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EFFECT OF GRAVITATIONAL AND AERODYNAMIC TORQUES ON A RIGID SKYLAB-TYPE SATELLITE

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16. ABSTRACT

A theoretical investigation of the influence of the aerodynamic torque on a Skylab-type satellite is presented. Only the case of free flow of uniform velocity and completely diffuse reflection is considered. It is found by linear stability analysis that the satellite has no stable equilibrium.

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EFFECT OF GRAVITATIONAL AND AERODYNAMIC TORQUES ON A RIGID SKYLAB-TYPE SATELLITE

I. INTRODUCTION

The behavior of a Skylab-type satellite under the influence of gravitational and aerodynamic torques is investigated. Only the simple case of free molecular flow of uniform velocity and completely diffuse reflection (i.e., the atmospheric particle transfers its total relative momentum to the satellite at impact) is considered. The aerodynamic torque on the satellite is expanded into a Fourier series with respect to the two angles characterizing the direction of the atmospheric velocity vector relative to the satellite. Making use of some invariance properties and symmetries of the satellite, and neglecting some relatively small effects of the torque, these Fourier series can be simplified; i.e., it can be shown that many terms vanish. The resulting expressions, suitably truncated, coincide with those given by Nurre [1], which he derived by numerical methods. Applying the standard methods of linear stability to the linearized equations of motion, it is found that the satellite does not have a stable equilibrium position. The presentation comprises sufficient details to make it a useful aid in similar studies.

II. ASSUMPTIONS

Throughout this study, the following assumptions are made:

1. The satellite is a rigid body.

2. The satellite orbits the central body in a circular orbit; the central body is represented by a point mass, orbital and rotational motion of the satellite are not coupled, and only the principal term of the gravitational torque is taken into account.

3. The atmosphere is resting with respect to a frame with inertial directions (i.e., the primary and its atmosphere do not rotate).

4. Only the case of completely diffuse reflection (accommodation coefficient equals 1) is studied, i.e., the atmospheric particle is at rest with respect to the satellite immediately after hitting the satellite; in other words, the atmospheric particle transfers its entire relative linear momentum to the satellite.

III. EQUATIONS OF MOTION

Three coordinate frames will be used: an inertial frame, an orbital frame, and a satellite-fixed body frame. All coordinate systems are right-handed cartesian systems.

A. Inertial Frame $\overline{Y}$

The inertial frame $\overline{Y}$ has its origin at the center of mass of the satellite, and the directions of the coordinate axes are space fixed. The Y_1-axis and Y_3-axis are in the plane of motion, and the Y_2-axis is in the direction of the orbital angular momentum vector. The inertial frame will not be used explicitly, but serves as a basic reference and starting point to develop coordinate transformations and equations of motion.

B. Orbital Frame $\overline{\xi}$

The orbital frame $\overline{\xi}$ has its origin at the center of mass of the satellite, and it rotates uniformly with respect to the inertial frame with angular speed Ω. The ξ_3-axis coincides with the line from the central body to the satellite, the ξ_1-axis is in the direction of the satellite's velocity, and the ξ_2-axis coincides with the Y_2-axis. We have

$$\overline{\xi} = W\overline{Y} = \begin{pmatrix} c\Omega t & 0 & -s\Omega t \\ 0 & 1 & 0 \\ s\Omega t & 0 & c\Omega t \end{pmatrix} \overline{Y}, \ \overline{\Omega}^* = \begin{pmatrix} 0 \\ \Omega \\ 0 \end{pmatrix} , \tag{1}$$

where $\overline{\Omega}^*$ is the constant angular velocity of the orbital frame with respect to the inertial frame, represented in the orbital frame (Fig. 1).

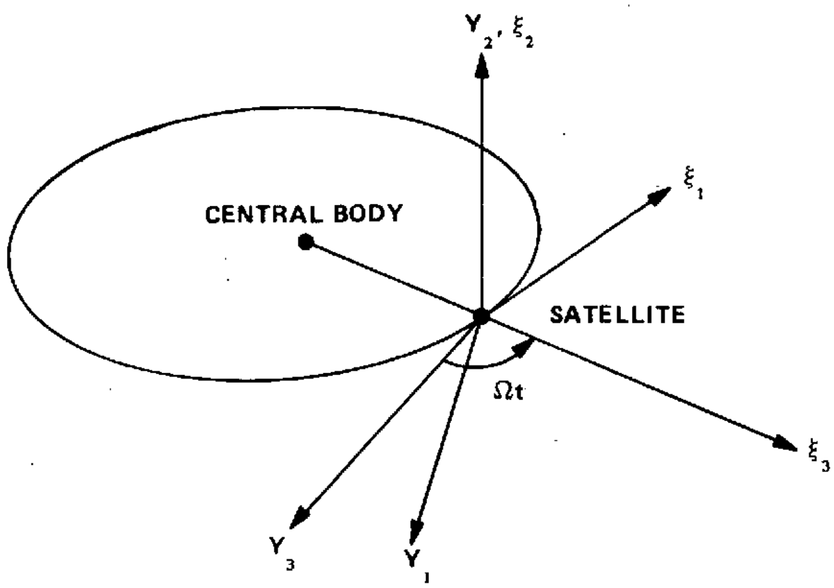

Figure 1. Inertial and orbital coordinate systems.

C. Body Frame

The body frame is fixed in the satellite; we will use the following four coordinate systems fixed in the body.

1. $\overline{\eta}$-Coordinate System. This is a general body fixed system, with its origin at the satellite's center of mass. Occasionally we will specialize it to utilize certain properties of the satellite, such as symmetries, etc. Many of our equations will be written in this system. The rotation from the orbital $\overline{\xi}$-system to the $\overline{\eta}$-system is represented as a product of three plane rotations: rotate through ψ_1 with fixed 1-axis, then through ψ_2 with fixed new 2-axis, then through ψ_3 with fixed new 3-axis. Thus

$$\overline{\eta} = \psi\overline{\xi} \quad , \psi = \begin{pmatrix} c\psi_3 & s\psi_3 & 0 \\ -s\psi_3 & c\psi_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\psi_2 & 0 & -s\psi_2 \\ 0 & 1 & 0 \\ s\psi_2 & 0 & c\psi_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\psi_1 & s\psi_1 \\ 0 & -s\psi_1 & c\psi_1 \end{pmatrix}$$

$$= \begin{pmatrix} c_2\, c_3 & c_1\, s_3 + s_1\, s_2\, c_3 & s_1\, s_3 - c_1\, s_2\, c_3 \\ -c_2\, s_3 & c_1\, c_3 - s_1\, s_2\, s_3 & s_1\, c_3 + c_1\, s_2\, s_3 \\ s_2 & -s_1\, c_2 & c_1\, c_2 \end{pmatrix} \tag{2}$$

$$\overline{\omega} = \begin{pmatrix} c_2\, c_3\, \dot{\psi}_1 + s_3\, \dot{\psi}_2 \\ -c_2\, s_3\, \dot{\psi}_1 + c_3\, \dot{\psi}_2 \\ s_2\, \dot{\psi}_1 + \dot{\psi}_3 \end{pmatrix} , \tag{3}$$

where we have written c_2 for $c\psi_2$, etc. $\overline{\omega}$ is the angular velocity of the $\overline{\eta}$-system with respect to the orbital $\overline{\xi}$-system, represented in the $\overline{\eta}$-system.

