Note on the Electrodynamics of Accelerated Systems[+]

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Abstract: Some preliminary results concerning the experimental testability of the free-space constitutive relations are discussed in connection with some recent theoretical developments.

A recent paper by T. C. Mo[1] on the electrodynamics of accelerated systems seems to make it desirable to recall some early experimentation that is relevant to free-space constitutive behavior.

There are few experimental tests on record which may be considered as a direct verification of the constitutive behavior observed on accelerated systems in free-space. The only experiments known to the authors of this note are the experiments performed some fifty years ago by Kennard[2] and by Pegram[3]; they constitute experimental tests for rotational motion.

The equipment that was used in both experiments consisted of a tubular cylindrical condensor which was being rotated in a coaxial magnetic field. Kennard found a potential to exist on the condensor when rotated, while Pegram's observation showed that a charge developed on the condensor when it is being shorted by a corotating short.

For both experiments it was found that the observations were independent of whether the solenoid generating the coaxial B field was stationary or rotating at the same angular velocity as the cylindrical condensor.

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Motivation for Proposed Experimentation in the Realm
of Accelerated E. M. Systems:
A Preliminary Design for an Experiment

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There has been some controversy surrounding these observations. Questions have been raised whether the observations were correct, and if so, how should they be interpreted in the light of the circumstance that the effects still exist even when the solenoid generating the coaxial B field rotates with the same angular velocity as the cylindrical condensor. It is the latter fact which in our opinion makes it desirable to consider these effects as observations of a constitutive nature concerning a frame of reference rotating in free-space.

To dispel any uncertainty concerning the reality of the mentioned observations, the authors of the present note constructed a piece of equipment similar to that of Kennard and Pegram. Our as yet preliminary observations show a qualitative agreement with those of Kennard and Pegram. The conditions of our observations were between those of Kennard and Pegram in the sense that our observations were made with an electrometer that had an impedance range intermediate between open circuit (Kennard) and complete short (Pegram).

To the extent that experimental results are available it seems that the observations can be consistently described by a constitutive relation of the following form (MKS units)

$$\overline{D} = \varepsilon_o \overline{E} + \varepsilon_o (\overline{\Omega} \times \overline{r}) \times \overline{B} \tag{1}$$

in which $\overline{D}$, $\overline{E}$, $\overline{B}$ and $\overline{r}$, defined on the rotating frame, have the usual meaning, ε_o is the free-space permittivity and $\overline{\Omega}$ is the angular velocity of the system with respect to inertial space.

For cylindrical symmetry and when using cylinder coordinates, one may write eq. (1) in the form

$$D_r = \varepsilon_o E_r + \varepsilon_o \Omega r B_z \tag{2}$$

It is a well-known riddle of E.M. theory that the second term of (1) [or (2) for that matter] has all by itself a nonvanishing divergence $\mathrm{div}(\overline{\Omega} \times \overline{r}) \times \overline{B} = 2\overline{\Omega} \cdot \overline{B} \neq 0$ (see for instance Sommerfeld[4] - last page). It would then appear as if an observation made from a rotating frame would record a space-charge where none was to begin with. We will make the elimination of this absurdity a cornerstone of the next following considerations; the basic idea being that divergences of individual electric field components $\overline{E}$ contributing to a total electric displacement $\overline{D}$ are not physically meaningful.

Let us instead take the divergence of the "surface" vector $\overline{D}$ and let us insist that its divergence vanishes also on the rotating system. We obtain then for conditions of cylindrical symmetry

$$\frac{1}{r}\frac{\partial}{\partial r} r\, D_r = 0 \tag{3}$$

Solving this equation, we have

$$D_r = A/r \tag{4}$$

with A as a constant of integration.

The ideal Kennard case (open circuit - no displacement) is now characterized by A = 0. The Kennard potential can then be obtained from (2) as

$$V_k = \int_{r_1}^{r_2} E_r dr = -\frac{1}{2}\,\Omega\, B_z(r_2^2 - r_1^2) \tag{5}$$

r_1 and r_2 being the radii of the inner and outer cylinder of the tubular condensor.

In the ideal Pegram case $A \neq 0$. Its value can be calculated from the condition that the potential $\int_{r_1}^{r_2} E_r dr = 0$. One then finds for the integration constant

$$A = \frac{1}{2}\, \varepsilon_o\, \Omega\, B_z (r_2^2 - r_1^2)/\ell n\; r_2/r_1 \tag{6}$$

The Pegram charge Q_p on the condensor is obtained by integrating D_r over the surface of the cylinder of length ℓ say,

$$Q_p = 2\pi\, A\ell \tag{7}$$

Substitution of (6) gives

$$Q_p = \varepsilon_o\, \pi\, \Omega\, \ell\, B_z (r_2^2 - r_1^2)/\ell n\; r_2/r_1 \tag{8}$$

One easily verifies that the ratio of the Pegram charge (8) and the Kennard potential (5) yields (in absolute value) the standard expression for the capacitance of a cylindrical capacitor

$$C = \left|\frac{Q_p}{V_k}\right| = \frac{2\pi\, \varepsilon_o\, \ell}{\ell n\; r_2/r_1} \tag{9}$$

In the light of the mentioned experimental observations and the simple interpretation of these observations in terms of a constitutive relation of the form (1), we summarize the following points as absolutely germane to any theoretical discussion involving accelerated systems in electrodynamics:

1. The Pegram and Kennard effects are realistic observations that cannot be discounted or disregarded.
2. A very simple constitutive relation of the form $\overline{D} = \overline{D}(\overline{E}, \overline{B})$ (see 1) directly accounts for these observations, rather than the customary relations $\overline{D} = \varepsilon_o \overline{E}$ or $\overline{D} = \overline{E}$ which only hold for inertial systems in matter-free space.
3. A constitutive relation of the form (1) for a rotating system

resolves the difficulty recorded by Sommerfeld that a rotation could give rise to an apparent space-charge $\mathrm{div}(\overline{\Omega} \times \overline{r}) \times \overline{B} = 2\overline{\Omega} \cdot \overline{B} \neq 0$.

In the recent paper by T. C. Mo we find that the existence of a free-space constitutive dependance of $\overline{D}$ on $\overline{B}$ is considered as a mistaken notion (last paragraph section 4). We feel that this statement is at variance with the experimental evidence presented by Kennard and Pegram as well as with our own observations. In fact a discussion of the constitutive nature of this evidence appears on p. 490 of reference 5 cited by Mo.

The fundamental issues touched upon here go well beyond Mo's paper. The question is not whether the method of "local" inertial tetrads, as used by Mo, can be made equivalent to a method of "global" noninertial references, as used in his reference 5. One would expect such an equivalence to exist, at least locally. Remarks to the contrary by Mo are out of context.

The fundamental issue is rather whether or not the method of local tetrads is a suitable mathematical expedient that enhances physical perspicuity such as claimed by its proponents. The presented evidence hardly supports such claims.

