FINAL REPORT

NASA CONTRACT NAS8-29900

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Analysis of the Relativistic Orbiting Gyroscope Experiment

Submitted by:

THE UNIVERSITY OF ALABAMA
P. O. BOX 2846
UNIVERSITY, ALABAMA 35486

Prepared by:

Dr. C. Alton Coulter
Associate Professor of Physics
P. O. Box 1921
University, Alabama 35486

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This report was prepared by The University of Alabama under Contract NAS8-29900, "Analysis of the Relativistic Orbiting Gyroscope Experiment," for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration.

FINAL TECHNICAL REPORT

I. SCOPE OF THE INVESTIGATION

The original contract Work Statement specified the following objectives:

"A. The contractor shall analyze the proposed relativistic orbiting gyroscope experiment to determine if an accuracy in the experiment sufficient to detect the predicted Lense-Thirring effect is justified in terms of providing additional physical information about gravitational theories not attainable from measurement of the geodetic precession alone.

"B. A lower limit shall be determined for the value of the parameter ω in the Brans-Dicke theory for which the scalar field contribution ceases to be of practical importance in gravitational and cosmological application of the theory.

"C. A suitable metric shall be developed incorporating a quadrupole moment for the source and such additional perturbations as prove to be relevant (drag, radiation pressure, etc.), and derive equations of motion which can be used for numerical analysis of orbital perturbations."

In addition, the principal investigator was subsequently requested to assist in the determination of the level of accuracy required in an Eötvös experiment in order to detect effects of the weak interaction.

II. METHODS EMPLOYED IN THE INVESTIGATION

In order to obtain the information required in Parts A and B of the Work Statement, an extensive survey was made of the literature relevant to the gyroscope experiment and the Brans-Dicke theory which could be located by reference to Physics Abstracts. (It is deemed unlikely that any significant relevant paper would be inaccessible by this approach.) Subject Indexes covering both the early and the recent literature on the gyroscope experiment and the Brans-Dicke theory were searched for papers which offered even a remote chance of proving to be related to the investigation, hundreds of abstracts obtained from these subject listings were scanned, and finally a group of papers was selected from the scanning of abstracts to be read in detail. An almost complete list of the literature treated in detail is included in this report under the heading "Literature Surveyed."

For Part C of the study the principal investigator used standard methods of describing an axially symmetric metric to derive the geodesic equations of motion for a particle moving in such a metric, and then obtained an expansion of the metric coefficients in this equation for the particular case of a monopole-quadrupole combination in general relativity from the exact expression for the metric in this case previously obtained by the principal investigator in collaboration with another worker.*

Finally, the principal investigator conducted a small literature search in order to obtain the information necessary to estimate the level of accuracy required in an Eötvös experiment in order to detect the effects of the weak interaction.

III. RESULTS OF THE INVESTIGATION

The conclusions obtained from the investigation are given below, organized by subject as in the Work Statement. Asterisks and daggers are used to indicate footnotes, while a superscript numeral refers to the paper or book of the corresponding number in the list of "Literature Surveyed."

A. Accuracy Desired in the Orbiting Gyroscope Experiment.

Accuracy requirements for the gyroscope experiment should be considered both in a theory-dependent framework and in a theory-independent one.

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*J. H. Young and C. A. Coulter, "Exact metric for a non-rotating mass with a quadrupole moment," Phys. Rev. 184, 1313 (1969).

Let us adopt the theory-dependent approach first. Most of the current gravitational theories of any interest are "metric theories," in which test particles follow geodesics in a metric space, and the significant features of most of these metric theories can be described by the parametrized post-Newtonian (PPN) formalism.[65,69,70,47] In the most recent version of the PPN formalism[75] the geodetic and Lense-Thirring (frame-dragging) precession rates for a gyroscope are expressed as

$$\Omega_{geod} = \frac{1}{3}(1 + 2\gamma)\Omega_{geod}^{GR} \quad , \tag{1}$$

$$\Omega_{LT} = \frac{1}{2}(1 + \gamma + \frac{1}{4}\alpha_1)\Omega_{LT}^{GR} \quad , \tag{2}$$

where Ω_{geod}^{GR} and Ω_{LT}^{GR} are the general-relativistic predictions for the geodetic and Lense-Thirring precessions, respectively, and γ and α_1 are parameters whose values depend on the theory considered. (In general relativity $\gamma = 1$ and $\alpha_1 = 0$.) It is thus clear that in principle the geodetic and Lense-Thirring precessions measure different characteristics of a metric theory, the parameter α_1 appearing in one of the above expressions but not the other. By consideration of experimental evidence on "solid-Earth tides," orbital motion of planets, and motion of the solar system relative to a "mean Universal rest frame," Nordtvedt and Will have concluded[52] that $|\alpha_1|<0.2$. This result is adequate to rule out all stratified theories with time-orthogonal, conformally flat space slices,[52] for which agreement with light-deflection and time-delay experiments requires $\alpha_1 \gtrsim -8$.[71] For the remaining theories of current interest $\alpha_1 = 0$, so that both the geodetic and the Lense-Thirring precessions depend only on the parameter γ. Thus within the theoretical framework assumed the geodetic and Lense-Thirring precessions in principle measure different characteristics of a gravitational theory; but in fact for the viable extant theories only a single parameter appears in the two predicted precession rates.