2. $\overline{X}$-Coordinate System. The origin of the $\overline{X}$-coordinate system is at the satellite's center of mass; the directions of the axes are chosen such that the inertia matrix is diagonal (principal axes system). The rotation from the $\overline{\xi}$-system to the $\overline{X}$-system and its matrix Θ are defined as for the $\overline{\eta}$ system, with ϑ_1, ϑ_2, ϑ_3 replacing ψ_1, ψ_2, ψ_3.

3. $\overline{Z}$-Coordinate System. The origin of the $\overline{Z}$-coordinate system is at the satellite's center of mass; the directions of the axes are chosen so as to exhibit geometrical symmetries, etc., of the satellite. The rotation from the $\overline{\xi}$-system to the $\overline{Z}$-system and its matrix Φ are defined as for the $\overline{\eta}$-system, with ϕ_1, ϕ_2, ϕ_3 replacing ψ_1, ψ_2, ψ_3.

4. $\overline{\zeta}$-Coordinate System. The axes of the $\overline{\zeta}$-coordinate system are parallel to those of the $\overline{Z}$-system; its origin is chosen so that the coordinates adapt to the geometry of the satellite.

If $\overline{Z}^*$ is the vector from the origin of the $\overline{Z}$-system to the origin of the $\overline{\zeta}$-system, then

$$\overline{\zeta} = \overline{Z} - \overline{Z}^* . \tag{4}$$

Figure 2 illustrates the various coordinate systems and their relations.

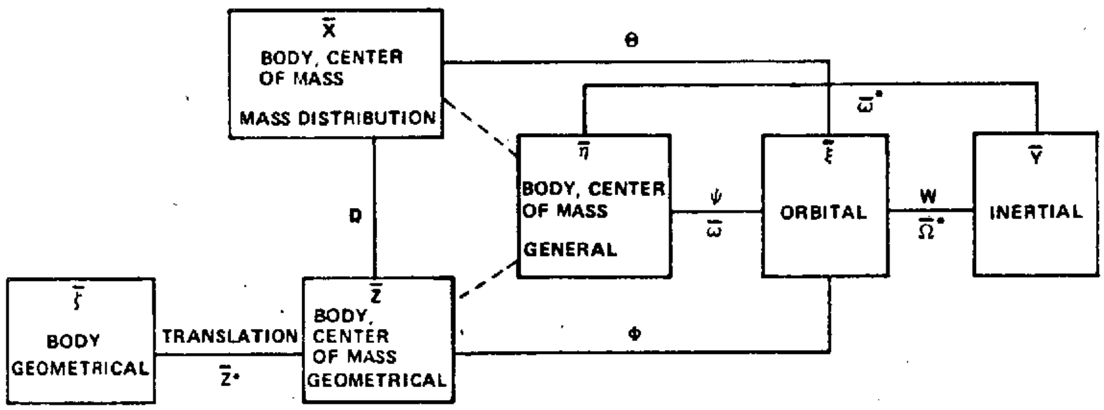

Figure 2. Diagram of coordinate systems and their relations.

The equation of rotational motion of the satellite, referred to a body frame, reads

$$I\dot{\bar{\omega}}^* + \bar{\omega}^* \times I\bar{\omega}^* - \bar{\ell} = 0 \quad , \tag{5}$$

where

I = (constant) moment of inertia matrix,

$\bar{\omega}^*$ = angular velocity of the body frame with respect to the inertial frame (and represented in the body frame),

$\bar{\ell} = \bar{\ell}_g + \bar{\ell}_a$ = gravitational torque plus aerodynamic torque.

For the subsequent discussions, we shall represent all quantities in the body frame; we then have

$$\bar{\omega}^* = \bar{\omega} + \bar{\Omega} \quad , \tag{6}$$

where, as stated before,

$\bar{\omega}$ = angular velocity of the body frame with respect to the orbital frame, represented in the body frame,

$\bar{\Omega}$ = $\psi\bar{\Omega}^*$; $\bar{\Omega}^*$ is the angular velocity of the orbital frame with respect to the inertial frame, represented in the orbital frame; for representation in the body frame it is multiplied by the transformation matrix ψ.

It follows that

$$\dot{\bar{\omega}}^* = \dot{\bar{\omega}} + \dot{\psi}\bar{\Omega}^* + \psi\dot{\bar{\Omega}}^* \quad ,$$

and with $\dot{\bar{\Omega}}^* = 0$ in our case,

$$\dot{\bar{\omega}}^* = \dot{\bar{\omega}} - \bar{\omega} \times \psi\bar{\Omega}^*$$

or

$$\dot{\bar{\omega}}^* = \dot{\bar{\omega}} - \bar{\omega} \times \bar{\Omega} \quad . \tag{7}$$

We thus arrive at the equation of motion

$$I(\dot{\bar{\omega}} - \bar{\omega} \times \bar{\Omega}) + (\bar{\omega} + \bar{\Omega}) \times I(\bar{\omega} + \bar{\Omega}) - \bar{\ell}_g - \bar{\ell}_a = 0 \quad , \tag{8}$$

where

$$\bar{\Omega} = \psi\bar{\Omega}^* = \Omega \begin{pmatrix} \psi_{12} \\ \psi_{22} \\ \psi_{32} \end{pmatrix} = \Omega \begin{pmatrix} c_1 s_3 + s_1 s_2 c_3 \\ c_1 c_3 - s_1 s_2 s_3 \\ -s_1 c_2 \end{pmatrix} \quad . \tag{9}$$