References:

1. T. C. Mo, Journal of Math. Phys., 11 (1970) 2589.
2. E. H. Kennard, Phil. Mag., 33 (1917) 179.
3. G. B. Pegram, Phys. Rev., 10 (1917) 591.
4. A. Sommerfeld, Electrodynamics, New York (1952).

Introduction

It is a well-known and a well-established fact that accelerated charges radiate in all conceivable classical configurations, while uniformly moving charges do not partake in this phenomenon of radiative energy emission. This fundamental observation then leads to a rather basic distinction between the fields surrounding a uniformly moving charge as contrasted with the fields surrounding an accelerated charge.

For a co-moving observer accompanying a uniformly moving (point) charge solely a centrally symmetric electric field would be noticeable, while no reason can be found for an accompanying magnetic field resulting from the motion.

For a co-moving and co-accelerated observer accompanying an accelerated moving charge one would have to assume that, in addition to the centrally symmetric electric field, also a magnetic field would have to appear; if only to account for the nonvanishing integral of the Pointing vector representing the power radiated by the accelerated charge.

It would be unreasonable to expect that the power radiated by the accelerated charge would depend on whether the observer finds himself in an inertial frame or in the accelerated frame itself. The radiation of energy brings about a change in the radiating system which could not possibly be a figment of the observer's imagination related to his choice of a co-accelerated reference.

Where the Pointing vector collects the rediative far-field contributions of the E and H fields generated by the accelerated charge, it would also be reasonable to inquire into the near-field situation of the accelerated charge, specifically for the co-accelerated observer. The following thought experiment clearly illustrates the nature of the near-field situation.

A Thought Experiment

Let us consider a circular disk condensor which can be spun around its axis of symmetry (see fig. 1). The condensor is charged to a high potential. It is then known that the surface charge on the condensor plates represents a convection current which generates a magnetic field for the stationary (inertial) observer. This fact was unambiguously established around the turn of the century through a long series of experiments by Röntgen, Wilson and Eichenwald.

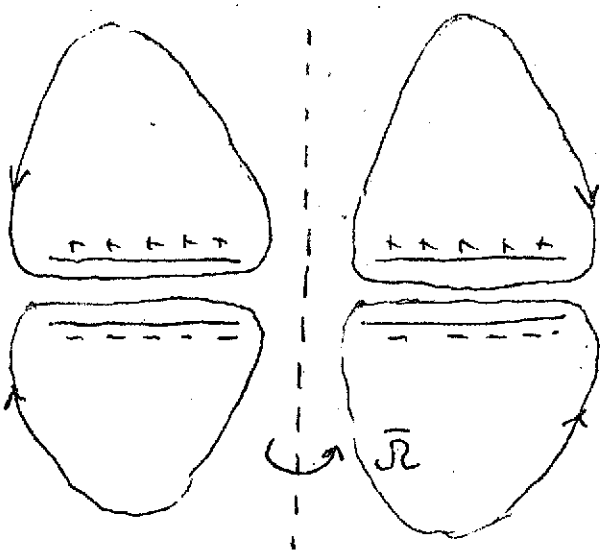

Fig. 1: Rotating charged disk condensor showing magnetic field lines generated by convected surface charges. Both plates rotate with same angular velocity $\bar{\Omega}$.

Now consider the same experiment except that the observer, instead of being in a stationary (inertial) frame, is now on the rotating system itself. Does the co-rotating observer still see a magnetic field? Or should the co-rotating observer conclude that the magnetic field vanishes, because with respect to the rotating system the convection current vanishes?

One may submit this question to any number of reputable and competent physicists and one can be sure to get a diversity of conflicting answers. In fact, they range from: there will be no magnetic field (because there is no current) - the field will be modified - the field will be the same as for the stationary observer.

There are a number of reasons to account for this disagreement among experts. The most important would probably be that people have to give themselves enough time to come up with a meaningful answer. A poll-taking is not very conducive to promote the right atmosphere for a more incisive discussion. If one takes the time for a thorough examination, the following two points seem to emerge as deserving further scrutiny.

1) Problems related to accelerated systems are only partly covered by the standard methods of the general theory of relativity. The general theory does not give unambiguous information about the induced transformation behavior of fields. Secondly, there is no general agreement about the "inducing" space-time transformation relating inertial and noninertial frames.

2) Questions arise what it means to measure a magnetic field. Does one measure the line-vector H or the surface-vector B. It seems necessary, even in free-space, to confront the possibility that H can vanish while B is different from zero. In the following discussion B and its associated magnetic flux will be regarded as the primary quantity that is measurable.

Where it does not seem conclusive to call on sophisticated theory to resolve the question presented by the thought experiment; one might take recourse to the alternative of attempting to obtain appropriate information through experimentation. Yet before doing so, one would like to have, at least, a preliminary indication of what to expect from such experimentation. For that purpose, I will present a simple argument, based on first principles, which indicates that a rotating observer in the configuration of Fig. 1 will measure the same B as the stationary observer.

Consider the rotationally symmetric B field of Fig. 1 and assume that we want to measure the nonuniformity of this field by an E M F loop vibrating up and down in the direction of the symmetry axis (see fig. 2).

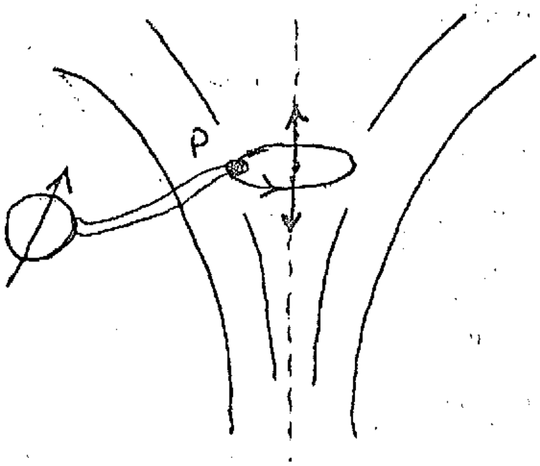

Fig. 2: Measuring the B field by a (vertically) vibrating test coil.

The E M F appearing on the test coil is taken out at the point P and then measured by an appropriate instrument. According to Faraday's law, the E M F solely depends on the change of flux through the coil. There is no indication whatsoever that the E M F would depend on a rotational motion of the coil "in itself", because the standard formulation of the induction law does not specify anything of this nature. The rotation of the coil "in itself" (P going around in the circle indicated in fig. 2) does not affect the flux through the coil. It appears that we can now state the following lemma as ensuing from the standard formulation of the induction law.

Lemma: The E M F developed on a test coil does not depend on a rotational motion of the coil in itself. Hence, the conclusions concerning the B field are not affected by this rotational motion in itsel . (P traveling around the circle in fig. 2).

