NASA TECHNICAL MEMORANDUM

NASA TM X-71977

COPY NO.

NASA TM X-71977

NUMERICAL SOLUTION OF THE STEADY-STATE NAVIER-STOKES EQUATIONS FOR HYPERSONIC FLOW ABOUT BLUNT AXISYMMETRIC BODIES

By E. C. Anderson
and James N. Moss

Langley Research Center
Hampton, Virginia

Presented at the 10th Anniversary Meeting
of the Society of Engineering Science, 1973

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
TM X-71977		

4. Title and Subtitle	5. Report Date
Numerical Solution of the Steady-State Navier-Stokes Equations for Hypersonic Flow About Blunt Axisymmetric Bodies	June 1974
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
E. C. Anderson and James N. Moss	
	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Langley Research Center Hampton, VA 23665'	
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Memorandum
NASA Langley	14. Sponsoring Agency Code

15. Supplementary Notes

Article proposed for publication in the Proceedings of the 10th Anniversary Meeting of the Society of Engineering Science, 1974.

16. Abstract

The steady-state Navier-Stokes equations are solved for hypersonic flow about blunt axisymmetric bodies. The equations of motion are solved by successive approximations using an implicit finite-difference scheme. The results are compared with viscous shock-layer theory, experimental data, and time-dependent solutions of the Navier-Stokes equations. It is demonstrated that viscous shock-layer theory is sufficiently accurate for the range of flight conditions normally encountered by entry vehicles.

17. Key Words (Suggested by Author(s)) (STAR category underlined)	18. Distribution Statement
Navier-Stokes Shock layer Hypersonic flow Blunt body Perfect gas Viscous	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	6	$3.00

*Available from { The National Technical Information Service, Springfield, Virginia 22151
STIF/NASA Scientific and Technical Information Facility, P.O. Box 33, College Park, MD 20740

NUMERICAL SOLUTION OF THE STEADY-STATE NAVIER-STOKES EQUATIONS FOR HYPERSONIC FLOW ABOUT BLUNT AXISYMMETRIC BODIES

E. C. Anderson
O.D.U. Research Foundation, Norfolk, Va.
and
James N. Moss
NASA Langley Research Center, Hampton, Va.

ABSTRACT

The steady-state Navier-Stokes equations are solved for hypersonic flow about blunt axisymmetric bodies. The equations of motion are solved by successive approximations using an implicit finite-difference scheme. The results are compared with viscous shock-layer theory, experimental data, and time-dependent solutions of the Navier-Stokes equations. It is demonstrated that viscous shock-layer theory is sufficiently accurate for the range of flight conditions normally encountered by entry vehicles.

INTRODUCTION

The solution for the flow field about blunt axisymmetric bodies at zero angle of attack has been determined for moderate to high Reynolds number conditions using the combination of inviscid and boundary-layer theories or viscous shock-layer theory. However, in a recent publication, Li [1] questions the validity of existing theories in the range of Reynolds numbers encountered by entry vehicles in the altitude range 200,000 to 300,000 feet. Li presents a time-dependent finite-difference solution of the Navier-Stokes equations and applies the method to typical shuttle trajectory points in this altitude range. This method of solution is severely restricted by storage requirements and excessive computing cost and, as a result, limited in application to the nose region of the body.

In a previous publication, Davis [2] presents a solution of the viscous shock-layer equations and shows good agreement with experimental data for Reynolds numbers as low as 22. The viscous shock-layer equations presented by Davis [2] contain all terms up to $O(\epsilon^2)$, where ϵ is the Van Dyke [3] expansion parameter defined as

$$\epsilon = [\mu(T_{ref})/\rho_\infty U_\infty R_N]^{1/2}$$

and

$T_{ref} = U_\infty^2/C_p$

R_N = body nose radius

U_∞ = free-stream velocity

μ = coefficient of viscosity

Consideration of the order of magnitude analysis of the Navier-Stokes equations given by Van Dyke [3] and the viscous shock-layer equations presented by Davis [2], shows that the solution of the two sets of equations will have significant differences only when $\epsilon \sim O(1)$. This suggests that a method of successive approximations in which the starting solution is given by the viscous shock-layer equations can be used to determine the solution of the steady-state Navier-Stokes equations for most flight conditions of practical interest.

The solutions of the Navier-Stokes equations presented herein are restricted to a perfect gas having a constant ratio of specific heats and the bow shock is assumed distinct. However, appropriate slip and jump boundary conditions are included. The solutions of the Navier-Stokes equations are compared with viscous shock-layer theory, experimental data, and the transient solution presented by Li [1].

METHOD OF SOLUTION

The Navier-Stokes equations are expressed in the shock-body oriented coordinate system defined by Davis [2]. After determining the solution of the viscous shock-layer equations using the implicit finite-difference scheme described in Reference [2], the terms of higher order are evaluated using these flow-field data. The higher order terms are held constant during the solution for the first approximation to the Navier-Stokes equation. After the flow-field data for the first approximation are determined, the higher order terms are reevaluated and held constant during the solution for the second approximation. This procedure is repeated until the flow-field data corresponding to successive approximations converge within a specified limit. For details of the numerical method, the reader is referred to the paper presented by Davis [2].

