAMMES
AVERAGED SECTIONAL INERTIA      5.000E-06 METRES**4
UNIFORM BLADE FREQUENCY   6.739E 01 RAD/SEC
```

NUMBER OF BLADE ELEMENTS = 18

PROGRAM OPTICN PRINT THE BLADE ELEMENT PARAMETERS

ELEMENT	LENGTH	RAD. DIST.	SPRING CTE.	CENTRAL MASS	TIP MASS
1	3.158E-01	6.308E-01	0.0	1.053E 01	5.264E 00
2	3.158E-01	9.467E-01	3.166E 06	1.053E 01	5.264E 00
3	3.158E-01	1.262E 00	3.166E 06	1.053E 01	5.264E 00
4	3.158E-01	1.578E 00	3.166E 06	1.053E 01	5.264E 00
5	3.158E-01	1.894E 00	3.166E 06	1.053E 01	5.264E 00
6	3.158E-01	2.210E 00	3.166E 06	1.053E 01	5.264E 00
7	3.158E-01	2.526E 00	3.166E 06	1.053E 01	5.264E 00
8	3.158E-01	2.842E 00	3.166E 06	1.053E 01	5.264E 00
9	3.158E-01	3.157E 00	3.166E 06	1.053E 01	5.264E 00
10	3.158E-01	3.473E 00	3.166E 06	1.053E 01	5.264E 00
11	3.158E-01	3.789E 00	3.166E 06	1.053E 01	5.264E 00
12	3.158E-01	4.105E 00	3.166E 06	1.053E 01	5.264E 00
13	3.158E-01	4.421E 00	3.166E 06	1.053E 01	5.264E 00
14	3.158E-01	4.737E 00	3.166E 06	1.053E 01	5.264E 00
15	3.158E-01	5.052E 00	3.166E 06	1.053E 01	5.264E 00
16	3.158E-01	5.368E 00	3.166E 06	1.053E 01	5.264E 00
17	3.158E-01	5.684E 00	3.166E 06	1.053E 01	5.264E 00
18	3.158E-01	6.000E 00	3.166E 06	1.053E 01	2.632E 00

PROGRAM OPTICN NO PRINT

PROGRAM OPTION EIGEN MODES OF THE BLADE

BLADE EIGENMODES AT OMEGA = 0.0 FOR 18 ELEMENTS

ROTOR FREQUENCY 0.0 CPS ROTOR RPM 0.0

**RESONANCE FREQUENCY OF UNIFORM BLADE (FUB) 10.72 CPS

MODE	EIGENVALUES (RAD/S)**2 REAL	IMAG	EIGENFREQUENCIES	F/FUB
FLAPPING 0	8.645E 01	0.0	1.48	0.14
BENDING 1	4.576E 03	0.0	10.77	1.00
BENDING 2	4.780E 04	0.0	34.80	3.24
BENDING 3	2.C81E 05	0.0	72.60	6.77
BENDING 4	6.C82E 05	0.0	124.12	11.57
BENDING 5	1.415E 06	0.0	189.32	17.65
BENDING 6	2.838E 06	0.0	268.11	25.00
BENDING 7	5.125E 06	0.0	360.29	33.59
BENDING 8	8.552E 06	0.0	465.42	43.40
BENDING 9	1.341E 07	0.0	582.78	54.34
BENDING 10	1.996E 07	0.0	711.09	66.30
BENDING 11	2.840E 07	0.0	848.18	79.08
BENDING 12	3.874E 07	0.0	990.63	92.37
BENDING 13	5.070E 07	0.0	1133.29	105.67
BENDING 14	6.357E 07	0.0	1268.97	118.32
BENDING 15	7.613E 07	0.0	1388.68	129.48
BENDING 16	8.679E 07	0.0	1482.74	138.25
BENDING 17	9.396E 07	0.0	1542.73	143.84

PROGRAM OPTICN EICEN MODES OF THE BLADE

BLADE EIGENMODES AT OMEGA = 33.30 FOR 18 ELEMENTS

ROTOR FREQUENCY 5.30 CPS ROTOR RPM 318.0

**RESONANCE FREQUENCY OF UNIFORM BLADE (FUB) 10.72 CPS

MODE	EIGENVALUES (RAD/S)**2 REAL	IMAG	EIGENFREQUENCIES	F/FUB
