U.S. DEPARTMENT OF COMMERCE
National Technical Information Service

N74-74138

Thermally Induced Oscillations in Cryogenic Systems

U.S. National Bureau of Standards, Boulder, Colo

Prepared for

National Aeronautics and Space Administration, Washington, D C

1 Apr 72

NBS REPORT

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THERMALLY INDUCED OSCILLATIONS IN CRYOGENIC SYSTEMS

by

R. S. Collier

NBS

(NASA-CR-138540) THERMALLY INDUCED N74-74138
OSCILLATIONS IN CRYOGENIC SYSTEMS
Technical Report, 1 Nov. 1970 - 31 Oct.
1971 (National Bureau of Standards) Unclas
30 p 00/99 16750

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
Institute for Basic Standards
Boulder, Colorado 80302

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT **NBS REPORT**

2750460 April 1, 1972 10 749

THERMALLY INDUCED OSCILLATIONS IN CRYOGENIC SYSTEMS

by

R. S. Collier

Work carried out at the National Bureau of Standards under the
sponsorship of National Aeronautics and Space Administration (SNSO)
Order No. W13, 300.

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U.S. DEPARTMENT OF COMMERCE
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CONTENTS

Part I. Temperature Oscillations of a Heated Surface in Slush Hydrogen

Part II. The Expansion Equations of Non-Equilbrium Thermodynamics

Part III. Surface Damping of Thermally Induced Oscillations in Cryogenic Systems

LIST OF FIGURES

THERMALLY INDUCED OSCILLATIONS IN CRYOGENIC SYSTEMS*

R. S. Collier

This work was done under NASA Order W 13,300 for the period November 1, 1970 to October 31, 1971 as a part of Task V – Slush Hydrogen Instrumentation.

One device – a carbon film sensor – has been found to possess a peculiar feature in addition to its demonstrated capability to discriminate among vapor, liquid, and slush phases. When a carbon film resistor is placed in an environment of slush hydrogen in a dc power mode, fluctuations of the carbon film temperature are detected which are unique to the slush environment.

These temperature oscillations can be correlated with fluctuations previously observed in research on the characterization of slush. This characteristic of slush hydrogen – the susceptability of the mixture under certain conditions to exhibit violent pressure excursions in valve stems, pressure gage lines, etc. – is probably the most singular difference between the technologies of slush and liquid hydrogen.

The similarity of the response of the carbon film sensor to thermally induced oscillations previously noted was suggested, and a study, both experimental and analytical, was proposed to:

1) Determine the nature of the temperature oscillations observed on the carbon thin films;

2) Proceed to perfect this point sensor as a slush hydrogen detector if these oscillations can be shown to have a firm physical and theoretical basis;

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3) Determine if there is a connection between the two observed phenomena – the temperature fluctuations of the carbon film and the so-called "thermal oscillations" observed in slush studies;

4) Assess whether these phenomena are potential problem areas in the extension of the use of slush hydrogen as a space fuel and propellant.

The first three goals have been pursued during this contracting period and the final results of this effort are contained in three separate papers contained in this report; the papers are being individually submitted to technical journals for publication and are titled as follows:

Part I	Temperature Oscillations of a Heated Surface in Slush Hydrogen.
Part II	The Expansion Equations of Non-Equilibrium Thermodynamics.
Part III	Surface Damping of Thermally Induced Oscillations in Cryogenic Systems.

Part I

Temperature Oscillations of a Heated Surface in Slush Hydrogen

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ABSTRACT

It was found that the surface temperature of a steadily heated cylinder oscillates with an amplitude of about 0.25 K and an oscillation frequency of about 8 Hz when immersed in slush hydrogen. The amplitude and the period of oscillation are relatively independent of heat flux at the surface in the range 0.25 - 1.5 watts/cm^2. The experimental results are consistent with a theoretical model which predicts oscillations in the thermal expansion of a liquid film located between the heated surface and the slush.

Key Words: Carbon thin film; slush hydrogen; thermal oscillations.

Introduction

A pyrex cylinder, approximately 1 cm long and 2 mm diameter, was coated with a vacuum deposited carbon film about 500 Å thick. The resistance of the carbon film was approximately 2300 Ω in slush hydrogen and was measured by attaching electrical leads to each end of the cylinder. The temperature sensitivity of the film in the liquid hydrogen range was approximately 50 Ω per kelvin. The carbon film was used as a constant current heat source with surface heat flux between 0.25 watt/cm^2 and 1.5 watt/cm^2. When this heat source was immersed in slush hydrogen, there was an 8 - 10 Hz voltage oscillation corresponding to a temperature oscillation about 0.25 K in amplitude. This temperature oscillation is consistent with a theoretical model of a "relatively incompressible, slowly accelerating fluid" which predicts oscillations in the thermal expansion of a fluid near a steadily heated surface[1]. When the heat source was immersed in liquid there were occasional oscillations at about 16 Hz and the amplitude of these oscillations was much smaller.

