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ROBUST DESIGN OF DYNAMIC OBSERVERS

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16. ABSTRACT

The two (identity) observer realizations $\dot{z} = Mz + Ky$ and $\dot{z} = \tilde{A}z + \tilde{K}(y - \tilde{C}z)$, respectively called the open loop and closed loop realizations, for the linear system $\dot{x} = Ax$, $y = Cx$ are analyzed with respect to the requirement of "robustness"; i.e., the requirement that the observer continue to regulate the error $x - z$ satisfactorily despite small variations in the observer parameters from the projected design values. The results show that the open loop realization is never "robust," that robustness requires a closed loop implementation, and that the closed loop realization is "robust" with respect to small perturbations in the gains $\tilde{K}$ if and only if the observer can be built to contain an exact replica of the unstable and underdamped dynamics of the system being observed. These results clarify the stringent accuracy requirements on both models and hardware that must be met before an observer can be considered for use in a control system.

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ROBUST DESIGN OF DYNAMIC OBSERVERS

I. INTRODUCTION

For the linear system

$$\dot{x}(t) = A\,x(t)$$
$$y(t) = C\,x(t) \quad , \quad x(0) = x_0 \tag{1}$$

the most common form of (identity) observer encountered in the literature (see Reference 1 and the references cited therein) is the system

$$\dot{z}(t) = M\,z(t) + K\,y(t) \quad , \quad z(0) = z_0 \qquad , \tag{2}$$

where

$$M = A - K_0\,C \qquad , \tag{3}$$

$$K = K_0 \qquad ; \tag{4}$$

and K_0 is chosen so that $A - K_0\,C$ is stable and the error $e(t) = x(t) - z(t)$ which is governed, subject to (3) and (4), by

$$\dot{e}(t) = (A - K_0\,C)\,e(t)$$

converges to zero at an acceptable rate. Thus, if (A,C) is an observable pair, the observer error $e(t)$ can be made to converge, subject to (3) and (4), with an arbitrary set of characteristic exponents, independent of the dynamics of the system (1).

If attention is focused on the observer as a device that is ultimately to be built with physical components (as opposed to idealized mathematical elements whose parameters can be set exactly), a problem of immediate interest is: what is the behavior of the observation error if the observer system matrix M and gain matrix K fail to satisfy equations (3) and (4) precisely and under what conditions can satisfactory convergence of the error be guaranteed under small parameter perturbations?

The above question is answered in this paper and it is shown that almost any perturbation of equations (3) and (4) will cause at least the observable modes of the system (1) to appear in the error response. Hence, if (1) contains unstable or underdamped modes (and these modes must be observable for satisfactory observer action), the error response of the observer will fail to converge or will decay slowly unless the observer parameters are adjusted to satisfy (3) and (4) with infinite accuracy. It is then shown that the acute sensitivity of the error to perturbations in the gain matrix K can disappear only if the observer is realized in the closed loop form (15); however, in this case, the action of the observer maps $\tilde{A}$ and $\tilde{C}$ are required to be identical to those of A and C at least on the eigenspace of unstable modes of A.

The results of the paper prove that "robust" observers cannot be constructed unless (a) the observer is built as a closed loop device, and (b) an accurate model of the unstable system dynamics is available and can be precisely duplicated. The treatment in this paper is restricted to (identity) full state observers; however, an immediate consequence of the results presented here is that the reduced order observers that have been reported in the literature (see Reference 1 and the references cited therein) fail to be "robust." The problem of designing "robust" low order observers is of great practical importance, and it is hoped that this paper will stimulate research into this area of observer theory.

II. MAIN RESULTS

Let $\mathcal{X}$, $\mathcal{Y}$ with dimension $\mathcal{X} = n$ and dimension $\mathcal{Y} = m$ denote real linear vector spaces associated with (1) and (2) with $A : \mathcal{X} \to \mathcal{X}$, $C : \mathcal{X} \to \mathcal{Y}$, $M : \mathcal{X} \to \mathcal{X}$ and $K : \mathcal{Y} \to \mathcal{X}$ being the corresponding linear maps. A, C, etc., will also denote matrix representations of these maps. The kernel (null space) of C will be denoted by ker C, and it is assumed that rank $C = m$. The minimal polynomial (m.p.) of A will be written $\alpha_A(\lambda) = \alpha_A^+(\lambda)\,\alpha_A^-(\lambda)$ where the zeroes of $\alpha_A^+(\lambda)$ $(\alpha_A^-(\lambda))$ lie in the closed right half (open left half) $\mathbb{C}^+(\mathbb{C}^-)$ of the complex plane. $\sigma(A)$ will denote the spectrum of A and A is stable if $\sigma(A) \subset \mathbb{C}^-$. $\mathcal{X}^+(A) = \ker \alpha_A^+(A)$ will denote the unstable eigenspace of A, and to highlight our results it is assumed (tacitly) that $\mathcal{X}^+(A) \neq 0$; i.e., A is not stable. If T and $\hat{T} = T + \delta T$ are real $p \times q$ matrices, $\hat{T}$ is the perturbed value and δT is a perturbation of T. For a given real number $\epsilon > 0$, the class of arbitrary ϵ perturbations of T, denoted by $\Omega_T(\epsilon)$, is defined to be the class of real $p \times q$ matrices $\{\delta T\}$, where each δT is constrained only by the requirement that the absolute value of each element be bounded by ϵ. A class of matrices $\{\delta T\}$ is defined to be a class of arbitrary "small" perturbations of T if $\{\delta T\} = \Omega_T(\epsilon)$ for some $\epsilon > 0$. In the sequel we use the well known fact [2-4] that if T is a real $n \times n$ stable matrix, there exists $\epsilon > 0$ such that $T + \delta T$ is stable for every δT in $\Omega_T(\epsilon)$.

