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# A METHOD OF PREDICTING THE OPTIMUM LUNAR LANDING AREA FOR A MANNED SPACECRAFT 

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## SUMMARY

From a safety aspect, our present knowledge of the moon is inadequate to make a fast decision concerning the best site for landing a manned spacecraft. To date, ten sites have been proposed from which we must select the safest. Additional data from the areas selected are necessary in order to compare the results with data obtained from terrestrial samples of the site where Surveyor has landed. This paper proposes a method of successive transformations of observational data obtained during the Surveyor program from which we can expect to acquire a considerable amount of information regarding the proposed sites.

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REPORT 1
A METHOD OF PREDICTING THE OPTTMUM LUNAR LANDING AREA FOR A MANNED SPACECRAFT
1.

INTRODUCTION
From a safety aspect, our present knowledge of the moon is inadequate to make a fast decision concerning the best site for landing a manned spacecraft. To date, ten sites have been proposed from which we must select the safest. Therefore, more lunar spectral-photometric, radiometric, and polarimetric observations are necessary to define the nature of the ground and, more precisely, the material composing the lunar surface. In other words, we need additional data from the areas selected in order to compare the results with data obtained from terrestrial samples.

At the same time, it would be useful to compare those areas proposed with respect to the site where Surveyor has landed. By extrapolating from the Surveyor data, we can expect to acquire a considerable amount of information about the other sites. The major problem, of course, is the method of extrapolation.

Direct comparison between the characteristics of areas observed and those of the reference source is not advisable if the aim of the research is to look at the landing safety aspects in all sites. In effect, with such a direct comparison, the discontinuities of the ground between the areas observed and the reference source would be neglected. In the same way, the observation of a few intermediate points does not help too much because we cannot satisfactorily discriminate between variations in the nature of the ground represented by such points. With a method such as the successive transformations of observational

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data, however, the extrapolation would not be difficult and it could be applied to any data reduction problem.
2. PROPOSED METHOD OF EXTRAPOLATION

As an example, let $\left(C_{i}\right)$ represent the color obtained from observations using the infrared filter; let $\left(C_{i}\right)$ represent the color corresponding to the point $A$ and $\left(C_{i}\right)_{1},\left(C_{i}\right)_{2},\left(C_{i}\right)_{3},\left(C_{i}\right)_{4}$ and $\left(C_{i}\right)_{5}$ be the colors corresponding to points $1,2,3,4,5$ for the same longitude but above $A$. Let $\left(C_{i}^{\prime \prime}\right)_{1},\left(C_{i}^{\prime \prime}\right)_{2},\left(C_{i}^{\prime \prime}\right)_{3},\left(C_{i}^{\prime \prime}\right)_{4}$ and $\left(C_{i}^{\prime \prime}\right)_{5}$ represent those which are at the same longitude but below $A$. In the same manner, $\operatorname{let}\left(C_{i}^{\prime}\right)_{1},\left(C_{i}^{\prime}\right)_{2} ;\left(C_{i}^{\prime}\right)_{3},\left(C_{i}^{\prime}\right)_{4}$ and $\left(C_{i}^{\prime}\right)_{5}$ represent the points which are situated at the same latitude as A.

Using the color of $A$ as a reference, we have the following relationships:

$$
\begin{aligned}
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}\right)_{1}}=a ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{1}}=a^{\prime} ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime \prime}\right)_{1}}=a^{\prime \prime} \\
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}\right)_{2}}=b ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{2}}=b^{\prime} ; \frac{\left(c_{i}\right)_{0}}{\left(C_{i}^{\prime \prime}\right)_{2}}=b^{\prime \prime} \\
& \frac{\left(c_{i}\right)_{0}}{\left(C_{i}\right)_{3}}=c ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{3}}=c^{\prime} ; \frac{\left(c_{i}\right)_{0}}{\left(C_{i}^{\prime \prime}\right)_{3}}=c^{\prime \prime} \\
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}\right)_{4}}=d ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{4}}=d^{\prime} ; \frac{\left(C_{i}\right)_{o}}{\left(C_{i}^{\prime \prime}\right)_{4}}=d^{\prime \prime} \\
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}\right)_{5}}=e ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{5}}=e^{\prime} ; \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime \prime}\right)_{5}}=e^{\prime \prime}
\end{aligned}
$$

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In other words, as is shown by Scheme No. 2 (see Appendix), we enlarge the area of $A$ in order to acquire additional data to serve as reference for the extrapolations which are shown by Scheme No. 1 (see Appendix). For better results, it is necessary to use the ratios $a, b, c, d, e, a^{\prime}, b^{\prime}, c^{\prime}, d^{\prime}, e^{\prime}$, and $a^{\prime \prime}, b^{\prime \prime}$, $c^{"}, d^{\prime \prime}, e^{"}$ as compared to direct use of the colors. This method is preferable because an extrapolation is not as accurate as an interpolation.

