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COHERENT OPTICAL INSTRUMENTATION FOR MEASUREMENTS OF PARTICLE PARAMETERS

FINAL TECHNICAL REPORT

By

William P. Chu

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Langley Research Center
Hampton, Virginia 23665

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William P. Chu[1]

SUMMARY

The objective of this research effort was to investigate the application of cross-beam laser doppler velocimeter (LDV) for sizing small particles. This report is a brief summary of research findings during the period from September 1973 to August 1974.

This report contains theoretical results obtained from analyzing the scattering characteristics of small particles in a cross-beam LDV system. Theoretical calculations based on scalar diffraction theory and Mie scattering theory have been performed. Experimental results are also obtained to compare with theoretical predictions. It is concluded that the forward scattering characteristics of small particles in a cross-beam LDV system can be used for particle sizing.

INTRODUCTION

Light scattering properties of small particles have been used extensively as a diagnostic tool for the measurements of particle size. In this report, we will investigate the scattering properties of small particles by two coherent cross-beams. The

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[1] Research Assistant Professor in Physics, School of Sciences, Old Dominion University, Norfolk, Virginia 23508.

problem is closely related to defining the characteristics of the doppler signals generated from a cross-beam LDV system. Previous investigation by Farmer [1] indicated that the modulation of the doppler signals is a unique function for different particle shapes and depends only on one parameter given by the ratio of particle geometric dimension to the fringe spacing formed in the probe volume of the LDV. In this report, we will show that Farmer's result is corrected only under some restrictive conditions. Only by measuring the forward scattering with a sufficiently large aperture system can particle size information then be obtained.

THEORETICAL ANALYSIS

Two approaches to analyzing the scattering of a spherical particle by two coherent cross-beams have been performed. The first approach is based on the scalar diffraction theory, which is an approximate theory valid for particle dimensions much larger than a wavelength. The second approach is based on an exact Mie scattering calculation which takes into account effects of index of refraction of the particles. The theoretical calculations will result in a doppler signal with the following general form

$$I(t) = \text{Const}\ (1 + V \cos 2\pi f_d t) \qquad (1)$$

where $I(t)$ is the doppler signal intensity, V is the visibility (or the modulation), and f_d is the doppler frequency. What we are interested in is the dependence of the visibility function V on parameters such as particle size, index of refraction, cross-beam angle, and the size of the receiving aperture.

1. Scalar Diffraction Theory

Consider in the input plane we have a spherical particle of radius a being illuminated by two unit amplitude plane waves with cross-beam angle of 2γ. The input field $U_{input}(x,y)$ can then be written as

$$U_{input}(x,y) = \left(e^{ik \sin \gamma x} + e^{-ik \sin \gamma x}\right) (1 - P(x - vt,y) \qquad (2)$$

where $P(x,y)$ is the transmission function of the particle, and v is the velocity assuming along the x-axis. For a spherical particle of radius a, $P(x,y) = \text{Circ}\,(r/a) = 1$ for $r \leq a$, zero otherwise. The diffracted wave at the far-field plane $(z = z_f)$ will then be given by the following Fourier transform expression [2],

$$U_{diff}(x_f,y_f) = (\text{Const phase})/\lambda z_f \iint_{-\infty}^{\infty} U_{input}(x,y)\, e^{-(2\pi i/\lambda z_f)(x_f x + y_f y)}\, dxdy \qquad (3)$$

Let us consider the integrated total scattering due to the particle in the forward half plane. As is well known in diffraction theory, most of the diffracted energy from a scattering object is contained within the first forward lobe of the diffraction pattern. This forward lobe extends to an angle approximately equal to λ/D, where D is the dimension of the object. Thus the total scattering signal should be approximately equal to the signal obtained by collecting the forward scattering within a cone of full angle greater than $2(\gamma + \lambda/D)$.

The total scattering intensity $I(t)$ can be calculated from equation (3). Using Parseval's theorem, we obtained the following result:

$$I(t) = 2\iint_{-\infty}^{\infty} dxdy\, (1 + \cos 2xk \sin \gamma)\, P^2\, (x - vt,y) \qquad (4)$$

Equation (4) is a very interesting result. It illustrates the fact that the integrated scattering signal is equivalent to the signal obtained by scanning a set of sinusoidal fringes of spacing $\delta = \lambda/2 \sin \gamma$ with a small aperture of a shape the same as the particle geometrical cross-section. For a spherical particle of radius a, we have $P^2(x,y) = \mathrm{Circ}\,(r/a)$. After some calculations, we then arrive at the visibility function of the doppler signal as

$$V_{\text{spherical particle}} = 2J_1\,(2a\pi/\delta)/(2a\pi/\delta) \tag{5}$$

where J_1 is the first order Bessel function. Figure 1 shows the visibility function versus the parameter $2a/\delta$. These results are similar to Farmer's results; however, they are corrected only for the total scattering signals.

Next we will consider the case of collecting the scattering along the forward direction with a finite size aperture. The resulting signals in this case can be calculated by integrating the scattering intensity across the aperture area. These can be done directly from equation (3), and the results are summarized in Figure 2. It shows the visibility function for spherical particles versus the size of the aperture for different values of $2a/\delta$. Notice that as the aperture sustains a cone of full angle greater than $2(\gamma + \lambda/2a)$, the values of V approach the limiting values as given by equation (5).

2. Mie Scattering Theory

The problem here is to solve the Maxwell's equations under the appropriate boundary conditions. For single beam scattering, the calculation is the well documented Mie calculation based on the multipole expansion of the electro-magnetic field. For the present case, we have two linearly polarized incident plane waves of the same polarization with cross-beam angle of 2γ. Assuming a coordinate system symmetric to the two incident plane waves, each plane wave can then be expanded into the following form [3]

$$\vec{E} = \hat{i}\, e^{i\vec{k}\cdot\vec{x}}$$

$$= \hat{i}\, 4\pi \sum_{\ell} i^{\ell}\, j_{\ell}(kr) \sum_{m} Y_{\ell m}(\Theta, \phi)\, Y_{\ell m}(\gamma, \pi/2) \qquad (6)$$

where $Y_{\ell m}$ is the spherical harmonic, and j_{ℓ} is the spherical Bessel function of order ℓ. Using equation (6) as the expansion for the incident plane waves, we can then construct the scattering fields and apply the appropriate boundary conditions for a spherical scattering center with radius a and index of refraction $n = n_r - i\beta$, where n_r is the real index and $-\beta$ is the imaginary index. The complete calculation is sufficiently lengthy and will not be presented here. We will try to summarize some of the results.

