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Abstract. The Earth Physics Satellite Systems error analysis program (EPSS) -- developed under NASA grant NGR 05-007-280 -- was applied to the problem of predicting the relative accuracy of station position determinations under varying orbital and observing geometries. The reference case consists of nine ground stations extending over 1500 km which laser ranged to a LAGEOS satellite, with simultaneous doppler tracking from a geosynchronous satellite for 16 days. Eleven variations from the reference case were tested.

The results showed little sensitivity to whether the LAGEOS altitude is 3700 or 5690 km. More significant were that the inclination was high, and that LAGEOS was tracked by a geosynchronous satellite.

1. Approach.

The position errors for stations relative to a single station were included in a solution vector with selected geopotential harmonic coefficients, surface mass points, and orbital elements for the high and the low satellites. Table 1 describes the reference and variation cases, including solution vector, a priori error sigmas and the station locations, orbit descriptions, and observation timing and limits which determine the observing geometry. The variation cases dealt with the following items:

- changes in the characteristics of the LAGEOS orbit, both altitude and inclination
- changes in the high satellite doing the doppler tracking, from the reference 1 geosynchonous satellite, to 3 geosynchronous, none, or a GEOPAUSE satellite
- changes in the number and configuration of tracking stations
- changes in the geopotential terms included, and in their apriori error sigmas.

Figure 1 illustrates the geometry of the station locations.

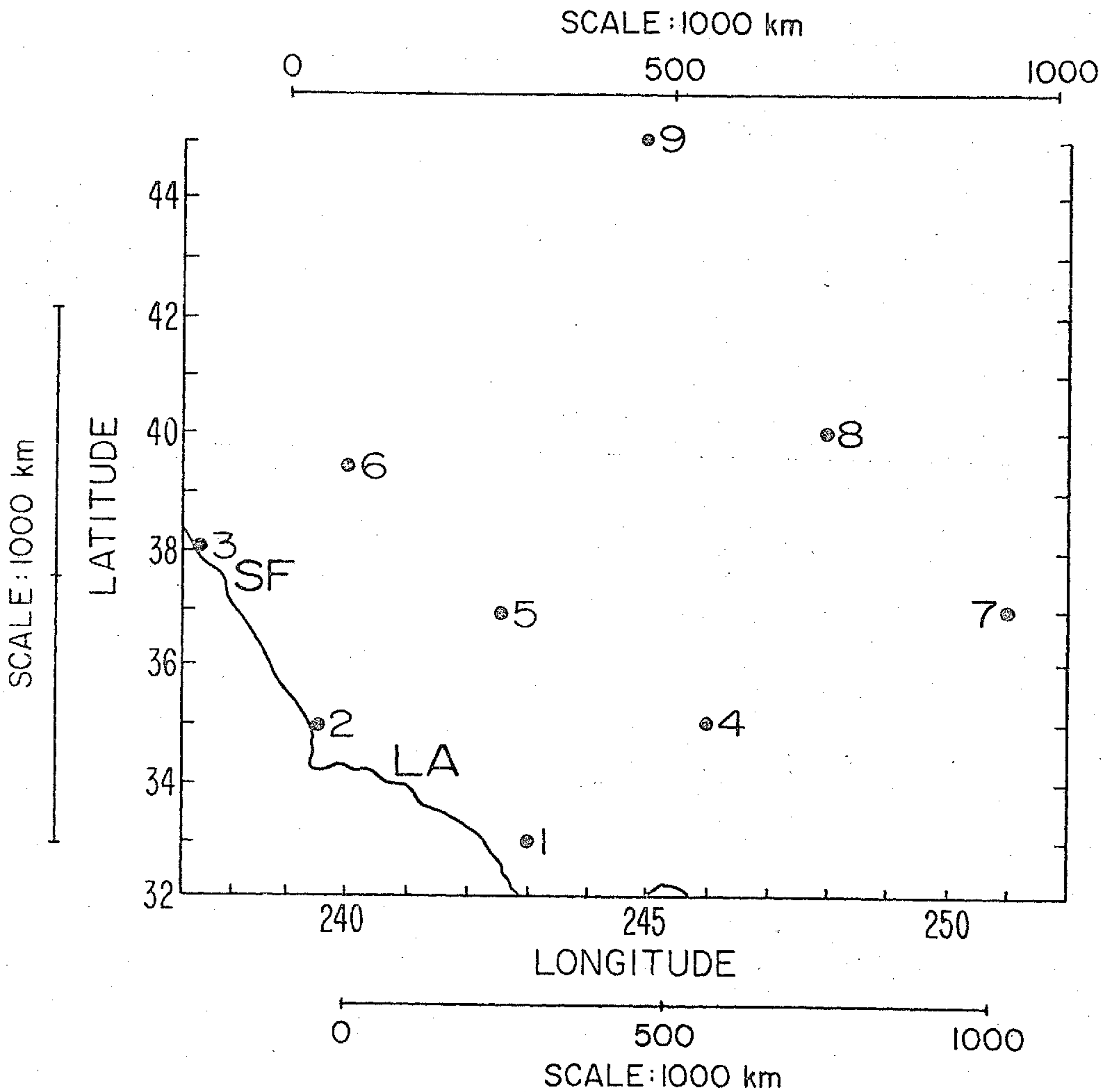

Fig. 1. Geometry of station locations.

2. Changes to the EPSS Program.

Modifications accomplished to the EPSS program were the following:

- Addition of multiple observations per pass to the ranging observations of the low satellite. The number permitted is either 1, 3, or 5 per pass: the first at the high point, the next two at the limiting zenith angle (near the horizon), and the last two at times halfway between the high point and the limiting zenith angle.
- Addition of GM as the zero order geopotential harmonic in the solution vector. This option is available by entering (0,0) in the list of geopotential coefficients desired.
- Addition of solution for relative station position errors as an option, as well as the original option for absolute station position errors. Station 1 is taken as the reference position for the remaining stations, and is treated as having an absolute position error.

Because of the large size of the program, the time required to make modifications and debug the program was more than might be expected. The application of the program to other Earth and Ocean Physics Satellites is planned to obtain a full return on the time invested in developing this tool.

3. Summary of Results.

The results are summarized here for clarity. First are tabular results, next are interpretations, and last are recommendations for LAGEOS orbit and viewing geometry considerations which are indicated from the results of the study. Detailed numerical results and extended discussion thereof are contained in sections 4 and 5, respectively. These sections are recommended when comparing results with other investigators. Preliminary results reported in a letter from William Kaula to Joseph Siry, dated March 15, 1974, suffered from a coding error and should be disregarded, although the conclusions drawn have not been changed by the results reported herein.

3.1. Tabulation of Results of Error Analysis.

Table 2 presents for all cases the average station position error components. The first station (the reference point for the relative positions) is not included in the average. Table 3 presents for selected cases the errors for those coefficients of the geopotential included in the solution vector.

