Feasibility of Coherent X-Ray Production by X-Ray Pumping

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It is suggested that coherent x rays can be produced by inverting the electron population in a suitable target, such as Li, through irradiation with x rays generated by fast electrons traversing an electromagnetic field (as in a storage ring). Conditions to be satisfied by target and radiation parameters are stated, and examples given.

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Coherent x rays could be produced through stimulated emission from a working substance containing an inverted electron population, e.g., from an assemblage of atoms with K-shell vacancies. Laser photons have been used for pumping, with varying success. Pumping with x rays of energy just above the K edge would offer the advantage of high efficiency because (1) the photoelectric cross section is large, and (2) the pumping radiation would be used directly, rather than through an intermediary plasma.

X rays radiated by fast electrons passing through an electromagnetic field could be suitable for pumping. The electrons may be circulating in a storage ring; the field may be the regular field of the ring or an intense magnetic field inserted in the path of the electrons to cause a sudden "bend" or "wave" in their orbit, or a radiation field shaped by an optical element called the "template" causing fast electron oscillations and associated x-ray emission.

We consider a target (Fig. 1) containing "active atoms" (the working substance) as well as other "inert atoms." The pumping x-ray beam travels parallel to the z axis, entering the target at \( z=0 \). Several conditions must be satisfied:

1. The coherence length of the emitted x rays (i.e., the distance they can travel before the various Fourier components in the beam begin to get significantly out of phase) must exceed the dimensions of the target; otherwise x rays produced in different parts of the target cannot be coherent:

\[
d_x, d_y, d_z \lesssim \tau_K c.
\]
Here, $\tau_K$ is the K-vacancy lifetime in active atoms, and $c$ is the velocity of light.

(2) Photons of the stimulated-emission energy $E_o$, traveling in the $z$ direction, are absorbed with a total cross section $\sigma_{ti}$ by inert atoms, of which there are $\rho_i$ per unit volume, with $\sigma_{ta}$ by active atoms in their ground state ($\rho_a$ per unit volume), and with $\sigma_{tk}$ by active atoms containing a K vacancy (of which there are $\rho_k$ per unit volume):

$$dN(E_o)/dz = -[\rho_i \sigma_{ti} + \rho_a \sigma_{ta} + \rho_k \sigma_{tk}],$$

where all cross sections are evaluated at $E_o$. If $\sigma_s$ is the stimulated-emission cross section, $N(E_o)$ will increase as

$$dN(E_o)/dz = \rho_k \sigma_s,$$

and there will be a net increase if (3) exceeds (2). If almost all active atoms contain a K vacancy ($\rho_a \approx 0$), this condition is

$$\rho_k \sigma_s = (\rho_i \sigma_{ti} + \rho_k \sigma_{tk})\beta; \quad \beta > 1.$$  

(3) The number of x-ray photons emitted per unit target volume in a short time interval $\Delta t$ by spontaneous K-vacancy decay is $\rho_k \Delta t/\tau_K$. Each of these photons can stimulate the emission of more photons, which in turn can cause further induced emission, etc. Photons from spontaneous decays are emitted randomly in all directions. A few, with momentum parallel to $z$, have a chance to produce a strong coherent x-ray pulse by deexciting atoms all along the target. Most, however, leave the target "sideways" after traveling only a short distance. They are undesirable, because by deexciting some atoms they reduce $\rho_k$. The number of these
undesirable deexcitations must be kept small. Then the exponential increase of photon number along the path of each undesired spontaneously emitted photon can be approximated by a linear increase, and the number per volume element of photons produced in $\Delta t$ by undesirable induced emission is $\left(\frac{\Delta t}{\tau_K}\right)\rho_K^2 \sigma_s d$ (where $d$ is the average distance in the target traveled by the undesirable photons). We wish to insure that this quantity is much smaller than $\rho_K$. For a long thin target, with $d$ of the order of $d_y$ we require $\left(\frac{\Delta t}{\tau_K}\right)\rho_K^2 \sigma_s d_y \ll 1$. We also want to keep the number of spontaneous deexcitations during $\Delta t$ small, i.e., $\Delta t/\tau_K \ll 1$. It follows that we need

$$\varepsilon \equiv \rho_K^2 \sigma_s d_y < 1. \quad (5)$$

(4) The pumping x-ray photons have momentum $k_o$ parallel to the $(y,z)$ plane, and the angle between $k_o$ and the $z$ axis is $\theta$ (Fig. 1). The path-length in the target of unabsorbed pumping photons is $d_x/\sin\theta$. We neglect spatial variations of $\rho_{\perp}$, $\rho_a$, and $\rho_K$ within the target and assume that only a small fraction of all pumping photons is absorbed. Then during one passage through the target, a pumping photon will augment $\rho_K$ by creating $\rho_a \sigma_p d_x/\sin\theta$ $K$ vacancies, where $\sigma_p$ is the appropriate photoionization cross section. Let $\tau_b$ be the duration of a pumping photon beam pulse, for which we assume the beam intensity to be a step function in time. The number of pumping photons per unit volume and unit time (during $\tau_b$) is $N_y (d_x d_y d_z \tau_b)^{-1}$, so that we have

