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AN APPLICATION OF QUEUEING THEORY TO THE DESIGN OF CHANNEL REQUIREMENTS FOR SPECIAL PURPOSE COMMUNICATIONS SATELLITES

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by

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ABSTRACT

The use of satellites for wide area direct broadcasts results in large power requirements for the satellite. Such satellites are or will be used to provide remote health care delivery and educational services to remote regions and many other new services including warnings to the public of impending natural disasters. These new special purpose satellites are very cost sensitive to the number of broadcast channels, usually will have Poisson arrivals, fairly low utilization (less than 35%), and a very high availability requirement. To solve the problem of determining the effects of limiting C the number of channels, the Poisson arrival, infinite server queueing model will be modified to describe the many server case. The model is predicated on the reproductive property of the Poisson distribution.

For small changes in the Poisson parameter under the assumptions stated previously, the resulting distribution of states or number in the system will be Poisson. A difference equation will be developed to describe the change in the Poisson parameter. When all initially delayed arrivals reenter the system a (C + 1) order polynomial must be solved to determine the new or effective value of the Poisson parameter. When less than 100% of the arrivals reenter the system the effective value must be determined by solving a transcendental equation.

The model will be used to determine the effects of limiting the number of channels for a disaster warning satellite. State probabilities and delay probabilities will be calculated for several values of the number of channels C for arrival and service rates obtained from disaster warnings issued by the National Weather Service. The results predicted by the queueing model will be compared with the results of a digital computer simulation.

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CHAPTER I

INTRODUCTION AND BACKGROUND

The use of geosynchronous orbit satellites for communications began in the 1960's with SYNCOM I. Since then a network has been established to provide international telecommunications via satellite to about 80 countries throughout the world. The use of modern telecommunications has become so commonplace that there is often a tendency to forget how recent are the technological achievements which make such usage possible (COMSAT Report 1972).

The presently used network of commercial traffic consists of four INTELSAT IV series satellites and over 70 earth stations all of which are used for point-to-point transmission of telephone and television traffic. Such a network requires large and expensive ground transmitter-receiver systems.

According to Podraczky and Kiesling (1972), INTELSAT is presently projecting annual growth rates of 15 to 35%. In 1965 INTELSAT I consisted of 480 communications channels. By 1971, each INTELSAT IV had a capacity of 10,000 channels. Podraczky and Kiesling (1972) estimate that each next generation satellite will have a capacity of 50 to 100 thousand channels and that by 1990 the Atlantic region alone will require 1,000,000 channels if the growth rate averages 35% annually.

In late 1971 an initiative study was conducted for the Executive office of the President primarily by NASA and included about 100 participants from various government agencies. This study, "Communications for Social Needs" (1971), was an effort to solve social problems through the use of advanced technology. One of the results of the study was the conclusion that there is an evolving need for what might be called special purpose communications satellites.

The initiative study concentrated on an area of communication satellite applications in which the ground receiver systems or terminals were very large in number and hence a driving cost parameter. Contrasting the commercial and the special purpose types of communication satellites, one finds the following differences:

Commercial	Special Purpose
Low Power	High Power
Point-to-Point Transmission	Wide Area Coverage
Small Number of Expensive Terminals	Large Number of Low Cost Terminals
Large Number of Channels	Small Number of Channels

In usage, the operation of the special purpose type generally consists of transmitting information to many receivers over a relatively large geographic area. The applications considered in the aforementioned study were the uses of communications for remote health care delivery, electronic mail handling, law enforcement, education and as a possibles means of warning the public of impending natural disasters, such as hurricanes or tornados.

The most important difference between the commercial and the special purpose satellites is that the major design objectives are radically opposed to one another. Herbert Raymond (1971) suggested that the most meaningful parameter for a commercial satellite system is probably the utilization factor. It should be high in order to maintain a profitable system. The special purpose satellite must be designed using availability as the major parameter. In the remote regions of the far northern hemisphere, for example, where neighboring villages may be separated by distances of hundreds of miles, the need for emergency medical care can be met through the use of paraprofessionals communicating with doctors via satellite. However, it is not reasonable to expect anyone to wait for service in this application. Should satellites be used in the future to warn the public of impending natural disasters, it would be essential to dispatch warnings as quickly as possible. A system which warns of tornados 15 minutes after their occurrence would be of little value to the public.

The National Aeronautics and Space Administration (NASA) and the National Oceanic and Atmospheric Administration (NOAA) have been conducting joint investigations of various technologies in order to examine the feasibility of using communication satellites for one of the applications mentioned previously, namely, to provide warnings to the general public in the event of an impending natural disaster. The various candidate systems for disaster warning which have been suggested for consideration include the mass ringing of telephones, microwave transmission of radio signals, terrestrial radio networks and the use of communication satellites.

Government organizations other than NASA are conducting studies of terrestrial systems and NASA is confining its investigation to the use of satellites. When completed, the studies will be used to determine the most cost effective system. At the present time, satellites offer a very viable alternative because several meteorological functions may be combined with the communication function, and the satellite system has the desirable property of being "hardened" against natural disasters. That is, satellites are not prone to destruction from an impending natural disaster.

The functions of a natural disaster warning system as reported by Hein and Stevenson (1972) and in the Federal Plan for Disaster Preparedness (1973) are:

1. Route disaster warnings to the general public.

2. Provide disaster communications among national, regional and local weather offices and affected areas.
3. Provide environmental information to the public.
4. Provide a system for collecting decision information for the dissemination of warnings.

The natural disasters which would be monitored by a DWS include tornados, severe thunderstorms, flash floods, tsunami, earthquakes,hurricanes, forest fires, winter storms, and a category called other.

The National Weather Service (NWS) is organized to monitor and predict weather locally, regionally and nationally. This organization as described in the Operations of the National Weather Service (1971), consists of about 300 offices and centers throughout the United States. There are national centers which specialize in certain types of weather phenomena, such as the National Hurricane Center in Miami, Florida. As part of the NWS network, 41 Weather Service Forecast Offices (WSFO's) are located throughout the contiguous United States.

The Weather Service transfers an enormous amount of data through its network of offices. In the event of an impending natural disaster, the NWS network is responsible for warning the public through the mass media. In recent years, it has been found that the present warning system tends to become saturated during large scale disasters such as Hurricane Agnes which occurred in 1972. In a report entitled The Agnes Floods (1972), the saturation of the present warning system was cited as reason for continuing research efforts to increase the capacity of the NWS to provide natural disaster warning to the public.

In order to alleviate problems and to deal with the projected data transfer of the future (1980's) the NWS is in the process of implementing the automation of routine weather data through a network called Automation of Field Operations and

Services (AFOS) (1972). When fully implemented, AFOS will be used to collect meteorological data through a data collection system called GOES (Geostationary Operational Environmental Satellite) (1972). Through the satellite system thousands of data collection platforms (DCP's) located throughout the United States will be interrogated periodically via a satellite-computer system in order to determine local weather conditions and whether or not there is an indication of any potential natural disaster occurring within any given populated region. For example, sensors might be used to determine water levels in areas subject to flash floods. If conditions indicate a danger level, a flood warning would be issued to the public in that particular area.

The system may include a direct broadcast capability. Warnings would be broadcast directly to home receivers or to local transmission points which would then be rebroadcast to home receivers. The home receivers would be designed so that they could be activated by a signal repeated through the satellite. A conceptual diagram of the functional system appears in Figure 1.1.

In September 1973, the Aerospace Systems Group of the Computer Sciences Corporation in Arlington, Virginia, began work on a feasibility study of using satellites for a natural disaster warning system. This study, funded by the NOAA and managed by the NASA Lewis Research Center, will provide the conceptual design of a satellite DWS.

As will be indicated in more detail, the primary problem with the design of such a satellite DWS is the determination of the minimum number of communication channels required to service the system needs. This determination requires an extensive analysis of the message traffic for the proposed system in the early 1980's.

A message traffic analysis and simulation was performed at

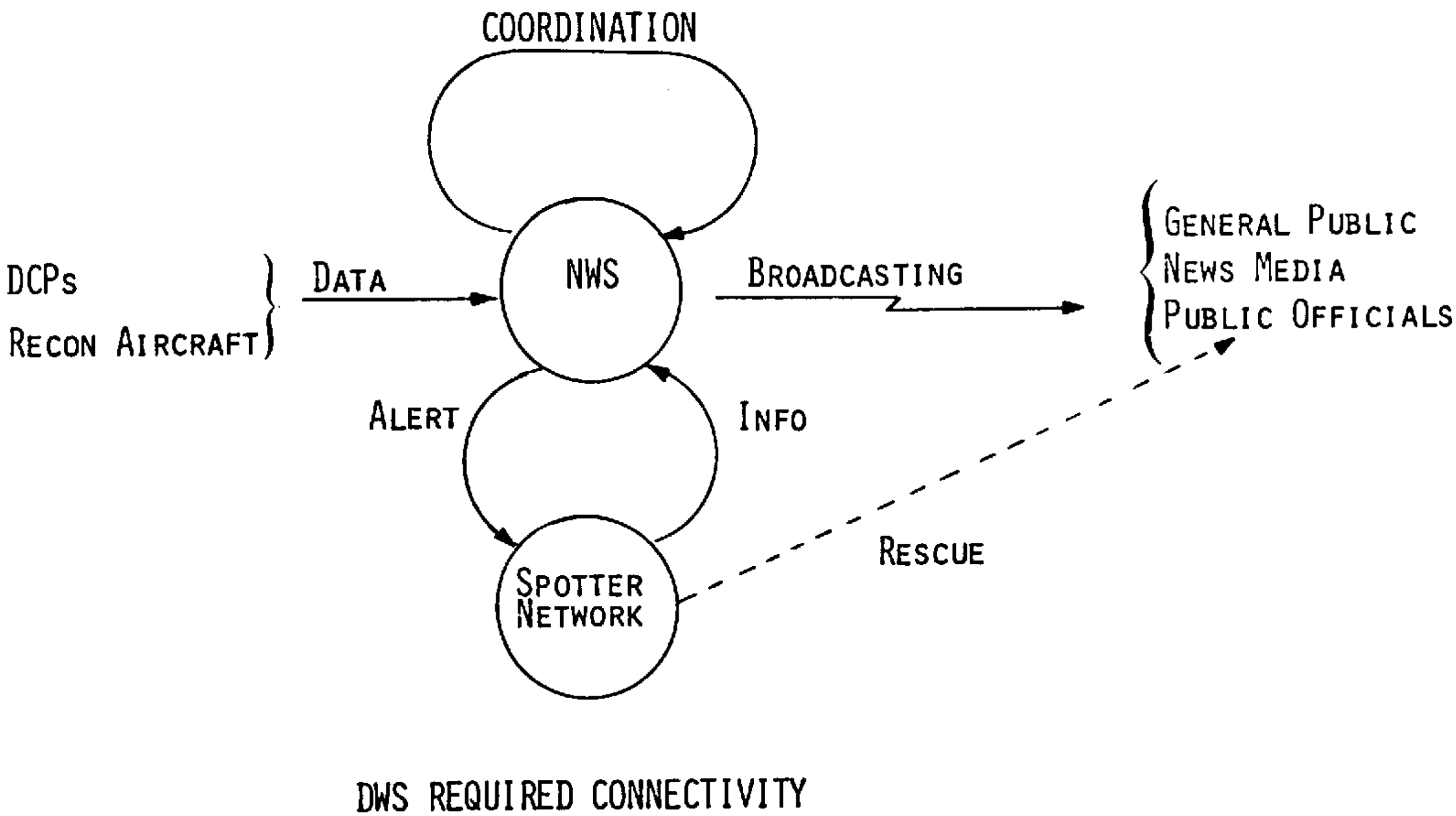

Figure 1.1 Conceptual diagram of a satellite DWS.

the NASA Lewis Research Center by Gerald F. Hein and Steven M. Stevenson with the assistance of NOAA's Environmental Satellite Service Administration and another was conducted by Vern Zurick of the Environmental Research Laboratory in Boulder, Colorado.

In the former report, a model of communications traffic was developed from the theory of multiserver queues. The accuracy of the model is dependent upon the assumption that the messages are requested to be sent according to a Poisson distribution and the time required to transmit the messages is distributed as an exponential density.

The data available at the time both studies were performed was somewhat incomplete in that warning messages issued by the NWS were recorded in terms of the number and type of messages per month. The only information available concerning the message length was the average message size. Thus a goodness-of-fit test for the assumption of Poisson arrivals could only be performed on the basis of number of messages per month. Since this was adequate at the time, a test was performed and the Poisson hypothesis could not be rejected at the 5% level of significance. Because no data were available, the hypothesis of the exponential density for processing times could not be tested. The studies reported by Hein and Stevenson (1972) and Zurick (1973) yielded valuable estimates of the number of channels required, but the problem was somewhat complicated by a NOAA requirement that the probability of message transmission would have to be at least 0.9999999. Some method was required for treating this specification. Hein and Stevenson (1972) interpreted the specification as "virtual certitude" that at least one channel would always be available to transmit a warning message. Such a concept can be manipulated with multiserver queueing theory if the arrivals are Poisson distributed and the departures are distributed exponentially. Thus the emphasis on Poisson arrivals and exponential departures in Hein and Stevenson

(1972) becomes obvious. The difficulty encountered with the aforementioned analysis is that the limiting distributions of service times do not seem to be exponential.

The satellite system will require a high Effective Isotropic Radiated Power (EIRP). Present total satellite power is on the order of 50 watts and designs of the mid to late seventies will be limited to less than 500 watts. As reported by the Computer Sciences Corporation (1973), DWS satellites will require more power than present designs can deliver. If the satellite is to be a viable alternative, the power will be limited to something less than 10 kilowatts which thus precludes the addition of large safety margins for the channel requirements.

There is a genuine need to determine the minimum number of channels for the DWS requirements. The application of satellite technology for the solution of social problems such as those discussed in the study "Communications for Social Needs" (1971) may be delayed if the problem discussed above is not solved. The requirements for the DWS satellite are very similar to the requirements for a large class of satellites which may be used to solve some social problems. For lack of a better description this class of satellites will be called special purpose communication satellites. The salient need is to determine the performance criteria required for the estimation of the channel requirements. Thus the motivation for the topic of this dissertation is the need for a solution of the problem discussed above. The alternatives are to overdesign which is not feasible here, use simulation techniques which may be expensive or use an analytical technique.

A search of the literature dealing with queueing theory revealed that general independent input and service distributions present many difficulties. Thomas Saaty has stated (1961):

> We shall give only the result. We refer readers to the works by Kiefer and Wolfowitz on this subject. The authors point out the desirability of solving their equation for special values of the input and service distributions but also state the task is likely to be difficult.

In another work Saaty (1966) stated that queueing models are rarely applied in practice. One of the main causes of this dearth of applications is that the theory has seldom been developed because of need. In the solution of problems a frequent approach is the use of fluid approximation techniques as discussed by Gordon Newell (1971). In the problem reported in this dissertation the development of the theoretical model was motivated by the application.

The potential solution of the channel requirements problem for special purpose communication satellites was thought to be in the problem class of the GI/G/C queue. An extensive analysis was made of the types of messages sent over the teletype network of the National Weather Service. It was found that the messages could be classified into six different groups of input. It was also demonstrated that the six types of messages could each be represented by a Poisson distribution, as will be reported later. Although such a simplification should be expected for large numbers of messages over the period of a month, the hypothesis testing was done for relatively short intervals such as four or five days because of requests by the people involved with the problem at NOAA. It is very important to know the distributions of what NOAA personnel have called "spikes" or short bursts of increased traffic intensity such as that experienced during the ten days of rampage by Hurricane Agnes from June 14-28, 1972 or the Palm Sunday tornados of April 11, 1965, so that the communication system can be designed to handle such traffic loads.

For this type of problem Saaty (1961) presented the distribution of the waiting time for the M/G/1 queue using an n-fold convolution. In the case of the $M/G/\infty$ queue Lajos Takács (1961) proved that the limiting distribution of the number in the system exists and is also a Poisson distribution. In the case of the many server process M/G/C, it is difficult to find useful expressions. Newell (1971) states that the case of infinite channel servers $M/M/\infty$ represents an idealized form of a queueing system in that the server itself causes no interaction between customers. He also states that useful approximations can be obtained for systems with sufficiently many channels, without stating what is meant by sufficiently many.

Recent work such as Chen (1970), Inglehart and Whitt (1969), Inglehart (1969), Ross (1970) and Yu (1971) represents contributions for certain classes of the many server problem, but nothing was found which provides an analytical solution for the problem of concern here, namely an estimation of the limiting distribution of the number of customers in the system or the waiting time distribution for Poisson arrivals and arbitrary service.

A general background of the queueing problem was presented in this chapter. In Chapter II there is a development of the applicable queueing model. To demonstrate the applicability of the theory, the work performed to classify the statistical distributions of six types of message arrivals and departures for the DWS is presented in Chapter III, and the third chapter is concluded with the application of the theory to determine the DWS channel requirements. In Chapter IV the predicted values and the results of a continuous digital simulation are compared in order to provide verification of the analytical model. This dissertation is concluded with a summary of results and suggestions for further research endeavors in Chapter V.

CHAPTER II

DEVELOPMENT OF THE APPLICABLE QUEUEING MODEL

In the previous chapter a rationale was presented for the development of an availability criterion for special purpose communication satellites rather than the utilization criterion which is so important to a commercial venture. In this chapter a queueing model will be developed for the type of problem discussed in Chapter I.