From a theory-independent viewpoint it seems fairly clear that the two predicted types of precession arise from physically distinguishable situations. In the geodetic case the gyroscope, from the viewpoint of its rest frame, sees the earth (or other object about which it is orbiting) nonrotating with respect to the distant stars, and both the earth and the distant stars in accelerated motion relative to itself. In the Lense-Thirring case the gyroscope sees itself (essentially) unaccelerated relative to the distant stars and the center of mass of the earth, and sees the earth rotating relative to the distant stars. Only by fairly detailed reasoning within a given theoretical framework could one relate one of these physical situations to the other. That it is dangerous to assume the validity of a theoretical relationship between physically distinguishable experimental situations prior to performing the experiments (or worse, in lieu of performing them) has been demonstrated by events in the area of weak interactions. Furthermore, well-known gravitational theorists such as Dicke,[16] Will, and Nordtvedt[*] have stressed the importance of not prejudging the outcome of gravitational experiments on the basis of existing theoretical frameworks.

Thus both from the standpoint of the PPN formalism and from theory-independent considerations one concludes that the geodetic precession and the Lense-Thirring precession are (in principle) independent effects which ultimately should both be measured. The remaining question to be answered is whether it is economically feasible and theoretically desirable to attempt to measure both in the near future. As is well-known, for a circular polar earth orbit of several hundred miles radius the geodetic precession is about 7 arc-seconds per year and the (integrated) Lense-Thirring precession is about 0.05 arc-second per year.[†] A study by Ball Brothers has indicated that the geodetic precession could be measured to about 0.1 arc-second per year with a non-drag-free satellite

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[*] Comments at conference of NASA and ESRO representatives held at Marshall Space Flight Center on November 7 and 8, 1973

[†] See, e.g., C. W. F. Everitt, "The Stanford gyroscope experiment," in Proceedings of the Conference on Experimental Tests of Gravitation Theories (JPL Technical Memorandum 33-499), p. 68.

in low non-polar earth orbit using a Scout launch vehicle.* Assuming the result of such a measurement agreed with the prediction of general relativity to within experimental error, it would serve to determine the parameter γ to about 2% and consequently provide a lower limit on the Brans-Dicke parameter ω of about 45. No evidence would be obtained about the existence or magnitude of the Lense-Thirring precession. The same satellite which could measure the gyroscope precession to 0.1 arc-second per year in the Scout mission could, if placed in a 500 mile high circular polar orbit by means of a Thor-Delta launch vehicle, measure the gyroscope precession to 0.01 arc-second per year.* Under the same assumption as before, this accuracy could determine γ to about 0.2% and place a lower limit on the Brans-Dicke parameter ω of about 450. Simultaneously the existence of the Lense-Thirring precession (at a magnitude comparable to that predicted by general relativity) could be checked on, and an independent criterion for deciding for or against the stratified theories with time-orthogonal conformally flat space slices would be obtained. The lower limit indicated for ω would effectively eliminate all basis for interest in the Brans-Dicke theory for the foreseeable future (see the discussion in the next section). The use of the more expensive Thor-Delta launch vehicle would increase the cost of the experiment by at most 20 to 25%, an increase amply justified by the much-improved return of scientific information to be expected from the more accurate experiment.

CONCLUSION: A gyroscope experiment designed to measure both the geodetic and the Lense-Thirring precessions to an accuracy of 0.01 arc-second per year appears economically and technically feasible and is scientifically highly desirable.

B. Lower Limit for ω Sufficient to Eliminate Interest in the Brans-Dicke Theory.

The original theoretical motivations for the introduction of the Brans-Dicke scalar-tensor theory were two: to provide a theoretical rather than

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*Peter B. Eby, private communication.

accidental basis for the observed relation

$$\frac{GM}{c^2R} \sim 1$$

between the mass M and radius R of the visible universe, the Newtonian gravitational constant G, and the speed of light c; and to attempt to incorporate Mach's principle into a gravitational theory in a more fundamental way than that in which it occurs in general relativity--that is, in the boundary conditions.[7] The theory attempts to achieve these ends by replacing G^{-1} by a scalar field ϕ and introducing a dimensionless parameter ω which was initially anticipated to be of the "general order of magnitude of unity."[7] We shall here consider the theory on both its experimental and theoretical merits.

The predictions of the theory which may be tested experimentally may be divided into the (somewhat overlapping) categories of cosmological, astrophysical, and orbital phenomena. We shall consider these in turn.

a) Cosmological phenomena. The scalar field exerts a dominant influence in the very early epochs of Brans-Dicke cosmologies.[43] This can result in changes in the predicted relative abundances of the elements[18,20] and on the time scales for development of galactic structures versus stellar structures,[14] but the available observational data are much too uncertain to make tests of the Brans-Dicke theory possible. The outlook is somewhat brighter for the test of a present-day prediction of Brans-Dicke theory, namely a time-varying gravitational "constant." Brans-Dicke cosmologies with $\omega \sim 6$ and a Hubble time consistent with current observations give a value of $|\dot{G}/G|$ of about 10^{-11}/yr. This is an order of magnitude smaller than present observational limits, but may be measureable in five to ten years.[42]

b) Astrophysical phenomena. One might hope to observe the effects of a Brans-Dicke gravitation rather than a general relativistic gravitation in the astrophysical situations where gravitational fields are strongest, namely in neutron stars and black holes. Calculations of the physics of cold neutron stars in the Brans-Dicke Theory have been carried out by Salmona[60] and by Matsuda,[37] and both find little difference between general relativity predictions and those of the Brans-Dicke theory with $\omega = 6$. It seems most doubtful that the accuracy and completeness of either observations or the theory of stellar interiors will be adequate to fix higher limits on ω in the next decade. As far as black holes are concerned, several people have inferred[66] and Hawking claims to have proved[25,26] that black holes in the Brans-Dicke theory must be Einstein black holes--i.e., have constant scalar field (at least in the neighborhood of the black hole). A consequence is that black holes in the Brans-Dicke theory will not move on geodesics in the conformal frame in which small test particles move on geodesics.[25] However, the possibility of making accurate determinations of the orbital parameters of one or more black holes in the near future seems remote. Finally, Shaviv and Bahcall[62,63] have calculated that for "fashionable" values of ω, the Hubble constant, and the heavy-element abundance, the solar neutrino flux predicted by the Brans-Dicke theory is about twice that predicted by constant-G theories. Until one succeeds in detecting a solar neutrino flux of any size the significance of this result is uncertain.