Table 1. Description of Solution Vector, Apriori Error Sigmas, and Observing Geometry for Reference and Variation Cases.

Orbits (standard)*		a	e	i	Ω	ω	$M(t_o)$	Altitude
LAGEOS Low		1.58	0.01	80.	0	0	0	3700 km
LAGEOS High		1.893	0.01	70.	0	0	0	5690 km
Geosynchronous	(1)	6.61	0.001	0.0573	0	0	0	
	(2)						120	
	(3)						240	
GEOPAUSE		4.60	0.001	89.9427	90	0	0	

Stations	Longitude	Latitude
1	243	33
2	239.5	35
3	237	38
4	246	35
5	242.5	37
6	240	39.5
7	251	37
8	248	40
9	245	45
10	120	-20
11	150	-25

Geopotential (standard)*

$C_{\ell m}, S_{\ell m}$; (ℓm):(0,0),(2,2),(3,0),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(4,4)

[No $S_{\ell m}$ when m = 0]

Observations

Range sigma	5 cm
Satellite-to-satellite Range rate sigma	0.5 mm/sec
Maximum zenith angle	75^o (standard)*

*Subject to variation in some cases as indicated in case summary.

Table 1. (continued)

Case Summary

Case	LAGEOS Orbit	Geosynchronous or GEOPAUSE	Maximum Zenith	Stations	Apriori Sigmas	3 Mass Points	Apriori Sigma	Geopotential Coefficients	Apriori Sigma
1	Low	Geosync(1)	75^{o}	1-9	6.371m	Yes	$.3x10^{-6}$	Standard	$.4x10^{-7}$
2	Low, i=50	X*	X	X	X	X	X	X	X
3	X	X	60^{o}	X	X	X	X	X	X
4	X	GEOPAUSE	X	X	X	X	X	X	X
5	X	X	X	X	X	X	X	X	$.1x10^{-6}$
6	X	X	X	1-11	X	X	X	X	X
7	X	None	X	X	X	X	X	X	X
8	X	Geosync(1)(2)(3)	X	X	X	X	X	X	X
9	X	X	X	2,1,3,5	X	X	X	X	X
10	X	X	X	X	X	X	X	**(7,6)(7,7)not (3,1)(4,1)	X
11	X	None	X	X	X	X	X	**(7,6)(7,7)not (3,1)(4,1)	X
1H	High	X	X	X	X	X	X	X	X
9H	High	X	X	2,1,3,5	X	X	X	X	X
11H	High	None	X	X	X	X	X	**(7,6)(7,7)not (3,1)(4,1)	X

* X signifies same as case 1; its replacement by another entry emphasizes where the case varies from the case 1 reference orbit.

**The 18 geopotential coefficients are the same as for the standard case, except (7,6) and (7,7) replace (3,1) and (4,1).

Table 2. Average Station Position Error Components (1 sigma)

Case	Short Description of Variation Orbit (of Table 1)	Average 1-Sigma Error in Centimeters for Stations		
		R	Longitude	Latitude
Low LAGEOS Orbit				
1	Reference	1.26	1.08	0.81
2	LAGEOS $i=50^{\circ}$	1.22	0.79	1.03
3	Max. zenith angle 60°	1.07	1.26	1.04
4	GEOPAUSE tracking	1.36	1.12	0.83
5	Apriori grav. sigma=$.1\times10^{-6}$	1.26	1.09	0.81
6	2 stations on farside	1.20*	1.06*	0.81*
7	No geosynchronous satellite	1.48	1.15	0.83
8	3 geosynchronous satellites	1.24	1.08	0.81
9	4 stations (~500 km square)	1.21	1.08	0.81
10	(7,6), (7,7) harmonics, not (3,1), (4,1)	1.26	1.07	0.81
11	like case 10 with no geosync. sat.	1.55	1.12	0.83
High LAGEOS Orbit				
1H	Reference, altitude=5690, $i=70^{\circ}$	1.34	1.04	0.85
9H	Same as 9, high LAGEOS	1.27	1.04	0.84
11H	Same as 11, high LAGEOS	1.54	1.07	0.88

*Average of stations 2-9 only; stations 10 and 11 on far side not included.

Table 3. Summary of Geopotential Errors

Case*	$\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$ 1-Sigma Errors (unit 10^{-8})(normalized coefficients**)									
ℓ,m	0,0	2,2	2,2	3,0	3,1	3,1	3,2	3,2	3,3	3,3
1	.66	.33	.36	.00047	.32	.28	.35	.34	.22	.23
1H	1.06	.39	.36	.00093	.59	.66	.85	.79	.59	.68
3	.71	.37	.38	.00049	.35	.30	.37	.35	.25	.25
10	.59	.29	.26	.00046	-	-	.33	.32	.22	.22
11	1.04	.41	.38	.00085	-	-	.81	.76	.45	.61
11H	.75	.54	.54	.00106	-	-	1.96	2.08	1.40	1.51
ℓ,m	4,1	4,1	4,2	4,2	4,3	4,3	4,4	4,4		
1	.54	.58	.65	.74	.35	.42	.25	.22		
1H	1.29	1.33	1.29	1.30	.75	.86	.45	.48		
3	.63	.66	.72	.78	.43	.57	.28	.28		
10	-	-	.62	.61	.33	.39	.21	.22		
11	-	-	.81	.87	.53	.85	.40	.32		
11H	-	-	1.60	1.86	1.63	1.58	.82	.78		
ℓ,m	7,6	7,6	7,7	7,7						
10	.41	.38	.15	.18						
11	.47	.43	.17	.21						
11H	.30	.31	.18	.21						

* Case 1: Reference, low LAGEOS orbit

Case 1H: Reference, high LAGEOS orbit

Case 3: Maximum zenith angle 60^{o}

Case 10: (7,6) and (7,7) harmonics replace (3,1), (4,1)

Case 11: Same as 10, with no geosynchronous satellite

Case 11H: Same as 11, high orbit.

**Apriori sigmas for all cases tabulated here were 4.0×10^{-8}.

3.2. Interpretations Summarized.

The system seems remarkably insensitive to plausible variations in its parameters of a wide variety, with only three changes having an influence of more than six percent on any of the average relative coordinate sigmas: (1) change of the LAGEOS inclination to 50^{o}; (2) omission of the Geosynchronous satellite tracking; and (3) decrease of the maximum zenith distance to 60^{o}. The lack of sensitivity holds true even for station 4, which is closest to station 1, 350 km away. Its results were little affected by the change in LAGEOS altitude from 3700 km to 5690 km: apparently the use of observations at a wide variety of look angles makes the geometry for a direct overhead pass (5.4^{o} vs. 3.5^{o} apex angle between stations) of little significance as an indicator.