To develop the framework of the model, reference will be made to the Poisson arrival model with an infinite number of servers discussed in Takács (1962). The infinite server case has been solved for Poisson arrivals. The basic hypothesis for Chapter II is that the state distribution for the many server case should be similar to that for the infinite server case. As will be seen from the Theorem of Takács, the state distribution for the infinite server model is Poisson. Because of the reproductive property of this distribution and the fact that the mean and variance are equal, a change in the Poisson parameter λ to $\lambda + \Delta\lambda$ results in a Poisson distribution with a mean and variance of $\lambda + \Delta\lambda$.

Thus the many server case should have a state distribution which is at least similar to a Poisson distribution. The hypothesis will be developed with a presentation of background material in 2.1 and the development of the model in 2.2. A general relationship will be presented in 2.3.

2.1 Prerequisite Queueing Theory

In the case of an infinite number of servers with Poisson arrivals, Takács (1962) proved that with an arbitrary service discipline, the limiting distribution of the states can be represented by a Poisson distribution. Moreover, the distribution is independent of the initial state. The proof of this theorem is given in Takács (1962), and is stated here.

$$\text{Let} \quad \alpha = \int_0^\infty \chi dH(\chi) \quad \text{and} \quad P\{\xi(t) = K\} = P_k(t)$$

where α is the average service time, $P_k(t)$ is the probability of being in state K at time t, θ is the Poisson arrival parameter, and $\xi(t)$ is the queue size or number in the system.

Theorem 1. If $\xi(0) = 0$ then

$$P_k(t) = \text{EXP}\left[-\theta\int_0^t [1-H(\chi)]d\chi\right] \frac{\left[\theta \int_0^t [1 - H(\chi)]d\chi\right]^K}{K!} \tag{1}$$

for $K = 0, 1, 2, \ldots$

and if $\alpha < \infty$ then

$$\lim_{t\to\infty} P_K(t) = P_K^* \qquad (K = 0, 1, 2, \ldots)$$

exists and we have

$$P_K^* = e^{-\theta\alpha} \frac{(\theta\alpha)^K}{K!} \tag{2}$$

The interpretation given to equation (2) is that it gives the long term proportion of time spent in each state, where state refers to the number in the system. Since there are an infinite number of servers available, an arrival is served immediately upon entry into the system. Because of the immediate service the servers do not cause any interaction between customers. There is input, processing and output. The expected values of the queue length and the waiting time (excluding service) in the queue are both zero.

2.2 Development of Applicable Model and Effective Values of State Parameters for the Many Server Queue

In the situation encountered in the special purpose communication satellite (which is also applicable to many other problems where availability is an important design criterion), the marginal cost of adding a server is usually large. At the same time the risk encountered by not adding a needed server is probably larger than the marginal cost of adding a server. Because of the intangibles involved it may be necessary to accept the risk of not adding a server. It is important to determine the effects of limiting the number of servers. In the infinite server case depicted in Figure 2.1 arrivals are served upon entry and depart after receiving service.

In the case of the many server queue without storage, arrivals will have a certain probability of being rejected if there is no server available when the arrival enters the system. In the case of the special purpose communication satellite, most of the arrivals will keep trying to enter the system until a server is available. In many problems, however, a fixed portion will leave the system and not return. The fixed fraction not returning will be specified by $(1 - \gamma)$ where $0 \leq \gamma \leq 1$. Figure 2.2 shows the effects of limiting the number of servers.

As stated previously, γ will usually be 1 in the application presented in this work. Thus $100\gamma\%$ return for service again.

In the theorem of Takács (1962), the state probabilities of the system can be described with a Poisson distribution. The parameter for this distribution is the Poisson parameter for the arrival rate times the average service time. That is $\lambda = \theta\alpha$. When the number of servers is constrained, the effect of the constraint may be described as an effective increase in the average number of arrivals. Since the service time remains constant, a

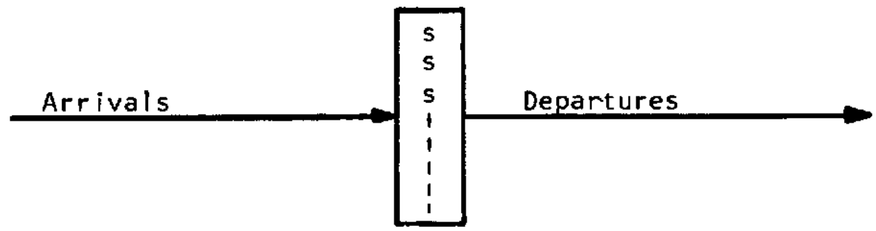

Figure 2.1 Infinite server model.

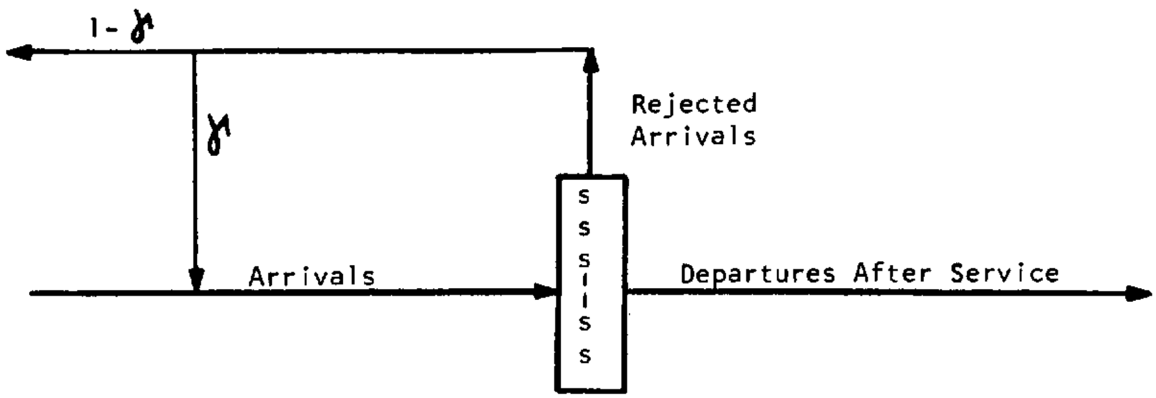

Figure 2.2 Reentry queueing model.

difference equation can be written to describe the effects of server-customer interaction. An arbitrary time interval will be selected so that the rejected arrivals will have a very low probability of entering the system more than once during the same time interval. For the first interval of time at start-up, the parameter $\lambda_i = \theta_i\alpha$ may be described as

$$\lambda_0 = \lambda_0 + 0 \tag{3}$$

During the second time interval

$$\lambda_1 = \lambda_0 + \gamma\lambda_0\Phi(C,\lambda_0) \tag{4}$$

where γ is the portion of rejects returning; Φ is the complementary cumulative of the Poisson distribution; λ_0 is the Poisson parameter and C is the number of servers in the system. The second term on the right side of equation 4 is the parametric increment caused by the rejected customers returning for service.

For the third interval

$$\lambda_2 = \lambda_0 + \gamma\lambda_1\Phi(C,\lambda_1)$$

$$= \lambda_0 + \gamma[\lambda_0\Phi(C,\lambda_1) + \gamma\lambda_0\Phi(C,\lambda_1)\Phi(C,\lambda_0)]$$

$$= \lambda_0[1 + \gamma\Phi(C,\lambda_1) + \gamma^2\Phi(C,\lambda_1)\Phi(C,\lambda_0)] \tag{5}$$

To simplify the notation, $\Phi(i)$ will be used to denote $\Phi(C,\lambda_i)$. For the next interval

$$\lambda_3 = \lambda_0 + \gamma\lambda_2\Phi(2)$$

$$= \lambda_0 + \gamma\lambda_0[1 + \gamma\Phi(1) + \gamma^2\Phi(1)\Phi(0)]\Phi(2)$$

$$= \lambda_0[1 + \gamma\Phi(2) + \gamma^2\Phi(2)\Phi(1) + \gamma^3\Phi(2)\Phi(1)\Phi(0)] \tag{6}$$

In general,

$$\lambda_j = \lambda_0[1 + \gamma\Phi(j-1) + \gamma^2\Phi(j-1)\Phi(j-2) + \ldots + \gamma^j\Phi(j-1)\ldots\Phi(0)]$$

$$= \lambda_0 \left[1 + \sum_{i=0}^{j=1} \gamma^{j-i}\left[\prod_{K=i}^{j-1} \Phi(K)\right]\right] \tag{7}$$

It is necessary to determine whether this sequence converges and if so, to determine the interval of convergence. In the event that $\{\lambda_j\}$ converges to some effective value, there will be an epsilon such that

$$\lambda_{N+1} = \lambda_N + \varepsilon$$

where epsilon can be made arbitrarily small.

The sequence $\{\lambda_j\}$ is a non-decreasing sequence. Each successive value of the sequence is the sum of positive terms because λ_0 is positive, γ lies between 0 and 1 and the $\{\Phi(i)\}$ are probabilities. Thus

$$\lambda_{N+1} \geq \lambda_N \qquad \text{for all} \quad N \geq 0$$

which implies

$$\lambda_N \geq \lambda_0 \qquad \text{for all} \quad N \geq 0$$

The complementary cumulative of the Poisson distribution for C servers is the probability of an arrival being rejected and is denoted by

$$\Phi(g) = \Phi(C,\lambda_g) = 1 - \sum_{n=0}^{C} \frac{\lambda_g^n \, e^{-\lambda_g}}{n!} \tag{8}$$

Substituting this relation into equation (7) yields

$$\lambda_j = \lambda_0 \left[1 + \sum_{i=0}^{j-1} \gamma^{j-i} \left[\prod_{K=i}^{j-1} \left(1 - e^{-\lambda_K} \sum_{n=0}^{C} \frac{\lambda_K^n}{n!} \right) \right] \right] \tag{9}$$

Expanding equation (9) yields

$$\lambda_j = \lambda_0 \left[1 + \gamma \left(1 - e^{-\lambda_{j-1}} \sum_{n=0}^{C} \frac{\lambda_{j-1}^n}{n!} \right) + \right.$$

$$+ \gamma^2 \left(1 - e^{-\lambda_{j-1}} \sum_{n=0}^{C} \frac{\lambda_{j-1}^n}{n!} \right) \left(1 - e^{-\lambda_{j-2}} \sum_{n=0}^{C} \frac{\lambda_{j-2}^n}{n!} \right) +$$

$$\left. + \ldots + \gamma^j \left(1 - e^{-\lambda_{j-1}} \sum_{n=0}^{C} \frac{\lambda_{j-1}^n}{n!} \right) \ldots \left(1 - e^{-\lambda_0} \sum_{n=0}^{C} \frac{\lambda_0^n}{n!} \right) \right] \tag{10}$$

The summations can be simplified using the relation

$$\lim_{C \to \infty} \sum_{n=0}^{C} \frac{\lambda^n}{n!} = e^{\lambda}.$$

Then the term

$$\gamma \left(1 - e^{-\lambda_{j-1}} \sum_{n=0}^{C} \frac{\lambda_{j-1}^n}{n!} \right)$$

where the number of servers is infinite, results in the following

$$\gamma \left(1 - e^{-\lambda_{j-1}} e^{\lambda_{j-1}} \right) = 0$$

In general, since

$$\sum_{n=0}^{C} \frac{\lambda_{j-1}^{n}}{n!} \geq 1$$

then

$$\gamma\left(1 - e^{-\lambda_{j-1}} \sum_{n=0}^{C} \frac{\lambda_{j-1}^{n}}{n!}\right) \leq \gamma \left(1 - e^{-\lambda_{j-1}}\right)$$

independently of the value of C. Using this inequality, equation (10) may be rewritten as an inequality

$$\lambda_j \leq \lambda_0\left[1 + \gamma\left(1 - e^{-\lambda_{j-1}}\right) + \gamma^2 \left(1 - e^{-\lambda_{j-1}}\right) \left(1 - e^{-\lambda_{j-2}}\right) + \right.$$
$$\left. + \ldots + \gamma^j \left(1 - e^{-\lambda_{j-1}}\right) \ldots \left(1 - e^{-\lambda_0}\right)\right] \qquad (11)$$

Using the fact that

$$\lambda_0 \leq \lambda_N \quad \text{for} \quad N \geq 0$$

it is then true that

$$\left(1 - e^{-\lambda_{j-1}}\right) \geq \left(1 - e^{-\lambda_0}\right)$$

for all $j \geq 1$.
It is also true that

$$\left(1 - e^{-\lambda_{j-1}}\right) \geq \left(1 - e^{-\lambda_{j-N}}\right)$$

for all N such that $1 \leq N \leq j$.

Substituting $1 - e^{-\lambda_{j-1}}$ for all $1 - e^{-\lambda_{j-N}}$ for all N in the interval $1 \leq N \leq j$ does not change the direction of the inequality in 11, and so it can be simplified to

$$\lambda_j \leq \lambda_0 \left[1 + \gamma\left(1 + e^{-\lambda_{j-1}}\right) + \gamma^2\left(1 - e^{-\lambda_{j-1}}\right)^2 + \right.$$
$$\left. + \dots + \gamma^j\left(1 - e^{-\lambda_{j-1}}\right)^j\right]$$

which can be expressed

$$\lambda_j \leq \lambda_0 \sum_{i=0}^{j} \gamma^i \left(1 - e^{-\lambda_{j-1}}\right)^i \tag{12}$$

As λ_{j-1} increases, $e^{-\lambda_{j-1}}$ approaches 0^+ and $\gamma\left(1 - e^{-\lambda_{j-1}}\right)$ is always less than one. The use of d'Alembert's Ratio Test guarantees the convergence of the right side of inequality (12) as j approaches ∞.

The interval of convergence for the sequence $\{\lambda_j\}$ is dependent upon the value of C, the number of servers. For the case where $C = \infty$, the sequence should provide results which are consistent with the theorem of Takács mentioned in the beginning of this chapter.

<u>$C = \infty$</u>

For $C = \infty$, during the interval of time for λ_j where $j > 0$

$$\lambda_j = \lambda_0 + \gamma\lambda_{j-1}\,\Phi(j-1)$$

$$= \lambda_0 + \gamma\lambda_{j-1}\left(1 - e^{-\lambda_{j-1}} \sum_{i=0}^{C} \frac{\lambda_{j-1}^i}{i!}\right) \tag{13}$$

Since there are an infinite number of servers,

$$e^{-\lambda_{j-1}} \sum_{i=0}^{\infty} \frac{\lambda_{j-1}^i}{i!} = e^{-\lambda_{j-1}}\, e^{\lambda_{j-1}} = 1$$

Thus equation (13) reduces to

$$\lambda_j = \lambda_0 + \gamma\lambda_{j-1} (1 - 1) = \lambda_0$$

which agrees with the theorem of Takács. In the case of the infinite server model, there are no limitations placed upon arrival rates or service times. In the limited or many server case, it is necessary that $\lambda_0 = \theta_0\alpha$ be less than the number of servers in order to have a stable queueing system (Takács 1962); otherwise the queue length increases without bound. The model developed in this chapter implies an additional constraint for stability in the many server case. Thus it is necessary to investigate the convergence interval for the model in order to determine the effects if any on the limiting value of $\lambda_0 = \theta_0\alpha$. If there exists a limit which is less than the number of servers, this value may be determined from the convergence interval. The maximum λ_0 will be given by Min $[C,\lambda_0]$.

The C = 1 case will be examined to determine the extent of the more stringent stability requirement for the reentry model for $\gamma = 1$. The $C = 2$, $\gamma = 1$ case will be discussed in the next section, and then a general case will be developed for $\gamma = 1$.

Single Server Case When $\gamma = 1$

Since $\lambda_j = \lambda_{j-1} + \varepsilon$ and from the proof that the sequence converges, ε can be made less than some arbitrary number. When this convergence occurs, λ_j and all $\{\lambda_K | K \geq j\}$ may be denoted by λ_* which will be referred to as the "Effective" λ. The limiting effective values will be developed for 2 values of C in this section and the next and then a general relation will be given.

C = 1. For the case when $C = 1$, assume that the Effective λ has been reached if it exists. Thus

$$\lambda_* = \lambda_0 + \lambda_* \left[1 - e^{-\lambda_*} (1 + \lambda_*) \right] \tag{14}$$

will be used rather than the exact relation

$$\lambda_{j-1} = \lambda_0 + \lambda_{j-1} \left[1 - e^{-\lambda_{j-1}} (1 + \lambda_{j-1}) \right] - \varepsilon$$

A proof is given in Appendix A that the two forms are equivalent if the sequence converges. Solving equation (14) for λ_0 yields

$$\lambda_0 = e^{-\lambda_*} \left(\lambda_* + \lambda_*^2 \right) \tag{15}$$

Considering λ_0 as a function of λ_* it is necessary to determine the value of λ_* which maximizes the function in the interval of convergence if such a value exists.

$$\frac{d\lambda_0}{d\lambda_*} = e^{-\lambda_*} (1 + 2\lambda_*) - e^{-\lambda_*} (\lambda_* + \lambda_*^2) = 0 \tag{16}$$

$$\frac{d^2\lambda_0}{d\lambda_*^2} = e^{-\lambda_*} (\lambda_*^2 - 3\lambda_*) \tag{17}$$

The second derivative is negative for all λ_* in the interval $0 < \lambda_* < 3$. Therefore the function has a maximum value in this same interval if there is a solution of equation (16) in the interval. Solving the equation yields

$$\lambda_* = \frac{1 \pm \sqrt{5}}{2}$$

Discarding the negative root which is physically meaningless

$$\lambda_{*MAX} = 1.618$$

which lies within the constraint. Solving equation (15) for the value of λ_0 yields

$$\lambda_{0MAX} = \text{MIN } [1, 0.8399] = 0.8399$$

The interpretation is that $\lambda_{0MAX} = (\theta_0\alpha)_{MAX}$ is the maximum value that will yield a controlled process if there is only a single server. If this value is exceeded, the average state of the process will eventually increase without bound because of the reentry mechanism. As an example, if α = .5 minutes, then θ_{0MAX} = 1.6798/minute is the maximum average arrival rate that will yield a stable system for the $C = 1$ case. Also, the proportion of time that the system will be in each state can be described by a Poisson distribution with a maximum parameter $\lambda_{*MAX} = 1.618$. A table of limiting values for $C = 1$ to 20 is given in Appendix C and the general equation for determining the values is presented in the next section of this chapter.