$\bar{\omega}$ is given in equation (3), and

$$\dot{\bar{\omega}} = \begin{pmatrix} c_2\, c_3 \\ -c_2\, s_3 \\ s_2 \end{pmatrix} \ddot{\psi}_1 + \begin{pmatrix} s_3 \\ c_3 \\ 0 \end{pmatrix} \ddot{\psi}_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ddot{\psi}_3 + \begin{pmatrix} -s_2\, c_3 \\ s_2\, s_3 \\ c_2 \end{pmatrix} \dot{\psi}_1\, \dot{\psi}_2 + \begin{pmatrix} -c_2\, s_3 \\ -c_2\, c_3 \\ 0 \end{pmatrix} \dot{\psi}_1\, \dot{\psi}_3$$

$$+ \begin{pmatrix} c_3 \\ -s_3 \\ 0 \end{pmatrix} \dot{\psi}_2\, \dot{\psi}_3 \quad . \tag{10}$$

IV. GRAVITATIONAL TORQUE

For a satellite orbiting the central body in a circular orbit with constant angular speed Ω, the principal term of the gravitational torque is

$$\bar{\ell}_g = 3\Omega^2\, \bar{r} \times I\bar{r} \quad , \tag{11}$$

where $\bar{r}$ is the unit vector from the central body to the satellite. For the orbital frame, denoting the vector by $\bar{r}^*$, we obviously have

$$\bar{r}^* = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad ; \tag{12}$$

hence

$$\bar{r} = \psi \bar{r}^* = \begin{pmatrix} \psi_{13} \\ \psi_{23} \\ \psi_{33} \end{pmatrix} \quad , \tag{13}$$

and, if we specialize the coordinate system such that

$$I = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \quad \text{(principal axes system)}, \tag{14}$$

we get

$$\bar{r} \times I\bar{r} = \begin{pmatrix} (I_3 - I_2) & \psi_{23} & \psi_{33} \\ (I_1 - I_3) & \psi_{13} & \psi_{33} \\ (I_2 - I_1) & \psi_{13} & \psi_{23} \end{pmatrix} \quad . \tag{15}$$

The matrix elements ψ_{jk} of ψ can be read from equation (2).

V. AERODYNAMIC FORCE AND TORQUE

The atmosphere is considered as a continuous medium which transfers its entire relative momentum to the surface of the satellite at impact.

A. Differential Force

We calculate the force exerted by the atmosphere on a surface element of the satellite in the body frame. The basic equation from which we start is

$$d\bar{\mu} = \bar{v}\, dm \quad , \tag{16}$$

where

$\overline{\mu}$ = (linear) momentum,

$\overline{v}$ = velocity of atmosphere with respect to the satellite,

m = mass of atmosphere.

The following fairly obvious sequence of equations (Fig. 3),

$$dm = \rho \, dV \quad , \tag{17}$$

$$dV = da \, dh \quad , \tag{18}$$

$$dh = ds \cos\alpha \quad , \tag{19}$$

$$\cos\alpha = \overline{e}_n \, \overline{e}_v \quad , \tag{20}$$

$$ds = v \, dt \quad , \tag{21}$$

where

ρ = density of atmosphere,

V = volume of atmosphere,

da = surface element of satellite,

$\bar{e}_n$ = normal unit vector of surface element, pointing toward the "outside,"

$\bar{e}_v$ = $\bar{v}/v$ = unit vector of $\bar{v}$,

yields upon substitution an equation for $d\bar{\mu}$.

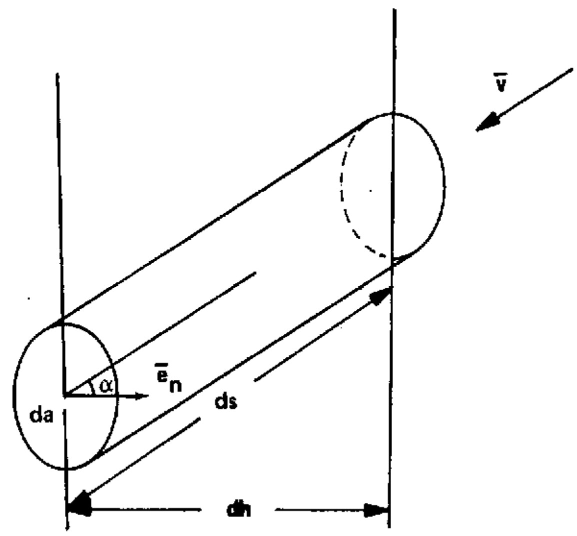

Figure 3. Atmospheric force on surface element.

Obviously, only those parts of the surface will contribute to the momentum $\bar{\mu}$ that satisfy the following two conditions:

1. The surface part is not shaded against the atmospheric stream by another surface part.

2. $\cos\alpha \geq 0$, i.e., $-\bar{e}_n \bar{e}_v \geq 0$. (22)

Combining these results, we find for the momentum transferred to the surface element da

$$d\bar{\mu} = \rho v^2 (-\bar{e}_n \bar{e}_v) \bar{e}_v \, da \, dt \quad , \quad \bar{e}_n \bar{e}_v \leq 0 \quad , \tag{23}$$

and for the force $d\overline{f} = d\overline{\mu}/dt$

$$d\overline{f} = \rho v^2(-\overline{e}_n \overline{e}_v)\, \overline{e}_v \, da \quad , \quad \overline{e}_n \overline{e}_v \leq 0 \quad . \tag{24}$$

For later use, we note that $\cos\alpha \; da = -\overline{e}_n \overline{e}_v \, da$ is the projection of the surface element da onto a plane orthogonal to the velocity vector $\overline{v}$.

B. Differential Torque

From the force $d\overline{f}$ we get the differential torque $d\overline{\ell}_p$ with respect to the point $\overline{P}$ as

$$d\overline{\ell}_p = (\overline{R} - \overline{P}) \times d\overline{f}$$

or

$$d\overline{\ell}_p = \rho v^2 (-\overline{e}_n \overline{e}_v)\, (\overline{R} - \overline{P}) \times \overline{e}_v \, da \quad , \; \overline{e}_n \overline{e}_v \leq 0 \quad , \tag{25}$$

where $\overline{R}$ is the location vector of the surface element da.

The total force $\overline{f}$ and torque $\overline{\ell}_p$ are calculated by integrating equations (24) and (25) over the permissible surface area; i.e., the area that satisfies the conditions (22).

VI. FOURIER EXPANSION OF AERODYNAMIC FORCE AND TORQUE

Except for very simply shaped bodies, the aerodynamic force and torque as given by (the integrated form of) equations (24) and (25) are complicated functions of the body shape and the direction of the velocity vector $\overline{v}$ of the atmosphere (via the integration over the body surface). It results, however, that both force and torque are, for a fixed given body, periodic functions of the two angles describing the direction of the atmospheric velocity vector.