3.3. Recommendations.

A. The dominant consideration in determining the LAGEOS altitude should be whether the lower altitude makes LAGEOS accessible to a significantly larger number of tracking stations. Otherwise, 3700 km and 5690 km both are near a rather mild optimization, where there are trade-offs between geometry and orbit perturbation sensitivity, which leave the errors about the same.

B. The LAGEOS inclination should be rather high, 70^{o} or more.

C. LAGEOS should be tracked by a geosynchronous satellite.

4. Detailed Results.

The station by station errors in all three components for each case are tabulated in Table 4. The first station has an absolute error, and all the remaining stations are referred to it. In Cases 9 and 9H, station 2 is listed first because it was the one to which the others were referred. Its location relative to the other three stations dictated the order (see Figure 1).

The correlation matrix for Case 1 is shown in Figure 2. The correlation matrix for Case 1H is shown in Figure 3.

Table 4. Station Errors (1-sigma in centimeters) for Radial, Longitudinal, Latitudinal Components

Station / Case	1 (Absolute error)			2 (Relative error)			3 (Relative error)		
1	1.06	.82	.59	1.18	1.07	.82	1.28	1.07	.81
2	1.02	.56	.70	1.03	.78	.97	1.25	.78	1.02
3	.81	.94	.76	1.02	1.26	1.05	1.08	1.27	1.04
4	1.15	.85	.60	1.25	1.11	.84	1.38	1.12	.83
5	1.08	.82	.59	1.17	1.08	.82	1.26	1.08	.81
6	1.02	.80	.59	1.15	1.07	.82	1.20	1.07	.81
7	1.21	.87	.61	1.31	1.12	.84	1.48	1.14	.83
8	1.07	.82	.59	1.17	1.07	.82	1.24	1.07	.81
10	1.06	.81	.59	1.18	1.07	.82	1.27	1.07	.81
11	1.23	.84	.61	1.35	1.11	.84	1.63	1.11	.83
1H	1.14	.78	.60	1.17	1.04	.84	1.31	1.03	.84
11H	1.28	.80	.62	1.32	1.07	.87	1.58	1.07	.87

Station / Case	2 (Absolute error)			1 (Relative error)			3 (Relative error)		
9	1.20	.83	.58	1.23	1.09	.82	1.23	1.07	.80
9H	1.22	.77	.61	1.25	1.06	.85	1.31	1.02	.84

Table 4. Station Errors (1-sigma in centimeters) for Radial, Longitudinal, Latitudinal Components (continued)

Station / Case	4 (Relative error)			5 (Relative error)			6 (Relative error)		
1	1.18	1.08	.82	1.17	1.07	.81	1.24	1.08	.80
2	1.01	.78	.97	1.06	.78	.99	1.26	.78	1.04
3	1.02	1.24	1.06	1.02	1.26	1.04	1.06	1.27	1.03
4	1.23	1.11	.84	1.27	1.10	.83	1.38	1.12	.82
5	1.16	1.08	.82	1.19	1.07	.81	1.27	1.09	.80
6	1.15	1.07	.82	1.16	1.06	.81	1.21	1.06	.80
7	1.30	1.12	.85	1.32	1.12	.83	1.46	1.15	.82
8	1.16	1.08	.83	1.18	1.07	.81	1.25	1.07	.80
10	1.17	1.07	.82	1.17	1.06	.81	1.24	1.07	.80
11	1.30	1.11	.85	1.33	1.10	.83	1.55	1.12	.82
1H	1.17	1.05	.84	1.22	1.04	.84	1.36	1.03	.84
11H	1.29	1.08	.87	1.36	1.07	.87	1.59	1.06	.87

Station / Case	5 (Relative error)		
9	1.18	1.07	.80
9H	1.24	1.04	.84

Table 4. Station Errors (1-sigma in centimeters) for Radial, Longitudinal, Latitudinal Components (continued)

Station / Case	7 (Relative error)			8 (Relative error)			9 (Relative error)		
1	1.35	1.09	.81	1.31	1.08	.80	1.42	1.10	.78
2	1.20	.80	1.01	1.28	.79	1.04	1.70	.79	1.19
3	1.10	1.25	1.05	1.08	1.25	1.03	1.18	1.30	1.02
4	1.33	1.13	.83	1.39	1.13	.81	1.64	1.14	.80
5	1.24	1.09	.81	1.28	1.09	.80	1.49	1.11	.79
6	1.18	1.07	.81	1.21	1.06	.80	1.34	1.05	.78
7	1.59	1.17	.84	1.59	1.17	.82	1.81	1.20	.81
8	1.21	1.09	.81	1.25	1.08	.80	1.43	1.08	.79
10	1.33	1.08	.81	1.29	1.07	.80	1.41	1.07	.78
11	1.65	1.14	.84	1.60	1.12	.82	1.95	1.14	.80
1H	1.32	1.05	.85	1.41	1.04	.85	1.73	1.04	.88
11H	1.54	1.08	.88	1.63	1.06	.87	2.07	1.06	.90

Station / Case	10 (Relative error)			11 (Relative error)			
*6	1.65	1.70	.97	1.50	1.72	1.08	(two stations on far side of Earth)

Figure 2, Case 1

CORRELATION COEFFICIENTS

```
 1  1.0
 2  0.1 1.0
 3 -0.1-0.1 1.0
 4 -0.6-0.0 0.0 1.0
 5 -0.1-0.7 0.0 0.1 1.0
 6  0.0 0.0-0.7-0.1-0.1 1.0
 7 -0.6-0.0 0.0 0.6 0.0-0.0 1.0
 8 -0.1-0.7 0.0 0.0 0.5-0.0 0.1 1.0
 9  0.1 0.0-0.7-0.0-0.0 0.5-0.0-0.1 1.0
10 -0.6-0.1 0.0 0.4 0.1-0.0 0.4 0.1-0.0 1.0
11 -0.0-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 1.0
12  0.0 0.0-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.1-0.1 1.0
13 -0.6-0.1 0.0 0.5 0.1-0.0 0.5 0.1-0.0 0.5 0.0-0.0 1.0
14 -0.1-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 0.5-0.0 0.1 1.0
15  0.1 0.0-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.1 1.0
16 -0.6-0.0 0.0 0.5 0.1-0.0 0.6 0.1-0.0 0.5 0.0-0.0 0.5 0.0-0.0 1.0
17 -0.1-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 1.0
18  0.1 0.0-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.1 1.0
19 -0.6-0.1 0.0 0.3 0.1-0.0 0.3 0.1-0.0 0.5 0.1-0.0 0.5 0.1-0.0 0.4 0.1-0.0 1.0
```

Key to Solution Vector Numbering:

1-27 Station position errors for 9 stations. Station 1 absolute, others relative to station 1. Radial, latitudinal and longitudinal error listed in meters for each station consecutively.