2.3 General Equation for Evaluating the Limiting State Parameters

In this section a presentation of the $C = 2, \gamma = 1$ case will be given and then the general equation for $C = n, \gamma = 1$ will be presented. When $\gamma = 0$, the solution is trivial and when $0 < \gamma < 1$, the resulting equation is transcendental; this case will be discussed later.

$C = 2, \gamma = 1$

For this case

$$\lambda_* = \lambda_0 + \lambda_*\left[1 - e^{-\lambda_*}\left(1 + \lambda_* + \frac{\lambda_*^2}{2}\right)\right]$$

Solving for λ_0 yields

$$\lambda_0 = e^{-\lambda_*}\left(\lambda_* + \lambda_*^2 + \frac{\lambda_*^3}{2}\right)$$

Differentiating with respect to λ_* and equating to 0,

$$\frac{d\lambda_0}{d\lambda_*} = \lambda_*^3 - \lambda_*^2 - 2\lambda_* - 2 = 0$$

yields

$$\lambda_{*MAX} = 2.2695$$

and

$$\lambda_{0MAX} = \text{MIN }[2,\ 1.3711] = 1.3711$$

In addition to the constraint that $\lambda_0 < 2$, the new constraint requires $\lambda_0 < 1.3711$ for a stable system when $\gamma = 1$.

<u>$C = n,\ \gamma = 1$</u>

For the general n server case

$$\lambda_0 = e^{-\lambda_*}\left(\lambda_* + \lambda_*^2 + \ldots + \frac{\lambda_*^n}{(n-1)!} + \frac{\lambda_*^{n+1}}{n!}\right) \tag{18}$$

Differentiating with respect to λ_* and equating to zero yields

$$\frac{\lambda_*^{n+1}}{n!} - \frac{\lambda_*^n}{n!} - \frac{\lambda_*^{n-1}}{(n-1)!} - \ldots - \lambda_* - 1 = 0 \tag{19}$$

According to the rule of signs developed by Descartes (1637), this polynomial has exactly one positive root. To obtain the roots of polynomials there are many computational methods available. One which is rather easy to use is the Newton-Raphson method discussed in Dorf (1967). According to this method the next estimate of the root of $f(x)$ is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

A computer program was written to search for the root of equation (19). The program and sample outputs are presented in Appendix C.

After the value of λ_{*MAX} is obtained λ_{0MAX} may be computed. If λ_{0MAX} is less than C, then λ_{0MAX} is the maximum λ_0 for that C; otherwise C is the limit.

<u>Case when $0 < \gamma < 1$</u>

In the case when $\gamma = 1$ a polynomial evaluation is required to determine the value of λ_* for a given C. Then the limiting value of λ_0 may be determined. When $0 < \gamma < 1$, only a fraction of the rejected arrivals return to the queue. Thus λ_{*MAX} increases without bound as γ approaches zero. As this happens, the new constraint on λ_0 is relaxed until $\lambda_{0MAX} = C$. For $C = 1$ this happens when $\gamma < 0.85$. The general constraint is $\lambda_{0MAX} = \text{MIN}\ [C, \lambda_0]$ so that $\gamma < 0.85$ yields a limit of 1 for λ_{0MAX}. The effect of varying γ for different values of C is given in more detail in Appendix D.

Summarizing the results of this chapter, an equation which describes the effective change in the Poisson parameter of the theorem of Takács was developed as a model for the many server case. It was proven that the sequence converged. The root of (19) is the effective value of the Poisson parameter for the many server case when $\gamma = 1$. Using the root of the equation a maximum allowable value of λ_0 can be determined using equation (18). If this value is less than the number of servers or channels, then a more stringent maximum value of $\lambda_0 = \theta_0\alpha$ exists in order to have a stable system. If the maximum λ_0 is not exceeded, then the resulting state probabilities may be described as a Poisson distribution with a parameter

λ_*. The constraint for stability is

$$\lambda_{0MAX} < \text{MIN}\ [C, \lambda_0]$$

where C is the number of channels and λ_{0MAX} for a given C may be calculated or determined for some values of C from the table in Appendix C.

To make use of the model presented in this chapter, it is necessary to evaluate λ_0 from data and then determine the feasible range for C. Thus if λ_0 is greater than .83 and $\gamma = 1$, the minimum feasible C is 2 rather than 1 which would be the minimum feasible C for $\gamma = 0$.

In the next chapter, an application of the theory presented in this chapter will be developed for a particular communication satellite system.

CHAPTER III

APPLICATION TO THE DESIGN OF SATELLITE CHANNEL REQUIREMENTS

In Chapter I a comparison was made between the commerical and the special purpose communication satellites. It was pointed out that the design philosophies of the two are in general quite different. As an example of this difference it was stated that high utilization is one of the most important parameters of the commerical venture whereas availability is one of the most important parameters for the special purpose type. One of the best examples of a special purpose satellite design being motivated by an availability criterion is the proposed use of a satellite system to provide warnings to the public in the event of an impending natural disaster such as a hurricane or a tornado.

The problem of determining the required number of communications channels for a natural disaster warning satellite has been selected to demonstrate the application of the theory presented in Chapter II. This demonstration requires a knowledge of the statistical patterns associated with the message arrival and processing times. As stated in Chapter I, the analysis performed by Hein and Stevenson (1972) was incomplete because the data were analyzed for intervals of a month over a period of six years to determine the arrival patterns. In order to demonstrate that a Poisson distribution could be used to describe the arrival patterns over a period of a few days or even a single day, it was necessary to take the output of the present NWS communications system and perform tests on these data.

Another shortcoming of the study by Hein and Stevenson (1972) is that the message lengths could not be categorized statistically since data were not available. In order to obtain this information, it was necessary to count the number of characters in each message.

This determination of arrival patterns and message lengths was performed for six categories of messages issued by the NWS.

These categories are:

Hurricane Warnings
Tornado and Severe Storm Warnings
Winter Storm Warnings
Small Craft Warnings
River Warnings
Other Warnings

The source of data for hurricane warnings consisted of the 819 messages sent during the events of Hurricane Agnes; these messages were published in the Preliminary Report on Hurricane Agnes (1972). The source of data for the remaining five types consisted of hundreds of feet of teletype output from the NWS in Silver Spring, Maryland. The origination time of each message was noted for every message of each particular category to determine whether Poisson distributions could be used to describe the arrival patterns of each category for the contiguous United States. Each message size was determined through a character count and the processing time was determined by allowing 5 characters per word at a nominal speaking rate of 137 words per minute. The categories "River" and "Other" were combined in the analysis because there was an inadequate sample for the "Other" category. A summary of the results of the analysis of the message data is presented in Figure 3.1. The procedure used for each message type is discussed in more detail below.

A presentation of the statistical procedures and analysis of the NWS warning data is given in Section 3.1 for each of the six categories of messages. Since the Poisson arrival pattern is required for the theory developed in Chapter II, the null hypothesis is that the arrival pattern may be represented by a Poisson distribution for each of the six types of message categories. Since the class marks are integers and the number of events for a Poisson

Message Category	Arrival Pattern	Broadcasting Time Density	Average Value
Hurricanes	Poisson	Log Normal	1.15 Mins.
Tornados	Poisson	Uniform	0.85 Mins.
Winter Storms	Poisson	Uniform	1.60 Mins.
Small Craft	Poisson	Uniform	1.00 Mins.
River & Other	Poisson	Uniform	1.10 Mins.

Figure 3.1 Statistical Patterns for Message Categories and Average Broadcasting Times.

distribution is an exclusive class (as discussed in Bradley (1968)), the Chi-squared goodness-of-fit test was selected to test the Poisson arrival hypothesis.

The message broadcasting time probability densities may be arbitrary according to the theory of Chapter II. In order to develop a computer simulation of the model, however, individual service densities were hypothesized and tested. For each category of warning message, the broadcast times were plotted as histograms with varying class sizes (.05 to .1 minutes). In the case of the hurricane warning messages, the density was immediately seen to resemble a log normal density as will be demonstrated in Section 3.1. For the remaining categories, the uniform density seemed to be the most logical choice. The goodness-of-fit test used for the determination of the acceptability of the hypothesized densities was the Kolmogorov-Smirnov because the distribution functions are continuous and because the Kolmogorov-Smirnov test is superior to the Chi-squared test (Bradley (1968)).

In Section 3.2 a projection of message traffic is made for 1985 since that will be the nominal traffic year for the first generation of DWS satellites if satellites are to be used. Then the theory of Chapter II is used to perform a comparative analysis for different values of C, the number of channels available for simultaneous broadcasts.

3.1 Analysis of Message Traffic Data

Hurricane Message Traffic

In order to determine whether hurricane warnings occur according to a Poisson arrival pattern as required for the theory of Chapter II, an analysis was made of all warning messages sent during Hurricane Agnes during the period June 14-28, 1972. Each of the 819 messages were used to determine the arrival pattern. The movement of Agnes up the Atlantic coast is shown in Figure 3.2. The maximum daily count of messages was 195; the date of occurrence was June 19,

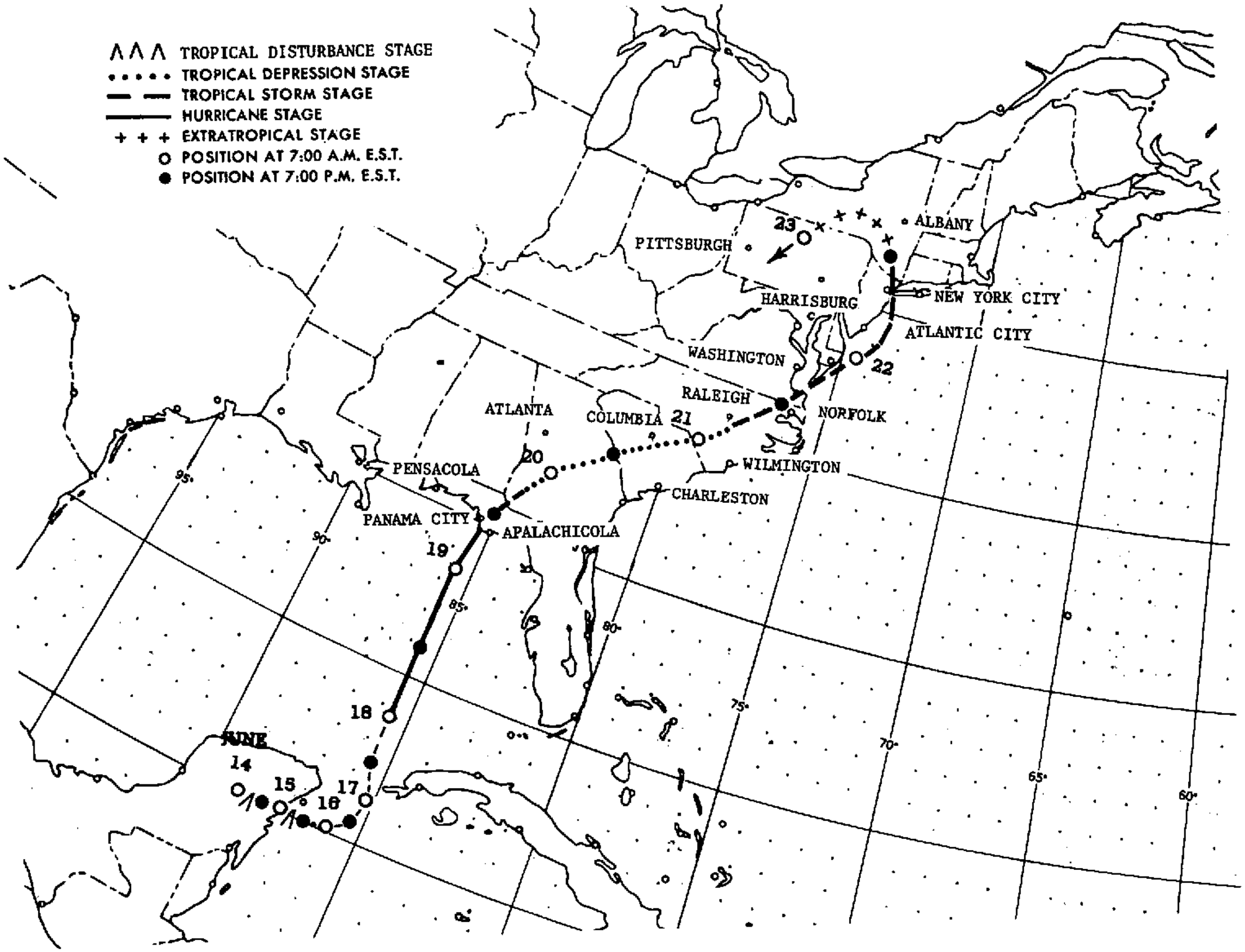

Figure 3.2 Path of hurricane Agnes.

which coincides with the approach of the hurricane to Panama City, Florida.

The days prior to the nineteenth saw relatively little warning message activity. Once the hurricane passed Panama City and started up the east coast the message intensity subsided with the hurricane as it passed over land. The next major problem was one of severe rain and floods around Washington, D.C. and in Pennsylvania. Thus the message intensity increased around the twenty-second of June and remained rather high until the twenty-fifth.

The data analysis for the hurricane warnings consists of testing the hypothesized statistical patterns. A goodness-of-fit test was performed for the Poisson arrival hypothesis and will be presented with the data and analysis for the broadcast time densities. Agnes was recommended for analysis by the NWS because it represents one of the worst hurricanes on record, and would have to be considered in the design of a satellite DWS.

In order to test the hypothesis of Poisson arrivals over the peak ten days of Agnes and over an interval of one day, two goodness-of-fit tests were performed. Some of the days during the June 14-28 period had relatively little message activity. As might be expected these periods are June 14-16 and June 25-28. The time of greatest interest to the NWS and to NASA would be June 19th because of the 195 messages transmitted that day. The design of the satellite requires a consideration of the worst spikes in the system. The 19th was divided into 48 periods of a half hour duration in order to perform a goodness-of-fit test. Any period smaller than one-half hour would degrade the arrival pattern to an exponential form because of the large frequency of zero events.

The 195 messages over 48 periods yielded a parameter estimate of 4.08 messages per half-hour. The frequencies were as follows:

No. of Messages	Frequency
0	3
1	3
2	5
3	11
4	8
5	7
6	3
7	4
8	2
9	1
10	0
11	0
12	1

Since the Chi-squared test requires that cell frequencies be equal to or greater than 5, the classes 0, 1 and 2 were combined into one class and 6 to 12 were combined. The Poisson probabilities were calculated for each class and multiplied by the total events to get expected cell frequencies. The experimental value of Chi-squared was calculated using the equation

$$X^2 = \sum_{i=1}^{K} \frac{(f_i - e_i)^2}{e_i}$$

where f_i is the observed frequency for cell i, e_i is the expected frequency and K is the number of cells or classes. For this test the experimental value was 0.59. In Miller and Freund (1965), the tabled value of Chi-squared for the 5% level of significance and 3 degrees of freedom is 7.815. Although the experimental value of Chi-squared was reduced through the combination of classes, the fit is very good; the null hypothesis could not be rejected.

Another distribution of considerable importance is that of the maximum number of messages during each 15 minute interval over the period June 14-28. This distribution is important because it represents an arrival compression (such as occurred in the tornados of April 3, 1974) of the traffic intensity. Another reason for the importance of this distribution is that it filters the effects of arbitrarily selecting a convenient time period for broadcasting warnings. For this distribution the parameter estimate was 2.96 per 15 minute interval or 6 per half hour. This rate is about 50% greater than that for June 19. The number and frequency of events for the distribution of the maxima is as follows:

Number of Messages	Frequency
0	5
1	23
2	20
3	13
4	13
5	12
6	4
7	4
8	1
9	0
10	1

Classes 6 to 10 were combined into one class in order to have five or more in each class for the Chi-squared test. The experimental value of Chi-squared was 10.09. The tabled value in Miller and Freund (1965) for the 5% level of significance and 5 degrees of freedom is 11.07. Thus the hypothesis that the distribution of the maxima may be represented as a Poisson distribution cannot be rejected.

The goodness-of-fit tests for the Poisson arrival hypothesis were required because the theory of Chapter II requires Poisson input.

The only requirement for the output, or broadcast times in this case, is a knowledge of the average broadcast time. In order to develop a computer simulation of the process so that the theory may be appropriately tested, the density functions for the broadcast times were determined.

The hurricane warning message broadcasting times were classified into cells of .05 minutes (3 seconds) on the interval 0.3 to 4.0 minutes. The frequency histogram of the broadcast times is shown in Figure 3.3. This frequency polygon possesses the shape of the log normal density and so the null hypothesis was that the broadcast times for hurricane warnings are distributed as a log normal density. Since the logarithm of a log normally distributed random variable is distributed normally, the logarithms of the boradcast times were used for the goodness-of-fit test. The test used was the Kolmogorov-Smirnov but the test was for a normal distribution and since the mean and variance were unknown, the test for normality as described by Lilliefors (1967) was used. If N is the sample size and D is the maximum deviation between the hypothesized and the actual distributions, then for a significance level of 5%, the critical value of $\sqrt{N}\,D$ is 0.886.

