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**A COMPUTING METHOD FOR SOUND PROPAGATION  
THROUGH A NONUNIFORM JET STREAM**

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A COMPUTING METHOD FOR SOUND PROPAGATION  
THROUGH A NONUNIFORM JET STREAM

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ABSTRACT

Understanding the principles of jet noise propagation is an essential ingredient of systematic noise reduction research. High speed computer methods offer a unique potential for dealing with complex real life physical systems whereas analytical solutions are restricted to sophisticated idealized models. The classical formulation of sound propagation through a jet flow was found to be inadequate for computer solutions and a more suitable approach was needed. Previous investigations selected the phase and amplitude of the acoustic pressure as dependent variables requiring the solution of a system of nonlinear algebraic equations. The nonlinearities complicated both the analysis and the computation. A reformulation of the convective wave equation in terms of a new set of dependent variables is developed with a special emphasis on its suitability for numerical solutions on fast computers. The technique is very attractive because the resulting equations are linear in nonwaving variables. The computer solution to such a linear system of algebraic equations may be obtained by well-defined and direct means which are conservative of computer time and storage space. Typical examples are illustrated and computational results are compared with available numerical and experimental data.

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Symbols

$a$	Ambient speed of sound
$A$	Amplitude
$d$	Jet exit diameter, 0.01905
$f$	Frequency in Hertz
$M$	Flow Mach number; $U_j/a$
$r, \theta, \phi$	Spherical polar coordinates as in figure 1
$\tilde{r}$	Non-dimensional $r$ w.r.t. $d$ ; $r/d$
$t$	Time
$U$	Longitudinal component of flow velocity
$U_j$	Jet exit velocity
$W$	Non-dimensional angular frequency; $\omega d/a$
$x$	Cartesian coordinate in the windward direction of the jet
$z$	Complex variable; $Ae^{i\psi_1}$
$\alpha$	Constant in equation 3; $\alpha^2 < 1$
$\zeta$	Coordinate transformation; $\tan^{-1}(\alpha^{-2} \tan \theta)$
$\eta$	Coordinate transformation; $\ln \tilde{r}$
$\Pi$	Quasi velocity potential
$\Psi, \Psi_1, \Psi_0$	Phase; $\Psi = \Psi_0 + \Psi_1/\tilde{r} + o(\tilde{r}^{-2})$
$\omega$	Angular frequency

## INTRODUCTION

This work is an extension of a program at NASA Langley Research Center concerning the investigation of noise generated by a jet. It improves upon previous work by Schubert<sup>1</sup> and by Liu and Maestrello<sup>2</sup> in which finite difference equations appropriate to the propagation of sound are solved by iterative methods for a sinusoidal point source on the axis of the potential flow region of a subsonic jet. A computer-oriented method is developed which solves typical sound propagation problems while using a minimum amount of costly computer resources. The method is tested for cases equivalent to those presented in reference 1 so that comparisons can be made with numerical as well as experimental results. The present results compare favorably with experimental measurements by Grande<sup>3</sup>, whereas the numerical computations in reference 1 overpredict the depth of the downstream "valley" in jet noise directivity. More notably, the present method drastically reduces the requisite computer time, computer storage space and peripheral usage. Furthermore, the current formulation can be solved by direct methods as opposed to the iterative procedure employed by Schubert. Direct methods are desirable because they are straightforward, they guarantee results and because suitable computer routines are often available as standard library subroutines.

The method presented here has been thoroughly tested using the idealized model outlined in reference 1. Immediate extension to more realistic models, e.g., reference 2, is possible and would presumably net even more reliable results.

### Formulation of Problem

The Nonuniform Jet Flow Field.— The present method of solution is tested for the case of an acoustic point source located on the centerline and contained within the potential core of a spreading jet, figure 1. The jet nozzle is cylindrical with diameter,  $d$ . The jet flow is axisymmetric but nonuniform such that the velocity profiles at any cross-section are determined from experimental and empirical data. The origin of the coordinate system is fixed at the source while the coordinate of the flow field is fixed at the virtual origin of the jet. Most of the calculations are done in the spherical polar coordinates,  $(r, \theta, \phi)$ .

