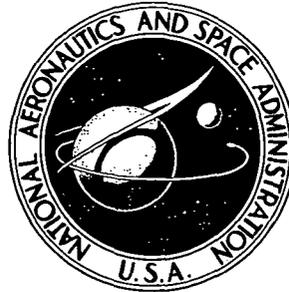


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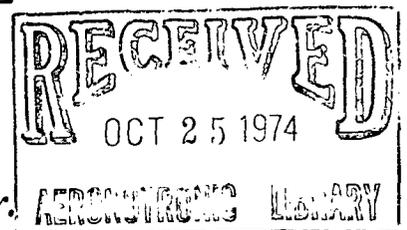
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A STREAMLINE CURVATURE METHOD  
FOR DESIGN OF SUPERCRITICAL  
AND SUBCRITICAL AIRFOILS

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# A STREAMLINE CURVATURE METHOD FOR DESIGN OF SUPERCRITICAL AND SUBCRITICAL AIRFOILS

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## SUMMARY

An airfoil design procedure, applicable to both subcritical and supercritical airfoils, is described. The method is based on the streamline curvature velocity equation. Several examples illustrating this method are presented and discussed.

## INTRODUCTION

A number of airfoil design methods are currently in existence. Most of these are confined within the framework of incompressible flow analysis. (See refs. 1 to 3 and the references therein.) Interest in supercritical airfoil sections has led to the development of compressible flow design procedures, such as those described in references 4 and 5.

The method described in this paper is also a compressible flow method but differs in some essential respects from the latter two analyses. First, it designs the airfoil from a prescribed pressure distribution rather than from a set of parameters. This feature enables one to design airfoils which are relatively insensitive to variations from the design conditions, since some types of pressure distributions have less tendency than others to develop shocks with small changes in Mach number or angle of attack. This method also permits the designer to work directly with the pressure distribution to alter performance parameters such as pitching moment.

Another advantage of the present method is that the calculations are performed in the physical plane rather than in the hodograph plane, and the pressure variations are related directly to the airfoil geometry. Thus, the designer has a physically intuitive insight into the nature of the variations at each stage in the design process.

Application of the method requires an initial airfoil with a pressure distribution that roughly approximates the desired one. The pressure distribution obtained is generally not exactly the one specified but is a significantly closer approximation than that of the unmodified airfoil.

The method described is implemented through a series of computer programs which share the common feature of being based on the analysis program of reference 6. This analysis program is used in successive modifications of the airfoil contour until the desired result is obtained.

There is no guarantee that, at some given angle of attack and Mach number, an arbitrary modification in the pressure distribution on the airfoil can be obtained by any practical modification of the contour. Only extensive use will reveal the actual limitations of the method.

### SYMBOLS

a	streamline curvature
C	empirical parameter
c	airfoil chord length
$c_m$	airfoil pitching-moment coefficient about the quarter-chord point
$c_p$	airfoil pressure coefficient
k	constant
M	Mach number
n	distance in direction normal to streamlines
P	point on airfoil surface
q	local velocity
R	local radius of curvature
t	airfoil thickness
x,y	airfoil coordinates

## Subscripts:

- i            ith point
- s            conditions at airfoil surface
- $\infty$         conditions at infinity

## ANALYSIS

### Basic Considerations

The design procedure requires as a starting point an airfoil of known contour. Convergence is improved if the characteristics of this airfoil approximate the desired characteristics. The pressure distribution of the initial airfoil need not be known with accuracy inasmuch as it is computed in the first phase of the design process. Aerodynamic characteristics of the new airfoil design are controlled by the specification of a desired pressure distribution which may display a considerable degree of arbitrariness. All the practical airfoil problems encountered thus far in applying this method have been in tailoring the pressure distribution of a given airfoil for which the performance had been somewhat unsatisfactory. Some examples of specified alterations are

- (1) Altering the inviscid upper surface shock structure at a supercritical Mach number to decrease the adverse shock effects
- (2) Increasing the upper surface suction near the nose and decreasing it at the crest to obtain a higher drag divergence Mach number at low angles of attack
- (3) Decreasing the upper surface velocity near the nose to obtain better performance at high angles of attack
- (4) Shifting the loading from the aft region toward the middle to decrease the magnitude of the pitching moment

### Calculation Procedure

The design calculation proceeds according to the following steps:

- (1) Compute the pressure distribution of the initial airfoil by the method of reference 6.
- (2) Compute the difference between this pressure distribution and the desired distribution.