28-45 Normalized Coefficients of Gravitational Harmonics

28	0,0	36-37	3,3
29-30	2,2	38-39	4,1
31	3,0	40-41	4,2
32-33	3,1	42-43	4,3
34-35	3,2	44-45	4,4

46-48 Surface Mass Points
49-54 Close Satellite Elements
55-60 Distant Satellite Elements (Node, i, perigee, a, e, M)

Figure 2, Case 1

CORRELATION COEFFICIENTS

1	1.0																		
2	0.1	1.0																	
3	-0.1	-0.1	1.0																
4	-0.6	-0.0	0.0	1.0															
5	-0.1	-0.7	0.0	0.1	1.0														
6	0.0	0.0	-0.7	-0.1	-0.1	1.0													
7	-0.6	-0.0	0.0	0.6	0.0	-0.0	1.0												
8	-0.1	-0.7	0.0	0.0	0.5	-0.0	0.1	1.0											
9	0.1	0.0	-0.7	-0.0	-0.0	0.5	-0.0	-0.1	1.0										
10	-0.6	-0.1	0.0	0.4	0.1	-0.0	0.4	0.1	-0.0	1.0									
11	-0.0	-0.7	0.0	0.0	0.5	-0.0	0.0	0.5	-0.0	0.1	1.0								
12	0.0	0.0	-0.7	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.1	-0.1	1.0							
13	-0.6	-0.1	0.0	0.5	0.1	-0.0	0.5	0.1	-0.0	0.5	0.0	-0.0	1.0						
14	-0.1	-0.7	0.0	0.0	0.5	-0.0	0.0	0.5	-0.0	0.1	0.5	-0.0	0.1	1.0					
15	0.1	0.0	-0.7	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.0	-0.1	1.0				
16	-0.6	-0.0	0.0	0.5	0.1	-0.0	0.6	0.1	-0.0	0.5	0.0	-0.0	0.5	0.0	-0.0	1.0			
17	-0.1	-0.7	0.0	0.0	0.5	-0.0	0.0	0.5	-0.0	0.1	0.5	-0.0	0.1	0.5	-0.0	0.1	1.0		
18	0.1	0.0	-0.7	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.0	-0.0	0.5	-0.0	-0.1	1.0	
19	-0.5	-0.1	0.0	0.3	0.1	-0.0	0.3	0.1	-0.0	0.5	0.1	-0.0	0.5	0.1	-0.0	0.4	0.1	-0.0	1.0

20 -0.0-0.7 0.0 0.0 0.5-0.0-0.0 0.5-0.0 0.1 0.5-0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 1.0

21 0.0 0.1-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 1.0

22 -0.5-0.1 0.0 0.4 0.1-0.0 0.4 0.1-0.0 0.5 0.1-0.0 0.5 0.1-0.0 0.5 0.1-0.0 0.6 0.1-0.0 1.0

23 -0.1-0.7 0.0 0.0 0.5-0.0-0.0 0.5-0.0 0.1 0.5-0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 0.5-0.0 0.1 1.0

24 0.1 0.0-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5 0.0-0.0 0.5-0.0-0.0 1.0

25 -0.7-0.1 0.0 0.4 0.1-0.0 0.5 0.1-0.0 0.5 0.0-0.0 0.5 0.1-0.0 0.6 0.1-0.0 0.5 0.0-0.0 0.6 0.1 0.0 1.0

26 -0.1-0.7 0.0 0.0 0.5-0.0-0.0 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.0 0.6-0.0 0.1 0.5-0.0 0.1 0.6-0.0 0.1
1.0

27 0.0 0.0-0.8-0.0-0.0 0.5 0.0-0.0 0.6-0.0-0.0 0.5 0.0-0.0 0.5 0.0-0.0 0.6 0.0-0.0 0.6 0.0-0.0 0.6 0.1
-0.0 1.0

28 0.2 0.0 0.0-0.1-0.0 0.0-0.1-0.1 0.0-0.1 0.0 0.0-0.1-0.0-0.0-0.2-0.1-0.0-0.1 0.0 0.0-0.2-0.0-0.0-0.3
-0.1-0.0 1.0

29 -0.0-0.0 0.0 0.0 0.0 0.0 0.1 0.1 0.0 0.0-0.0-0.0 0.0 0.0 0.0 0.1 0.0 0.0-0.0-0.1-0.0 0.0-0.0-0.0 0.1
0.0-0.0-0.1 1.0

30 -0.1-0.0-0.0-0.0 0.0-0.0-0.1 0.0-0.0 0.1 0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.2 0.0 0.0 0.2 0.0 0.0 0.1
0.1 0.0-0.2 0.1 1.0

31 -0.2 0.0-0.0 0.1 0.0-0.0 0.2 0.0 0.0 0.0-0.0 0.0 0.1-0.0 0.0 0.2-0.0 0.0 0.1-0.0 0.0 0.2-0.0 0.0 0.3
-0.0 0.1-0.2-0.2-0.1 1.0

32 0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.1-0.1-0.0-0.1-0.1-0.0-0.0
-0.1-0.0 0.5 0.1-0.2 0.0 1.0

33 0.0 0.1-0.0 0.0-0.0 0.0 0.1-0.1 0.0-0.0-0.0 0.0-0.0-0.1 0.0 0.0-0.1 0.0-0.1-0.1-0.0-0.1-0.1-0.0-0.1
-0.2-0.0-0.0-0.1 0.2-0.2-0.2 1.0

34 -0.1-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0 0.1 0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0 0.3 0.1 0.0 0.2 0.0 0.0 0.1
0.0 0.0 0.4 0.1-0.0-0.1 0.1 0.1 1.0

35 -0.1 0.0-0.0 0.2-0.0 0.0 0.3-0.0 0.0-0.1-0.0-0.0 0.1-0.0 0.0 0.2-0.0 0.0-0.2-0.0-0.0-0.1-0.0-0.0 0.0
-0.0 0.0 0.1-0.1 0.2 0.1-0.2 0.2-0.1 1.0