Let us apply this lemma to the configuration of the thought experiment of fig. 1. It then follows that the E M F, and consequently the B field, would be the same, regardless whether the point P (where the lead wires are extracted) is at rest in an inertial frame or co-rotating with the charged condensor. This observation would be unexplainable if the B field on the rotating system would vanish. It follows that the co-rotating observer measures the same B field as the stationary (inertial) observer q. e. d.

One can now conclude that the charged rotating disk condensor of fig. 1 plus its attached co-rotating test coil and measuring instrument is an absolute rotation sensor. It provides a purely electrical criterion to decide whether or not the total instrument is in an inertial frame.

In an earlier proposal, which led to an investigation supported by NASA Grant NGR 30-002-061, it was pointed out that an effect of this nature could be expected on the basis of its dual relation to the Kennard-Pegram effect. The latter effect was reconfirmed (see forthcoming publication in J. of Math. Physics by E. J. Post and D. Bahulikar). The two effects together are intimately related to a deeper understanding of the ring-laser effect.

However, apart from the, for the time being, weak potential of these effects for guidance and control, a deeper understanding of them would be consequential for a reassessment of observations concerning terrestrial, planetary and solar magnetism. Hence, an experimental pursuit that can help to delineate the true nature of the mentioned acceleration effects would be quite germane for a meaningful data evaluation in the space sciences.

Preliminary Design Sketch for an Experiment

There are several possible physical realizations of the thought experiment. For a constant rotation one would have a constant magnetic induction which would require a test-coil performing a motion superimposed on the already existing rotational motion. One can avoid this complexity by choosing a nonuniform rotation--for instance, a torsional vibration. A torsional vibration of the charged condensor then generates an alternating convection current and a corresponding alternating flux in the inertial frame as well as in the co-accelerated frame.

Fig. 3 shows a schematic of such a torsionally vibrating arrangement. One may close the field-lines by mounting the condensor inside a ferrox-cube pot-core. The input is provided by a high voltage source to charge the condensor and a mechanical excitation device for the torsional motion. The output reading is obtained from a coil wound on the central leg of the pot-core. The output is an a. c. voltage with a frequency that equals the frequency of the torsional pendulum arrangement. A low-noise narrow band amplifier will be necessary to prevent the small output signal from being immersed in the thermal noise level. The E M F that can be extracted from the test-coil on the central leg should obey the following relation.

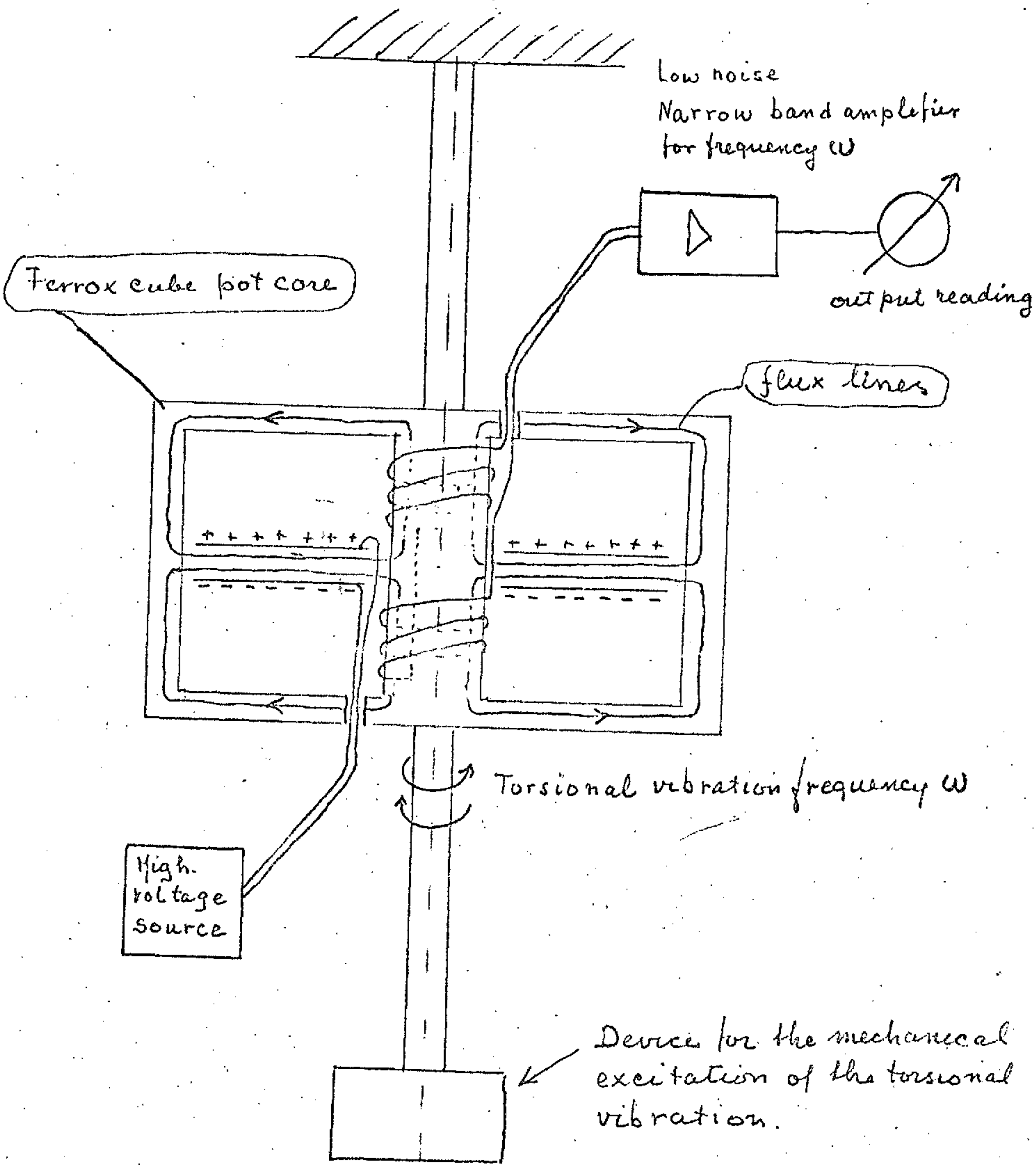

Fig 3: Schematic of a possible realization of the Thought-experiment in terms of a torsional pendulum arrangement.

$$\text{output E M F} = \left|\frac{4\pi^3 n}{c^2}\;\frac{r_2^2 - r_1^2}{\ln(r_2/r_1)}\;\nu^2 \phi_0 V\right| \qquad \text{(volts)}$$

where

n = number of turns of the coil

c = light velocity (Meters/sec)

r_2 = outer radius of disk condensor (meters)

r_1 = inner radius of disk condensor (meters)

ν = $\omega/2\pi$ = torsional vibration frequency ($\sec^{-1}$)

ϕ_0 = angular amplitude of vibration (radians)

V = potential of the condensor in volts

This expression for the output E M F can be calculated in two different ways:

1) For the stationary observer one can calculate the total sinusoidal convection current which then gives the line integral $\oint \overline{H}\cdot d\overline{l}$. The total flux can then be obtained as $\phi = B\,S = \mu_0 HS$ where S is a cylindrical surface of radius r and height d equal to the distance of the condensor plates. One finds that r drops out of the end result, hence the flux is the same at r_1 and r_2 (no magnetic sources inside the condensor space).

