# A Note on the Annual Cycles of Surface Heat Balance And Temperature over a Continent[1,2]

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# Abstract

A surface heating function, defined as the ratio of the time derivative of the mean annual temperature curve to the surface heat balance, is computed from the annual temperature range and heat balance data for the North American continent. An annual cycle of the surface heat balance is then reconstructed from the surface heating function and the annual temperature curve, and an annual cycle of evaporative plus turbulent heat loss is recomputed from the annual cycles of radiation balance and surface heat balance for the continent. The implications of these results for long-range weather forecasting are discussed.

## Introduction

Reliable prediction of monthly mean surface temperature anomalies (i.e., departures from the climatological normals) is one of the major goals of long-range weather forecasting. To be of practical use, such forecasts should probably depict the spatial distribution of expected anomalies with a resolution no coarser than $10^2$ - $10^3$ km. However, accurate long-range prediction on a scale that small appears rather remote at this time. It therefore seems reasonable to accept a more modest objective and to investigate the possibility of forecasting monthly mean surface temperatures averaged over much larger areas, e.g., areas of continental scale. While such forecasts can be of little practical value, continental scale surface temperature prediction may provide a useful test problem for the development of physical methods of long-range weather prediction.

The problem of forecasting the surface ground temperature of a continent is formally simpler than that of forecasting the surface air temperature, as no advective terms appear explicitly in the thermodynamic equation for the ground. In view of the crude state of the art, it will therefore be assumed that the ground and surface air are in thermal equilibrium on a monthly time scale, and that the two temperatures are identical.

The physical law governing the variation of surface temperature with time, t, may be written in simplest form as

$$\frac{\partial u}{\partial t} = k h, \tag{1}$$

where u is the daily[4] surface temperature, h is the daily surface heat balance, and k is a corresponding surface heating function. If h is expressed in langleys per unit time, k is in units of degrees per langley ($\mathrm{deg\,ly}^{-1}$). The surface heating function, k, may be interpreted as the reciprocal of the product of an average heat capacity and an "equivalent depth of thermal influence" of the continental surface layer. (See Appendix).

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[4] In place of the "daily" temperature one may substitute the temperature observed on any observational schedule, e.g., hourly, twice-daily, etc.

However, it is probably simpler merely to consider equation (1) as the definition of k. The surface heat balance, h, may be written as

$$h = r - e - p \tag{2}$$

where r is the daily surface radiation balance, e the evaporative heat loss by the surface, and p the turbulent ("sensible") heat loss. (See, e.g., Sellers, 1965.)

Let a bar denote the monthly mean value, and a prime the daily deviation from the monthly mean. Also, let a capital letter with an asterisk denote a climatological monthly normal, while a capital letter without asterisk denotes the monthly anomaly. Then,

$$\begin{array}{lll} u = \bar{u} + u', & h = \bar{k} + k', & h = \bar{h} + h', \\ \bar{u} = U + U^{*}, & \bar{k} = K + K^{*}, & \bar{h} = H + H^{*}, \\ \text{etc.} & & \end{array} \tag{3}$$

From (1) and (3)

$$\frac{\partial U^{*}}{\partial t} + \frac{\partial U}{\partial t} = K^{*} H^{*} + KH^{*} + K^{*}H + K\,H + (\overline{k'h'}). \tag{4}$$

It will be assumed that, for the climatological normal values,

$$\frac{\partial U^{*}}{\partial t} = K^{*} H^{*} , \tag{5}$$

which may, in fact, be considered to be a definition of $K^{*}$. Furthermore, it is probably safe to assume that $(\overline{k'h'})$ may be neglected in (4) and that $K << K^{*}$. Thus, an approximate prognostic equation for the monthly surface temperature anomaly is

$$\frac{\partial U}{\partial t} = K H^{*} + K^{*} H , \tag{6}$$

which might perhaps provide a basis for the prediction of monthly surface temperature anomalies by one-step time extrapolation. (Experimental one-step predictions of monthly temperature anomalies, based on a more elaborate physical model, have been described by Adem (1965, 1970).).