(3) Compute an airfoil variation, based on this difference, with a pressure distribution closer to that desired.

(4) Iterate: Replace the initial airfoil with the variation obtained in step (3).

This process is terminated by prescribing the number of iterations. It is not practical to prescribe a maximum error, inasmuch as it is not known initially how closely the prescribed distribution can be approximated.

### Calculation of Airfoil Change

Step (3) in the preceding section requires a mathematical expression relating a change in the airfoil contour to a change in the pressure distribution. This expression can be a perturbation type of expression, inasmuch as it represents a relation between variations or increments. Furthermore, it need not be extremely accurate because the iteration process will generally improve the approximation.

To obtain such an expression, one may start with the momentum equation in the streamline curvature form. This equation gives, for the velocity distribution along a streamline normal,

$$\frac{dq}{q} = -a(n) \, dn \quad (1)$$

In equation (1) the curvature  $a$  of a streamline is simply the reciprocal of its local radius of curvature. Consider a point on the airfoil surface where the local curvature is  $a_s$ . The streamlines transversed by a normal emanating from this point have curvature  $a_s$  at the surface and curvature approaching zero as  $n$  approaches infinity (in subsonic flow). This curvature distribution is arbitrarily assumed to have a negative exponential form

$$a(n) = a_s e^{-kn} \quad (2)$$

for some  $k$  which is constant for a given normal but may vary from one normal to another.

Such a variation is certainly a reasonable approximation for those normals that emanate from points near the crest of the airfoil. At the surface where  $n = 0$ ,  $a = a_s$  and the curvature of the streamlines rapidly decreases as  $n$  increases. On the other hand, the curvature decrease along normals emanating, say near the leading edge, is clearly not exponential, since the curvature distribution along these normals changes sign. One is motivated, however, to explore the possibility of applying equation (2) over the entire airfoil for several reasons:

(1) Since only the values at the surface are required, the importance of the incorrect curvature variation away from the surface is minimized.

(2) As a result of assuming equation (2), a relation between the surface geometry and the surface velocity can be obtained without a calculation of the entire flow field.

(3) The parameter  $k$ , which may vary along the surface, can be varied to provide some compensation for the error involved in the assumed form of the curvature variation.

Therefore, proceeding to explore the use of equation (2), one substitutes equation (2) into equation (1) and integrates the resulting equation outward along a normal. Thus,

$$\int_s^\infty \frac{dq}{q} = -a_s \int_0^\infty e^{-kn} dn$$

which results in the relation

$$\ln \frac{q_s}{q_\infty} = \frac{a_s}{k} \quad (3)$$

Now, if one postulates a small local variation in surface curvature  $\Delta a_s$ , the resulting change in  $q_s$  is obtained from equation (3) as

$$\frac{\Delta q_s}{q_s} = \frac{\Delta a_s}{k} \quad (4)$$

If the parameter  $k$  in equation (4) is obtained from equation (3), the result is

$$\frac{\Delta q_s}{q_s} = \frac{\Delta a_s \ln \frac{q_s}{q_\infty}}{a_s}$$

or, if the change in velocity  $\Delta q_s$  is specified, the corresponding increment in surface curvature is

$$\Delta a_s = \left( \frac{a_s}{\ln \frac{q_s}{q_\infty}} \right) \frac{\Delta q_s}{q_s} \quad (5)$$

It is necessary to examine this expression in some detail. Near the crest of the airfoil where  $q_s > q_\infty$ , the logarithm varies gradually, and the expression gives a near proportion between  $\Delta a_s/a_s$  and  $\Delta q_s/q_s$ . On the other hand, where  $q_s = q_\infty$  (that is, where the pressure coefficient is zero) the logarithm is zero, and the expression becomes unusable. This problem arises because of the error involved in the assumption of equation (2). However, there is no physical reason why the flow at the surface should be qualitatively different at a point where  $c_p = 0$  than at any other point. Therefore, the possibility exists, as suggested earlier, of altering the parameter  $k$  to compensate for the error involved in assuming the exponential variation in equation (2). The most obvious possibility is to set

$$k = Ca_s$$

which will give a simple proportion between  $\Delta a_s/a_s$  and  $\Delta q_s/q_s$  everywhere on the airfoil. Equation (5) is replaced with an equation of the form

$$\Delta a_s = Ca_s \frac{\Delta q_s}{q_s} \quad (6)$$

which gives a variation approximately similar to equation (5) where  $q_s > q_\infty$  and a much more reasonable variation where  $q_s \leq q_\infty$ . Thus  $C$  is an empirical parameter which is, in general, a function of local Mach number or of local curvature. In actual practice it has been found to be sufficient to let  $C$  vary with free-stream Mach number. A value that is often used initially is

$$C \approx 10(1 - M_\infty^2)$$

If the step size on each iteration then appears to be too large or too small,  $C$  is adjusted accordingly. Normally, no more than one such adjustment is required.