The Convective Wave Equation.— Sound propagation through a flow field can be described most simply by the following convective wave equation:

$$\left[ \frac{1}{a^2} \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 - \nabla^2 \right] \Pi = 0 \quad (1)$$

Here  $a$  is the ambient speed of sound,  $U$  is flow velocity, and  $\Pi$  is Obukhov's "quasi-potential" variable<sup>1</sup>. The formulation in terms of  $\Pi$  is selected over the usual pressure formulation since in this way relatively good results are possible without the addition of corrective terms to the right-hand side of the equation.

Assuming a harmonic source,  $\Pi$  can be expressed as  $\Pi = A \exp \left[ i(\Psi \tilde{r} - \omega t) \right]$  where  $A$  &  $\Psi$  are the amplitude and phase of the radiated sound waves and  $\tilde{r}$  denotes the nondimensional radial distance with respect to the nozzle diameter,  $d$ . Substituting  $\Pi$  into equation 1 results in a time independent partial differential equation which is nonlinear in  $A$  and  $\Psi$ . Schubert<sup>1</sup> chooses to solve this equation by application of the finite difference

method. This necessitates solving a system of simultaneous nonlinear algebraic equations.

There is no direct method for solving such a system of nonlinear equations. Iterative computer methods are available but they have a number of drawbacks. First, they are highly specific. The coding is dependent on the problem to be solved and must be rewritten and retested for each new equation. Secondly, the accuracy of the results depends upon how well and how quickly the process converges. In the case of sound propagation through a flow, it remains to be shown that the process does in fact converge for all instances.

The problems inherent in iterative methods can be avoided. The variable  $\Psi$  can be represented by the expansion,  $\Psi = \Psi_0 + \Psi_1/\tilde{r} + o(\tilde{r}^{-2})$  so that  $\Psi$  approaches  $\Psi_0$  as  $\tilde{r}$  approaches infinity. Then  $\Pi = Z \exp \left[ i(\Psi_0 \tilde{r} - \omega t) \right]$ .

Thus,  $Z$  is a single complex variable and equation 1 can be rewritten as a linear partial differential equation in  $Z$ . In this way, the solution by finite difference method involves merely the solution of a system of simultaneous linear algebraic equations. Most computer subroutine libraries contain routines adequately suited to this task.

### Numerical Solution

Underlying Assumptions.— The basic assumptions and basic scheme for solving the problem follow directly from previous authors<sup>1,2</sup>. Figure 2 shows the spreading jet superimposed on a polar grid. An antijet is assumed in order to avoid problematic boundary conditions along the rigid walls of the jet nozzle.

The value of the complex variable,  $Z$ , changes most rapidly near the point source and in the region of significant flow. It is desirable to apply the finite difference method to an unequally spaced grid which has a

concentration of points in the areas of greatest change, figure 2. To avoid the difficulties posed by an uneven grid, an even grid is specified in some new coordinates,  $\eta$  and  $\zeta$ , figure 3. This new grid in  $(\eta, \zeta)$  maps onto the desired uneven grid in  $(\tilde{r}, \theta)$  according to the following transformation;

$$\begin{aligned}\eta &= \ln \tilde{r} \\ \zeta &= \tan^{-1} (\alpha^{-2} \tan \theta).\end{aligned}\tag{2}$$

For illustration purposes, the polar grid in  $(\eta, \zeta)$  is thought of as a rectangular grid with  $n \times m$  intersecting lines.

The Finite Difference Method.— The partial differential equation in  $Z$  should be satisfied at each interior point of the region of interest. If the partial derivatives in this equation are replaced by their central difference approximations, a nine-point difference equation results. Writing this difference equation at each interior grid point gives  $(n-1) \times (m-1)$  equations in  $n \times m$  unknowns. Boundary conditions are the additional equations needed to specify a unique solution.

Boundary Conditions.— The equation in  $Z$  is an elliptic type partial differential equation, thus, the boundary value problem is well-posed.

The region,  $D$ , is bounded by two concentric half circles centered on the point source and by two line segments on the axis of symmetry, figure 4. The inner circle has a radius of  $\frac{1}{2} d$  and the outer circle has a radius of  $100 d$ , where  $d$  is the diameter of the jet nozzle. In this way, the inner boundary is contained in the potential core of the jet, and the outer boundary approximates the far field.

