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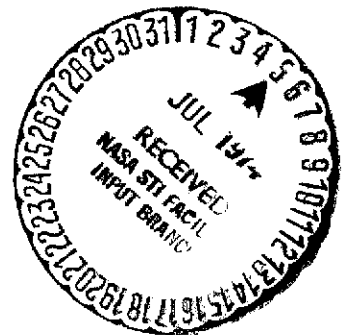
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OPTICAL RADIATION FROM THE CRAB PULSAR

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ABSTRACT

Possible mechanisms for producing the optical radiation from the Crab pulsar are proposed and discussed. There are severe difficulties in interpreting the radiation as being produced by an incoherent process, whether it be synchrotron radiation, inverse-Compton radiation or curvature radiation. It is proposed therefore that radiation in the optical part of the spectrum is coherent. In the polar cap model, a small bunch of electrons and positrons forms near each primary electron as a result of the pair-production cascade process. Ambient electric fields give rise to energy separation, as a result of which either the electrons or positrons will dominate the radiation from each bunch. Calculations, which involve a number of simplifying assumptions, indicate that the optical radiation from the Crab pulsar can be understood in this way if the mass of the star is approximately  $0.3 M_{\odot}$ . The roll-off in the infrared is ascribed to synchrotron absorption by electrons and positrons located between the surface of the star and the force-balance radius. Various consequences of this model, which may be subjected to observational test, are discussed.

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†We regret to inform our readers that Steve Turk died on May 2, 1973

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## 1. Introduction

Pulsar models in which radiation occurs at the magnetic polar caps near the surface of the neutron star (Radhakrishnan and Cooke, 1969; Komesaroff, 1970; Sturrock, 1970; Sturrock, 1971\*; Tademaru, 1971) have properties which compare favorably with radio observations. "Polar-cap" models appear to explain the polarization properties and also the observed  $\nu^{-1/4}$  dependence of the spacing of components of a double pulse profile (Tademaru, 1971). The observed period-pulse-width distribution and the number of pulsars with interpulses are both in good accord with theoretical calculations (Roberts and Sturrock, 1972a, 1972b, 1973<sup>†</sup>) if it is further assumed that the transition from closed to open field lines occurs at the "force balance" radius, i.e. the radius at which a corotating neutral particle is subject to zero net force (PCFB model), rather than at the light cylinder (PCLC model).

Calculations were previously made for the PCLC model (I), the results of which appeared to be compatible with the radio, optical, x-ray, gamma ray and particle flux data for the Crab pulsar. However, it has been pointed out that the discussion of optical radiation in that article was incorrect, since it did not use formulas appropriate for synchrotron radiation by electrons with small pitch angles (O'Dell and Sartori, 1970a, 1970b; Epstein, 1972). This topic has been discussed recently by Epstein and Petrosian (1973) who conclude that, if optical

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\*This article will be referred to briefly as "I".

†These articles will be referred to collectively as "II".

radiation from the Crab pulsar occurs by the synchrotron mechanism, the radiation must originate near the light cylinder. This topic will be discussed further in Section 2, where we now conclude that even this possibility is not open. It appears impossible for the optical radiation from the Crab pulsar to be produced by the synchrotron mechanism anywhere within the light cylinder. This rules out not only our earlier model, but also the suggestion of Shklovsky (1970), that radiation originates at the light cylinder, and the suggestion of Smith (1970), that radiation occurs at a radius of about  $0.9R_L$ , where  $R_L$  (cm) is the radius of the light cylinder, given by

$$R_L = \frac{cT}{2\pi} \approx 10^{9.7} T \quad (1.1)$$

where  $T$  (sec) is the rotation period.

This leaves us with the problem of finding alternative mechanisms for producing the optical radiation. Elitzur (1973) has recently proposed that the optical radiation is produced by the inverse-Compton process, due to high-energy electrons in the polar caps scattering RF photons in this region. This topic is discussed in Section 3, where we conclude that this mechanism gives too small a luminosity. An important aspect of this calculation is the fact that the Compton process is greatly modified by the strong magnetic field. In Section 4, we discuss the possibility that the optical radiation is due to curvature radiation, but once again we find that the luminosity would be too small. As a result of the studies presented briefly in Section 2, 3 and 4, we have come to question the implicit assumption that the optical radiation is due to an incoherent process. (We have also questioned the possibility

that radiation is due to electrons, but the hypothesis that radiation is due to ions appears to be very unpromising.) At first sight, one might suppose that the constancy of the optical luminosity argues against the possibility that the radiation is due to a coherent process, since it is known that the radio luminosity (which is generally believed to be coherent) is variable. However, the degree of variation depends critically on the degree of coherence -- as measured, for instance, by the number of independent "bunches"  $N_B$  responsible for the radiation, where each bunch is assumed to radiate coherently but different bunches have an incoherent phase relationship. If the standard deviation of the luminosity per bunch is comparable with the mean value of this luminosity, the fractional variation in the luminosity will be of order  $N_B^{-1/2}$ . This implies that  $N_B$ , for the RF radiation process, is a small number. It also implies that if  $N_B$  is large, for some model of coherent optical radiation, this model may be consistent with the lack of detectable variability of the optical radiation from the Crab pulsar, despite the fact that the radiation is assumed to be coherent.

In Section 5, we investigate the possibility that optical radiation is due to coherent curvature radiation. As a result of the cascade process due to the conversion of gamma rays into electron-positron pairs, a large number of secondary electrons and positrons are created close to each primary electron. One finds that the dimensions and orientation of each bunch are such that optical radiation could be produced coherently. On the other hand, each bunch is almost charge-neutral so it appears, at first sight, that the coherent radiation from each bunch will be no more than that from a single electron.

The above argument would be valid if each bunch were moving in a region free from electric field. However, strong DC and RF electric fields develop in the polar-cap region in the context of our model. It follows that, although electrons and positrons of each bunch may be closely positioned in space, they may be separated in energy. A small separation in energy is sufficient to lead to coherent radiation from either the electrons or the positrons in a bunch, whichever have the higher energy.