For total scattering, where we integrated over 4π steradian, the visibility function has the following interesting expression

$$V = \frac{\sum_{\ell} \frac{(2\ell + 1)}{\ell(\ell + 1)} \left[|a_{\ell}|^2\, \pi_{\ell}\, (\cos 2\gamma) + |b_{\ell}|^2\, \tau_{\ell}\, (\cos 2\gamma) \right]}{\sum_{\ell} (2\ell + 1)(|a_{\ell}|^2 + |b_{\ell}|^2)} \qquad (7)$$

where a_{ℓ} and b_{ℓ} are the standard Mie A and B coefficients, π_{ℓ} and τ_{ℓ} are similarly the Mie angular function. Notice the similarity of equation (7) to the single beam Mie scattering field expressions [4].

Numerical solutions to equation (7) have been obtained and some of the results are shown in Figures 3 to 6. Figures 3 and 4 are results obtained for a cross-beam angle of 0.2 radian, whereas Figures 5 and 6 are for a cross-beam angle of 0.1 radian. The visibility functions show strong oscillation for no absorption. The oscillation decreases rapidly as absorption is introduced or as the cross-beam angle is decreased. The general shape of the visibility functions is very similar to Figure 1 with results obtained from scalar diffraction theory.

Calculations of the visibility function for a detector receiving a small cone angle of the forward scattering have also been done. The results for varying particle size with a full cone angle of 0.5 radian are shown in Figure 7. The results are similar to those shown in Figure 3.

Figure 8 shows the change of the visibility function for a fixed diameter particle with different receiver's cone angles in both the forward and backward directions. For forward scattering, the similarity with results from scalar diffraction calculations is obvious. However, in the back scattering case, no information on sizing can be obtained from measurements of the visibility function.

EXPERIMENTAL RESULTS

A standard cross-beam LDV system has been constructed to measure the visibility function for spherical particles of different diameters. The glass particles are deposited on a good quality AR-coated glass plate. The glass plate with different particles on it was then slowly scanned across the probe volume of the LDV system. Measurements of the visibility functions have been done with this LDV system for different fringe spacings, particles with different sizes, and different detector's receiving cone angles. All experiments were performed with detector's optics in the forward scattering direction. Figure 9 shows the resulting values of visibility versus $2a/\delta$ for a receiver's full cone angle of 0.64 radian. Equation (5) is also shown for comparison. Figure 10 shows the effect of varying the receiving cone angle on the value of the visibility function for a fixed size particle. Theoretical results identical to those shown in Figure 8 are also shown for comparison.

CONCLUSION

Theoretical calculations based on both scalar diffraction theory and the exact Mie scattering theory show that the forward scattering characteristics of a small particle in a cross-beam LDV system can be used for particle sizing. The exact Mie calculations indicate that effects due to index of refraction are small provided the cross-beam angle is small. Both theories predict similar scattering characteristics with varying receiver's aperture size.

The materials in this report are being prepared for publication in technical journals. The journal articles will provide a detailed description of the calculations involved in deriving most of the results presented in this report.

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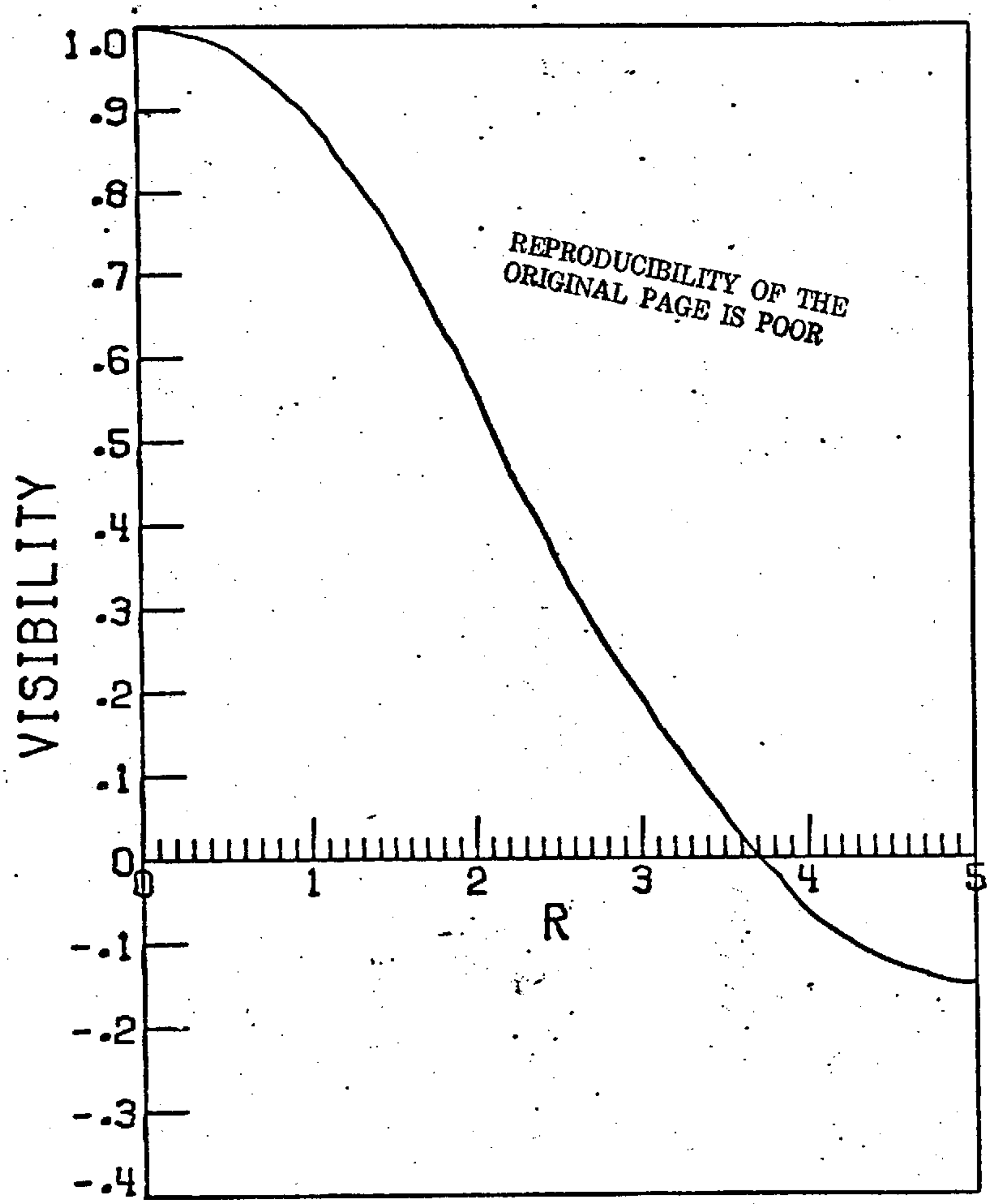