### Airfoil Profile Modification

The curvature distribution of the modified airfoil is known from the curvature distribution of the original airfoil plus the change in curvature given by equation (6). The profile coordinates can then be computed by using simple geometric concepts, such as the fact that a circle is determined by any three noncollinear points on its circumference or by its radius and two points on the circumference.

If the modification is to the upper surface, the new contour is constructed as follows: If  $P_i$  denotes the first point of the prescribed region of change, then a circle is determined which contains  $P_{i-1}$  and  $P_i$  on the airfoil and has radius  $R_i = \frac{1}{a_i}$ .

(See fig. 1(a).) At the next specified x-location, the y-coordinate of a point on this circle is found. This x,y pair represents the coordinates on the new contour which now has the desired curvature at  $P_i$ . The next point  $P_{i+2}$  is now constructed so that the contour will have the prescribed curvature at  $P_{i+1}$ . This procedure is continued, with the use of the new curvature distribution, to the end of the region of change. Then it is continued to the trailing edge, with the use of the curvature distribution of the original airfoil in this region.

The trailing-edge point of the new contour computed in this manner will usually be above or below the trailing edge of the original airfoil. The resulting gap is closed with a linear adjustment starting at the leading edge. (See fig. 1(b).) Since this adjustment alters the ordinates linearly, it changes the slopes by a constant and leaves the curvature distribution unchanged between the leading edge and the trailing edge. This adjustment changes the pressure distribution somewhat outside of the prescribed region of change and results in a generally small distributed error there.

If the modification is prescribed for the lower surface instead of the upper surface, a similar procedure is followed. However, since the calculation proceeds from the trailing edge to the leading edge along the lower surface, the gap must be closed at the leading edge instead of the trailing edge.

The examples shown in figures 2, 3, and 4 were computed by use of the method just discussed. However, an alternative method of handling the discontinuity at the end of the region for which changes in the local radius of curvature are specified was used in some cases. The gap in the coordinates at this point was closed by a linear transformation pivoted at the first point of the region. This method left discontinuities in the slope at the end points of the region of change, but these were eliminated by a simple fairing algorithm. This alternative method was used to generate the final curve of figure 5, and it can be seen that no discontinuities appear at the end points of the region of change, 5.3 percent and 92.6 percent of the profile chord.

### Boundary-Layer Considerations

Although the procedure described is an inviscid calculation, some allowance for the boundary layer can be made. One way of doing this would be to perform the calculation for a composite profile consisting of the basic airfoil plus its boundary-layer displacement thickness. In the design process, the trailing-edge thickness would be adjusted to just equal the original boundary-layer displacement thickness. After the completion of the design, the original boundary-layer displacement thickness would be subtracted to give the actual airfoil ordinates. For this approximation, the boundary layers on the original and modified airfoils are assumed to be essentially the same.

An alternate procedure would be as follows:

- (1) Calculate both the viscous and inviscid pressure distributions for both airfoils.
- (2) Specify the desired viscous pressure distribution and obtain the difference between the original and desired viscous distributions.
- (3) Specify the desired inviscid distribution as the original inviscid pressure plus the difference obtained in step (2).
- (4) Obtain the resulting design and compute its viscous pressure distribution. If the viscous pressure distribution on the new airfoil is not sufficiently close to that desired, the process can be iterated.

A third possibility, of course, would be to replace the inviscid analysis phase of the iteration process with a calculation that includes the boundary layer.

### EXAMPLES

The first example, shown in figure 2, illustrates a modification to the upper surface of a supercritical airfoil, intended to decrease the leading-edge suction peak and to increase the suction somewhat aft of the peak. It is seen that, with two iterations, the desired distribution is obtained approximately. This type of change results in a somewhat thicker airfoil.