From these relationships, we have:

$$
\left.\begin{array}{l}
a\left(C_{i}\right)_{1}-2 b\left(C_{i}\right)_{2}-c\left(C_{i}\right)_{3}+d\left(C_{i}\right)_{4}+e\left(C_{i}\right)_{5}=0 \\
a^{\prime}\left(C_{i}^{\prime}\right)_{1}-2 b^{\prime}\left(C_{i}^{\prime}\right)_{2}-c^{\prime}\left(C_{i}^{\prime}\right)_{3}+d^{\prime}\left(C_{i}^{\prime}\right)_{4}+e^{\prime}\left(C_{i}^{\prime}\right)_{5}=0  \tag{1}\\
a^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{1}-2 b^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{2}-c^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{3}+d^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{4}+e^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{5}=0
\end{array}\right\}
$$

So, from one circle to another, we have

$$
\begin{align*}
& {\left[a\left(C_{i}\right)_{1}-2 a^{\prime}\left(C_{i}^{\prime}\right)_{1}+a^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{1}\right] } \\
- & {\left[2 b\left(C_{i}\right)_{2}-4 b^{\prime}\left(C_{i}^{\prime}\right)_{2}+2 b^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{2}\right] } \\
- & {\left[c\left(C_{i}\right)_{3}-2 c^{\prime}\left(C_{i}^{\prime}\right)_{3}+c^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{3}\right] } \\
+ & {\left[d\left(C_{i}\right)_{4}+d^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{4}\right] } \\
+ & {\left[e\left(C_{i}\right)_{5}-2 e^{\prime}\left(C_{i}^{\prime}\right)_{5}+e^{\prime \prime}\left(C_{i}^{\prime \prime}\right)_{5}\right]=0 } \tag{2}
\end{align*}
$$

Then, from the relationships among $\left(C_{i}\right)_{1},\left(C_{i}^{\prime}\right)_{1},\left(C_{i}^{\prime \prime}\right)_{1}$, we obtain the following for the first circle:

$$
\begin{aligned}
& a=\frac{a^{\prime}\left(1+a^{\prime}\right)}{a^{\prime \prime}}+\frac{a^{\prime \prime}+1}{2} ; a^{\prime}=\frac{1}{2}(a-1)-2 a^{\prime \prime} \text { and } \\
& a^{\prime \prime}=-\frac{1+2 a^{\prime}}{4}
\end{aligned}
$$

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which reduces to:

$$
(a-1)-2 a^{\prime}-4 a^{\prime \prime}=0
$$

Next, if we replace the ratios by their respective values, we

$$
\begin{aligned}
& \text { obtain the following identity: } \\
& \qquad\left(c_{i}^{\prime}\right)_{1}\left(c_{i}^{\prime \prime}\right)_{1}-2\left(c_{i}\right)_{1}\left(c_{i}^{\prime}\right)_{1}-4\left(c_{i}\right)_{1}\left(c_{i}^{\prime \prime}\right)_{1} \equiv \frac{\left(c_{i}\right)_{1}\left(c_{i}^{\prime}\right)_{1}\left(c_{i}^{\prime \prime}\right)_{1}}{\left(c_{i}\right)_{0}}
\end{aligned}
$$

or:

$$
\begin{aligned}
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}\right)_{1}}=1=a-1 \equiv\left(C_{i}^{\prime}\right)_{1}\left(C_{i}^{\prime \prime}\right)_{1} \\
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime}\right)_{1}}=a^{\prime} \equiv\left(C_{i}\right)_{1}\left(C_{i}^{\prime}\right)_{1} \\
& \frac{\left(C_{i}\right)_{0}}{\left(C_{i}^{\prime \prime}\right)_{1}}=a^{\prime \prime} \equiv\left(C_{i}\right)_{1}\left(C_{i}^{\prime \prime}\right)_{1} \\
& \\
& \frac{\left(C_{i}\right)_{1}\left(C_{i}^{\prime}\right)_{1}\left(C_{i}^{\prime \prime}\right)_{1}}{\left(C_{i}\right)_{0}} \equiv 0
\end{aligned}
$$