Figure 1. Visibility function versus the parameter $R = 2a/\delta$ for spherical particles of radius a, and δ is the fringe spacing.

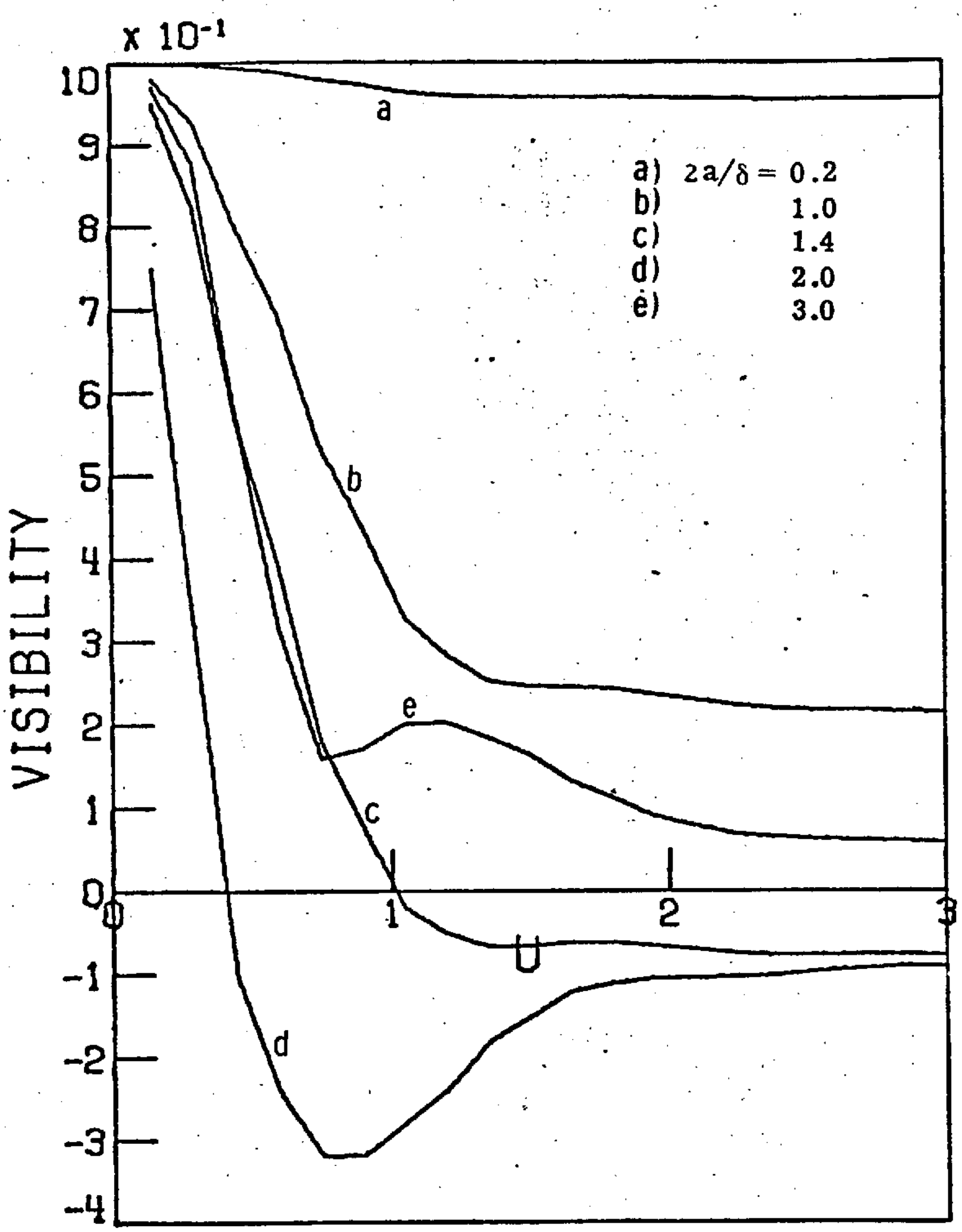

Figure 2. The visibility functions for spherical particle versus the detector's aperture size for five different values of $2a/\delta$. $U = 2ar_f/\lambda z_f$, where r_f is the radius of the detector's aperture.

