NGR 01-001-020

ANALYTICAL STUDIES OF ULTRASHORT OPTICAL PULSE PROPAGATION IN A RESONANT ABSORBING MEDIUM

FINAL REPORT

Covering the Period September 1972-August 1974

Prepared for

The National Aeronautics and Space Administration

Washington, D. C.

By

Ching Tsung Lee

Department of Physics and Mathematics

Alabama Agricultural and Mechanical University

Normal, Alabama 35762

The NASA Technical Officer for this Grant:

Dr. Robert L. Kurtz

NASA-Marshall Space Flight Center, Alabama 35812

I. INTRODUCTION

One of the major causes for the degradation of laser pulses propagating through the atmosphere is the absorption of energy by the medium, especially the resonant absorption. It was recently discovered [1] that when a pulse is very short and intense, and of hyperbolic secant envelope, the resonant absorption can be reduced to zero. This is possible because, while the leading part of the pulse is absorbed to invert the atomic or molecular population, the trailing part induces stimulated emission and returns the absorbed energy back to the pulse, hence, there is no net absorption at all. This kind of effect is called Self-Induced Transparency (SIT).

Since the discovery of SIT, much attention has been given to the analytic descriptions of short pulse propagation in a resonant medium [2]. However, most of these works are based on Slowly Varying Envelope Approximation (SVEA) and in linear media. During the two year period covered in this report, we have extended the studies of SIT along two principal directions: First, to improve the SVEA by introducing what we call Short Pulse Approximation (SPA); Second, to consider the effect of nonlinear refractive index. These will be described in more detail in the following sections.

II. ULTRASHORT PULSE STUDY

For the study of picosecond pulses, the usual SVEA is no longer a good approximation. So we introduce the SPA by assuming

$$(c/V - 1)/\omega\tau \ll 1 \tag{1}$$

where c is the phase speed, V is the pulse speed, ω is the carrier frequency, and τ is the pulse duration.

Under the SPA and in the steady state, the coupled Maxwell-Bloch equations reduce to the following five first order differential equations:

$$\dot{\varepsilon} = -\alpha\omega v \tag{2}$$

$$\dot{\phi} = \alpha\omega u/\varepsilon + \alpha\kappa w \tag{3}$$

$$\dot{u} = \dot{\phi} v \tag{4}$$

$$\dot{v} = -\dot{\phi} u + \kappa\varepsilon w \tag{5}$$

$$\dot{w} = -\kappa\varepsilon v \tag{6}$$

In the above equations, ε is the envelope of the pulse, ϕ is the phase, u and -v are dimensionless dispersion and absoption components of the dipole moments, and w is the atom energy divided by $\hbar\omega/2$. These quantities depend on their arguments t and z only through the combination $\zeta = t - z/V$. The derivatives are with respect to ζ. The constants α and κ are defined as follows:

$$\alpha = \frac{2\pi Np}{c/V - 1} \qquad \kappa = 2p/\hbar \tag{7}$$

where N is the number of interacting atoms per unit volume and p is the transition dipole moment.

The analytic expressions of all possible solutions to the set of equations (2)-(6) are listed as follows:

A. Single Pulses

(a) $\varepsilon(\zeta) = (2/\kappa\tau)[\cosh^2(\zeta/\tau) - 1/2 + (1 - 1/\omega^2\tau^2)^{1/2}/2]^{-1/2}$

(b) $\varepsilon(\zeta) = (2/\kappa\tau)[\sinh^2(\zeta/\tau) + 1/2 - (1 - 1/\omega^2\tau^2)^{1/2}/2]^{-1/2}$

(c) $\varepsilon(\zeta) = (4\omega/\kappa)[4\omega^2\tau^2 + 1]^{-1/2}$

B. Pulse Trains

(a) $\varepsilon(\zeta) = (\sqrt{2}/\kappa\tau)[(1 + 1/4\omega^2\tau^2)^{1/2} - \cos(\zeta/\tau)]^{-1/2}$

(b) $\varepsilon(\zeta) = A\{[1 - \ell^2 \mathrm{sn}^2(\zeta/\tau,k)]/[1 + n^2 \mathrm{sn}^2(\zeta/\tau,k)]\}^{1/2}$

(c) $\varepsilon(\zeta) = A\{[1 + \ell^2 \mathrm{sn}^2(\zeta/\tau,k)]/[1 + n^2 \mathrm{sn}^2(\zeta/\tau,k)]\}^{1/2}$

(d) $\varepsilon(\zeta) = A\{[1 - \ell^2 \mathrm{sn}^2(\zeta/\tau,k)]/[1 - n^2 \mathrm{sn}^2(\zeta/\tau,k)]\}^{1/2}$

