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A PERTURBATION ANALYSIS OF THE PROPAGATION AND INSTABILITY OF WAVES IN DUCTS WITH VARYING CROSS SECTIONS

FINAL REPORT
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I. Published Reports

1.1 Nayfeh, A. H., Kaiser, J. E., and Telionis, D. P., "The Acoustics of Aircraft Engine-Duct Systems," AIAA Paper No. 73-1153; AIAA J., 1974 (in Press).

Noise generated in aircraft engines is usually suppressed by acoustically treating the engine ducts. The optimization of this treatment requires an understanding of the transmission and attenuation of the acoustic waves. A critical review is presented of the state of the art regarding methods of determining the transmission and attenuation parameters and the effect on these parameters of (1) acoustic properties of liners, (2) the mean velocity, including uniform and shear profiles and nonparallel flow, (3) axial and transverse temperature gradients, (4) slowly and abruptly varying cross sections, and (5) finite-amplitude waves and nonlinear duct liners.

1.2 Nayfeh, A. H. and Telionis, D. P., "Algebraically Growing Waves in Ducts with Sheared Mean Flow," J. Acoust. Soc. Am., 55, 16-18, 1974.

Standing or traveling waves which vary algebraically with the axial distance in uniform ducts with sheared mean velocity profiles are investigated. The results show that such waves are not possible for ducts with uniform cross sections and fully developed mean flows.

1.3 Nayfeh, A. H., "Effect of the Acoustic Boundary Layer on the Wave Propagation in Ducts," J. Acoust. Soc. Am., 54, 1737-1742, 1973.

An analysis is presented for the wave propagation in two-dimensional and circular lined ducts taking into account the effects of viscosity in both the mean and the acoustic problems. The method of composite expansions is used to express each acoustic flow quantity as the sum of an inviscid part and a boundary layer part insignificant outside a thin layer next to the wall. The problem is reduced to solving a second-order ordinary differential equation for the pressure perturbation, as in the inviscid acoustic case, but with a modified specific wall admittance. An analytic expression is presented for the variation of the modified admittance with the wall and flow parameters, such as, the mean-flow Reynolds number, the mean velocity and temperature gradients at the wall, the frequency of oscillation, and the wave length.

1.4 Nayfeh, A. H., Kaiser, J. E., and Shaker, B. S., "Effect of Mean-Velocity Profile Shapes on Sound Transmission through Two-Dimensional Ducts," J. Sound and Vib., 34, 413-423, 1974.

Acoustic propagation through a lined rectangular duct is examined in order to assess the influence of the shape of the mean-velocity profile on the attenuation rate. Four mean-velocity profiles are considered:

linear, parabolic, Pohlhausen, and a 1/7th power law with a linear sublayer. It is shown that when the attenuation rate is tabulated as a function of the boundary-layer thickness, as is usually done, substantially different results are obtained from the four mean profiles. However, when the displacement thickness is used, a considerable collapse is achieved in the attenuation curves that are obtained from the various profiles. For downstream propagation, all profiles produce essentially the same results over a reasonable range of boundary-layer thickness. However, for many cases of upstream propagation, the results from the "turbulent" boundary-layer profile differ significantly from the results of the other profiles even when compared on the basis of displacement thickness. For these cases, another scaling parameter such as the shape factor must be used in addition to the displacement thickness. The effectiveness of the use of the shape factor as a scaling parameter is demonstrated by the results from a linear profile with slip at the wall. In all cases, in the limit as the boundary-layer thickness vanishes, the numerical results approach those obtained from a uniform profile with continuity of particle displacement at the wall.

1.5 Nayfeh, A. H. and Sun, J., "Effect of Transverse Velocity and Temperature Gradients on Sound Attenuation in Two-Dimensional Ducts," J. Sound and Vib., 34, 505-517, 1974.

An investigation is presented for the effect of transverse mean-velocity and temperature gradients on sound attenuation in acoustically treated rectangular ducts. The results show that cooling the cut walls leads to channelling the sound toward the walls for both downstream and upstream propagation. The effect of mean-temperature gradients on the attenuation rates of the lowest three modes can be as important as the effect of mean-velocity gradients.