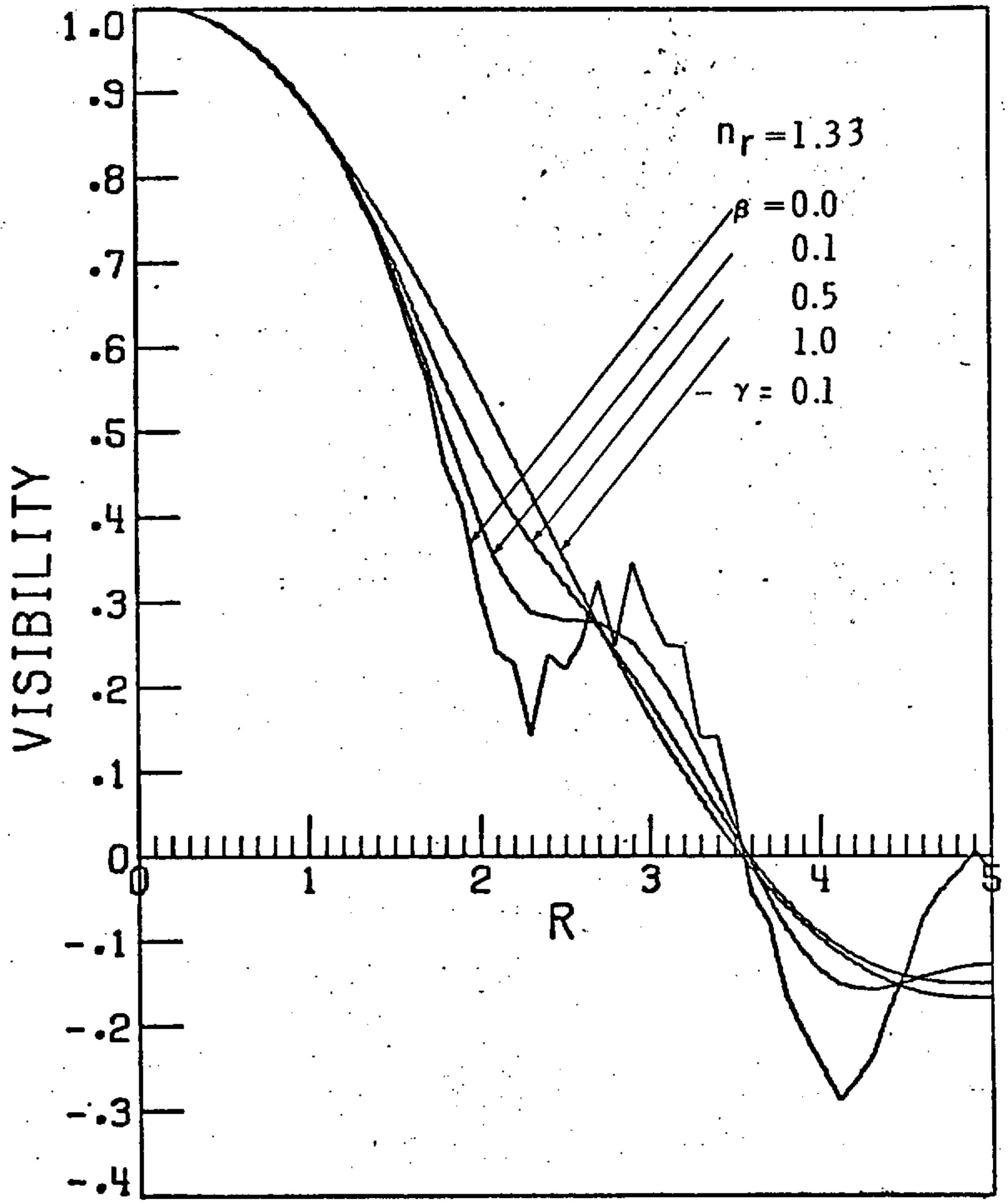

Figure 3. Visibility functions calculated from Mie scattering theory versus parameter $2a/\delta$ for real index $n_r = 1.33$ and four different imaginary indices. The cross-beam angle $2\gamma = 0.2$ radian.

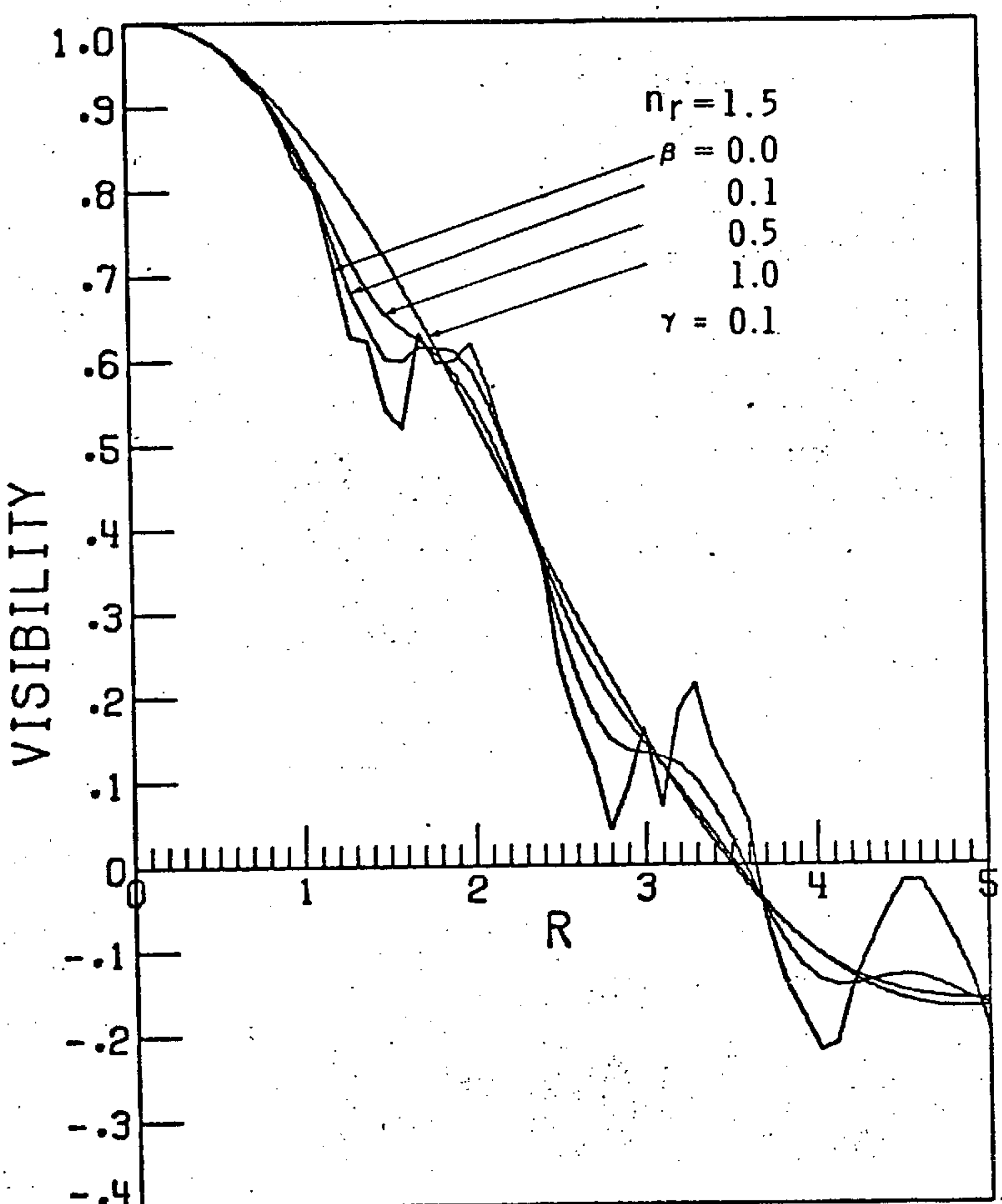

Figure 4. Visibility functions calculated from Mie scattering theory versus parameter $2a/\delta$ for real index $n_r = 1.5$ and four different imaginary indices. The cross-beam angle $2\gamma = 0.2$ radian.