Experimental

The carbon film used in this experiment is similar to those developed for fluid phase discrimination[2, 3], where the film is subjected to a constant current pulse and the transient response is analyzed to determine whether or not the sensor is in the liquid or gas phase. When the sensor was immersed in slush hydrogen, the transient response as it appears on an oscilloscope trace is shown in figures 1-3 for three different power levels. The initial part of the transient is the same in the liquid and in the slush, indicating that the sensor is surrounded by a layer of liquid; the surface temperature then becomes slightly warmer in the slush than in the liquid and steady state oscillations appear which persist indefinitely.

Theoretical Interpretation

It has been shown that these steady state oscillations can be described as approximate solutions to the conservation equations of non-equilibrium thermodynamics[1]. The period of oscillations, τ, is given by

$$\tau = \frac{10.2d^2}{\alpha\beta \Delta T_m} \qquad (1)$$

where α is the thermal diffusivity, β is the Bulk Coefficient of Thermal Expansion, ΔT_m is the peak to peak surface temperature oscillation, and d is the thickness of the liquid film on the surface of the sensor. The thickness, d, is the only quantity which is not directly measurable.

For liquid hydrogen at 14 K, $\alpha\beta = 2 \times 10^{-4} \text{cm}^2/\text{sec K}$. It is seen in figures 1-3 that ΔT_m ($\approx 0.2 - 0.3$ K) is relatively independent of the power input which theoretically predicts, by equation (1), that the period of oscillation, τ, is relatively independent of power input. This is seen to be true, $1/\tau \approx 8 - 10$ Hz for all three power levels. From these numbers the liquid film thickness is calculated to be $d \approx 0.8 \times 10^{-3}$cm.

Acknowledgements

The author wishes to acknowledge Larry Lowe, Daniel Weitzel, and Lewis O. Mullen for the participation in the experimental aspects of this work.

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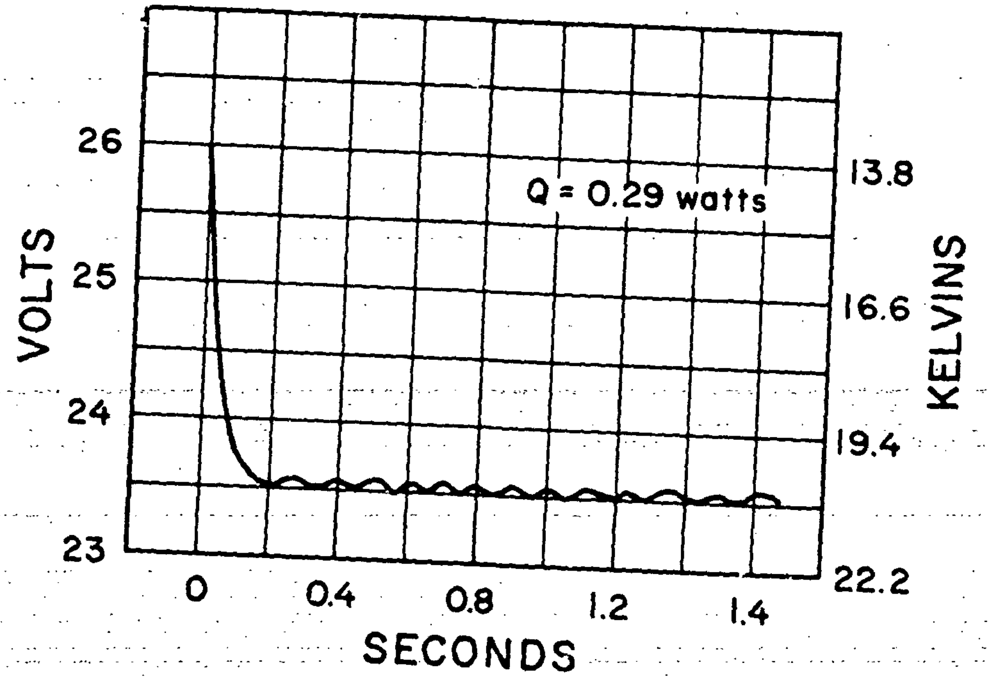
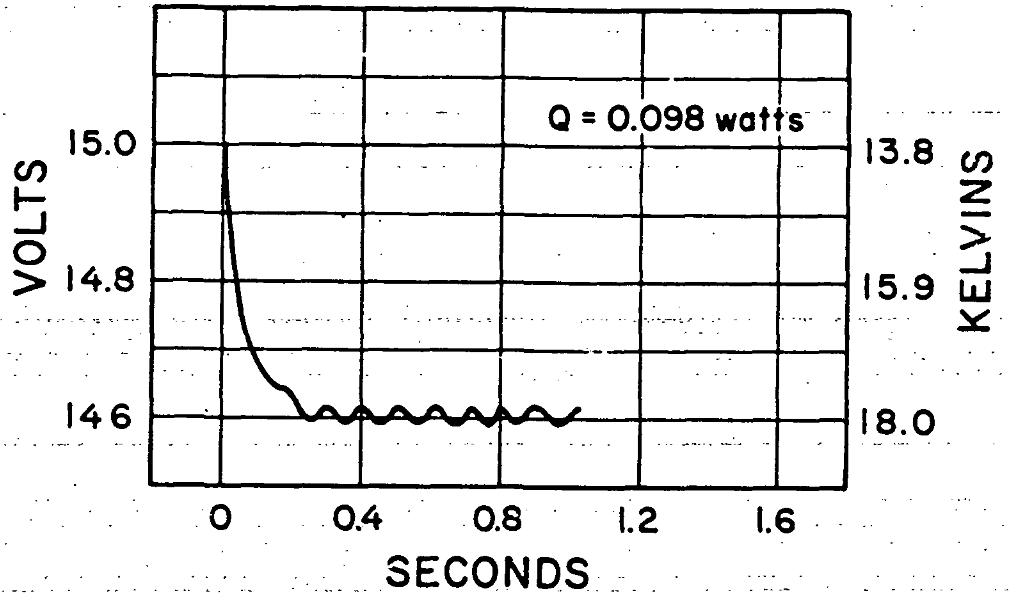