c) Orbital phenomena. Because the effects of scalar and tensor fields on a test particle are different, and because the scalar field does not contribute in the same way to the gravitational and the inertial masses of a massive body, the detailed trajectories of orbiting bodies differ somewhat between the Brans-Dicke theory and general relativity. Attempts to observe this effect in planetary precessions have been thwarted thus far because of the unknown size of the solar quadrupole moment. Light-deflection and time-delay measurements are currently not capable of deciding definitively between general relativity and the Brans-Dicke theory with $\omega = 6$, but might achieve roughly a five-fold increase in precision over the next several years. Lunar laser ranging might achieve an accuracy sufficient to distinguish between general relativity and the

Brans-Dicke theory with $\omega = 6$ in roughly the same time period,* but is unlikely to be able to provide an accuracy great enough to push ω beyond the range 25-30 in the foreseeable future.

The current experimental staus of the Brans-Dicke theory is then the following. The predictions of the Brans-Dicke theory with $\omega = 6$ are indistinguishable from those of general relativity with current experimental accuracies. Only refined orbit determinations within the solar system (including light-deflection and time-delay experiments) appear to offer hope of distinguishing between these predictions in the next decade.

It is now necessary to determine the theoretical merits of the Brans-Dicke theory, and for this purpose it seems reasonable to examine the extent to which the theory achieves the two objectives which first prompted its introduction.

a) Incorporation of Mach's principle. The connection between the Brans-Dicke theory and Mach's principle has been investigated by several workers. Toton[67,68] concludes that "...the scalar-tensor theory is no more compatible with Mach's principle than is general relativity," and Katz[31] suggests that Mach's principle should be incorporated into the Brans-Dicke theory in the same way it is incorporated into general relativity--i.e., through the boundary conditions. It appears that the Brans-Dicke theory offers no improvement over general relativity in this respect.

b) Explanation of the relationship between "cosmic numbers." The Brans-Dicke theory "explains" the relation

$$\frac{GM}{c^2R} \sim 1 \qquad (3)$$

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*P. O. Bender et al., "The lunar laser ranging experiment," Proceedings of the Conference on Experimental Tests of Gravitation Theories (JPL Technical Memorandum 33-499), p. 178.

by (as stated earlier) replacing the gravitational constant G by the inverse of a space-time dependent scalar field ϕ and introducing a dimensionless parameter ω whose value may be adjusted to yield agreement with experimental results. However, relation (3) is only "explained" by this treatment if the parameter ω turns out to be roughly unity. In fact, Brans and Dicke state in their original paper[7] that "in any sensible theory ω must be of the general order of magnitude of unity." Otherwise one is simply shifting the unknown origin of relation (3) from one parameter to another. Since in the limit $\omega \to \infty$ the predictions of the Brans-Dicke theory approach those of general relativity, a sequence of experiments favoring the predictions of general relativity to higher and higher accuracies would necessitate the assignment of higher and higher values to ω. If this were to occur one would ultimately have to conclude that the Brans-Dicke theory failed to explain relation (3) but simply replaced this puzzle with another one, that of the magnitude of ω. As a general guide to the size of ω at which one could conclude that this failure had occurred one might assume this to be the case once ω differed from unity by roughly as many powers of ten as other dimensionless coupling constants whose size is considered as not understood and as requiring explanation by a future more complete theory. An example of such a coupling constant is the electromagnetic fine structure constant $\alpha = e^2/\hbar c \approx 1/137$.

CONCLUSION: The Brans-Dicke scalar-tensor theory can be considered to have no more experimental or theoretical interest for the foreseeable future once it has been established that ω is at least of magnitude 10^2-10^3.

Comment 1. Will has also indicated that a lower limit of 100 for ω would be adequate to eliminate interest in the Brans-Dicke theory.*

Comment 2. A lower limit for ω in the range 10^2-10^3 could be established by a measurement of the geodetic precession to 0.01 arc-second per year, but not by a measurement to 0.1 arc-second per year.

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*Clifford M. Will (private communication).

C. Equations of Motion Valid for a Source with Quadrupole Moment.

It is known* that by an appropriate choice of coordinate system the metric for a static axially-symmetric body can be brought to the form indicated by the invariant interval squared

$$ds^2 = g(r,\theta)c^2dt^2 - f(r,\theta)dr^2 - r^2f(r,\theta)d\theta^2 - r^2\sin^2\theta\ h(r,\theta)d\phi^2, \quad (4)$$

where r is a radial coordinate, ϕ an azimuthal angle coordinate running 0 to 2π, and θ an angular coordinate running 0 to π. Outside a sphere about the origin completely enclosing the source the functions g, f, and h would, for all known theories, be representable in the forms

$$g(r,\theta) = \sum_{\substack{n=0\\ \ell=0}}^{\infty} \rho_{n\ell} r^{-n} P_\ell(\cos\theta), \quad (5)$$

$$f(r,\theta) = \sum_{\substack{n=0\\ \ell=0}}^{\infty} \sigma_{n\ell} r^{-n} P_\ell(\cos\theta), \quad (6)$$

$$h(r,\theta) = \sum_{\substack{n=0\\ \ell=0}}^{\infty} \tau_{n\ell} r^{-n} P_\ell(\cos\theta), \quad (7)$$

where $P_\ell(x)$ is the Legendre polynomial of order ℓ. By standard techniques one derives from this metric the following four equations for the geodesic motion of a test particle:

$$\frac{d^2t}{ds^2} + \frac{1}{g}\frac{\partial g}{\partial r}\frac{dt}{ds}\frac{dr}{ds} + \frac{1}{g}\frac{\partial g}{\partial \theta}\frac{dt}{ds}\frac{d\theta}{ds} = 0, \quad (8)$$