2) For the co-accelerated observer the convection current vanishes. The expression for the finite flux is then obtained from the modified constitutive relation (cylindrical coordinates).

$$H_r = \frac{1}{\mu_0} B_r + \varepsilon_0 \Omega r\, E_z$$

in which $\Omega = \omega\phi_0$ the amplitude of the angular velocity of the torsion pendulum. It follows from div B = 0 that B = C/r with C a constant of integration which can be determined from the condition $\oint H\, dl = 0$ (no current). The alternating flux can thus be calculated and the expression for the output E M F results.

The two calculations lead to identical results as already suggested by the conclusion of the thought-experiment discussed in the previous section.

Inserting numbers in the given formula for the output E M F one finds that the experiment is not easy, yet with the help of modern facilities it should be possible to obtain a conclusive answer one way or the other.

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A Possible Solution to Sommerfeld's Riddle+

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Abstract: An application of standard electromagnetic theory to noninertial systems leads to fundamental problems that culminate in a riddle concerning space charges that cannot possibly exist. A solution is proposed by adapting the constitutive relations so that they become also applicable to noninertial frames. The adaption has been constructed to be consistent with a number of classical experiments and with more recent ring laser experiments. New experimentation is proposed that can further substantiate or refute the proposed adaptation.

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Sommerfeld's Riddle

After an unusually rich career as scientist and teacher, Sommerfeld bequeathed to the world of physics a monumental treatise covering a major part of theoretical physics. One of the five volumes is devoted to what may well be the most-well-rounded of physical theories: the theory of the electromagnetic field. The book draws on the author's wide experience in both applied and pure aspects of the theory. It differs in a few remarkable respects from the traditional presentation of E.M. theory by physicists.

First of all Sommerfeld aligned himself with the engineers. He reversed his position with respect to the touchy question of units; a change of heart that was resented by several of his fellow physicists.

Then secondly, after expounding the beauty and consistency of relativity he confronts the reader (on the last page of his book) with a harmless looking but nevertheless disturbing difficulty occurring in the theory of rotating systems.[1]

Although it is unlikely that Sommerfeld first noticed this beauty defect, for the purpose of this article we will call it the Sommerfeld riddle, because Sommerfeld elevated the problem to a more prominent position in the textbook literature. In commemoration and honor of this healthy precedent of not evading obscure passages of well-established theory, the content of this riddle shall now be discussed.

Consider a rectangular (inertial) frame of reference in a matter and charge free region of space. Assume a uniform magnetic field of induction $\overline{B}$ in the z direction. Now rotate the frame around the z axis with an angular velocity Ω. In the rotating frame one will then observe an electric field of magnitude $\overline{E} = (\overline{\Omega} \times \overline{r}) \times \overline{B}$, in which $\overline{r}$ is the position vector of the point of observation.

The riddle comes about if we take the divergence of $E = (\overline{\Omega} \times \overline{r}) \times \overline{B}$: $\operatorname{div} \overline{E} = 2\overline{\Omega} \cdot \overline{B} \neq 0$. It seems as if the rotation produces a charge density in a region of space that was assumed to be free of matter and charge. The conclusion is obviously absurd, yet the formula for $\overline{E}$ is known to be valid for regions of space filled with conducting matter (e.g. electric machinery).

The major objective of this article is: 1) to propose a possible resolution of this apparent contradiction, 2) to show how the proposed resolution interrelates a number of otherwise disconnected classical experiments, 3) to discuss new experimentation that can further substantiate or refute the proposed resolution.

The Theoretical Nature of the Riddle

In diagnosing the nature of the problem one can firstly take cognizance of the fact that the apparent space charge does not appear if one makes a transition from one inertial frame to another inertial frame. The velocity term $\overline{\Omega} \times \overline{r}$ can then be replaced by a constant velocity $\overline{V}$. The divergence of $\overline{E}$ vanishes in the new frame if it vanishes in the original frame; in fact div E = 0 in the whole family of inertial frames. Hence one concludes that the occurrence of the apparent space charge is typically associated with a noninertial situation.

The mentioning of moving systems immediately brings to bear the question whether or not the contradiction can be resolved within the realm of the theory of relativity.

Traditionally the theory of relativity separates into two parts: the special theory of relativity and the general theory of relativity. These two theories are also known under the misnomers: theory of special relativity and theory of general relativity.[+]

The special theory of relativity is restricted to the description of physical phenomena with respect to inertial frames only. One may consider accelerations of objects with respect to these inertial frames. However, the description of phenomena as seen from noninertial frames is not covered by the special theory of relativity!

The general theory of relativity is, by contrast, a theory in which the physical phenomenon of gravitation is related to a conceivable

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[+] German is more permissive with compound nouns than English e.g. Relativitätstheorie = theory of relativity. It follows that an adjective (e.g. general) applying to such a compound noun is affected by an ambiguity in translation. This language technicality still creates much confusion in writing and discussion.

non-Euclidian structure of the space-time manifold.

It thus appears that the special theory of relativity is not suited to approach the problem at hand while the general theory seems irrelevant. What is needed is a theory that permits a description of physical phenomena as seen from noninertial frames.

Fortunately, there is a principle which permits us to relate gravity and accelerated frames of reference. It is known as the principle of "local" equivalence. It expresses the "local" indistinguishability of gravitational and kinematic acceleration.

The term local, in this context, is meant to convey the idea that the observational indistinguishability only holds if one refrains from exploring the environment of the point of acceleration. Putting it in less abstract terms: looking out of the window of one's confinement one would soon be able to tell whether or not one is affected by gravitational acceleration, kinematic acceleration or by both accelerations simultaneously. The principle of equivalence has been a key point in the development of the general theory.

One thus sees that the general theory of relativity is relevant. The mathematical formalism that accommodates gravitation also accommodates accelerated systems of reference. There is the added advantage that the description of kinematic acceleration does not require the validity of the gravitational field equations.

The mathematical implementation of the principle of equivalence draws on another principle that was also instrumental for the development of the general theory. This principle is known as the general principle of covariance or the principle of general covariance. Again, the position of the adjective "general" causes some confusion about the precise content of the principle. It suffices to mention that the name was obviously meant to create a contrast with the more restricted concepts of Lorentz covariance and Lorentz invariance.

The conceptual obscurity surrounding these principles has unfortunately led to a situation in physics where the word general covariance is hardly respectable. Nevertheless the initial attempts to understand and to describe coherently physical observations in accelerated systems mostly start in some way or another as applications of the principle of general covariance.