After the broadcast times were determined and classed the mean and standard deviation of the logarithms were estimated from the data to be 0.13 (1.15 minutes) and 0.6 (1.8 minutes) respectively. The theoretical and experimental distributions were then compared to determine the maximum deviation between the two. The largest deviations occurred in the right tail of the density functions in such a way that the theoretical distribution function lagged the experimental distribution function. The two distributions differed by less than 0.02 up to the 0.909 value of the experimental distribution. The maximum deviation of 0.039 occurred when the value of the logarithm of the broadcast time equalled 1.08. At this value the observed and expected values of the distribution functions were

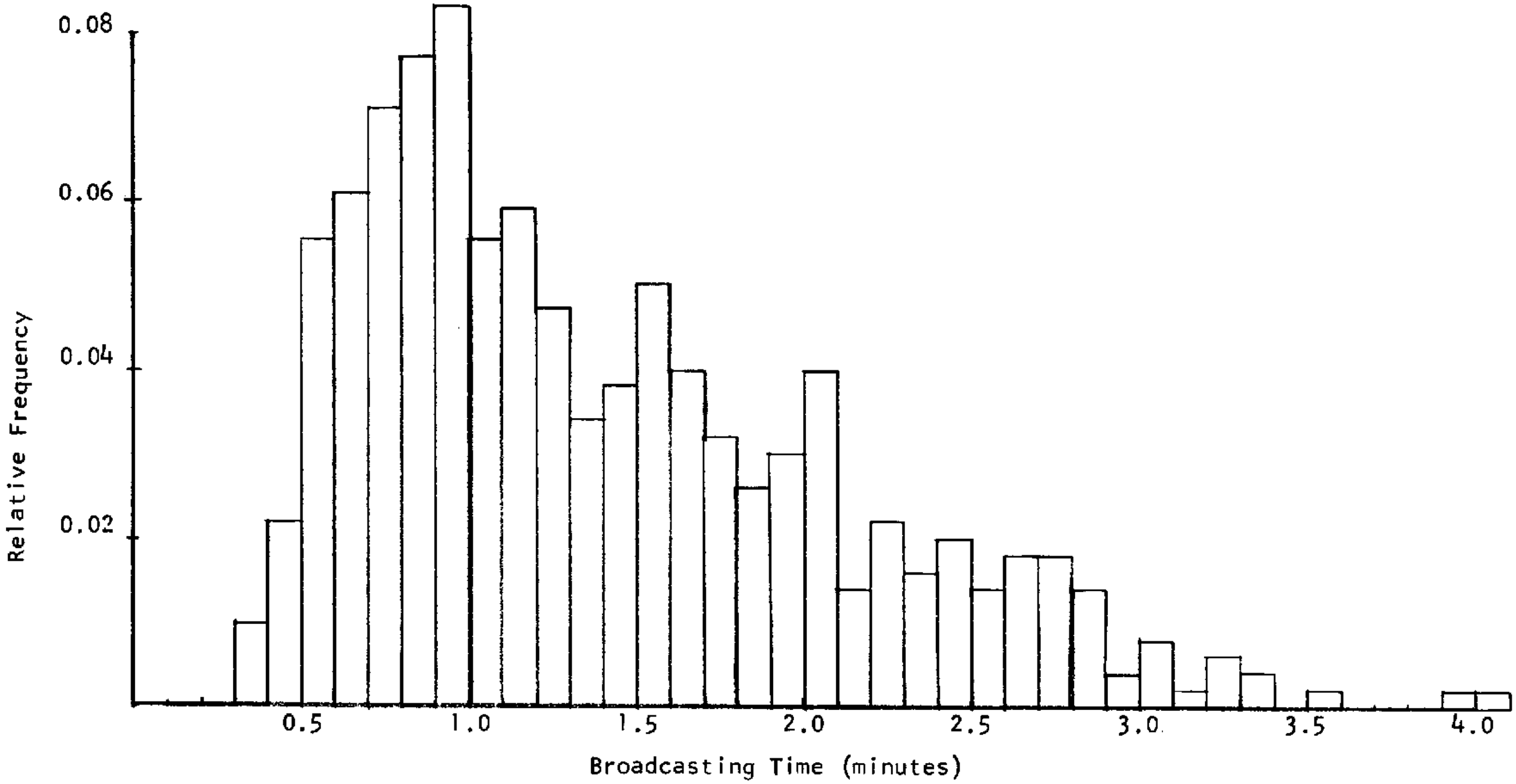

Figure 3.3 Frequency polygon for hurricane message broadcast times.

0.982 and 0.943 respectively. The calculated value of $\sqrt{N}$ D was 0.866 which is less than the critical value of 0.886. Thus the log normal hypothesis could not be rejected at the 5% level of significance. Moreover, since the theoretical distribution function lags the experimental one, there is a safety margin added in the simulation program because the dispersion of broadcast times is larger than is actually necessary, as indicated by the fact that the experimental density tapers off more quickly than the hypothesized density function.

Winter Storm Warning Message Traffic

The data sample for the winter storm warnings consisted of the NWS teletype rolls for the period December 15-17, 1973. There were 89 messages in the sample. The maximum number over a period of one hour occurred from noon to 1 p.m. on December 16th. At that time 7 messages originated at 12 noon. To test the Poisson arrival hypothesis, the Chi-squared test was used. The experimental value of Chi-squared was calculated to be 1.59. The tabled value of Chi-squared for the 5% level of significance and 2 degrees of freedom is 5.991. Thus the Poisson distribution seems to be quite acceptable.

The message broadcasting times varied from .55 minutes to 3.81 minutes. The 9 messages with durations greater than 2.7 minutes originated primarily in New York and consisted of regional warnings for the eastern USA (Maine to Georgia, Lake Erie to the Atlantic for Special Weather Bulletin 15 originating in New York at 6 p.m. December 17, 1973 and lasting 2.8 minutes).

The broadcast time probability density function for the winter storm warnings seemed to be uniform on the interval from .55 to 2.7 minutes. Since the hypothesized density was continuous, the Kolmogorov-Smirnov test was used. The hypothesized interval was 0.5 to 2.7 minutes. The maximum deviation between the observed and expected distribution functions was 0.13 which occurred twice at 2.03 and 2.06 minutes. The expected distribution lagged the observed distribution; the theoretical values of the distribution were 0.70 and 0.71 and the sample values were 0.83 and 0.84. Thus the theoretical distribution is more conservative than what was actually encountered in the sample.

In Beyer (1966), the asymptotic critical deviation for the Kolmogorov-Smirnov test at the 5% level of significance is given as $1.36/\sqrt{N}$ where N is the sample size. The sample size for the winter storm warnings was 80 (9 regional broadcasts excluded from sample) and the critical value was 0.15. Since the maximum deviation between the distribution functions was 0.13 the uniform density hypothesis for the winter storm warning boradcast times could not be rejected.

Tornado and Severe Storm Warning Message Traffic

The data sample for tornado and severe storm warnings consisted of the NWS teletype rolls for December 3, 4 and 13, 1973, when there was considerable meteorological activity of this type. Although the occurrences were small compared to the April 3, 1974 tornados which occurred on a line from Huntsville, Alabama to Toronto, Ontario, the analysis was performed to determine if the statistical arrival pattern could be modeled with a Poisson distribution and to obtain information regarding the broadcast time densities.

The data consisted of 100 messages originating over the 3 days in the states of Georgia, Tennessee, South Carolina, Louisiana, Texas, Arkansas, Mississippi, Missouri, Alabama, Florida, Indiana, Illinois and Kentucky.

The Chi-squared statistic was used to test the Poisson hypothesis using the number of messages originating in each 15 minute interval over the 3 days of data. The calculated value of Chi-squared was 2.31. The tabled value of the statistic for the 5% significance level and 2 degrees of freedom is 5.991. The hypothesis could not be rejected at the 0.05 level of significance.

The broadcast times for the 100 messages ranged from .45 to 1.23 minutes. A uniform density from .45 to 1.25 was hypothesized and tested using the Kolmogorov-Smirnov test. The asymptotic critical value of the maximum deviation between the observed and expected distributions at the 0.05 level is $1.36/\sqrt{N} = 0.136$. The maximum deviation of 0.11 occurred at the 4 times 1.04, 1.05, 1.06 and 1.07 minutes. The observed and expected values were .85, .86, .87, .89 and .74, .75, .76, .78 respectively. The expected values of the distribution lagged the observed values yielding conservative estimates of processing times. The uniform hypothesis could not be rejected at the 0.05 level of significance.

The density type and bounds are not surprising in view of the necessity for extremely rapid dispatch for this category of message. The body of the messages consists primarily of an alert, watch or warning and the expected duration of the hazard.

Small Craft Warning Message Traffic

The data sample for the small craft warnings was taken from the NWS teletype rolls for December 5, 11, 17, 1973 and covered a geographic region from Maine to Texas.

The three days were divided into 15 minute intervals to test the hypothesis of Poisson arrivals. The tabled value of the Chi-squared statistic for the 5% level of significance and 2 degrees of freedom is 5.991. The calculated value of Chi-squared was 3.33. Therefore, the Poisson hypothesis could not be rejected at the 0.05 level of significance.

The message sample consisted of 35 messages with a range of broadcast times from 0.25 minutes to one minute. A uniform distribution was hypothesized for the message durations and tested with the Kolmogorov-Smirnov goodness-of-fit test. The critical value at the 0.05 significance level and a sample size of 35 is 0.23. The maximum deviation between the observed and expected distributions was 0.13. This deviation occurred when the expected and observed values of the distribution functions were 0.73 and 0.86 respectively. Again, a lag exists between the expected and observed values causing the simulation to produce somewhat conservative results.

The goodness-of-fit test for the small craft warning broadcast times was performed by excluding the 3 data points 1.70, 1.85 and 1.87. However, all calculations and simulations were performed using the interval 0.25 to 1.75 for the uniform distribution of small craft warning broadcast times.

River and Other Warning Message Traffic

The river and other categories were combined because there was an inadequate sample of the "other" category. The data for these categories came from the teletype rolls of the NWS for the period December 3-7, 1974 and the messages examined covered the 48 contiguous states. The Poisson arrival hypothesis was tested in the same manner as the other warnings. The tabled Chi-squared value for the 5% significance level and 1 degree of freedom is 3.841. The experimental Chi-squared value was calculated to be 1.88. The null hypothesis could not be rejected at the 0.05 level of significance.

A uniform distribution was hypothesized for the broadcast times on the interval 0.6 to 1.6 minutes and was tested using Kolmogorov-Smirnov. For the 5% significance level the critical value of a sample size of 35 is 0.23. The maximum deviation between the observed and expected distributions was 0.20 at the value 1.21 minutes. The expected distribution lagged the observed distribution over the entire interval which causes the data to yield conservative time estimates.

3.2 Expected Message Traffic and Channel Requirements for 1985

The NASA and NOAA joint working group for the examination of the feasibility of implementing a satellite DWS has been in existence since 1971. Before the results of detailed traffic analyses were obtained, it was estimated by the joint group that a satellite DWS would be required to have ten simultaneous broadcast channels for disaster warnings to the public. As the feasibility study by CSC progressed during early 1974, it became obvious that the ten channel requirement would result in very heavy and expensive satellites. Therefore it was necessary to examine the ten channel requirement to determine whether or not it could be relaxed. This examination required a detailed traffic analysis of the type presented in Section 3.1. At the same time, it was necessary to obtain estimates of what might be expected as the worst case occurring in 1985 when the satellite system would be in operation.

A linear regression analysis was performed using six years of weather warning data and was reported by Hein and Stevenson (1972). A more detailed analysis was performed by the Computer Science Corporation (1973). In the latter analysis, each message category was analyzed to determine trends and seasonal variations. The upper 95% confidence interval was used as an upper bound for traffic estimates. These data were then extrapolated to 1985. Although this estimate may seem unrealistic, very definite linear growth patterns have been experience during the last eight years. The linear correlation coefficient for the regression analysis of total monthly warnings was greater than 0.95. However, it is believed that the traffic load estimates by CSC (1973) for 1985 are conservative. During a joint NOAA-NASA review of the satellite feasibility study on April 23, 1974, Dr. John Townsend, Deputy Administrator of NOAA, stated that the primary factor for the growth in numbers of warnings during recent years has been the increased capability of spotting severe weather conditions. This growth in capability has been multilateral. There are thousands of spotters throughout the U.S. at the local community level who report sightings of tornados to the NWS within minutes, and sometimes seconds, after the sighting. At the same time, the use of advanced technology has enhanced the capability of the NWS. A vast communications network exists to relay data from all parts of the country, and to transfer satellite imagery which also has greatly enhanced meteorological capability. Dr. Townsend said that he expected the growth in warnings to subside as the network reaches a point of diminishing returns. Thus linear extrapolation to 1985 is fairly conservative.

The maximum estimated monthly load of 21,370 messages will occur during the month of December. The estimates for each category are as follows:

Category	Number of Messages Per Month
River	1608
Tornado and Severe Storms	548
Winter Storms	5521
Small Craft	8727
Other	4966

The effect of a hurricane such as Agnes will be included below.

The average number of messages occurring in December for the six years of data examined is about 7500. The estimate of 21370 for 1985 will probably not be achieved. In this situation such conservatism is preferable to underestimating the traffic flow because of the availability requirement. Using the 21370 estimate and the evidence that the arrival patterns can be described by a Poisson distribution, an estimate of the parameter is 0.495 messages per minute for the arrival rate. The Poisson parameter obtained for Hurricane Agnes was 0.035. Thus the parameter for the total arrival rate is 0.53 per minute.

A weighted average broadcasting time was determined from the data analyzed and is summarized in Figure 3.4.

The estimate of the average broadcasting time is 1.18 minutes. This estimate is for the month of December which is weighted heavily by the effect of winter storm warnings. The 1.18 minute estimate is the longest average broadcast time since the largest number of messages as well as those longest in duration occur in December.

Category	Relative Frequency	Average Broadcast Times (mins.)
Hurricanes	0.066	1.15
Tornados	0.024	0.85
Winter Storms	0.242	1.60
Small Craft	0.381	1.00
River & Other	0.287	1.10

Figure 3.4 Relative Frequencies and Average Broadcasting Times for Each Message Category for the Month of December.

The Poisson parameter and the average broadcasting time may be used to determine the initial state parameter:

$$\lambda_0 = \theta_0 \alpha = (0.53)\ (1.18) = 0.6254$$

Once the value of λ_0 has been determined, it is necessary to determine the feasible range for the number of channels. Since λ_0 is less than λ_{0MAX} for one channel, the range of feasibility is $1 \leq C \leq \infty$.

The equation

$$\lambda_* = \lambda_0 + \lambda_* \left[1 - e^{-\lambda_*}\left(1 + \lambda_* + \frac{\lambda_*^2}{2!} + \ldots + \frac{\lambda_*^C}{C!}\right)\right]$$

must be solved for λ_* over the range of values of interest for C, the number of simultaneous broadcast channels in the satellite. After determining λ_* for the range of C, the state probabilities must be determined from the equation

$$P_K^* = \frac{\lambda_*^K}{K!} e^{-\lambda_*} \quad K = 0, 1, 2, \ldots$$

These state probabilities for the effective lambda (λ_*) may be used to calculate the probability of a delay, or to determine the probability of a delay exceeding a given time value. Now the determination of the effects of limiting C becomes a relatively simple analytical procedure and does not require the use of simulation. The procedure may be used to determine the required number of channels for any queueing system with Poisson arrivals and arbitrary service. A criterion may be established in advance and then the implications of the criterion may be analyzed. Moreover, the results are quite realistic as will be demonstrated in Chapter IV.

Using the value of $\lambda_0 = 0.6254$, the values of λ_* and the state probabilities are given for a range of C in Figure 3.5.

The maximum number of warning messages in a single month for the NWS data from 1966 to 1972 was about 10,000. The data in Figure 3.5 allow for a 130% growth in message traffic by 1985. Whether such a growth is realistic can only be judged by the National Weather Service.

In order to provide information regarding various growth rates, the state probabilities for λ_0 = 0.4, 0.5 and 0.6254 are presented for comparison in Figure 3.6. These values of λ_0 correspond to monthly traffic loads of 14688, 18317 and 22896 messages, respectively, with average broadcasting times of 1.18 minutes.

It is believed that the data presented in Figures 3.5 and 3.6 are more realistic than the results originally reported by Hein and Stevenson (1972) because the Poisson arrival assumption has been justified, and the general nature of the short interval message processing characteristics have been determined through an analysis of several thousand messages sent by the NWS.

In order to determine the delay or queueing time for messages, it is necessary to introduce a time factor. The probability of a delay is the probability that W_q (queueing time) is greater than zero:

$$P\ (W_q > 0) = \sum_{n=C+1}^{\infty} P_n$$

where C is the number of simultaneously accessible channels in the satellite system. Thus

$$P\ (W_q > 0) = 1 - \sum_{n=0}^{C} P_n$$

The average service rate per channel is $1/\alpha$. For C channels this rate is C/α. But this is true only when there are C channels available for service. The average availability is (1 - Utilization Factor) which is

C	1	2	3	4	5	$6 \to \infty$
λ_*	0.7597	0.6432	0.6279	0.6257	0.62543	0.62540
State						
0	0.4678	0.5256	0.5337	0.5349	0.5350	0.5350
1	0.3554	0.3381	0.3357	0.3347	0.3346	0.3346
2	0.1350	0.1087	0.1052	0.1047	0.1046	0.1046
3	0.0342	0.0233	0.0220	0.0218	0.0218	0.0218
4	0.0065	0.0037	0.0035	0.0034	0.0034	0.0034
5	0.0010	0.0005	0.0004	0.0004	0.0004	0.0004
6	0.0001	0.00005	0.00005	0.00004	0.00004	0.00004
7	$1X10^{-5}$	$5X10^{-6}$	$4X10^{-6}$	$4X10^{-6}$	$4X10^{-6}$	$4X10^{-6}$
8	$1X10^{-6}$	$4X10^{-7}$	$3X10^{-7}$	$3X10^{-7}$	$3X10^{-7}$	$3X10^{-7}$
>8	0	0	0	0	0	0

Figure 3.5 Estimated State Probabilities for λ_0 Equal to 0.6254 and $C = 1$ to ∞.