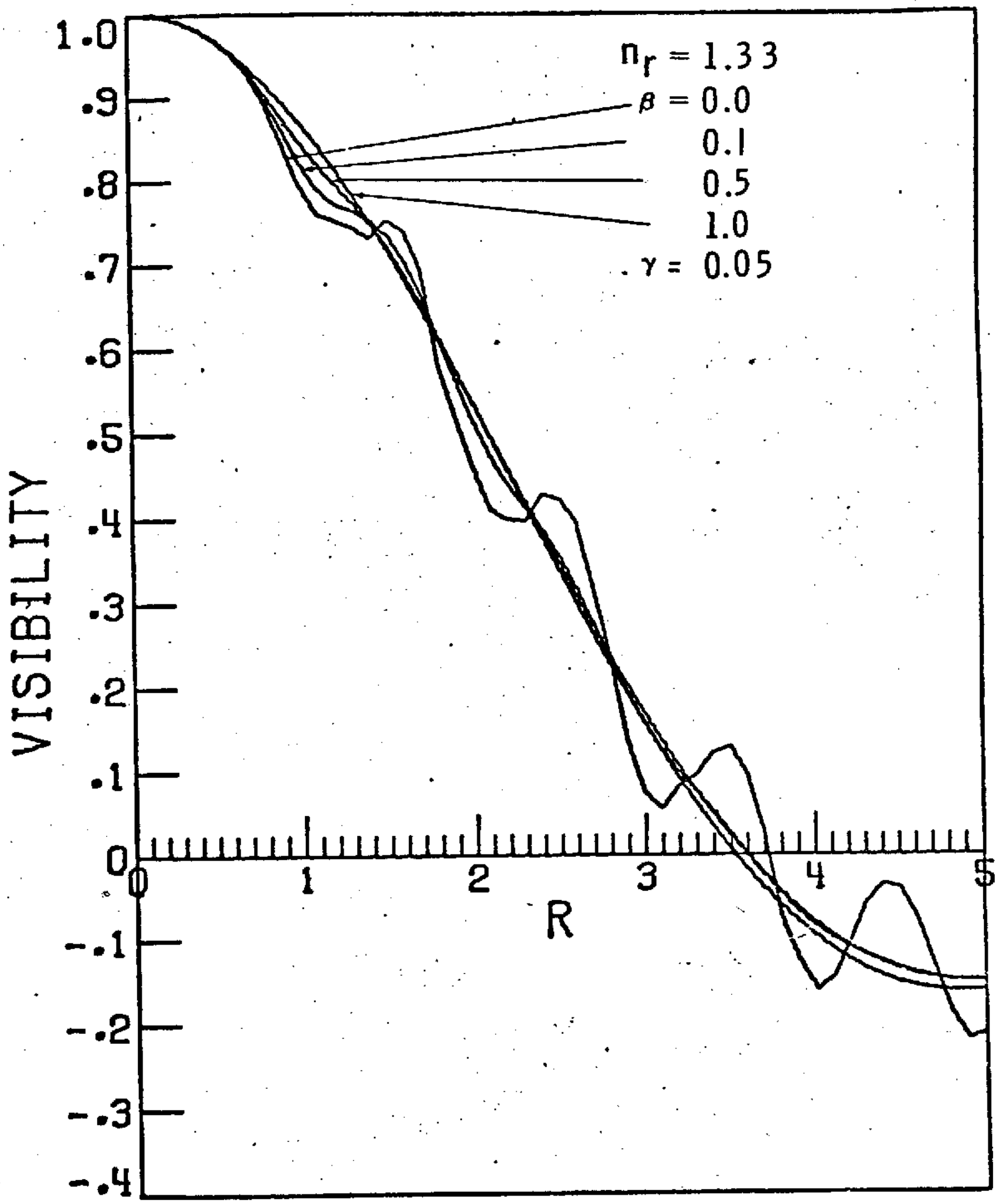

Figure 5. Visibility functions calculated from Mie scattering theory versus parameter $2a/\delta$ for real index $n_r = 1.33$ and four different imaginary indices. The cross-beam angle $2\gamma = 0.1$ radian.