The next consideration is any point $a_{x}$ between a and a' or any point $a_{y}$ between $a^{\prime}$ and $a^{\prime \prime}$. The relationships between $a_{x}$ and a, a or between $a_{y}$ and $a^{\prime}, a^{\prime \prime}$ must be established in the following manner:
(1) Theoretically, there is no variation of color with respect to the points belonging to a same longitude. However, there are small variations of color with longitude but they are due only to the variations of the ground's nature.

According ry, $a_{x} \approx a_{m}$

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(2) The color varies with the latitude. For this reason, when enlarging area $A$ in order to obtain a larger quantity of data for reference, we must not proceed too far from $A$, otherwise, we will accrue systematic errors in our evaluation of the ratios $a, b, c, d, e ;$ $a^{\prime}, b^{\prime}, c^{\prime}, d^{\prime}, e^{\prime}$ and $a^{\prime \prime}, b^{\prime \prime}, c^{\prime \prime}, d^{\prime \prime}, e^{\prime \prime}$

Under the circumstances mentioned in steps (1) and (2), we can establish a correlation between the ratio $\left(C_{i}^{\prime}\right)_{1} /\left(C_{i}\right)_{1}$ of $a, a^{\prime}$ and the ratio $\left(C_{i}^{n}\right)_{1} /\left(C_{i}^{x}\right)_{1}$ of any point located between $a$ and $a$. The same consideration is applicable to the point $a_{y}$ with respect to $a^{\prime}$ and $a^{\prime \prime}$. Therefore, $a_{x}=\frac{a a_{n}}{a^{\prime}}$ and $a_{y}=\frac{a a_{0}}{a^{\prime}}$

From equation (2):

$$
\begin{align*}
& a=\frac{a^{\prime}\left(1+a^{\prime}\right)}{a^{\prime \prime}}+\frac{a^{\prime \prime}+1}{2} ; a^{\prime}=\frac{1}{2}(a-1)-2 a^{\prime \prime} ; \\
& a^{\prime \prime}=-\frac{1+2 a^{\prime}}{4} \text { for the 1st circle } \\
& b=\frac{b^{\prime}\left(1+b^{\prime}\right)}{b^{\prime \prime}}+\frac{b^{\prime \prime}+1}{2} ; b^{\prime}=\frac{1}{2}(b-1)-2 b^{\prime \prime} ; \\
& b^{\prime \prime}=-\frac{1+2 b^{\prime}}{4} \text { for the 2nd circle }  \tag{3}\\
& c=\frac{c^{\prime}\left(1+c^{\prime}\right)}{c^{\prime \prime}}+\frac{c^{\prime \prime}+1}{2} ; c^{\prime}=\frac{1}{2}(c-1)-2 c^{\prime \prime} ; \\
& c^{\prime \prime}=-\frac{1+2 c^{\prime}}{4} \text { for the } 3 r d \text { circle } \\
& d^{\prime}=\frac{a^{\prime}\left(1+d^{\prime}\right)}{d^{\prime \prime}}+\frac{a^{\prime \prime}+1}{2} ; d^{\prime}=\frac{1}{2}(d-1)-2 d^{\prime \prime} ; \\
& d^{\prime \prime}=-\frac{1+2 d^{\prime}}{4} \text { for the } 4 \text { th circle }
\end{align*}
$$

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$$
\begin{align*}
& e=\frac{e^{\prime}\left(1+e^{\prime}\right)}{e^{\prime \prime}}+\frac{e^{\prime \prime}+1}{2} ; e^{\prime}=\frac{1}{2}(e-1)-2 e^{\prime \prime} ;  \tag{3}\\
& e^{\prime \prime}=-\frac{1+2 e^{\prime}}{4} \text { for the 5th circle }
\end{align*}
$$

It is evident then that the ratios $a, a^{\prime}, a^{\prime \prime} ; b, b^{\prime}, b^{\prime \prime} ; \ldots$; $e, e^{\prime}, e^{"}$ correspond to the points which are chosen (when observing the Moon) for analysing region $A$ going from its center $a_{0}$ to the limit $e_{x}$. The ratios $\left(a_{x}, a_{y}\right),\left(b_{x}, b_{y}\right), \ldots,\left(e_{x}, e_{y}\right)$ are the intermediate points for studying the lunar surface with respect to those other ratios serving as reference.