FLAPPING 0	1.260E 03	0.0	5.65	0.53
BENDING 1	1.220E 04	0.0	17.58	1.64
BENDING 2	6.934E 04	0.0	41.91	3.91
BENDING 3	2.521E 05	0.0	79.92	7.45
BENDING 4	6.841E 05	0.0	131.64	12.27
BENDING 5	1.533E 06	0.0	197.05	18.37
BENDING 6	3.009E 06	0.0	276.09	25.74
BENDING 7	5.362E 06	0.0	368.54	34.36
BENDING 8	8.870E 06	0.0	473.99	44.20
BENDING 9	1.382E 07	0.0	591.74	55.17
BENDING 10	2.049E 07	0.0	720.44	67.17
BENDING 11	2.906E 07	0.0	857.98	80.00
BENDING 12	3.955E 07	0.0	1000.89	93.32
BENDING 13	5.167E 07	0.0	1144.00	106.67
BENDING 14	6.469E 07	0.0	1280.08	119.36
BENDING 15	7.739E 07	0.0	1400.12	130.55
BENDING 16	8.817E 07	0.0	1494.46	139.34
BENDING 17	9.553E 07	0.0	1555.58	145.04

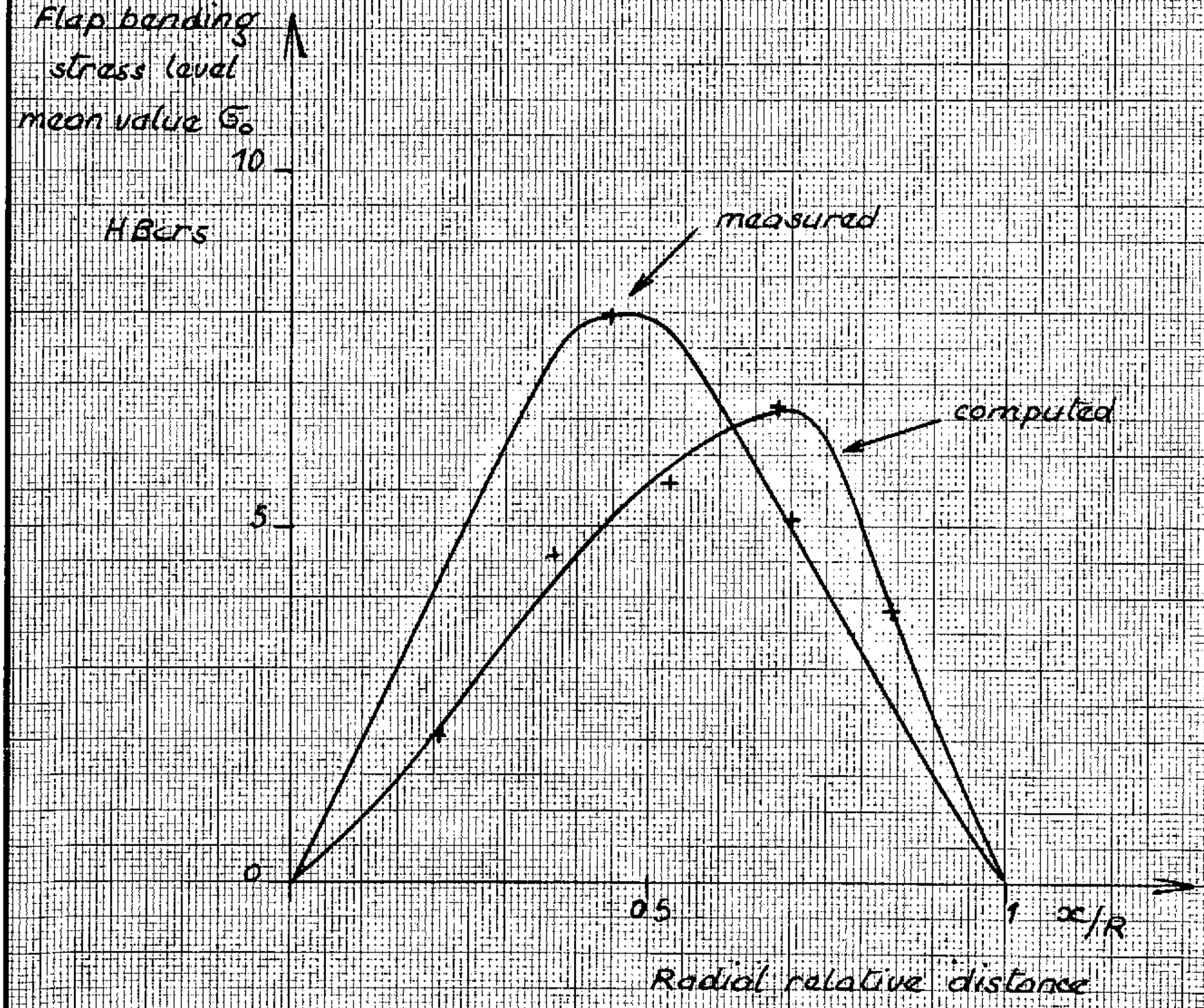
PROGRAM OPTION CONTINUE

PROGRAM OPTION END OF THE COMPUTATIONS

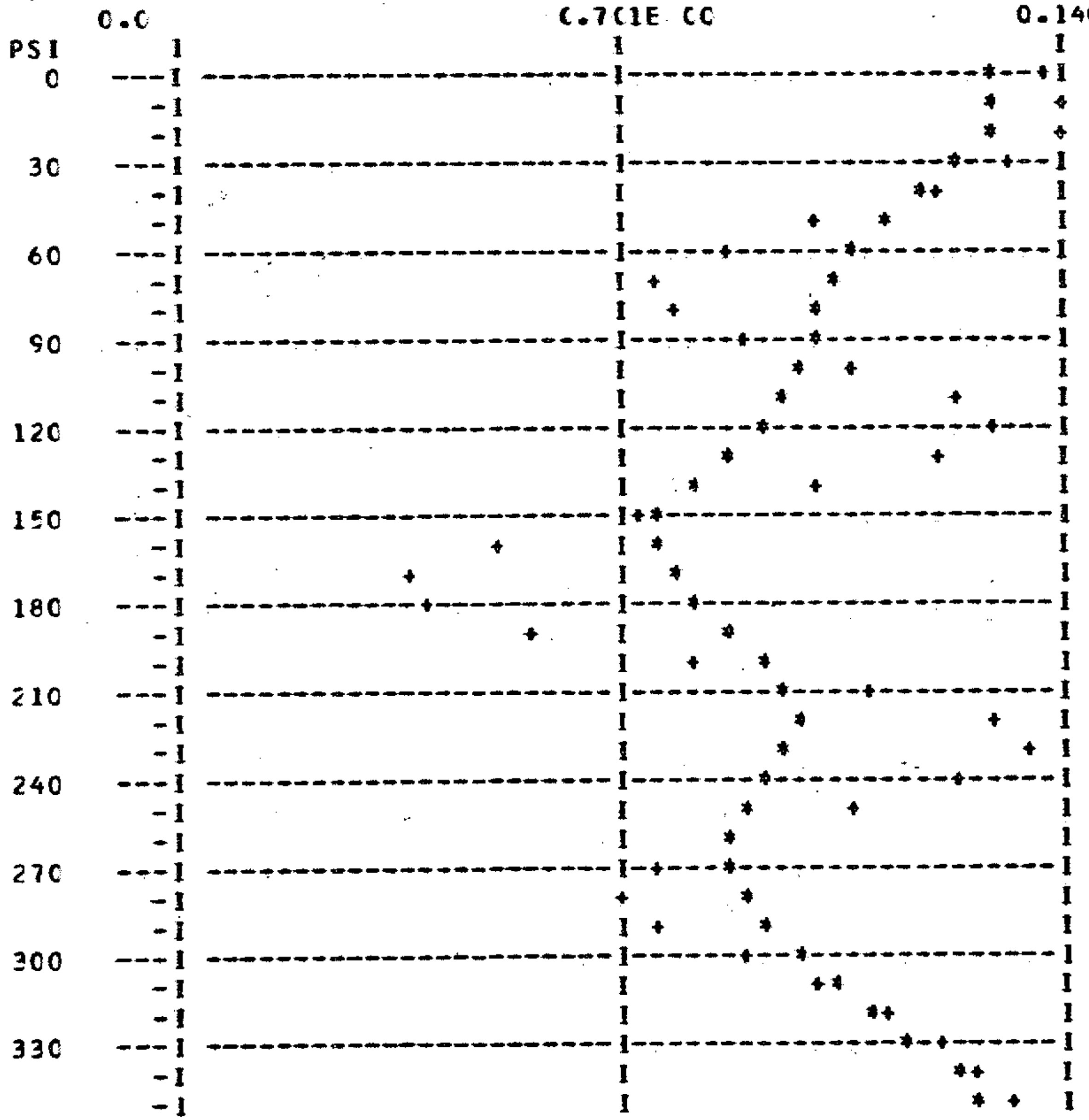
FIG. 13 _ COMPARISON OF FLAP BENDING STRESS DISTRIBUTION BETWEEN COMPUTED AND MEASURED CASES _

RUN PT = 9.03

(SIM 11 FOR COMPUTATION)



RUN 9.03
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R

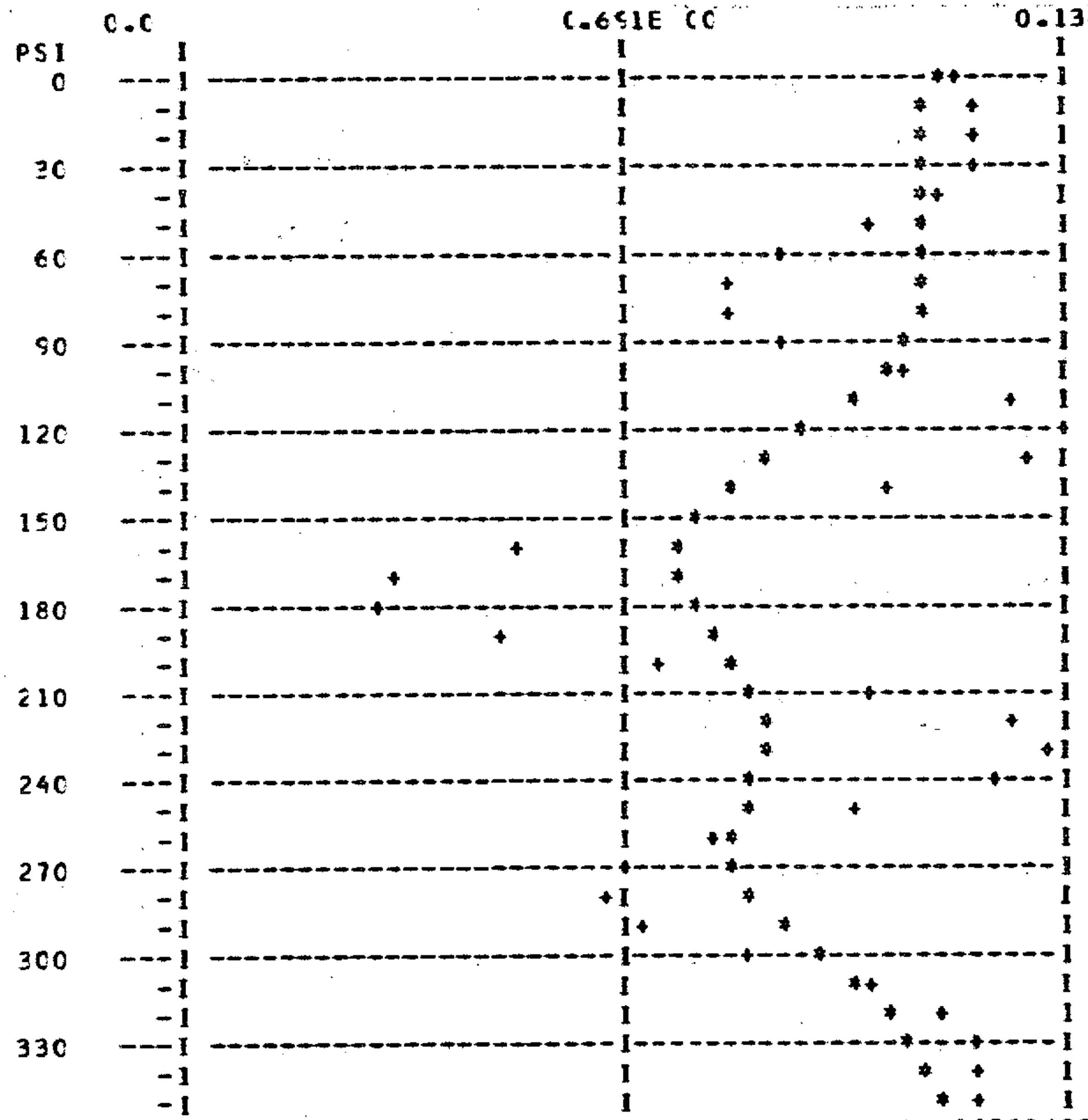


COMPUTED	MEASURED
0.130E 01	0.137E 01
0.130E 01	0.140E 01
0.128E 01	0.139E 01
0.123E 01	0.132E 01
0.117E 01	0.119E 01
0.111E 01	0.102E 01
0.106E 01	0.859E 00
0.103E 01	0.766E 00
0.101E 01	0.777E 00
0.100E 01	0.893E 00
0.986E 00	0.107E 01
0.957E 00	0.122E 01
0.913E 00	0.128E 01
0.857E 00	0.121E 01
0.803E 00	0.997E 00
0.766E 00	0.718E 00
0.756E 00	0.464E 00
0.777E 00	0.326E 00
0.824E 00	0.355E 00
0.881E 00	0.543E 00
0.934E 00	0.825E 00
0.967E 00	0.111E 01
0.974E 00	0.130E 01
0.958E 00	0.134E 01
0.928E 00	0.125E 01
0.897E 00	0.107E 01
0.876E 00	0.875E 00
0.873E 00	0.743E 00
0.891E 00	0.708E 00
0.927E 00	0.767E 00
0.977E 00	0.885E 00
0.104E 01	0.102E 01
0.110E 01	0.113E 01
0.116E 01	0.122E 01
0.122E 01	0.127E 01
0.127E 01	0.132E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.790HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.930HECTOBARS

FIG. 14

RUN 9.04
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R

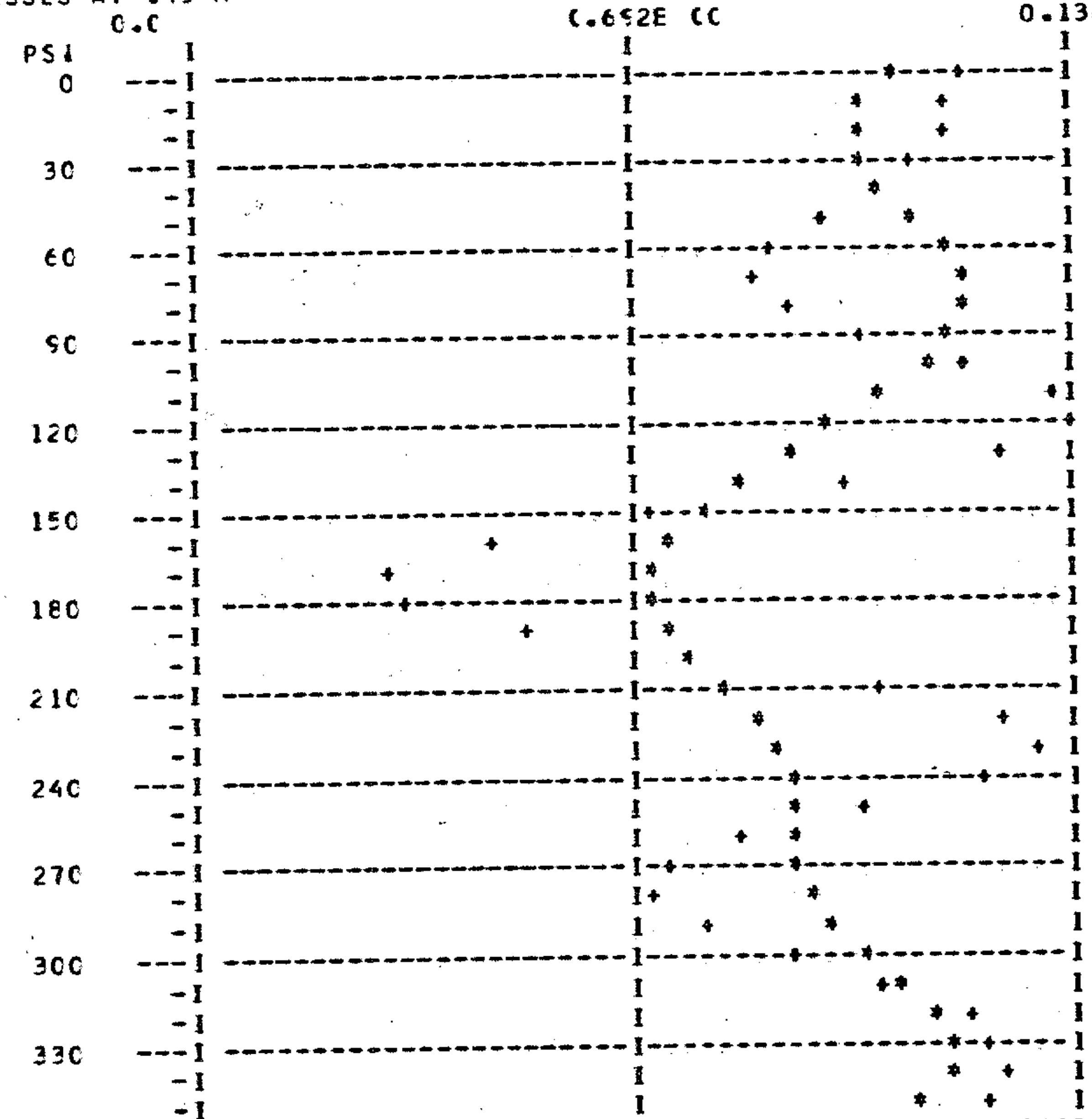


COMPUTED	MEASURED
0.118E 01	0.122E 01
0.117E 01	0.123E 01
0.116E 01	0.125E 01
0.116E 01	0.124E 01
0.116E 01	0.118E 01
0.116E 01	0.107E 01
0.116E 01	0.944E 00
0.116E 01	0.855E 00
0.115E 01	0.852E 00
0.113E 01	0.951E 00
0.109E 01	0.112E 01
0.104E 01	0.129E 01