Observer action requires that

$$\lim_{t\to\infty} e(t) \equiv \lim_{t\to\infty} \big(x(t) - z(t)\big) = 0 \;\forall\; x_0 \in \mathcal{X},\, z_0 \in \mathcal{X} \quad . \tag{5}$$

A necessary condition for (5) to hold is that there exist K_0, such that $A - K_0 C$ is stable; it is assumed that such a choice of K_0 exists and has been made. To consider the effect of observer parameter perturbations on (5), assume that (3) and (4) are not necessarily satisfied and define

$$\delta M \equiv M - (A - K_0C) \tag{6}$$

$$\delta K \equiv K - K_0 \quad . \tag{7}$$

Since M and K in the realization (2) represent physically distinct entities, we consider only independent classes of perturbations in M and K; thus, for example, the classes of perturbations we consider must include the possibility that $\underline{\delta K = 0 \text{ when } \delta M \neq 0}$ and vice versa.*

Theorem 1. With the observer configuration (2), the necessary conditions for observer action (5) to be preserved under independent perturbations δM and δK are that these perturbations satisfy

$$\mathcal{X}^{+}(A) \subset \ker \delta M \tag{8}$$

and

$$\mathcal{X}^{+}(A) \subset \ker \delta KC \quad . \tag{9}$$

If (8) and (9) hold, there exists $\epsilon > 0$, such that (5) holds for every δM in $\Omega_M(\epsilon)$ that satisfies (8).

Proof. From (1), (2), (6), and (7)

$$\dot{e}(t) = (A - K_0C + \delta M)\, e(t) - \delta M\, x(t) - \delta KC\, x(t) \quad , \tag{10}$$

and therefore (5) requires, since δM and δK are independent, that

* A more precise definition of independence of the classes $\{\delta M\}$ $\{\delta K\}$ is not made because this would entail detailed consideration of a (perhaps stochastic) model for generating these perturbations. For the purposes of the paper the operational statement (underlined) above suffices.

$$\lim_{t\to\infty} \delta M\, x(t) = 0 \;\; \forall\; x_0 \in X \tag{11}$$

and

$$\lim_{t\to\infty} \delta K\, C\, x(t) = 0 \;\; \forall\; x_0 \in X \quad , \tag{12}$$

which is equivalent [5] to (8) and (9). When (11) and (12) hold, (5) is true if and only if $(A - K_0 C + \delta M)$ is stable. Therefore, the last statement of the theorem follows from the fact that $A - K_0 C$ is stable. ■

To state the next result let the matrices M, $A - K_0C$, δM and K, K_0, δK be parameterized in $\mathcal{R}^{n^2}$ and $\mathcal{R}^{nm}$ respectively by the points $m = (M)$, $m_0 = (A - K_0C)$, $\delta m = (\delta M)$ and $k = (K)$, $K_0 = (K_0)$, $\delta k = (\delta K)$.

Theorem 2. Under the conditions of Theorem 1, (5) fails, if A is not stable, for almost every value of M and K that differ respectively from $A - K_0C$ and K_0; i.e., (5) fails for every point $m(k)$ in $\mathcal{R}^{n^2}$ $(\mathcal{R}^{nm})$ except $m = m_0$ $(k = k_0)$ and possibly for values of m (k) lying on a proper algebraic variety in $\mathcal{R}^{n^2}$ $(\mathcal{R}^{nm})$. ■

Corollary. If $\sigma(A) \subset \mathbb{C}^+$, then (5) fails, under the conditions of Theorem 1, for every independent perturbation of equations (3) and (4).