26
25
24
23
VOLTS
13.8
16.6
19.4
22.2
KELVINS
Q = 0.29 watts
0
0.4
0.8
1.2
1.4
SECONDS

Figure 1.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14.8
14 6
KELVINS
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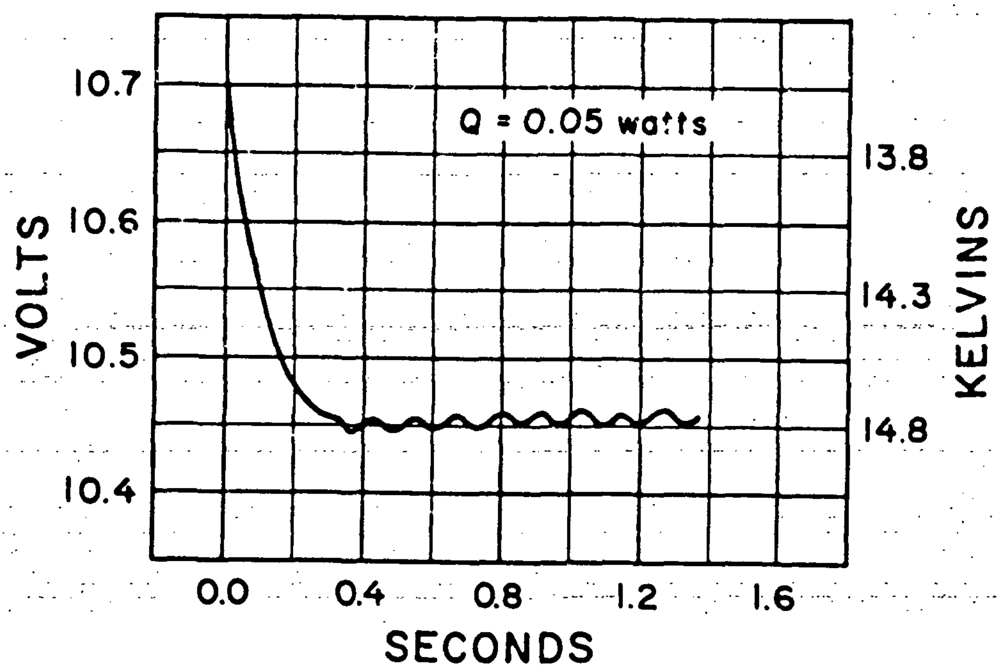
15.9
18.0
0
0.4
0.8
1.2
1.6
SECONDS

Figure 2.


10.7
10.6
10.5
10.4
VOLTS
Q = 0.05 watts
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14.3
14.8
KELVINS
0.0
0.4
0.8
1.2
1.6
SECONDS

Figure 3.

Part II

The Expansion Equations of Non-Equilibrium Thermodynamics

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ABSTRACT

Definitions of "Relatively Incompressible" and "Slowly Accelerating" are used to convert the differential forms of the conservation equations to a pair of coupled non-linear partial differential equations in the density, ρ, and the thermal expansion velocity, v. An approximate solution to these "expansion equations" indicates that oscillatory solutions can exist for the thermal expansion of a fluid near a steadily heated surface. This example suggests several conjectures which are posed subject to a more exact treatment of the expansion equations.

Key words: Expansion equations; non-equilibrium thermodynamics; slush mixtures; thermal oscillations.