The evaluation of (6) requires a knowledge of the monthly anomalies of the surface heat balance and surface heating function, as well as their monthly climatological values. However, an effort (described below) to evaluate $K^*$, as well as $H^*$, for each month of the year from climatological data was not successful, and indeed it proved necessary to assume a constant value of $K^*$ all year round in order to derive a consistent set of monthly $H^*$ values. In view of the need for such crude approximations, it appears futile, at this time, to attempt to evaluate K. The approximate prognostic equation is thus reduced to

$$\frac{\partial U}{\partial t} = K^* H, \tag{7}$$

where $K^*$ is treated as a constant all year round.

The practical applications of (7), including the evaluation of H and the extrapolation of U, are beyond the scope of this paper. We have, however, attempted to evaluate monthly values of $H^*$ and a mean annual value of $K^*$ over a continental area, namely the greater part of North America. The results of that effort are described in this note.

## Calculations and Results

Maps of mean monthly surface (air) temperature are available in a variety of climatological atlases. For the continental mean monthly surface temperature calculations we have used the Northern Hemisphere atlas of Crutcher and Meserve (1970). The most comprehensive source of global surface heat balance data is the atlas of Budyko (1963), in which are presented global maps of the mean monthly components of the surface heat balance, including the surface radiation balance and the evaporative and turbulent heat losses.

Mean monthly values of the temperatures and surface heat balance components were numerically integrated with respect to area over North America (based on interpolations between isopleths and summations over a 5 degree latitude - longitude grid) between latitudes 30 N and 60 N. Budyko (1963) has computed the average annual values of the heat balance components over all continental and oceanic areas, but does not present the monthly values.

The continental average monthly values of the surface radiation balance, $R^*$, the evaporative heat loss, $E^*$, and the turbulent heat loss, $P^*$, were computed separately, and the surface heat balance, $H^*$, was calculated from

$$H^* = R^* - (E^* + P^*) \quad . \tag{8}$$

The monthly heat balances were then added to obtain the annual heat balance, which should be zero. As shown in Table 1, this sum was not quite zero. A monthly correction was therefore computed by dividing the sum by twelve, and this constant factor was then subtracted from each monthly value to obtain a set of adjusted monthly heat balances for which the annual sum is effectively zero (except for roundoff). The correction is undoubtedly smaller than the errors of estimation, such as those resulting from interpolation between isopleths, and does not exceed 10% for any month. The computed values of $R^*$, $E^* + P^*$, $H^*$, and the adjusted value, $H^*_a$, together with the monthly values of $U^*$, are shown in Table 1. The annual values of $R^*$ and

$E^* + P^*$ in Table 1 are about 10 percent higher than those computed by Budyko (1963) for North America, probably due to our use of 60° N as the northern limit of integration.

The original intention of this study was to compute $K^*$ for each month from equation (5), with $H^*$ calculated from (8) and the temperature derivative computed from the annual temperature curve; but this was not possible. On physical grounds, $K^*$ should be positive and finite for all months. Therefore, $H^*$ should be zero at the times of maximum and minimum monthly mean temperature, with positive maximum during the spring warming and negative minimum during the fall cooling, i.e., approximately one season out of phase with the temperature curve. However, as shown in Table 1, the pnase relations between $H^*$ and $\partial U^*/\partial t$ appear to be unrealistic, and it is impossible to compute a set of reasonable monthly values of $K^*$ from equation (5) and the tabulated data. For example, a direct computation of $K^*$ would give values of zero twice a year, at the times of minimum and maximum temperature, and infinity twice a year, when $H^*$ reverses sign. Furthermore, negative values of $K^*$ would be computed for some months. Part of the difficulty undoubtedly arises because $H^*$ is computed as the relatively small difference, in most months, between two quantities, $R^*$ and $E^* + P^*$, which are, in most months, of the same sign and approximately equal magnitude. The computed annual cycle of $R^*$ appears to be realistic, but that of $E^* + P^*$ is far less reliable. Thus, the difference, $H^*$, exhibits an unrealistic annual cycle.

To calculate a constant annual value of $K^*$, it was assumed that the integral of the computed surface energy balance over the 6-month period from the January temperature minimum to the July temperature maximum is a reliable quantity. $K^*$ was thus assumed to be equal to the ratio of the annual temperature range, in degrees Celsius, divided by the total surface heat balance, in kilolangleys (kly), over the same half-year period. (Since the temperature range is in effect calculated between mid-months, the half-year heat balance is computed as the sum of the monthly values for February through June plus half the sum of the values for January and July.) From Table 1, the mean annual temperature range from mid-January to mid-July over North America is 28.61°C, and the half-year surface heat balance is 2.39 kly. Thus, $K^*$ is found to be 12 deg C $\text{kly}^{-1}$. (The same numerical value is, of course, obtained if the calculation is done for the 6-month cooling period from July to January, as the annual sum of $H_a^*$ is effectively zero.)