By investigating the cascade process in detail for the PCFB model, we find that we can understand the optical luminosity of the Crab pulsar on the assumption that this radiation is due to coherent curvature radiation. Similar calculations have also been made for the PCLC model, but these calculations do not yield results consistent with observational data.

In principle one might consider also the possibility that the optical radiation is due to a similar coherent process, involving bunches of electrons or positrons, based on inverse-Compton radiation rather than curvature radiation. However, since the calculations of Section 3 and 4 show that incoherent inverse-Compton radiation is much less effective than incoherent curvature radiation, we infer that coherent inverse-Compton radiation would be less effective than coherent curvature radiation. These and other points are discussed further in Section 6, where certain new observations are proposed which would test the suggestion advanced in this article.

## 2. Synchrotron Radiation

The possibility that optical radiation from the Crab pulsar is produced by the synchrotron mechanism has recently been discussed by Epstein and Petrosian (1973). This section represents a slight extension

of their discussion.

It has been found that the optical pulse shape appears cusp-like, at the pulse maximum, when measured with a resolution of  $32 \mu\text{s}$  (Papaliolios, Carleton and Horowitz, 1970). Since the period of the Crab pulsar is 33 ms, the variations imply that the pulsar beam has a total angular scale of at most  $10^{-2.2}$  radian. It follows immediately that if the optical radiation is due to electrons with energy factor  $\gamma$  and pitch angle  $\xi$ , then

$$\gamma \geq \gamma_c = 10^{2.5}, \quad \xi \leq \xi_c = 10^{-2.5}. \quad (2.1)$$

If the radiation is due to normal (large-angle) synchrotron radiation, the emitted spectrum peaks at the frequency given by

$$\nu = \frac{3}{2} \nu_G \gamma^2 \xi, \quad (\gamma \xi \geq 4/3) \quad (2.2)$$

where  $\nu_G$  (Hz) is the gyro frequency given by

$$\nu_G = 10^{6.5} B. \quad (2.3)$$

If, on the other hand, the product  $\gamma \xi$  is such that the small-angle theory is appropriate, the spectrum peaks at

$$\nu = 2 \nu_G \gamma \quad (\gamma \xi \leq 4/3). \quad (2.4)$$

On using the inequalities of (2.1), (2.2) and (2.4), we find that, for either case,

$$\nu > 2 \nu_G \gamma_c. \quad (2.5)$$



Since the observed optical spectrum peaks at about  $10^{14.8}$  Hz, we find from (2.1), (2.3) and (2.5) that

$$B \leq 10^{5.5} \text{ gauss.} \quad (2.6)$$

One may obtain information concerning the magnetic field strength in the magnetosphere of the Crab pulsar by discussion of the power budget and slow-down rate. If  $S$  ( $\text{erg s}^{-1}$ ) is the total power budget, this is related to the torque  $\Theta$  (dyne cm) by

$$S = \Theta \omega = 2\pi T^{-1} \Theta. \quad (2.7)$$

The magnetic torque has elsewhere (I.2.6) been estimated to be

$$\Theta = \frac{1}{2} R_L^3 B_L^2. \quad (2.8)$$

For the Crab pulsar,  $S \approx 10^{38}$  and  $T = 10^{-1.5}$  so that  $R_L = 10^{8.2}$ ; hence  $B_L = 10^{5.7}$ . Since the magnetic-field strength at the light cylinder is larger than that allowed by equation (2.6), it appears that the optical radiation from the Crab pulsar cannot be produced by the synchrotron mechanism anywhere at or within the light cylinder.

The torque is related to the "age"  $\tau$  (s), defined by

$$\tau = \frac{T}{dT/dt}, \quad (2.9)$$

by

$$-I \frac{d\omega}{dt} = 2\pi I T^{-1} \tau^{-1} = \Theta. \quad (2.10)$$

This equation shows that, for the Crab pulsar, the moment of inertia  $I$  ( $\text{g cm}^2$ ) has the value  $I \approx 10^{43.9}$ . According to the neutron-star

model of Baym, Pethick and Sutherland (1971), this indicates that the Crab pulsar has a mass close to  $M = 0.24 M_{\odot}$ .

### 3. Inverse-Compton Radiation

We next consider the possibility that optical radiation is produced in the polar cap regions due to the scattering of RF photons by high-energy electrons. In this context, it is important to note that electrons are predominantly streaming along magnetic field lines, since any transverse motion is rapidly damped by synchrotron radiation. The magnetic field effectively constrains electrons to one-dimensional motion so that radiation is produced only by the component of acceleration parallel to the electron velocity vector. If  $\chi$  is the (small) angle between the electron velocity and the wave vector of the (randomly polarized) RF electromagnetic wave, the resulting radiation occurs at frequency

$$\nu_o = \frac{1}{2} \chi^2 \gamma^2 \nu_r \quad (3.1)$$

and the power emitted per electron  $P_1$  ( $\text{erg s}^{-1}$ ) is given by

$$P_1 = \frac{1}{2} \chi^2 c \sigma_T w_r \quad (3.2)$$

where  $\nu_r$  is the RF frequency,  $\nu_o$  is the resulting optical frequency,  $w_r$  is the RF energy density and  $\sigma_T$  is the Thomson cross section.

In applying the above formulas to the polar-cap model, we note that the maximum angle between electron velocity vectors and RF photon vectors may be calculated in the same way as we calculated the maximum angle between gamma ray velocity vectors and magnetic field vectors in I.

From the equations of I, Section 4, we obtain the following estimate

for the maximum value of  $\chi$  :

$$\chi = 10^{-0.7} \theta_p , \quad (3.3)$$

where  $\theta_p$  is the polar cap angle. Since this maximum value of  $\chi$  occurs at a height of  $\frac{1}{3}R$ , where  $R$  (cm) is the star radius, we assume that the important contribution to the radiation occurs over an arc length of  $\frac{2}{3}R$ . Then the total optical luminosity (per polar cap) is given approximately by

$$L_o = \frac{2}{3} \frac{R}{c} J_{ic} P_1 , \quad (3.4)$$

where  $J_{ic}$  ( $s^{-1}$ ) is the rate of flow of "inverse-Compton" electrons in each polar cap.