Introduction

Six independent fluid variables of non-equilibrium thermodynamics are given by the density, ρ, the temperature, T, the pressure, P, and the velocity, v, which is a three dimensional vector. These variables are related by the conservation equations of mass, momentum, and energy and by the equation of state. The conservation equations have several forms which are equivalent under certain conditions. The differential forms, neglecting viscous and body forces, are given in rectangular coordinates as follows[1]:

A. Conservation of Mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho v = 0 \tag{1}$$

or

$$D\rho + \rho \nabla \cdot v = 0. \tag{2}$$

B. Conservation of Momentum:

$$\rho D v = -\nabla P \tag{3}$$

or

$$\frac{\partial}{\partial t}(\rho v) + \nabla(\pi) = 0. \tag{4}$$

C. Conservation of Energy:

$$\rho C_\rho DT = \rho Q + \nabla \cdot k \nabla T - T\left(\frac{\partial P}{\partial T}\right)_\rho \nabla \cdot v \tag{5}$$

or

$$\rho C_P DT = \rho Q + \nabla \cdot k \nabla T - \frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P DP. \tag{6}$$

In these equations, ∇ is the vector differential operator, D is the convective differential operator given by $D = \partial/\partial t + v \cdot \nabla$, C_ρ and C_P are the specific heats of the fluid at constant volume (density) and constant pressure, respectively. Q represents the heat input per unit mass, and π is the momentum flux density tensor which is given in this case by $(\pi)_{ij} = P\delta_{ij} + \rho v_i v_j$; here, v_i and v_j are velocity components and δ_{ij} is the Kronecher Delta. The viscous and body forces are neglected for simplicity and do not have a material effect on the arguments presented here, except that it will normally be much harder to obtain solutions to the resulting equations if these forces are included.

A Relatively Incompressible Fluid

The equation of state for small density excursions about ρ_o can be written as

$$\rho = \rho_o - \rho\beta(T - T_o) + \rho K(P - P_o) \qquad (7)$$

where $\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$ is the Bulk Coefficient of Thermal Expansion

and $K = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T$ is the Isothermal Compressibility Coefficient.

In the remainder of this discussion, the situation is considered where the last term in equation (7) can be neglected; that is, to a good approximation, ρ and its derivatives can be given by

$$\left.\begin{aligned} \rho &= \rho_d - \rho\beta(T - T_d) \\ \frac{\partial \rho}{\partial t} &= -\rho\beta\frac{\partial T}{\partial t}, \quad \nabla\rho = -\rho\beta\nabla T \end{aligned}\right\} \qquad (8)$$

where ρ_d and T_d are constants. In this case, the fluid is considered incompressible relative to its thermal expansion. This situation is of considerable importance in many applications involving gases and cryogenic fluids where the heat transfer is high, β is large, and the pressure effects are small.

Under equations (8), the energy equation can be easily simplified. Combining equations (5) and (2), it follows that

$$\rho C_\rho DT - T\left(\frac{\partial P}{\partial T}\right)_\rho \frac{D\rho}{\rho} = \rho Q + \nabla\cdot k\nabla T, \qquad (9)$$

and using (8), equation (9) becomes

$$\left(\rho C_\rho + T\left(\frac{\partial P}{\partial T}\right)_\rho \beta\right) DT = \rho Q + \nabla\cdot k\nabla T. \qquad (10)$$

The coefficient of DT in equation (10) is equal to ρC_P. This follows from the basic thermodynamic identities[2]

$$\left(\frac{\partial P}{\partial T}\right)_\rho \left(\frac{\partial T}{\partial \rho}\right)_P \left(\frac{\partial \rho}{\partial P}\right)_T = -1 \qquad (11)$$

and

$$C_P - C_\rho = \frac{T}{\rho}\frac{\beta^2}{K}. \qquad (12)$$

Hence, the energy equation becomes

$$\rho C_P DT = \rho Q + \nabla\cdot k\nabla T. \qquad (13)$$

It is also possible to eliminate DT and ∇T from this equation by using (8) and (2), and it follows that

$$\rho Q = \frac{\rho C_P}{\beta}\nabla\cdot v + \nabla\cdot\frac{k}{\beta}\nabla\rho. \qquad (14)$$

This appears to be the most useful form of the energy equation. The physical content of equation (14) is that the power input density at a point is divided between the thermal conduction output and the convective output due to thermal expansion.

It is seen by comparing equations (6) and (13) that the hypothesis of "relative incompressibility," equations (8), implies DP = 0 or

$$\frac{\partial P}{\partial t} + v \cdot \nabla P = 0. \tag{15}$$

The final equation to complete this set is obtained by operating on equation (1) by $\partial/\partial t$, switching the time and space derivatives in the second term; also operating on equation (4) by $\nabla\cdot$. These two results may be combined to give

$$\frac{\partial^2 \rho}{\partial t^2} - \nabla\cdot\nabla(\pi) = 0 \tag{16}$$

where

$$\nabla\cdot\nabla(\pi) = \nabla^2 P + \sum_{ij} \frac{\partial^2}{\partial x^i \partial x^j} \rho v_i v_j.$$

Equation (16) is normally used to predict non-linear effects associated with sound waves; it is relevant to note that equation (16) predicts the existence of shock waves, but does not describe them[3].