C	$4 \to \infty$	$4 \to \infty$	4	$5 \to \infty$
λ_0	0.4	0.5	0.6254	0.6254
State				
0	0.6703	0.6065	0.5340	0.5350
1	0.2681	0.3033	0.3347	0.3346
2	0.0536	0.0758	0.1047	0.1046
3	0.0072	0.0126	0.0218	0.0218
4	0.0007	0.0016	0.0034	0.0034
5	$6X10^{-5}$	0.0002	0.0004	0.0004
6	$1X10^{-7}$	$1X10^{-5}$	$4X10^{-5}$	$4X10^{-5}$
7	$5X10^{-8}$	$9X10^{-7}$	$4X10^{-6}$	$4X10^{-6}$
8	$3X10^{-8}$	$6X10^{-8}$	$3X10^{-7}$	$3X10^{-7}$
9	$1X10^{-9}$	$3X10^{-9}$	$2X10^{-8}$	$2X10^{-8}$
>9	0	0	0	0

Figure 3.6 Estimated State Probabilities for λ_0 Equal to 0.4, 0.5, and 0.6254, and $C = 1$ to ∞.

$$1 - \frac{\lambda_*}{C}$$

The average service rate is then

$$\frac{C}{\alpha}\left(1 - \frac{\lambda_*}{C}\right)$$

which is analogous to the constant hazard rate in reliability theory. The complementary waiting time distribution is then given by

$$P\ \{W_q > t\} = \sum_{n=0}^{\infty} P_n\ P\ \{W_q > t | E_n\}$$

$$P\ \{W_q > t\} = e^{-\frac{C}{\alpha}\left(1 - \frac{\lambda_*}{C}\right)t}\ P\ \{W_q > 0\}$$

and

$$P\ \{W_q \leq t\} = 1 - P\ \{W_q > t\}$$

Using the λ_0's from Figure 3.6, which corresponded to 14688, 18317 and 22896 messages per month, respectively, for 0.4, 0.5, 0.6254, the probabilities of delays exceeding various times are given in Figure 3.7 for 6 channels. For the most adverse value of 0.6254, a delay of more than 30 seconds would occur on the average only once in 4.3 years.

A summary of the effects of 4, 6 and 10 channels are presented for 23,000, 18,000 and 15,000 messages per month in Figures 3.8 to 3.10.

As a result of the model developed in Chapter II, it is possible to determine the required number of channels to meet design availability requirements for queueing systems with Poisson arrivals and a knowledge of the average service time. In Chapter IV, a comparison will be made between the analytical results and the results obtained from a computer simulation.

Delay \ λ_0	0.4	0.5	0.6254
> 5 seconds	13 years	3 years	0.7 years
>10 seconds	19 years	4 years	1.0 years
>15 seconds	27 years	6 years	1.4 years
>30 seconds	95 years	21 years	4.3 years

Figure 3.7 Mean Time Between Delays Exceeding Certain Durations for λ_0 Equal to 0.4, 0.5, and 0.6254 for 6 Channels.

Case 1: 23,000 Per Month

Channels	4	6	10
Utilization %	15.6	10.4	6.3
Frequency of Delays			
>30 sec	1/week	1/4.3 yrs	never
> 1 min	1/month	1/41 yrs	never

Figure 3.8 Utilization and Expected Delays for 23,000 Messages Per Month.

Case 2: 18,000 Per Month

Channels	4	6	10
Utilization %	12.5	8.3	5.0
Frequency of Delays			
>30 sec	1/3 weeks	1/20 yrs	never
> 1 min	1/3 months	never	never

Figure 3.9 Utilization and Expected Delays for 18,000 Messages Per Month.

Case 3: 15,000 Per Month

Channels	4	6	10
Utilization %	10	6.7	4
Frequency of Delays			
> 30 sec	1/2 months	never	never
> 1 min	1/8 months	never	never

Figure 3.10 Utilization and Expected Delays for 15,000 Messages Per Month.

CHAPTER IV

RESULTS OF THE COMPUTER SIMULATION

In order to verify the theory of Chapter II and the predicted results of Chapter III, a computer program was written to simulate the warning communications traffic for a DWS satellite. In the report by Hein and Stevenson (1972), a digital simulation of the same process was reported using Poisson input and exponential service disciplines.

The simulation process reported here differs in several respects from the one mentioned above. Both are digital, and both use Poisson arrivals but the service disciplines reported here are not exponential. In the Hein and Stevenson report, there were three arrival categories, whereas here there are six. The timing here is asynchronous; in the other report, the timing is synchronous. Finally, this simulation process is in continuous time, whereas that reported by Hein and Stevenson was performed in discrete time. The basic unit of time for the process reported here is one minute.

Because the basic time unit is small, many problems normally associated with obtaining a steady state have been avoided. The parameter of prime importance here is the number of message requests in the system at any given time. Short simulation runs of one or two days were more than adequate to demonstrate that the process cycles "infinitely often" through a single state (namely zero) as described by Crane and Inglehart (1972). When such a cycling occurs, the sample sizes required to reduce intervals of uncertainty are significantly smaller than what would normally be required because steady state is achieved upon startup.

For the cases where an interval estimation is made and the variance is unknown, the following relationship will be used:

$$- t_{\alpha/2} < \frac{\bar{x} - \mu}{s/\sqrt{n}} < t_{\alpha/2}$$

The interval for μ is

$$\bar{x} - t_{\alpha/2}\, s/\sqrt{n} < \mu < \bar{x} + t_{\alpha/2}\, s/\sqrt{n}$$

where $\pm\, t_{\alpha/2}$ is the Student - t distribution curve area to the right or left equal to $\alpha/2$; $\bar{x}$ is the estimated mean; s is the sample standard deviation; and n is the sample size.

This chapter will consist of a description of the computer software used for the simulation and a comparison of the simulation results with those predicted in Chapter III.

4.1 Description of Computer Programs

The simulation computer program was written in FORTRAN IV for the IBM 360/67 running under TSS (Time Sharing System) at the NASA Lewis Research Center. The program consists of a main routine and four subroutines and is designed to operate in an on-line interactive mode. A functional description will be given of each segment and more explicit documentation may be obtained from the program and sample outputs in Appendix B.

MAIN ROUTINE - Initializes all variables and requests the user to specify the duration of the simulation run in days, the number of communication channels and a random number seed between 1 and 100,000. The seed allows reproduction of the same sequence or the generation of unique sequences. The main routine generates the sequence of Poisson arrivals and then calls subroutines to generate service times, calculates the run statistics and then generates the report.

SUBROUTINE RAND - This subroutine generates a sequence of uniformly distributed random numbers on the interval zero to one. The technique used is the multiplicative congruential as described in Carnahan, Luther and Wilkes (1969). The period of the generator is greater than 500,000 and the autocorrelation is less than 10^{-6}. An integer is required to start the generator and allows reproducibility or the generation of large numbers of unique sequences.

SUBROUTINE MMIN - This subroutine determines the channel which is available in the shortest period of time. In the event more than one are available simultaneously, the one with the lowest index is chosen.

SUBROUTINE MMPROC - This subroutine contains the probability distribution functions for the message broadcast times of the six message types.

SUBROUTINE CHAN - This subroutine manages the channel traffic and accumulates statistics pertaining to arrival and departure times as well as any delays in processing.

The report produced for each simulation includes the number of channels, the duration of the simulation and the number of arrivals. Means and standard deviations are given for the interarrival time, broadcast time and number of messages in the system. Other information given includes the maximum delay, if any, number of delays, the distribution of states over the duration of the simulation at a sampling rate of once per minute and a log of delayed messages. Various sampling rates were tried and it was found by trial and error that the one minute rate provides about the same information as rates of every 15 or 30 seconds but with much less computer processing time.

4.2 Comparison of Results

In Chapter III the values for λ_* were calculated when $\lambda_0 = 0.6254$ for C ranging from 1 to 6. These values are given in Figure 3.5. As may be seen in the figure, there is no arrival interaction causing an increase in λ_0 for 6 channels. For $C = 6$, $\lambda_0 = 0.6254$. The interaction begins causing an increase in λ_0 for C equal to 5 and continues to the limiting value of C equal to 1.

Simulations were performed to test whether the effective lambda's (λ_*) approach the predicted values, and to determine whether the predicted distribution functions describe the process adequately. The cases for C equal to 2, 4 and 6 were selected for evaluation because the satellite DWS will consist of a pair of satellites. In order to meet the DWS operational requirements by satellite, two satellites separated by 20 degrees are required because of the eclipsing caused by the earth. The total shadow time each year is about 1% with the maxima of 70 minutes per day occurring at the vernal and autumnal equinoxes. The shadowing begins about 20 days before each equinox and gradually builds up to a maximum and then tapers off to zero about 20 days after each equinox.

The simulation period was 18 days or 25,920 minutes. The procedure used for sampling was to simulate 9 periods of 2 days each for the values of C. There is no need to test values of C greater than 6 since the state probabilities would be equal to the 6 channel case. The tests performed and the data are summarized below for each case. The λ_0's and the expected λ_*'s for the 3 cases are:

C	λ_0	λ_*
2	0.62540	0.64315
4	0.62540	0.62569
6	0.62540	0.62540

<u>Case for C = 6</u>. A summary of the results of the 9 simulation runs for this case is presented in Figure 4.1. The observations were made at the rate of one per each simulation minute. The Poisson probabilities were calculated from a Poisson distribution with the

State	Observations	Poisson Probability	Expected Observations
0	14001	0.5400	13996.7
1	8647	0.3327	8623.6
2	2620	0.1025	2656.8
3	545	0.0211	546.9
4	87	0.0032	82.9
5	18	0.0005	13.0
6	2	0.00004	1.0

$\hat{\lambda}_* = 0.6162$ $\hat{\sigma}^2 = 0.6213$

Figure 4.1 Summary of Results for the C = 6 Case Simulation.

parameter estimate $\hat{\lambda}_* = 0.6162$ which was the estimate obtained from the 9 simulation runs. A Chi-squared goodness of fit test was performed on the data in Figure 4.1.

States 5 and 6 were combined because of the small number of observations of state 6. The critical values of Chi-squared for significance levels of 5%, 10% and 25% are given in Beyer (1966) for 4 degrees of freedom as follows:

$$X^2_{.05,4} = 9.49$$

$$X^2_{.10,4} = 7.78$$

$$X^2_{.25,4} = 5.39$$

The calculated value of Chi-squared obtained from the goodness of fit tests was 4.55 which is less than any of the critical values. The Poisson hypothesis must be accepted for the $C = 6$ case. The next tests conducted were on the mean and variance. The student-t distribution was used for the test on the mean, and the Chi-squared test was used for the variance test. The null hypotheses are that the mean $\lambda_* = 0.6254$ and that the variance $\lambda_* = 0.6254$. The test statistics are obtained from:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$X^2 = \frac{(n-1)s^2}{\sigma^2}$$

where $\bar{x}$ is the sample mean, μ is the hypothesized mean, s is the sample standard deviation and n is the sample size. The calculated statistics are:

$$t = \frac{0.6162 - 0.6254}{0.7882/\sqrt{9}}$$

$$= -0.03$$

$$X^2 = \frac{(8)(0.6213)}{0.6254}$$

$$= 7.95$$

The critical values of t and X^2 for 8 degrees of freedom and a significance level of 10% are given by Beyer (1966) as 1.397 and 13.362, respectively. The null hypotheses cannot be rejected. There were no delayed messages for the $C = 6$ case. The final tests conducted were to develop 95% confidence intervals for the state probabilities. These intervals are presented in Figure 4.2. In all cases the intervals include the expected and observed values.

Case for C = 4. A summary of the results for this case is presented in Figure 4.3. The data consist of the same number of samples, but the random number sequences were varied to ensure results which were independent of the 6 channel case. The observations were again made at the rate of one per each simulation minute. The parameter estimate for the 4 channel case was $\hat{\lambda}_* = 0.6145$ which was the estimate obtained from the 9 simulation runs. The Chi-squared test for goodness-of-fit was performed on the data in Figure 4.3. Because there are only two observations of state 6, states 5 and 6 were combined. Since there are 4 degrees of freedom, the critical values for this case are identical to the 6 channel case. The calculated value of Chi-squared for this test was 3.76 which is less than the critical values. Again, the Poisson hypothesis cannot be rejected. The parameter estimate of $\hat{\lambda}_*$ is lower in this case than in the 6 channel case. This is attributed to randomness. The tests for the mean and variance were conducted with the null hypotheses being that the mean and variance were each equal to 0.6257, the value predicted from the theory in Chapter II. Again the student-t statistic was used for the mean and the Chi-squared was used for the variance.

State	Observed Relative Frequency	Expected Relative Frequency	95% Confidence Interval
0	0.5402	0.5350	0.5301 to 0.5503
1	0.3336	0.3346	0.3265 to 0.3407
2	0.1011	0.1046	0.0963 to 0.1061
3	0.0210	0.0218	0.0175 to 0.0245
4	0.0034	0.0034	0.0029 to 0.0039
5	0.00069	0.00043	0.00035 to 0.00101
6	0.000077	0.000044	0.000006 to 0.00015

Figure 4.2 Observed and Expected Relative Frequencies and 95% Confidence Intervals for $C = 6$ and $\hat{\lambda}_* = 0.6162$.

State	Observations	Poisson Probability	Expected Observations
0	14035	0.5409	14020.4
1	8596	0.3327	8615.6
2	2648	0.1021	2647.1
3	548	0.0209	542.2
4	76	0.0032	83.3
5	15	0.0005	10.2
6	2	0.00004	1.0

$\hat{\lambda}_* = 0.6145$ $\qquad$ $\hat{\sigma}^2 = 0.6172$

Figure 4.3 Summary of Results for the C = 4 Case Simulation.

The calculated statistics are:

$$t = \frac{0.6145 - 0.6257}{.7856/\sqrt{9}}$$

$$= - 0.04$$

$$X^2 = \frac{(8)(0.6172)}{0.6254}$$

$$= 7.90$$

Both values are less than the critical values for 8 degrees of freedom as given above. Thus the null hypotheses cannot be rejected. The relative observed and expected frequencies are given in Figure 4.4 along with the 95% confidence intervals. In all cases, the intervals include the expected and observed values.

In the 4 channel case, it was mentioned previously that the increase in λ_* over λ_0 is caused by the customer-server interaction. A more valid test of the model is whether or not delays encountered correspond to the predicted delays. There were no delays encountered in the simulations of the 6 channel case. In the 4 channel case, however, delays began occurring with regularity. The delays varied from 0.01 minutes to 0.64 minutes. Since the standard deviations of the broadcast times are usually on the order of 0.5 minutes, a delay of 0.01 minutes is meaningless. From a human factor's point of view, a delay becomes substantial when it reaches a certain magnitude. Since the definition of intolerable delays is beyond the scope of this work and really lies with those individuals who must use the system, it was arbitrarily decided that only those delays greater than 30 seconds would be noted. The frequency of these delays in the simulations would then be compared with the predicted delay frequencies. For the 4 channel case, there were 4 delays exceeding 30 seconds. The durations were 33, 34, 34

State	Observed Relative Frequency	Expected Relative Frequency	95% Confidence Interval	
0	0.5414	0.5409	0.5302	to 0.5528
1	0.3316	0.3324	0.3227	to 0.3405
2	0.1022	0.1021	0.0990	to 0.1054
3	0.0211	0.0209	0.0193	to 0.0229
4	0.0029	0.0032	0.0023	to 0.0035
5	0.00058	0.00039	0.00028	to 0.00088
6	0.00008	0.00004	0.000006	to 0.000150

Figure 4.4 Observed and Expected Relative Frequencies and 95% Confidence Intervals for $C = 4$ and $\hat{\lambda}_* = 0.6145$.

and 38 seconds. Using the equations for delays exceeding T from Chapter III:

$$P\ \{W_q > t\} = e^{-\frac{C}{\alpha}\left(1 - \frac{\lambda_*}{C}\right)t}\ P\ \{W_q > 0\}$$

$$P\ \{W_q > 0\} = 1 - \sum_{n=0}^{C} P_n$$

For $C = 4$, $\lambda_* = 0.6257$ and $\alpha = 1.18$,

$$P\ \{W_q > 30 \text{ seconds}\} = 1.13 \times 10^{-4}$$

There were 4 delays greater than 30 seconds during a simulated period of 25920 minutes. The relative frequency of delays encountered is 1.54×10^{-4} or a 36% difference from the expected occurrence. The sample mean of expected delays was 1.54×10^{-4}. The sample standard deviation was 1.24×10^{-2}. Using the t statistic for comparison of means

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

yields a value of 0.532. The critical values for an infinite sample size at the 10%, 5% and 1% levels of significance are given in Beyer (1966) as follows:

$$t_{.10,\infty} = 1.282$$

$$t_{.05,\infty} = 1.645$$

$$t_{.01,\infty} = 2.326$$

The test demonstrates that there is no significant difference between the predicted value and that encountered in the simulations at the

0.05 level of significance.

It was predicted that there should be about 3 delays in 18 days and the simulations produced 4 delays in 18 days. In order for there to have been a significant difference between expected and predicted delays at the 10%, 5% and 1% levels of significance, there would have to be at least 6, 7 and 8 delays, respectively, in a simulation of 18 days.