36 -0.0-0.1 0.0 0.1 0.0 0.0 0.1 0.0 0.0-0.0 0.0-0.0 0.0 0.1 0.0 0.1 0.1 0.0-0.0 0.0-0.0 0.0 0.1-0.0 0.1
0.2-0.0-0.1-0.1 0.4-0.0-0.3 0.1-0.3 0.6 1.0

```
37  0.1-0.0 0.0-0.0-0.0 0.0-0.1-0.0-0.0-0.0 0.0-0.0-0.1 0.0-0.0-0.1-0.0-0.0-0.1 0.1-0.0-0.1 0.1-0.0-0.
    0.1-0.0-0.4-0.3-0.0 0.0-0.3-0.1-0.7-0.2 0.1 1.0

38  0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.1 0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.1 0.0-0.1 0.0-0.0-0.1 0.0-0.0-0.0
    0.0 0.0 0.2-0.2-0.7 0.1 0.2-0.3-0.1-0.3-0.2-0.1 1.0

39  0.0-0.1 0.0-0.0 0.0 0.0-0.0 0.1-0.0-0.0 0.0-0.0-0.0 0.1-0.0-0.0 0.1-0.0-0.0 0.1-0.0-0.0 0.1-0.0-0.0
    0.2-0.0 0.0 0.4-0.4-0.0 0.1-0.4 0.1-0.2-0.2-0.0 0.4 1.0

40  0.1-0.0 0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.0 0.0-0.0-0.1 0.0-0.0-0.1-0.0-0.0 0.0 0.1 0.0-0.0 0.1-0.0-0.1
    0.0-0.0 0.0-0.8-0.0 0.1-0.1-0.1-0.1-0.1 0.0 0.4 0.0-0.3 1.0

41  0.1 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.1-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1
   -0.1-0.0 0.2-0.0-0.8 0.1 0.3-0.1 0.1-0.1-0.4-0.1 0.5 0.3-0.1 1.0

42 -0.0-0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.1 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0
    0.1 0.0 0.0 0.1-0.3 0.0 0.1-0.1-0.0 0.2 0.2-0.1 0.4 0.5-0.3 0.5 1.0

43 -0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0
    0.0-0.0-0.3 0.2 0.3 0.0-0.2-0.0-0.4 0.0 0.2 0.3-0.4 0.2-0.4-0.4-0.1 1.0

44 -0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0-0.0
    0.0-0.0-0.4 0.4 0.6-0.1-0.3 0.0-0.3-0.1 0.1 0.4-0.5 0.0-0.5-0.6-0.4 0.7 1.0

45  0.0 0.0-0.0-0.0-0.0-0.0-0.1-0.0-0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0 0.0 0.1 0.0-0.0 0.0-0.0
   -0.1 0.0 0.1-0.5 0.2 0.0 0.1-0.0 0.1-0.2-0.3-0.1 0.1-0.1 0.6-0.2-0.4-0.3-0.1 1.0

46  0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0
   -0.0-0.0 0.0-0.0-0.2-0.1-0.1 0.1 0.1-0.0-0.2-0.0 0.1-0.0-0.0 0.1-0.1-0.1-0.1-0.0 1.0

47  0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.1-0.0 0.0-0.0-0.0-0.0-0.0
   -0.1-0.0-0.0 0.1-0.1 0.1 0.2-0.1-0.1-0.1-0.2 0.1 0.0-0.0-0.0 0.2-0.1-0.0-0.0 0.1-0.2 1.0

48 -0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.1 0.0-0.0 0.1 0.0 0.0 0.1
    0.1 0.0-0.0 0.0 0.2 0.0-0.1 0.1 0.0 0.1 0.3-0.1-0.1-0.0-0.0-0.2 0.1 0.1 0.0-0.0-0.5-0.7 1.0

49  0.0-0.0 0.0 0.1 0.0 0.0 0.1 0.1 0.0-0.1 0.0-0.0-0.0 0.0-0.0 0.1 0.1 0.0-0.2-0.0-0.0-0.2 0.0-0.0-0.1
    0.0-0.0 0.0 0.2-0.5 0.0 0.2-0.2-0.2 0.0-0.1-0.0 0.7 0.8-0.3 0.5 0.5 0.0-0.2 0.0 0.0 0.0-0.1 1.0

50  0.1 0.0 0.0-0.0 0.0 0.0-0.0 0.1 0.0-0.1-0.0-0.0-0.1 0.0 0.0-0.1 0.0 0.0-0.1-0.1-0.0-0.1-0.1-0.0-0.1
   -0.0-0.0 0.2 0.2-0.2-0.1 0.3-0.1-0.2-0.2-0.1-0.3 0.5-0.0-0.2 0.1 0.1-0.3-0.2 0.0-0.0 0.1-0.0 0.2 1.0

51  0.2 0.0 0.0-0.1-0.0 0.0-0.2-0.0-0.0-0.1 0.0 0.0-0.2-0.0-0.0-0.3-0.0-0.0-0.1 0.0-0.0-0.3-0.0-0.0-0.4
   -0.0-0.1 0.5 0.2 0.1-0.2-0.1 0.0 0.0-0.0 0.0 0.0-0.1-0.0-0.1-0.1-0.0-0.0 0.2-0.1 0.0-0.0-0.0-0.1 0.4
    1.0
```

52 0.2 0.0 0.0-0.1-0.0 0.0-0.1-0.1 0.0-0.1 0.0 0.0-0.1-0.0-0.0-0.2-0.1-0.0-0.1 0.0 0.0-0.2-0.0-0.0-0.3
-0.1-0.0 1.0-0.1-0.2-0.2 0.5-0.0 0.4 0.1-0.1-0.4 0.1-0.0 0.0 0.2 0.0-0.3-0.4 0.1 0.1-0.0-0.0 0.0 0.2
0.5 1.0

53 0.2 0.1 0.0-0.0-0.0 0.0-0.1-0.1 0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.2-0.1-0.0-0.2-0.1-0.0-0.3-0.1-0.0-0.4
-0.1-0.1 0.4 0.0 0.3-0.7 0.0 0.6 0.1 0.1 0.1-0.1-0.3-0.3 0.0-0.3-0.3-0.0 0.2 0.1 0.1-0.0 0.0-0.2 0.2
0.4 0.4 1.0

54 -0.2-0.0-0.0 0.1 0.0-0.0 0.2 0.0 0.0 0.1-0.0 0.0 0.2 0.0 0.0 0.3 0.0 0.0 0.1-0.0 0.0 0.3-0.0 0.0 0.4
0.0 0.1-0.5-0.2-0.1 0.2 0.1 0.0-0.1 0.0 0.0-0.0 0.0-0.0 0.1 0.0-0.0 0.0-0.1 0.1-0.0 0.0 0.0 0.0-0.4
-1.0-0.5-0.4 1.0

55 0.0-0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.1
0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.1-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0 0.0 0.0
-0.1 0.0 0.0 0.0-1.0

56 -0.0 0.0 0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.1 0.0 0.0 0.0-0.0 0.0 0.0
-0.0 0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.1-0.1-0.1-0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0 0.0
0.0 0.0-0.0-0.0 0.2 1.0

57 -0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.1
-0.0-0.0-0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.0
0.1-0.0-0.0-0.0 1.0-0.1-1.0

58 0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0-0.0-0.0 0.0 0.0
-0.0-0.0-0.0 0.0-0.0 0.0-0.0 0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0 0.0-0.0
-0.0-0.0-0.0 0.0-0.0-0.0 0.0 1.0

59 0.0-0.0-0.0-0.0 0.0 0.0-0.1 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.1 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.1
0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.1-0.0 0.0 0.1 0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0
-0.0 0.0 0.0 0.0-1.0 0.5 1.1 0.0-1.0