The relative incompressibility hypothesis thus implies the following equations for non-equilibrium thermodynamics:

$$\left.\begin{aligned} \rho Q &= \frac{\rho C_P}{\beta} \nabla\cdot v + \nabla\cdot \frac{k}{\rho\beta} \nabla\rho \\ \frac{\partial^2 \rho}{\partial t^2} &- \nabla\cdot\nabla(\pi) = 0 \\ DP &= 0 \text{ or } \frac{\partial P}{\partial t} + v\cdot\nabla P = 0. \end{aligned}\right\} \tag{17}$$

A Slowly Accelerating Fluid

The pressure P can be eliminated from these equations if we are willing to concentrate on the dynamics of the thermal expansion process near a heat transfer surface. In this situation the expansion is controlled by thermal diffusion which is a relatively slow process as compared to the propagation of sound waves. Hence, we can neglect the effect of sound waves by assuming that the fluid is slowly accelerating and that the pressure and velocity are related by Bernoulli's equation

$$P = P_\infty - \tfrac{1}{2}\rho v^2 \tag{18}$$

where P_∞ is a constant ullage pressure.

In this case, the first two non-equilibrium equations (17) become a pair of coupled non-linear equations in ρ and v

$$\left.\begin{aligned} \rho Q &= \frac{\rho C_P}{\beta} \nabla\cdot v + \nabla\cdot \frac{k}{\rho\beta} \nabla\rho \\ \frac{\partial^2 \rho}{\partial t^2} - \nabla\cdot\nabla(\pi) &= 0 \end{aligned}\right\} \tag{19}$$

where

$$\nabla\cdot\nabla(\pi) = -\tfrac{1}{2}\nabla^2(\rho v^2) + \sum_{ij} \frac{\partial^2}{\partial x_i \partial x} \rho v_i v_j.$$

Equations (19) will be referred to as "the expansion equations" and it will be shown by an example that the thermal expansion described by these equations can have oscillatory, wave-like properties near a heat transfer surface.

Thermal Oscillations

Although sound waves have been neglected, they have not been forgotten. Indeed, an important effect is the coupling of the thermal expansion waves to the possible acoustic modes of the system. These are the effects known as thermo-acoustic oscillations or thermally induced pressure oscillations[4]. In describing this coupling, both hypotheses made here will have to be relaxed far enough to include, at least, linear sound propagation. This discussion is relevant to a consideration of these effects, however, since the thermal expansion considered here is a possible driving mechanism for the thermo-acoustic oscillations.

Example – Heat Transfer in Slush

As an example of an approximate integration of the expansion equations, consider a slab (film) of liquid between a flat heater at $x = 0$ with heat flux $q(t)$ and a porous surface at $x = d$ kept at a constant temperature T_d. In this case the expansion equations for $0 < x < d$ become

$$\frac{\rho C_P}{\beta} \frac{\partial v}{\partial x} + \frac{\partial}{\partial x} \frac{k}{\rho\beta} \frac{\partial \rho}{\partial x} = 0 \qquad (20)$$

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{1}{2} \frac{\partial^2}{\partial x^2} \rho v^2 = 0 \qquad (21)$$

where v is described by a single component in the x direction. The boundary conditions are given by

$$v = 0 \text{ and } \frac{k}{\rho\beta} \frac{\partial \rho}{\partial x} = q(t) \qquad \text{at } x = 0 \qquad (22)$$

$$\rho(d, t) = \rho_d \text{ (a constant)} \qquad \text{at } x = d. \qquad (23)$$

The quantities $\frac{\rho C_P}{\beta}$ and $\frac{k}{\rho\beta}$ will be assumed constant so that equation (2) becomes

$$\frac{\partial v}{\partial x} + \frac{\alpha}{\rho} \frac{\partial^2 \rho}{\partial x^2} = 0 \qquad (24)$$

where $\alpha = \frac{k}{\rho C_P}$ is the Thermal Diffusivity. Equation (21) has a first integral given by

$$v + \frac{\alpha}{\rho}\frac{\partial \rho}{\partial x} = f(t) \qquad (25)$$

where f is an arbitrary function of t. The application of the boundary conditions (22) to equation (25) gives

$$v = \frac{\beta q(t)}{\rho C_P} - \frac{\alpha}{\rho}\frac{\partial \rho}{\partial x}. \qquad (26)$$

If the spacial derivatives of v^2 are neglected, the system of equations become

$$v = \frac{\beta q(t)}{\rho C_P} - \frac{\alpha}{\rho}\frac{\partial \rho}{\partial x} \qquad (27)$$

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{1}{2} v^2 \frac{\partial^2 \rho}{\partial x^2} = 0. \qquad (28)$$

This is a quasi-linear, coupled, set of equations and there exist separable solutions of these equations which satisfy the boundary conditions. We look for solutions of the form

$$\frac{\alpha}{\rho}\rho(x,t) = \frac{\alpha}{\rho}\rho_o(t) + \frac{\beta q(t) x}{\rho C_P} - R(t)\, S(x) \qquad (29)$$

where $\rho_o(t)$ is the density at $x = 0$, which is determined by measuring the surface temperature and using the relation (8); R and S are functions to be determined. It is convenient to consider $\rho_o(t)$ as the sum of two quantities

$$\rho_o(t) = \rho_{oo}(t) + \Delta\rho_o(t) \qquad (30)$$

where $\rho_{oo}(t)$ depends only on the nature of the heat source, q(t), and

$\Delta\rho_o(t)$ is the density (and hence temperature) change caused by the dynamics of the expansion process. If $\rho_{oo}(t)$ is given by

$$\frac{\alpha}{\rho}\rho_{oo}(t) = \frac{\alpha}{\rho}\rho_d - \frac{\beta q(t)d}{\rho C_P}, \tag{31}$$

then it follows from (23) and (29) that

$$\frac{\alpha}{\rho}\Delta\rho_o(t) = R(t)\,S(d). \tag{32}$$

Also by substituting (29) into (27) it follows that

$$v(x,t) = R(t)\,S'(x) \tag{33}$$

where the prime, ′, indicates differentiation with respect to the argument.