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*See, e.g., J. L. Synge, Relativity: The General Theory (North-Holland Publishing Co., Amsterdam, 1964), p. 310. The particular form of the metric given here is chosen to insure that the metric assumes an isotropic form when the source becomes not only axially but spherically symmetric.

$$\frac{d^2r}{ds^2} + \frac{1}{2f}\frac{\partial f}{\partial r}\left(\frac{dr}{ds}\right)^2 + \frac{1}{f}\frac{\partial f}{\partial \theta}\frac{dr}{ds}\frac{d\theta}{ds} + \frac{c^2}{2f}\frac{\partial g}{\partial r}\left(\frac{dt}{ds}\right)^2$$
$$- \frac{1}{2f}\frac{\partial(r^2 f)}{\partial r}\left(\frac{d\theta}{ds}\right)^2 - \frac{1}{2f}\frac{\partial(r^2\sin^2\theta h)}{\partial r}\left(\frac{d\phi}{ds}\right)^2 = 0, \qquad (9)$$

$$\frac{d^2\theta}{ds^2} + \frac{1}{r^2 f}\frac{\partial(r^2 f)}{\partial r}\frac{dr}{ds}\frac{d\theta}{ds} + \frac{1}{2f}\frac{\partial f}{\partial \theta}\left(\frac{d\theta}{ds}\right)^2 + \frac{c^2}{2r^2 f}\frac{\partial g}{\partial \theta}\left(\frac{dt}{ds}\right)^2$$
$$- \frac{1}{2r^2 f}\frac{\partial f}{\partial \theta}\left(\frac{dr}{ds}\right)^2 - \frac{1}{2f}\frac{\partial(\sin^2\theta h)}{\partial \theta}\left(\frac{d\phi}{ds}\right)^2 = 0, \qquad (10)$$

$$\frac{d^2\phi}{ds^2} + \frac{1}{r^2 h}\frac{\partial(r^2 h)}{\partial r}\frac{dr}{ds}\frac{d\phi}{ds} + \frac{1}{\sin^2\theta h}\frac{\partial(\sin^2\theta h)}{\partial \theta}\frac{d\theta}{ds}\frac{d\phi}{ds} = 0\,. \qquad (11)$$

Eqs. (8) and (11) have the immediate first integrals

$$g\,\frac{dt}{ds} = \frac{T}{c}\,, \qquad (12)$$

$$r^2\sin^2\theta h\,\frac{d\phi}{ds} = L, \qquad (13)$$

where L and T are constants of integration. Another first integral

$$c^2 g\left(\frac{dt}{ds}\right)^2 - f\left(\frac{dr}{ds}\right)^2 - r^2 f\left(\frac{d\theta}{ds}\right)^2 - r^2\sin^2\theta h\left(\frac{d\phi}{ds}\right)^2 = 1 \qquad (14)$$

can be obtained by simply dividing the expression (4) for the invariant interval squared by ds^2. By using (12) and (13) in (9) and (14) one can reduce the problem of integrating Eqs. (8-11) to the integration of the system of two equations

$$\frac{d^2r}{ds^2} + \frac{1}{2}\frac{f_1}{f}\left(\frac{dr}{ds}\right)^2 + \frac{f_2}{f}\frac{dr}{ds}\frac{d\theta}{ds} - \frac{r^2}{2}\left[\frac{2}{r} + \frac{f_1}{f}\right]\left(\frac{d\theta}{ds}\right)^2$$
$$+ \frac{1}{2}\frac{g_1}{f}\left(\frac{T}{g}\right)^2 - \frac{1}{2}\left[\frac{2}{r} + \frac{h_1}{h}\right]\frac{L^2}{hfr^2\sin^2\theta} = 0, \qquad (15)$$

$$f\left(\frac{dr}{ds}\right)^2 + r^2 f\left(\frac{d\theta}{ds}\right)^2 + \frac{L^2}{hr^2\sin^2\theta} - \frac{T^2}{g} + 1 = 0, \tag{16}$$

where

$$g_1 \equiv \frac{\partial g}{\partial r} = - \sum_{\substack{n=0\\ \ell=0}}^{\infty} n\rho_{n\ell} r^{-n-1} P_\ell(\cos\theta), \tag{17}$$

$$f_1 \equiv \frac{\partial f}{\partial r} = - \sum_{\substack{n=0\\ \ell=0}}^{\infty} n\sigma_{n\ell} r^{-n-1} P_\ell(\cos\theta), \tag{18}$$

$$h_1 \equiv \frac{\partial h}{\partial r} = - \sum_{\substack{n=0\\ \ell=0}}^{\infty} n\tau_{n\ell} r^{-n-1} P_\ell(\cos\theta) \tag{19}$$

$$f_2 \equiv \frac{\partial f}{\partial \theta} = - \frac{1}{\sin\theta} \sum_{\substack{n=0\\ \ell=0}}^{\infty} (\ell + 1)\sigma_{n,\ell+1} r^{-n} P_\ell(\cos\theta)$$

$$+ \cot\theta \sum_{\substack{n=0\\ \ell=0}}^{\infty} \ell\sigma_{n\ell} r^{-n} P_\ell(\cos\theta). \tag{20}$$

After the solution of this system of differential equations one can obtain t and ϕ as functions of s by straightforward definite integration of the two equations

$$\frac{dt}{ds} = \frac{T}{c\ g}, \tag{21}$$

$$\frac{d\phi}{ds} = \frac{L}{hr^2\sin^2\theta}. \tag{22}$$

A knowledge of the parameters $\rho_{n\ell}$, $\sigma_{n\ell}$, and $\tau_{n\ell}$ will allow one to use the above equations to determine the trajectory of a test particle for an axially symmetric stationary metric in any metric theory.