0.979E 00	0.138E 01
0.914E 00	0.133E 01
0.854E 00	0.112E 01
0.807E 00	0.807E 00
0.780E 00	0.501E 00
0.775E 00	0.304E 00
0.792E 00	0.288E 00
0.822E 00	0.460E 00
0.858E 00	0.758E 00
0.888E 00	0.108E 01
0.906E 00	0.130E 01
0.909E 00	0.137E 01
0.898E 00	0.127E 01
0.882E 00	0.109E 01
0.869E 00	0.826E 00
0.871E 00	0.669E 00
0.891E 00	0.638E 00
0.930E 00	0.729E 00
0.985E 00	0.896E 00
0.104E 01	0.107E 01
0.110E 01	0.120E 01
0.115E 01	0.125E 01
0.117E 01	0.125E 01
0.118E 01	0.123E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 5.114HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 8.250HECTOBARS

FIG. 15-

RUN 9.05
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



COMPUTED	MEASURED
0.110E 01	0.121E 01
0.106E 01	0.120E 01
0.104E 01	0.118E 01
0.105E 01	0.114E 01
0.108E 01	0.107E 01
0.113E 01	0.986E 00
0.116E 01	0.909E 00
0.122E 01	0.882E 00
0.122E 01	0.934E 00
0.120E 01	0.106E 01
0.115E 01	0.122E 01
0.108E 01	0.135E 01
0.101E 01	0.138E 01
0.928E 00	0.127E 01
0.856E 00	0.103E 01
0.795E 00	0.724E 00
0.749E 00	0.446E 00
0.724E 00	0.289E 00
0.721E 00	0.308E 00
0.741E 00	0.495E 00
0.779E 00	0.786E 00
0.826E 00	0.108E 01
0.873E 00	0.128E 01
0.909E 00	0.134E 01
0.929E 00	0.125E 01
0.935E 00	0.106E 01
0.935E 00	0.866E 00
0.941E 00	0.735E 00
0.963E 00	0.713E 00
0.100E 01	0.795E 00
0.104E 01	0.940E 00
0.112E 01	0.109E 01
0.117E 01	0.120E 01
0.119E 01	0.126E 01
0.118E 01	0.126E 01
0.115E 01	0.124E 01

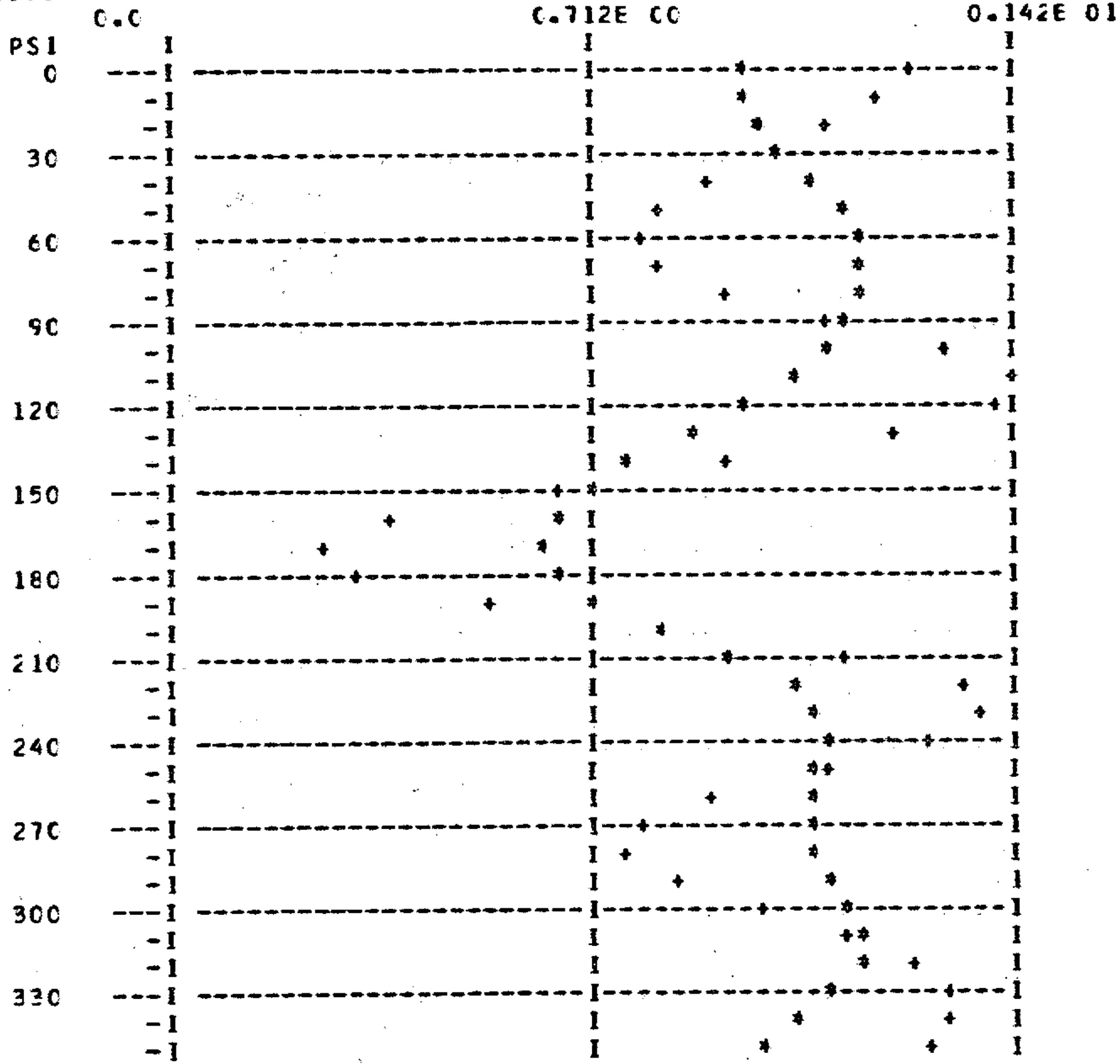
COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.804HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.970HECTOBARS

GIRAVIONS DORAND

FIG. 15

Doc^t : DH-2011 C - E 1
Page : 79

RUN 9.06
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



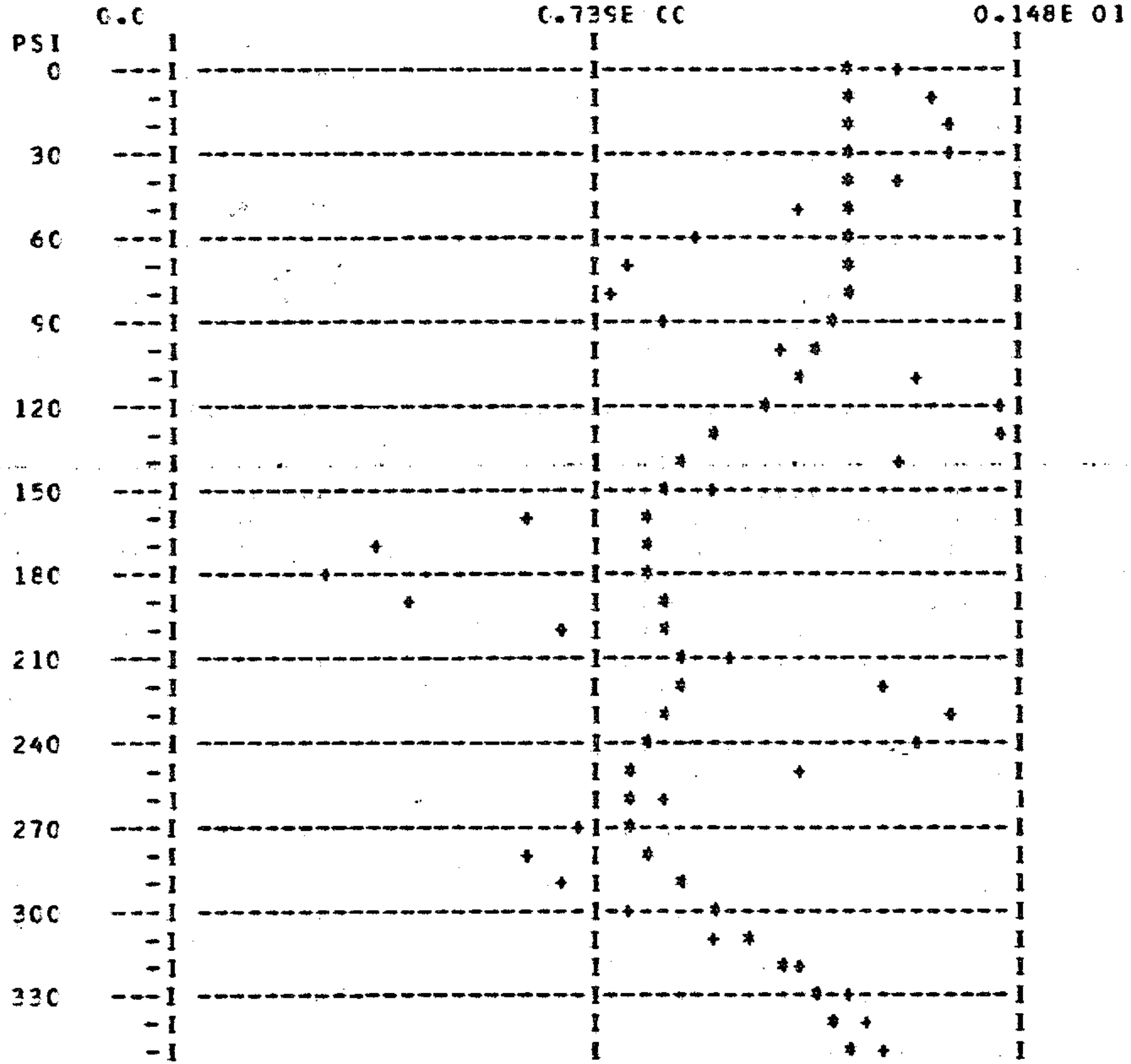
COMPUTED	MEASURED
0.969E 00	0.124E 01
0.963E 00	0.118E 01
0.987E 00	0.111E 01
0.103E 01	0.102E 01
0.109E 01	0.924E 00
0.113E 01	0.835E 00
0.116E 01	0.791E 00
0.117E 01	0.823E 00
0.117E 01	0.939E 00
0.115E 01	0.112E 01
0.111E 01	0.130E 01
0.106E 01	0.141E 01
0.978E 00	0.140E 01
0.881E 00	0.123E 01
0.777E 00	0.948E 00
0.682E 00	0.621E 00
0.617E 00	0.352E 00
0.601E 00	0.228E 00
0.640E 00	0.290E 00
0.726E 00	0.515E 00
0.839E 00	0.830E 00
0.952E 00	0.113E 01
0.104E 01	0.133E 01
0.109E 01	0.138E 01
0.110E 01	0.128E 01
0.109E 01	0.110E 01
0.107E 01	0.915E 00
0.107E 01	0.794E 00
0.109E 01	0.777E 00