If the heat input is constant, $q(t) = q_o$, then ρ_{oo} is just the static equilibrium surface density. This case will now be considered:

It follows that equation (28) becomes

$$R''(S(d) - S(x)) - \tfrac{1}{2}R^3(S(d) - S(x))'^2\,(S(d) - S(x))'' = 0 \tag{34}$$

where we note in passing that $(S(d) - S(x)'^2 = (S'(s))^2$. Defining $\Delta S(x) \equiv S(d) - S(x)$, it follows that the boundary conditions for $\Delta S(x)$ are

$$\Delta S(0) = S(d) \equiv S_d, \quad \Delta S(d) = 0. \tag{35}$$

Here we note that $S(0) = 0$.

Separating variables in (34), ordinary differential equations for R and ΔS are obtained

$$R'' - bR^3 = 0 \tag{36}$$

$$(\Delta S)'^2(\Delta S)'' - 2b(\Delta S) = 0 \tag{37}$$

where b is a separation constant. These equations are directly integrable in terms of Elliptic Functions[5].

The first integrals of (36) and (37) are, respectively,

$$R'^2 = \tfrac{1}{2} b \, (R^4 + E) \tag{38}$$

$$(\Delta S)'^4 = 4 \, b \, ((\Delta S)^2 + F) \tag{39}$$

where E and F are arbitrary constants. Equations (22), (33), and (35) imply from the conditions at $x = 0$ that $F = - S_d{}^2$, and equation (39) becomes

$$S'^4 = 4b \, S(S - 2S_d). \tag{40}$$

A graph of $S(S - 2S_d)$ shows that $S(S - 2S_d) < 0$ for S between zero and S_d no matter whether sign of S is positive or negative; hence, the reality conditions of equation (40) dictate that $0 \geq b \equiv - \kappa^2$, $\kappa \geq 0$. If $b = 0$, equations (33) and (40) indicate static equilibrium; if $b < 0$ then an analysis of equation (36) shows that the solution is oscillatory as far as the time development goes. In this case, equations (38) and (40) become

$$R'^2 = \tfrac{1}{2} \kappa^2 \, (R_m{}^4 - R^4) \tag{41}$$

$$S'^4 = 4\kappa^2 S(2S_d - S) \tag{42}$$

where $|v_m(x)| = |R_m S'(x)|$ is the maximum speed of expansion at the point x and serves to define the constant, R_m. The solution to equation (41) is given by

$$R = R_m \mathrm{cn} \, (R_m \kappa t, \ 1/\sqrt{2}) \tag{43}$$

where $\mathrm{cn} \, (R_m \kappa t, \ 1/\sqrt{2})$ is the elliptic cosine of $(R_m \kappa t)$ of modulus $1/\sqrt{2}$. This is similar in form to the cosine function except in this case, the oscillations are more sawtooth in shape. A graph of this function is given in figure 4. The period of oscillation, τ, is given by

$$\tau = \frac{4}{R_m \kappa} F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) = \frac{4 \times 1.85}{R_m \kappa} \tag{44}$$

where $F(\pi/2, 1/\sqrt{2})$ is the Elliptic Integral of the First Kind of modulus $1/\sqrt{2}$ and argument $\pi/2$ (Complete Elliptic Integral). As is normal for non-linear oscillations, the period depends on the amplitude, R_m, and in this case is inversely proportional to R_m.

The separation constant κ can be evaluated by solving (42) and applying the boundary conditions at $x = d$. In this case, it is interesting to do this in order to see how the period of oscillation, τ, depends on the other parameters of the problem.

The solution to equation (42) is given by

$$\pm\sqrt{2\kappa}x = \sqrt{2S_d}\left[\left(F(\pi/2, 1/\sqrt{2}) - F(\varphi, 1/\sqrt{2})\right) + 2\left(E(\varphi, 1/\sqrt{2}) - E(\pi/2, 1/\sqrt{2})\right)\right] \tag{45}$$

where F and E are Elliptic Integrals of the First and Second Kind respectively, of modulus $1/\sqrt{2}$; the argument φ is given by

$$\varphi(x) = \cos^{-1}\sqrt{\frac{S(x)(2S_d - S(x))}{S_d^2}} \tag{46}$$

and it is seen that $\varphi(d) = 0$. Therefore, the separation constant, κ, is given by

$$\left.\begin{aligned} \kappa &= \frac{S_d}{d^2}\left[F(\pi/2, 1/\sqrt{2}) - 2E(\pi/2, 1/\sqrt{2})\right]^2 \\ &\text{or} \\ \kappa &= \frac{0.723\, S_d}{d^2}. \end{aligned}\right\} \tag{47}$$