60 -0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0 0.1 0.0 0.0 0.0-0.0 0.0 0.0
-0.0 0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.1-0.1-0.1-0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0 0.0
0.0 0.0-0.0-0.0 0.3 1.0-0.2-0.1 0.6 1.0

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60															

Figure 3, Case 1H

CORRELATION COEFFICIENTS

1 1.0

2 0.1 1.0

3 -0.1-0.1 1.0

4 -0.6-0.1 0.1 1.0

5 -0.1-0.7 0.0 0.1 1.0

6 0.1 0.0-0.7-0.1-0.1 1.0

7 -0.7-0.1 0.1 0.6 0.1-0.1 1.0

8 -0.1-0.7 0.1 0.1 0.5-0.0 0.1 1.0

9 0.1 0.0-0.7-0.1-0.0 0.5-0.1-0.1 1.0

10 -0.5-0.1 0.1 0.5 0.1-0.1 0.5 0.1-0.1 1.0

11 -0.0-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 1.0

12 0.1 0.1-0.7-0.1-0.0 0.5-0.1-0.0 0.5-0.1-0.1 1.0

13 -0.6-0.1 0.1 0.5 0.1-0.1 0.6 0.1-0.1 0.5 0.0-0.1 1.0

14 -0.1-0.7 0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 1.0

15 0.1 0.0-0.7-0.1-0.0 0.5-0.1-0.0 0.5-0.1-0.0 0.5-0.1-0.1 1.0

16 -0.7-0.1 0.1 0.5 0.1-0.1 0.6 0.1-0.0 0.5 0.0-0.1 0.6 0.1-0.0 1.0

17 -0.1-0.7 0.1 0.0 0.5-0.0 0.0 0.6-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 1.0

18 0.1 0.0-0.7-0.0-0.0 0.5-0.0-0.0 0.5-0.1-0.0 0.5-0.0-0.0 0.5-0.1-0.1 1.0

Key to Solution Vector Numbering:

1-27 Station position errors for 9 stations. Station 1 absolute, others relative to station 1. Radial, latitudinal and longitudinal erro[rs] listed in meters for each station consecutively.

28-45 Normalized Coefficients of Gravitational Harmonics

28	0,0	36-37	3,3
29-30	2,2	38-39	4,1
31	3,0	40-41	4,2
32-33	3,1	42-43	4,3
34-35	3,2	44-45	4,4

46-48 Surface Mass Points

49-54 Close Satellite Elements

55-60 Distant Satellite Elements (Node, i, perigee, a, e, M)

Figure 3, Case 1H

CORRELATION COEFFICIENTS

```
 1   1.0
 2   0.1  1.0
 3  -0.1 -0.1  1.0
 4  -0.6 -0.1  0.1  1.0
 5  -0.1 -0.7  0.0  0.1  1.0
 6   0.1  0.0 -0.7 -0.1 -0.1  1.0
 7  -0.7 -0.1  0.1  0.6  0.1 -0.1  1.0
 8  -0.1 -0.7  0.1  0.1  0.5 -0.0  0.1  1.0
 9   0.1  0.0 -0.7 -0.1 -0.0  0.5 -0.1 -0.1  1.0
10  -0.6 -0.1  0.1  0.5  0.1 -0.1  0.5  0.1 -0.1  1.0
11  -0.0 -0.7  0.0  0.0  0.5 -0.0  0.0  0.5 -0.0  0.1  1.0
12   0.1  0.1 -0.7 -0.1 -0.0  0.5 -0.1 -0.0  0.5 -0.1 -0.1  1.0
13  -0.6 -0.1  0.1  0.5  0.1 -0.1  0.6  0.1 -0.1  0.5  0.0 -0.1  1.0
14  -0.1 -0.7  0.0  0.1  0.5 -0.0  0.1  0.5 -0.0  0.1  0.5 -0.0  0.1  1.0
15   0.1  0.0 -0.7 -0.1 -0.0  0.5 -0.1 -0.0  0.5 -0.1 -0.0  0.5 -0.1 -0.1  1.0
16  -0.7 -0.1  0.1  0.5  0.1 -0.1  0.6  0.1 -0.0  0.5  0.0 -0.1  0.6  0.1 -0.0  1.0
17  -0.1 -0.7  0.1  0.0  0.5 -0.0  0.0  0.6 -0.0  0.1  0.5 -0.0  0.1  0.5 -0.0  0.1  1.0
18   0.1  0.0 -0.7 -0.0 -0.0  0.5 -0.0 -0.0  0.5 -0.1 -0.0  0.5 -0.0 -0.0  0.5 -0.1 -0.1  1.0
```

19 -0.6-0.1 0.1 0.4 0.1-0.1 0.4 0.1-0.1 0.5 0.1-0.1 0.5 0.1-0.1 0.5 0.1-0.0 1.0

20 -0.0-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 0.5-0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 1.0

21 0.1 0.1-0.7-0.1-0.0 0.5-0.1-0.0 0.5-0.1-0.0 0.5-0.1-0.0 0.5-0.0-0.0 0.5-0.1-0.1 1.0

22 -0.7-0.1 0.1 0.5 0.1-0.1 0.6 0.1-0.0 0.5 0.1-0.1 0.6 0.1-0.0 0.6 0.1-0.0 0.6 0.1-0.0 1.0

23 -0.1-0.7 0.0 0.0 0.5-0.0 0.0 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 1.0

24 0.1 0.1-0.7-0.1-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.0 0.5-0.0-0.1 1.0

25 -0.7-0.1 0.0 0.5 0.1-0.0 0.6 0.1-0.0 0.5 0.0-0.0 0.6 0.1-0.0 0.7 0.1-0.0 0.6 0.0-0.0 0.7 0.1 0.0 1.

26 -0.1-0.8 0.1 0.0 0.5-0.0 0.0 0.6-0.0 0.1 0.5-0.0 0.1 0.5-0.0 0.1 0.6-0.0 0.1 0.6-0.0 0.1 0.6-0.0 0. 1.0

27 0.0 0.0-0.7-0.0-0.0 0.5 0.0-0.0 0.5-0.0-0.0 0.5 0.0-0.0 0.5 0.1-0.0 0.5 0.0-0.0 0.5 0.1-0.0 0.5 0. -0.0 1.0

28 0.3 0.0-0.0-0.1-0.0 0.0-0.3-0.0 0.0-0.1-0.0 0.0-0.2-0.0 0.0-0.3-0.0-0.0-0.2-0.0 0.0-0.3-0.0-0.0-0. -0.1-0.1 1.0

29 0.1 0.0 0.1-0.0 0.0-0.0-0.1 0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1 0.0-0.1-0.2-0.1-0.0-0.2-0.1-0.1-0. -0.1-0.2 0.1 1.0