The parameters $a_{n}$ and $a_{0}$ are very important due to the fact that they are useful for the constant verification of the values obtained from observations of $\left(a_{x}, a_{y}\right),\left(b_{x}, b_{y}\right), \ldots,\left(e_{x}, e_{y}\right)$. For each circle of equation (3), these parameters are:

$$
\left.\begin{array}{l}
a_{x}=\frac{a a_{n}}{a^{\prime}} ; a_{y}=\frac{a^{\prime \prime} a_{o}}{a^{\prime}} \quad \text { for the 1st circle } \\
b_{x}=\frac{b b_{n}}{b^{\prime}} ; b_{y}=\frac{b^{\prime \prime} b_{o}}{b^{\prime}} \quad \text { for the 2nd circle } \\
c_{x}=\frac{c c_{n}}{c^{\prime}} ; c_{y}=\frac{c^{\prime \prime} c_{o}}{c^{\prime}} \quad \text { for the 3rd circle } \\
d_{x}=\frac{d^{\prime} d_{n}}{d^{\prime}} ; d_{y}=\frac{d^{\prime \prime} d_{o}}{d^{\prime}} \quad \text { for the 4th circle } \\
e_{x}=\frac{e e_{n}}{e^{\prime}} ; e_{y}=\frac{e^{\prime \prime} e_{o}}{e^{\prime}} \quad \text { for the 5th circle }
\end{array}\right\}
$$

Because of the importance of $a_{n}$ and $a_{0}$, we must proceed, when observing the Moon, in the following way:
(1) Obtain the colors for the points serving as reference and then compute the corresponding ratios with respect to the color of point A.

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(2) Complete a similar step for the intermediate points $\left(a_{x}, a_{y}\right),\left(b_{x}, b_{y}\right), \ldots,\left(e_{x}, e_{y}\right)$; then derive the corresponding values ( $a_{n_{x}}, a_{y_{x}}$ ) for proceeding to the verification cited by using equation (4).

Actually, equation (4) is a continuous transformation of data for any point considered with respect to the color of point $A$. In effect, when establishing the ratios $a, b, c, a, e ; a^{\prime}, b^{\prime}, c^{\prime}$, $d^{\prime}, e^{\prime}$ and $a^{\prime \prime}, b^{\prime \prime}, c^{\prime \prime}, d^{\prime \prime}, e^{\prime \prime}$, we are transforming ( $\left.C_{i}\right), C_{i}^{\prime}$ ), $\left(c_{i}^{\prime \prime}\right)$ with respect to $\left(C_{i}\right)_{0}$ in order to obtain the index of the color variation for inferring the physical properties of the new points which are compared with $A$. Also, for points $a_{x}, b_{x}, c_{x}$, $d_{x}, e_{x}$ and $a_{y}, b_{y}, c_{y}, d_{y}, e_{y}$, we are doing a second transformation with respect to $A$ and $a, ~ a, ~ a, \ldots ; e, ~ e, ~ e ; ~ e t c . ~ F i n a l l y, ~$ with the points $\left(a_{n_{x}}, a_{m_{x}}\right)$ and $\left(a_{o_{x}}, a_{n_{x}}\right)$, we are doing a third transformation with respect to $A ; a, a^{\top}, a, \ldots ; e, e, e$; etc., and $\left(a_{x}, a_{y}\right), \ldots,\left(e_{x}, e_{y}\right)$. Because $a_{x} \approx a_{m}$ and $a_{y} \approx a_{n}$, the third transformations $a_{n}$ and $a_{0}$ are sufficient to give us the precise information which is desired about the physical properties for the points considered other than A. This is the physical meaning of the identities that we have seen before.