RESULTS AND DISCUSSION

A comparison of the temperature profiles along the stagnation streamline and at a distance of 0.7 nose radii downstream of the stagnation point is shown in Figure 1 for solutions corresponding to the steady-state Navier-Stokes equations, the viscous shock-layer equations, and Li's transient solution of the Navier-Stokes equations. The flight conditions for this case correspond

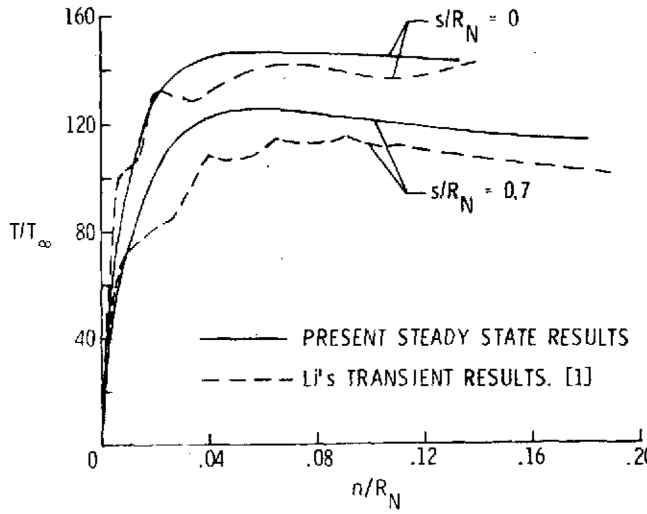

Figure 1. Comparisons of temperature profiles for transient and steady-state solutions.

to a Mach number of 24 at an altitude of 250,000 feet, a free-stream Reynolds number of 14,400 based on a 2-foot nose radius, and an ϵ of 0.04. The body considered is a 20° hyperboloid. For these conditions, the present solution of the Navier-Stokes equations and the solution of the viscous shock-layer equations differed by less than 0.5% and these results are shown as a single curve. The solutions given by Li are seen to exhibit large oscillations at both body locations. Since the maximum number of node points across the shock layer in Li's solution is less than 35, these oscillations can probably be removed by increasing the number of node points. For the present analysis, a minimum of 50 grid points was necessary for adequate resolution of this problem. As indicated by Li, this is apparently a prohibitive requirement in his solution. It is noted that the temperature behind the shock at the downstream station computed using the transient method of solution is approximately 15% lower than predicted by the present method. The temperature predicted by Li closely corresponds with the value obtained assuming that the local shock angle is the same as that at the body. The comparison of the present solution of the steady-state Navier-Stokes equations and the viscous shock-layer equations with the transient solution of Reference [1] indicates that the transient solution is an unacceptable alternate method for solving the viscous blunt-body problem.

The present solution of the Navier-Stokes equations is compared with the solution of the viscous shock-layer equations and experimental data for Reynolds numbers in the range 90 to 31,160 in Figure 2. The experimental data are given by Little [4]. The comparisons of the total drag coefficient, C_D, shown in Figure 2 are representative of the differences noted in heat transfer,

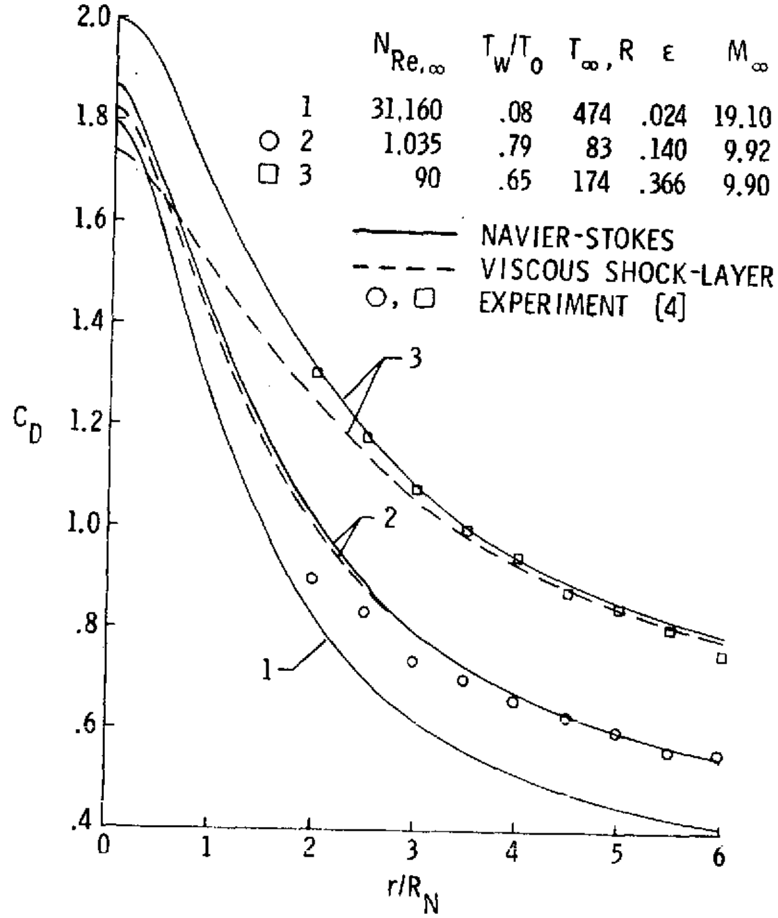

Figure 2. Total drag coefficient for a 45° hyperboloid.