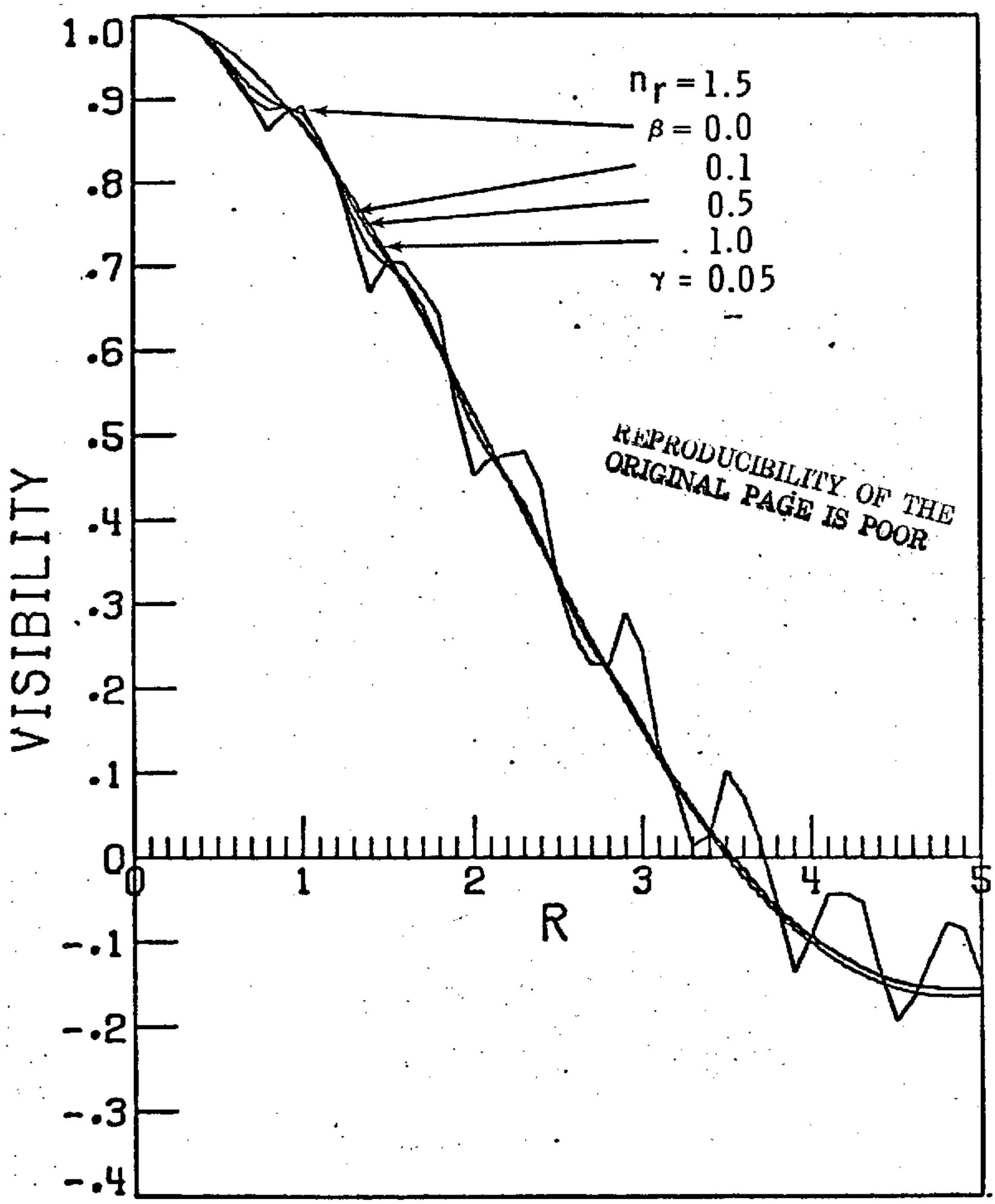

Figure 6. Visibility functions calculated from Mie scattering theory versus parameter $2a/\delta$ for real index $n_r = 1.5$ and four different imaginary indices. The cross-beam angle $2\gamma = 0.1$ radian.

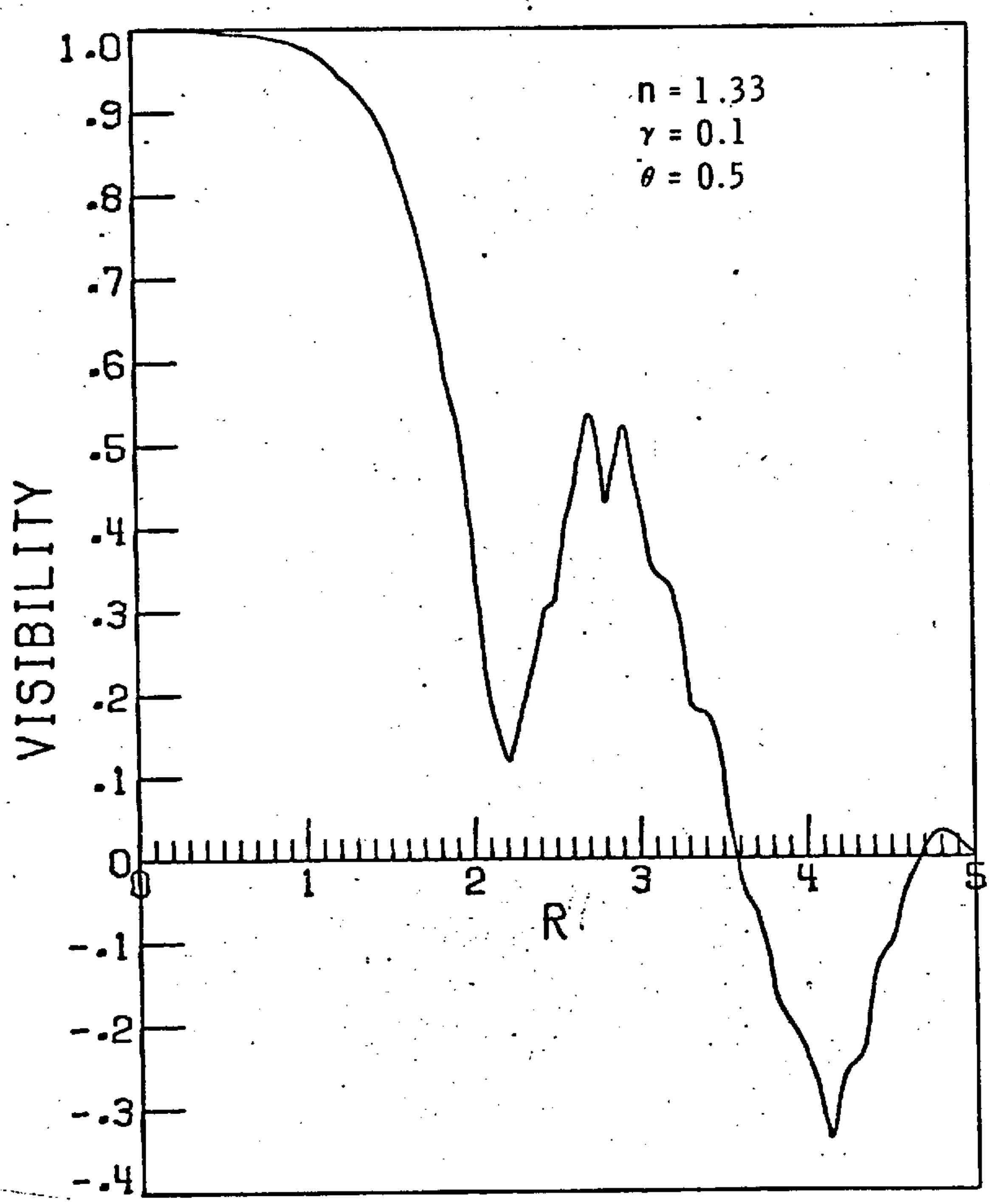

Figure 7. The visibility function calculated from Mie theory for a detector with full cone angle of 0.5 radian in the forward direction versus the parameter $R = 2a/\delta$. The real index is 1.33 and imaginary index is zero.