Now, consider in a given circle of region $A$, the different points $a_{x}$ and $a_{y}$. For example, consider $\left(a_{x=1}, a_{x=2}, a_{x=3}\right)$, and $\left(a_{y=1}, a_{y=2}, a_{y=3}\right)_{1}$, which are some of the points situated between a, $a^{\prime}$ and $a^{\prime}, a^{\prime \prime}$ in the first circle. In this case, we have:

$$
\begin{aligned}
& \left(a_{x=1}+a_{x=2}+a_{x=3}\right)_{1}=\frac{a}{a}\left(a_{n_{x=1}}+a_{n_{x=2}}+a_{n_{x=3}}\right)_{1} \\
& \left(a_{y=1}+a_{y=2}+a_{y=3}\right)_{1}=\frac{a^{\prime \prime}}{a^{\prime}}\left(a_{0=1}+a_{0=2}+a_{0=3}\right)_{1}
\end{aligned}
$$

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Consequently, for all points X and Y situated in the first circle between $a, ~ a ~ a n d ~ a, ~ a ", ~ w e ~ c a n ~ e x p r e s s ~ t h e ~ f o l l o w i n g: ~$

$$
\begin{align*}
& \left(a_{x=1}+a_{x=2}+a_{x=3}+\ldots+a_{x=x}\right)_{1} \\
& =\left(\frac{a}{a}\right)\left(a_{n_{x=1}}+a_{n_{x=2}}+a_{n_{x=3}}+\ldots+a_{x=x}\right)=\left(\frac{a}{a}\right) \sum_{x=a_{0}}^{x=x=a^{\prime}} a_{n_{x}}  \tag{5}\\
& \left(a_{y=1}+a_{y=2}+a_{y=3}+\cdots+a_{y=Y}\right)_{1} \\
& =\left(\frac{a^{\prime \prime}}{a}\right)\left(a_{0=1}+a_{0=2}+a_{0=3}+\ldots+a_{0=Y}\right)=\left(\frac{a^{\prime \prime}}{a}\right) \sum_{x=a_{0}}^{x=Y=a} a_{0}
\end{align*}
$$

In going from one circle to another, we obtain the following for region $A$ : ,

$$
\begin{aligned}
& \left(\frac{a}{a}, \sum_{x=a_{0}}^{x=a^{\prime}} a_{n_{x}}=\alpha P_{1} \quad ; \quad\left(\frac{a_{1}^{\prime \prime}}{a^{\prime}} \sum_{x=a_{0}^{\prime}}^{\prime} \quad a_{o_{Y}}=\alpha^{\prime} Q_{1}\right.\right. \\
& \underset{b}{\left(\frac{b}{b},\right)} \sum_{x=a_{0}}^{x=b^{\prime}} b_{n_{x}}=\beta_{2} \quad ; \quad\left(\frac{b^{\prime \prime}}{b^{\prime}} \sum^{x=b^{\prime}} \sum_{x=a_{0}}^{b_{0}}=\beta^{\prime} Q_{2}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left(\frac{d}{d}\right)^{x=d^{\prime}} \sum_{x=a_{0}} d_{n_{x}}=\delta P_{4} ; \quad\left(\frac{d^{\prime \prime}}{d^{\prime}} \sum_{x=a_{0}}^{x=d^{\prime}} \quad d_{o_{Y}}=\dot{\delta}^{Q_{4}}\right.
\end{aligned}
$$

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$$
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Where: $\alpha, \beta, \gamma, \delta, \epsilon$, and $\alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}, \delta^{\prime}, \epsilon^{\prime}$, are the transformation coefficients, respectively, of the points $P_{1}, P_{2}, P_{3}, P_{4}, P_{5}$ and $Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}$. These points represent the observational data which must be successively transformed with respect to $P_{o}$, i.e., the point where Surveyor landed. Thus, if such points are the source of observational data that we constantly compare with the physical parameters of $P_{0}$, then the transformation coefficients tell us how these physical parameters are varying with location from $P_{0}$ to other points on the lunar surface.