Proof of Theorem 2. Since A is not stable, $X^+(A) \neq 0$, and since $A - K_0C$ is stable, it follows from detectability of the pair (A,C) that $C\,X^+(A) \neq 0$. Hence, the relations

$$\delta M\, X^+(A) = 0 \tag{13}$$

$$\delta K\, C\, X^+(A) = 0 \tag{14}$$

determine <u>proper</u> subspaces S and Q of $\mathcal{R}^{n^2}$ and $\mathcal{R}^{nm}$ with the property that (13) and (14) are satisfied if and only if δm and δk lie respectively in S and Q. Thus (8) and (9), and hence by Theorem 1, (5) fail unless m lies on the proper variety $m_0 + S$ and k lies on the proper variety $k_0 + Q$. ■

<u>Proof of the Corollary</u>. If $\sigma(A) \subset \mathbb{C}^+$, $X^+(A) = X$ and since rank $C = m$, $CX = Y$. Hence, (13) and (14) imply that $\delta M = 0$ and $\delta K = 0$; i.e., the subspaces S and Q shrink to zero dimensional subspaces. ■

By Theorem 2 almost any difference between the observer gain matrix K and the matrix K_0 that enters equation (3) will cause the observer action to fail. This difficulty clearly cannot possibly disappear unless the same physical elements that realize K also <u>actually realize</u> K_0. This can only be accomplished if the observer is realized as

$$\dot{z}(t) = \widetilde{A}\, z(t) + \widetilde{K}\left(y(t) - \widetilde{C}\, z(t)\right) \qquad , \tag{15}$$

where

$$\widetilde{A} = A \qquad , \tag{16}$$

$$\widetilde{K} = K_0 \qquad , \tag{17}$$

$$\widetilde{C} = C \qquad , \tag{18}$$

and as before $A - K_0C$ is stable. Although (15) through (18) are mathematically equivalent to (2) through (4), construction of the observer according to (15) through (18) involves a feedback implementation as opposed to the open loop implementation of (2); also (15) involves reproduction of the possibly unstable matrix A, whereas in (2) M is always stable. In the remainder of the paper we analyze the observer (15) with respect to perturbations in (16) through (18). Assume, therefore, that (16) through (18) are not necessarily satisfied and let

$$\delta A = \widetilde{A} - A \tag{19}$$

$$\delta\widetilde{K} = \widetilde{K} - K_0 \tag{20}$$

$$\delta C = \widetilde{C} - C \quad . \tag{21}$$

As before, only independent perturbations in $\widetilde{A}, \widetilde{K}$ and $\widetilde{C}$ are considered.

Theorem 3. With the observer configuration (15) the necessary conditions for observer action (5) to be preserved under independent perturbations $\delta\widetilde{A}$, $\delta\widetilde{K}$, and $\delta\widetilde{C}$ with $\delta\widetilde{K}$ arbitrary are that

$$X^+(A) \subset \ker \delta A \tag{22}$$

and

$$X^+(A) \subset \ker \delta C \quad . \tag{23}$$

Subject to (22) and (23), observer action is preserved under arbitrary "small" perturbations in $\delta\widetilde{K}$ and admissible [i.e., satisfying (22) and (23)] "small" perturbations δA and δC; i.e., there exists $\epsilon > 0$ such that (5) holds for all matrices $\delta\widetilde{K}$ in $\Omega_{\widetilde{K}}(\epsilon)$ and all matrices δA and δC in $\Omega_A(\epsilon)$ and $\Omega_C(\epsilon)$ that satisfy (22) and (23).

Corollary. If $\sigma(A) \subset \mathbb{C}^+$, observer action is preserved, under the conditions of the theorem, for a class of arbitrary "small" perturbations $\delta\widetilde{K}$, if and only if

$$\widetilde{A} = A \tag{24}$$

$$\widetilde{C} = C \quad . \tag{25}$$

Proof of Theorem 3. From (1) and (15) through (21)

$$\dot{e}(t) = (A - K_0 C + \delta\Delta)\, e(t) - \delta A\, x(t) + K_0\, \delta C\, x(t) + \delta\widetilde{K}\, \delta C\, x(t) \quad , \tag{26}$$

where

$$\delta\Delta = \delta A - K_0\, \delta C - \delta\widetilde{K} C - \delta\widetilde{K}\, \delta C \quad . \tag{27}$$

Convergence of $e(t)$ requires, since the perturbations are to be independent, that

$$\lim_{t\to\infty} \delta A\, x(t) = 0 \quad \forall\, x_0 \in X \quad , \tag{28}$$

$$\lim_{t\to\infty} K_0\, \delta C\, x(t) = 0 \quad \forall\, x_0 \in X \quad , \tag{29}$$

and

$$\lim_{t\to\infty} \delta\widetilde{K}\, \delta C\, x(t) = 0 \quad \forall\, x_0 \in X \quad . \tag{30}$$

Equation (30) holds for arbitrary $\delta\widetilde{K}$ if and only if

$$\lim_{t\to\infty} \delta C\, x(t) = 0 \quad \forall\, x_0 \in X \quad . \tag{31}$$

Equation (31) implies that (29) and (28) through (30) are now equivalent to (22) and (23).