Case for $C = 2$. A summary of the results for this case is presented in Figure 4.5. The observations were made at the rate of one per minute of simulated time. An anomaly appears in the data in that the expected number of observations is lower than what was observed for state zero, but the reverse is true for state one. This suggests that for this case a faster sampling rate would be more appropriate. To do so, however, would require much more storage in the computer. Since the program runs in real time on a virtual storage machine, a faster sampling rate would require an enormous increase in computer time because execution time is a function of storage on a virtual memory machine.

Since states zero and one do not impose any delays, the 2 states may be combined into an aggregate state. Another point about the results is that the degradation from a Poisson to a discrete-type of exponential distribution has begun to occur. One would expect that the relative frequency of the higher states would increase as λ_* approaches λ_{*MAX}. The parameter estimates obtained from the 9 runs of 2 days each was $\hat{\lambda}_* = 0.6508$. The Chi-squared test for goodness of fit was performed on the data of Figure 4.5. Because of the anomaly in states 0 and 1, these states were combined. Also the same type of anomaly appears in states 4, 5 and 6; these states were combined because of the small number of observations in state 6 and because of the anomaly. In Beyer (1966) the critical value of Chi-squared for 2 degrees of freedom at the one percent level of significance is 9.21. The calculated value of Chi-squared was 8.82. The Poisson hypothesis

State	Observations	Poisson Probability	Expected Observations
0	13693	.5216	13519.3
1	8548	.3395	8799.7
2	2864	.1105	2863.9
3	681	.0240	621.4
4	122	.0039	101.1
5	11	.0005	13.2
6	1	.000055	1.6

$\hat{\lambda}_* = 0.6508$ $\qquad$ $\hat{\sigma}^2 = 0.6721$

Figure 4.5 Summary of the Results for the $C = 2$ Case Simulation.

cannot be rejected; but it would have been at any higher level of significance or without the state combinations described above. The tests for the mean and variance were conducted with the null hypotheses being that the mean and variance were each equal to 0.6432, the value predicted from Chapter II. The procedure was identical to the two previous cases. The calculated statistics are:

$$t = \frac{0.6508 - 0.6432}{.8198/\sqrt{9}}$$

$$= .0278$$

$$X^2 = \frac{8(0.6720)}{0.6508}$$

$$= 8.26$$

Both values are less than the critical values given above for the C = 6 case. The null hypotheses cannot be rejected. The relative observed and expected frequencies are given in Figure 4.6 along with the 99% confidence intervals rather than the 95% intervals. The wider interval does not include the expected relative frequencies for state 1 and state 4. This exclusion is not as important for state 4 as for state 1 because of the magnitude of the values. The standard deviations of the samples for all states were as follows:

State	Standard Deviation
0	0.0080
1	0.0033
2	0.0053
3	0.0028
4	0.0012
5	0.00029
6	0.000117

State	Observed Relative Frequency	Expected Relative Frequency	99% Confidence Interval	
0	0.5283	0.5216	0.5194	to 0.5372
1	0.3298	0.3395	0.3261	to 0.3335
2	0.1105	0.1105	0.1046	to 0.1164
3	0.0263	0.0240	0.0232	to 0.0294
4	0.0047	0.0039	0.0044	to 0.0050
5	0.00042	0.00051	0.00011	to 0.00075
6	0.000039	0.000055	0.0	to 0.00017

Figure 4.6 Observed and Expected Relative Frequencies and 99% Confidence Intervals for $C = 2$ and $\hat{\lambda}_* = 0.6508$.

The expected standard deviation for state 2 should lie between 0.0053 and 0.0080. This anomaly in the standard deviation supports the combination of states 0 and 1 in the goodness-of-fit test above. States 0 and 1 were combined to obtain a total confidence interval with an expected relative frequency of 0.8611 and an observed frequency of 0.8581. The combined standard deviation is 0.0087 and the combined 99% confidence interval is 0.8484 to 0.8678.

Large numbers of delays were encountered in this case. Only those delays longer than 30 seconds were included. The delay equations for $C = 2$, $\lambda_* = 0.6508$ and $\alpha = 1.18$ yield

$$P\ \{W_q > 0\} = 0.0284$$

$$P\ \{W_q > 30 \text{ seconds}\} = 0.0160$$

$$P\ \{W_q > 1 \text{ minute}\} = 0.0091$$

For an 18-day period, the number of delays greater than zero is expected to be 736. Over the same period, 416 delays are expected to exceed 30 seconds and 235 are expected to exceed 1 minute. When the number of channels is limited to 2, a new phenomenon is encountered. The server-customer interaction increases greatly so that many more small delays are encountered than would be expected. Also, when the interaction increases, clusters of delays occur. One delay may be the cause of 5 or 6 other delays. The expected and observed delays for queueing times greater than zero, 30 seconds and 1 minute are shown in Figure 4.7. For this case it is obvious that the delay equations are no longer adequate for delays in the zero to 30 second range. For delays exceeding 30 seconds the equations yield results which may be unacceptable in many cases. However, for delays exceeding one minute, the predicted results become conservative again. Also, for delays greater than 30 seconds, if each cluster is counted as a single delay, the results of observed and expected delays become compatible again.

The delay results for the 2 channel case demonstrate that the model begins to break down as the utilization and server-customer interaction increases. The interaction effect is very noticeable for delays of 30 seconds or less but disappears completely for delays exceeding 1 minute.

Summary of Simulation Results. The results of the simulations demonstrate that the model developed in Chapter II provides excellent results as long as the original assumption remains valid; namely, that utilization is relatively low. Thus, the model is still valid for many realistic applications. Some useful results may be obtained for higher utilizations but the model would have to be improved in order to provide acceptable results in all cases. In the next chapter, the satellite system design will be summarized to demonstrate how the model may be used for the satellite DWS and application to other areas will be discussed.

	Expected Delays	Observed Delays	Observed Clusters
$W_q > 0$	736	1885	NA
$W_q > 30$ seconds	416	770	519
$W_q > 1$ minute	235	167	None

Figure 4.7 Expected and Observed Delays for $C = 2$, $\hat{\lambda}_* = .6508$ for a Simulated Period of 18 days.

CHAPTER V

SUMMARY OF RESULTS AND CONCLUSIONS

In Chapter I, the rationale for special purpose communication satellites was developed with particular emphasis on the need for a satellite based DWS. The model for this type of queueing system was developed in Chapter II. The major assumptions were that utilization would be relatively low because of the availability criterion and that the arrivals are Poisson distributed. Large amounts of weather warning data were obtained from the National Weather Service in order to analyze the arrival and broadcast distributions for warning messages. These data were also used to analyze the effects of limiting the number of satellite broadcast channels in Chapter III. The expected results were compared with the results of a computer simulation in Chapter IV.

The results obtained from the model developed in Chapter II provided excellent approximations for the 6 and 4 channel cases. Statistically there was no significant difference between the predicted results and those obtained from the simulations. When the traffic intensity was increased through a reduction in service channels to 2, the customer-server interaction caused a degradation in the quality of the predicted results. The interaction causes the variance of the Poisson distribution to increase through a distortion of the relative state frequencies. This distortion causes a change in the state distribution; it deviates from the predicted Poisson distribution. Although the 2 channel case may provide useful results, it also demonstates that the model is close to the limits of usefulness. In order to demonstrate why and how the model developed in Chapter II may be used, a summary of the feasibility study and conceptual design by the Computer Science Corporation will be included in this Chapter. The summary is taken from the report to Dr. John Townsend, Deputy Administrator of the National Oceanic and Atmospheric Administration. The presentation was made on May 23, 1974 at NOAA Headquarters in Rockville, Maryland and included the work done by CSC and the results of the 4, 6 and 10 channel cases using the model developed in Chapter II.

Although the message traffic analysis constitutes only a small part of the study, the cost of doing the traffic analysis using computer simulation required 50 to 100 hours of computer time. At a nominal cost of $1000 per hour for computing time, the advantages of using an analytical technique rather than simulation are immediately obvious.

The operational concepts of the terrestrial and the satellite systems for disaster warning are shown in Figures 5.1 and 5.2. In the terrestrial system, the NWS network would consist of ground-based transmission systems connecting spotters with WSO's (Weather Service Offices), WSFO's (Weather Service Forecast Offices) and such specialized centers as the National Hurricane Center (NHC) in Miami. The operational concept for both systems is shown in Figure 5.3. The design criteria for the dissemination of warnings is shown in Figure 5.4, and the operational requirements are shown in Figure 5.5. The geographical coverage pattern required for the DWS is denoted by the shaded portion of Figure 5.6. In order to meet the operational requirements by satellite, two satellites separated by 20 degrees are required as described in Chapter IV. In Figure 5.7 a qualitative comparison is made between the satellite and the terrestrial systems, and the major cost drivers for each system are shown in Figure 5.8. The original satellite requirements were that 10 simultaneous channels would be required for broadcasting warning messages. The delays expected with 4, 6 and 10 channels were shown in Figures 3.8, 3.9 and 3.10.

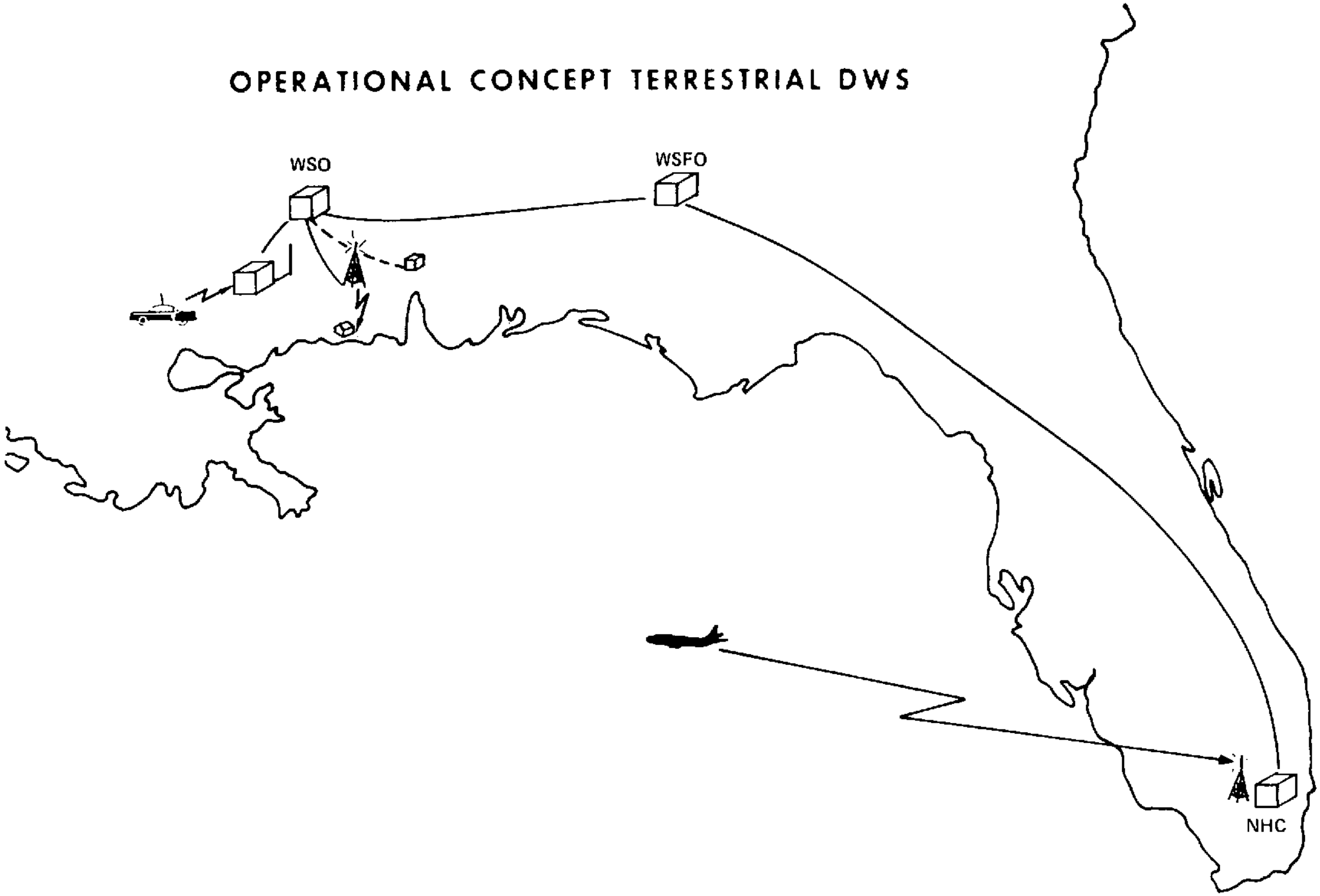

Figure 5.1 Terrestrial based DWS.

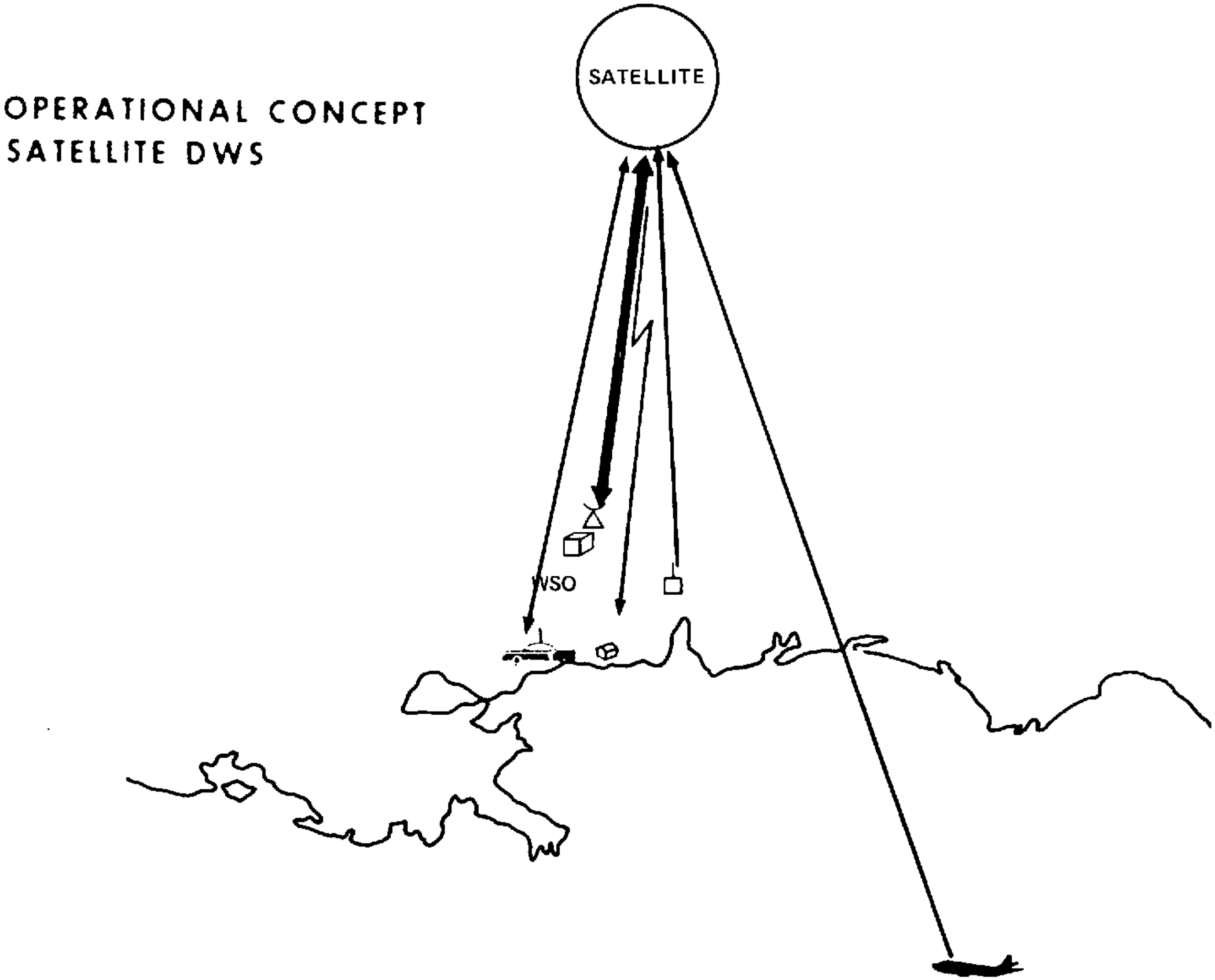

Figure 5.2 Satellite based DWS.