The inner boundary conditions come from a solution derived by Moretti and Slutsky<sup>4</sup>. The solution is specified for a point source in a uniform



flow. It gives the sound pressure level for points a small distance from the source. Since the acoustic source and the inner boundary are contained in the potential core of the jet, they are in a locally uniform flow and the Moretti and Slutsky solution is approximately valid.

The outer boundary conditions at the far field are essentially the same as the Sommerfeld radiation conditions. Along the centerline, three-point symmetry conditions are used.

### Results and Discussion

Figures 5-7, show sample results at a variety of flow velocities, acoustic source frequencies and radial distances from the source. In each graph, sound pressure level relative to SPL at  $\theta = 90^\circ$  is plotted against  $\theta$ . These figures facilitate comparisons between the experimental data of reference 3 and the numerical results of this paper and reference 1.

Figure 5 shows comparable results for the flow velocities Mach 0.3, Mach 0.5, and Mach 0.9. Here radial distance from the source and source frequency remain constant at 100 diameters and 3000 Hertz, respectively. Notice that the correspondence between experimental results and the current numerical results is especially good at the lower Mach numbers. Also notice the discrepancies between the numerical results of reference 1 and experimental results at these same Mach numbers.

Figure 6 shows graphs, similar to those above, where radial distance and flow velocity are constant ( $r = 100 d$  and  $M = 0.3$ ) while frequency changes from 3000 Hz to 5000 Hz to 7000 Hz. In all cases, numerical results obtained by the present method are superior to those obtained in reference 1 with respect to their agreement with experiment.

Figure 7 illustrates the differences between the current method and

the previous one. Sound pressure levels for Mach 0.7 and frequency 3000 Hz are plotted at a variety of radial distances from the source. It is clear that the differences between the graphs increase with the radial distance. Even greater discrepancies are evident when phase is plotted against  $\theta$  as in figure 8. Again, results for Mach 0.7 and frequency 3000 Hz are plotted at a variety of radial distances from the source. In this case, the graphs from reference 1 are markedly different from those produced by the present method.

It should be emphasized that the present method and the method of reference 1 use the same sound propagation model for testing purposes. Theoretically, they should produce identical results. Differences arise when theory is translated into practical computer programs. Iterative methods, such as the one described in reference 1, are often inaccurate because the iterative process must be terminated after a reasonable amount of computer time has elapsed. Furthermore, an iterative method is more susceptible to round-off errors which tend to multiply as computing time increases. Direct methods, such as the method presented here, largely avoid these problems and generally produce more reliable results. This explains why graphs of the direct method's results match graphs of experimental data so closely.

The present method has other advantages besides accuracy. It is quite straightforward and relatively easy to program. A typical program will execute in well under 1 minute of central memory time in a CDC 6600 computer. The computer storage space requirements are reasonable so that transfer of data to and from peripheral storage devices is unnecessary. None of these things can be said about the method of reference 1. Far from guaranteeing results in under 1 minute, the iterative method can not

guarantee convergence to a solution for the general case. Furthermore, the algorithm for solving a system of nonlinear equations is quite complex and requires large amounts of storage space.

### Conclusion

It has been shown that the present method for sound propagation problems has distinct advantages over previous methods. It is a direct rather than iterative method and, consequently, it is faster, more accurate, and less complicated. The method is conservative of computer resources, relatively easy to program, and makes use of standard library subroutines. Since the method itself is relatively simple, increasingly complex and realistic models can be tested.