We shall subsequently establish the Fourier series representing these periodic functions and in the next part specialize it to a Skylab-type satellite. Consider the atmospheric force and torque in the body-fixed coordinate system $\overline{\eta}$; then, $\overline{\eta} = \psi \overline{\xi}$. In the orbital frame, the atmospheric velocity vector $\overline{v}_\xi$ is

$$\overline{v}_\xi = -v \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} , \tag{26}$$

and for the $\overline{\eta}$-coordinate system

$$\overline{v}_\eta = -v\psi \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= -v \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -v \begin{pmatrix} c\psi_2 \; c\psi_3 \\ -c\psi_2 \; s\psi_3 \\ s\psi_2 \end{pmatrix} . \tag{27}$$

Comparing this with the representation

$$\begin{pmatrix} c\alpha \;\; c\beta \\ c\alpha \;\; s\beta \\ s\alpha \end{pmatrix}$$

of a unit vector in spherical coordinates where α is the latitude and β is the longitude, we see that we can interpret ψ_2 as latitude and ψ_3 as negative longitude in the $\bar{\eta}$-coordinate system.

In general, the area of the satellite exposed to impact of atmospheric particles is a discontinuous function of the direction of the atmospheric velocity vector; i.e., of the components $c_2 c_3$, $-c_2 s_3$, s_2 of $\bar{e}_V$, and thus of the angles ψ_2 and ψ_3 (think, for example, of a planar part of the satellite that changes from not being exposed to being exposed to impact instantly at certain values of ψ_2 and ψ_3). Nevertheless, the projection of the area onto a plane orthogonal to the atmospheric velocity vector and the projection's boundary are continuous functions (of the body shape and) of the components of $\bar{e}_V$.

As mentioned previously, this projection of the surface element da is

$$da^* = -\bar{e}_n \bar{e}_V \, da \quad (\bar{e}_n \bar{e}_V \leq 0) \quad ; \tag{28}$$

hence

$$\text{force } \bar{f}(\psi_2, \psi_3) = \rho v^2 \bar{e}_V \int_{\substack{\text{projected}\\ \text{area}}} da^* \tag{29}$$

and

$$\text{torque } \bar{\ell}_p(\psi_2, \psi_3) = \rho v^2 \int_{\substack{\text{projected}\\ \text{area}}} (\bar{R} - \bar{P}) \times \bar{e}_V \, da^* \tag{30}$$

$$= \rho v^2 \left(\int_{\substack{\text{projected}\\ \text{area}}} \bar{R} \, da^* \right) \times \bar{e}_V - \rho v^2 \bar{P} \times \bar{e}_V \int_{\substack{\text{projected}\\ \text{area}}} da^* \ , \tag{31}$$

where the second term on the right of equation (31) is just $-\bar{P} \times \bar{f}$. All integrals and thus $\bar{f}$ and $\bar{\ell}_p$ are continuous functions of ψ_2 and ψ_3; to repeat, the angles enter the functions $\overline{f}$ and $\overline{\ell}_p$ only through the components of $\overline{e}_v$ in the combinations $c\psi_2\, c\psi_3$, $c\psi_2\, s\psi_3$, $s\psi_2$. We can now write the following Fourier series:

$$\overline{f} = \sum_{\mu,\nu=0}^{\infty} (\overline{A}_{\mu\nu}\; c\mu\,\psi_2\, c\nu\,\psi_3 + \overline{B}_{\mu\nu}\; c\mu\psi_2\; s\nu\psi_3 + \overline{C}_{\mu\nu}\; s\mu\psi_2\; c\nu\psi_3$$

$$+ \overline{D}_{\mu\nu}\; s\mu\psi_2\; s\nu\psi_3) \tag{32}$$

$$\overline{\ell} = \overline{\ell}_p = \sum_{\mu,\nu=0}^{\infty} (\overline{E}_{\mu\nu}\; c\mu\psi_2\; c\nu\psi_3 + \overline{F}_{\mu\nu}\; c\mu\psi_2\; s\nu\psi_3 + \overline{G}_{\mu\nu}\; s\mu\psi_2\; c\nu\psi_3$$

$$+ \overline{H}_{\mu\nu}\; s\mu\psi_2\; s\nu\psi_3) \tag{33}$$

or, in easily understood abbreviation,

$$\begin{aligned} \overline{f} &= \Sigma f\,cc + \Sigma f\,cs + \Sigma f\,sc + \Sigma f\,ss \\ \overline{\ell} &= \Sigma \ell\,cc + \Sigma \ell\,cs + \Sigma \ell\,sc + \Sigma \ell\,ss \quad . \end{aligned} \tag{34}$$

Equations (34) state that each component of $\overline{f}$ and $\overline{\ell}$ is of the indicated form, where, for example, Σ fcs stands for Σ (appropriate coefficient for $\overline{f}$-term)• $c\mu\psi_2\; s\nu\psi_3$, and all sums are taken over the appropriate values of the indices, as later stated in each case.

By various invariance considerations, some general and some specialized to a Skylab-type satellite, we will show that "most" of the coefficients in equations (32) and (33) actually vanish.

By previous remarks the Fourier series must remain unchanged under all transformations of the angles ψ_2 and ψ_3 that leave the components $c\psi_2\, c\psi_3$, $c\psi_2\, s\psi_3$, and $s\psi_2$ of $\overline{e}_v$ unchanged. The set of equations

$$
\begin{aligned}
c\psi_2^* \, c\psi_3^* &= c\psi_2 \, c\psi_3 \\
c\psi_2^* \, s\psi_3^* &= c\psi_2 \, s\psi_3 \\
s\psi_2^* &= s\psi_2
\end{aligned}
\tag{35}
$$

has the two solutions

$$
\begin{aligned}
&\text{(Trivial) I:} \quad \psi_2^* = \psi_2 \quad , \quad \psi_3^* = \psi_3 \\
&\text{II:} \quad \psi_2^* = \pi - \psi_2, \quad \psi_3^* = \pi + \psi_3 \quad ;
\end{aligned}
\tag{36}
$$

hence we must have

$$
\begin{aligned}
\bar{f}(\psi_2, \psi_3) &= \bar{f}(\pi - \psi_2, \pi + \psi_3) \\
\bar{\ell}(\psi_2, \psi_3) &= \bar{\ell}(\pi - \psi_2, \pi + \psi_3) \quad .
\end{aligned}
\tag{37}
$$