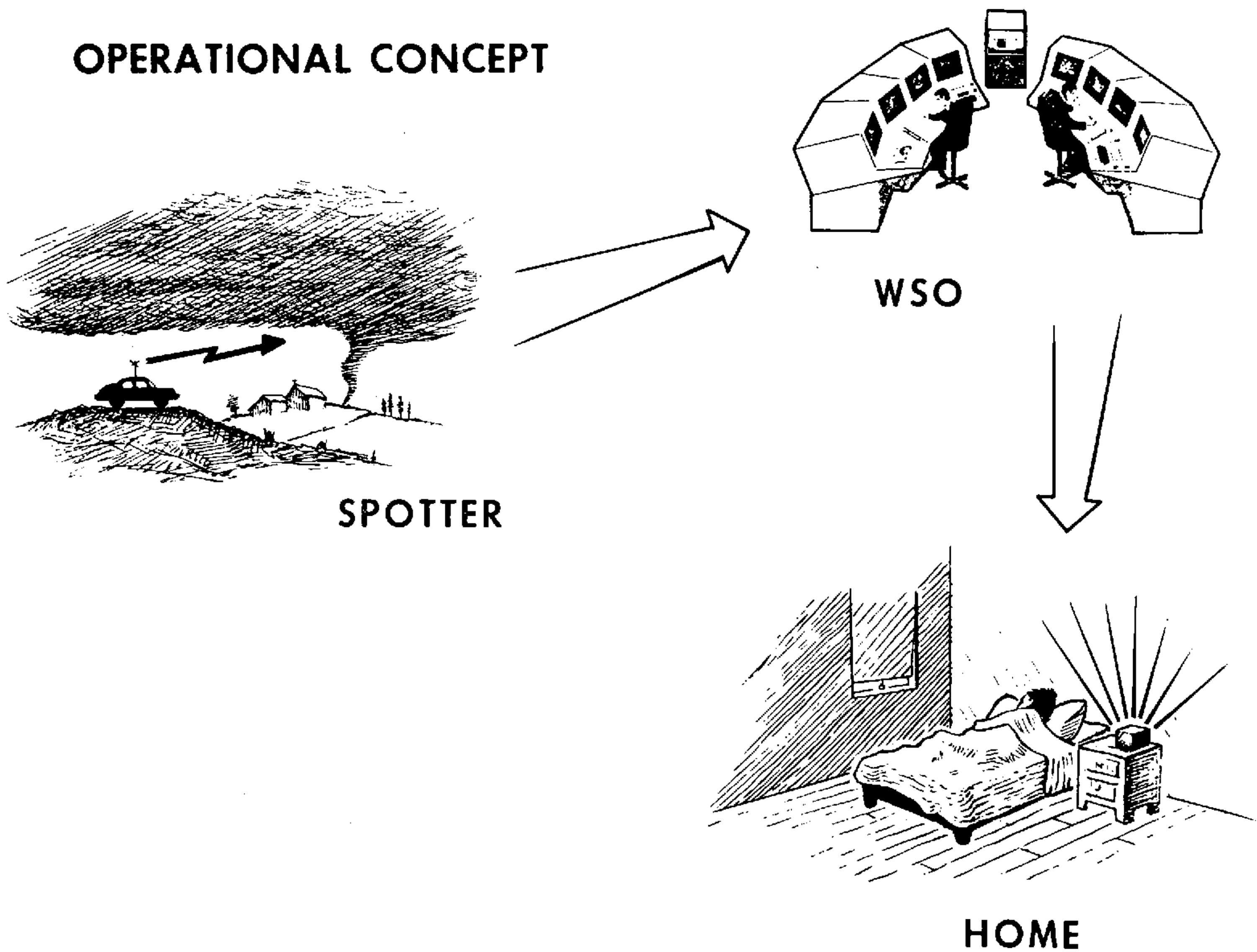

Figure 5.3 Operational concept of a DWS.

Required Response To Disaster Types

Disaster Type	Smallest Area Warned	Message On-Line Upper Bound
Tornado or Severe Storm	Part of County	1-5 Min
Hurricane	Part of Coast	1-15 Min
River Flood	Part of State	15 Min-1Hr
Small Craft	Part of Coast (Lake)	15 Min-1Hr
Winter Storm	Part of State	15 Min-1Hr
Others	Part of County	1 Min-1Hr

Figure 5.4 Functional Design Requirements for a DWS.

OPERATIONAL REQUIREMENTS

System

24-Hour Operation

Immune From Natural Disasters

Autonomous Power Source

Simultaneous Warning Capability

Home Receiver

Inside Antenna

Activate $\leq$ 15 Seconds

Selective Addressing

On-Off Option

Figure 5.5 Operational Design Requirements for a DWS.

GEOGRAPHICAL COVERAGE REQUIREMENT

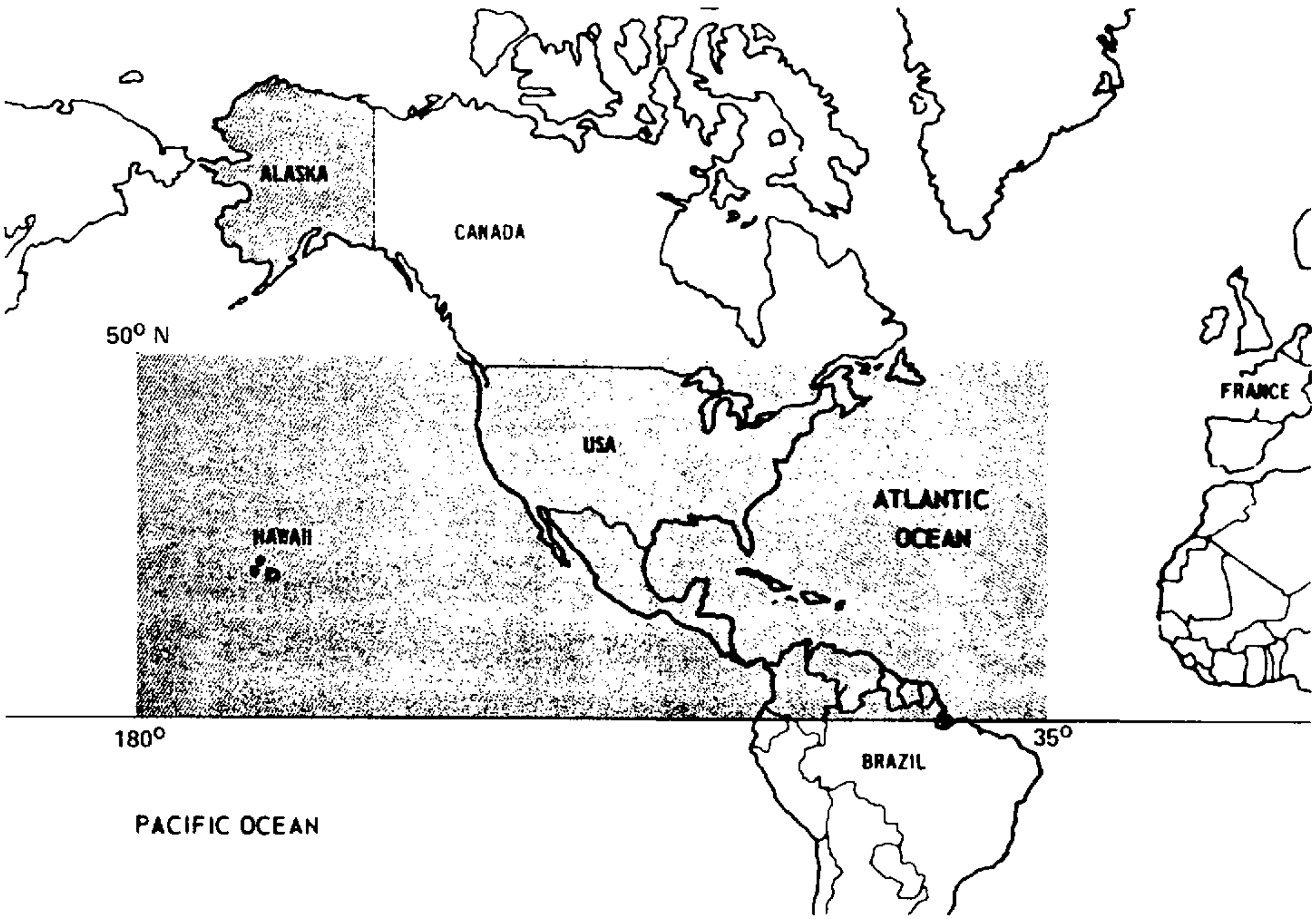

Figure 5.6 Coverage requirements for a DWS.

Although this document is not the proper forum for a cost comparison, it must be acknowledged that the satellite system with 10 channels would not be competitive with the terrestrial system. Four channels may be adequate to meet NOAA requirements. If so, reducing the channel requirements from 10 to 4 would make the satellite and terrestrial systems approximately equal in cost. If it were possible to use a 2 channel system, the satellite system would probably be more advantageous than the terrestrial system. Such a reduction would require a reduction in the present warning traffic or the use of priority queueing.

As a result of the model developed in Chapter II, a more realistic appraisal method of the channel requirements for a satellite DWS was obtained. The model developed here may also be used to determine channel requirements for other types of special purpose communications satellites. Remote health care delivery systems via satellite have been in existence for several years. This application will increase as a result of the recent launch of ATS-F (Applications Technology Satellite-F) by NASA. This satellite will be used to provide high powered television signals to remote areas in the western states, Alaska, and to remote villages in India. The smallest antennas will be 15 feet in diameter and the receivers will cost about 5 or 6 thousand dollars. This size and cost represents an enormous reduction compared to the commercial satellite ground stations and renders feasible many applications of communications in regions which previously could not afford anything beyond the essentials for existence. As the cost of receivers is reduced through an increase in the satellite power, the many applications discussed in Communications for Social Needs (1971) will become realities. All of these applications are characterized by relatively low utilization and a high availability factor. Some of these applications are the DWS noted previously, remote health care delivery, educational television for remote regions, electronic mail handling, finger-

SATELLITE/TERRESTRIAL COMPARISON

Terrestrial		Satellite
Uncertain	Immunity To Natural Disasters	Good
Cost Dependent	Response Time	Fast
Complex	System Control	Good

Figure 5.7 Qualitative Comparison of Terrestrial and Satellite Based Systems.

MAJOR COST DRIVERS

Terrestrial	Satellite
Extensive Coverage	Simultaneous Transmissions
Complete Connectivity	Small Ground Terminals
Fast Response Time	Real-Time Voice Communications

Figure 5.8 Major Cost Sensitive Parameters for Terrestrial and Satellite Systems.

printing transmissions for law enforcement agencies, and many others. The model developed in this dissertation is applicable to these new applications of communications satellites whenever the input can be shown to be Poisson distributed.

There are many areas of application beyond usage of the model for communications satellites. For example, stock brokers must be available by phone to their clients. When line utilization increases beyond some point, customers will begin going elsewhere. The model could be used to determine the optimal number of telephone lines for a broker to have. Police departments might also use such a model to meet some pre-specified telephone line availability requirement. Applications of the model to queueing systems where low utilization/high availability is the predominant characteristic are more numerous than can be mentioned here. Moreover, the necessity of minimizing waiting time will become even more important in the future as world economies become dominated by service industries. Market strategies will include tradeoff analyses of the value of minimizing waiting versus the cost of adding more service channels.

The major shortcoming of the model developed here is that when utilization increases beyond a certain point, the reliability of the results is questionable. Extension of the model to heavy traffic would allow many new applications of queueing theory. Urban ground transportation and traffic is a subject which might use an extension of the model. Those service industries and markets where traffic flow is heavy and utilization is high have often demonstrated that adding service channels was seldom part of their planning. Rather, many organizations simply expanded their hours of operation. Examples of such a policy are gasoline service stations, supermarkets, discount department stores, airports, and many others. With the advent of the petroleum shortage of the 1970's, service station operators found they could shorten operating hours; the effect was a large decrease in operating expenses and overhead.

In order to conserve energy, many public buildings and stores changed operating hours and increased the traffic intensity in the process. In order to maintain customer satisfaction and to keep the customer, consideration should be given to the effect of operational changes on the customer whether the motive is profit or public service.

In Chapter I, it was mentioned Thomas Saaty wrote in 1966 that queueing models are seldom applied to real situations. Saaty's lament pointed to the very genuine problem of allowing need to dictate the direction of technological change and innovation. The decade since 1966 has witnessed new developments and applications of queueing theory. Operational needs have been the driving force behind the development of many applications. Such was the motivation here.

APPENDIX A

Proof Of The Equivalence Of λ_{j-1} And λ_* When The Sequence Converges

It was stated that the forms for

$$\lambda_* = \lambda_0 + \gamma\lambda_* \Phi(\lambda_*) \qquad \text{A.1}$$

and

$$\lambda_{j-1} = \lambda_0 + \gamma\lambda_{j-1}\Phi(\lambda_{j-1}) \qquad \text{A.2}$$

are equivalent if the sequence $\{\lambda_i\}$ converges. If the sequence converges then λ_j differs from λ_{j-1} at most by some value ε which can be made arbitrarily small. Thus for some j greater than N

$$\lambda_j = \lambda_{j-1} + \varepsilon \qquad \text{A.3}$$

Substituting A.3 into A.1 yields

$$\lambda_{j-1} = \lambda_0 + \gamma\lambda_{j-1}\Phi(\lambda_{j-1}) - \varepsilon$$

Since ε can be made arbitrarily small

$$\lim_{\varepsilon\to 0} \lambda_{j-1} = \lambda_0 + \gamma\lambda_{j-1}\Phi(\lambda_{j-1})$$

This procedure may be repeated for j, j+1, j+2, ... so that for some N and all integers greater than N, the relationship for λ_N may be expressed as