0.112E 01	0.858E 00
0.115E 01	0.998E 00
0.117E 01	0.115E 01
0.116E 01	0.126E 01
0.112E 01	0.131E 01
0.106E 01	0.132E 01
0.101E 01	0.129E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.448HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.590HECTOBARS

GIRAVIONS DORAND

FIG. 17-

RUN 12.10
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R

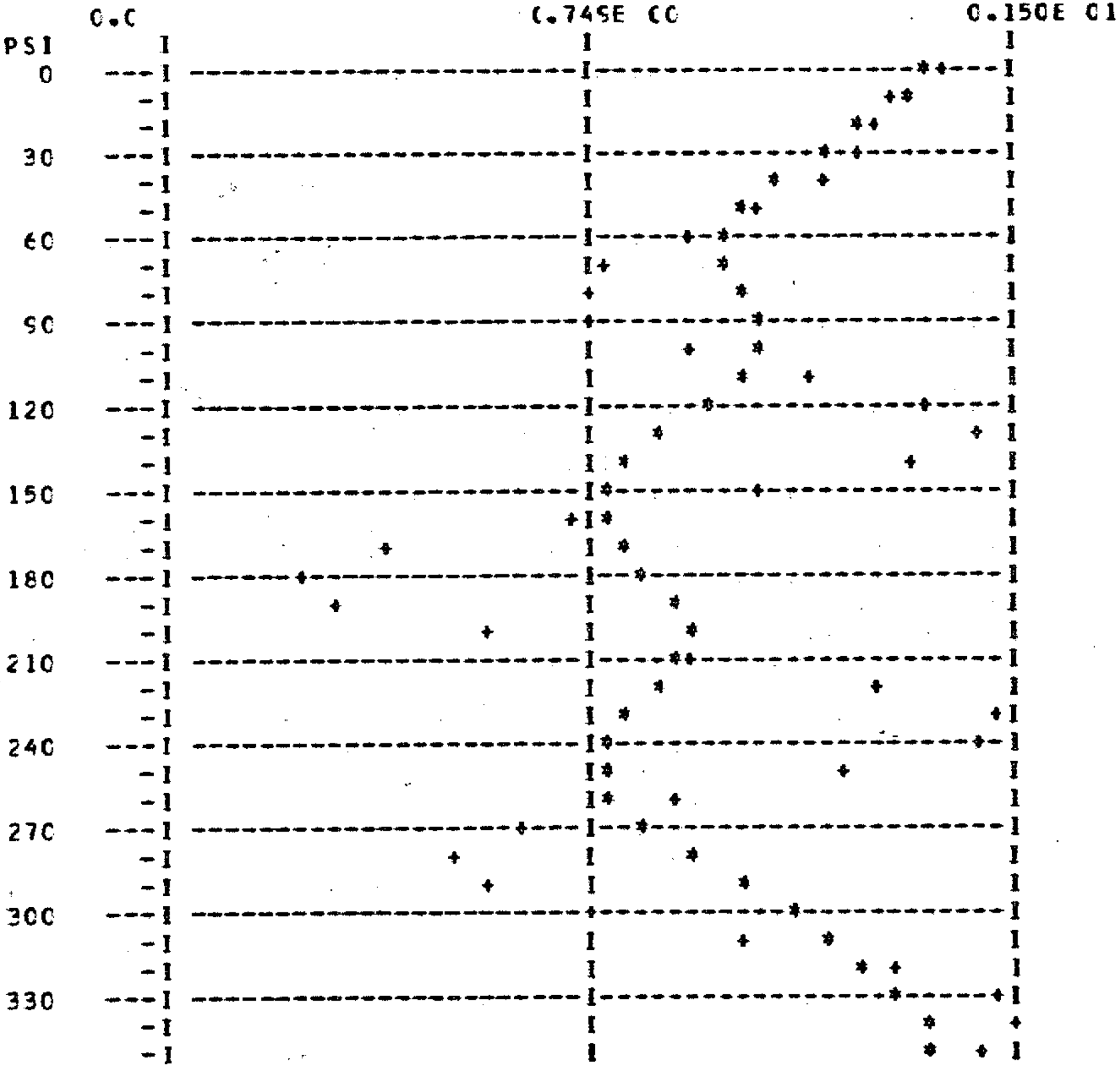


PSI	COMPUTED	MEASURED
0	0.119E 01	0.128E 01
	0.119E 01	0.133E 01
	0.118E 01	0.136E 01
30	0.118E 01	0.135E 01
	0.118E 01	0.126E 01
	0.118E 01	0.110E 01
60	0.118E 01	0.927E 00
	0.119E 01	0.793E 00
	0.118E 01	0.764E 00
90	0.116E 01	0.868E 00
	0.113E 01	0.107E 01
	0.108E 01	0.130E 01
120	0.102E 01	0.146E 01
	0.955E 00	0.145E 01
	0.896E 00	0.127E 01
150	0.850E 00	0.947E 00
	0.824E 00	0.595E 00
	0.819E 00	0.331E 00
180	0.830E 00	0.248E 00
	0.850E 00	0.371E 00
	0.868E 00	0.691E 00
210	0.877E 00	0.983E 00
	0.873E 00	0.125E 01
	0.856E 00	0.136E 01
240	0.832E 00	0.129E 01
	0.811E 00	0.110E 01
	0.800E 00	0.861E 00
270	0.807E 00	0.674E 00
	0.835E 00	0.601E 00
	0.882E 00	0.652E 00
300	0.943E 00	0.790E 00
	0.101E 01	0.956E 00
	0.107E 01	0.109E 01
330	0.112E 01	0.118E 01
	0.116E 01	0.122E 01
	0.118E 01	0.125E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.604HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.770HECTOBARS

FIG.18

UN 12.11
OMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
TRESSES AT .45*R



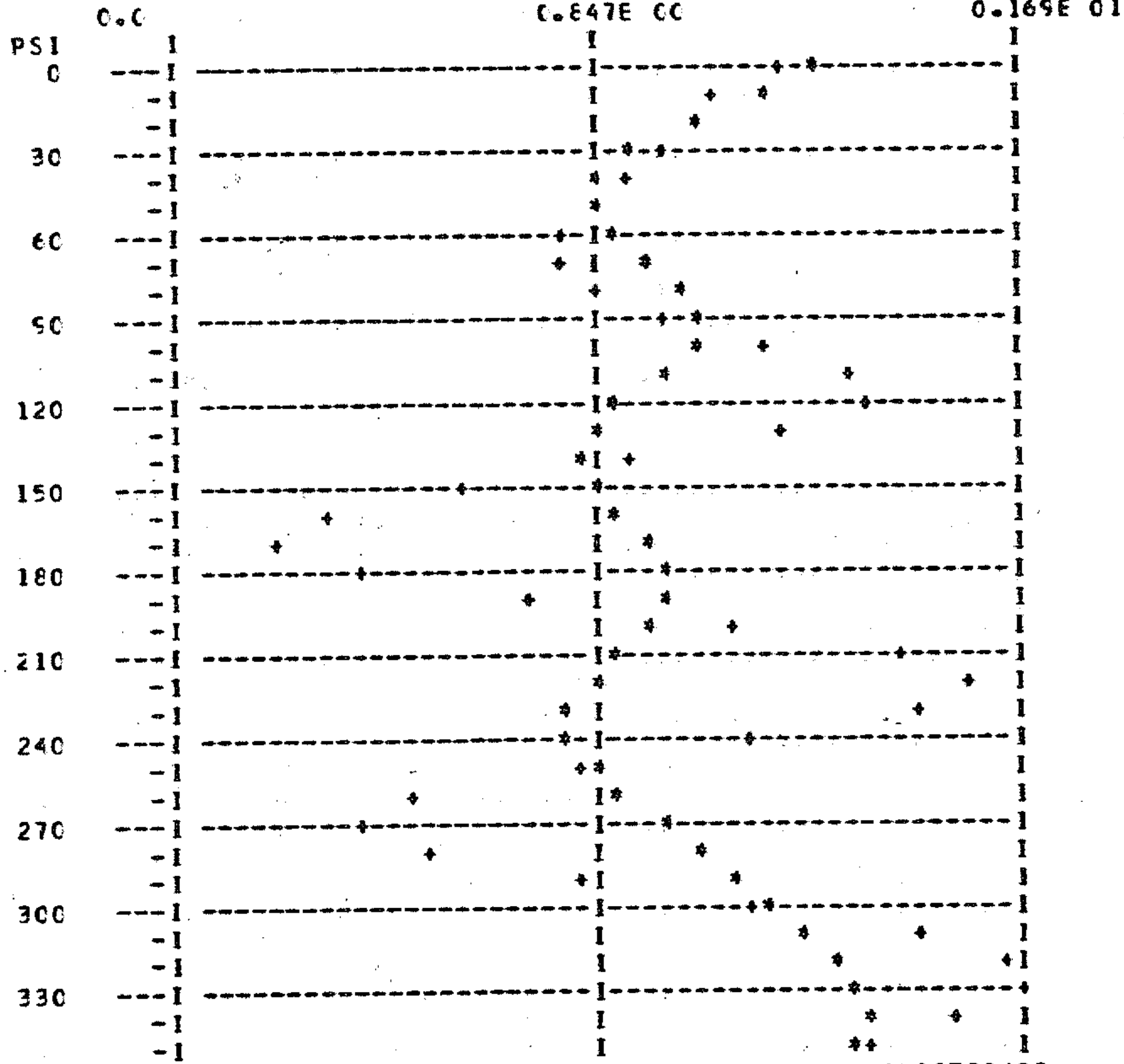
COMPUTED	MEASURED
0.135E 01	0.137E 01
0.131E 01	0.130E 01
0.124E 01	0.125E 01
0.116E 01	0.122E 01
0.107E 01	0.116E 01
0.101E 01	0.106E 01
0.984E 00	0.914E 00
0.988E 00	0.775E 00
0.101E 01	0.765E 00
0.103E 01	0.750E 00
0.104E 01	0.915E 00
0.101E 01	0.115E 01
0.944E 00	0.135E 01
0.870E 00	0.143E 01
0.803E 00	0.132E 01
0.766E 00	0.104E 01
0.767E 00	0.678E 00
0.801E 00	0.356E 00
0.851E 00	0.198E 00
0.895E 00	0.268E 00
0.914E 00	0.543E 00
0.900E 00	0.924E 00
0.861E 00	0.127E 01
0.812E 00	0.146E 01
0.775E 00	0.143E 01
0.767E 00	0.120E 01
0.793E 00	0.888E 00
0.851E 00	0.610E 00
0.929E 00	0.476E 00
0.101E 01	0.530E 00
0.110E 01	0.743E 00
0.117E 01	0.103E 01
0.124E 01	0.129E 01
0.129E 01	0.145E 01
0.133E 01	0.150E 01
0.135E 01	0.145E 01

OMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.592HECTOBARS
EASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.650HECTOBARS

GIRAVIONS DORAND