skin friction, and surface pressure distribution using the two sets of governing equations. For the higher Reynolds number case, the two solutions differed by less than 0.1% and are shown as a single curve. At a Reynolds number of 1035, the maximum difference in the two solutions is less than 5%, and at a Reynolds number of 90, the two solutions differ by approximately 15% in the stagnation region and less than 2% in the downstream region. The solution of both the viscous shock-layer equations and the Navier-Stokes equations is in good agreement with the experimental data. Figure 3 shows the skin friction and heat-transfer distributions for the case of a Reynolds number of 90.

The solutions of the Navier-Stokes equations presented herein are seen to reduce to the solution of the viscous shock-layer equations for high Reynolds number flow and significant differences occur only at very low Reynolds numbers. These results are in agreement with the requirements of Van Dyke's [3] order of magnitude analysis. The cases presented were solved to a distance of 8 nose radii downstream of the stagnation point

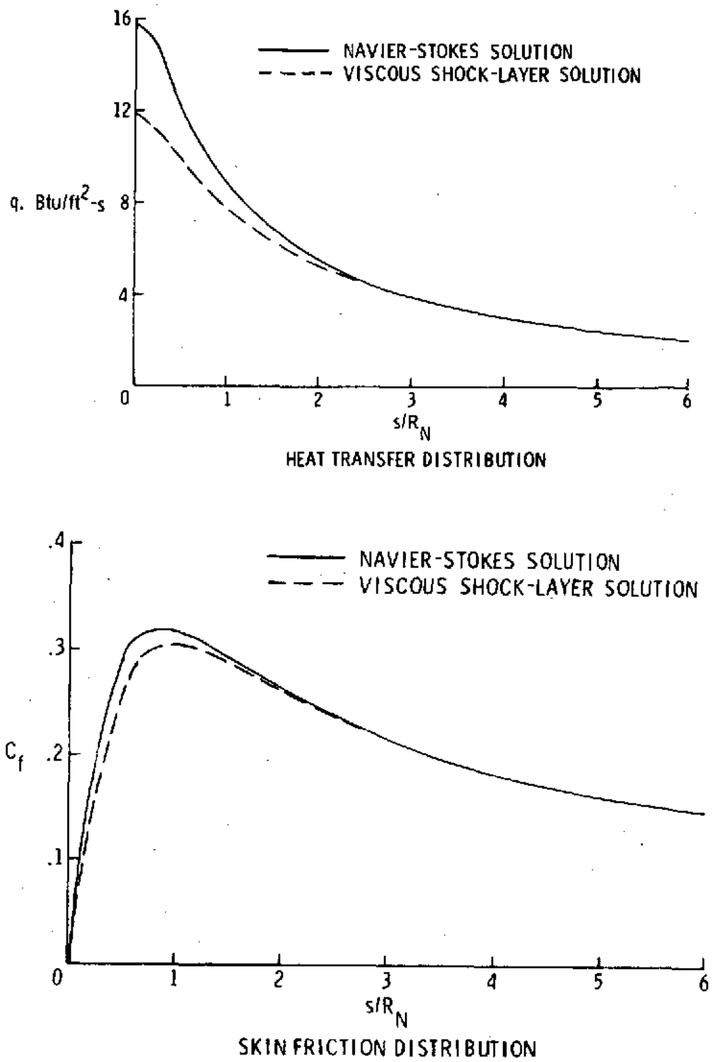

Figure 3. Comparisons of results for a 45° hyperboloid.

using a CDC-6600 digital computer. The maximum computing time was less than 3 minutes and the storage requirement is less than 50,000$_8$.

The results of this investigation reveal that there is little need for a solution of the Navier-Stokes equations for viscous blunt-body flows in the range of flow conditions of practical interest. However, a method which is accurate and computationally efficient has been developed for their solution. The numerical method with proper modifications can be extended to merged layer problems and reacting gas mixtures.

REFERENCES

[1]C. P. Li, "Numerical Solution of Viscous Reacting Blunt Body Flows of a Multicomponent Mixture," AIAA Paper No. 73-202, 1973.

[2]R. T. Davis, AIAA J. 8, 843-851 (1970).

[3]M. Van Dyke, "A Review and Extension of Second-Order Hypersonic Boundary-Layer Theory," Rarefied Gas Dynamics, Fluid Symposium Supplement 2, Vol. II, Academic Press, New York, 1963.

[4]H. R. Little, "An Experimental Investigation of Surface Conditions on Hyperboloids and Paraboloids at a Mach Number of 10," M. S. Thesis, University of Tennessee, 1969.