By a routine process of comparing coefficients we find the reduced series

$$
\bar{f} = \sum_{\mu+\nu \text{ even}} fcc + \sum_{\mu+\nu \text{ even}} fcs + \sum_{\mu+\nu \text{ odd}} fsc + \sum_{\mu+\nu \text{ odd}} fss \tag{38}
$$

$$
\bar{\ell} = \sum_{\mu+\nu \text{ even}} \ell cc + \sum_{\mu+\nu \text{ even}} \ell cs + \sum_{\mu+\nu \text{ odd}} \ell sc + \sum_{\mu+\nu \text{ odd}} \ell ss \quad . \tag{39}
$$

The second consideration applies rigorously to the force $\bar{f}$ only. It is obvious that the projected area $\int da^*$ is the same for $\bar{e}_v$ and $-\bar{e}_v$; therefore [see equation (30)] the substitution $\bar{e}_v \to -\bar{e}_v$ implies $\bar{f} \to -\bar{f}$ The equations

$$
\begin{aligned}
c\psi_2{}^* \, c\psi_3{}^* &= -c\psi_2 \, c\psi_3 \\
c\psi_2{}^* \, s\psi_3{}^* &= -c\psi_2 \, s\psi_3 \\
s\psi_2{}^* &= -s\psi_2
\end{aligned}
\tag{40}
$$

have the two solutions

$$
\begin{aligned}
\text{I:} \quad & \psi_2{}^* = -\psi_2 \quad , \quad \psi_3{}^* = \pi + \psi_3 \\
\text{II:} \quad & \psi_2{}^* = \pi + \psi_2 \quad , \quad \psi_3{}^* = \psi_3 \quad ;
\end{aligned}
\tag{41}
$$

both these solutions lead to the reduction of equation (38) to

$$
\overline{f} = \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{odd}}} fcc + \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{odd}}} fcs + \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{even}}} fsc + \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{even}}} fss \quad . \tag{42}
$$

VII. SPECIALIZATION TO A SKYLAB-TYPE SATELLITE

We assume now that the satellite is symmetric with respect to the η_1, η_3-plane. This implies that for a surface element da at η_1, η_2, η_3 with $\overline{e}_n{}^T = (e_{n1} \; e_{n2} \; e_{n3})$ there is a surface element at η_1, $-\eta_2$, η_3 with $\overline{e}_n{}^T = (e_{n1} \; -e_{n2} \; e_{n3})$ and the same area. Write $\overline{f}$ as a sum of integrals over the "right" ($\eta_2 \leq 0$) and the "left" ($\eta_2 \geq 0$) parts R and L of the satellite:

$$\overline{f} = \rho v^2 \left\{ \int_R | e_{n1} e_{v1} + e_{n2} e_{v2} + e_{n3} e_{v3} | \begin{pmatrix} e_{v1} \\ e_{v2} \\ e_{v3} \end{pmatrix} da \right.$$

$$\left. + \int_L | e_{n1} e_{v1} - e_{n2} e_{v2} + e_{n3} e_{v3} | \begin{pmatrix} e_{v1} \\ e_{v2} \\ e_{v3} \end{pmatrix} da \right\} \quad . \tag{43}$$

Under the reflection

$$(\eta_1 \, \eta_2 \, \eta_3) \rightarrow (\eta_1 \, -\eta_2 \, \eta_3) \quad , \quad (e_{v1} \, e_{v2} \, e_{v3}) \rightarrow (e_{v1} \, -e_{v2} \, e_{v3}) \tag{44}$$

corresponding to the η_1, η_3-plane symmetry, the integrals change according to

$$\int_R \rightarrow \int_L | e_{n1} e_{v1} - e_{n2} e_{v2} + e_{n3} e_{v3} | \begin{pmatrix} e_{v1} \\ -e_{v2} \\ e_{v3} \end{pmatrix} da$$

and (45)

$$\int_L \rightarrow \int_R | e_{n1} e_{v1} + e_{n2} e_{v2} + e_{n3} e_{v3} | \begin{pmatrix} e_{v1} \\ -e_{v2} \\ e_{v3} \end{pmatrix} da \quad .$$

Hence, under this reflection the force $\overline{f}$ is changed according to

$$\overline{f}^T = (f_1 \, f_2 \, f_3) \rightarrow (f_1 \, -f_2 \, f_3) \quad , \tag{46}$$

as we would expect intuitively. Similarly, we find that the torque $\bar{\ell}$ is changed according to

$$\bar{\ell}^T = (\ell_1\ \ell_2\ \ell_3) \rightarrow (-\ell_1\ \ell_2\ -\ell_3) \quad . \tag{47}$$

(Observe that for equation (47) to hold we assume that the reference point $\bar{P}$ lies in the symmetry plane $\eta_2 = 0$.)

We now use the just discussed symmetry for a further reduction of the Fourier series (39) and (42). The transformations of the angles ψ_2 and ψ_3 corresponding to the transformation (44) are solutions of

$$\begin{aligned} c\psi_2{}^* \, c\psi_3{}^* &= c\psi_2 \, c\psi_3 \\ c\psi_2{}^* \, s\psi_3{}^* &= -c\psi_2 \, s\psi_3 \\ s\psi_2{}^* &= s\psi_2 \quad ; \end{aligned} \tag{48}$$

the two solutions are

$$\begin{aligned} &\text{I:} \quad \psi_2{}^* = \psi_2 \quad , \quad \psi_3{}^* = -\psi_3 \\ &\text{II:} \quad \psi_2{}^* = \pi - \psi_2 \quad , \quad \psi_3{}^* = \pi - \psi_3 \quad . \end{aligned} \tag{49}$$

Taking into account the change of $\bar{f}$ and $\bar{\ell}$ according to (46) and (47), both solutions (49) lead to the same further reduced Fourier series

$$f_1 \, , \, f_3 = \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} fcc \;+\; \sum_{\substack{\mu \text{ odd} \\ \nu \text{ even}}} fsc \tag{50}$$

$$f_2 = \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} fcs + \sum_{\substack{\mu \text{ odd} \\ \nu \text{ even}}} fss \tag{51}$$

$$\ell_1, \ell_3 = \sum_{\mu+\nu \text{ even}} \ell cs + \sum_{\mu+\nu \text{ odd}} \ell ss \tag{52}$$

$$\ell_2 = \sum_{\mu+\nu \text{ even}} \ell cc + \sum_{\mu+\nu \text{ odd}} \ell sc \quad . \tag{53}$$