Now, assuming that (22) and (23) are true, it follows from (26) and (28) through (30) that $e(t)$ converges if and only if $A - K_0\, C + \delta\Delta$ is stable. Since $A - K_0\, C$ is stable, there exists $\epsilon^* > 0$ such that for every $\delta\Delta$ in $\Omega_{\Delta}(\epsilon^*)$, $(A - K_0 C + \delta\Delta)$ is stable. From (27) it follows that there exists ϵ (depending on ϵ^*) such that for each δA in $\Omega_A(\epsilon)$, δC in $\Omega_C(\epsilon)$, and $\delta\widetilde{K}$ in $\Omega_{\widetilde{K}}(\epsilon)$, the corresponding $\delta\Delta$ is in $\Omega_\Delta(\epsilon^*)$. ■

The proof of the corollary is identical to the proof of the corollary to Theorem 2.

Conditions (22) and (23) imply that the observer maps $\widetilde{A}, \widetilde{C}$ must be such that for each $x \in X^+(A)$,

$$\widetilde{A}\, x = A\, x$$

and

$$\widetilde{C}\, x = C\, x \quad .$$

If A and C are represented, in a suitable coordinate system, as

$$A = \begin{bmatrix} A^+ & 0 \\ 0 & A^- \end{bmatrix} \qquad C = [C^+ \quad C^-] \quad ,$$

where A^+ and C^+ represent the restriction of A and C respectively to $X^+(A)$, then (22) and (23) imply that in the same coordinate system $\widetilde{A}$ and $\widetilde{C}$ must have the forms

$$\widetilde{A} = \begin{bmatrix} A^+ & A^0 \\ 0 & \widetilde{A}^- \end{bmatrix} \qquad \widetilde{C} = [C^+ \quad \widetilde{C}^-] \quad .$$

Therefore, the theorem states that the observer (15) must contain a faithful copy of at least the unstable part of the system. Subject to this requirement, however, the realization (15) is superior to the realization (2). To emphasize this important fact, we state the following theorem, the proof of which is obvious from the previous results.

Theorem 4. Let $\sigma(A) \subset \mathbb{C}^+$ and assume that the open loop observer (2) can be constructed so that $M = A - K_0 C$ is satisfied exactly, and the closed loop observer is constructed so that $\widetilde{A} = A$ and $\widetilde{C} = C$ are satisfied exactly. Then, with the observer (2), observer action fails for every gain matrix K that is not precisely equal to K_0, and for the observer (15) observer action is preserved for every gain matrix $\widetilde{K} = K_0 + \delta\widetilde{K}'$ with $\delta\widetilde{K}$ contained in $\Omega_{\widetilde{K}}(\epsilon)$ for some $\epsilon > 0$.

III. EXAMPLE

To illustrate the above results we consider observer design for the discrete system,

$$x(k+1) = A\,x(k) \qquad y(k) = C\,x(k), \qquad k = 0, 1, \ldots,$$

with

$$A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}, \qquad C = [0 \quad 1] \quad ,$$

and $x_1(0) = 1$, $x_2(0) = 2$. Assuming that a deadbeat error response is desired, we have

$$K_0 = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad ;$$

thus, the open loop observer is the discrete analog of (2), with

$$M = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad K = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad ,$$

and the closed loop observer is the discrete analog of (15), with

$$\widetilde{A} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix} , \quad \widetilde{C} = [0 \quad 1] \quad , \text{ and } \widetilde{K} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

The initial state of the observer is assumed to be zero in each case; i.e., $z_1(0) = z_2(0) = 0$. The error responses $e_1(k) = x_1(k) - z_1(k)$ and $e_2(k) = x_2(k) - z_2(k)$, $k = 0, 1, 2, \ldots$, for the open and closed loop observer with various perturbations, are displayed in Tables 1 through 5 (correspondingly, Figs. 1 through 5). The observer error is seen to diverge for the open loop observer with $\delta K \neq 0$ (Fig. 2), and the closed loop observer with $\delta A \neq 0$ (Fig. 5), in accordance with what is expected from Theorem 4; on the other hand the closed loop observer (with $\delta A = 0, \delta C = 0$) regulates the error in the face of $\delta\widetilde{K} \neq 0$ (Figs. 3 and 4), although in Fig. 4 where the nonzero element of $\delta\widetilde{K}$ is 25 percent of the nominal, the error response while convergent is noticeably different from deadbeat.