A case of particular interest is that of the motion predicted by general relativity for a test particle moving in the gravitational field of a body with mass M and quadrupole moment M_2. The parameters $\rho_{n\ell}$, $\sigma_{n\ell}$, $\tau_{n\ell}$ have been calculated for this case for all values of n from 0 to 9 (i.e., to large enough values of n to permit calculation of the gravitational motion of a particle in the solar system down to the surface of the sun to an accuracy of at least forty-five significant figures). In terms of the quantities

$$m = \frac{GM}{c^2}, \qquad Q = \frac{G M_2}{c^2 m^3}$$

the nonvanishing parameters for $n \leq 9$ are those given below. (Note that all parameters with ℓ odd as well as all parameters with $n \leq \ell$ vanish).

1. Parameters for g.

$\rho_{00} = 1$

$\rho_{10} = -2m$

$\rho_{20} = 2m^2$

$\rho_{30} = -(3/2)m^3$ $\qquad$ $\rho_{32} = -2Qm^3$

$\rho_{40} = m^4$ $\qquad$ $\rho_{42} = 4Qm^4$

$\rho_{50} = -(5/8)m^5$ $\qquad$ $\rho_{52} = -(59/14)Qm^5$

$\rho_{60} = [(3/8) + (2/5)Q^2]m^6$ $\qquad$ $\rho_{62} = [(24/7)Q + (4/7)Q^2]m^6$

$\rho_{64} = (36/35)Q^2m^6$

$\rho_{70} = -[(7/32) + (4/5)Q^2]m^7$ $\qquad$ $\rho_{72} = -[(69/28)Q + (8/7)Q^2]m^7$

$\rho_{74} = -(72/35)Q^2m^7$

$\rho_{80} = [(1/8) + (31/35)Q^2]m^8$ $\qquad$ $\rho_{82} = [(23/14)Q + (62/49)Q^2]m^8$

$\rho_{84} = (558/245)Q^2m^8$

$$\rho_{90} = -\,[(9/128) + (27/35)Q^2 + (8/105)Q^3]m^9$$

$$\rho_{92} = -\,[(3853/3696)Q + (54/49)Q^2 + (4/7)Q^3]m^9$$

$$\rho_{94} = -\,[(486/245)Q^2 + (144/385)Q^3]m^9$$

$$\rho_{96} = -\,(24/77)Q^3m^9$$

2. Parameters for f.

$$\sigma_{00} = 1$$

$$\sigma_{10} = 2m$$

$$\sigma_{20} = (3/2)m^2$$

$$\sigma_{30} = (1/2)m^3 \qquad \sigma_{32} = 2Qm^3$$

$$\sigma_{40} = (1/16)m^4 \qquad \sigma_{42} = (16/7)Qm^4$$

$$\sigma_{44} = (12/7)Qm^4$$

$$\sigma_{50} = 0 \qquad \sigma_{52} = -(3/14)Qm^5$$

$$\sigma_{54} = (24/7)Qm^5$$

$$\sigma_{60} = (8/35)Q^2m^6 \qquad \sigma_{62} = [-(4/3)Q + (2/7)Q^2]m^6$$

$$\sigma_{64} = [(186/77)Q + (72/385)Q^2]m^6$$

$$\sigma_{66} = [(80/231)Q + (100/77)Q^2]m^6$$

$$\sigma_{70} = -(8/35)Q^2m^7 \qquad \sigma_{72} = [-(127/168)Q + (4/7)Q^2]m^7$$

$$\sigma_{74} = [(6/11)Q - (192/385)Q^2]m^7$$

$$\sigma_{76} = [(160/231)Q + (320/77)Q^2]m^7$$

$$\sigma_{80} = -(18/35)Q^2m^8 \qquad \sigma_{82} = [-(19/77)Q + (162/539)Q^2]m^8$$