Table 1. Monthly computed values of surface radiation balance ($R^*$), evaporative plus turbulent heat loss ($E^* + P^*$), unadjusted surface heat balance, $H^*$, and adjusted heat balance, $H_a^*$, in kilolangleys per month ($\text{kly mo}^{-1}$), for North America between latitudes 30 N and 60 N, derived from Budyko (1963). Also shown are monthly mean temperatures, $U^*$, in degrees Celsius (°C), derived from Crutcher and Meserve (1970), for the same region.

| Month | $R^*$ | $(E^* + P^*)$ | $H^*$ | $H_a^*$ | $U^*$ |
|---|---|---|---|---|---|
| Jan | - 0.28 | + 0.72 | - 1.00 | - 0.96 | - 9.27 |
| Feb | + 0.73 | + 1.49 | - 0.76 | - 0.72 | - 7.57 |
| Mar | + 2.60 | + 2.75 | - 0.15 | - 0.11 | - 1.45 |
| Apr | + 4.75 | + 3.72 | + 1.03 | + 1.08 | + 5.47 |
| May | + 6.74 | + 5.73 | + 1.01 | + 1.06 | + 11.55 |
| Jun | + 8.07 | + 6.86 | + 1.21 | + 1.26 | + 16.04 |
| Jul | + 7.90 | + 7.35 | + 0.55 | + 0.60 | + 19.34 |
| Aug | + 6.45 | + 5.85 | + 0.60 | + 0.65 | + 18.80 |
| Sep | + 4.29 | + 4.69 | - 0.40 | - 0.36 | + 13.82 |
| Oct | + 2.19 | + 3.07 | - 0.88 | - 0.84 | + 8.21 |
| Nov | + 0.63 | + 1.43 | - 0.80 | - 0.76 | + 0.21 |
| Dec | - 0.04 | + 0.91 | - 0.95 | - 0.91 | - 6.03 |
| Sum | 44.03 | 44.57 | - 0.54 | - 0.01 | |

Correction: 0.54/12 = 0.045

On the assumption that $K^*$ is constant throughout the year, it is now possible to calculate new monthly values of $H^*$ from equation (5) and the monthly mean temperatures. For this calculation, the results of which are shown in Table 2, $\partial U^*/\partial t$ was evaluated for each month by centered differences over two months. From the new surface heat balance, $H_c^*$, and the presumably reliable radiation balances in Table 1, it is now possible to calculate a new set of values of $E^* + P^*$. These estimates, designated as $(E^* + P^*)_c$, are also listed in Table 2. The annual cycle of $(E^* + P^*)_c$ in Table 2 appears to be at least as reasonable as that of $(E^* + P^*)$ in Table 1.

The results of the calculations are illustrated in Figure 1, showing the annual cycles of $U^*$, $R^*$, $H_a^*$, and $H_c^*$, and in Figure 2, in which are plotted the annual cycles of $R^*$, $H_c^*$, and $(E^* + P^*)_c$. It should be noted that the scales for $R^*$ and $H^*$ are not the same in Figure 1, whereas a uniform scale is used in Figure 2.

From the new values of $H_c^*$ in Table 2, the half-year surface heat balance is found to be 2.20 kly rather than 2.39 kly. If this value is used to calculate $K^*$, the heating function is found to be 13 deg $\text{kly}^{-1}$ rather than 12 deg $\text{kly}^{-1}$.

## Discussion

Equation (7) represents, in its simplest form, the basis for thermodynamic prediction of monthly surface temperature anomalies over a large continental area. If the estimated value of $K^*$ (13 deg C $\text{kly}^{-1}$) is correct, an anomaly of about $\pm 0.1$ kly $\text{mo}^{-1}$ in the surface heat balance is required to produce a surface temperature anomaly tendency of $\pm$ 1 deg C $\text{mo}^{-1}$. This energy flux represents at least 20 percent of the normal monthly surface heat balance over North America. However, a heat balance anomaly of 0.1 kly $\text{mo}^{-1}$ also represents less than 5 percent of the normal monthly surface radiation balance (or the normal monthly evaporative plus turbulent heat loss) over the continent in any of the months from March through October.