IV. CONCLUDING REMARKS

In the treatment above, satisfactory observer action was considered to be equivalent to the minimal requirement that the observer error merely converge asymptotically. Thus, the theorems are directed at specifying those classes of perturbations under which the observer error remains uncoupled from the unstable system modes; in fact, for an arbitrary set S of the modes of A with eigenspace $X_S(A)$, the theorem statements with $X^+(A)$ replaced by $X_S(A)$ specify those perturbations that guarantee that the error will not contain characteristic exponents corresponding to the modes S. If the observer is used in a control configuration, the error will usually be required to converge faster than the fastest decaying modes of A; in this case $X_S(A) = X$ and from Theorem 4 and the two corollaries it follows that the observer (2) cannot tolerate any perturbations at all and that the observer (15) cannot tolerate perturbations in $\widetilde{A}$, $\widetilde{C}$ but can accommodate arbitrary "small" perturbations in $\widetilde{K}$. However, it should be noted that the number of fixed parameters in either realization is $n^2 + nm$.

The results of this paper have direct application to the servomechanism problem [5-10] where the disturbance and command signals are generated as the outputs of autonomous, unstable linear systems. In various treatments of this problem [5, 9, 10], observers have been proposed for predicting the states of the disturbance and command signal generators required for implementing the control law. The results of this paper prove that such designs require highly accurate models of these generators and equally precise hardware to duplicate these models. Of course these facts are intuitively clear from classical feedback theory, if the observer is regarded as a servomechanism tracking the state of system (1).

It should be pointed out that from Theorem 2 and its corollary, it follows that the reduced order observers that have been reported in the literature to date (see Reference 1 and the references cited therein) are never "robust"; i.e., they cannot tolerate the slightest perturbation in their parameters. This is true since all these observers are realized in the open loop form (2). In fact the general conditions under which a reduced order observer can be realized in feedback form do not seem to have been reported in the literature and it may well be that minimal order observers will fail to be "robust" in many cases. In any event, it should be clear that the problem of designing low order "robust" observers is a fruitful and important area of research.

Finally. the paper should generate some rethinking on the "observed state feedback" design philosophy so widely accepted by control theorists and stimulate renewed efforts to systematize design of classical type compensators for multivariable systems. The least one can say about these compensators is that they are "robust" and provide, via gain and phase margin, a good degree of parametric stability.

TABLE 1. IDEAL (PERTURBATION-FREE) OBSERVER

k	0	1	2	3	4	5	6	7	8	9	10
e_1	1	0	0	0	0	0	0	0	0	0	0
e_2	2	1	0	0	0	0	0	0	0	0	0

TABLE 2. OPEN LOOP OBSERVER WITH $\delta M = 0$ and $\delta K = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}$

k	0	1	2	3	4	5	6	7	8	9	10
e_1	1.00	-0.02	-0.01	-0.17	-0.37	-0.77	-1.57	-3.57	-6.37	-12.77	-25.57
e_2	2.00	1.00	-0.02	-0.01	-0.17	-0.37	-0.77	-1.57	-3.57	-6.37	-12.77

TABLE 3. CLOSED LOOP OBSERVER WITH $\delta A = 0$, $\delta C = 0$, and $\delta\widetilde{K} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}$

k	0	1	2	3	4	5	6	7	8	9	10
e_1	1.00	-0.02	-0.01	2×10^{-4}	1×10^{-4}	-2×10^{-6}	-1×10^{-6}	2×10^{-8}	1×10^{-8}	-2×10^{-10}	-1×10^{-10}
e_2	2.00	1.00	-0.02	-0.01	2×10^{-4}	1×10^{-4}	-2×10^{-6}	-1×10^{-6}	2×10^{-8}	1×10^{-8}	-2×10^{-10}

TABLE 4. CLOSED LOOP OBSERVER WITH $\delta A = 0$, $\delta C = 0$, and $\delta\widetilde{K} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$

k	0	1	2	3	4	5	6	7	8	9	10
e_1	1.00	-1.00	-0.50	0.50	0.25	-0.25	-0.12	0.12	0.06	-0.06	0.03
e_2	2.00	1.00	-1.00	-0.50	0.50	0.25	-0.25	-0.12	0.12	0.06	-0.06

TABLE 5. CLOSED LOOP OBSERVER WITH $\delta A = \begin{bmatrix} 0 & 0.01 \\ 0 & 0 \end{bmatrix}$, $\delta\widetilde{K} = 0$, AND $\delta C = 0$

k	0	1	2	3	4	5	6	7	8	9	10
e_1	1.00	0.00	-0.06	-0.17	-0.37	-0.77	-1.57	-3.18	-6.38	-12.80	-25.83
e_2	2.00	1.00	0.00	-0.06	-0.17	-0.37	-0.77	-1.57	-3.18	-6.38	-12.80