In terms of the same quantities, the RF luminosity is given by

$$L_r = \pi \theta_p^2 R^2 c w_r . \quad (3.5)$$

Hence we find

$$\frac{L_o}{L_r} = 10^{-37.1} R^{-1} J_{ic} . \quad (3.6)$$

If the radio and optical flux densities at a distance  $d$  from the star are, respectively,  $F_{\nu,r}$  and  $F_{\nu,o}$ , then

$$L_r \approx \nu_r F_{\nu,r} \pi \psi_p^2 d^2 \quad (3.7)$$

$$L_o \approx \nu_o F_{\nu,o} \pi \psi_p^2 d^2 \quad (3.8)$$

where  $\psi_p$  is the half angle of the radiation pattern given, in this model, by

$$\psi_p = \frac{3}{2} \theta_p \quad . \quad (3.9)$$

Hence

$$\frac{L_o}{L_r} = \frac{\nu_o F_{\nu,o}}{\nu_r F_{\nu,r}} \quad . \quad (3.10)$$

We compare these calculations with observational data by referring to Rankin et.al. (1970) for radio data and Oke (1969) for optical data. The radio spectrum is rising toward low frequencies but observations are not available below about 10 MHz. For our purposes, we therefore adopt  $\nu_r = 10^7$  for which the time-averaged flux ( $\text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$ ) is approximately  $\bar{F}_{\nu,r} = 10^{-19.0}$ . After correcting for reddening, Oke finds that the optical spectrum peaks at about  $\nu_o = 10^{14.8}$ , for which  $\bar{F}_{\nu,o} = 10^{-25.3}$ . Equation (3.10) now shows that

$$\frac{L_o}{L_r} = 10^{1.5} \quad . \quad (3.11)$$

On referring to equation (3.6) and noting that, for a neutron star of mass  $0.24M_\odot$ , the radius is  $10^{6.1}$  cm, we see that, to explain the observed optical luminosity of the Crab pulsar by the inverse-Compton process, we would need a flux of high-energy electrons  $J_{ic} = 10^{44.7}$ . This is larger by a factor of about  $10^{11.6}$  than the primary electron flux from the star, and larger by a factor of about  $10^{3.7}$  than the total flux of secondaries resulting from the pair-production cascade (I).

The PCFB model (II) for the Crab pulsar gives the estimate  $\theta_p = 10^{-0.5}$ . Equation (3.3) then shows that  $\chi = 10^{-1.2}$ . Hence, from (3.1), the required energy factor of electrons for the inverse-Compton process is  $\gamma = 10^{5.3}$ . The relation

$$S_p = 10^{-5.7} \gamma J \quad (3.12)$$

between the particle flux  $J$  and the corresponding energy flux  $S_p$  ( $\text{erg s}^{-1}$ ) shows that, for this model, we require  $S_p = 10^{44.3}$ , which is larger by a factor  $10^{6.3}$  than the total power budget of the Crab pulsar.

#### 4. Curvature Radiation

Since, in the polar-cap model, high-energy electrons are moving along curved field lines, some radiation will be produced in the optical part of the spectrum. We now estimate the particle flux required to produce the observed luminosity in this way.

Electrons of energy  $E_e$  (eV) moving along field lines with radius of curvature  $R_c$  (cm) produce a radiation spectrum which peaks at the frequency

$$\nu = 10^{-7.8} E_e^3 R_c^{-1} \quad (4.1)$$

with total power

$$P_l = 10^{-31.1} E_e^4 R_c^{-2} \quad (4.2)$$

(See equations (I.3.8), (I.3.9).) For the PCFB model (II),

$$R_c = 10^{-1.4} M^{1/6} R^{1/2} T^{1/3} \quad (4.3)$$

which, for the Crab pulsar, gives  $R_c \approx 10^{6.6}$ . Since the radius of curvature varies only slowly with the radius, we assume that electrons radiate on the average for a time  $R c^{-1}$  so that the optical luminosity is given by

$$L_o = R c^{-1} J_{cr} P_1 \quad (4.4)$$

where  $J_{cr} (s^{-1})$  is the flux of "curvature radiation" electrons which produce optical radiation by the curvature radiation mechanism.

We see from equation (4.1) that radiation at the frequency  $\nu = 10^{14.8}$  will be produced by electrons of energy  $E_e = 10^{9.7}$ , considerably lower than the energy ( $10^{11.0}$  eV) required for the inverse-Compton radiation.

If the distance to the Crab nebula is taken to be 1700 pc, data given by Neugebauer et al. (1969) for the main pulse leads one to estimate the optical luminosity per polar cap as  $L_o = 10^{33.2}$ . Since, for the values quoted,  $P_1 = 10^{-5.5}$ , we find from equation (4.4) that the required flux of high-energy electrons is  $J_{cr} = 10^{43.1}$ . This is a factor 10 less than that required by the inverse-Compton process, but is still considerably larger than either the primary electron flux or the secondary electron-positron flux. Moreover, the power represented by this particle flux is  $S_p = 10^{41.0}$ , a factor  $10^{3.0}$  larger than the power budget of the Crab pulsar. Hence optical radiation from the Crab pulsar cannot be produced by incoherent curvature radiation.

## 5. Coherent Curvature Radiation

In this section we consider the possibility that optical radiation from the Crab pulsar is due to coherent curvature radiation. We ascribe the coherency to the fact that a small cloud of electrons and positrons forms near each primary electron as a result of the pair-production cascade process. Our first step will be to calculate the luminosity due to a certain stage of the cascade process, assuming that the electrons radiate independently of the positrons, or vice-versa.

Whether this assumption is justifiable will be discussed later in this section. The physical processes involved are those outlined in I. However, calculations will be made for the PCFB model rather than the PCLC model since we believe that the former is to be preferred to the latter on grounds which have already been discussed elsewhere (II). Formulas for the PCFB model, to be quoted below, are taken from unpublished work by Turk which will be published at a later date.