It is not difficult to remove formally the Sommerfeld riddle by defining an invariant divergence that vanishes in inertial frames as well as in noninertial frames.[2]

A discussion of relevant experimentation performed in rotating systems and a review of attempts at correlating these observations in the spirit of some principle of general covariance has been given by the author in a recent article.[3] Probably the principal conclusion of the latter study is the conceivable existence of a generic relation between the optical Sagnac effect (now better known as ringlaser effect) and some little known effects pertaining to the phenomenon of unipolar induction. The latter have been studied elaborately by Barnett, Bateman, Kennard, Pegram, Swann, Tate and several others.[4]

Notwithstanding the somewhat discredited position of the principle of general covariance in physics it can hardly be denied that some version of general covariance will have to be the tool for approaching problems in accelerated systems. The present author retains a personal confidence in appropriate discussions based on such principles.

However, considering the controversy surrounding these principles there is also a real need for a physically more deductive method of understanding basic phenomena in accelerated systems. The next section is devoted to such an approach. The salient features of accelerated systems are delineated in a manner that is as much as possible independent of the controversial aspects of covariance. The discussion will be restricted to uniformly rotating systems.

A further central point in the following considerations is that physical observables such as potential differences, charges and currents appear in the theory as the result of integrations. The so-called field quantities are the integrants of these integrals. These integrants are not necessarily uniquely determined by those integrals representing the observations. It is then reasonable to give a primary operational role to the integrals rather than to the integrants. Yet one can at least retain in the field quantity as much as possible the original operational qualities pertaining to the integrals they come from. The latter point is basic in the following discussions.

E.M. Relations in Rotating Systems

A few points of major concern for the developments to be presented now will appear to be rather formal in nature. It may thus be difficult to escape an impression that covariance related concepts are still coming in through the back door. But even so, if that happens, should there be any objection against a reminder of a handwriting that has been on the wall for some time?

For almost a century it has been common practice in textbook literature to have curls and divergences operate on the same vector fields. It is known that this peculiarity is a unique feature of vector analysis in three-space. A by-product of this coincidental situation is that one and the same vector field can be considered as the integrant of a line integral as well as the integrant of a surface integral.

If one searches the literature for opinions of leading physicists about this subject matter, one finds that Maxwell was among the first to have pangs of conscience about the possibly deceptive consequences of a too freely used mathematical opportunism offered by the traditional system of vector analysis. In an article specially devoted to this subject Maxwell insisted on the existence of four different vector-species in three-space. His arguments, although mathematical in nature, were motivated by physical needs.

Maxwell introduced the names force and flux vectors to correspond to the notions of line vector and surface vector. Each of these vectors can have the property of being polar or axial. A pairwise combination of these properties leads to four basic vector species in three-space. A classification of the fundamental field vectors of electromagnetic theory then leads to the following diagram.

Space Vectors	Polar	Axial
force	$\overline{E}$	$\overline{H}$
flux	$\overline{D}$	$\overline{B}$

in which $\overline{E}$ and $\overline{D}$ are the electric field and electric displacement while $\overline{H}$ and $\overline{B}$ are the magnetic field and the magnetic induction.[+]

For matter-free space, it is known that there is a very simple relation between these field vectors. In fact by choosing a somewhat ad-hoc system of mixed units one can further promote the simplicity of the relation and bring about an actual identification of the electric and magnetic field vectors respectively: $\overline{E} = \overline{D}$ and $\overline{H} = \overline{B}$. It is not normally explicitly stated whether this identification is good for inertial as well as for noninertial frames of reference. Standard texts are usually tacitly restricted to an inertial frame treatment of E.M. theory. To substantiate this statement I refer to a particularly authoritative text in which this obscurity is not swept under the rug. Feynman[6] simply declares E.M. theory not to be valid outside the family of inertial frames[++]; a drastic point of view which surely guarantees the avoidance of Sommerfeld's riddle.

However, in this article we address ourselves to the problem of presenting a possible solution, not an avoidance, of the Sommerfeld riddle. Hence the position taken by Feynman is of no help, in fact it is unnecessarily restrictive if we consider the classical applications of E.M. theory in rotating machinery. Feynman rejects the

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\+ A more elaborate discussion of the four vector species in three-space and their relation to transformational properties in three-space and four-space is given in chapters II and III of ref. 5.

++ I quote from Feynman, ref. 6, section 14.4: "We must be sure to use equations of electromagnetism only with respect to inertial coordinate systems."

problem out of hand, Sommerfeld takes a more constructive position.

Proceeding in the spirit of Sommerfeld by applying E.M. relations to rotating systems, we may now make the observation that there is no ground whatsoever for assuming the field identification $\overline{E} = \overline{D}$ and $\overline{H} = \overline{B}$ to hold and to be meaningful outside the realm of inertial frames. In fact unless we want to bereave ourselves from the onset of any possible solution, we do well in maintaining the distinction of vector species as originally indicated by Maxwell. The latter statement is not only meant to apply to macrophysical situations concerning material media. It will also be necessary to maintain the distinction in matter-free space if the system of reference is noninertial.

Having thus eliminated the most obvious inertial frame characteristics from the commonly presented form of E.M. theory, we can now turn to the question how an accelerated frame, and a rotating frame in particular, affects the fundamental E.M. relations. Does a rotation affect the Maxwell equations, does it affect the constitutive equations or does it affect both simultaneously?

The position taken in the present attempt at resolving the Sommerfeld riddle is the following:

1. The Maxwell equations retain their form on accelerated frames, provided they are expressed in terms of the four distinct field quantities $\overline{E}$, $\overline{B}$ and $\overline{D}$, $\overline{H}$.
2. The criterion, whether or not a frame is accelerated depends completely on the constitutive relations between $\overline{E}$, $\overline{B}$ and $\overline{D}$, $\overline{H}$.

The choice presented here is not arbitrary, because nothing in the fundamental observations leading to the Maxwell equation restricts them to inertial frames. The first set of equations follows from the

Faraday induction law and from the absence of magnetic charge. There is nothing in Faraday's observation that restricts the law to be valid in inertial frames only! Furthermore one would not expect the absence of magnetic charge to depend on whether or not one observes from an inertial or from a noninertial frame! Similar conclusions can also be extracted from the equations denoting continuity of charge and the Biot-Savart relations.

It was necessary to waste some time in combat against established inertial frame habits in order to set the stage for meaningful non-inertial work. We can now concentrate on how to modify the constitutive relations so as to include the treatment of rotating frames.

Let us examine the consistency of the following set of constitutive equations for a frame rotating with angular velocity $\overline{\Omega}$ with respect to inertial space. Note that all field vectors are referred to one and the same noninertial frame; these equations are not transformational relations!+

$$\overline{D} = \varepsilon_o \overline{E} + \varepsilon_o (\overline{\Omega} \times \overline{r}) \times \overline{B} \qquad \text{(a)}$$

$$\overline{H} = \frac{1}{\mu_o} \overline{B} + \varepsilon_o (\overline{\Omega} \times \overline{r}) \times \overline{E} \qquad \text{(b)} \qquad \text{I}$$

Retaining the Maxwell distinction of four field vectors, it was only an appropriate expedient to use MKS units; ε_o and μ_o are the usual free-space permittivity and permeability. One may consider ε_o and μ_o as operators needed to convert force-vectors into flux-vectors.