30 0.0-0.0 0.0-0.1 0.0-0.0-0.1 0.1-0.1 0.0-0.0 0.0-0.1 0.0-0.0-0.1 0.0-0.1 0.0-0.0 0.0-0.0-0.0-0.0-0. 0.0-0.1-0.0 0.1 1.0

31 -0.1 0.0-0.0 0.0-0.0-0.0 0.1-0.0-0.0 0.1 0.0 0.0 0.1-0.0 0.0 0.1-0.0-0.0 0.1 0.0 0.0 0.1-0.0 0.0 0. -0.0 0.0-0.0 0.1-0.1 1.0

32 0.1 0.0 0.0-0.0 0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1-0.1-0.0-0.1-0.1-0.0-0. -0.1-0.1 0.3 0.4 0.4-0.0 1.0

33 -0.1 0.0-0.0 0.1-0.0 0.0 0.1-0.0 0.0 0.0-0.0 0.0 0.1-0.0 0.0 0.1-0.0 0.0-0.0-0.0 0.0 0.0-0.0 0.0 0. -0.1 0.1-0.2-0.3 0.0-0.1 0.0 1.0

34 -0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.1-0.0 0.0 0.0-0.0-0.0 0. -0.0-0.0 0.0 0.3-0.2-0.1-0.1 0.3 1.0

35 -0.1-0.0-0.0 0.1 0.0-0.0 0.1 0.0 0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.1 0.0 0.0-0.0-0.0 0.0 0.0-0.0 0.0 0. 0.0 0.0-0.1-0.1 0.4 0.2-0.2 0.1-0.2 1.0

36 -0.1-0.0-0.0 0.1 0.0 0.0 0.2 0.0 0.1-0.1 0.0-0.0 0.1 0.0 0.0 0.1 0.0 0.0-0.1-0.0-0.1-0.0 0.0-0.0 0.
0.0 0.0-0.1 0.0 0.0 0.1-0.1-0.0-0.3 0.6 1.0

37 0.1-0.0 0.0-0.0-0.0-0.0-0.1-0.0-0.0-0.0 0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1 0.0-0.1-0.1 0.0-0.0-0.
0.0-0.1-0.2-0.1-0.1 0.0 0.0-0.2-0.6-0.3 0.0 1.0

38 -0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0 0.
-0.0 0.1 0.6-0.3-0.2-0.2-0.2-0.3-0.1-0.1-0.0-0.2 1.0

39 0.1 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.1-0.0-0.0-0.0-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.
-0.0-0.0-0.2 0.1 0.1-0.3-0.1-0.6-0.2-0.1-0.1 0.2 0.2 1.0

40 0.1-0.0 0.0-0.0-0.0 0.0-0.1-0.0-0.0-0.1 0.0-0.0-0.1 0.0-0.0-0.1 0.0-0.0-0.1 0.1-0.0-0.1 0.1-0.0-0.
0.0-0.1-0.1-0.2-0.2 0.0-0.1 0.1-0.0-0.1-0.1 0.3-0.1 0.1 1.0

41 0.1 0.1 0.0-0.1-0.1-0.0-0.1-0.1-0.0-0.0-0.0 0.0-0.1-0.1-0.0-0.1-0.1-0.0-0.0 0.0 0.0-0.1-0.0-0.0-0.
-0.1-0.0-0.0-0.1-0.2-0.0-0.1-0.0-0.0-0.1-0.3 0.2 0.1 0.0-0.1 1.0

42 -0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.
0.0 0.0-0.0-0.2 0.3 0.1-0.2-0.1-0.3 0.6 0.4-0.0 0.2 0.1-0.4 0.5 1.0

43 -0.1-0.0-0.0 0.0 0.0 0.0 0.1-0.0 0.0 0.0 0.0 0.0 0.1 0.0 0.0 0.1-0.0 0.0 0.1 0.0-0.0 0.1 0.0 0.0 0.
0.0 0.1-0.1-0.1 0.1-0.0 0.2-0.2-0.7-0.2 0.2 0.5-0.1 0.2-0.4-0.2 0.1 1.0

44 0.0-0.0 0.0-0.0 0.0-0.0-0.1 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.1 0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.
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45 0.0-0.0-0.0-0.1 0.0-0.0-0.1 0.0-0.1 0.1 0.0 0.0-0.0 0.0-0.0-0.1 0.0-0.0 0.2 0.1 0.1 0.1 0.1 0.1 0.
0.1 0.0 0.1-0.3 0.3-0.1 0.0 0.2 0.2-0.1-0.6-0.3 0.1-0.1 0.4-0.2-0.4-0.3-0.0 1.0

46 -0.0-0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.
0.0 0.0-0.0-0.0-0.1-0.2-0.1-0.0 0.1-0.1-0.1-0.0 0.1 0.1 0.0-0.0-0.1-0.0-0.0 0.1 1.0

47 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.
-0.0-0.0 0.1 0.0 0.1 0.1 0.2 0.1-0.1-0.1-0.1 0.1-0.1-0.2-0.0-0.0-0.1 0.1 0.1 0.0-0.3 1.0

48 -0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0 0.0 1.
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49 0.0 0.1-0.0 0.1-0.0-0.0 0.1-0.0-0.0-0.1-0.0 0.0 0.0-0.0 0.0 0.0-0.1-0.0-0.1-0.1 0.1-0.1-0.1 0.0-0.
-0.1 0.0-0.1-0.2 0.4-0.2 0.0 0.1-0.5 0.5 0.2 0.1 0.2 0.3-0.3 0.2 0.5 0.3 0.1-0.1-0.1-0.0 0.1 1.0

50 0.3 0.0 0.0-0.1-0.0 0.0-0.2 0.0-0.0-0.1-0.0-0.0-0.2-0.0-0.0-0.3-0.0-0.0-0.2-0.0-0.0-0.3-0.1-0.1-0.
-0.1-0.2 0.2 0.5 0.3-0.0 0.1-0.1-0.1-0.0 0.1-0.0-0.2 0.1-0.2-0.2-0.1 0.2 0.3-0.1-0.1 0.1 0.0 0.1 1.