The colatitude of each polar cap  $\theta_p$  is given by

$$\theta_p = 10^{1.5} M^{-1/6} R^{1/2} T^{-1/3} \quad (10^{-0.45}) \quad (5.1)$$

so that the radius of each polar cap is given by

$$R_p = 10^{1.5} M^{-1/6} R^{3/2} T^{-1/3} \quad (10^{5.59}) \quad (5.2)$$

The magnetic torque is given by

$$\dot{\Omega} = 10^{-4.2} M^{-2/3} B^2 R^6 T^{-7/3} \quad (5.3)$$

When this equation is coupled with equation (2.10), we see that

$$\tau = 10^{5.0} I M^{2/3} B^{-2} R^{-6} T^{4/3} \quad (5.4)$$

so that the magnetic field strength at the surface of the star may be estimated in terms of the period and age as follows:

$$B = 10^{2.5} I^{1/2} M^{1/3} R^{-3} T^{2/3} \tau^{-1/2} \quad (10^{10.90}) \quad (5.5)$$

For the Crab pulsar,  $T = 10^{-1.48}$  and  $\tau = 10^{10.90}$ . For reasons to be given later, we consider a neutron star of mass  $M = 10^{32.77}$  ( $\sim 0.3M_\odot$ ); then (Baym, Pethick and Sutherland, 1971)  $R = 10^{6.04}$  and

$I = 10^{44.06}$ . Estimates of various quantities will be given for this particular case in brackets following each formula, as in equations (5.1), (5.2) and (5.5). These estimates are given to accuracy  $10^{0.01}$  simply to ensure consistency to accuracy  $10^{0.1}$ .

The current (in e.m.u.) flowing through each zone (EPZ and IPZ) is given by

$$J = 10^{-7.1} M^{-1/3} B R^3 T^{-5/3} \quad (10^{13.46}) \quad (5.6)$$

so that the flux ( $s^{-1}$ ) of "primary electrons" in each EPZ is given by

$$J_{pe} = 10^{12.7} M^{-1/3} B R^3 T^{-5/3} \quad (10^{33.26}) \quad (5.7)$$

The maximum electric field (in e.s.u.) in each polar cap is given by

$$\mathcal{E}_M = 10^{-7.9} M^{-1/6} B R^{3/2} T^{-4/3}, \quad (10^{8.54}) \quad (5.8)$$

and the accelerating potential (e.s.u.) by

$$\phi_M = 10^{-6.8} M^{-1/3} B R^3 T^{-5/3} \quad (10^{13.76}) \quad (5.9)$$

The radius of curvature  $R_c$  (cm) of the outermost field lines of each polar cap is given by

$$R_c = 10^{-1.4} M^{1/6} R^{1/2} T^{1/3} \quad (10^{6.59}) \quad (5.10)$$

The maximum energy  $E_{pe}$  (eV) of each primary electron is limited by radiation reaction to the value

$$E_{pe} = 10^{5.4} M^{1/24} B^{1/4} R^{5/8} T^{-1/6} \quad (10^{13.51}) \quad (5.11)$$

In moving through the accelerating region, each primary electron emits



gamma rays, due to curvature radiation, of energy  $E_{p\gamma}$  (eV):

$$E_{p\gamma} = 10^{-4.6} M^{-1/24} B^{3/4} R^{11/8} T^{-5/6} \cdot (10^{11.75}) \quad (5.12)$$

The number of such primary gamma rays emitted by each primary electron,  $\Sigma_{1\gamma}$ , is found by converting  $\phi_M$  into eV and dividing by  $E_{p\gamma}$ . The result is

$$\Sigma_{1\gamma} = 10^{0.3} M^{-7/24} B^{1/4} R^{13/8} T^{-5/6} \cdot (10^{4.48}) \quad (5.13)$$

Each primary gamma ray produces one secondary electron and one secondary positron. We see from equation (I.4.7) that the secondary particles are born where the transverse field strength  $B_{\perp}$  is given by

$$B_{\perp} = 10^{18.6} E_Y^{-1} \cdot (10^{6.85}) \quad (5.14)$$

We see from equation (5.12) that the pitch angle  $\xi$  of the newly produced particles is given by

$$\xi = \frac{B_{\perp}}{B} = 10^{23.2} M^{1/24} B^{-7/4} R^{-11/8} T^{5/6} \cdot (10^{-4.05}) \quad (5.15)$$

These electrons and positrons will radiate by the large-angle synchrotron-radiation mechanism until their energy factor  $\gamma$  reaches  $\xi^{-1}$ , after which the energy will remain constant and the pitch angle will tend to zero (O'Dell and Sartori, 1970a, 1970b; Epstein, 1972). Hence the secondary electrons and positrons rapidly reach the stage at which they are moving with zero pitch angle and energy given by

$$E_{se} = 10^{5.7} \xi^{-1} = 10^{-17.5} M^{-1/24} B^{7/4} R^{11/8} T^{-5/6} (10^{9.75}) \quad (5.16)$$

The spectrum of radiation emitted by the secondary particles is seen

from equation (4.1) to peak at the frequency

$$\nu_s = 10^{-58.9} M^{-7/24} B^{21/4} R^{29/8} T^{-17/6}. \quad (5.17)$$

The power produced per electron by incoherent curvature radiation is given by equation (4.2). On assuming that all electrons (or all positrons) of each bunch act coherently, and that the radiation is effective over an arc length of order  $R$  (since  $R_c$  varies only slowly with radius), we obtain the following estimate of the luminosity ( $\text{erg s}^{-1}$ ) produced by coherent curvature radiation:

$$L_{cc} = R c^{-1} J_{pe} \sum_{1\gamma}^2 P_1. \quad (5.18)$$

On substituting the appropriate formulas, this becomes

$$L_{cc} = 10^{-95.5} M^{-17/12} B^{17/2} R^{47/4} T^{-22/3}. \quad (10^{32.55}) \quad (5.19)$$