In this last instance, it should be noted that equation (6) is good for studying any kind of data we need with respect to comparable data related to $P_{0}$. In effect, it would be sufficient to define $\left(P_{1}, Q_{1}\right),\left(P_{2}, Q_{2}\right), \ldots$, according to our needs for comparing subsequent points with the reference point $P_{0}$.
For example, let $P_{0}=0.18, P_{1}=0.23, P_{2}=0.21, P_{3}=0.14$, $P_{4}=0.09$ and $P_{5}=0.17$ be arbitrary values to illustrate the subsequent computations. Let $N$ be the number of successive transformations done for any point $P$ with respect to $P_{0}$. If we consider the circles of the region $A$, then we have:

For the 1st circle:
$P_{0}=\alpha P_{1}+\frac{N}{100}$ and the error $\Delta P=\left(\alpha P_{1}+\frac{N}{100}-P_{0}\right.$
For the 2nd circle:
$P_{0}=\beta P_{2}+\frac{N}{100}$ and the error $\Delta P=\left(\beta^{P_{2}}+\frac{N}{100}-P_{0}\right\}$
For the 3rd circle:
$P_{0}=\gamma P_{3}+\frac{N}{100}$ and the error $\Delta P=\left(\gamma P_{3}+\frac{N}{100}-P_{0}\right.$

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For the 4th circle:

$$
\begin{aligned}
& P_{0}=\delta P_{4}+\frac{N}{100} \text { and the error } \Delta P=\left(\delta P_{4}+\frac{N}{100}\right)-P_{0} \\
& \text { For the } 5 \text { th circle: } \\
& P_{0}=\epsilon P_{5}+\frac{N}{100} \text { and the error } \Delta P=\left(\varepsilon P_{5}+\frac{N}{100}\right)-P_{0}
\end{aligned}
$$

In these examples, all computations are carried out and shown in the final pages of this same section. From the equations listed above, however, we can see that errors must be on the same order of magnitude. This is due to the fact that, when transforming data from a given $P$ to $P_{0}$, we are not going directly from one to another but successively passing through the series of points situated between them. Also, as is shown by Scheme 4 (see Appendix), the variation of observational data with respect to $P_{o}$ is obtained with a high degree of accuracy because the correlations $a / a^{\prime}, b / b^{\prime}, \ldots, e / e^{\prime}$ are constantly providing the exact location of points whose data is transformed.

It is not practical to proceed past area $A$ because the required matrix would be excessively long and involved. Consequently, it is desirable to use an equation capable of inducing data within the same error $(-0.02<\Delta P<0.02)$ shown in developing the previous five examples. Accordingly, consider the last number of example No. 5:

$$
\frac{P_{5}}{P_{0}}\left[\left(\frac{P_{1}}{P_{4}}\right)^{5} \cdot \frac{\left(P_{3}\right)^{7}}{\left(\mathrm{P}_{2}\right)^{10}}\right]
$$

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At first, we see that the number $N$ of successive transformation is a function of the number of points considered in the series $P_{0}, P_{1}, \ldots, P_{n}$ while the transformation coefficient is a function of $P$ itself. For this reason, the last number of the example mentioned is composed as follows:
(1) The ratio $P_{5} / P_{0}$ corresponding to the extremities of the series
(2) A product of ratios whose number is depending on $N$
(3) Each such ratio is a new series, smaller than that which precedes it
(4) The powers simulataneously depend on $N$ and $P$. From this analysis of the last number of example No. 5, it is then possible to define the final equation with the same shape as equation (7), proceeding in the following way:
(1) We establish the condition "sine quanon" that points considered be of equal distance between them. Then $N=0$ for $P_{o, ~} N=1$ for $P_{1}, N=2$ for $P_{2}$ and so on if the distances $P_{0} \longrightarrow P_{1}=P_{1} \longrightarrow P_{2}=P_{2} \longrightarrow P_{3}, \cdots$
(2) To avoid systematic errors in the reduction of data to the maximum extent, do not take distances greater than $2^{\circ}$ in the lunar coordinates when observing.
(3) We determine $a, b, c, d, e ; a_{\prime \prime}^{\prime \prime}, b^{\prime}, c^{\prime}, d^{\prime}, e^{\prime}$ and $a ", b ", c^{",}, d^{\prime \prime}, e^{" ~ f r o m ~ t h e ~ o b s e r v a t i o n a l ~ d a t a ~ i n ~ o r d e r ~}$ to define the $P_{n}$ and $Q_{n}$.
(4) We adopt $\eta_{p}, \eta_{a}$ as the summation of the successive transformation coefficients $(\alpha, \beta, \gamma, \ldots)_{P_{-} \rightarrow P}$ and $\left(\alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}, \ldots\right)_{a_{0 \rightarrow a}}$. So for any $P$, for example, we can now write as follows:

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## First Case

$$
\begin{gather*}
P_{0}=\eta P+\frac{N}{100} \text { and the error is } \Delta P=\left(\eta P+\frac{N}{100}\right)-P_{0} \\
\text { when } \frac{N}{100}<P_{0} \tag{8}
\end{gather*}
$$

Second Case

$$
\begin{aligned}
& P_{0}=2\left(\frac{N}{100}-\frac{2}{100}\right)-\eta P=0.02(N-2)-\eta P \\
& \text { and the error is } \Delta P=[0.02(N-2)-\eta P]-P_{0}
\end{aligned}
$$

$$
\text { when } \frac{N}{100} \equiv P_{0}
$$

We have checked these equations by developing successive transformations of 45 examples, including the five examples previously cited. The results are given in Table I (see Appendix). Since the points were 46 with $P_{0}$ when doing the successive transformations, then the enumeration $N$ on Table I corresponds to the subsequent computations.

For the lunar observations done with more than a filter, we make the same computations after having defined the $P, Q$ which correspond to the different colors observed. In the same way, if we desire to study other data obtained from the moon by means different than that of the Photometry, we first select the lunar coordinates which correspond to our N and proceed to the subsequent definitions of $P, Q$ according to the nature of the physical parameters obtained by such other means.

Concerning the analysis of observational data after their reduction within the successive transformation method, the following work must be accomplished: NASA requests the study of fourteen areas from which the safest for landing a manned spacecraft must be selected. To recognize the safest among the many areas to be considered, the following method is used:

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(1) The sites considered would be included in the band covered by Scheme 1 so we proceed to observe, with respect to the $P_{0}$ of Surveyor, the whole band at $N=1$, $\mathrm{N}=2, \mathrm{~N}=3$... until we reach an N beyond the last of the areas proposed. This is for the purpose of satisfying the condition which requires that the points being observed must be equally spaced. For avoiding, at best, the systematic errors, we would adopt a distance among the points not exceeding $2^{\circ}$.
(2) For each filter used, $a, a^{\prime}, a^{\prime \prime} ; b^{\prime}, b^{\prime}, b^{\prime \prime} ; \ldots$; $e, e^{\prime}, e^{\prime \prime}$ are defined and the corresponding correlations are computed by using equations (3) and (4).
(3) The quantities $P$ or $Q$ are determined, the point observed being above or below the longitude of $P_{0}$, and the transformation coefficient $\eta$ is computed by using equation (8).

The plotting of the result using this method consists of the following steps:
(1) As shown by Scheme 4, the plotting $\eta$, $N$ informs us of the variation of the physical parameter considered in the lunar band observed. With respect to this, the curve of Scheme 4 apparently shows nothing new when we compare its shape with curves obtained from other procedures. As we can see by the following example, however, this conclusion is not realistic.

In Table 1, $\mathrm{P}_{25}=\mathrm{P}_{0}=0.18$. If we assume that the number 0.18 represents a white color, for instance, the equality $P_{25}=P_{0}$ would mean that both areas have the same color. But in Scheme 4 the ordinate of $P_{25}$ is too high. In terms of successive transformations,

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this would mean that both areas effectively have the same color with the difference being; for example, that one of them is a flat white surface while the other is a granular white surface.
(2) As shown by Figure 1 (see Appendix), plotting 7, N, and slope of $P_{0}$ gives a direct answer to the question about safety previously referenced. If we assume, for example, that point $P_{0}$ of Surveyor corresponds to an area which except for being too hot, is in all respects satisfactory for landing, other points placed on its slope would have the same nature as $P_{0}$. The only difference being that the temperature varies as a function of latitude.

In our examples, Figure 1 shows that $P_{22}$ is the safest point of those whose data were studied within the successive transformations method. The points $\mathrm{P}_{1}, \mathrm{P}_{2}$, $P_{3}$, and $P_{4}$, but especially $P_{1}$, appear also as good as $P_{o}$, but this is not surprising because they are the closest points to $\mathrm{P}_{\mathrm{o}}$.

There now remains only a final question: "How to define exactly those surface properties identified as the safest for landing a manned spacecraft on the moon?" After identification, the area will be observed again to establish the variation of $\eta$ with the different phases of the moon. The systematic errors will be eliminated by observing that area at least three times, with each observation corresponding to a different lunation. The mean of these three observations will be used for plotting $\eta$, phases of the moon and to obtain, in this way, the new photometric function given by the successive transformations method.