The second example (fig. 3) represents a more practical problem. It shows a modification to the lower surface designed to decrease the pitching moment without decreasing the lift. The thinning of the airfoil is a result of increasing the loading in the middle portion of the lower surface. Such a change could be combined with that of the first example to compensate for the thickening tendency of that type of modification. For this particular modification, the prescribed change was almost exactly obtained.

The third example (fig. 4) shows a different type of modification to the original airfoil used in the first example. This example has two complicating factors. First, the airfoil is designed for performance at an angle of attack of  $2^\circ$ . Second, the pressure distribution of the original airfoil contains a fairly strong shock wave within the prescribed region of change. After three iterations, the pressure distribution obtained still deviates somewhat from the desired distribution, especially in the vicinity of the leading-edge suction peak; but the extreme velocities have been reduced, the shock ceases to be a problem, and the desired distribution is closely approximated over most of the airfoil.

In this example, the thickening of the airfoil is significant. As a matter of fact, if the iteration process were continued to reduce the discrepancy near the leading edge further, an even thicker airfoil would result. Not all significant modifications in the pressure

distribution result in significant thickness changes, however, as is demonstrated in the next example.

The fourth example, illustrated in figure 5, involves a large region of supersonic flow on the upper surface. The problem was to replace a pressure distribution having two moderately strong shock waves with one having a single weak wave, and the desired pressure distribution was nearly obtained after six iterations, as shown in the figure. No attempt was made to retain the original lift coefficient, and the lift coefficient of the modified profile is slightly lower. As mentioned previously, the change in thickness ratio was insignificant.

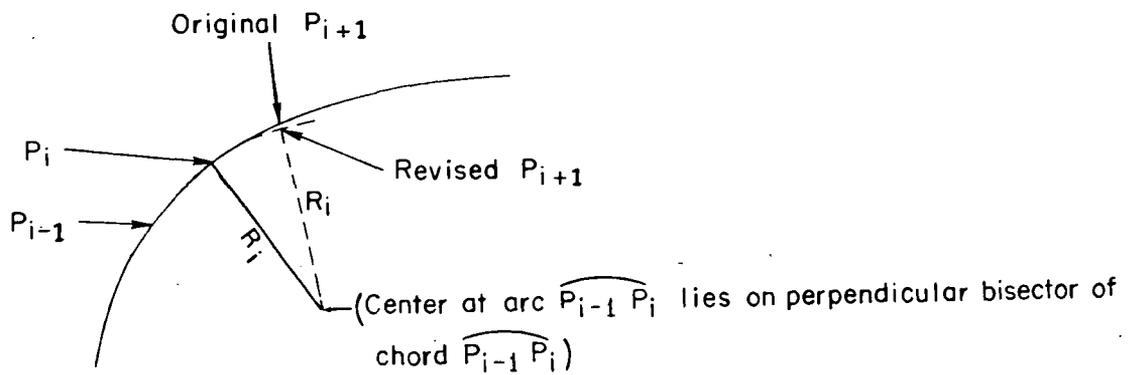
### CONCLUDING REMARKS

An airfoil design procedure applicable to both subcritical and supercritical airfoils has been described. The method is based on the streamline curvature velocity equation. This method does not include the boundary-layer calculation directly, but suggestions for applying the method to the viscous case were given. Several examples illustrating this method were presented and discussed.