From equations (44) and (47), the period, τ, is given by

$$\tau = \frac{10.2\,d^2}{R_m S_d}, \tag{48}$$

or in terms of the maximum dynamic density excursion at the surface, $\Delta\rho_{om}$ (see equation (32)),

$$\tau = \frac{10.2\,d^2}{\frac{\alpha}{\rho}\,\Delta\rho_{om}}, \tag{49}$$

and finally, in terms of the maximum dynamic surface temperature excursion, $\Delta T_m = \beta\Delta\rho_{om}/\rho$ (see equation (8)),

$$\tau = \frac{10.2\,d^2}{\alpha\beta\Delta T_m}. \tag{50}$$

This expression has been experimentally verified as being approximately correct for slush hydrogen[6]; it is seen that the oscillation period is also relatively independent of the amount of heat input, q_o, and this has also been verified[6]. (In a more accurate treatment of the expansion equations it is likely that ΔT_m will have a weak dependence on q_o.)

Discussion

The example outlined above, specialized as it may be, allows the formulation of several conjectures which should be checked by a more exact treatment of the expansion equations and also could form the basis of an experimental program:

1) Oscillations in the thermal expansion can occur in any fluid within a distance of the order of a thermal diffusion length of the heat transfer surface, provided the thermal expansion coefficient is high enough.

2) The frequency of the oscillation is relatively independent of the magnitude of the heat flux at the surface.

3) It is seen that the initial conditions of the example are formulated assuming that the fluid is in a condition of steady state oscillation; it is quite likely that the existence of oscillations depends on the nature of the initial conditions; and in particular, what is q(t)? From this point of view it is also quite likely that mechanical perturbations can initiate oscillations; this has been seen experimentally[7].

4) It is possible that the one dimensional nature of the example is essential for the existence of oscillations. If the expansion velocity has more than one component, then the tensorial properties of π make it quite unlikely that separable solutions could be obtained. Although separability is not, a priori, necessary for the existence of oscillations, it certainly enhances the possibility. This conjecture has been checked experimentally[8] by cutting machine threads into the inside of siphon tubes[9], and also by threading a valve stem[10]. In these cases the oscillations are damped, possibly because of a three component expansion velocity.

5) It is likely that viscous forces, if they are large, could dampen the oscillations both from the point of view of separability and also as an energy absorbing mechanism.

6) An analysis of equation (45) near $x = 0$ shows that there is a weak singularity in the density at the heated surface suggesting that the oscillations in an extended fluid may be associated either with shock waves or weak shock waves. If this is the case, it may be necessary to use the integral form of the conservation equations in order to obtain a complete solution to the problem.

7) It is likely that these expansion oscillations may excite possible acoustic modes of a system (thermo-acoustic oscillations). In this case,

the oscillation frequency will probably be determined by the acoustic mode. The expansion oscillations may be at a different frequency, but since this oscillation is non-linear it would have a broad excitation spectrum as a forcing mechanism for the acoustic mode.

Acknowledgements

In formulating this problem and the resulting conjectures, the author has been benefited on many occasions by discussion with his colleagues; Vincent D. Arp, Patricia J. Giarratano, Michael C. Jones, Richard H. Kropschot, Douglas B. Mann, Alan F. Schmidt, Charles F. Sindt, W. Gene Steward, Donald B. Sullivan, and Daniel H. Weitzel.

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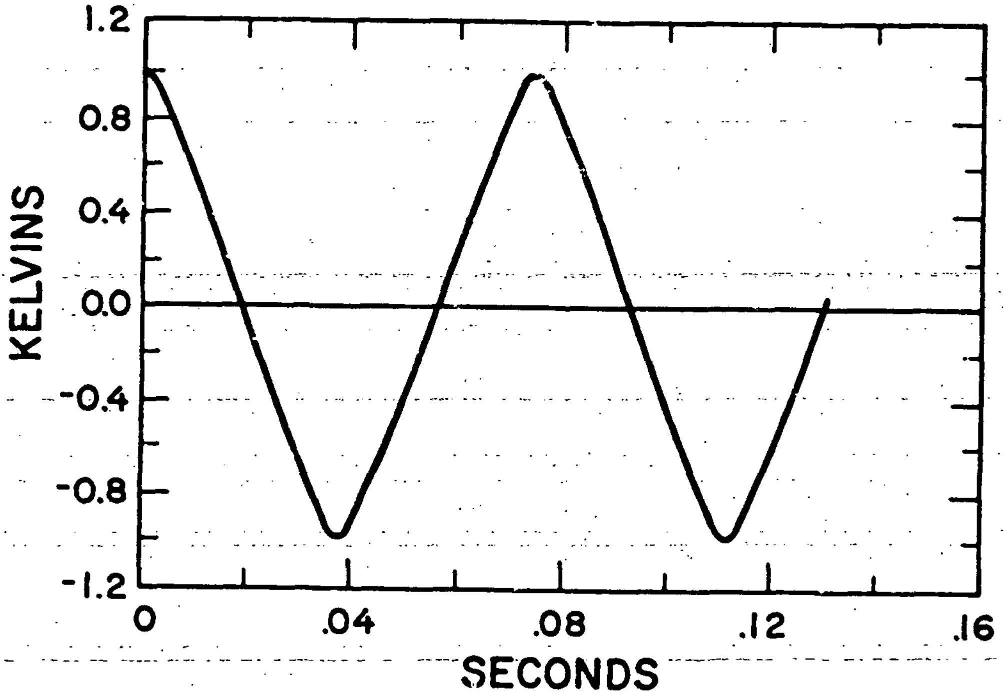