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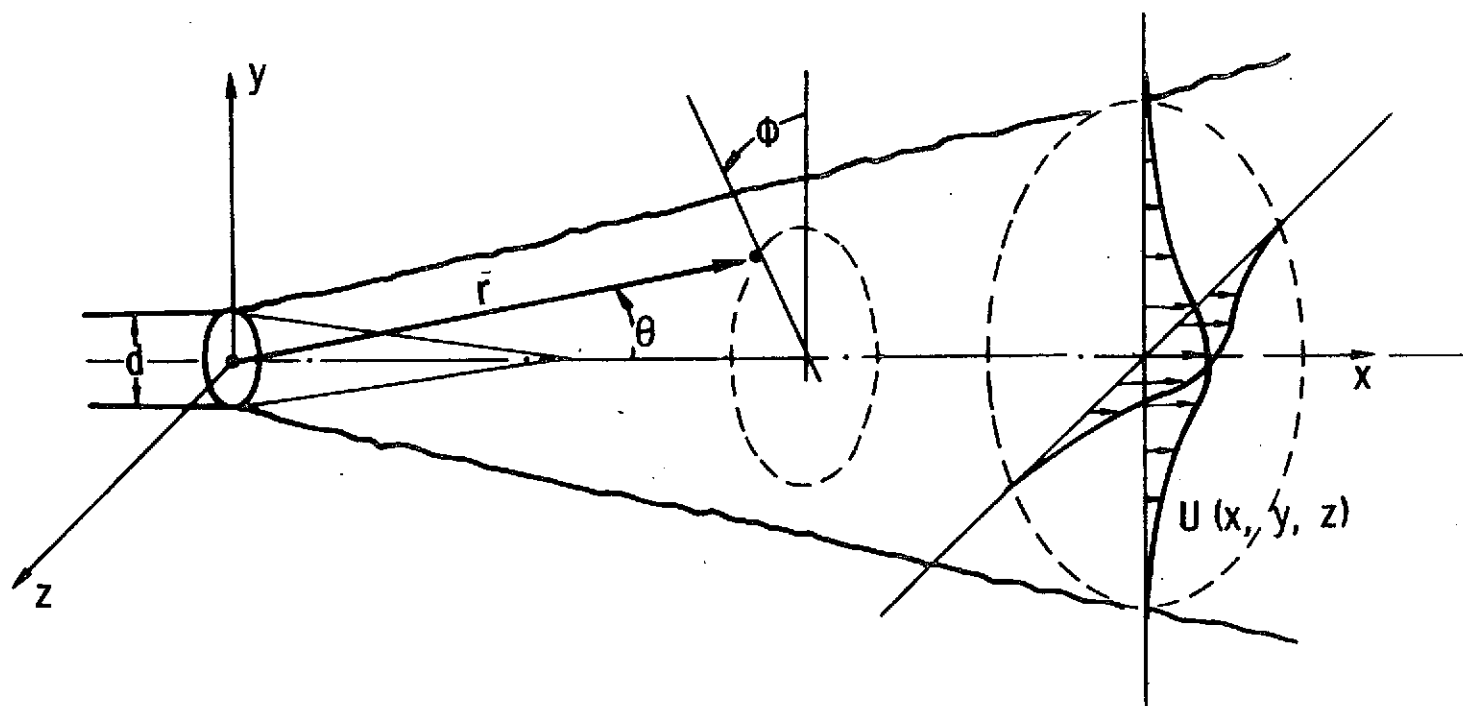


Figure 1. Jet flow configuration.