We see from equations (5.17) and (5.19) that the peak value of the luminosity spectrum may be estimated to be

$$L_{cc,\nu} = 10^{-36.6} M^{-9/8} B^{13/4} R^{65/8} T^{-9/2}. \quad (10^{17.69}) \quad (5.20)$$

In order to obtain estimates for the peak frequency and peak luminosity spectrum for various neutron-star models, it is convenient to eliminate the field strength from equations (5.17) and (5.20) by using equation (5.5). This leads to the formulas

$$\nu_s = 10^{-45.8} M^{35/24} I^{21/8} R^{-97/8} T^{2/3} \tau^{-21/8}, \quad (10^{14.80}) \quad (5.21)$$

$$L_{cc,\nu} = 10^{-28.5} M^{-1/24} I^{13/8} R^{-13/8} T^{-7/3} \tau^{-13/8}. \quad (10^{17.45}) \quad (5.22)$$

These quantities are plotted as a function of  $\rho_0$ , the central density, in Figure 1 for the Baym, Pethick, Sutherland (1971) model and for the values of  $T$  and  $\tau$  appropriate to the Crab pulsar. It is seen that fair agreement between observation and theory is obtained for a central density in the range  $\rho_0 = (6-7) \cdot 10^{14}$ , corresponding to a mass  $M = 10^{32.69} - 10^{32.81}$ , radius  $R = 10^{6.06} - 10^{6.03}$  and moment of inertia  $I = 10^{43.95} - 10^{44.11}$ . The specific model which we have evaluated corresponds to a central density  $\rho_0 = 6.67 \cdot 10^{14}$ , and has a mass ( $0.3 M_\odot$ ) close to that ( $0.24 M_\odot$ ) indicated (Section 2) by comparison of the power budget of the Crab pulsar and the age.

Many simplifying assumptions have been made in obtaining this estimate of the optical radiation due to the coherent curvature mechanism. We now discuss the validity of some of these assumptions, and ask whether each bunch is in fact sufficiently small to behave coherently. First we consider the "thickness" of the bunch, which is determined by the fact that gamma rays emitted by the primary electron travel in a straight line before annihilating, whereas the primary electrons continue along curved trajectories. The thickness, which we term  $b_z$  (cm) is estimated to be

$$b_z = \frac{1}{3} \xi^3 R_c \quad (5.23)$$

which is approximately  $10^{-6.1}$ , much smaller than the wavelength ( $\lambda = 10^{-4.3}$ ) of the optical radiation.

The extent  $b_x$  along the direction normal to the magnetic field lines may be found by determining the distance traveled by a gamma ray before annihilation. It is given approximately by

$$b_x = \frac{1}{2} \xi^2 R_c \quad (5.24)$$

which is  $10^{-1.8}$ . However, since the half angle of the polar diagram of radiation is approximately  $\gamma_s^{-1}$ , i.e.  $\xi$ , this length should be compared with  $\gamma_s \lambda$  which is approximately  $10^{-0.2}$ .

The largest extent of each bunch, according to the model presented, is in fact along the binormal; we denote this length by  $b_y$ . The primary electron in fact does not precisely follow a magnetic field line but drifts in the direction of the binormal. For a highly relativistic electron, this drift velocity ( $\text{cm s}^{-1}$ ) is given by

$$v_D = \frac{\gamma m c^3}{e B R_c} \approx 10^{8.5} E_e B^{-1} R_c^{-1} \quad (5.25)$$

which is found to be about  $10^{4.5}$ . Since electrons have the energy given by equation (5.11) only for a time  $R_p c^{-1}$ , we see that

$$b_y \approx v_D R_p c^{-1}, \quad (5.26)$$

which leads to the estimate  $b \approx 10^{-0.4}$ . This is close to, but still smaller than,  $\lambda \gamma_s$ .

In the model of this section, we have assumed that the radiation is due either to the electrons or to the positrons of a bunch. We have also assumed that the terminal energy of electrons and positrons is determined by the synchrotron radiation process. These assumptions are contradictory, and in fact neither assumption can be strictly correct. There must be a very complicated electromagnetic field distribution in each polar cap. The mean DC electric field in each polar zone has been estimated in terms of the current which must be drawn from each zone. However, this field can be neither steady nor

uniform. The cascade process gives rise to a large plasma density, such that a very small charge separation would offset the accelerating electric field. It is our belief that both plasma and electromagnetic field are in a highly "turbulent" state. Estimates of the particle flux into the Crab nebula (Shklovsky, 1968) indicate that most of the particles produced during the cascade leave the pulsar, so that most of the cascade-produced particles can see only a small average electric field.

An upper limit to the average electric field seen by a typical secondary particle may be found by considering the final energy of electrons and positrons produced during the cascade. This is given by the PCFB analog of equation (I.5.4), which is

$$E_{eF} = 10^{18.1} M^{1/6} B^{-1} R^{-1/2} T^{1/3} \quad (10^{9.15}) \quad (5.27)$$

the value of which is shown, for the present model, in parentheses. Hence the path integral of the average electric field seen by a typical secondary particle must be less than  $10^{9.2}$  V ( $10^{6.7}$  e.s.u.). However, even this electric field is sufficient to produce substantial energy separation of electrons and positrons in a bunch; in the absence of such an electric field, the energies would be about  $6 \cdot 10^9$  eV, but in the presence of this electric field the energies would be separated to  $4 \cdot 10^9$  eV and  $8 \cdot 10^9$  eV. Since the spectrum and power of curvature radiation depend sensitively on the electron energy, even this separation is more than enough to imply that one of the two species will be dominant in the coherent radiation process.

It must be stressed that the present calculations are offered simply to test whether coherent curvature radiation is possible.

If it appears that the case is made, estimates of the spectrum and luminosity will require more detailed calculations based on a more complete model of the polar-cap region. It may turn out, for instance, that the important part of the phenomenon occurs in small cells of high electric field strength. Within these cells, primary and secondary electrons are accelerated to high energies, so that the cascade process will be more intense than is implied by our estimates, which ignore acceleration of secondary particles. We hope to discuss this point in more detail in a subsequent article.