FIG.19 -

RUN 12.12
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



COMPUTED	MEASURED
0.129E 01	0.121E 01
0.118E 01	0.110E 01
0.105E 01	0.104E 01
0.928E 00	0.986E 00
0.855E 00	0.919E 00
0.843E 00	0.831E 00
0.888E 00	0.793E 00
0.962E 00	0.733E 00
0.103E 01	0.810E 00
0.106E 01	0.981E 00
0.104E 01	0.119E 01
0.965E 00	0.135E 01
0.885E 00	0.138E 01
0.817E 00	0.122E 01
0.793E 00	0.909E 00
0.819E 00	0.546E 00
0.881E 00	0.264E 00
0.948E 00	0.179E 00
0.991E 00	0.338E 00
0.989E 00	0.694E 00
0.942E 00	0.112E 01
0.869E 00	0.147E 01
0.797E 00	0.160E 01
0.756E 00	0.148E 01
0.759E 00	0.116E 01
0.807E 00	0.765E 00
0.884E 00	0.453E 00
0.971E 00	0.341E 00
0.105E 01	0.467E 00
0.112E 01	0.779E 00
0.119E 01	0.116E 01
0.125E 01	0.149E 01
0.130E 01	0.167E 01
0.135E 01	0.168E 01
0.138E 01	0.155E 01
0.136E 01	0.138E 01

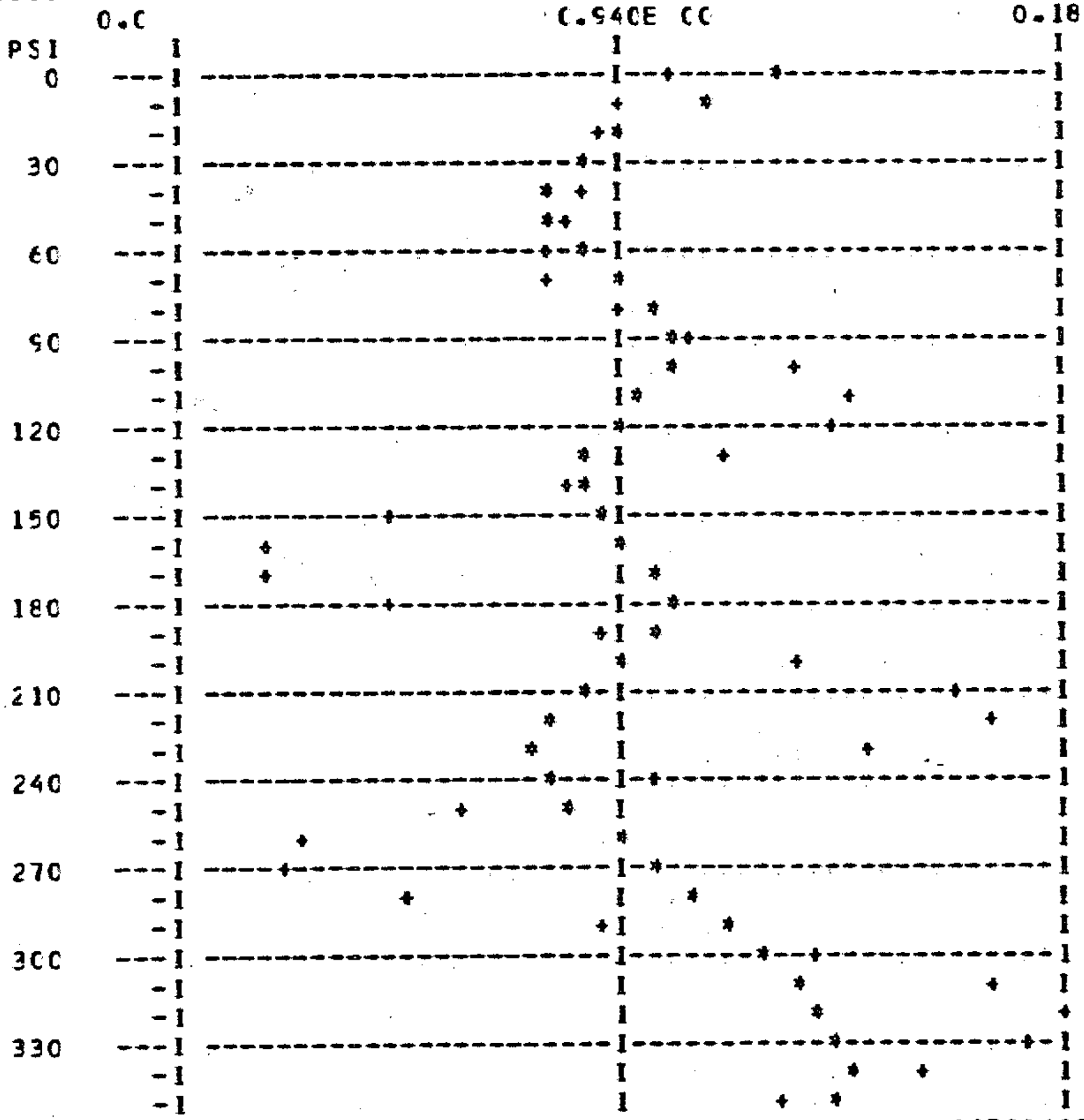
COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.792HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.510HECTOBARS

FIG.20 -

GIRAVIONS DORAND

FIG. 21.

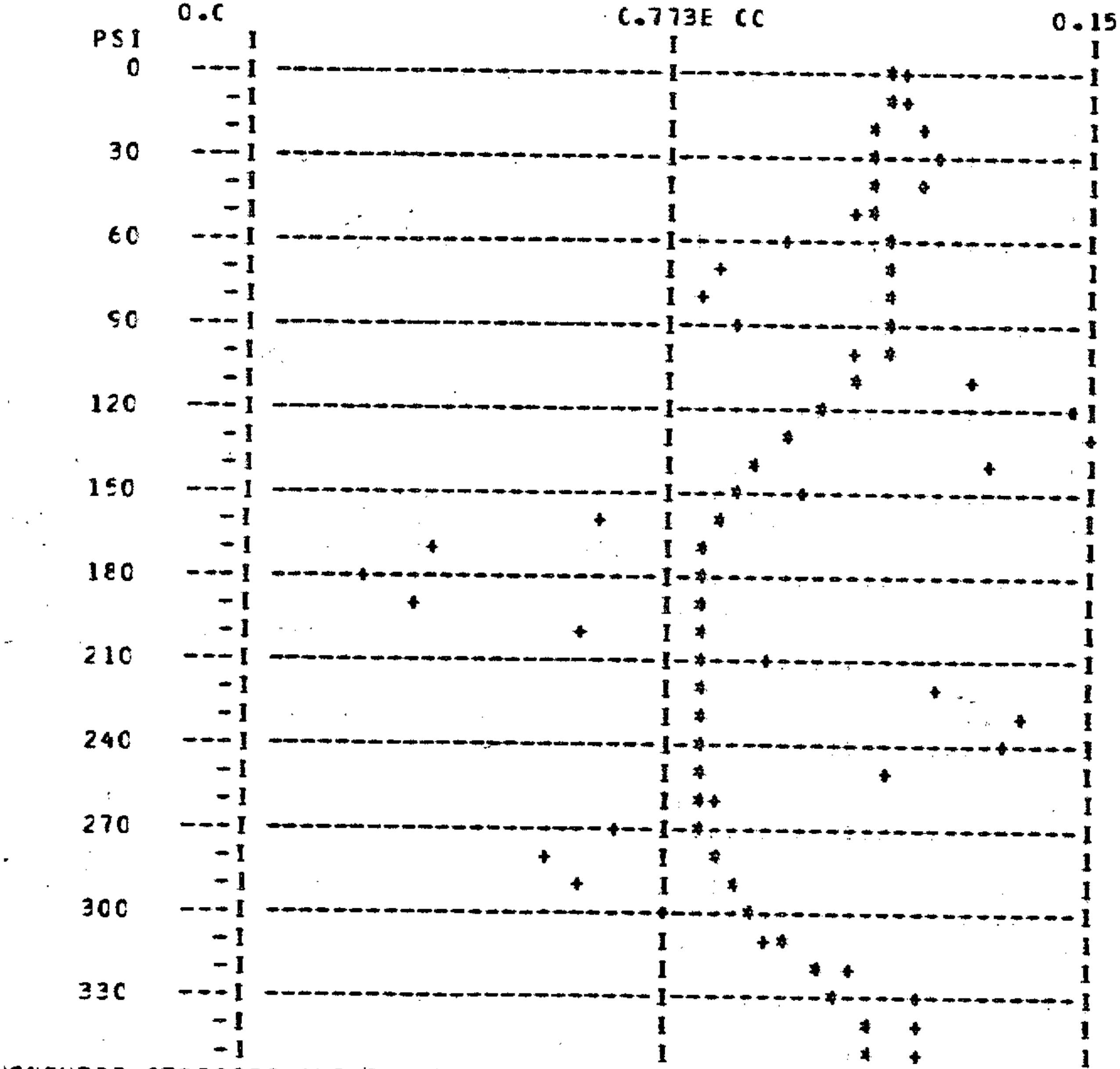
RUN 12.13
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



COMPUTED	MEASURED
0.128E 01	0.106E 01
0.113E 01	0.924E 00
0.956E 00	0.866E 00
0.817E 00	0.845E 00
0.740E 00	0.819E 00
0.744E 00	0.775E 00
0.817E 00	0.742E 00
0.924E 00	0.767E 00
0.102E 01	0.885E 00
0.107E 01	0.108E 01
0.105E 01	0.130E 01
0.978E 00	0.144E 01
0.893E 00	0.140E 01
0.831E 00	0.117E 01
0.819E 00	0.863E 00
0.862E 00	0.412E 00
0.937E 00	0.154E 00
0.101E 01	0.141E 00
0.104E 01	0.401E 00
0.102E 01	0.850E 00
0.943E 00	0.133E 01
0.844E 00	0.166E 01
0.759E 00	0.171E 01
0.718E 00	0.147E 01
0.736E 00	0.103E 01
0.804E 00	0.559E 00
0.902E 00	0.238E 00
0.101E 01	0.194E 00
0.110E 01	0.440E 00
0.117E 01	0.882E 00
0.124E 01	0.136E 01
0.130E 01	0.173E 01
0.136E 01	0.188E 01
0.141E 01	0.180E 01
0.142E 01	0.157E 01
0.138E 01	0.130E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.768HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.540HECTOBARS

RUN 14.10
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



COMPUTED	MEASURED
0.117E 01	0.121E 01
0.116E 01	0.122E 01
0.115E 01	0.129E 01
0.114E 01	0.126E 01
0.114E 01	0.123E 01
0.115E 01	0.113E 01
0.116E 01	0.985E 00
0.118E 01	0.863E 00
0.119E 01	0.829E 00
0.119E 01	0.911E 00
0.116E 01	0.111E 01
0.112E 01	0.134E 01
0.106E 01	0.151E 01
0.999E 00	0.153E 01