1.6 Nayfeh, A. H., Sun, J., and Telionis, D. P., "Effect of Bulk-Reacting Liners on Wave Propagation in Ducts," AIAA J., 12, 838-843, 1974.

An analysis is presented for the effect of bulk-reacting liners on the wave propagation and attenuation in two-dimensional and circular ducts with sheared flow. The results show that the attenuation increases as the resistance, thickness of the liner, or frequency of oscillation increases, attains a maximum, and then decreases to a small value. Bulk-reacting liners attenuate the second mode more than point-reacting liners at all frequencies. However, bulk-reacting liners attenuate the lowest mode more than point-reacting liners except at low frequencies. The results suggest that an optimum liner consists of a combination of a bulk-reacting liner to attenuate the low frequency noise and a point-reacting liner to attenuate the high frequency noise.

1.7 Kaiser, J. E., Shaker, B. S., and Nayfeh, A. H., "Influence of Liner Thickness on Wave Propagation in Ducts," VPI&SU Report No. E-74-8; J. Sound and Vib., 1974 (in Press).

The acoustic characteristics of a point-reacting duct liner that consists of a porous facing sheet backed by cellular cavities are derived by examining the wave propagation within the liner. The relation between the derived expression for the liner acoustic admittance and a semi-empirical formula that is widely used in the literature is discussed. The influence of the liner on acoustic propagation in a duct is examined for the case of a plane duct that carries a uniform mean flow. Numerical results for the attenuation rates vs. frequency are presented. These results are of three types: (1) comparisons with previously published results for no backing cavity and no mean flow are mode, and these results are extended to include the effects of the mean flow; (2) results of parametric variations of the liner dimensions are presented to assess the relative influence of the facing-sheet thickness and the cavity depth; (3) results from the derived expression for liner specific admittance and from the semi-empirical formula are compared in order to determine the significance of the wave propagation within the porous material and to determine the range of validity of the semi-empirical formula.

1.8 Nayfeh, A. H. and Tsai, M.-S., "Nonlinear Acoustic Propagation in Two-Dimensional Ducts," J. Acoust. Soc. Am., 55, 1166-1172, 1974.

The method of multiple scales is used to obtain a second-order uniformly valid expansion for the nonlinear acoustic wave propagation in a two-dimensional duct whose walls are treated with a nonlinear acoustic material. The wave propagation in the duct is characterized by the unsteady nonlinear Euler equations. The results show that nonlinear effects tend to flatten and broaden the absorption vs. frequency curve, in qualitative agreement with the experimental observations. Moreover, the effect of the gas nonlinearity increases with increasing sound frequency, whereas the effect of the material nonlinearity decreases with increasing sound frequency.

1.9 Nayfeh, A. H. and Tsai, M.-S, "Nonlinear Wave Propagation in Acoustically Lined Circular Ducts," J. Sound and Vib., 35(3), 1974 (in press).

A second-order uniformly valid expansion is obtained by using the method of multiple scales for the nonlinear wave propagation in a circular duct whose walls are lined with a nonlinear acoustic material. The acoustic material is characterized by an empirical, nonlinear impedance in which the instantaneous resistance is a nonlinear function of the acoustic velocity while the instantaneous reactance is a nonlinear function of both the frequency and the acoustic velocity. The results show that for each mode there exists a threshold frequency above which the nonlinearity has a favorable effect and below which it has an adverse effect on the absorption coefficient, in qualitative agreement with the experimental observations. Moreover, the effect of the gas nonlinearity increases with increasing sound frequency, whereas the effect of the material nonlinearity decreases with increasing sound frequency.

1.10 Nayfeh, A. H. and Tsai, N.-S., "Finite Amplitude Waves in Two-Dimensional Lined Ducts," J. Sound and Vib., 36(4), 1974 (in press).