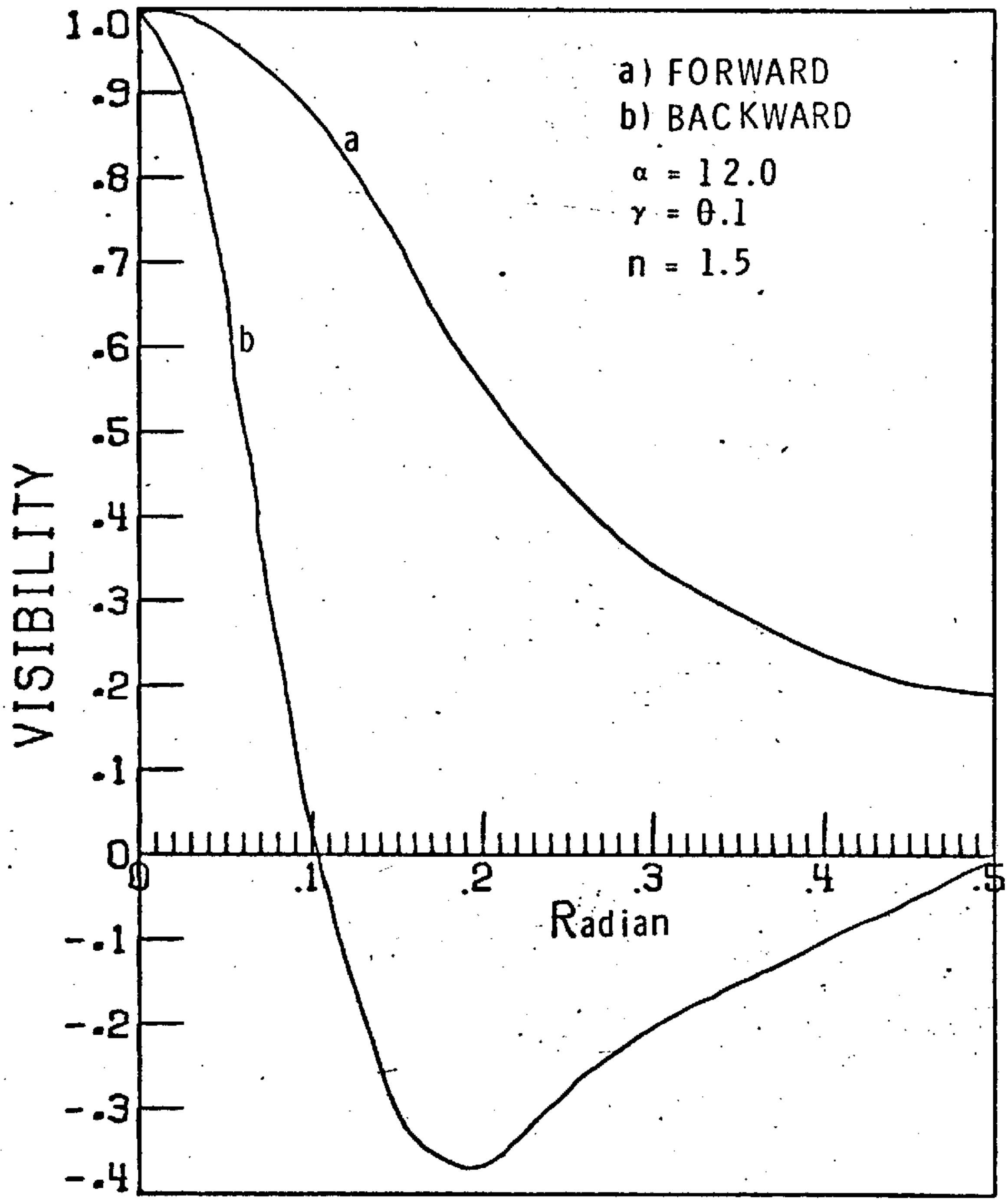

Figure 8. The visibility function versus collecting optics half cone angle in radian for both the forward and back scattering directions with a spherical particle of $ka = 12.0$ and index of refraction $n = 1.5$. The cross-beam angle in this case is $2\gamma = 0.2$.