0.938E 00	0.135E 01
0.889E 00	0.102E 01
0.855E 00	0.624E 00
0.836E 00	0.306E 00
0.831E 00	0.175E 00
0.834E 00	0.277E 00
0.839E 00	0.572E 00
0.842E 00	0.549E 00
0.843E 00	0.127E 01
0.840E 00	0.143E 01
0.835E 00	0.138E 01
0.832E 00	0.116E 01
0.834E 00	0.876E 00
0.844E 00	0.636E 00
0.865E 00	0.528E 00
0.897E 00	0.575E 00
0.940E 00	0.741E 00
0.991E 00	0.950E 00
0.104E 01	0.113E 01
0.109E 01	0.122E 01
0.113E 01	0.125E 01
0.116E 01	0.123E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 5.020HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 8.110HECTOBARS

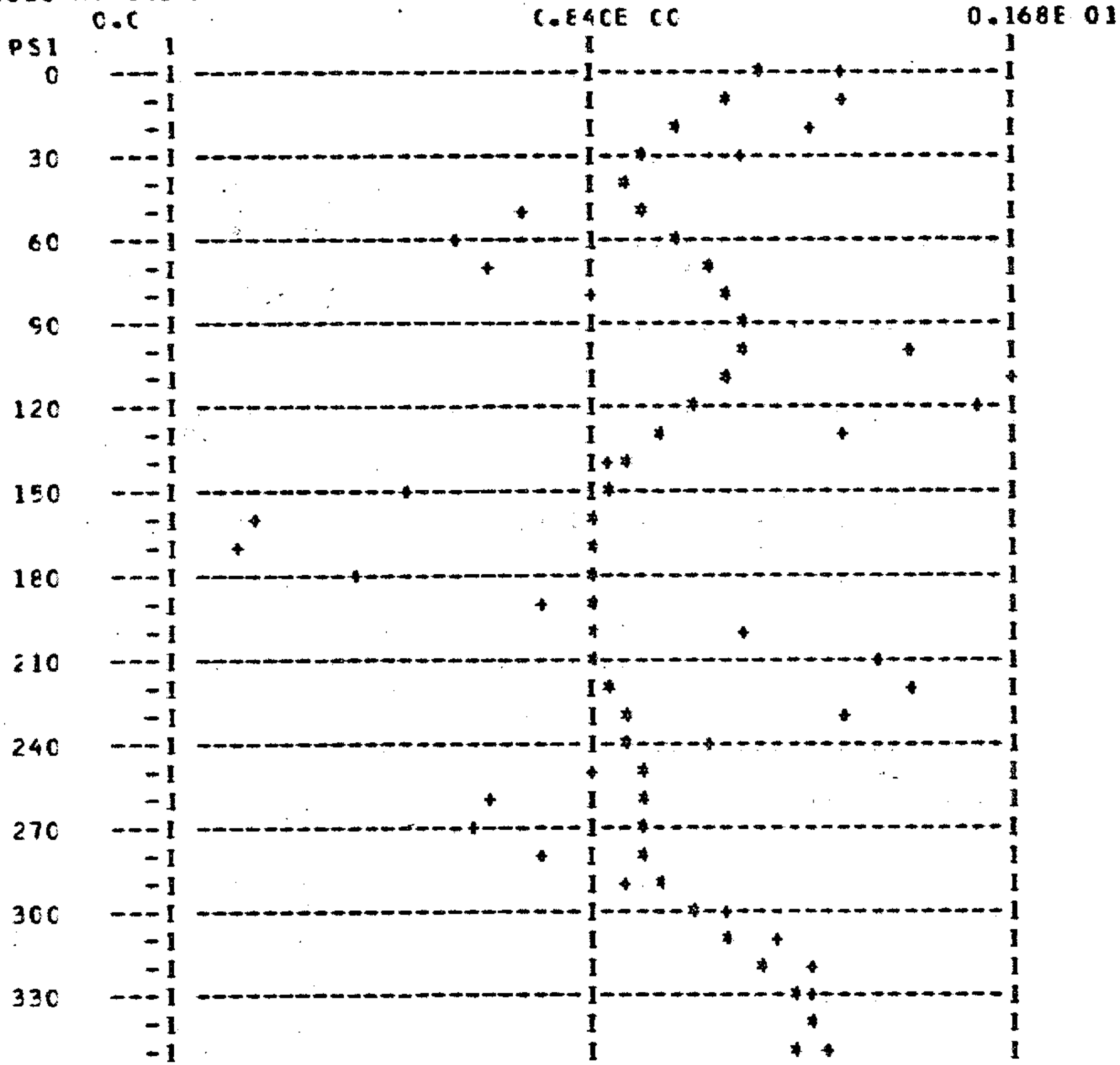
GIRAVIONS DORAND

FIG. 22

RUN 14.11

COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)

STRESSES AT .45*R



PSI	COMPUTED	MEASURED
0	0.119E 01	0.133E 01
	0.110E 01	0.134E 01
	0.100E 01	0.129E 01
30	0.937E 00	0.113E 01
	0.913E 00	0.909E 00
	0.936E 00	0.680E 00
60	0.993E 00	0.551E 00
	0.106E 01	0.592E 00
	0.112E 01	0.815E 00
90	0.115E 01	0.115E 01
	0.114E 01	0.149E 01
	0.110E 01	0.167E 01
120	0.104E 01	0.162E 01
	0.978E 00	0.133E 01
	0.924E 00	0.884E 00
150	0.884E 00	0.434E 00
	0.857E 00	0.139E 00
	0.839E 00	0.995E-01
180	0.829E 00	0.322E 00
	0.825E 00	0.715E 00
	0.831E 00	0.113E 01
210	0.846E 00	0.142E 01
	0.870E 00	0.149E 01
	0.897E 00	0.135E 01
240	0.921E 00	0.108E 01
	0.936E 00	0.792E 00
	0.941E 00	0.609E 00
270	0.942E 00	0.585E 00
	0.951E 00	0.705E 00
	0.979E 00	0.903E 00
300	0.103E 01	0.110E 01
	0.110E 01	0.122E 01
	0.118E 01	0.128E 01
330	0.124E 01	0.128E 01
	0.127E 01	0.127E 01
	0.125E 01	0.129E 01

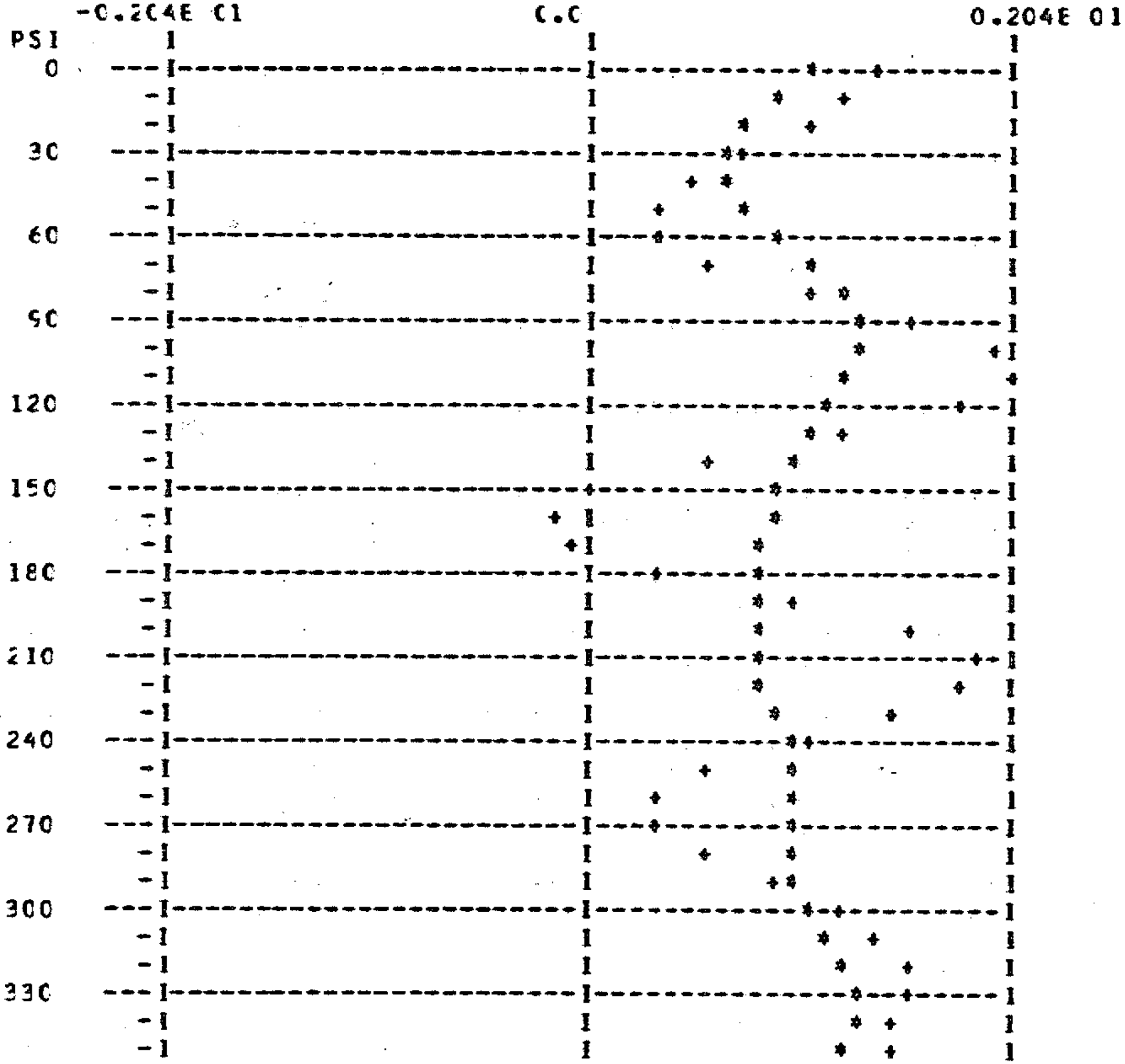
COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 5.064HECTOBARS

MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.830HECTOBARS

GIRAVIONS DORAND

FIG. 23

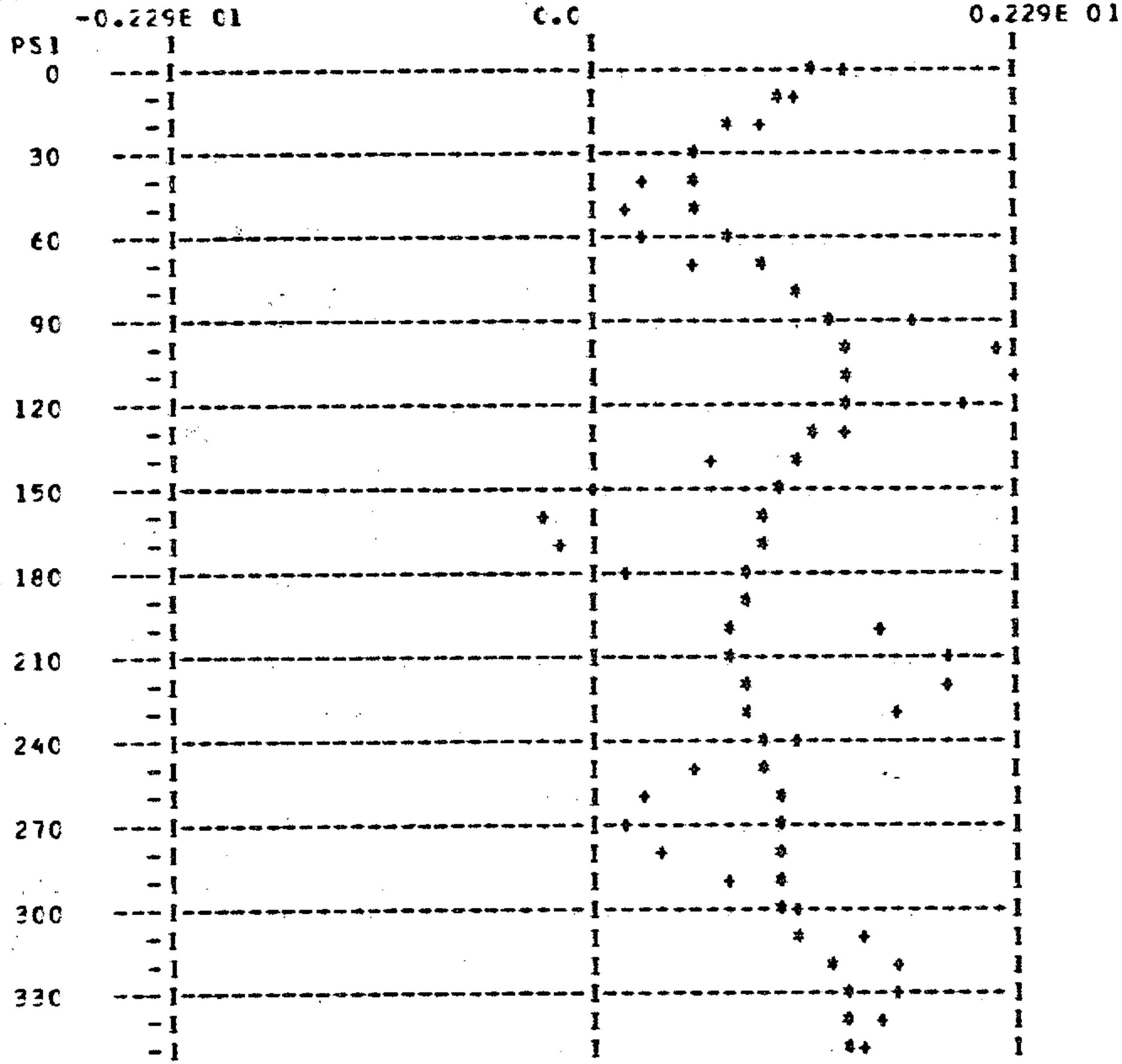
RUN 14.12
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



PSI	COMPUTED	MEASURED
0	0.110E 01	0.136E 01
	0.920E 00	0.123E 01
	0.760E 00	0.103E 01
30	0.665E 00	0.758E 00
	0.664E 00	0.479E 00
	0.759E 00	0.297E 00
60	0.910E 00	0.313E 00
	0.108E 01	0.571E 00
	0.123E 01	0.103E 01
90	0.131E 01	0.154E 01
	0.132E 01	0.194E 01
	0.126E 01	0.204E 01
120	0.118E 01	0.179E 01
	0.108E 01	0.124E 01
	0.998E 00	0.569E 00
150	0.939E 00	-0.578E-02
	0.887E 00	-0.274E 00
	0.849E 00	-0.150E 00
180	0.818E 00	0.319E 00
	0.798E 00	0.947E 00
	0.794E 00	0.152E 01
210	0.812E 00	0.184E 01