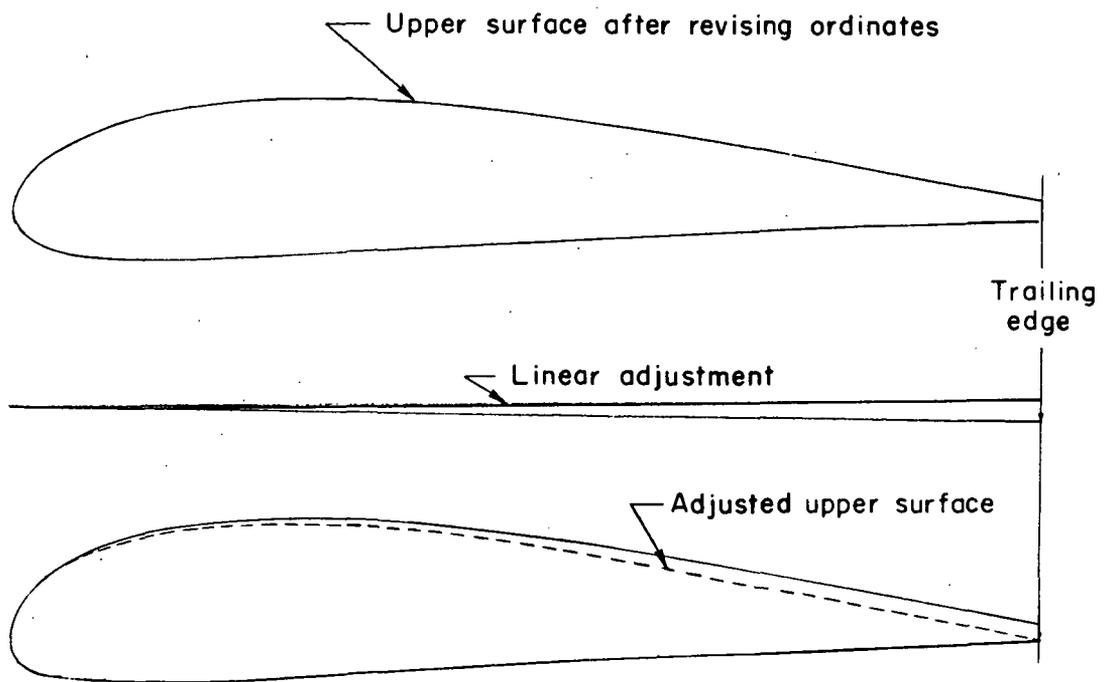
Langley Research Center,  
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(a) Method of defining revised ordinates.



(b) Method of correcting trailing-edge closure.

Figure 1.- Illustration of methods used in the definition of redesigned airfoil sections.