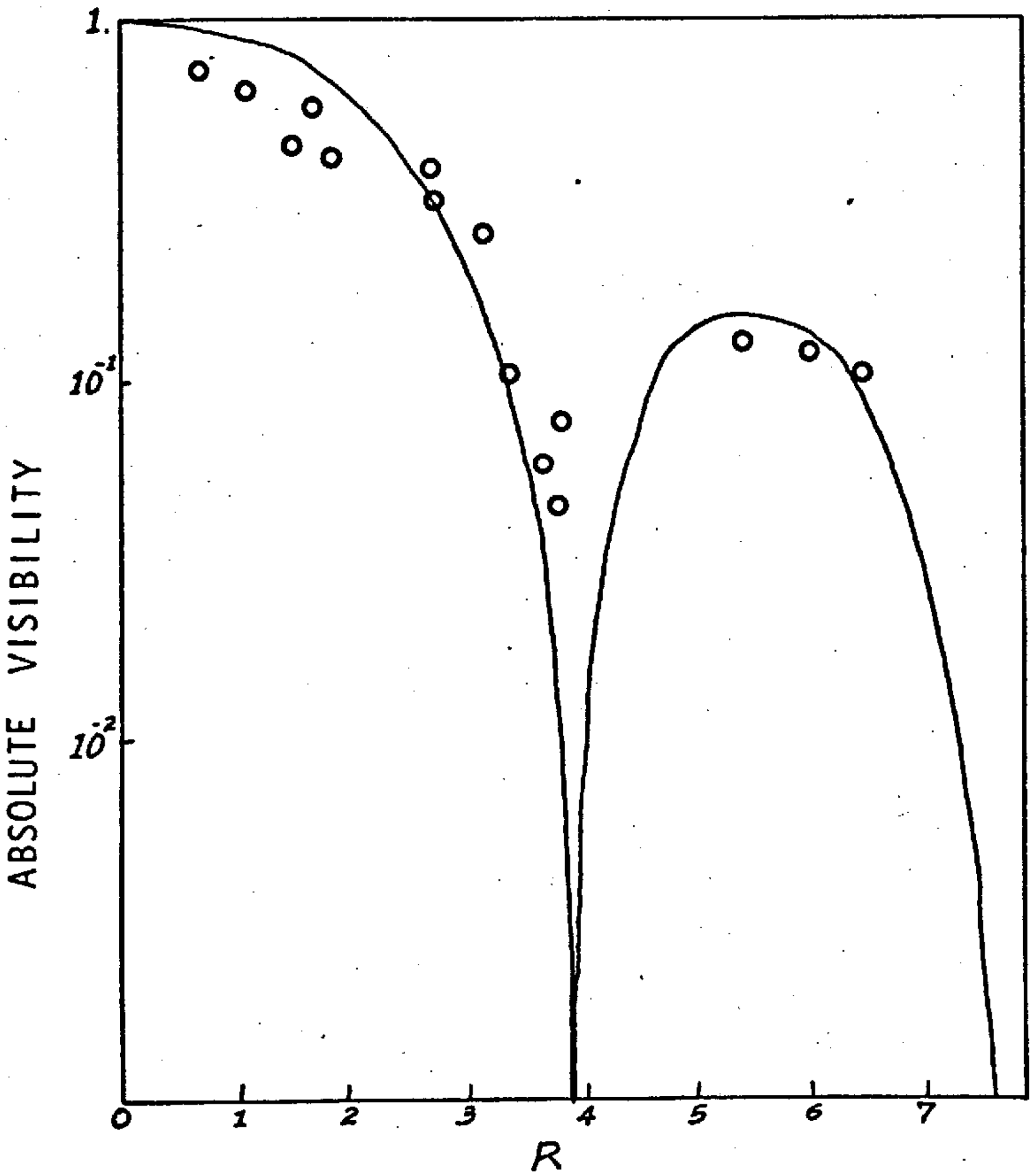

Figure 9. Experimental results from the measurements of the absolute visibility for different values of $R = 2a/\delta$. Equation (5) is shown for comparison.

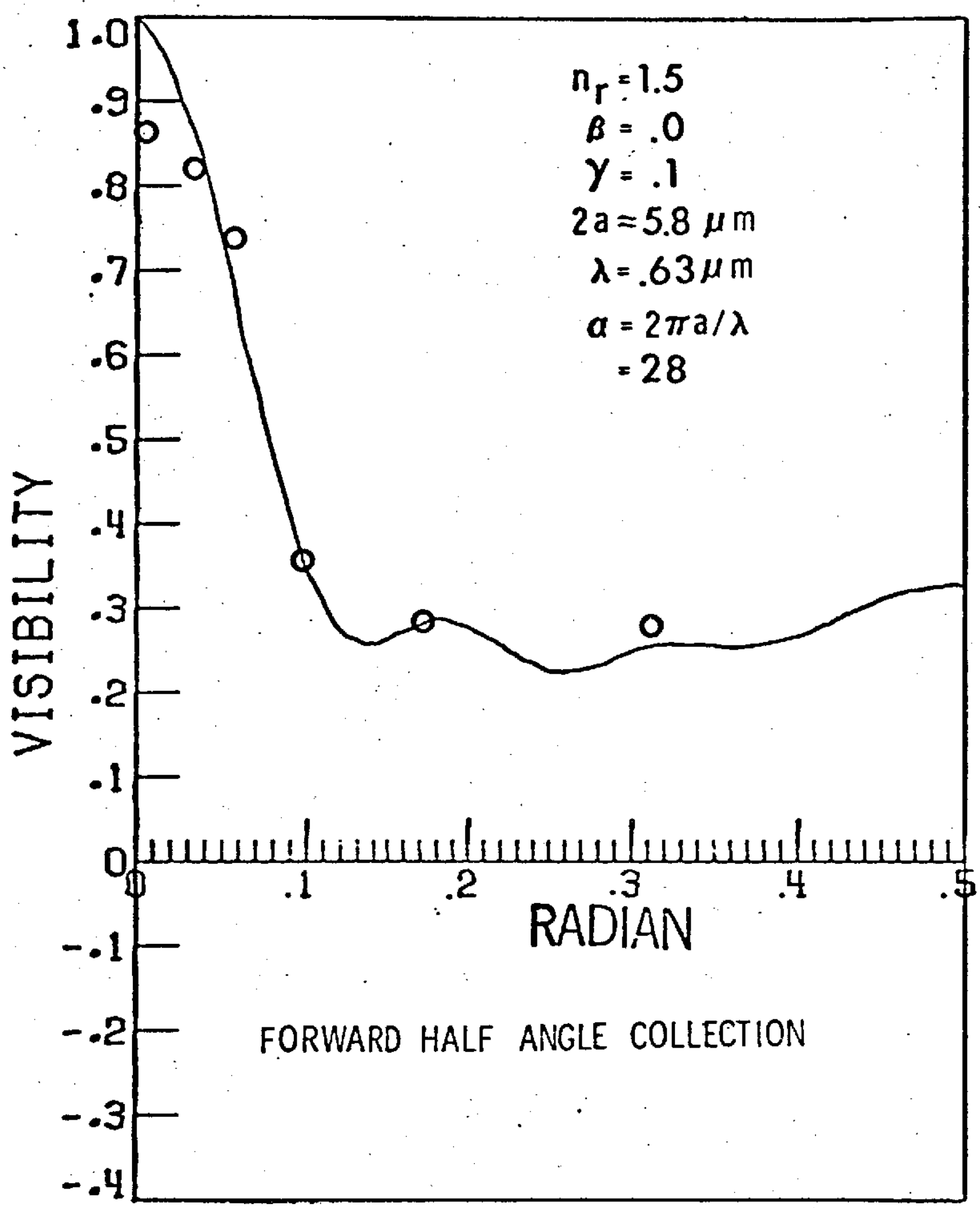

Figure 10. Experimental results on the change of visibility as a function of receiving optics half cone angle in radian.