	0.852E 00	0.182E 01
	0.902E 00	0.150E 01
240	0.947E 00	0.103E 01
	0.974E 00	0.593E 00
	0.978E 00	0.343E 00
270	0.970E 00	0.343E 00
	0.967E 00	0.597E 00
	0.990E 00	0.801E 00
300	0.109E 01	0.120E 01
	0.114E 01	0.142E 01
	0.124E 01	0.151E 01
330	0.131E 01	0.152E 01
	0.132E 01	0.148E 01
	0.124E 01	0.143E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 5.020HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.750HECTOBARS

RUN 14.12
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EPC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



PSI	COMPUTED	MEASURED
0	0.119E 01	0.135E 01
	0.979E 00	0.114E 01
	0.756E 00	0.873E 00
30	0.582E 00	0.565E 00
	0.508E 00	0.289E 00
	0.553E 00	0.150E 00
60	0.702E 00	0.238E 00
	0.912E 00	0.587E 00
	0.112E 01	0.114E 01
90	0.129E 01	0.173E 01
	0.138E 01	0.217E 01
	0.138E 01	0.229E 01
120	0.133E 01	0.200E 01
	0.123E 01	0.138E 01
	0.113E 01	0.607E 00
150	0.104E 01	-0.584E-01
	0.999E 00	-0.400E 00
	0.889E 00	-0.307E 00
180	0.826E 00	0.174E 00
	0.779E 00	0.664E 00
	0.793E 00	0.152E 01
210	0.756E 00	0.191E 01
	0.790E 00	0.194E 01
	0.846E 00	0.162E 01
240	0.908E 00	0.110E 01
	0.957E 00	0.594E 00
	0.981E 00	0.264E 00
270	0.983E 00	0.206E 00
	0.979E 00	0.403E 00
	0.999E 00	0.756E 00
300	0.105E 01	0.114E 01
	0.114E 01	0.143E 01
	0.126E 01	0.160E 01
330	0.135E 01	0.165E 01
	0.139E 01	0.160E 01
	0.134E 01	0.150E 01

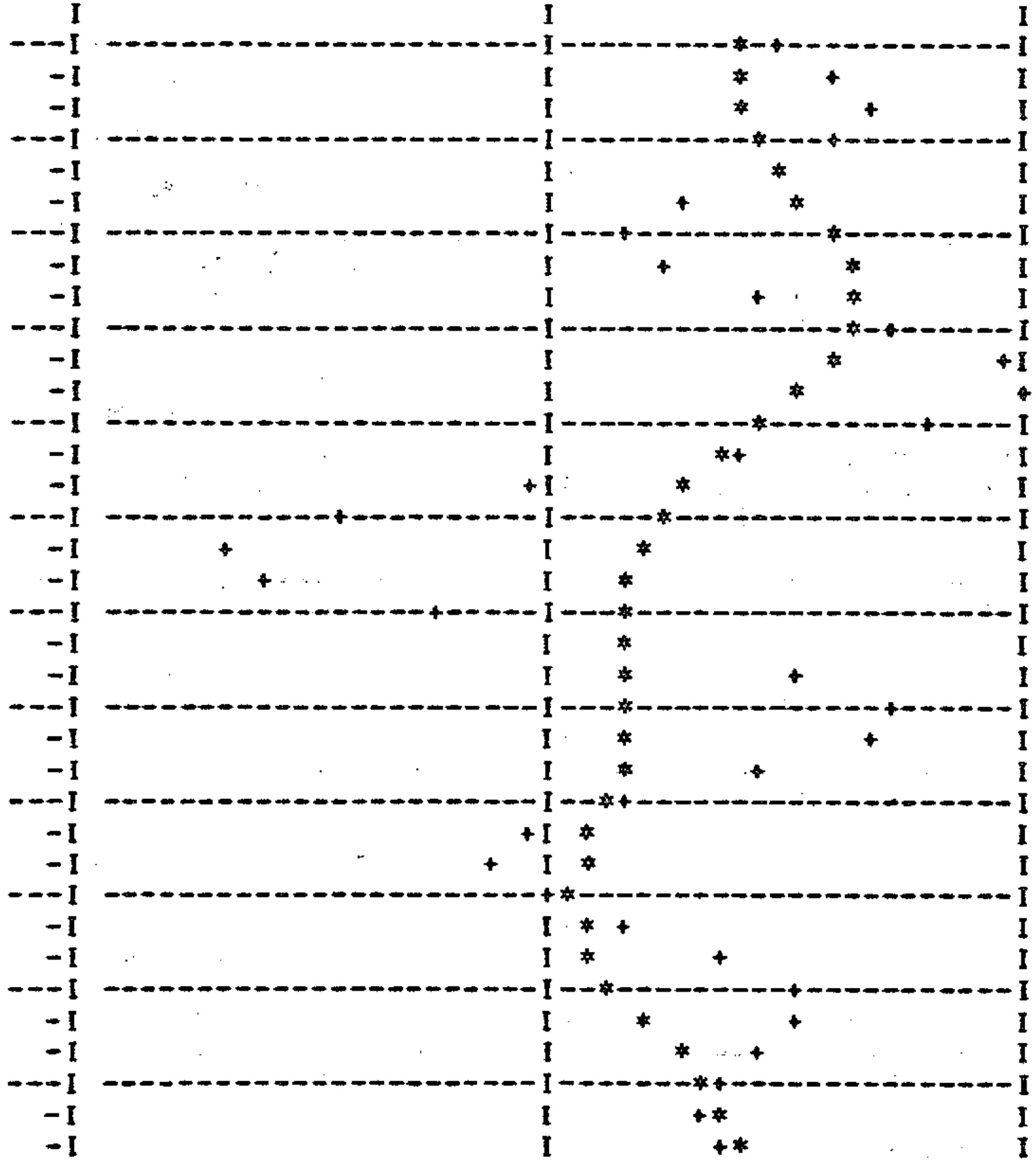
COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 5.112HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.750HECTOBARS

FIG. 25 -

GIRAVIONS DORAND

Doc t : DH-2011 C - E 1
Page : 88

RUN 16.08
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R

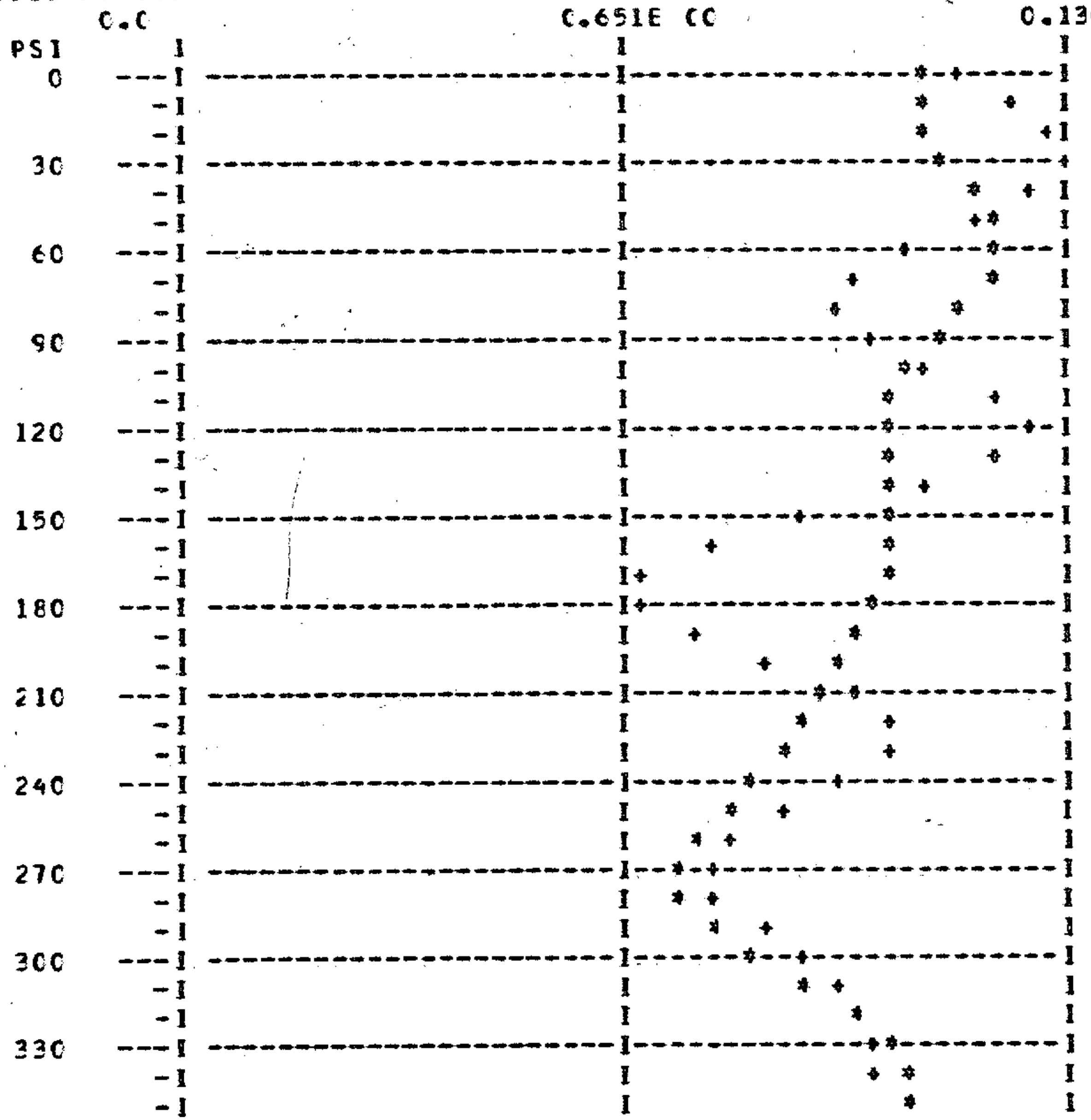


PSI	COMPUTED	MEASURED
0	0.107E 01	0.113E 01
	0.107E 01	0.123E 01
	0.108E 01	0.128E 01
30	0.109E 01	0.124E 01
	0.112E 01	0.112E 01
	0.116E 01	0.980E 00
60	0.121E 01	0.901E 00
	0.125E 01	0.937E 00
	0.127E 01	0.109E 01
90	0.126E 01	0.131E 01
	0.123E 01	0.148E 01
	0.118E 01	0.153E 01
120	0.111E 01	0.139E 01
	0.104E 01	0.108E 01
	0.979E 00	0.705E 00
150	0.934E 00	0.383E 00
	0.907E 00	0.226E 00
	0.896E 00	0.288E 00
180	0.895E 00	0.536E 00
	0.899E 00	0.871E 00
	0.902E 00	0.117E 01
210	0.899E 00	0.132E 01
	0.888E 00	0.129E 01
	0.871E 00	0.111E 01
240	0.850E 00	0.885E 00
	0.828E 00	0.708E 00
	0.812E 00	0.651E 00
270	0.805E 00	0.727E 00