$$\lambda_* = \lambda_0 + \gamma\lambda_*\Phi(\lambda_*)$$

APPENDIX B

Computer Simulation Programs And Sample Output

```
      IMPLICIT REAL*8 (A-F)
      DIMENSION A(50),B(50),C(50),D(50),E(50),F(50)
      DATA E/50*1.0/
      DMAX=100.0
      INDEX=0
      DFAC=1.0
      WRITE (6,1000)
 1000 FORMAT (' ',T5,'THIS PROGRAM CALCULATES THE MAXIMUM VALUES OF'/
     1T5,'LAMBDA0 AND LAMBDA* FOR A GIVEN NUMBER OF CHANNELS'//T5,'NO. CHANN-
     2ELS     LAMBDA0      LAMBDA*'/)
      WRITE (6,1003)
 1003 FORMAT (' ',T5,5X,' 1            0.839962     1.618034')
      DO 500 J=2,20
      DX=DMAX
      DO 100 I=1,20
      DFAC=DFAC*I
  100 A(I)=-1.0/DFAC
      A(J+1)=-A(J)
      DFAC=1.0
      JM1=J-1
      DO 200 I=1,JM1
  200 B(I)=A(I)
      B(J)=A(J+1)*(J+1)
  250 DFN=-1.0
      DFNPR=-1.0
      DO 300 I=1,J
      DFN=DFN+A(I)*(DX**I)
  300 DFNPR=DFNPR+B(I)*(DX**I)
      DFN=DFN+A(J+1)*(DX**J)*DX
      DTEMP=DX-DFN/DFNPR
      INDEX=INDEX+1
      DEL=DABS(DTEMP-DX)
      IF (DEL.LT.0.000000001) GO TO 500
      DX=DTEMP
      GO TO 250
  500 C(J)=DTEMP
      DO 600 J=1,20
      C1=C(J)
  600 D(J)=DEXP(-C1)*C1
      DO 800 J=1,20
      DFAC=1.0
      DO 700 I=1,J
      DFAC=DFAC*I
  700 E(J)=E(J)+ (C(J)**I)/DFAC
  800 F(J)=D(J)*E(J)
      DO 900 J=2,20
  900 WRITE (6,1001) J,F(J),C(J)
 1001 FORMAT (' ',T5,5X,I2,9X,F9.6,3X,F9.6)
      STOP
      END
```

```
C           THIS PROGRAM IS A CONTINUOUS TIME ASYNCHRONOUS
C           SIMULATION PROGRAM.  THE PRIMARY USE IS INTENDED
C           FOR THE SIMULATION OF THE COMMUNICATIONS OF A
C           DISASTER WARNING SATELLITE SYSTEM.  HOWEVER, THE
C           PROGRAM MAY BE USED TO SIMULATE ANY QUEUEING
C           SYSTEM WITH POISSON ARRIVALS.
C
C           THE PROGRAM CONSISTS OF A MAIN ROUTINE AND FOUR
C           SUBROUTINES.  THE FUNCTIONS OF EACH MODULE ARE
C           DESCRIBED BELOW AND EACH MODULE CONTAINS SOME
C           DOCUMENTATION FOR THE MAJOR FUNCTIONS.
C
C
C           MAIN ROUTINE************************************
C           SOURCE PROGRAM: SOURCE.CONSIM
C           OBJECT PROGRAM: CONSIM
C           THE MAIN ROUTINE INITIALIZES SIX POISSON ARRIVAL
C           PARAMETERS AND THEN PROMPTS THE USER TO SPECIFY
C           THE NUMBER OF DAYS IN THIS RUN, THE NUMBER OF
C           CHANNELS AND AN INTEGER SEED FOR THE RANDOM
C           NUMBER GENERATOR.  THE PROGRAM THEN GENERATES A
C           SEQUENCE OF ARRIVALS AND CALLS THE PROCESSOR TO
C           DETERMINE THE PROCESSING TIMES INCLUDING WAITING
C           TIMES IF ANY.  THE PROGRAM THEN CALLS THE CHANNEL
C           ASSIGNMENT SUBROUTINE TO DETERMINE WHERE THE
C           ARRIVAL WILL BE PROCESSED.  THE STATISTICS ARE
C           COMPUTED AND THEN THE REPORT IS GENERATED.
C
C
C           SUBROUTINE RAND*********************************
C           SOURCE PROGRAM: SOURCE.URANNO
C           OBJECT PROGRAM: URANNO
C           THIS SUBROUTINE GENERATES UNIFORMLY DISTRIBUTED
C           RANDOM NUMBERS OF THE INTERVAL FROM ZERO TO ONE.
C           THE VARIABLE IGESS IS AN INTEGER SEED WHICH ALLOWS
C           THE REPRODUCTION OF A SEQUENCE OR THE GENERATION OF
C           100,000 UNIQUE SEQUENCES.  THE TECHNIQUE USED IS
C           THE MULTIPLICATIVE CONGRUENTIAL.
C
C
C           SUBROUTINE MMIN*********************************
C           SOURCE PROGRAM: SOURCE.SMIN
C           OBJECT PROGRAM: SMIN
C           THIS SUBROUTINE DETERMINES THE NEXT ARRIVAL FROM THE
C           SIX DIFFERENT TYPES OF ARRIVALS.  EACH OF THE SIX
C           ARRIVAL PATTERNS ARE GENERATED INDEPENDENTLY.  THIS
C           SUBPROGRAM COMPARES THE ARRIVAL TIMES WITH THE CLOCK.
C
C
C           SUBROUTINE MMPROC*******************************
```

```
C          SOURCE PROGRAM: SOURCE.SMPROC
C          OBJECT PROGRAM: SMPROC
C          THIS SUBROUTINE GENERATES AND TRANSFORMS THE
C          UNIFORMLY DISTRIBUTED RANDOM NUMBERS TO THE
C          DISTRIBUTIONS REQUIRED FOR THE SIX TYPES OF
C          POISSON ARRIVALS IN ORDER TO OBTAIN THE
C          PROCESSING TIMES.
C
C
C          SUBROUTINE CHAN*********************************
C          SOURCE PROGRAM:  SOURCE.SCHAN
C          OBJECT PROGRAM:  SCHAN
C          THIS SUBROUTINE IS THE CHANNEL PROCESSOR.  IT
C          MAINTAINS A RECORD OF THE USAGE AND THE
C          SOONEST AVAILABLE CHANNEL.  WAITING TIMES ARE
C          ALSO CALCULATED IF THERE ARE ANY.
C
C
C
C
      DIMENSION A(6),LAMDA(6),C(100),IFREQ(101),ARR(3000),AWAIT(3000),ARRON(3000)
      DIMENSION PROC(3000),ARROFF(3000),ISTATE(3000),IAR(3000),IARC(3000)
      REAL LAMDA
      INTEGER*2 IAR,IARC
      DATA LAMDA/0.035,0.0135,0.0372,0.1278,0.2015,0.1150/
      T=0.0
      ISW=0
C
C
C            PROMPT FOR ENTRY OF TIME,CHANNELS AND
C            THE RANDOM NUMBER SEED
C
C
      WRITE (6,1000)
      READ  (5,1001) D
      TMAX=60.0*24.0*D
      WRITE (6,1002)
      READ  (5,1003) NOCHAN
      WRITE (6,1004)
      READ  (5,1005) IGESS
    0 DO 100 J=1,6
      CALL RAND(Z,IGESS,IA,IX,IXX,ISW)
  100 A(J)=ALOG(Z)/(-LAMDA(J))
C
C
C            THE FIRST SIX ARRIVALS ARE GENERATED WITH
C            THE TRANSFORMATION AT STATEMENT 100
C
C
       I=0
```

```
200   CALL MMIN(A,T,J)
      T=A(J)
      IF (T.GE.TMAX)  GO TO 300
      I=I+1
      ARR(I)=T
      IAR(I)=J
      CALL RAND(Z,IGESS,IA,IX,IXX,ISW)
      A(J)=T+(ALOG(Z)/(-LAMDA(J)))
      GO TO 200
C
C
C           ARRIVALS ARE GENERATED WITH THE CLOCK TIME
C           DISPLACEMENT ADDED ON
C
C
C           I = NUMBER OF ARRIVALS UP TO TIME D DAYS
C           ARR(J) = ARRIVAL TIME OF ARRIVAL J
C           IAR(J) = TYPE OF ARRIVAL (1-6)
C
C
300   DO 400 J=1,I
      CALL MMPROC(IAR(J),PTIM,Z,IGESS,IA,IX,IXX,ISW)
400   PROC(J)=PTIM
C
C
C           CALL THE CHANNEL PROCESSOR
C
C
      DO 500 J=1,I
500   CALL CHAN(NOCHAN,ARR(J),PROC(J),ARRON(J),AWAIT(J),ARROFF(J),IARC(J),C)
C
C
C           CALCULATE STATISTICS FOR THIS RUN
C
C
      IUN=TMAX
      DO 600 ICTR=1,IUN
      TCTR=ICTR
      DO 600 INDEX=1,I
      IF ((ARR(INDEX).LE.TCTR).AND.(ARROFF(INDEX).GE.TCTR)) ISTATE(ICTR)=ISTATE(ICTR)+1
600   CONTINUE
C
C
C
      DMAX=0.0
      SUMAR=ARR(1)
      SUMAR2=ARR(1)*ARR(1)
      SUMPR=PROC(1)
      SUMPR2=PROC(1)*PROC(1)
      DO 700 J=2,I
```

```
      JM1=J-1
      IF (AWAIT(J).GT.0.0) NODEL=NODEL+1
      DELA=ARR(J)-ARR(JM1)
      SUMAR=SUMAR+DELA
      SUMAR2=SUMAR2+DELA*DELA
      IF (AWAIT(J).GE.DMAX) DMAX=AWAIT(J)
      SUMPR=SUMPR+PROC(J)
  700 SUMPR2=SUMPR2+PROC(J)*PROC(J)
      SNO=NOCHAN
      UTIL=SUMPR/(TMAX*SNO)
      UTIL=UTIL*100.0
      TI=I
      ARRBAR=SUMAR/TI
      PRBAR=SUMPR/TI
      TI2=TI*(TI-1.0)
      ARSTD=SQRT((TI*SUMAR2-SUMAR**2)/TI2)
      PRSTD=SQRT((TI*SUMPR2-SUMPR**2)/TI2)
      TIUN=IUN
C
C
C
C
      DO 800 ICTR=1,IUN
      SUMST=SUMST+ISTATE(ICTR)
      SUMST2=SUMST2+ISTATE(ICTR)**2
      ICT1=ISTATE(ICTR)+1
      IF (ICT1.GT.100) ICT1=100
  800 IFREQ(ICT1)=IFREQ(ICT1)+1
C
C
C
      TIUNM1=TIUN-1.0
      STBAR=SUMST/TIUN
      STSTD=SQRT((TIUN*SUMST2-SUMST*SUMST)/(TIUN*TIUNM1))
C
C
C
C
C           GENERATE REPORT
C
C
C
C
      WRITE (6,1010)
      WRITE (6,1011) NOCHAN,D,I
      WRITE (6,1010)
      WRITE (6,1012) ARRBAR,ARSTD
      WRITE (6,1013) PRBAR,PRSTD
      WRITE (6,1014) STBAR,STSTD
      WRITE (6,1010)
```

```
      WRITE (6,1015) DMAX
      WRITE (6,1016) NODEL,UTIL
      WRITE (6,1010)
      WRITE (6,1017)
      WRITE (6,1018)
      DO 900 ICT=1,3
      ICT1=ICT+3
      ICT2=ICT+6
      ICT3=ICT+9
      ICTM1=ICT-1
      ICT1M1=ICT1-1
      ICT2M1=ICT2-1
      ICT3M1=ICT3-1
  900 WRITE (6,1019) ICTM1,IFREQ(ICT),ICT1M1,IFREQ(ICT1),ICT2M1,IFREQ(ICT2),ICT3M1,IFREQ(ICT3)
      WRITE (6,1010)
      WRITE (6,1020)
      DO 950 J=1,I
      IF (AWAIT(J).LE.0.50) GO TO 950
      WRITE (6,1021) J,IAR(J),ARR(J),PROC(J),AWAIT(J)
  950 CONTINUE
      STOP
 1000 FORMAT (' ',T5,'ENTER NO. DAYS TO BE SIMULATED IN F10.2')
 1001 FORMAT (F10.2)
 1002 FORMAT (' ',T5,'ENTER NO. COMM. CHANNELS IN I2')
 1003 FORMAT (I2)
 1004 FORMAT (' ',T5,'ENTER A RANDOM INTEGER BETWEEN 1 AND 100,000 IN FORMAT I6')
 1005 FORMAT (I6)
 1010 FORMAT (' ',T10,'        '/)
 1011 FORMAT (' ',T17,I2,' CHANNELS SIMULATED FOR',F4.1,' DAYS WITH ',I5,' ARRIVALS')
 1012 FORMAT (' ',T24,'AV. ARR. TIME',3X,F7.4,4X,'ST. DEV.',6X,F5.2)
 1013 FORMAT (' ',T24,'AV. SER. TIME',3X,F7.4,4X,'ST. DEV.',6X,F5.2)
 1014 FORMAT (' ',T24,'AV. STATE',7X,F7.4,4X,'ST. DEV.',6X,F5.2)
 1015 FORMAT (' ',T24,'MAX DELAY',7X,F5.2)
 1016 FORMAT (' ',T24,'NO. DELAYS',6X,I5,6X,'UTIL. %',6X,F6.2)
 1017 FORMAT (' ',T17,'DISTRIBUTION OF STATES'/)
 1018 FORMAT (' ',T21,4('STATE   FREQ   '))
 1019 FORMAT (' ',T22,4(I2,I8,5X))
 1020 FORMAT (' ',T17,'LOG OF DELAYS LONGER THAN 30 SECONDS'/)
 1021 FORMAT (' ',T17,'MSG. NO.',I6,'  MSG. TYPE',I6,'  ARR. TIME',F10.2,'  PROC. TIME',F10.2,'
     1'  WAIT TIME',F10.2)
      END
```

```
      SUBROUTINE RAND(Z,IGESS,A,X,I,ISW)
      INTEGER A,X
      M=2**20
      FM=M
      IF (I.EQ.1) GO TO 100
      I=1
      X=566387
      A=2**10 + 3
100   X=MOD(A*X,M)
      FX=X
      Z=FX/FM
      IF (ISW.EQ.1) GO TO 300
      DO 200 K=1,IGESS
      X=MOD(A*X,M)
      FX=X
      Z=FX/FM
200   CONTINUE
      ISW=1
300   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MMIN(A,T,J)
      DIMENSION A(6)
      J=1
      XMIN=A(1)
      DO 100 K=2,6
      IF (XMIN.LE.A(K)) GO TO 100
      XMIN=A(K)
      J=K
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MMPROC(K,PTIM,Z,IGESS,IA,IX,IXX,ISW)
      INTEGER*2 K
      CALL RAND(Z,IGESS,IA,IX,IXX,ISW)
      GO TO (100,200,300,400,500,600),K
  100 V=Z
      DO 150 JV=1,11
      CALL RAND (Z,IGESS,IA,IX,IXX,ISW)
  150 V=V+Z
      V1=0.6*V-3.47
      SMU=EXP(V1)
      GO TO 800
  200 SMU=0.80*Z + 0.45
      GO TO 800
  300 SMU= Z + 0.60
      GO'TO 800
  400 SMU= 2.20*Z + 0.50
      GO TO 800
  500 SMU= 1.50*Z + 0.25
      GO TO 800
  600 SMU= Z + 0.60
  800 PTIM=SMU
      RETURN
      END
```

```
      SUBROUTINE CHAN(NOCHAN,ARR,PROC,ON,WAIT,OFF,ICHAN,C)
      DIMENSION C(100)
      INTEGER*2 ICHAN
      MIN=1
      DO 100 J=2,NOCHAN
      IF(C(MIN).LE.C(J)) GO TO 100
      MIN=J
100   CONTINUE
      ICHAN=MIN
      IF (ARR.GT.C(MIN)) GO TO 200
      WAIT=C(MIN)-ARR
      ON=C(MIN)
      OFF=C(MIN)+PROC
      GO TO 300
200   ON=ARR
      OFF=ARR+PROC
300   CONTINUE
      C(MIN)=OFF
      RETURN
      END
```

```
consim
   ENTER NO. DAYS TO BE SIMULATED IN F10.2
2.0
   ENTER NO. COMM. CHANNELS IN I2
04
   ENTER A RANDOM INTEGER BETWEEN 1 AND 100,000 IN FORMAT I6
034421


         4 CHANNELS SIMULATED FOR 2.0 DAYS WITH  1537 ARRIVALS

              AV. ARR. TIME     1.8723    ST. DEV.       1.81
              AV. SER. TIME     1.2108    ST. DEV.       0.56
              AV. STATE         0.6503    ST. DEV.       0.81


              MAX DELAY         0.64
              NO. DELAYS          12      UTIL. %       16.15

         DISTRIBUTION OF STATES

             STATE   FREQ   STATE   FREQ   STATE   FREQ   STATE   FREQ
               0     1487     3      64      6       1      9       0
               1     1015     4      11      7       0     10       0
               2      298     5       4      8       0     11       0

         LOG OF DELAYS LONGER THAN 30 SECONDS

         MSG. NO.   215  MSG. TYPE    6  ARR. TIME   406.80  PROC. TIME     1.32  WAIT TIME     0.64
```

APPENDIX C

Limiting Values Of λ_0 And λ_* For C = 1 To 20

```
      IMPLICIT REAL*8 (A-F)
      DIMENSION A(50),B(50),C(50),D(50),E(50),F(50)
      DATA E/50*1.0/
      DMAX=100.0
      INDEX=0
      DFAC=1.0
      WRITE (6,1000)
 1000 FORMAT (' ',T5,'THIS PROGRAM CALCULATES THE MAXIMUM VALUES OF'/-
     1T5,'LAMBDA0 AND LAMBDA* FOR A GIVEN NUMBER OF CHANNELS'//T5,'NO. CHANN-
     2ELS     LAMBDA0      LAMBDA*'/)
      WRITE (6,1003)
 1003 FORMAT (' ',T5,5X,' 1            0.839962     1.618034')
      DO 500 J=2,20
      DX=DMAX
      DO 100 I=1,20
      DFAC=DFAC*I
  100 A(I)=-1.0/DFAC
      A(J+1)=-A(J)
      DFAC=1.0
      JM1=J-1
      DO 200 I=1,JM1
  200 B(I)=A(I)
      B(J)=A(J+1)*(J+1)
  250 DFN=-1.0
      DFNPR=-1.0
      DO 300 I=1,J
      DFN=DFN+A(I)*(DX**I)
  300 DFNPR=DFNPR+B(I)*(DX**I)
      DFN=DFN+A(J+1)*(DX**J)*DX
      DTEMP=DX-DFN/DFNPR
      INDEX=INDEX+1
      DEL=DABS(DTEMP-DX)
      IF (DEL.LT.0.000000001) GO TO 500
      DX=DTEMP
      GO TO 250
  500 C(J)=DTEMP
      DO 600 J=1,20
      C1=C(J)
  600 D(J)=DEXP(-C1)*C1
      DO 800 J=1,20
      DFAC=1.0
      DO 700 I=1,J
      DFAC=DFAC*I
  700 E(J)=E(J)+ (C(J)**I)/DFAC
  800 F(J)=D(J)*E(J)
      DO 900 J=2,20
  900 WRITE (6,1001) J,F(J),C(J)
 1001 FORMAT (' ',T5,5X,I2,9X,F9.6,3X,F9.6)
      STOP
      END
```

THIS PROGRAM CALCULATES THE MAXIMUM VALUES OF
LAMBDA0 AND LAMBDA* FOR A GIVEN NUMBER OF CHANNELS

NO. CHANNELS	LAMBDA0	LAMBDA*
1	0.839962	1.618034
2	1.371102	2.269531
3	1.942381	2.945186
4	2.543534	3.639547
5	3.168185	4.349048
6	3.812021	5.071184
7	4.471954	5.804110
8	5.145672	6.546411
9	5.831388	7.296973
10	6.527684	8.054895
11	7.233412	8.819440
12	7.947624	9.589989
13	8.669525	10.366021
14	9.398444	11.147089
15	10.133803	11.932806
16	10.875103	12.722834
17	11.621909	13.516878
18	12.373837	14.314675
19	13.130548	15.115990
20	13.891741	15.920615

APPENDIX D

Examples Of Effect Of Varying γ For Certain Values Of λ_0 And C

Whenever $0 < \gamma < 1$, the resulting equation is transcendental and must be solved numerically. The solution of the equation is dependent upon the values of λ_0, γ, and C. The equation for the effective lambda is

$$\lambda_j = \lambda_0 + \gamma\lambda_{j-1}\Phi(C,\lambda_{j-1})$$

Repeatedly solving this equation until it either diverges or converges for given values of λ_0, γ, C yield the effective value λ_*. Three sets of values were selected for λ_0, γ, and C and a convergence criterion of 10^{-5} was used to demonstrate the effects of varying γ for fixed values of λ_0 and C.

In the first example $\lambda_0 = 1.5$ and $C = 2$. If $\gamma = 0$ then the λ_* equals λ_0 since no rejects reenter the system. As γ is increased, λ_* increases nonlinearly. For $\lambda_0 = 1.5$ and $\gamma = 1$, the minimum feasible value of C for a controlled process is given in Appendix C. This value is $C = 3$. For $\lambda_0 = 1.5$ and $C = 2$, the value of λ_* begins to increase very rapidly for $\gamma > 0.8$ until the process diverges at some γ in the interval $0.9 < \gamma \leq 1.0$. The second and third sets of values demonstrate the effects of varying γ for $\lambda_0 = 5.5$ and $C = 6$ and 7.

λ_0	C	γ	λ_*
1.5	2	0.0	1.50
		0.1	1.53
		0.2	1.56
		0.3	1.60
		0.4	1.65
		0.5	1.71
		0.6	1.78
		0.7	1.89
		0.8	2.00
		0.9	15.00
		1.0	∞
5.5	6	0.0	5.5
		0.1	5.7
		0.2	6.0
		0.3	6.4
		0.4	7.1
		0.5	9.3
		0.6	13.3
		0.7	18.3
		0.8	27.5
		0.9	55.0
		1.0	∞
5.5	7	0.0	5.5
		0.1	5.6
		0.2	5.8
		0.3	5.9
		0.4	6.2
		0.5	6.7
		0.6	12.4
		0.7	18.2
		0.8	27.5
		0.9	55.0
		1.0	∞

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