We now discuss briefly possible explanations of the rapid roll-over of the optical spectrum towards the infrared. This cannot be due to self-absorption by the bunches, since the energy of each bunch is  $\Sigma_{ly} E_{se}$  which is  $10^{14.22}$  eV corresponding to a temperature of  $10^{18.2}$  °K. If radiation in the optical part of the spectrum were black-body limited at this temperature, the luminosity spectrum from each polar cap would be  $L_{\nu} = 10^{23.3}$  which is larger, by a factor of about  $10^5$ , than is indicated by observations. Since coherent curvature radiation is more important than incoherent curvature radiation, incoherent self-absorption will be less important than coherent self-absorption. Hence this effect also is negligible.

It is possible that the roll-over is due to the electron energy spectrum, but this seems unlikely. It would require a number flux which increases with energy rather than decreases. Furthermore, below the critical frequency, the intrinsic spectrum of a single particle increases only slowly with frequency as  $\nu^{1/3}$ . One would expect this part of the spectrum to become less steep rather than more steep if

the radiation is coherent, since the wave-length restriction on the bunch size becomes weaker at lower frequencies.

We are left with the possibility that the roll-over is due neither to self-absorption nor to the intrinsic radiation spectrum, but is caused by an absorption process which is separate from the radiation process. It is most natural to consider the possibility that this absorption is due to the synchrotron mechanism since the pulsar magnetosphere contains both magnetic field and electrons and positrons. We now discuss briefly the possibility that the absorption is due to electrons located at a radius  $R_a$ .

The magnetic field strength  $B_a$  in this region depends on the location of this region. It may be estimated approximately as follows

$$\left. \begin{aligned} B_a &\approx B R^3 R_a^{-3} & , & \quad R \leq R_a \leq R_{FB} \\ B_a &\approx B R^3 R_{FB}^{-1} R_a^{-2} & , & \quad R_{FB} \leq R_a \leq R_L \end{aligned} \right\} \quad (5.28)$$

Following Scheuer (1967), we suppose that the intensity of radiation is limited by the black-body intensity appropriate to a gas, the temperature of which is set by the energy  $E_{ea}$  of the absorbing electrons. Then

$$F_\nu = 10^{-32.5} E_{ea}^2 \nu^2 . \quad (5.29)$$

Since, for synchrotron radiation,

$$\nu = 10^{-5.3} E_a^2 B_a , \quad (5.30)$$

we see that

$$F_{\nu} = 10^{-29.3} B_a^{-1/2} \nu^{5/2} \quad (5.31)$$

At the radius  $R_a$ , the area intercepted by the radiation pattern from the polar caps is given by

$$A_a = \pi \psi_p^2 R_a^2 \quad (5.32)$$

so that the absorbed part of the radiation will be limited to the spectrum

$$L_{\nu,A} = \pi \psi_p^2 R_a^2 \pi \psi_p^2 F_{\nu} \quad (5.33)$$

which becomes

$$L_{\nu,A} = 10^{-28.3} \psi_p^4 R_a^2 B_a^{-1/2} \nu^{5/2} \quad (5.34)$$

Study of the infrared part of the spectrum (Neugebauer et.al., 1969) leads to the estimate

$$\nu^{-5/2} L_{\nu,A} = 10^{-17.4} \quad ; \quad (5.35)$$

With  $\psi_p = 10^{-0.25}$ , this leads to

$$R_a^2 B_a^{-1/2} = 10^{10.9} \quad (5.36)$$

This required combination of  $R_a$  and  $B_a$  is compared, in Figure 2, with the values given by equation (5.28), for the parameters derived earlier in this section, for various values of  $R_a$ . It is found that an approximate match is found at a radius  $R_a \approx 10^{7.3}$ . This indicates that absorption occurs in the neighborhood of the force balance radius ( $R_{FB} \approx 10^{7.0}$ ).



## 6. Discussion

It was mentioned in the Introduction that the optical radiation could be due to a coherent mechanism, even though the optical flux shows no detectable variation, provided the number of independent "bunches"  $N_B$  is sufficiently large. For the model discussed in Section 5, one finds that  $N_B$  is of order  $10^{29}$  so that  $N_B^{-1/2} = 10^{-14.5}$ . Fractional variations of this magnitude would be undetectable.

Although this initial "trial" calculation seems to indicate the possibility of coherent curvature radiation, it is possible that more detailed investigation of the electrodynamics of the polar-cap region will bring to light objections to the currently proposed model. Since the calculations made in Sections 2, 3 and 4 seem to rule out the possibility that the optical radiation is due to an incoherent mechanism (synchrotron, inverse-Compton or curvature), one should then investigate the possibility that there is some mechanism other than that proposed which would lead to coherent radiation. In this context, the instability discussed by Goldreich and Keeley (1971) may be important in indicating another mechanism for particle bunching which may lead to coherent curvature radiation. Goldreich and Keeley were concerned with the possibility that a plasma-type instability might be responsible for bunching which gives rise to coherent radio emission. It would be necessary to determine whether the same, or a similar, instability might lead to sufficiently tight bunching to produce coherent radiation in the optical part of the spectrum. More detailed theoretical study of the proposed mechanism should be based on a more complete model for the plasma-electromagnetic-field configuration in the polar cap regions.

This would be essential, for instance, to determine the theoretical spectrum for comparison with observational data. If our conjecture is correct, that the low-frequency roll-over is due to absorption by the synchrotron mechanism, discussion of this part of the spectrum will involve more detailed understanding of the low-energy plasma in the polar cap regions at a few star-radii. Calculation of the polarization may be less sensitive to the plasma properties. It would be particularly interesting to determine whether this model indicates that polarization should be different in the infrared part of the spectrum from that in the visible part of the spectrum.

It appears unlikely that the mechanism proposed for optical radiation would be effective also for x-ray emission. Since the x-ray spectrum is continuous to gamma-ray energies, it seems more likely that the x-ray and gamma-ray radiation is due to another mechanism, presumably the synchrotron mechanism.