To achieve another reduction of the force terms, we can argue as follows. Under the transformation $\psi_2 \rightarrow -\psi_2$ (reflection on the η_1, η_2-plane) the projected areas remain unchanged (if we neglect shading effects, which are considered to be relatively small), since every part of the satellite is (assumed to be) symmetric with respect to some plane parallel to the η_1, η_2-plane (Fig. 4). Hence, if we change

$$\bar{e}_v^T = (e_{v1}\ e_{v2}\ e_{v3}) \rightarrow (e_{v1}\ e_{v2}\ -e_{v3}) \quad , \tag{54}$$

which corresponds to $\psi_2 \rightarrow -\psi_2$, we get

$$\bar{f}^T = (f_1\ f_2\ f_3) \rightarrow (f_1\ f_2\ -f_3) \tag{55}$$

(see, for example, the foregoing discussion of the η_1, η_3-plane symmetry), and it follows that f_1 and f_2 are (approximately) even functions of ψ_2, and f_3 is (approximately) an odd function of ψ_2. Applied to equations (50) and (51), this leads to

$$f_1 \cong \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} fcc \tag{56}$$

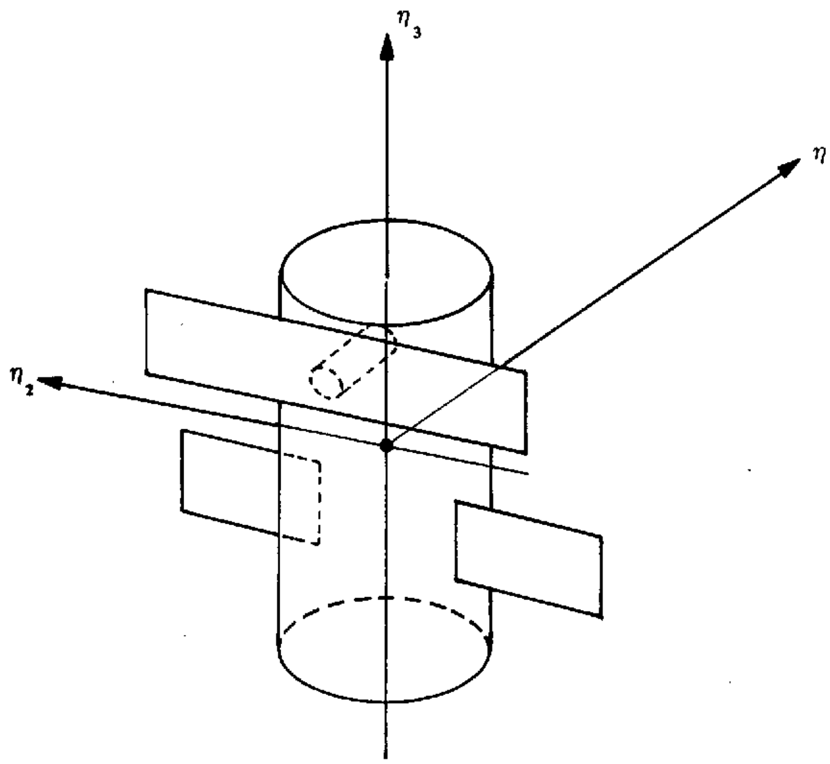

Figure 4. η-coordinate system with respect to satellite.

$$f_2 \cong \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{odd}}} \text{fcs} \tag{57}$$

$$f_3 \cong \sum_{\substack{\mu\ \text{odd} \\ \nu\ \text{even}}} \text{fsc} \quad . \tag{58}$$

To further reduce the Fourier series for the torque $\overline{\ell}$, we have to use the specific geometry of the satellite and must restrict ourselves to only approximately valid arguments and conclusions. Thus, we assume that the satellite is composed of a main cylindrical part 1, of circular cross section and with orthogonal planar end panels 2a and 2b, and of rectangular flat panels 3a, 3b, and 5. The part 4, connecting the cylindrical mantle 1 and panel 5, will be neglected (Figs. 4 and 5).

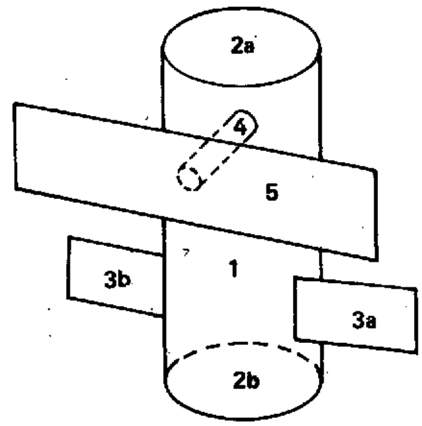

Figure 5. Shape and parts of idealized satellite.

For the following considerations, the origin of the coordinate system is at the center of the main cylinder 1, and the η_3-axis is its centerline. The panel 2 = 2a + 2b is parallel to the η_1, η_2-plane, and the panels 3 = 3a + 3b and 5 are parallel to the η_2, η_3-plane. Both panels 3 and 5 are symmetric with respect to the η_1, η_3-plane (assumed basic symmetry). [We argue here in an elementary geometrical and mechanical manner. A more precise derivation can easily be supplied by the method used to derive equation (46).] Shading effects are neglected.

First, we proceed as before in a general fashion; we get, however, results only for ℓ_3. Consider the transformation $\bar{e}_v \rightarrow -\bar{e}_v$. It is easily seen that the torque $\bar{\ell}$ with respect to the origin of the panels 3 = 3a + 3b and 5 transforms like $\bar{\ell} \rightarrow -\bar{\ell}$. For the mantle 1 and panel 2 = 2a + 2b, the torque transforms like $\bar{\ell} \rightarrow \bar{\ell}$ (observe, for example, that for $e_{v3} > 0$ only 2a, but not 2b, which is shaded by 1 and 2a, experiences torque). Because of symmetry, both for 1 and 2, we have $\ell_3 = 0$; thus

$$\ell_3 \cong \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} \ell cs + \sum_{\substack{\mu \text{ odd} \\ \nu \text{ even}}} \ell ss \quad . \tag{59}$$

Under the transformation $\psi_2 \rightarrow -\psi_2$, i.e., $(e_{v1}, e_{v2}\ e_{v3}) \rightarrow (e_{v1}\ e_{v2}\ -e_{v3})$, for both panels 3 and 5 we see that $\ell_3 \rightarrow \ell_3$ (also $\ell_1 \rightarrow \ell_1$, which we cannot use); thus, ℓ_3 must be an even function of ψ_2, and we get

$$\ell_3 \cong \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} \ell cs \quad . \tag{60}$$