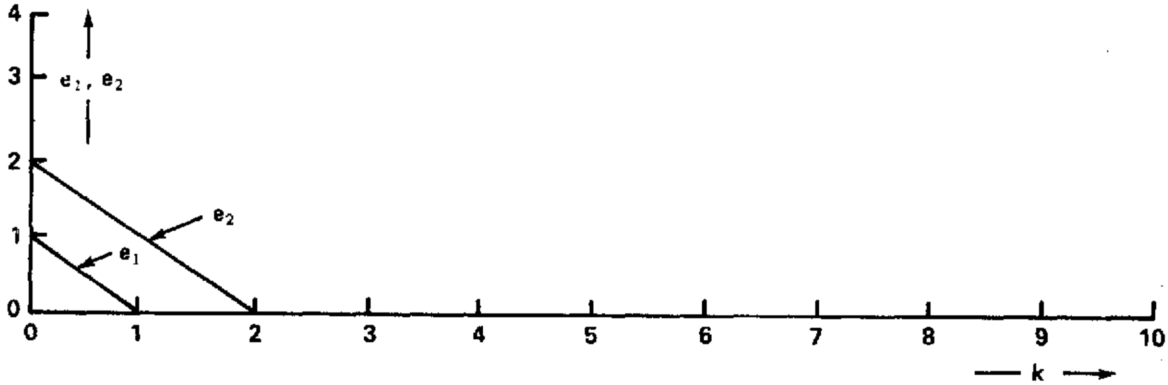

Figure 1. Error response of ideal (perturbation-free) deadbeat observer.

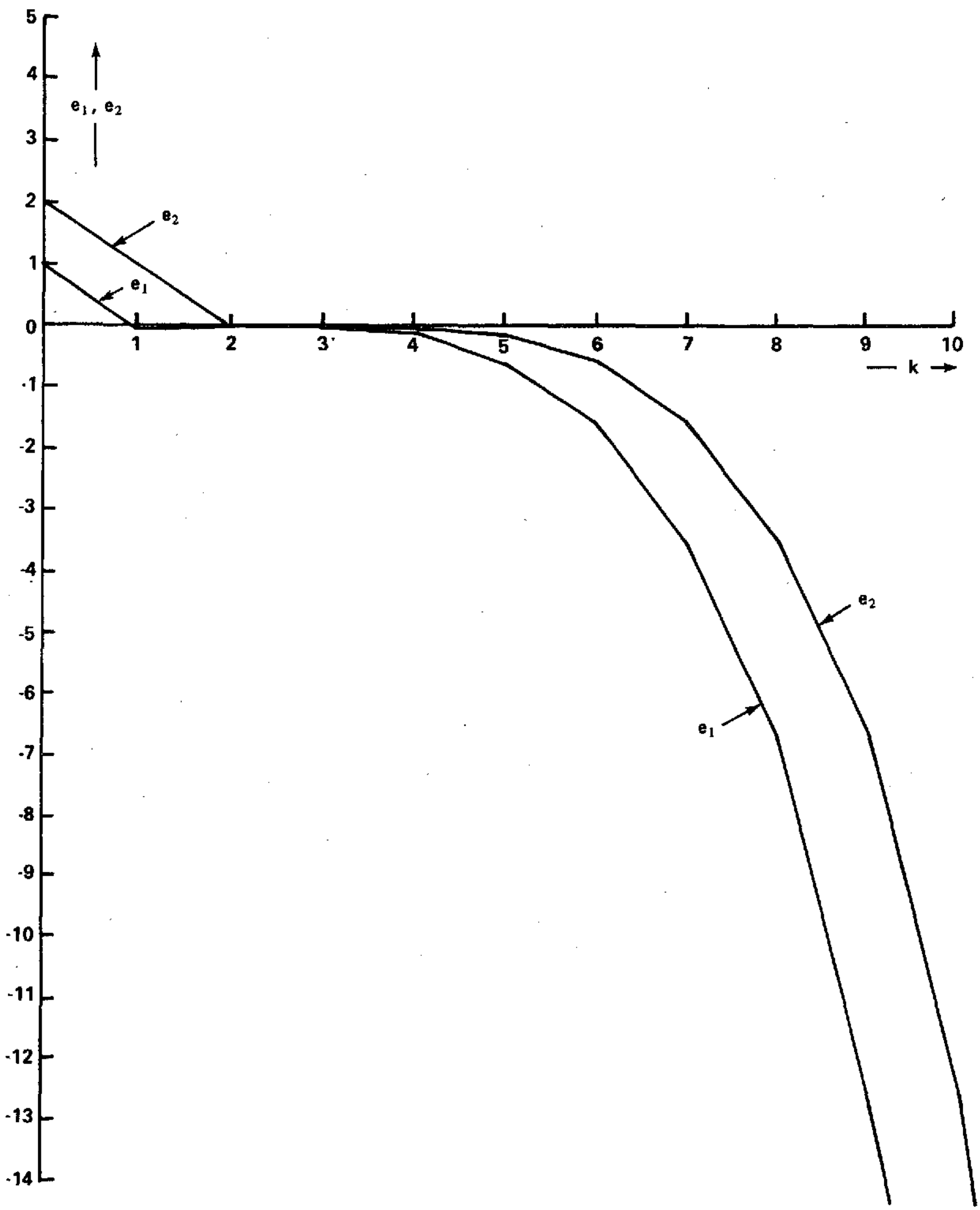

Figure 2. Error response of the open loop observer with

$$\delta M = 0 \text{ and } \delta K = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}.$$