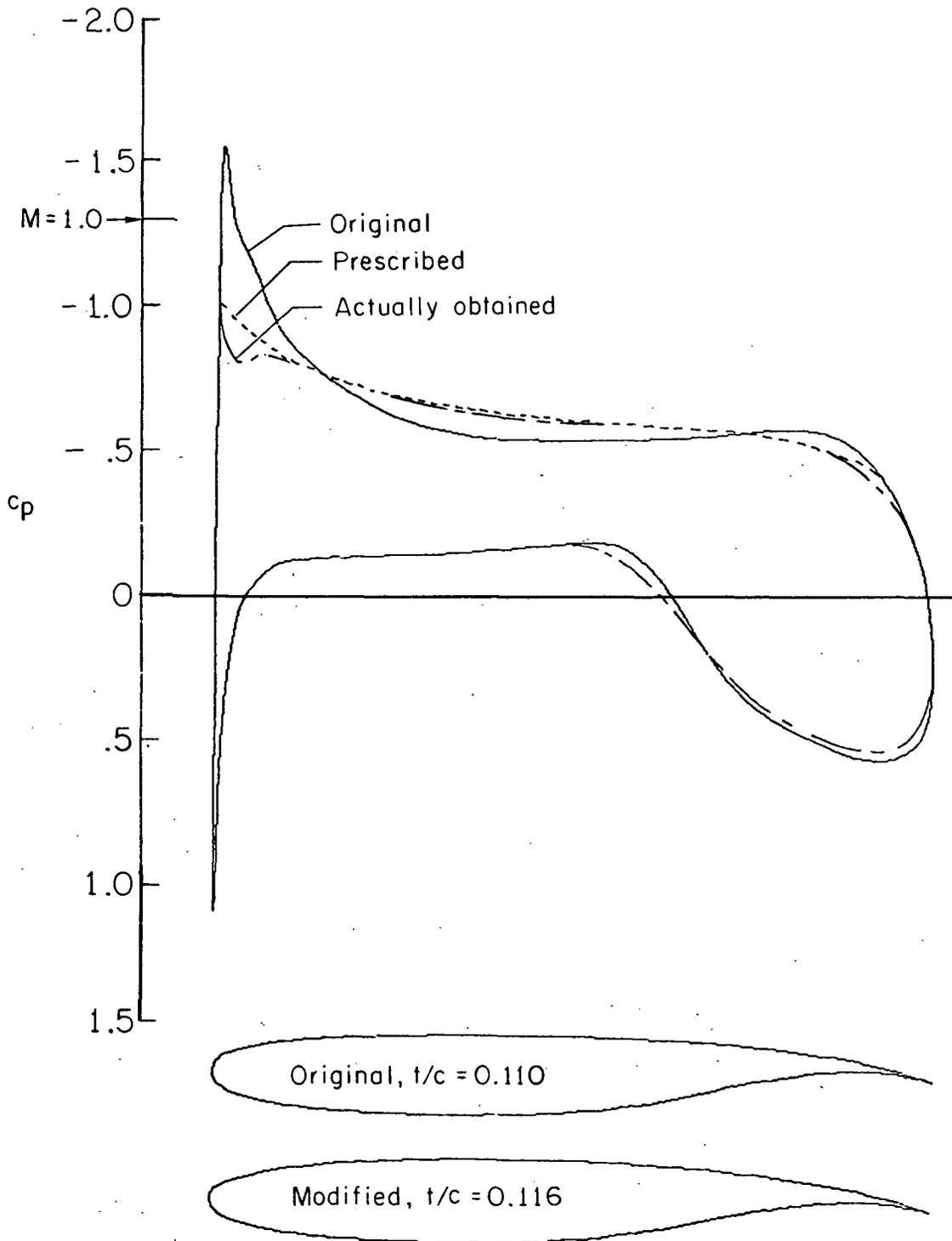


Figure 2.- Compressible flow design method applied to a supercritical airfoil at zero angle of attack.  $M_\infty = 0.6$ ; two iterations.