$$-\frac{d\rho_a}{dt} = \frac{d\rho_K}{dt} = \left(\frac{N_y}{\tau_b}\right) (d_x d_y d_z)^{-1} \sigma_p a_y (\sin\theta)^{-1}, \quad (6)$$
where $\bar{\sigma}_p$ is averaged over the energy of the pumping photons. We have neglected unwanted spontaneous decay, in accordance with condition (5). Let pumping start at $t=0$, then we have the boundary condition $\rho_K(0)=0$, with which the solution of Eq. (6) is

$$\rho_K(t) = \rho_a(0) \left[ 1 - \exp\left(-N \sigma t / \tau_p \right) \right].$$

(7)

The condition $\rho_K(0)=0$ implies

$$\rho_a(t) + \rho_K(t) = \rho_a(0).$$

(8)

If at some time $\Delta t$ a $K$ vacancy exists in almost all active atoms, then $\rho_a(\Delta t)/\rho_K(\Delta t) \ll 1$. Using Eqs. (7) and (8), this can be rewritten

$$\delta \equiv \exp\left(-N \sigma \Delta t / \tau_p \right) \ll 1, \text{ if } dy / \sin \theta \ll dz.$$

(9)

(5) After one photon of the right energy $E_o$ enters the target with momentum $\vec{k}_o$ parallel to the $z$ axis, the number of photons with the same energy and momentum will increase exponentially due to induced emission. At $z=d$, we have

$$N(E_o, \vec{k}_o, z) = \exp \left\{ -\rho_i \sigma_{ti}(E_o) - (\rho_a \sigma_{ta}(E_o) - \rho_K \sigma_{tK}(E_o) + \rho_K \sigma_s) d_z \right\}.$$

(10)

If $\rho_a$ can be neglected, then

$$N(E_o, \vec{k}_o, z) = \exp \left\{ [\rho_K \sigma_s - \sigma_{tK}(E_o)] - \rho_i \sigma_{ti}(E_o) d_z \right\}.$$

(11)

The device will produce significant amplification if

$$f \equiv \ln N(E_o, \vec{k}_o, z) = [\rho_K \sigma_s - \sigma_{tK}(E_o)] - \rho_i \sigma_{ti}(E_o) \gg 1.$$

(12)

The cross section for stimulated emission is

$$\sigma_s = (\lambda_o^2 / 2\pi) \omega_K.$$

(13)
where \( \omega_K \) is the K-shell fluorescence yield. To satisfy conditions (1), (4), (5), (9), and (12), we look for an atom with a relatively long K-vacancy lifetime \( \tau_K \). For atoms with more than 3 electrons, we have \( \tau_K < 10^{-14} \) s. However, an isolated Li atom with a K-shell vacancy has a mean life of 500 \( \mu \)s for the two-photon 60.7-eV El decay from the \( 1S_0 \) state and a mean life of 50 s for the 54.8-eV Ml decay from the \( 3S_1 \) state; Auger transitions are impossible, whence \( \omega_K = 1 \).

We find \( \sigma \approx 5 \times 10^{-18} \) cm\(^2\) near 60 eV. 5 When the atom is not isolated, the effect of neighboring atoms must be taken into account. We concentrate on the 54.8-eV transition and consider three special cases:

(A) In a pure Li gas, \( \tau_K \) will be governed by the collisional deexcitation rate. We estimate that one would need very low density,

\[ \rho_K < 10^{13} \text{ cm}^{-3} \]

[\( \rho \) as in Eq. (4)], and a very long device, \( d > 10^5 \) cm for \( f > 2 \) [\( f \) as in Eq. (12)].

(B) In a metal (pure Li or an alloy), a 10% to 90% p-state admixture in band electrons might lead to a radiative lifetime of the order of 1 ns; \( \tau_K \) would be governed by radiationless transitions; one could expect \( \omega_K \approx 10^{-4} \).

(C) In most crystals \( \tau_K \) will be short, as can be inferred from LiF Auger spectra. However, in LiH one may expect \( \omega_K \approx 10^{-3} \) and \( \tau_K \approx 10^{-13} \) s, possibly longer. We consider LiH in some detail.