Thus, a forecasting system based on the monitoring of radiative fluxes requires very high precision in the radiation measurements. An even greater difficulty lies in the evaluation of the evaporative plus turbulent heat losses.

It is apparent from Figure 2 that the monthly normal values of $R^*$ and $(E^* + P^*)_c$ are highly correlated and just slightly out of phase. The annual temperature cycle is seen to be the result of only a slight imbalance between surface heat sources and sinks. At the present time no program exists for monitoring these surface energy fluxes on a large scale. Even meteorological satellites are, at best, capable of measuring only the radiation balances at satellite altitude. Thus, the direct evaluation of monthly anomalies of surface heat balance over a continent is clearly a formidable problem, and does not appear to represent a viable approach to routine long-range temperature prediction. However, the thermodynamic method may be useful when anomalies in the surface energy fluxes constitute large fractions of the normal surface fluxes. Such extreme anomalous events are most likely to occur in winter. Unfortunately, very little is known about month-to-month and year-to-year variations in the surface energy fluxes over continents. Thus, there is as yet little or no empirical basis for thermodynamic prediction by the method described above.

Table 2. Monthly surface heat balance, $H^*_c$, computed from estimated $K^*$ (12 deg $kly^{-1}$) and mean annual temperature curve for North America. Also shown are monthly estimates of $E^* + P^*$ computed from $(E^* + P^*)_c = R^* - H^*_c$ [illegible]ts are kly $mo^{-1}$.

| Month | $H^*_c$ | $(E^* + P^*)_c$ |
|---|---|---|
| Jan | - 0.06 | - 0.22 |
| Feb | + 0.33 | + 0.40 |
| Mar | + 0.54 | + 2.06 |
| Apr | + 0.54 | + 4.21 |
| May | + 0.44 | + 6.30 |
| Jun | + 0.32 | + 7.75 |
| Jul | + 0.12 | + 7.78 |
| Aug | - 0.23 | + 6.68 |
| Sep | - 0.44 | + 4.73 |
| Oct | - 0.57 | + 2.76 |
| Nov | - 0.59 | + 1.22 |
| Dec | - 0.40 | + 0.36 |
| Sum | 0.00 | 44.03 |

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# Figures

Fig. 1 Annual cycles of mean monthly temperature, $T^*$ (°C), surface radiation balance, $R^*$ (kly $\text{mo}^{-1}$), surface heat balance, $H_a^*$ (kly $\text{mo}^{-1}$), derived from Budyko (1963) atlas only, and recomputed surface heat balance, $H_c^*$ (kly $\text{mo}^{-1}$) derived from Budyko data and annual temperature cycle for the North America continent between latitudes 30N and 60N.

Fig. 2 Annual cycles of surface radiation balance, $R^*$ (kly $\text{mo}^{-1}$), recomputed surface heat balance, $H_c^*$ (kly $\text{mo}^{-1}$), and recomputed evaporative plus turbulent heat loss, $(E^* + P^*)$ c for the North American continent between latitudes 30N and 60N.

## Appendix

The heat conduction equation for the surface layer of a continent may be written as

$$\frac{\partial T}{\partial t} = \frac{1}{C} \frac{\partial q}{\partial z}$$

where T, the earth temperature, and q, the vertical heat flux, are both functions of depth, z, as well as time, t, and C is the heat capacity in the layer. The heat equation may be integrated from the earth's surface, where q = h, down to a depth, L, where q = 0. Thus, for a homogeneous layer,

$$\int_0^L \frac{\partial T}{\partial t}\, dz = \frac{1}{C} \int_0^L \frac{\partial q}{\partial z}\, dz = \frac{h}{C}$$

For the mean value theorem,

$$\int_0^L \frac{\partial T}{\partial t}\, dz = \left( \overline{\frac{\partial T}{\partial t}} \right) \cdot L$$

where the bar denotes a mean value with respect to depth. Let an "equivalent depth of thermal influence", D, be defined such that