A direct calculation of the torques on the various parts and addition, with shading effects neglected as before, yields considerably sharper results. Applying formula (31) for the torque $\bar{\ell}$, it can be shown that the torque of the complete cylinder, mantle 1, and panels 2a and 2b is always zero. Thus, we need to consider only the panels 3 = 3a + 3b and 5. Both 3 and 5 are symmetric with respect to the plane $\eta_2 = 0$; therefore, the vector $\bar{q}$ from the origin to the geometric center of 3 or 5 is

$$\bar{q} = \begin{pmatrix} q_1 \\ 0 \\ q_3 \end{pmatrix} \quad . \tag{61}$$

The torque of each panel with respect to its geometric center vanishes, and the force $\bar{f}$ can easily be computed from equation (29) to be

$$\bar{f} = \rho v^2 A \, | e_{v1} | \, \bar{e}_v \quad , \tag{62}$$

where A is the total area of the panel. Thus, the torque $\bar{\ell}$ on each panel is of the form

$$\bar{\ell} = \rho v^2 A \, | e_{v1} | \, \bar{q} \times \bar{e}_v = \rho v^2 A \, | e_{v1} | \begin{pmatrix} -q_3 e_{v2} \\ q_3 e_{v1} - q_1 e_{v3} \\ q_1 e_{v2} \end{pmatrix} \quad . \tag{63}$$

We see that the total torque with respect to the geometric center of the cylindrical main body 1 + 2a + 2b on the satellite is of the same form.

Generally, the Fourier series for $|\cos\alpha|$ is a cosine series containing only even multiples of α; using this fact, we find

$$\begin{aligned} \ell_1, \ \ell_3 \sim |c\psi_2|| c\psi_3 | c\psi_2 \, s\psi_3 &= \left[\sum_{\mu \text{ odd}} (\text{coeff}) \, c\mu\psi_2\right]\left[\sum_{\nu \text{ odd}} (\text{coeff}) \, s\nu\psi_3\right] \\ &= \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} \ell cs \end{aligned} \tag{64}$$

$$\ell_2 \sim |\, c\psi_2 \,\|\, c\psi_3 \,|\, (q_3\ c\psi_2\ c\psi_3 - q_1\ s\psi_2) = \sum_{\substack{\mu \text{ odd} \\ \nu \text{ odd}}} \ell cc + \sum_{\substack{\mu \text{ odd} \\ \nu \text{ even}}} \ell sc \quad . \tag{65}$$

We assume further that the center of mass of the satellite lies in the plane $\eta_2 = 0$ and that the axes of the geometric coordinate system (the specialized $\overline{\eta}$-system used last) and the $\overline{X}$-system (principal axes system) are parallel. If $\overline{p}$ is the vector from the origin of the $\overline{X}$-system to the origin of the $\overline{\eta}$-system, we get for the total torque $\overline{\ell}_{c.o.m.}$ with respect to the center of mass

$$\overline{\ell}_{c.o.m.} = \overline{\ell} + \overline{p} \times \overline{f} = \begin{pmatrix} \ell_1 - p_3\, f_2 \\ \ell_2 + p_3\, f_1 - p_1\, f_3 \\ \ell_3 + p_1\, f_2 \end{pmatrix} \quad , \tag{66}$$

and using f_1, f_2, and f_3 from expressions (56) through (58), we see that the components of $\overline{\ell}_{c.o.m.}$ have the same structure [expressions (64) and (65)] as those of $\overline{\ell}$.

The expressions (56) through (58) for the force components and (64) and (65) for the torque components have, suitably truncated, the form given by Nurre [1], p. 1047, equations 5] (there, in the last term, read $c3\Theta_2$ for $s3\Theta_2$).

Now we use the angles ϑ_j, and for the form of the final expressions for $\overline{f}$ and $\overline{\ell}$, we take that given by Nurre [1] [for the notation, see expressions (32) and (33)]:

$$\left.\begin{aligned} f_1 &= A_{111}\, c\vartheta_2\, c\vartheta_3 + A_{131}\, c\vartheta_2\, c3\vartheta_3 + A_{311}\, c3\vartheta_2\, c\vartheta_3 + A_{331}\, c3\vartheta_2\, c3\vartheta_3 \\ f_2 &= B_{112}\, c\vartheta_2\, s\vartheta_3 + B_{132}\, c\vartheta_2\, s3\vartheta_3 + B_{312}\, c3\vartheta_2\, s\vartheta_3 + B_{332}\, c3\vartheta_2\, s3\vartheta_3 \\ f_3 &= C_{103}\, s\vartheta_2 + C_{123}\, s\vartheta_2\, c2\vartheta_3 + C_{303}\, s3\vartheta_2 + C_{323}\, s3\vartheta_2\, c2\vartheta_3 \end{aligned}\right\} \tag{67}$$

$$\left.\begin{aligned}
\ell_1 &= F_{111}\, c\vartheta_2\, s\vartheta_3 + F_{131}\, c\vartheta_2\, s3\vartheta_3 + F_{311}\, c3\vartheta_2\, s\vartheta_3 + F_{331}\, c3\vartheta_2\, s3\vartheta_3 \\
\ell_2 &= E_{112}\, c\vartheta_2\, c\vartheta_3 + E_{132}\, c\vartheta_2\, c3\vartheta_3 + E_{312}\, c3\vartheta_2\, c\vartheta_3 + E_{332}\, c3\vartheta_2\, c3\vartheta_3 \\
&\quad + G_{102}\, s\vartheta_2 + G_{122}\, s\vartheta_2\, c2\vartheta_3 + G_{302}\, s3\vartheta_2 + G_{322}\, s3\vartheta_2\, c2\vartheta_3 \\
\ell_3 &= F_{113}\, c\vartheta_2\, s\vartheta_3 + F_{133}\, c\vartheta_2\, s3\vartheta_3 + F_{313}\, c3\vartheta_2\, s\vartheta_3 + F_{333}\, c3\vartheta_2\, s3\vartheta_3
\end{aligned}\right\} . \quad (68)$$

VIII. EQUILIBRIUM

The equilibria of a system $\dot{\overline{x}} = \overline{g}(\overline{x})$ of differential equations are defined to be the special solutions with all derivatives set to zero; i.e., the solutions of $\overline{g}(\overline{x}) = 0$. The equilibria of the equation of motion (8) are the solutions of

$$\overline{\Omega} \times \mathbb{I}\overline{\Omega} - \overline{\ell}_g - \overline{\ell}_a = 0 \quad , \qquad (69)$$

where $\overline{\ell}_g$ is given by (11) and (15), $\overline{\ell}_a$ by (68), and $\overline{\Omega}$ by (9).