Let the pumping radiation have an energy spectrum that is constant from 55 to 75 eV and zero elsewhere, and angular divergence \( \Delta \theta \approx 0.5 \times 10^{-3} \) rad, as for synchrotron radiation from a storage ring like SPEAR when the energy maximum is near 60 eV. With appropriate mirrors
and windows the step-function like energy spectrum can be achieved.
(We keep $E > E_0$ so that the pumping radiation cannot induce stimulated emission.) Then we have $\sigma_p = 5 \times 10^{-18}$ cm$^2$ and $\sigma_{ti} = 1.5 \times 10^{-19}$ cm$^2$. Let the LiH be deposited on an inactive surface in a nearly monomolecular layer, $d \approx 10^{-8}$ cm. (Dilution of the layer with a low-$\sigma_{ti}$ substance or empty space may be desirable.)

With essentially all Li atoms pumped up, we have $\rho_K \approx \rho_i$ and condition (4) becomes

$$\beta = \sigma_s [\sigma_{ti}(E_0) + \sigma_{tk}(E_0)]^{-1} > 1.$$  

With Eq. (13) and assuming $\sigma_{tk} \approx \sigma_{ta}$, this requirement becomes

$$\omega_K > 1.1 \times 10^{-6}.$$  

It further follows [Eq. (5)] that $\Delta t/\tau_K \ll 1$, and

$$\rho_K \omega_K \ll 1.7 \times 10^{20} \text{ cm}^{-3}$$  

are required. Let the average $\theta \approx \sin \theta \approx 10^{-3}$ rad, then condition (9) becomes

$$\delta = \exp\left[-(\Delta t/\tau_b)(N_\gamma/d_x d_z)5 \times 10^{-15} \text{ cm}^2\right] \ll 1, \quad d_z > 10^3 d_y,$$

and with $\rho_i \approx \rho_K$, condition (12) is

$$f = \rho_K \left(6.0 \times 10^{-13} \omega_K - 6.5 \times 10^{-19}\right) d_z \gg 1.$$  

We take the inequality $\omega_K > 1.1 \times 10^{-6}$ to be satisfied and need to insure that the remaining conditions listed in the preceding paragraph, as well as Eq. (1), are met. Let $\omega_K = \mu x 10^{-3}$; we expect $10^{-2} < u < 10$. The upper limit on $\rho_K \omega_K$ then is obeyed if $\rho < 10^{-2} u^{-1}$, whence the LiH must be diluted by a volume factor of at least $5u$. The second term in parentheses in Eq. (16) can be neglected compared with the first, and we find $d_z \gg 1.5 \times 10^{-5}$ cm. To satisfy $\Delta t/\tau_K \ll 1$, we choose $\Delta t = \tau_K/5$. 
We assume that the pumping radiation is produced by synchrotron radiation from ~400-MeV electrons in a magnetic field as in SPEAR, where the vertical diameter of the electron beam in the interaction region is ~10^{-3} cm and the angular divergence is ~10^{-4} rad. Remembering that the beam diameter scales linearly with energy, and focusing the pumping radiation on a spot of diameter equal to that of the beam (not less, so as not to increase \( \theta \) and hence requiring a larger \( N_x \)), we choose \( d_x = 10^{-4} \) cm. We assume \( T_B = 10^{-10} \) s, as planned for SPEAR. Under these circumstances, Eq. (15) leads to \( N_x \approx 10d_z^2/T_K \), with \( d_z \) in cm and \( T_K \) in s. For \( 10^{-14} < T_K < 10^{-12} \) s, Eq. (1) requires that \( d_x, d_y, \) and \( d_z \) be less than a length between \( 3 \times 10^{-4} \) and \( 3 \times 10^{-2} \) cm.

For example, choosing \( d_z = 3 \times 10^{-4} \) cm, our target will contain \( \approx 3 \times 10^6 \) Li atoms. Most of these will have a K vacancy at time \( \Delta t = T_K/5 \), at which time they can be triggered by a photon of energy \( E_\alpha \) traversing the target with \( \vec{k}_\alpha \) parallel to the z axis, resulting in a coherent pulse.

If \( T_K = 10^{-14} \) s, then the pumping pulse must contain \( \gtrsim 10^{11} \) photons in the range from 55 to 75 eV. If \( T_K = 10^{-12} \) s, this number only needs to be \( \gtrsim 10^9 \). The number of synchrotron radiation photons within the required energy range and 4 mrad angle delivered at SSRP per 50-mA pulse at 2.5 GeV is \( \approx 10^9 \). It appears that this method of producing coherent x rays may be feasible, particularly if equipment specifically designed for this purpose \(^3 - 5\) were available.

For materials with shorter \( \lambda_\alpha \), the same considerations hold, but because of the shorter \( T_K \), a shorter \( \Delta t \) is required, implying higher \( N_x/T_B \).
This line of thinking should encourage detailed study of $\omega_k$ and $\tau_k$ for elements such as Li, in various chemical surroundings.

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References


3. H. Winick, private communication.


FIG. 1. Orientation of the target and momentum $\vec{k}_o$ of incident pumping photons, in the x, y, z coordinate frame.