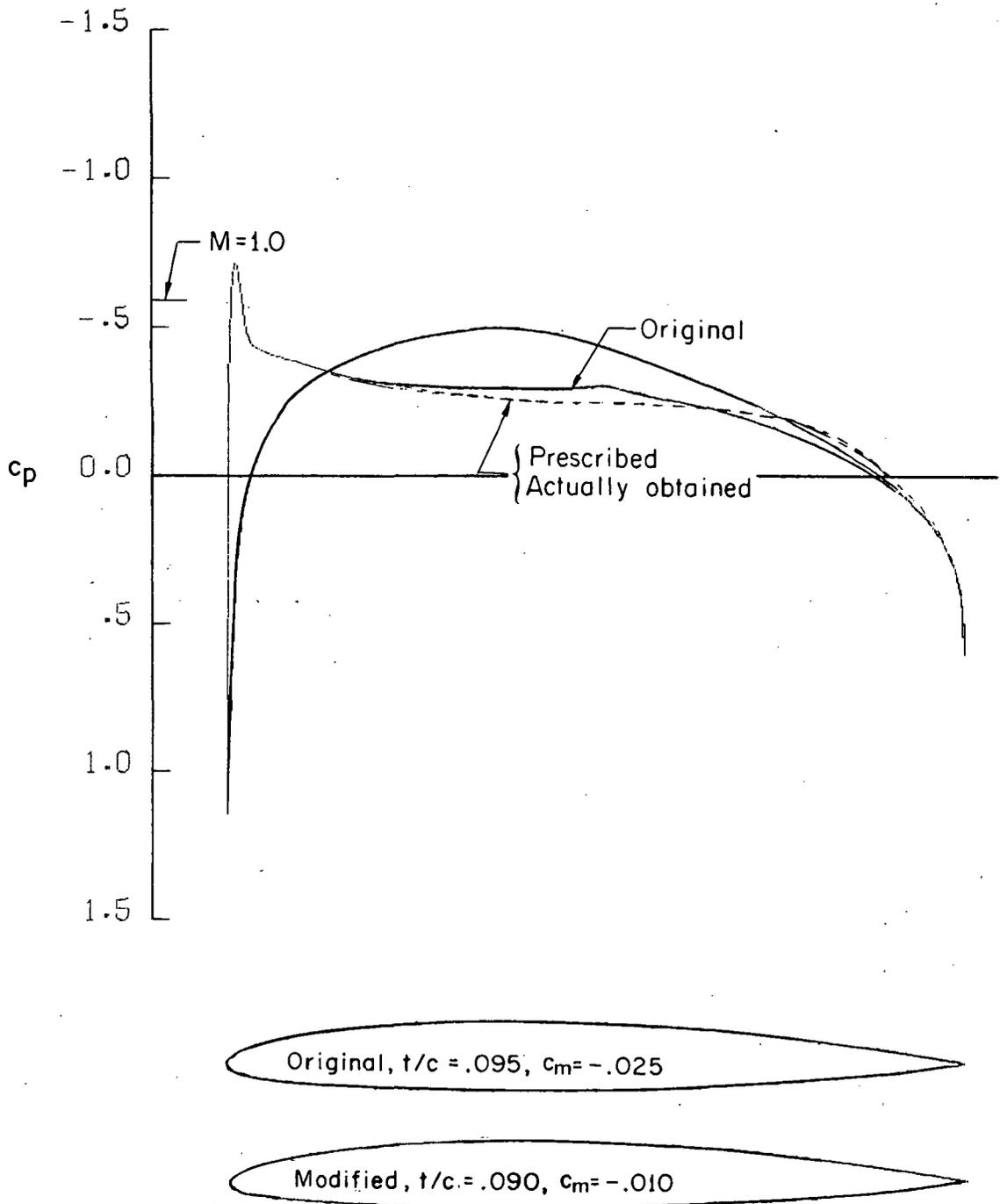


Figure 3.- Modification to lower surface designed to decrease pitching moment without decreasing lift.  $M_\infty = 0.75$ ; angle of attack =  $0.8$ ; five iterations.

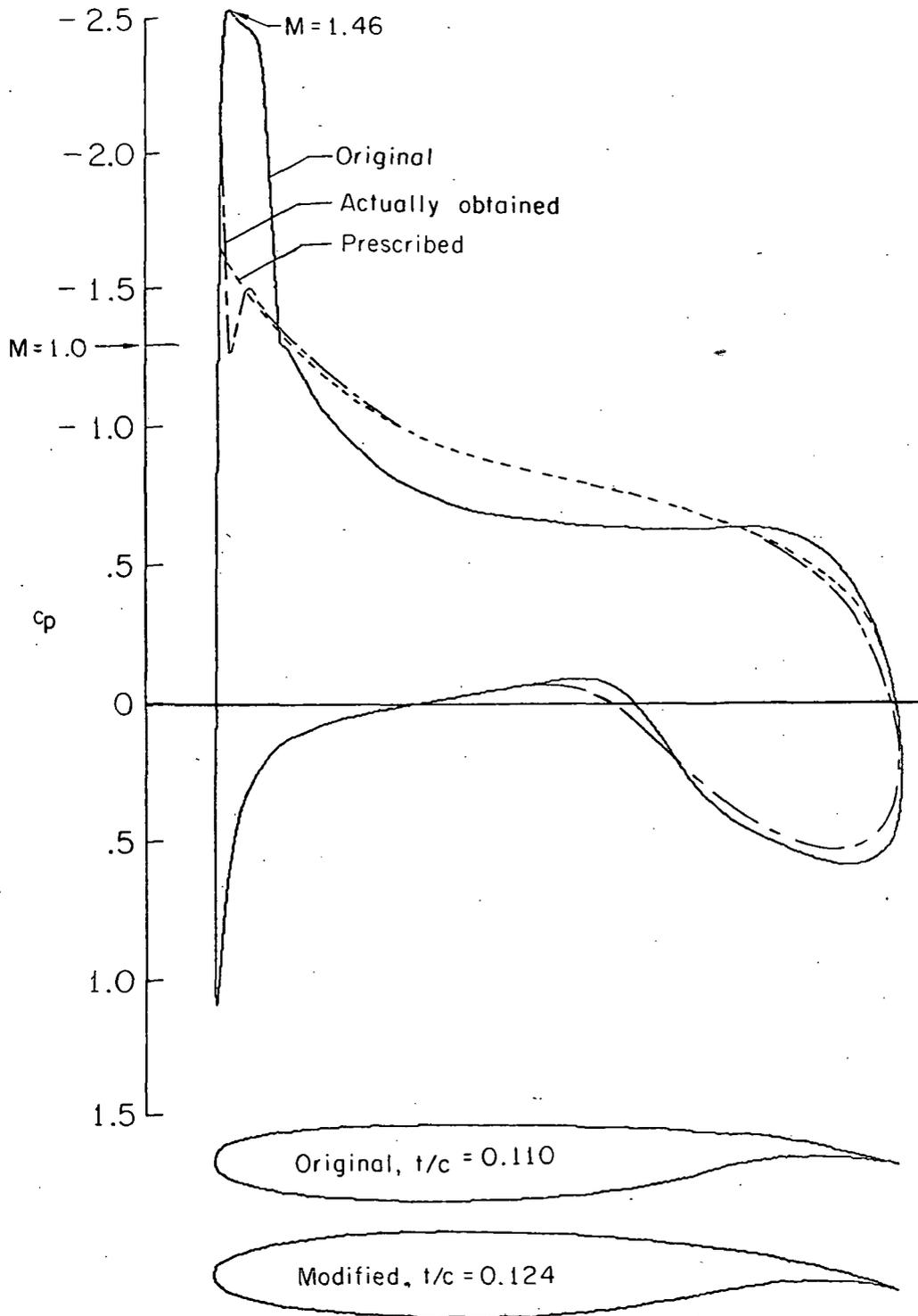


Figure 4.- Compressible flow design method applied to a supercritical airfoil at  $2^\circ$  angle of attack.  $M_\infty = 0.6$ ; three iterations.