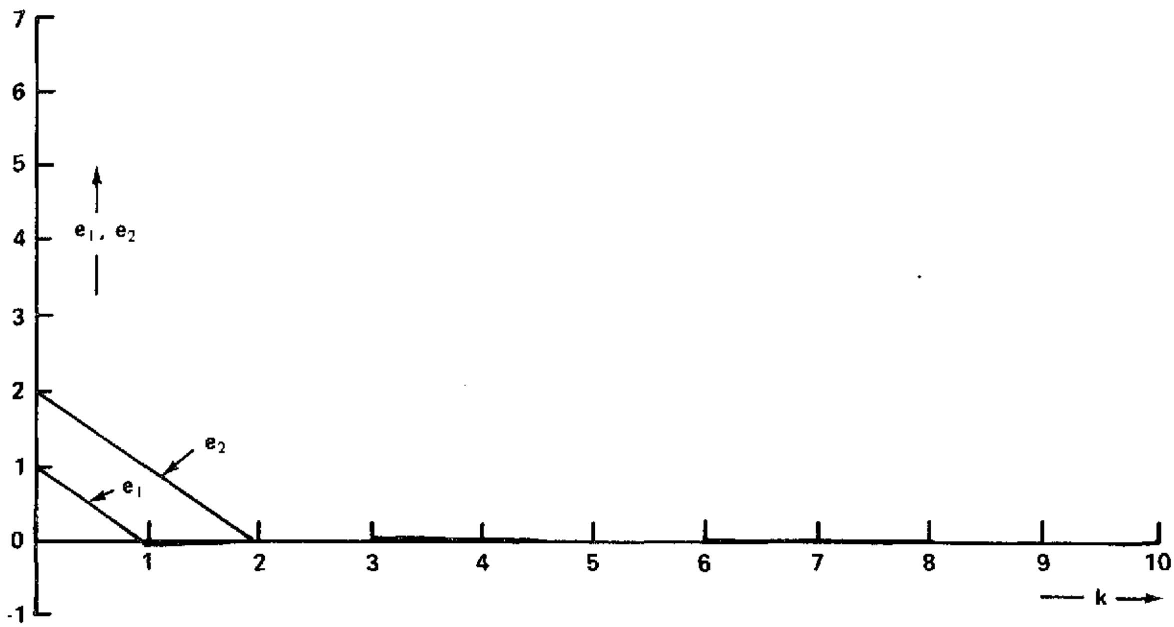

Figure 3. Error response of closed loop observer with

$$\delta A = 0,\ \delta C = 0, \text{ and } \ \delta \tilde{K} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}.$$

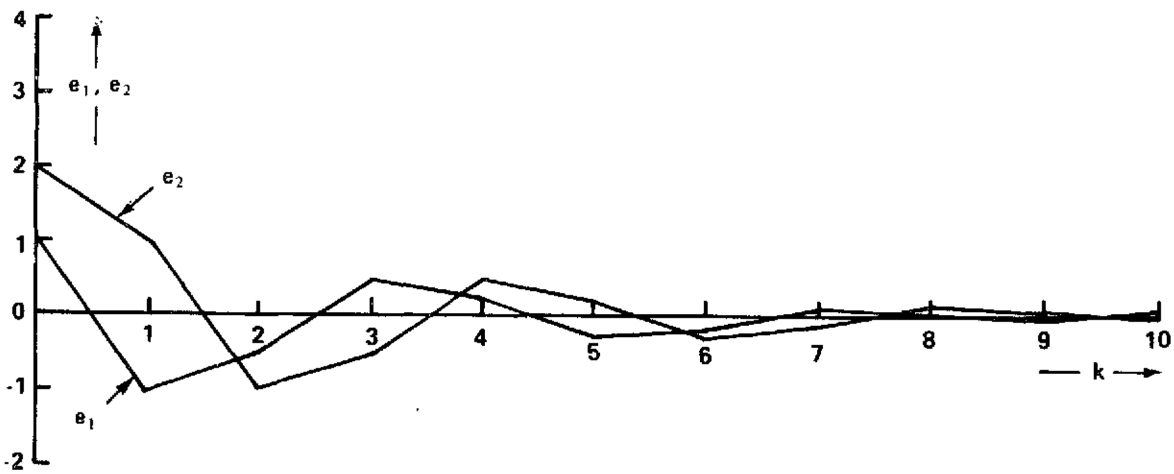

Figure 4. Error response of closed loop observer with

$$\delta A = 0,\ \delta C = 0, \text{ and } \ \delta \tilde{K} = \begin{bmatrix} 0.50 \\ 0 \end{bmatrix}.$$