In the proposed model, the electric vector at pulse maximum should be parallel to the projection of the rotation axis on the plane of the sky. If the magnetic field at the center of the nebula is wrapped toroidally around the rotation axis, the position angle of the rotation axis should be the same as that of the electric vector of optical polarization at the center of the nebula. Kristian et.al. (1970) find this position angle to be  $\theta_{\text{NEB}} = 159^\circ \pm 2$ . They also find that, at the "center of symmetry" of the main pulse, the position angle of the optical electric vector is  $\theta_{\text{OPT}} = 160^\circ \pm 7$ . These position angles are therefore related as one would expect in the current model.

If the radio emission is produced by coherent curvature radiation,

the coherency being due to much larger bunches than are responsible for the optical radiation, one would expect the polarization in the optical and the radio to be similar. It appears that they are not: the variation of position angle through the pulse is opposite in these two parts of the spectrum; furthermore, Manchester et al. (1972) find that, at pulse maximum,  $\theta_{\text{RAD}} \approx 45^\circ$ . However, Manchester et al. emphasized that it is difficult to determine the intrinsic polarization profile since the observed data is strongly affected by "pulse smearing". They suggest, for instance, that the position angle may in fact increase through the intrinsic pulse, whereas the position angle decreases in the observed pulse. It is clear that more information is necessary before we can make a definitive comparison between polarization angles of electric vectors in the optical and radio parts of the spectrum.

If, as seems likely, the x-ray emission is due to synchrotron radiation in the polar cap regions, the electric vectors in the optical and x-ray parts of the spectrum should be orthogonal. It would be a valuable test of this model to check this prediction. Another important consequence of the proposal, that the optical and x-ray parts of the spectrum are due to different mechanisms, is that the spectrum may be discontinuous between these two regions. It is therefore important to try to extend the observations of the Crab spectrum into the ultraviolet, from both the optical and x-ray regions.

Our discussion of the infrared absorption raises the possibility that absorption may in fact be due to comparatively cool plasma in the region of the force-balance radius. Since fluctuations in pulse timing have been ascribed to variations in the amount of plasma trapped in the

magnetosphere (II), this hypothesis raises the possibility that the luminosity may be variable in the infrared part of the spectrum.

Another observation which would be very helpful in clarifying possible mechanisms of optical radiation concerns the cusp of the main pulse. The sharpness of this cusp plays a key role in the discussion of Section 2. If more detailed observations show that the cusp is in fact sharper than is presently believed, this would greatly strengthen the argument of Section 2.

Calculations have been presented in this article only for the PCFB model, since our discussion (II) of the braking index and period-pulse-width distribution support the PCFB model as compared with the PCLC model. However, calculations made in Section 5 have in fact been repeated for the PCLC model. We were not able to match simultaneously the frequency and luminosity at the peak of the optical spectrum in this model.

Bunches of particles, such as those hypothesized in Section 5, would give rise to coherent inverse-Compton radiation as well as to coherent curvature radiation. However, since the calculations presented in Sections 3 and 4 show that incoherent curvature radiation gives a higher luminosity, for lower-energy particles, than does incoherent inverse-Compton radiation, it seems likely that coherent curvature radiation will be more important than coherent inverse-Compton radiation. One may also note that, since the radio emission from the Crab pulsar is variable, optical radiation produced by the inverse-Compton process (coherent or incoherent) should also be variable. The fact that the optical radiation shows no variability therefore implies that this

radiation is not produced by the inverse-Compton mechanism.

We see from equation (5.20) that the flux from the Crab pulsar should be decreasing by approximately 0.2% per year. Equation (5.17) indicates that the peak frequency should be decreasing by approximately 0.1% per year.

The upper limit of optical radiation from the Vela pulsar (Kristian, 1970) is shown in Figure 3. Within the context of the present model, the absence of detectable optical radiation from this pulsar indicates that this star has a mass substantially less than  $0.5 M_{\odot}$ . If this is the correct interpretation, it is still possible that one may detect radiation in the far infrared part of the spectrum from the Vela pulsar which is the analog of optical radiation from the Crab pulsar.

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# FIGURE CAPTIONS

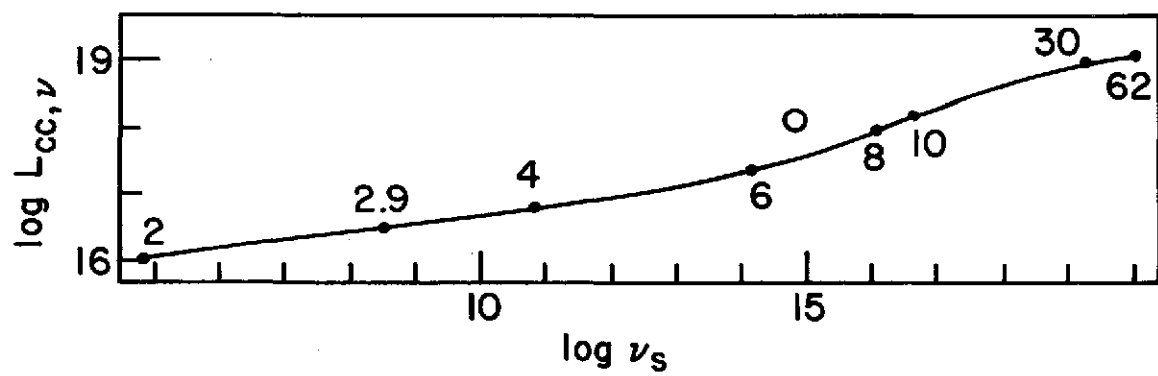
Figure 1.  $\log \nu_s$  and  $\log L_{cc,\nu}$  evaluated for the Crab pulsar for the PCFB model. The solid points are evaluated from formulas (5.21) and (5.22) and applied to the neutron star model of Baym et al. (1971). Figures attached to these points denote  $10^{-14} \rho_c$ , where  $\rho_c$  ( $\text{g cm}^{-3}$ ) is the central density. The open point denotes the maximum flux from the Crab pulsar.