	0.812E 00	0.887E 00
	0.834E 00	0.105E 01
300	0.871E 00	0.116E 01
	0.917E 00	0.117E 01
	0.966E 00	0.111E 01
330	0.101E 01	0.103E 01
	0.104E 01	0.997E 00
	0.106E 01	0.104E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.796HECTOBARS

FIG. 26.

RUN 16.09
COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EPC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)
STRESSES AT .45*R



COMPUTED	MEASURED
0.108E 01	0.114E 01
0.109E 01	0.122E 01
0.110E 01	0.128E 01
0.113E 01	0.130E 01
0.116E 01	0.126E 01
0.119E 01	0.117E 01
0.120E 01	0.106E 01
0.119E 01	0.980E 00
0.116E 01	0.961E 00
0.111E 01	0.101E 01
0.107E 01	0.110E 01
0.104E 01	0.120E 01
0.103E 01	0.124E 01
0.103E 01	0.120E 01
0.104E 01	0.109E 01
0.105E 01	0.923E 00
0.104E 01	0.769E 00
0.103E 01	0.676E 00
0.101E 01	0.670E 00
0.986E 00	0.747E 00
0.963E 00	0.869E 00
0.939E 00	0.984E 00
0.913E 00	0.105E 01
0.880E 00	0.104E 01
0.839E 00	0.974E 00
0.795E 00	0.882E 00
0.756E 00	0.805E 00
0.735E 00	0.773E 00
0.740E 00	0.753E 00
0.776E 00	0.850E 00
0.837E 00	0.917E 00
0.911E 00	0.968E 00
0.981E 00	0.995E 00
0.103E 01	0.101E 01
0.107E 01	0.103E 01
0.108E 01	0.107E 01

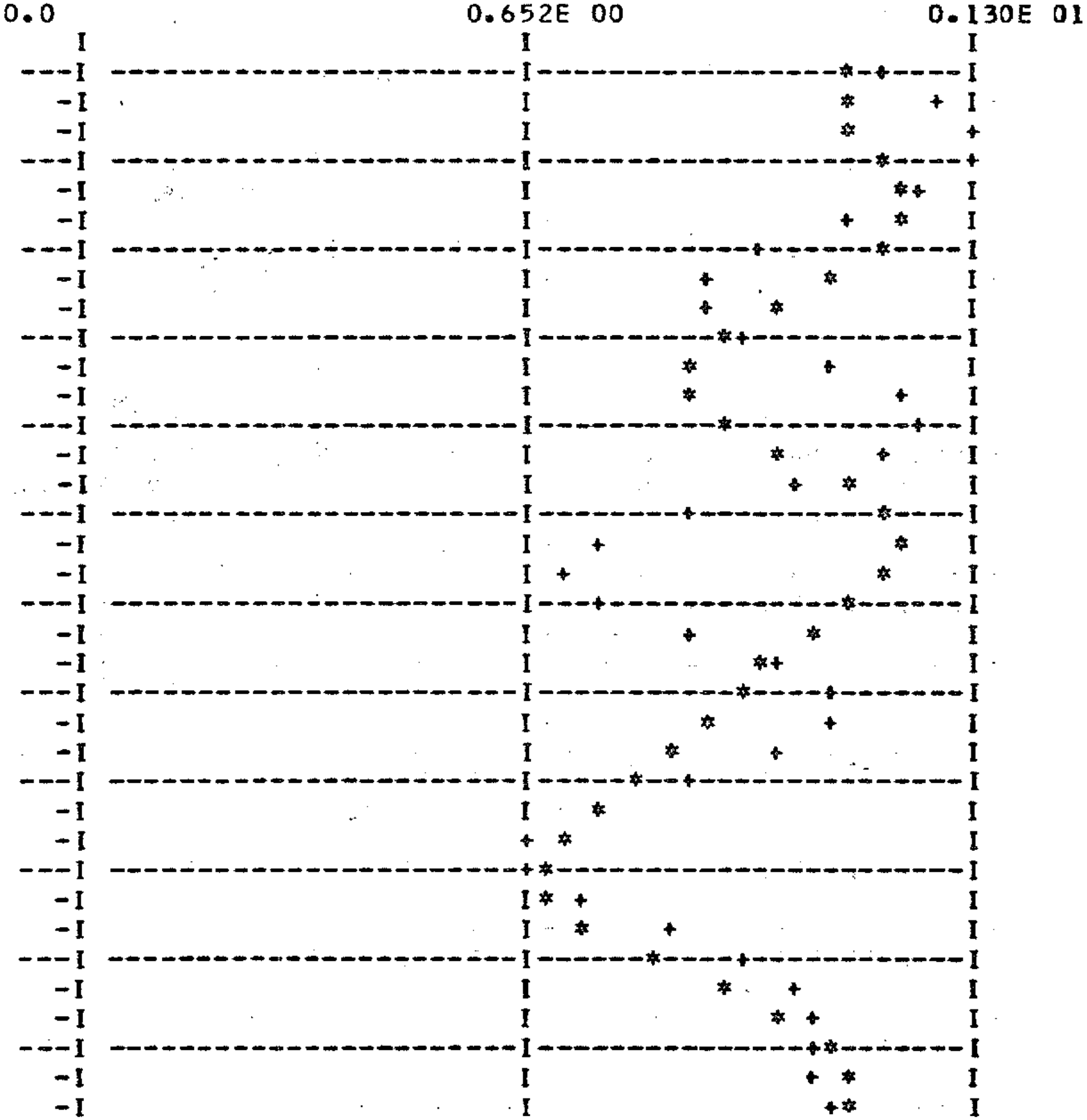
COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.716HECTOBARS
MEASURED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 7.830HECTOBARS

FIG. 27

RUN 16.10

COMPARISON BETWEEN COMPUTED STRESSES(*)(FROM EMC SIMULATIONS) AND MEASURED STRESSES(+)(MARCH 71 TESTS)

STRESSES AT .45*R



PSI	COMPUTED	MEASURED
0	0.111E 01	0.117E 01
	0.111E 01	0.124E 01
	0.113E 01	0.130E 01
30	0.117E 01	0.130E 01
	0.120E 01	0.123E 01
	0.120E 01	0.112E 01
60	0.117E 01	0.995E 00
	0.110E 01	0.915E 00
	0.102E 01	0.909E 00
90	0.937E 00	0.978E 00
	0.890E 00	0.109E 01
	0.893E 00	0.119E 01
120	0.946E 00	0.123E 01
	0.103E 01	0.118E 01
	0.112E 01	0.105E 01
150	0.118E 01	0.895E 00
	0.120E 01	0.767E 00
	0.118E 01	0.717E 00
180	0.113E 01	0.761E 00
	0.106E 01	0.875E 00
	0.100E 01	0.101E 01
210	0.953E 00	0.109E 01
	0.911E 00	0.110E 01
	0.868E 00	0.102E 01
240	0.817E 00	0.882E 00
	0.760E 00	0.747E 00
	0.709E 00	0.664E 00
270	0.681E 00	0.663E 00
	0.690E 00	0.737E 00
	0.742E 00	0.852E 00
300	0.829E 00	0.965E 00
	0.930E 00	0.104E 01
	0.102E 01	0.107E 01
330	0.109E 01	0.107E 01
	0.111E 01	0.108E 01
	0.112E 01	0.111E 01

COMPUTED STRESSES ARE DIVIDED BY THEIR MEAN VALUE= 4.620HECTOBARS

FIG. 28-