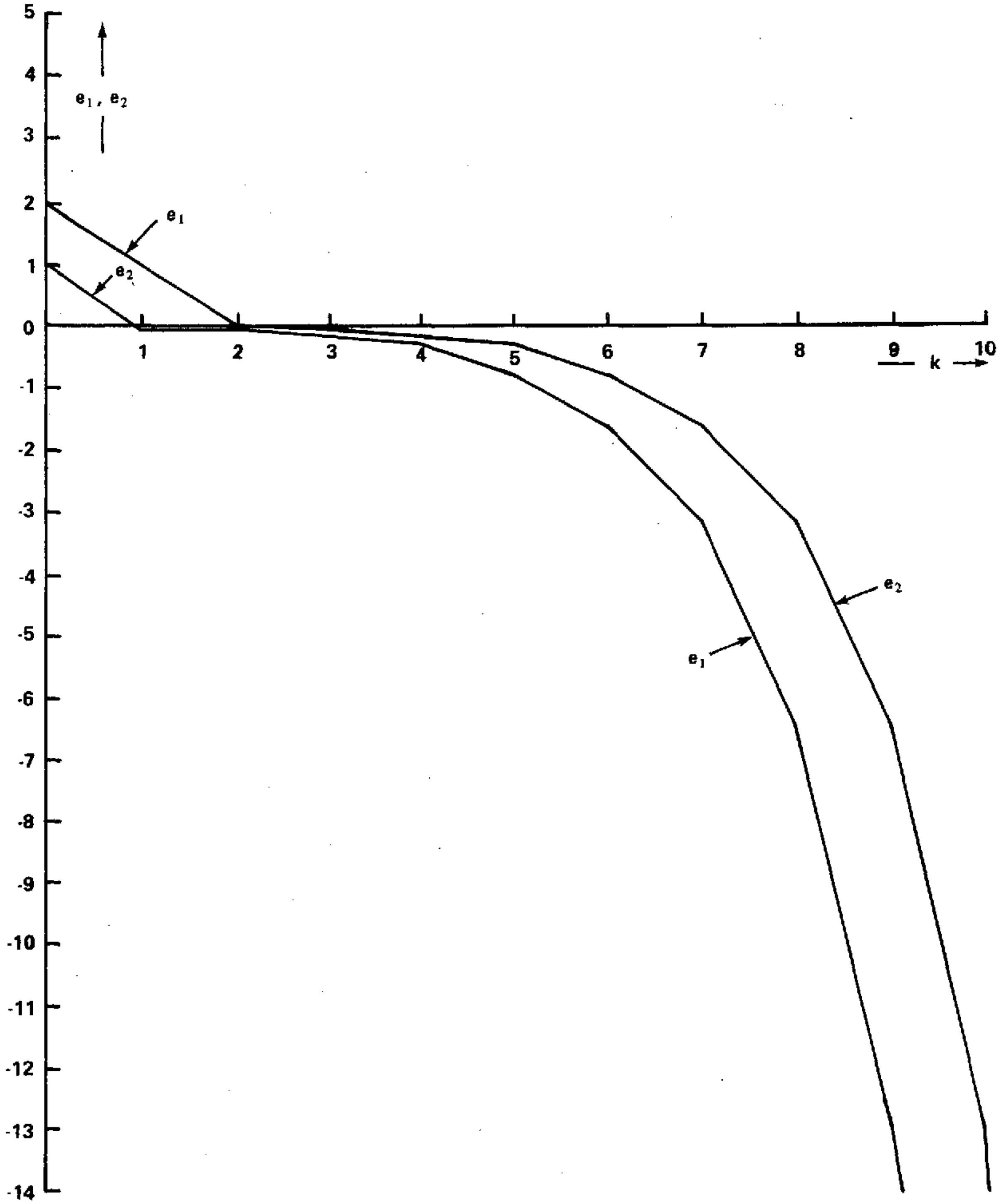

Figure 5. Error response of closed loop observer with

$$\delta C = 0,\ \delta\widetilde{K} = 0, \text{ and } \delta A = \begin{bmatrix} 0 & 0.01 \\ 0 & 0 \end{bmatrix}.$$

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APPROVAL

ROBUST DESIGN OF DYNAMIC OBSERVERS

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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