$$\sigma_{84} = -[(537/4004)Q + (38292/35035)Q^2]m^8$$

$$\sigma_{86} = [(16/33)Q + (300/77)Q^2]m^8$$

$$\sigma_{88} = [(32/429)Q + (120/143)Q^2]m^8$$

$$\sigma_{90} = -[(26/735)Q^2 + (4/105)Q^3]m^9$$

$$\sigma_{92} = -[(213/2464)Q + (503/1617)Q^2 + (32/77)Q^3]m^9$$

$$\sigma_{94} = -[(93/1001)Q + (21864/35035)Q^2 - (2304/5005)Q^3]m^9$$

$$\sigma_{96} = [(8/77)Q + (1376/1617)Q^2 + (16/77)Q^3]m^9$$

$$\sigma_{98} = [(64/429)Q + (848/429)Q^2 + (160/143)Q^3]m^9$$

3. Parameters for h.

$$\tau_{00} = 1$$

$$\tau_{10} = 2m$$

$$\tau_{20} = (3/2)m^2$$

$$\tau_{30} = (1/2)m^3 \qquad \tau_{32} = 2Qm^3$$

$$\tau_{40} = (1/16)m^4 \qquad \tau_{42} = 4Qm^4$$

$$\tau_{50} = 0 \qquad \tau_{52} = (45/14)Qm^5$$

$$\tau_{60} = (2/5)Q^2m^6 \qquad \tau_{62} = [(10/7)Q + (4/7)Q^2]m^6$$

$$\tau_{64} = (36/35)Q^2m^6$$

$$\tau_{70} = (4/5)Q^2m^7 \qquad \tau_{72} = [(27/56)Q + (8/7)Q^2]m^7$$

$$\tau_{74} = (72/35)Q^2m^7$$

$$\tau_{80} = (24/35)Q^2m^8 \qquad \tau_{82} = [(5/28)Q + (48/49)Q^2]m^8$$

$$\tau_{84} = (432/245)Q^2m^8$$

$$\tau_{90} = [(13/35)Q^2 + (8/105)Q^3]m^9$$

$$\tau_{92} = [(545/7392)Q + (26/49)Q^2 + (4/7)Q^3]m^9$$

$$\tau_{94} = [(234/245)Q^2 + (144/385)Q^3]m^9$$

$$\tau_{96} = (24/77)Q^3m^9$$

Accuracy Required in the Eötvös Experiment to Detect the Weak Interaction.

The theory of weak interactions is not yet in a definitive form, and consequently any estimate of the contribution of this interaction to the rest masses of atomic nuclei must necessarily be approximate and tentative. However, it happens that rather widely differing theoretical models for the weak interaction yield remarkably similar values for the ratio of weak to strong potential energies in nuclei, and that there is some experimental evidence to support a weak interaction-strong interaction nuclear energy ratio of about this magnitude. There thus seems to be a reasonable basis for supposing that the present theoretical estimates are correct at least as far as order of magnitude is concerned.

Calculations of an effective "weak interaction potential" between nucleons have been made by Blinstoyle[3] using a direct current-current form for the weak interaction and by Blinstoyle and Herczeg[4] utilizing an intermediate vector boson model for the interaction. The analytical forms of the potentials derived from these two models are rather different; however, when the value of the internucleon potential is evaluated at the average internucleon distance found in nuclei, both potentials yield a ratio of weak to strong internucleon potential energies of about 10^{-7}. The reason for the similar results of these two different models is basically that the range of the weak interaction is so short for either the current-current interaction or the intermediate vector boson interaction (because of the large mass of the latter) that the interaction strength is determined primarily by the strong-interaction form factors of the nucleons (which give them a finite "size") and the weak-interaction coupling constant, which are both known reasonably well experimentally.

Experiments to detect a weak-interaction component of the nucleon-nucleon forces in nuclei have been carried out by looking for parity-violating events in certain nuclear processes such as gamma emission. The experiments are difficult[21] because of the small size of the effects looked for, but parity-violating effects corresponding to a parity-nonconserving/parity-conserving force ratio of order 10^{-7} have been observed[35] (and even larger ratios have been reported from other experiments,[6] though the results have been questioned[35]).

It thus seems reasonable to assume that the ratio of weak interaction binding energy to strong interaction binding energy in a nucleus is of order 10^{-7}. The mean binding energy per nucleon from the strong interaction is in the range 8-9 MeV from about A (total number of nucleons) = 20 onward, increasing (on the average) slowly with the number of nucleons. (The total binding energy per nucleon <u>decreases</u> with increasing A beyond about A = 52, but this is believed to be an effect of the coulomb rather than the strong interaction energy.) Since the rest mass energy per nucleon is somewhat less than 1 GeV, the strong interaction makes a contribution of slightly less than 1% to the rest mass of the nucleus. Using the above ratio of weak to strong interaction energies in the nucleus, one concludes that the weak interactions contribute about one part in 10^9 to the rest mass of the nucleus, and thus of the total atom (since the nucleus contains more than 99.97% of the atomic mass).

However, the usual Eötvös experiment compares the ratio of gravitational and inertial masses for two different materials, rather than measuring the ratio directly for a single material. For the usual choices of materials (an element near the beginning of the periodic table, with an A in the 20's, and a stable element near the end of the table, with A near 200) the difference in strong interaction binding energy per nucleon may be one to several per cent, so that an accuracy of about one part in 10^4 in an Eötvös experiment would be required to detect a total failure of the strong interaction to contribute to gravitational mass, and correspondingly higher accuracies to detect the situation where the strong interaction contributes to both inertial and gravitational masses but to different extents. The variation of the strong interaction binding energy per nucleon with increasing A is primarily due to a surface energy effect, because of the short range of the strong interaction effective inter-nucleon potential. As mentioned earlier, the range of the weak interaction potential between nucleons is primarily determined by the strong interaction form factors, and thus is about the same as the strong interaction range. It then seems reasonable to assume that the contribution of the weak interaction to the average binding energy per nucleon has a variation with A that rather closely parallels that of the strong interaction binding energy, but which is a factor 10^{-7} smaller on an absolute scale.

CONCLUSION. On the basis of currently available theoretical and experimental evidence, and assuming the usual techniques used in an Eötvös experiment, it is estimated that an accuracy of about one part in 10^{11} in an Eötvös experiment would be required to detect a situation in which the weak interaction contributed to the inertial masses but not the gravitational masses of nuclei, and a correspondingly higher accuracy in the experiment to detect the situation where the weak interaction contributed to the gravitational masses some fraction between zero and one (excluding the endpoints) of its contribution to the inertial masses.

Comment. This estimate coincides with that given by Chapman.*

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*P. K. Chapman and A. J. Hanson, "An Eötvös experiment in earth orbit," Proceedings of the Conference on Experimental Tests of Gravitation Theories (JPL Technical Memorandum 33-499), p.228.