The following observations can be made about these proposed constitutive equations:

\+ The equations I and II can be obtained by a transformational procedure - see ref. 3, formulae 74 and 76.

a. They reduce to the familiar relations $\overline{D} = \varepsilon_o \overline{E}$ and $\overline{B} = \mu_o \overline{H}$ for an inertial frame if $\overline{\Omega} = 0$.

b. The second term in the righthand member of equation Ia resembles the induction field that led to the Sommerfeld riddle. The total displacement D is generated by the sum of two electric fields: a source related field and an emf. The divergence of D does not lead to a contradiction now.

c. The existence of the second term in the equation Ib can be inferred from the assumption that the Lagrangian should be a total differential in the field variables $\overline{E}$ and $\overline{B}$. The term has the characteristics of an H field generated by a convection current.

It may be mentioned that the constitutive equations I bear some resemblance to the constitutive equations of a uniformly translating material medium. The extra terms in the latter vanish if the product of relative permittivity ε_r and the relative permeability μ_r approach unity.

Let us next consider the assignment of extending the equations I for a corotating dielectric of relative permeability ε_r. It is then obvious how the first terms in the righthand member of equation I will be affected. To arrive at a conclusion of what happens to the second terms it is useful to consider the following thought experiment.

For a rotating system it is natural to examine a coaxial capacitor that is being rotated about its axis of symmetry. First consider the case without a comoving dielectric. Assume the capacitor to be charged, say the outer cylinder is positive and the inner cylinder is negative (see Figure 1).

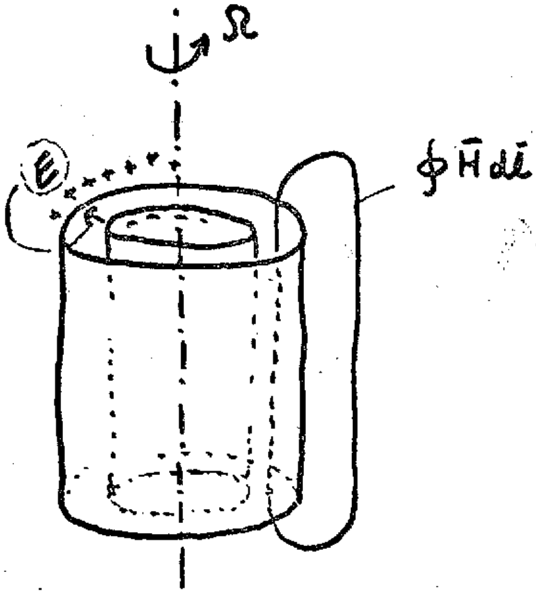

Figure 1. Rotating charged coaxial condensor.

When examined from an inertial frame of reference one will notice that the convected surface charges represent closed current loops that will result in a residual magnetic moment as seen by the inertial observer. The surface charge per unit area (on the outer cylinder, say $r = r_2$) is $\varepsilon_o E$. A calculation of the total current $\mathfrak{J}$ enclosed by the line integral of H (see Figure 1) leads to the expression

$$\mathfrak{J} = \Omega' \; r_2 \; \varepsilon_o \; E' \ell = \oint H' d\ell' = \oint \frac{1}{\mu_o} B' d\ell' \tag{1}$$

ℓ is the length of the tubular condensor. Primed symbols refer to quantities in the inertial frame.

Let us now consider an observation on the rotating frame. The convection current as seen from the rotating frame vanishes. Hence the line integral $\oint H d\ell$ when taken on the rotating system also vanishes, because the Maxwell equations, in this case the Biot-Savart integral law, is not affected by going to a rotating system (consult the set of underlying assumptions). It then follows from Ib that

$$\oint \frac{1}{\mu_o} \overline{B} \cdot d\overline{\ell} = -\oint \varepsilon_o (\overline{\Omega} \times \overline{r}) \times \overline{E} \cdot d\overline{\ell} \tag{2}$$

The righthand member of (2) reduces for the case of the cylindrical arrangement of Figure 1 to

$$\oint \frac{1}{\mu_o} \overline{B} \cdot d\overline{\ell} = - \Omega \, r_2 \, \varepsilon_o \, E\ell \qquad (2a)$$

A comparison with (1) shows that the B' in the primed system is equal in magnitude to the B in the unprimed system if E = E'. The E and E' would only be different if there had been a strong B field to begin with ($E' = \overline{E} + (\overline{\Omega} \times \overline{r}) \times \overline{B}$). Generated by E itself the B in this case is already higher order small. Hence the assumption that E = E' is well justified ipso facto $\overline{B}' = \overline{B}$.

The only field variable that changes drastically in transformation is H'. Indeed $\overline{H} = \overline{H}' - (\overline{\Omega} \times \overline{r}) \times D' = 0$, follows in essence from (1). The change in sign is properly resolved by a consistent convention for the sign of Ω i.e. $\Omega' = -\Omega$.

We are thus confronted with the remarkable situation that B is unaffected but H goes to zero when going from the inertial frame to the rotating frame. Of course, <u>locally</u> one can always define an H equal to B/μ_o provided its closed loop line integral vanishes. This local H is derivable from a potential and has at most an ad hoc physical meaning.

It is now a simple matter to see what happens to what may be appropriately called the (non) inertial terms in I if the capacitor is being filled with a corotating dielectric. There is now the additional surface charge of polarization, P rotating along with the surface charge D of the capacitor. It follows that the net charge per unit area carried around is still $\varepsilon_o E$, because $\varepsilon_o \overline{E} = \overline{D} - \overline{P}$ (see Figure 2). Hence the inertial term in Ib is not affected. The usual thermodynamic type argument can then be invoked to argue that also the inertial term in Ia is not affected. The constitutive equations for a corotating dielectric may thus be written:

$$\overline{D} = \varepsilon_r \varepsilon_o \overline{E} + \varepsilon_o(\overline{\Omega} \times \overline{r}) \times \overline{B} \qquad \text{(a)}$$

II

$$\overline{H} = \frac{1}{\mu_o}\overline{B} + \varepsilon_o(\overline{\Omega} \times \overline{r}) \times \overline{E} \qquad \text{(b)}$$

Figure 2. Convected charges in a coaxial condensor filled with a corotating dielectric.

A further, not yet mentioned, assumption underlying the equations II is that ε_r is not significantly affected by the rotational accelerations. This assumption is well supported for most practical purposes, because the inter-molecular binding forces of the polarization charges prevail over the acceleration forces.