A second-order uniform expansion is obtained for nonlinear wave propagation in a two-dimensional duct lined with a point-reacting acoustic material consisting of a porous sheet followed by honeycomb cavities and backed by the impervious wall of the duct. The waves in the duct are coupled with those in the porous sheet and the cavities. An analytical expression is obtained for the absorption coefficient in terms of the sound frequency, the physical properties of the porous sheet, and the geometrical parameters of the flow configuration. The results show that the nonlinearity flattens and broadens the absorption vs. frequency curve, irrespective of the geometrical dimensions or the porous material acoustic properties, in agreement with experimental observations.

1.11 Nayfeh, A. H. and Tsai, M.-S., "Finite Amplitude Waves in Cylindrical Lined Ducts," AIAA Paper No. 74-553.

A second-order uniformly valid expansion is obtained for nonlinear waves propagating in a cylindrical duct lined with a point-reacting acoustic material that consists of a porous sheet followed by honey-comb cavities and backed by the impervious walls of the duct. The effect of the liner is taken into account by coupling the waves in the duct with those in the liner. As in the two-dimensional case, the nonlinearity increases the attenuation rate at all frequencies except in narrow bandwidths around the resonant frequencies, irrespective of the geometrical dimensions of the liner or the acoustic properties of the porous sheet.

1.12 Nayfeh, A. H. and Tsai, M.-S., "High Intensity Sound in Lined Ducts," Proceedings of the Second Interagency Symposium on University Research in Transportation Noise, June 1974, North Carolina State University.

The objective of the project is to analyze the nonlinear effects of the gas motion as well as the acoustic lining material on the transmission and attenuation of sound in two-dimensional and circular ducts with uniform cross sections. Approximate solutions have been obtained for small but finite amplitudes by using the method of multiple scales. The effect of the acoustic material is included either by characterizing the material by an empirical nonlinear impedance or by coupling the waves in the duct with those in the liner. The results show that the nonlinearity flattens and broadens the absorption vs. frequency curves. Moreover, the effect of the gas nonlinearity increases with increasing sound frequency, whereas the effect of the material nonlinearity decreases with increasing sound frequency.

1.13 Nayfeh, A. H. "Nonlinear Propagation of a Wave Packet in a Hard-Walled Circular Duct," VPI&SU Report E-74-9.

The method of multiple scales is used to derive a nonlinear Schrödinger equation for the temporal and spatial modulation of the amplitudes and the phases of waves propagating in a hard-walled circular duct. This equation is used to show that monochromatic waves are stable and to determine the amplitude dependance of the cut-off frequencies.

1.14 Nayfeh, A. H. and Telionis, D. P., "Acoustic Propagation in Ducts with Varying Cross Sections," J. Acoust. Soc. Am., 54, 1654-1660, 1973.

The method of multiple scales is used to derive the equations that describe the spatial and temporal variation of the amplitudes and phases of a wave packet propagating in slowly varying hard-walled or lined ducts. The analysis is carried out for rectangular as well as circular ducts. These equations are statements of the conservation of energy. For large admittance or high-frequency modes, an approximate expression is obtained for the attenuation. This expression shows that all possible acoustic modes are attenuating. The results also show that decreasing the cross-sectional area can lead to elimination of some of the acoustic modes.

1.15 Nayfeh, A. H., Telionis, D. P., Lekoudis, S. H., "Acoustic Propagation in Ducts with Varying Cross Sections and Sheared Mean Flow," AIAA Paper No. 73-1008, 1973.

An analysis is presented of the wave propagation and attenuation in two-dimensional ducts with slowly varying cross sections which carry sheared mean flow. A uniformly valid asymptotic expansion of the acoustic waves is obtained in terms of the maximum slope of the wall by using the method of multiple scales. The result is a first-order differential equation describing the amplitude variation with the axial distance. This equation is used to evaluate the effects of variations of the duct cross section and growing boundary layers on the different acoustic modes.