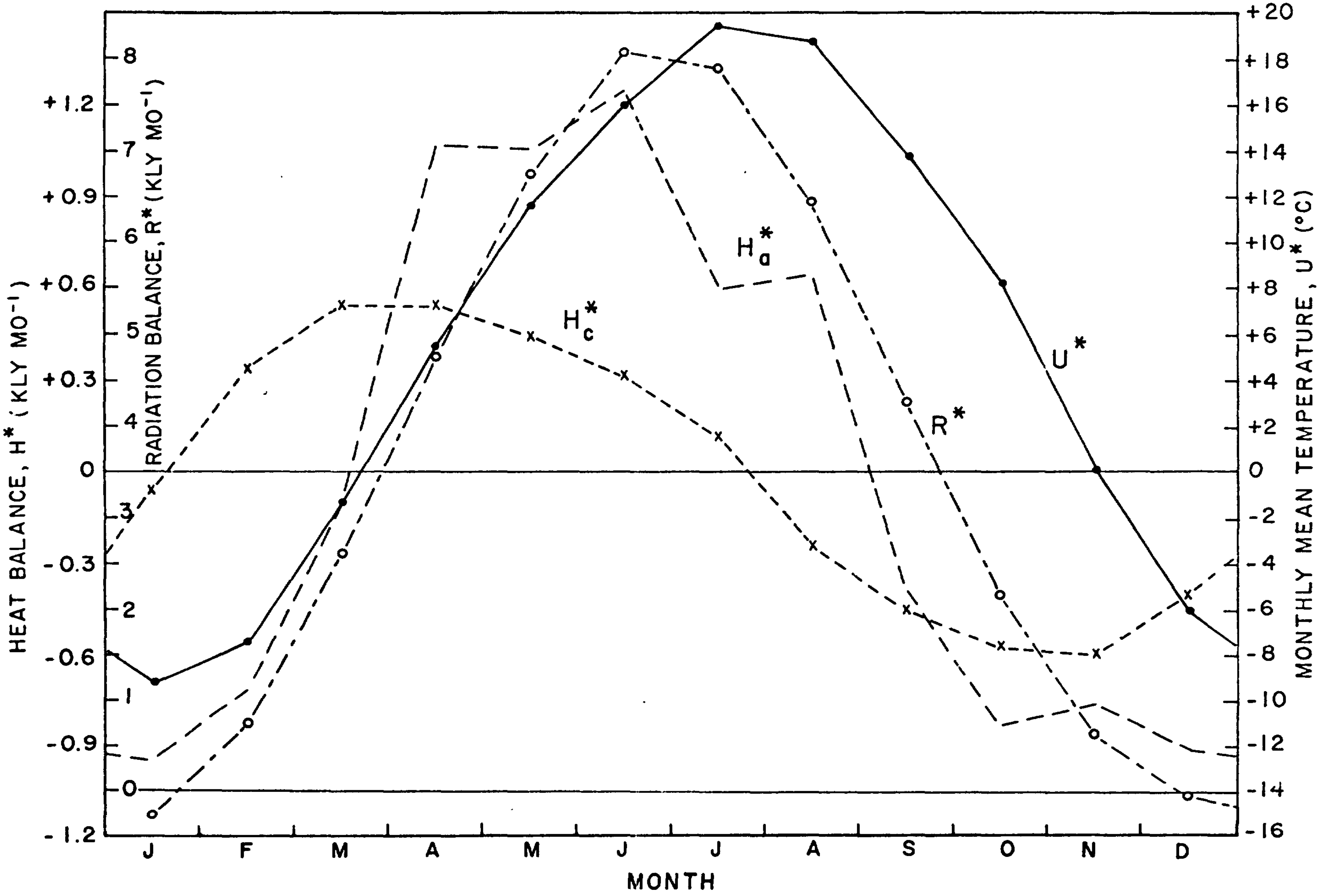
$$\left( \overline{\frac{\partial T}{\partial t}} \right) L = \frac{\partial u}{\partial t} D$$