A Calculation of the Kennard and Pegram Effects

The most striking experiments that support, at least in the sense of a compatibility, the constitutive equations Ia are probably the Kennard[7] and Pegram[8] experiments.

Both experiments utilize a coaxial rotating capacitor as in Figure 1. A strong axial magnetic field is generated by an energized coil coaxial with the capacitor.

In the Kennard experiment one measures a potential difference between the plates of the capacitor when the capacitor is rotating.

In the Pegram experiment one measures a charge on the capacitor when the capacitor is being shorted by a corotating short during the rotation.

For both experiments it seems to be immaterial whether the coil generating the B field is stationary or corotating with the capacitor. The two effects thus depend solely on the rotation of the capacitor with respect to inertial space. The mutual motion of coil and capacitor does not affect the observation.

So far the experiments have been performed without a corotating dielectric (air dielectric). A beauty defect of the Kennard experiment is that the potential difference is measured in the stationary frame via a pair of sliprings.

The early explanation of these effects have been a point of much theoretical discussion[4]. The interpretations tended to be oriented towards an e.m.f. effect asymptotically related to Faraday's induction law.

Let us now calculate the effects on the basis of the constitutive equations I and II. We may consider right away the case of a corotating dielectric. The result should reduce to the free-space case for $\varepsilon_r = 1$. We then use the relation IIa

$$\overline{D} = \varepsilon_r \varepsilon_o \overline{E} + \varepsilon_o(\overline{\Omega} \times \overline{r}) \times \overline{B} \qquad \text{IIa}$$

For cylindrical symmetry it is natural to use cylindrical coordinates and to consider only the radial components of $\overline{E}$ and $\overline{D}$ and the z component of B. The expression IIa then becomes

$$D_r = \varepsilon_r \varepsilon_o E_r + \varepsilon_o r_2 B_z \Omega \qquad (3)$$

The absence of a free charge in the dielectric shall now be made the cornerstone of the discussion. (For matter-free space that is the absence of the Sommerfeld riddle). Hence

$$\text{div } \overline{D} = \frac{1}{r} \frac{\partial}{\partial r} r D_r = 0. \qquad (4)$$

Solving this equation we find

$$D_r = A/r, \qquad (5)$$

in which A is a constant of integration.

In the (ideal) Kennard case the constant of integration A is zero, because $\overline{D} = 0$. It then follows from (3) that

$$E_r = \frac{-1}{\varepsilon_r} \Omega r B_z \qquad (6)$$

The potential difference V between the plates of the coaxial capacitor becomes if r_1 and r_2 are the radii of the inner and outer conductor

$$V = \int_{r_1}^{r_2} E_r dr = - \frac{\Omega B_z}{\varepsilon_r} \frac{(r_2{}^2 - r_1{}^2)}{2} \qquad (7)$$

In the (ideal) Pegram case $\overline{D} \neq 0$ but the potential between the plates of the capacitor is zero. It then follows from (3) and (5) that

$$E_r = \frac{1}{\varepsilon_r} \Omega \, r \, B_z - \frac{A}{\varepsilon_o \, \varepsilon_r \, r} \tag{8}$$

The potential difference V is now zero. Hence from (8) we have that the constant of integration A is

$$A = \varepsilon_o \, r \, B_z \frac{r_2^{\,2} - r_1^{\,2}}{2} \Big/ \ln \frac{r_2}{r_1} \tag{9}$$

From (5) and (9) one obtains as the total charge Q on the capacitor

$$Q = 2\pi\varepsilon_o r \, B_z \, \ell \frac{r_2^{\,2} - r_1^{\,2}}{2} \Big/ \ln(r_2/r_1) \tag{10}$$

where ℓ is the length of the cylinder.

The capacitance C of a tubular cylindrical capacitor of length ℓ is given by the expression

$$C = \frac{2\pi \, \varepsilon_o \, \varepsilon_r \, \ell}{\ln(r_2/r_1)} \tag{11}$$

It follows from (7) and (10) that the ratio of the Pegram charge and the Kennard potential still reproduces the conventional capacitance of a cylindrical capacitor (11).

However, in marked contrast with an emf based interpretation of the Kennard and Pegram effects, we find that the Pegram charge Q is independent of the relative permittivity ε_r of the corotating dielectric, while the Kennard potential decreases in the ratio $1/\varepsilon_r$; see equations (10) and (7). For an emf based interpretation one would obtain a Kennard potential independent of ε_r while the Pegram charge Q would increase in the ratio ε_r. Hence here is a point susceptible to experimental check.

A Survey of Relevant Experimentation

The Kennard and the Pegram experiments discussed in the previous section are among the important and crucial ones that can be considered to test the proposed solution of the Sommerfeld riddle. However, the experiments that have been performed so far all utilized a tubular capacitor with a free-space (air) dielectric. A simple comparison with the earlier emf based explanations of these effects shows that the predicted result is exactly the same as the ones obtained in the previous section, because the relative permittivity equals unity; $\varepsilon_r = 1$. It would be a different matter if $\varepsilon_r \neq 1$.

In order to stipulate precisely what experimentation would be indicated to resolve the matter more conclusively, let us reiterate the basic premises of the presently proposed approach for noninertial frames: 1) the Maxwell equations retain their usual form in noninertial frames provided we retain the distinction of four basic field quantities; the reduction to two field quantities was found to impose an unmentioned hidden restriction to inertial frames. 2) The behavior in noninertial frames is solely and completely describable by an appropriate modification of the constitutive equation with so-called "inertial terms" (see eq. I and II). Note that the modification also occurs in matter-free space.

It follows that the burden of experimental proof must be sought in experimentation testing constitutive behavior in noninertial frames. These experiments may include static constitutive behavior (Kennard and Pegram) as well as dynamic constitutive behavior (Sagnac and ring laser effects). It has been shown in a previous study (see ref. 3) that the inertial terms in the constitutive equations are indeed essential for describing the Sagnac effect.

Summarizing, the following three points can then be cited as directly supporting the salient features of the proposed procedure for treating noninertial systems:

1. The Sommerfeld riddle can be resolved.
2. Within experimental precision, the correct values for the free-space Kennard and Pegram experiments can be calculated.
3. The inertial terms in the constitutive equations yield a connection between the static Kennard and Pegram effects and the dynamic Sagnac or ring-laser effect. This latter relation also holds for $\varepsilon_r \neq 1$.

The point (2) at this stage is at most a compatibility check, because the Kennard and Pegram experiments have not been performed as yet with a corotating dielectric. Sagnac experiments have been performed with the light beam traversing a comoving refracting medium, so here is a positive but not yet unique support for the equations II.

To obtain more conclusive evidence to support or refute the proposed procedure of treating noninertial systems, the following three points may be considered. They contain suggestions for further experimentation that conceivably could swing the evidence more clearly pro or con.