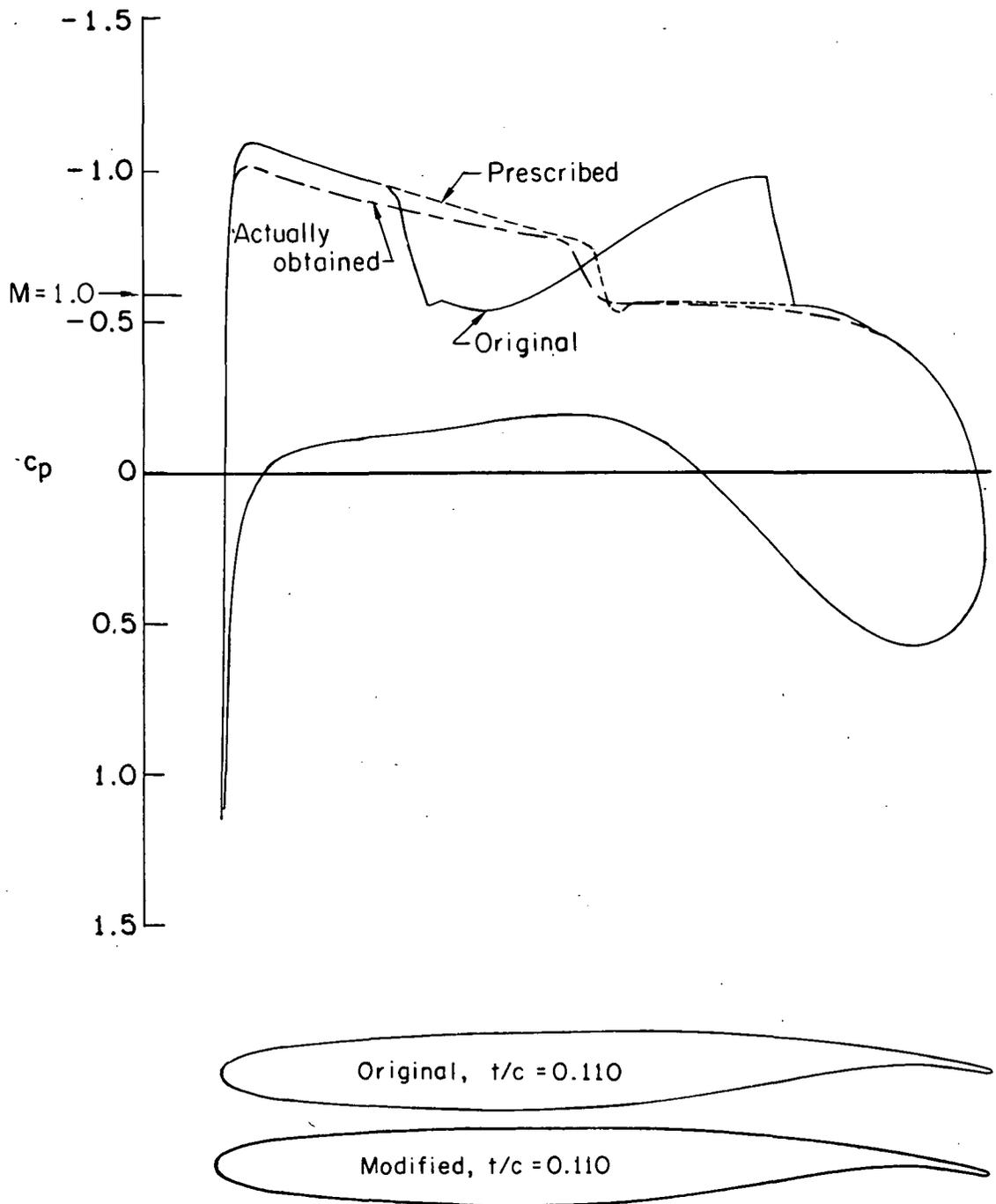


Figure 5.- Compressible flow design method applied to a supercritical airfoil at zero angle of attack.  $M_\infty = 0.75$ ; six iterations.



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