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51  0.2 0.0-0.0-0.2-0.0 0.0-0.3-0.0-0.0-0.1 0.0 0.0-0.3-0.0 0.0-0.4-0.0-0.0-0.2 0.0 0.0-0.3-0.0-0.0-0.
   -0.0-0.1 0.8 0.2-0.0 0.2 0.0-0.5-0.1-0.1-0.1 0.1 0.5 0.1-0.1 0.1 0.1 0.0 0.1-0.0-0.0 0.0 0.0-0.1 0.
    1.0

52  0.3 0.0-0.0-0.1-0.0 0.0-0.3-0.0 0.0-0.1-0.0 0.0-0.2-0.0 0.0-0.3-0.0-0.0-0.2-0.0 0.0-0.3-0.0-0.0-0.4
   -0.1-0.1 1.0 0.1-0.0-0.0 0.3-0.2 0.0-0.1-0.1-0.2 0.6-0.2-0.1-0.0-0.0-0.1-0.1 0.1 0.0 0.1-0.0-0.1 0.
    0.8 1.0

53  0.1 0.0-0.0-0.0-0.0 0.0-0.1-0.0 0.0-0.1-0.0 0.0-0.1-0.0-0.0-0.1-0.0-0.0-0.2-0.1 0.0-0.2-0.1-0.0-0.
   -0.1-0.1 0.2-0.0 0.1-0.3 0.4 0.7 0.1 0.0 0.0-0.1-0.1-0.5 0.0-0.2-0.2-0.1-0.0 0.1-0.0 0.1-0.1 0.1 0.2
   -0.2 0.3 1.0

54 -0.3-0.0 0.0 0.2 0.0-0.0 0.3 0.0 0.0 0.1-0.0-0.0 0.3 0.0-0.0 0.4 0.0 0.0 0.2-0.0-0.0 0.4 0.0 0.0 0.
    0.0 0.1-0.6-0.2 0.0-0.1-0.0 0.5 0.1 0.1 0.1-0.1-0.5-0.1 0.1-0.1-0.1 0.0-0.1 0.0-0.0-0.0-0.0 0.1-0.4
   -1.0-0.8 0.1 1.0

55  0.0-0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.
    0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0
    0.0 0.0 0.0-0.0-1.0

56 -0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0
   -0.0-0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.1-0.1-0.1-0.1-0.0-0.0-0.0 0.0-0.1-0.0-0.0 0.0-0.0 0.0-0.0-0.1 0.0
    0.0 0.0-0.0-0.0 0.3 1.0

57 -0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.
   -0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0
    0.0-0.0-0.0 0.0 1.0-0.1-1.0

58  0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0-0.0-0.0 0.0 0.
   -0.0-0.0-0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0 0.0 0.0-0.0
   -0.0-0.0 0.0 0.0-0.0-0.0 0.0 1.0

59  0.0-0.0-0.0-0.0 0.0 0.0-0.1 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.1 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.
    0.0 0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.1-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0
    0.0 0.0 0.0-0.0-1.0 0.6 1.1 0.0-1.0

60 -0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.
   -0.0-0.0-0.0 0.1-0.1 0.0-0.0 0.0 0.1-0.1-0.1-0.1-0.0-0.0-0.0 0.0-0.1-0.0-0.0 0.0-0.0 0.0-0.0-0.1 0.
    0.0-0.0-0.0-0.0 0.3 1.0-0.2-0.1 0.6 1.0
     1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25
    26  27  28  29  30  31  32  33  34  35  36  37  38  39  40  41  42  43  44  45  46  47  48  49  50
    51  52  53  54  55  56  57  58  59  60
```

5. Discussion of Results.

The dominant conclusion from examining the LAGEOS tracking error analysis results is that for the magnitudes of the changes considered, the station errors are rather insensitive to change. The results are here considered and interpreted case by case, and station by station when appropriate. For this discussion refer to Tables 2, 3, and 4.

Case 1: Reference, Low LAGEOS Orbit.

For an assumed 1-sigma error of 5 cm in laser ranging observations, and 0.5 mm/sec in range-rate satellite-to-satellite tracking, the resulting errors in station position are about 1 cm in each coordinate. The radial uncertainty is about 20% larger, and the latitudinal error about 20% smaller than the longitudinal error. The radial error increased with distance from station 1. These results are optimistic because of the simplifications inherent in an error analysis, which omits many biases that are difficult to model, or that are unpredictable. However, the relative changes in errors between this case and later cases can be considered indicative of the real situation. The absolute position error (for station 1) was of slightly smaller magnitude than the relative position errors for the other stations -- a result of the fact that all stations' observations contribute information to the absolute position, and hence it is better determined than the relative position of a single station, which is determined only from its own observations. This holds true also in Case 9, when only four close stations (within 500 km) were included in the solution. The geopotential coefficient errors were reduced by an order of magnitude from their a-priori values.

Case 2: LAGEOS Inclination = 50°

A shift in inclination from 80° to 50° for the low LAGEOS orbit resulted in a shift of error from the latitudinal to the longitudinal component. This results from the larger variety of geometries (for obtaining the longitudinal component) inherent in an inclined orbit as opposed to a near polar orbit, and the lesser coverage in latitude for an orbit not going north of 50° latitude.

Case 3: Maximum Zenith Angle = 60°

The radial component was better determined than in Case 1 with a zenith limit of 75°, and the other components were less well determined (by 20% each). Because of the limitation to 5 observations per pass, the radial component was improved because the 5 range observations included larger radial components. In the real problem of continuous tracking to the zenith limit, the radial component would be as well determined for a zenith angle limit of 75°. Hence the significant result is that better angular locations are obtained when tracking to within 15° of the horizon. The bad trend for radial error components through all cases but Case 3 apparently results from the 5 observation per pass limit.

Case 4: GEOPAUSE Tracking

Replacing the geosynchronous satellite by a GEOPAUSE satellite resulted in a slight increase in all components of the error. This is accountable, at least in part, to a decrease in the number of observations of the high satellite because it is not always in view.

Case 5: Apriori Geopotential Errors Larger ($.1x10^{-6}$, not $.4x10^{-7}$)

No significant change occurred.

Case 6: Two Stations in Australia Added

The radial error for stations 1000 km or so from station 1 improved significantly. The errors for the stations in Australia (stations 10 and 11) were about half again as large as those for nearby stations.

Case 7: No Geosynchronous Satellite

Leaving out the satellite-to-satellite tracking increased the station position error by about 20% in the radial component, and by less in other components.

Case 8: 3 Geosynchronous Satellites

No significant change occurred.

Case 9: 4 Stations (~ 500 km square)

The average error improved slightly, because the far stations (especially 9) have larger errors.

Case 10: (7,6) and (7,7) Harmonics Replace (3,1), (4,1)

No significant change occurred in station position error. The geopotential coefficient errors improved slightly.

Case 11: Same as Case 10 Without Satellite-to-Satellite Range Rate

The station errors were slightly worse than in the similar Case 7, indicating that the (7,6) and (7,7) harmonics require the satellite-to-satellite tracking more than do the (3,1), (4,1).

Case 1H: Reference, High LAGEOS Orbit

There was a slight (6%) increase in station error over that for the low orbit. The geopotential errors were doubled. The advantage of a low orbit is not strong, based on this factor alone.

Case 9H: 4 Stations, High LAGEOS Orbit

There was a smaller increase in station error, than in the case of 1H compared to 1.

Case 11H: No Satellite-to-Satellite Range Rate, High LAGEOS Orbit

The station errors were about the same as in Case 11. The geopotential errors were only slightly improved from their apriori values of $0.4x10^{-7}$.

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