1. An improved Kennard experiment should be performed with the potential measuring probe on the rotating system itself.
2. The (improved) Kennard experiment and the Pegram experiment should then be repeated with a corotating dielectric ($\varepsilon_r \neq 1$).
3. Then, for the sake of completeness, here is the dual of the Sommerfeld riddle: does the magnetic moment generated by a convection current still exist for the comoving observer?

(Section III of this article). Some classical experiments have confirmed that the magnetic moment indeed exists in the inertial frame. So far there is no explicit experimental verification that this is still the case for the comoving observer.

Finally there is an indirect experimental check on the constitutive relations I and II by virtue of the fact that Kennard and Pegram effects are brother and sister to the so-called ring-laser effect. To see how this family relation can be, note that the latter is related to the resonance splitting of a closed optical circuit, when the mirrors and beam splitter determining that circuit are at rest in a frame that is rotating with respect to inertial space. The magnitude $\delta\omega$ of the resonance splitting can be calculated from the Boltzmann-Ehrenfest relation for an adiabatic change of state. The energy density changes are obtainable from II and the corresponding energy changes in the optical circuit are obtainable by integration over the light path. One finds after some simple reductions, assuming energy conservation in the light beam

$$\frac{\delta\omega}{\omega} = \pm \frac{\oint \bar{\Omega} \times \bar{r} \cdot d\bar{r}}{c \oint n \, ds} \tag{12a}$$

The integral in the denominator is the Fermat integral with $n = \sqrt{\varepsilon_r}$, the index of refraction in the light path. The path of integration is the closed optical circuit; $c^{-1} = \sqrt{\varepsilon_o \mu_o}$.

From the invariance of phase (see ref. 3) one obtains a similar relation

$$\frac{\delta\omega}{\omega} = \pm \frac{\oint \bar{V} \cdot d\bar{r}}{c \oint n \, ds} \tag{12}$$

in which $\bar{V}$ is the velocity field denoting the motion (and possible deformation) of the optical light path.

A comparison shows that eq. (12a) is contained in eq. (12). It follows in addition from eq. (12) that the ring-laser effect is independent of the center of rotation. A situation of potential experimental interest occurs ehen the index of refraction approaches zero such as in a wave guide near the cut-off frequency. Note also that $\delta\omega = 0$ if $\overline{V}$ = constant, relativity of uniform motion.

An interesting difficulty arises when we permit also magnetic permeable media in the light path $\mu_r \neq 1$. What happens experimentally is a rhetoric question, because in the optical range the permeability of all materials approaches unity except perhaps for the fourth or fifth decimal place. The theoretical aspects, however, are of some interest. The obvious extrapolation of the eq. II for $\mu_r \neq 1$ yields the following expression for the resonance splitting

$$\frac{\delta\omega}{\omega} = \pm \frac{\oint \mu_r \, \overline{\Omega} \times \overline{r} \cdot d\overline{r}}{c \oint \sqrt{\varepsilon_r \mu_r} \, ds} \tag{12b}$$

It is still true that $\delta\omega = 0$ for $\Omega = 0$, however, the eq. (12b) is not any longer contained in the "kinematical" result (12), such as was the case for eq. (12a). It thus follows that the effect (12b) does not in general share the property of being independent of the center of rotation.

The premise underlying all relations (12) but in particular (12b) assumes that ε_r, μ_r and n are not subject to dynamical changes due to acceleration forces experienced in noninertial frames. It is easy to show that this assumption is well justified for ε_r, because gravitational interaction is very weak compared to electric interaction. It is more difficult to justify the same statement for μ_r; the well-known Larmor theorem clearly illustrates the nature of that limitation.

Summarizing one does well to conclude that the equations II are not to be considered as the last word in constitutive behavior in rotating frames. Their extrapolation to media with $\mu_r \neq 1$ should be considered with due caution.

The claim of some authors[9] of having resolved this difficulty purely through the use of transformational procedures concerning the observer should be also considered with due reservation. Their result hinges on their assumption - quote - that they do not see the experimental need for considering derivatives of the four-velocity - unquote. It is dangerous to infer a general conclusion from such a highly specialized assumption. Moreover it is easy to think of realistic experimental situations where the assumption does not hold.

Perhaps experimentation of the Kennard-Pegram type could help to resolve this matter further, although the fact that almost all magnetic media have an appreciable loss angle is a serious handicap in performing such experimentation.

APPENDIX I

There exists a theoretical approach which is just about the opposite of the one suggested in this article. Instead of using a single global frame that is accelerated with the system, one uses a method that is akin to one common in fluid dynamics (Lagrangian coordinates). In every point of the system a local inertial tetrad is defined. These tetrads move uniformly with the instantaneous velocity at that point and at that time.

The constitutive relations with respect to such local inertial tetrads are, of course, the same as in any inertial system. By contrast the Maxwell equations now change their form when going from an inertial to a noninertial situation. The noninertial situation is represented by the fact that the local inertial tetrads change their orientation from space-time point to space-time point.

The mathematical implementation of this method of local inertial tetrads is very cumbersome. The curl, divergence and gradient expressions require additional terms related to the so-called linear connection between the local tetrads.

Historically the method of local inertial tetrads is a natural extension of the method of local cartesian triads. The latter is normally used when introducing curvilinear coordinates in three dimensional space.

The method of local inertial tetrads does not lend itself to a lucid discussion of the Sommerfeld riddle or the Pegram and Kennard effects. In fact the chances are that one does not recognize and identify these matters as realistic physical issues (see last paragraph of section 4 of ref. 10).

The physical advantages claimed for the method of local inertial frames is that it enables one to work in a coordinate environment with which one believes to be familiar. Indeed most of contemporary physics emerged from inertial frame considerations. Many of its notions do not permit a simple extrapolation to noninertial situations. Fortunately noninertial situations can be frequently evaded in practice.

In cases where the noninertial situation cannot be circumvented the method of local inertial tetrads seems to be the wrong mathematical tool for the job. Its use is prompted by psychological rather than practical considerations. Additional references concerning origin and use of the method can be found in reference 10.

APPENDIX II

Readers who may have taken the trouble of consulting reference 3 may have found that the definition of field quantities used in that article differs in a perhaps disturbing manner from the conventional definitions. The unconventional choice was made to open the possibility of using mathematical methods related to the exterior differential calculus. The unconventional field quantities are the pure coefficients of differential forms. For the conventional definition one separates out a scale factor to retain the dimensional homogeneity of the field components. More details of this procedure can be found in references 2 and 5. For the purpose of translating the unconventional into the conventional field quantities (and vice versa) for the case of cylindrical coordinates the following table is provided:

UNCON.	CON.	UNCON.	CON.
E_r	E_r	D_r	rD_r
E_ϕ	E_ϕ/r	D_ϕ	D_ϕ
E_z	E_z	D_z	rD_z
H_r	H_r	B_r	rB_r
H_ϕ	H_ϕ/r	B_ϕ	B_ϕ
H_z	H_z	B_z	rB_z

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