(e) $\varepsilon(\zeta) = B\{[1 + \ell^2 \mathrm{cn}(2\zeta/\tau,k)]/[1 - n^2 \mathrm{cn}(2\zeta/\tau,k)]\}^{1/2}$

where k is the modulus of the elliptic functions, ℓ and n are two parameters that determine the maxima and minima of the envelopes, and

$$A = (2/\kappa\tau)[2\omega\tau/(\ell^2 + n^2)]^{1/2}[n^2(k^2 + n^2)(1 + n^2)]^{1/4}$$

$$B = (2/\kappa\tau)[2\omega\tau/(\ell^2 + n^2)]^{1/2}[(1 - n^4)(k^2 + n^4 - k^2n^4)]^{1/4}$$

III. EFFECT OF NONLINEAR REFRACTIVE INDEX

The problem of the propagation of coherent pulses in absorbers with quadratic nonlinearity in the refractive index:

$$\eta_a = \eta_o(1 + \beta\varepsilon^2) \tag{1}$$

was first studied by Eberly and Matulic in 1969 [3]. We have reinvestigated the same problem and found that a term quadratic in optical Kerr constant β neglected by Eberly and Matulic can be very important for picosecond pulses. When this term is included, the pulse shape is different from the famous hyperbolic secant; namely

$$\varepsilon = (2/\kappa\tau)\{(1 + 64\gamma)^{1/2}[\cosh^2(\zeta/\tau) - 1/2] + 1/2\}^{-1/2} \qquad (2)$$

where

$$\gamma = (\beta\eta_0^2\hbar^3/64\pi N p^4)^2\tau^{-6} \qquad (3)$$

The peak value of the envelope is found to be

$$\varepsilon_0 = (1/\sqrt{8}\kappa\tau)\{[(1 + 64\gamma)^{1/2} - 1]/\gamma\}^{1/2} \qquad (3)$$

which is no longer inversely proportional to τ and will not increase indefinitely; but will reach a maximum and then decrease as τ shortens. This means that the nonlinearity of the refractive index imposes a limit to the peak intensity of a coherent pulse. We can locate this maximum point to be at

$$\bar{\tau} = (\hbar/\sqrt{8})[\beta\eta_0^2/\pi N p^4]^{1/3} \propto N^{-1/3} \qquad (4)$$

and calculate the maximum value of ε_0 to be

$$\bar{\varepsilon}_0 = (8\pi N p/\beta\eta_0^2)^{1/3} \propto N^{1/3} \qquad (5)$$

It should be very interesting to test experimentally the validity of our calculations, especially the dependence of $\bar{\tau}$ and $\bar{\varepsilon}_0$ on N.

The stability of our steady-state solution is still an open question. We have tried to investigated it but without any success so far because the mathematical problem involved is somewhat formidable. We suspect that the pulses might be unstable when $\tau < \bar{\tau}$. If this proves to be true, then the nonlinear refractive index also imposes a lower limit to the pulse duration.

IV. PUBLICATIONS

Three papers based on the results of this research project have been published in the "open" literature. They are listed as follows:

1. "Self-Induced Transparency of an Extremely Short Pulse"
   Optics Communications, Vol. 9, No. 1, PP. 1-3, September 1973.

2. "Four Possible Types of Pulses for Self-Induced Transparency"
   Optics Communications, Vol. 10, No. 2, PP. 111-113, February 1974.

3. "Limit Pulses in Passive Nonlinear Absorbers"
   Applied Physics Letters, Vol. 25, No. 4, PP. 222-224, 15 August 1974.

We plane to prepare two more papers for publication.

V. CONTINUATION

This research will be continued under a new number:

NSG-8011

We will continue to attack the stability problem of pulse propagation in nonlinear absorbers. We will also study the superradiant effects on pulse propagation in resonant media.

REFERENCES

[1] S. L. McCall and E. L. Hahn, Phys. Rev. Letters 18, 908 (1967); Phys. Rev. 183, 457 (1969).

[2] See the review article by G. L. Lamb, Jr., Rev. Mod. Phys. 43, 99 (1972), and some more recent papers such as: L. Matulic and J. H. Eberly, Phys. Rev. A6, 822 (1972); R. A. Marth, D. A. Holmes, and J. H. Eberly, Phys. Rev. A9, 2733 (1974); R. K. Bullough, P. J. Caudrey, J. C. Eilbeck, J. D. Gibbon, Opto-electronics 6, 121 (1974); Z. Bialynicka-Birula, Phys. Rev. A10, 999 (1974), etc.

[3] J. H. Eberly and L. Matulic, Opt. Commun. 1, 241 (1969).