where u denotes the surface temperature. Then,

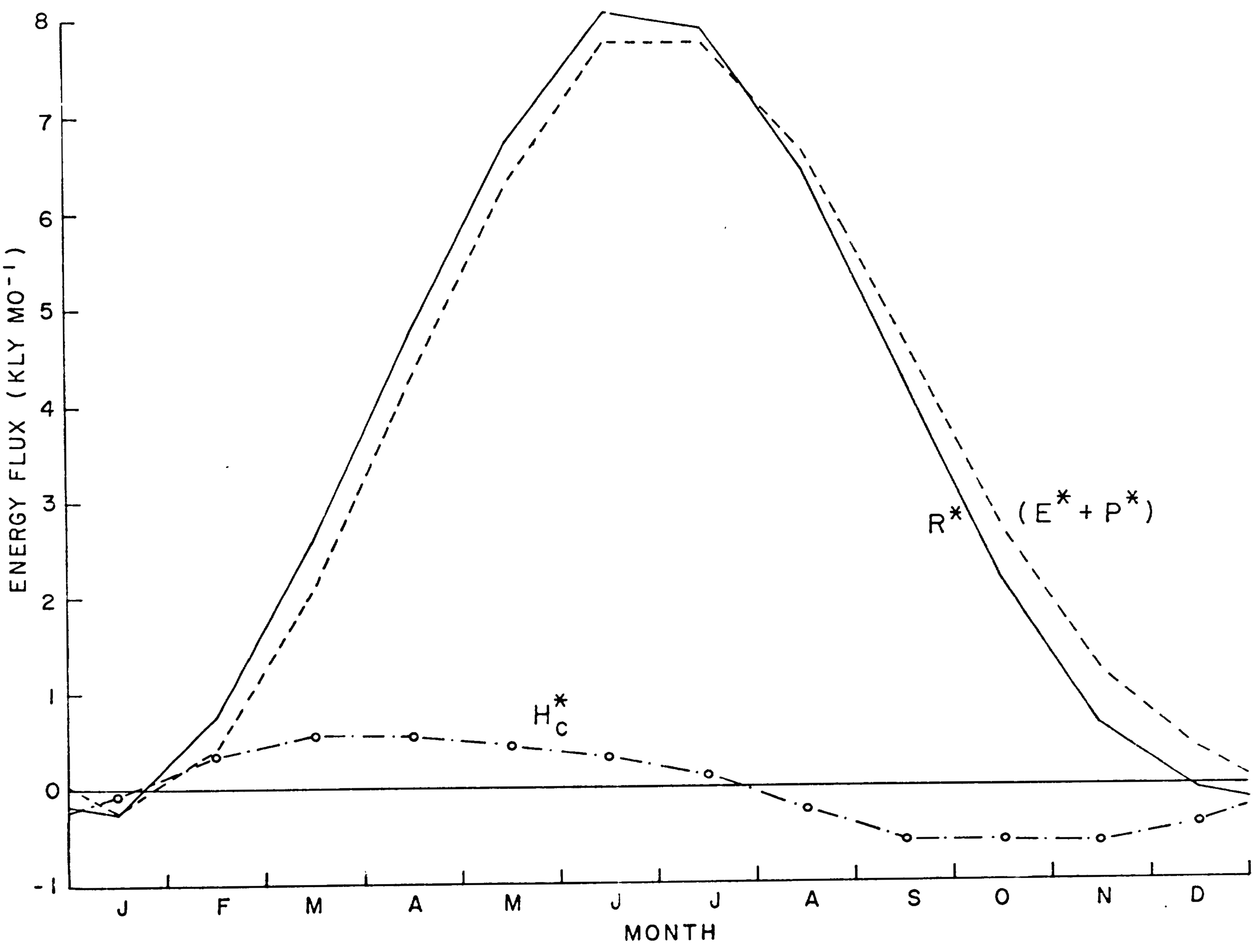
$$\frac{\partial u}{\partial t} = \frac{h}{CD} = kh \,.$$

In an isothermal mixed layer such as may be found in the ocean, L = D, while for a continental surface layer, L is greater than D. For example, if the amplitude of the temperature variation were a linear function of depth, vanishing at the depth L, then L would be equal to 2 D. However, in the more realistic case of an exponentially damped temperature variation (which follows from the solution of the heat conduction equation for a layer of uniform thermal diffusivity), L is greater than 2 D.

From the annual cycles of temperature and surface heat balance averaged over the North American continent, a mean annual heating function is computed in this paper to be about 12 degrees Celsius per kilolangley. The average heat capacity of soils is roughly 1/2 cal $cm^{-3}$ deg $C^{-1}$ (Sellers, 1965). Thus, the mean value of D is approximately 1 1/2 meters and L is greater than 3 meters for the annual cycle. Sellers (1965) indicates that the annual temperature cycle over land penetrates to depths of 5-20 meters.



SPAR + CRANE Fig. 1


ENERGY FLUX (KLY MO⁻¹)
8
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J
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MONTH
R*
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H*c