With

$$\mathbb{I} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \quad , \qquad (70)$$

we have

$$\overline{\Omega} \times \mathbb{I}\overline{\Omega} = \Omega^2 \begin{pmatrix} (I_3 - I_2)\,\theta_{22}\ \theta_{32} \\ (I_1 - I_3)\,\theta_{12}\ \theta_{32} \\ (I_2 - I_1)\,\theta_{12}\ \theta_{22} \end{pmatrix} \quad . \qquad (71)$$

The vector equation (69) represents a system of three scalar equations for the three quantities ϑ_1, ϑ_2, and ϑ_3 and in general there exist finitely many solutions mod 2π.

In our case, the solution of most interest is $\vartheta_1 = \vartheta_3 = 0$; the first and third components of (69) vanish identically, and the second is of the form (A, B, ..., E are constant coefficients)

$$
A c\vartheta_2 + B s\vartheta_2 + C c3\vartheta_2 + D s3\vartheta_2 + E c\vartheta_2\, s\vartheta_2 = 0 \quad , \tag{72}
$$

which can be solved for ϑ_2; denote a solution of interest by ϑ_2^*.

IX. LINEARIZED EQUATIONS OF MOTION

For the investigation of the linear stability of the previously described equilibrium $\vartheta_1 = \vartheta_3 = 0$, $\vartheta_2 = \vartheta_2^*$, we set

$$
\vartheta_1 = \delta_1 \ , \ \vartheta_2 = \vartheta_2^* + \delta_2 \ , \ \vartheta_3 = \delta_3 \tag{73}
$$

and linearize the equations of motion (8) with respect to the δ_j and their time derivatives. Using

$$
\begin{aligned}
c(\vartheta_2^* + \delta_2) &\cong c\vartheta_2^* - \delta_2\, s\vartheta_2^* = c^* - \delta_2\, s^* \\
s(\vartheta_2^* + \delta_2) &\cong s\vartheta_2^* + \delta_2\, c\vartheta_2^* = s^* + \delta_2 c^* \quad ,
\end{aligned} \tag{74}
$$

we easily derive

$$
\ddot{\delta}_1 + \frac{I_1 + I_3 - I_2}{I_1}\frac{s^*}{c^*}\dot{\delta}_1 + \frac{I_1 + I_3 - I_2}{I_1}\frac{1}{c^*}\dot{\delta}_3 - 4\frac{I_3 - I_2}{I_1}\delta_1 - (M_1 + 3\frac{I_3 - I_2}{I_1}s^*)\,\delta_3 = 0 \tag{75}
$$

$$\ddot{\delta}_2 + \left[3\frac{I_1 - I_3}{I_2}(c^{*2} - s^{*2}) - M_2\right]\delta_2 = 0 \tag{76}$$

$$\ddot{\delta}_3 + s^*\ddot{\delta}_1 + \frac{I_2 - I_3 - I_1}{I_3} c^*\dot{\delta}_1 + 4\frac{I_2 - I_1}{I_3} s^*\delta_1 + \left[\frac{I_2 - I_1}{I_3}(1 + 3s^{*2}) - M_3\right]\delta_3 = 0 \quad , \tag{77}$$

where M_1, M_2, and M_3 are certain constants.

X. LINEAR STABILITY

Equation (76) for δ_2 is not coupled to the other two equations; its stability behavior is determined by the coefficient of δ_2. The solution is unstable if

$$3\frac{I_1 - I_3}{I_2}(c^{*2} - s^{*2}) - M_2 < 0 \quad ; \tag{78}$$

for ≥ 0 in (78) the solution can be represented by trigonometric functions.

Of more interest is the system (75), (77); it is of the form

$$A\ddot{\delta} + B\dot{\delta} + C\delta = 0 \quad , \tag{79}$$

where A, B, and C are 2 by 2 matrices and $\delta^T = (\delta_1 \; \delta_2)$. Introduce $\partial = \dot{\delta}$; then

$$\begin{aligned} &\dot{\delta} - \partial = 0 \\ &A\dot{\partial} + C\delta + B\partial = 0 \end{aligned} \tag{80}$$

or in matrix form,

$$\begin{pmatrix} U & 0 \\ 0 & A \end{pmatrix} \begin{pmatrix} \dot{\delta} \\ \partial \end{pmatrix} + \begin{pmatrix} 0 & -U \\ C & B \end{pmatrix} \begin{pmatrix} \delta \\ \partial \end{pmatrix} = 0 \quad , \tag{81}$$

where U is the 2 by 2 unit matrix. Since

$$\begin{pmatrix} U & 0 \\ 0 & A \end{pmatrix}^{-1} = \begin{pmatrix} U & 0 \\ 0 & A^{-1} \end{pmatrix} \quad ,$$

we get the standard form

$$\begin{pmatrix} \dot{\delta} \\ \partial \end{pmatrix} + \begin{pmatrix} 0 & -U \\ A^{-1}C & A^{-1}B \end{pmatrix} \begin{pmatrix} \delta \\ \partial \end{pmatrix} = 0 \quad . \tag{82}$$

The coefficient of the cubic term in the characteristic polynomial of (82) is the trace of the coefficient matrix, which is here the trace of $A^{-1}B$; since

$$A^{-1} = \begin{pmatrix} 1 & 0 \\ -s^* & 1 \end{pmatrix} \quad , \quad B = \frac{I_1 + I_3 - I_2}{I_1} \begin{pmatrix} \frac{s^*}{c^*} & \frac{1}{c^*} \\ -\frac{I_1}{I_3} c^* & 0 \end{pmatrix} \quad , \tag{83}$$

we conclude that this trace vanishes. Moreover, this coefficient is the sum of the four roots of the characteristic polynomial, and since it vanishes, not all roots can have a negative real part (the sum of the real parts must vanish) and the solutions cannot be stable.

XI. CONCLUSIONS

The discussions and results of Sections IX and X indicate that the conclusion concerning the instability of a Skylab-type satellite caused by aerodynamic torque is rather generally valid; neither the specific values of the coefficient in the Fourier expansions of the aerodynamic torque nor the specific value of the equilibrium angle ϑ_2^* enter into the final conclusion. Of course, the form of the expansions (i.e., the set of the nonvanishing coefficients) and the form of the equilibrium solution do influence the final result. Furthermore, it is a general experience that perturbations of the considered "pure" problem, such as asymmetries of the satellite, a noncircular orbit, the basic nonlinearity of the problem, etc., will increase the instability. Thus, we can state with reasonable justification that a satellite of Skylab's type is not stable under the influence of high altitude aerodynamic torques.

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APPROVAL

EFFECT OF GRAVITATIONAL AND AERODYNAMIC TORQUES ON A RIGID SKYLAB-TYPE SATELLITE

By Hans J. Sperling

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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