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This new photometric function will give us the precise information desired about the physical properties of the area mentioned. Concerning the observations themselves, a correction will be introduced with regard to the extinction coefficient of our atmosphere. For this, an "Air Mass Table" will be prepared by adopting the least square solutions in order to obtain the extinction coefficient pertaining to the dates of observations, geographical coordinates of the observatory and instrument used. This will be the part of this research for the Computer Program.

APPENDIX

$i_{o}=$ Landing site of Surveyor
1 = Landing site selected for a Manned Spacecraft
$P_{1}, P_{2}, P_{3}, \ldots, P_{45}$ Mamples of successive transformations for observational data


Enlargement of the site where Surveyor has landed
Definition of the points $\left(a_{x}, a_{y}\right),\left(b_{y}, b_{y}\right), \ldots,\left(e_{x}, e_{y}\right)$ with respect to ( $a, a^{\prime}, a^{\prime \prime}$ ), (b, $\left.b^{\mp}, b^{\prime \prime}\right), \ldots,\left(e, e^{\prime}, e^{\prime \prime}\right)$.

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TABIE I
Ixamples of successive transfomations for some poirts above
$P_{0}=0.18$ in the SCHENE 1





SCHiNE 4
fronefometon of obervational data and related transformation coefficierts


FTCTRE 1.

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```
F.O2 A-6
0. --8
Z.cument: - i-4-03)
```

EXAMPIES NO. 1, NO. 2 AND NO. 3
Let, in the regior $A, P_{0}=0.18$ the data coacerning the point where Sur :ayor ia: u.d. Iet $\equiv=0.23, P_{2}=0.21$ ard $P_{3}=0.14$ the corresponding data of those points si-uiated an, reajectively, the $1 \mathrm{st}, 2 \mathrm{nd}$, and 3rd circles.

What are tile transformation coefficients $\alpha, \beta$ and $\gamma$ with respect to $\ddot{z} \because$
What is the efficiency on the result obtain ${ }^{\text {a }}$, or error committed in tri succes.a e thans1 urnations, whish must be considered when analyzirg data to select an area ci the Yo.


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EXAMPLE TO. 4
 $P=0.23, P_{2}=0.21, F_{3}=0.14$ and $P_{4}=0.09$ the corresponding data of those points i..thated 0 , respectivel, the 1st, and, 3 rd and 4 th circies.

What is the transformation coefficient, $\mathcal{O}$, of $P_{4}$ with respect vo $P_{0}$ ?
What is tin efficiency on the result obtain, or error committed in th, succes. - ق firnations, whish must be considered when analyzing data to select ar area c. the ic


EXPLAIAMION
Eegiraire from $P_{0}$ of the point where Survey: landed, and coritiruing the poi: $P_{2}$, of the



$N I_{0}=P_{L}+\frac{100}{100}=(1.31)(0.09)+\frac{5}{100}=3 .: 7$,
and the eyrir is $\left(\delta_{P_{4}}+\frac{1}{160}\right)-\ldots, 0 .-0 . i=\ldots$

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EXANPIE NO. 5

$=0.23, P_{2}-0.21, I_{3}=0 . i 4, P_{4}=0.09$ and $P_{j}=0.17$ the correspording $\dot{G}, i$ of ti., e $x$ nts E-tiated aj, i三 pective ly, the 1 st , 2nd, $3 \mathrm{rd}, 4 \mathrm{ti}$ and 5 th circles.

What is thi transiormation coefficient $\in$, of $P_{5}$ with respect to $P_{0}$ ?
What is $\hat{i n}$ efficiency on the result obtain $i$, or error committe in tr. succes. e enans$i$ iormations, whish must be considered when analyzir. data to select an area i. the jicu:


## EXPLALAAIION

Beginaine from $F$, of the point where Surveron landed, and contiruing $t$ : whe poi.: $F_{y}$, of the $\therefore$ cicci: 0 : he rege on $A$, we need to do in the case a number $N$ of succes re tra: orusituan.


$\dot{\sim} \cdot=\Gamma_{i}+\frac{-}{1 . j}=(0.33)(0.17)+\frac{6}{100}=3.19$,
and the er: 3 is $\left(\in P_{5}+\frac{1}{100}\right)-?_{0}=0.0$.