Figure 2. Plot of the function  $R_a^2 B_a^{-1/2}$ , which appears in the synchrotron absorption formulas, evaluated for a neutron star of mass  $0.3 M_\odot$ . The broken line indicates the value required to fit the infrared part of the spectrum.

Figure 3.  $\log \nu_s$  and  $\log L_{cc,\nu}$  evaluated for the Vela pulsar for the PCFB model. The solid points are evaluated from formulas (5.21) and (5.22) as applied to the neutron star model of Baym et al. (1971). Figures attached to these points denote  $10^{-14} \rho_c$ , where  $\rho_c$  ( $\text{g cm}^{-3}$ ) is the central density. The open point and arrow indicates the upper limit on flux from the Vela pulsar in the optical part of the spectrum.



Figure 1



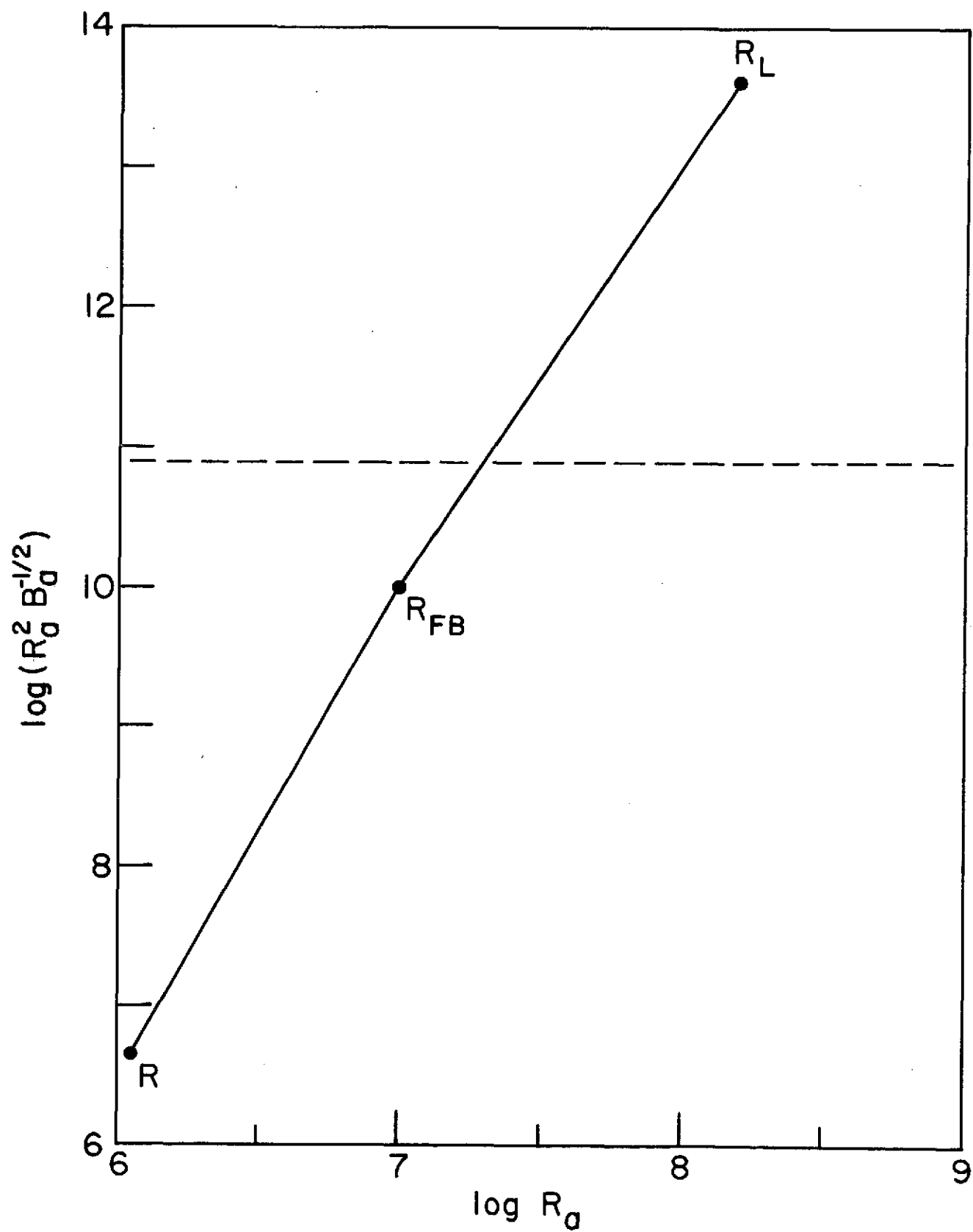


Figure 2

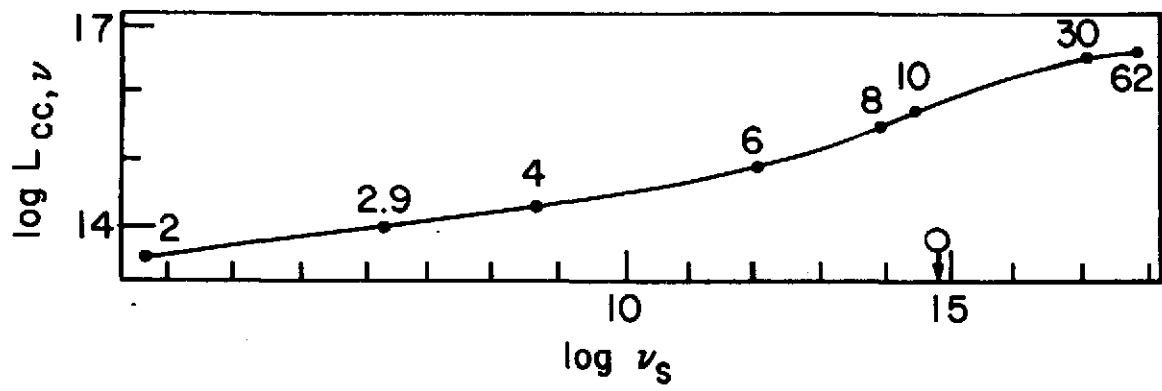


Figure 3