Figure 4.

Part III

Surface Damping of Thermally Induced Oscillations in Cryogenic Systems

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ABSTRACT

Theoretical and experimental investigations have shown that thermal oscillations originating at a steadily heated surface depend on the nature of the thermal expansion processes at the surface as described by the expansion equations of non-equilibrium thermodynamics. These equations indicate that thermal oscillations are stronger and easier to initiate when the heat transfer surface has a smooth, regular geometry; and are weak and hard to initiate and maintain if the heat transfer surface is rough or irregular. Simple experiments are described which substantiate these findings.

Key words: Liquid helium; siphon tubes; thermal oscillations.

Introduction

The existence of thermally induced oscillations is a recurring problem in liquid helium and slush hydrogen cryogenic systems. These low frequency oscillations are normally observed as pressure fluctuations and high heat transfer in gauge lines, valve stem clearances, relief valves, siphon tubes, etc. It is possible that large scale oscillations which are occasionally observed in cryogenic systems may have the same origin. They have also been observed as surface temperature fluctuations, using a resistive carbon film sensor immersed in slush and liquid hydrogen[1].

These oscillations are always associated with a region of the system where there is heat conduction from a solid constraint into the system; hence, the term "thermally induced oscillations." Various methods have been proposed for controlling or damping the oscillations which include drilling small vent holes in closed tubes[2], teeing-in Helmholtz resonators to external lines[3], and inserting woolen threads into siphon tubes[4]. In this note, a method is proposed which could be applied to many different situations.

The conservation equations of non-equilibrium thermodynamics suggest that thermally induced oscillations originate as non-linear oscillations in the thermal expansion of the fluid near the heat transfer surface[5]. These non-linear oscillations can occur if 1) there is a weak singularity in the expansion velocity at the heat transfer surface and 2) there is a long range coherence of the expansion velocity within a distance, d, from the surface. The distance, d, depends on the heat transport properties of the fluid and the geometry of the surface; for example, for a flat surface in liquid hydrogen, $d \approx 10^{-3}$ cm.

It is tempting to suggest that a surface roughness on the order of d will excite random components of the expansion velocity destroying the long range coherence and thus damping the oscillations at the source. Accordingly, it was proposed[6] that a screw thread cut into the inside of a siphon tube might be of sufficient roughness to dampen the oscillations.

Four brass tubes, 1/4-inch OD and 5 inches long, were fabricated as follows: three of them were drilled with a #46 (0.0810 inch) drill and two of these were tapped with a 3-56 tap (75% full thread); the fourth tube was drilled smooth with a #39 (0.0995 inch) drill corresponding to the OD of the tap. These brass tubes were soldered to four stainless steel siphon tubes 1/8-inch OD, 0.010 inch wall, and three feet long. They were inserted (after precooling) into a 25 liter liquid helium storage dewar which was about half full of liquid. Oscillations were detected as pressure oscillations in a small cup which was soldered to the other end of the stainless steel tube. For the two smooth drilled brass tubes, the oscillations were very strong, easy to excite, and persisted indefinitely. For the two tapped brass tubes, the oscillations were very weak, hard to excite, and occasionally would damp out completely.

The principles outlined above were applied to an existing problem involving a valve stem clearance in a slush hydrogen system[7]. The valve stem was a 3/32-inch stainless rod which had a small flat filed along its 4-foot length. The stem traveled through a 1/8-inch, 0.010-inch wall stainless steel tube which was inserted into a 1/4-inch stainless steel tube. The 1/4-inch tube was a vacuum jacket and the space between the 1/4-inch tube and the 1/8-inch tube was filled with helium gas at ~ 5 psi. The oscillations were observed as vapor bubbles streaming from the 1/8-inch tube whenever the end of the tube was inserted into the slush, thus indicating an abnormally high heat transfer. Various efforts to open, close, change the gas or pressures at the other end of the tube failed to suppress the bubbles although the bubbling would occasionally stop after a period of about 20 minutes.

The valve stem was then threaded along its entire length with a 3-56 die. It was found that the bubbling damped completely in less than one minute after inserting the 1/8-inch tube into the slush.

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