LITERATURE SURVEYED

1. Anderson, J. L., "Scale invariance of the second kind and the Brans-Dicke scalar-tensor theory," Phys. Rev. D3, 1689 (1971).

2. Barker, B. M., and R. F. O'Connell, "Derivation of the equations of motion of a gyroscope from the quantum theory of gravitation," Phys. Rev. D2, 1428 (1970).

3. Blinstoyle, R. J. "Parity nonconserving internucleon potentials," Phys. Rev. 118, 1605 (1960).

4. Blinstoyle, R. J., and P. Herczeg, "Second-class currents and the weak nucleon-nucleon interaction," Nuclear Phys. B5, 291 (1968).

5. Blokland, A., "Oscillating gyroscope in a satellite," Astrophys. Space Sci. (Netherlands) 6, 352 (1970).

6. Boehm, F., and E. Kankleit, "Experimental evidence for parity inpurity in a nuclear gamma transition," Phys. Rev. Lett. 14, 312 (1965).

7. Brans, C., and R. H. Dicke, "Mach's principle and a relativistic theory of gravitation," Phys. Rev. 124, 925 (1961).

8. Brans, C. H.,"Mach's principle and a relativistic theory of gravitation. II," Phys. Rev. 125, 2194 (1962).

9. Cooper, D. H., et al., "A proposed test of the Einstein theory of gravitation by means of an unshielded orbiting gyro using passive telemetry," Report NASA-CR-94933, Illinois Univ., Coordinated Science Lab., Urbana, Ill. (May 1968).

10. Deser, S., and J. Higbie, "Essential singularities in general relativity," Phys. Rev. Lett. 23, 1184 (1969).

11. Dicke, R. H., "The earth and cosmology," Science 138, 653 (1962).

12. Dicke, R. H., "Mach's principle and invariance under transformations of units," Phys. Rev. 125, 2163 (1962).

13. Dicke, R. H., "Long-range scalar interaction," Phys. Rev. 126, 1875 (1962).

14. Dicke, R. H., "Implications for cosmology of stellar and galactic evolution rates," Rev. Mod. Phys. 34, 110 (1962).

15. Dicke, R. H., "Cosmology, Mach's principle and relativity," I.C.S.U. Review 5, 40 (1963).

16. Dicke, R. H., "Experimental relativity" in Relativity, Groups and Topology (Gordon and Breach, New York, 1964), p. 166.

17. Dicke, R. H., "The Richtmeyer Memorial Lecture: Gravitation and cosmic physics," Amer. J. Phys. 35, 559 (1967).

18. Dicke, R. H., "Scalar-tensor gravitation and the cosmic fireball," Astrophys. J. 152, 1 (1968).

19. Gamow, G., "Does gravity change with time?", Proc. Natl. Acad. Sci. 57, 187 (1967).

20. Greenstein, G. S., "Brans-Dicke cosmology," Astrophys. Lett. (GB) 1, 139 (1968).

21. Hamilton, W. D., "Parity violation in electromagnetic and strong interaction processes," in Brink and Mulvey, Eds., Progress in Nuclear Physics, Vol. 10 (Pergamon Press, Oxford, 1969), p. 1.

22. Harrison, E. A., "The cosmic numbers," Physics Today 25, No. 12, 30 (Dec. 1972).

23. Harrison, E. R., "Scalar-tensor theory and general relativity," Phys. Rev. D6, 2077 (1972).

24. Hart, H. B., "Conservation laws and symmetry properties of scalar-tensor gravitational theories," Phys. Rev. D5, 1256 (1972).

25. Hawking, S. W., "Black holes in the Brans-Dicke theory of gravitation," Commun. Math. Phys. 25, 167 (1972).

26. Hawking, S. W., and G.F.R. Ellis, The Large-Scale Structure of Space-Time (Cambridge University Press, Cambridge, 1972).

27. Hönl, H., and H. Dehnen, "Does the 3°K radiation permit conclusions to be drawn regarding a constant or variable gravitational constant?", Z. Astrophys. (Germany) 68, 181 (1968).

28. Jackiw, R. "Planetary precession gravitation theory," Nuovo Cimento 54B, 11 (1968).

29. Janis, A. I., "Comments on Einstein scalar solutions," Phys. Rev. 186, 1729 (1969).

30. Jordan, P., "Remarks on the works of H. Hönl and H. Dehnen: Does the 3°K radiation permit conclusions to be drawn regarding a constant or variable gravitational constant?", Z. Astrophys. (Germany) 68, 201 (1968).

31. Katz, A., "Boundary conditions that enforce Mach's principle in general relativity," Nuovo Cimento 51B, 502 (1967).

32. Kaufmann, W. J. III, "Radiating spheres in the scalar-tensor theory of general relativity," J. Math. Phys. 9, 1053 (1968).

33. Krogh, C., and R. Baierlein, "Lunar laser ranging and the Brans-Dicke theory," Phys. Rev. 175, 1576 (1968).

34. Lightman, A. P., and David L. Lee, "Restricted proof that the weak equivalence principle implies the Einstein equivalence principle.", Phys. Rev. D8, 364 (1973).

35. Lobashov, V. M., et al., "Parity non-conservation in the gamma decay of ^{181}Ta," Phys. Lett. 25B, 104 (1967).

36. Mahanta, M. N., and D.R.K. Reddy, "An approximate solution of the vacuum static case of spherical symmetry in Brans-Dicke theory," J. Math. Phys. 12, 929 (1971).