1.16 Nayfeh, A. H. , Telionis, D. P., and Kaiser, J. E., "Transmission of Sound Through Annular Ducts of Varying Cross Sections," AIAA Paper No. 74-58; accepted for publication in the AIAA J.

An asymptotic solution is presented for the transmission and attenuation of acoustic waves in an annular duct of slowly varying cross section which carries a sheared mean flow. The analysis also takes into account the growth of the boundary layer as well as any slow variations in the acoustic liner properties. The problem is reduced to the solution of a first-order ordinary differential equation for the amplitude. The analysis is used to estimate the effects of the variations of the duct cross section and boundary layer growth on the different acoustic modes.

1.17 Nayfeh, A. H., "Sound Waves in Two-Dimensional Ducts with Sinusoidal Walls," J. Acoust. Soc. Am. 55, 768-770, 1974.

The method of multiple scales is used to analyze the wave propagation in two-dimensional hard-walled ducts with sinusoidal walls. Resonance occurs whenever the wall wavenumber k_w is equal to the difference or the sum of the wavenumbers k_n and k_m of any two duct acoustic modes. In the first case, the waves are bounded with axial distance; however, neither of the resonant modes could exist without the other. When $k_w \approx k_n + k_m$, the resonance may cause the resonant modes to grow exponentially with axial distance, depending on the nearness of k_w to $k_n + k_m$.

1.18 Nayfeh, A. H. Kaiser, J. E., and Telionis, D. P., "Transmission of Sound Through Ducts with Varying Cross Sections," Proceedings of the Second Interagency Symposium on University Research in Transportation Noise, June 1974, North Carolina State University.

The main objective of the project is to analyze the transmission and attenuation of sound in two-dimensional and circular ducts with slowly varying cross sections with or without mean flow, taking into account any slow variations in the wall admittance and the growth of the boundary layer. Approximate solutions for incompressible mean flows have been obtained by using the method of multiple scales. A parametric study is underway to determine the effect of the different flow and liner properties on the attenuation of sound, and the analysis is being extended to compressible mean flows. A secondary objective is to analyze the wave propagation in ducts with sinusoidal wall undulations.

II. Talks

2.1 "Acoustic Propagation in Ducts with Varying Cross Sections," Acoustical Society of America Meeting, November 28 - December 1, 1972 in Miami Beach.

2.2 "Effect of Bulk-Reacting Liners on Wave Propagation in Ducts," AIAA Aerospace Sciences Meeting, January 10-12, 1973, Washington, D. C.

2.3 "Nonlinear Acoustic Propagation in Rectangular Ducts," 1973 Symposium on Finite-Amplitude Wave Effects in Fluids, Technical University of Demark, Aug. 20-22, 1973.

2.4 "Acoustic Propagation in Ducts with Varying Cross Sections and Sheared Mean Flow," AIAA Aeroacoustics Specialist Conference, Seattle, Washington, October 15-17, 1973.

2.5 "The Acoustics of Aircraft Engine-Duct Systems," CASI-AIAA Aeronautical Meeting, Montreal, Canada, Oct. 29-30, 1973.

2.6 "Transmission of Sound Through Annular Ducts of Varying Cross Sections and Sheared Mean Flow," AIAA 12th Aerospace Sciences Meeting, Washington, D. C., Jan. 31, 1974.

2.7 "Transmission and Attenuation of Sound Through Ducts with Varying Cross Sections," presented at Sandia Laboratories, Albuquerque, New Mexico, April 11, 1974.

2.8 "Transmission and Attenuation of Sound Through Ducts with Varying Cross Sections," presented at Stanford University, Palo Alto, California, April 9, 1974.

2.9 "Transmission of Sound Through Ducts with Varying Cross Sections," presented at the Second Interagency Symposium on University Research in Transportation Noise, N. C. State Univ., Raleigh, North Carolina, June 5-7, 1974.

2.10 "High Intensity Sound in Lined Ducts," presented at the Second Interagency Symposium on University Research in Transportation Noise, N. C. State Univ., Raleigh, North Carolina, June 5-7, 1974.