37. Matsuda, T., "Cold neutron star model in Brans-Dicke theory of gravity," Progr. Theor. Phys. 48, 341 (1972).

38. Matsuda, T., "On the gravitational collapse in Brans-Dicke theory of gravity," Progr. Theor. Phys. 47, 738 (1972).

39. Morganstern, R. E., and H.-Y. Chiu, "Scalar gravitational fields in pulsating stars," Phys. Rev. 157, 1228 (1967).

40. Morganstern, R. E., "Brans-Dicke theory under transformation of units and its relation to the Jordan theories," Phys. Rev. D1, 2969 (1970).

41. Morganstern, R. E., "Brans-Dicke theory under a transformation of units and the three tests," Phys. Rev. D3, 2946 (1971).

42. Morganstern, R. E., "Cosmological upper limit on the time variation of G," Nature 232, 109 (1971).

43. Morganstern, R. E., "Exact solutions to Brans-Dicke cosmologies in flat Friedmann Universes," Phys. Rev. D4, 946 (1971).

44. Nariai, H. "On the Brans solution in the scalar-tensor theory of gravitation," Progr. Theor. Phys. 42, 742 (1969).

45. Nariai, H., "Grantiational instability in the Brans-Dicke cosmology," Progr. Theor. Phys. 42, 544 (1969).

46. Newton, R. R., "Experimental evidence for a secular decrease in the gravitational constant G," J. Geophys. Research 73, 3765 (1968).

47. Ni, Wei-Tou, "Theoretical frameworks for testing relativistic gravity. IV. A compendium of metric theories of gravity and their post-Newtonian limits," Astrophys. J. 176, 769 (1972).

48. Noerdlinger, Peter D., "Theoretical necessity of a positive value for the Brans-Dicke coupling constant ω," Phys. Rev. 170, 1175 (1968).

49. Nordtvedt, Kenneth Jr., "Equivalence principle for massive bodies. I. Phenomenology," Phys. Rev. 169, 1014 (1968); "II. Theory," Phys. Rev. 169, 1017 (1968).

50. Nordtvedt, Kenneth Jr., "Testing relativity with laser ranging to the moon," Phys. Rev. 170, 1186 (1968).

51. Nordtvedt, Kenneth Jr., "Solar system Eötvös experiments," Icarus 12, 91 (1970).

52. Nordtvedt, Kenneth Jr., and C. M. Will, "Conservation laws and preferred frames in relativistic gravity. II. Experimental evidence to rule out preferred-frame theories of gravity.", Astrophys. J. 177, 775 (1972).

53. Nutku, Yavuz, "The post-Newtonian equations of hydronamics in the Brans-Dicke theory," Astrophys. J. 155, 999 (1969).

54. O'Connell, R. F., "Gyroscope test of gravitation: an analysis of the important perturbations," in Proceedings of the Conference on Experimental Tests of Gravitation Theories (JPL Technical Memorandum 33-499), p. 82.

55. O'Connell, R. F., "Schiff's proposed gyroscope experiment as a test of the scalar-tensor theory of general relativity," Phys. Rev. Lett. 20, 69 (1968).

56. Ohanian, H. C., "Inertial and gravitational mass in the Brans-Dicke theory," Ann. Phys. (U.S.) 67, 648 (1971).

57. O'Hanlon, J., and K. K. Tam, "Stellar ages and an extended theory of gravitation," Progr. Theor. Phys. 43, 684 (1970).

58. O'Hanlon, J., and B.O.J. Tupper, "Vacuum-field solutions in the Brans-Dicke Theory," Nuovo Cimento 7B, 305 (1972).

59. O'Hanlon, J., "Mach's principle and a new gauge freedom in Brans-Dicke theory," J. Phys. A5, 803 (1972).

60. Salmona, A., "Effect of gravitational scalar field on high-density star structure," Phys. Rev. 154, 1218 (1967).

61. Salpeter, E. E., "The oblate sun and relativity," Comments Nuclear and Particle Phys. 1, 99 (1967).

62. Shaviv, G., and J. N. Bahcall, "Brans-Dicke cosmology and the solar neutrino flux," Publ. Astron. Soc. Pacific 79, 438 (1967).

63. Shaviv, G., and J. N. Bahcall, "The effect of the Brans-Dicke cosmology on solar evolution and neutrino fluxes," Astrophys. J. 155, 135 (1969).

64. Thorne, Kip S., C. M. Will, and Wei-Tou Ni, "Theoretical frameworks for testing relativistic gravity--a review," in Proceedings of the Conference on Experimental Tests of Gravitation Theories (JPL Technical Memorandum 33-499), p. 10.

65. Thorne, Kips S., and C. M. Will, "Theoretical frameworks for testing relativistic gravity. I. General theory," Astrophys. J. 163, 595 (1971).

66. Thorne, Kips S., and J. J. Dykla, "Black holes in the Brans-Dicke-Jordan theory of gravity," Astrophys. J. 166, L35 (1971).

67. Toton, E., "Report on the investigation of Mach's principle in the scalar-tensor theory of gravity," Phys. Rev. Lett. 21, 1401 (1968).

68. Toton, E. T., "Scalar-tensor theory in canonical form. I. The question of Mach's principle.", J. Math. Phys. 11, 1713 (1970).

69. Will, Clifford M., "Theoretical frameworks for testing relativistic gravity. II. Parametrized post-Newtonian hydrodynamics, and the Nordtvedt effect," Astrophys. J. 163, 611 (1971).

70. Will, Clifford M., "Theoretical frameworks for testing relativistic gravity. III. Conservation laws, Lorentz invariance, and values of the PPN parameters," Astrophys. J. 169, 125 (1971).

71. Will, Clifford M., and Kenneth Nordtvedt, Jr., "Conservation laws and preferred frames in relativistic gravity. I. Preferred-frame theories and an extended PPN formalism," Astrophys. J. 177, 757 (1972).