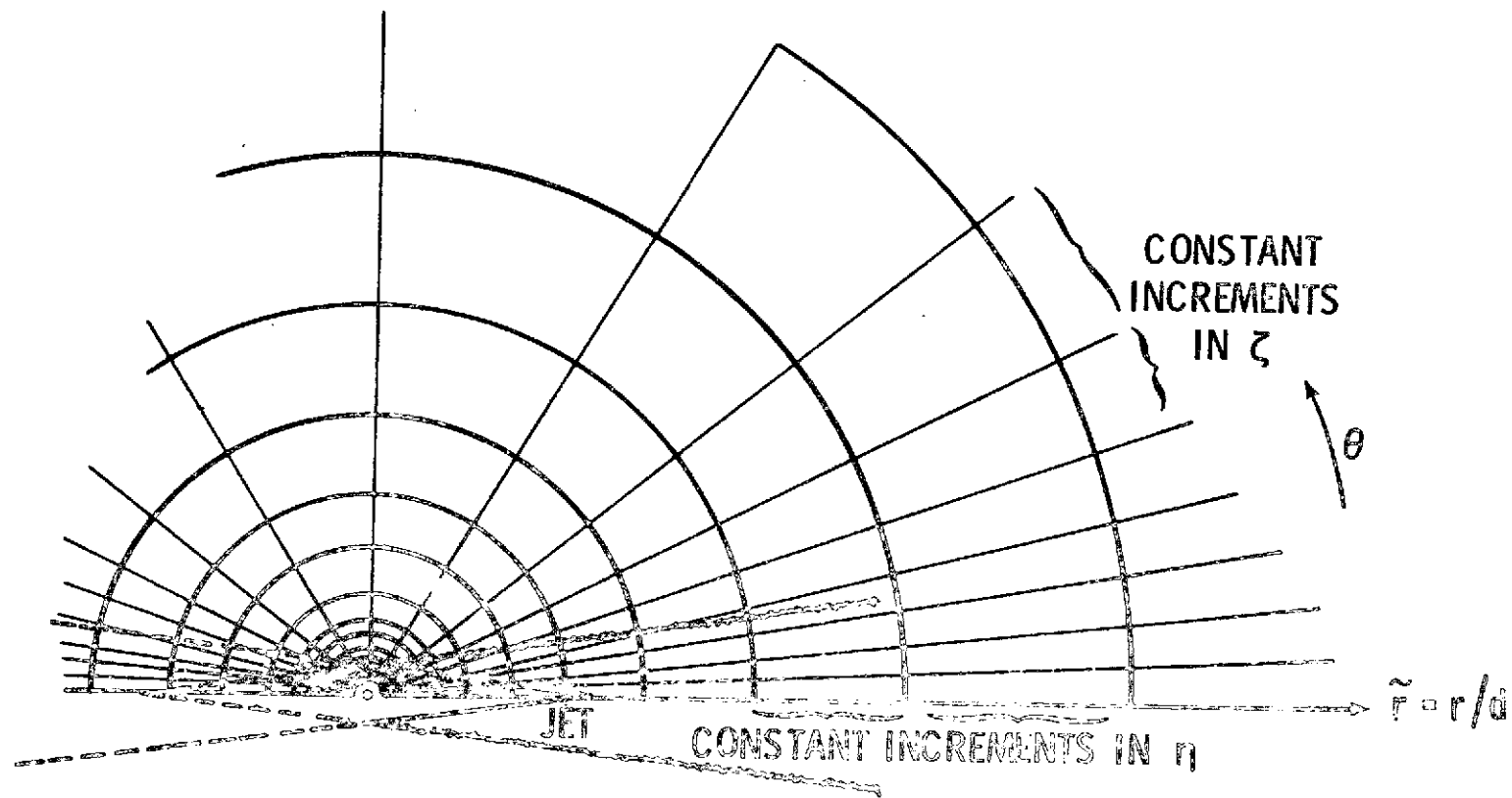


Figure 2. The finite difference grid as seen in the physical plane

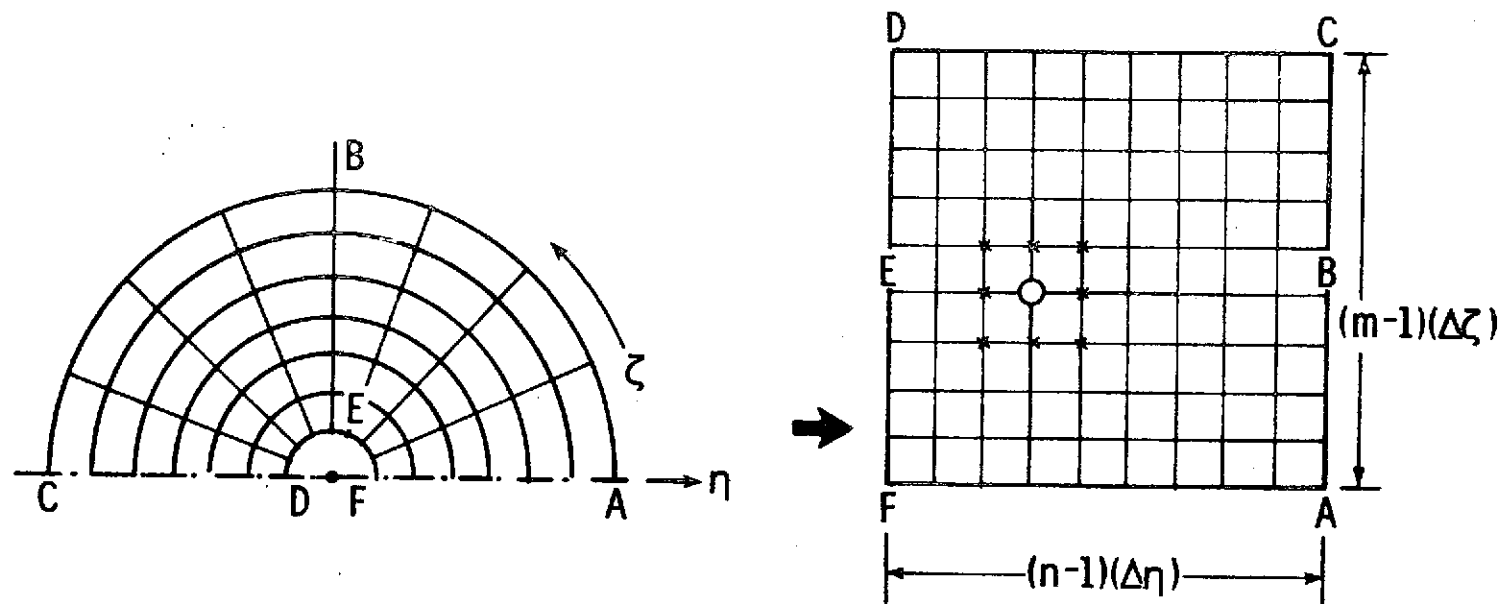


Figure 3. A grid for the finite difference method  
in the  $(\eta, \zeta)$  plane

INNER BOUNDARY:

MORETTI AND SLUTSKY'S SOLUTION (1959)

OUTER BOUNDARY:

SOMMERFELD RADIATION CONDITIONS

CENTER LINE:

THREE-POINT SYMMETRY CONDITIONS

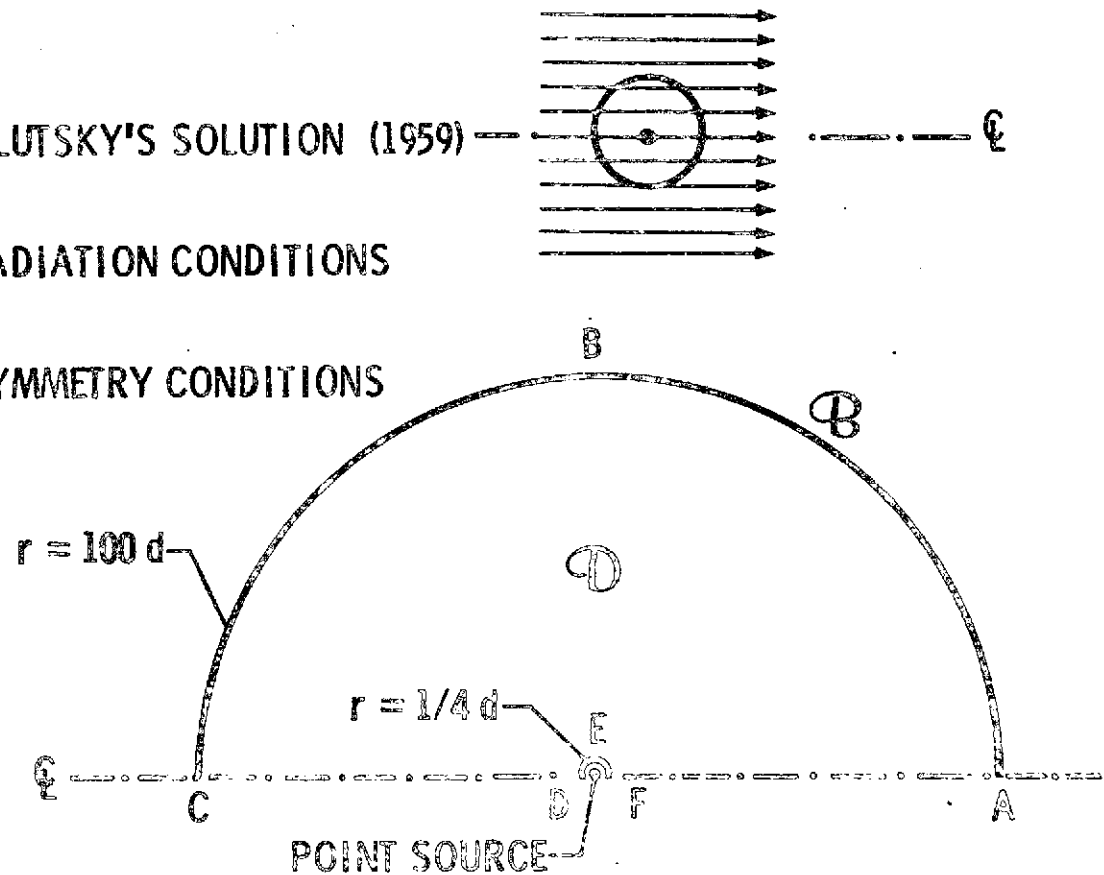


Figure 4. Boundary conditions

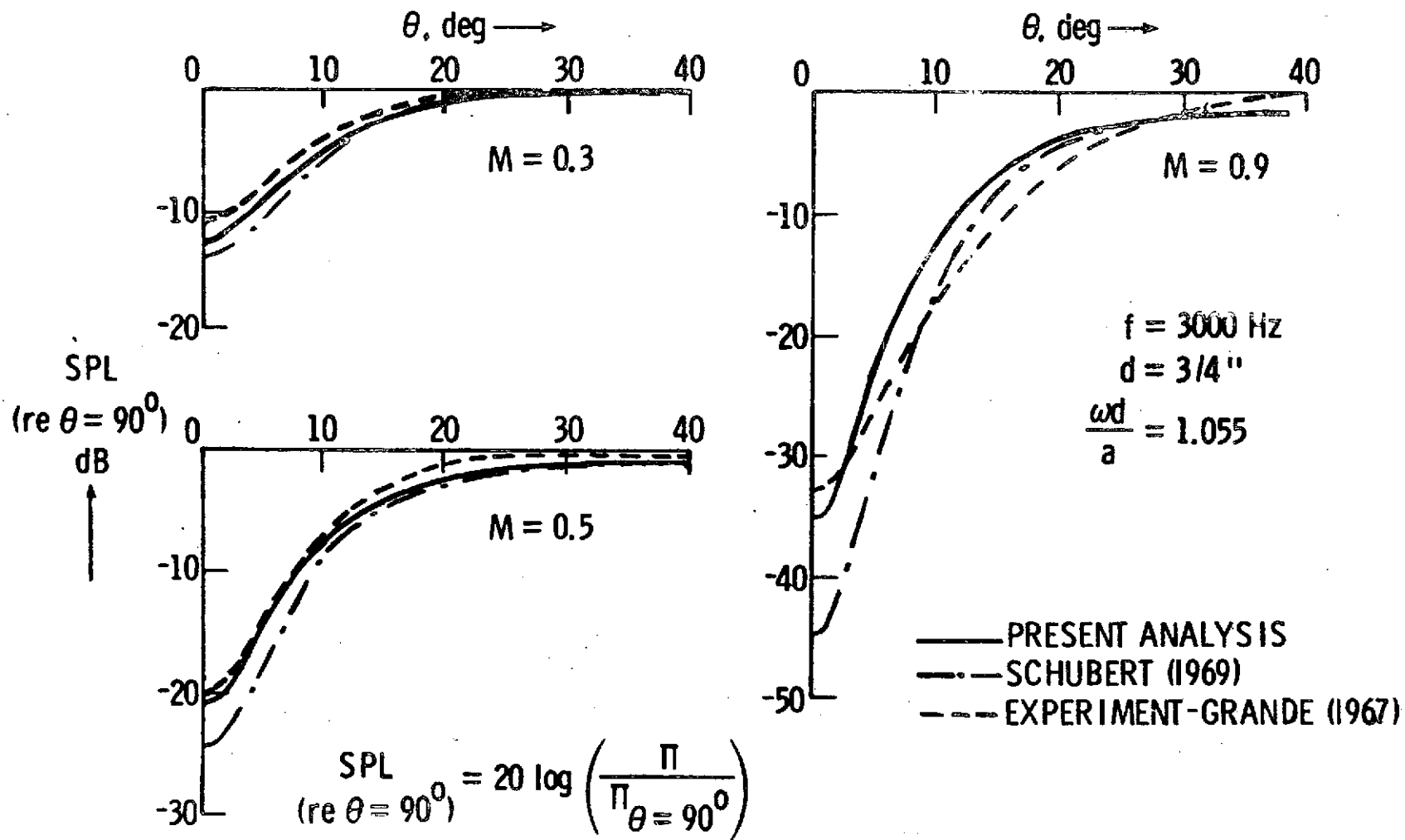


Figure 5. Sound pressure level at 100 d from the source for several Mach numbers. Source at 2 d.



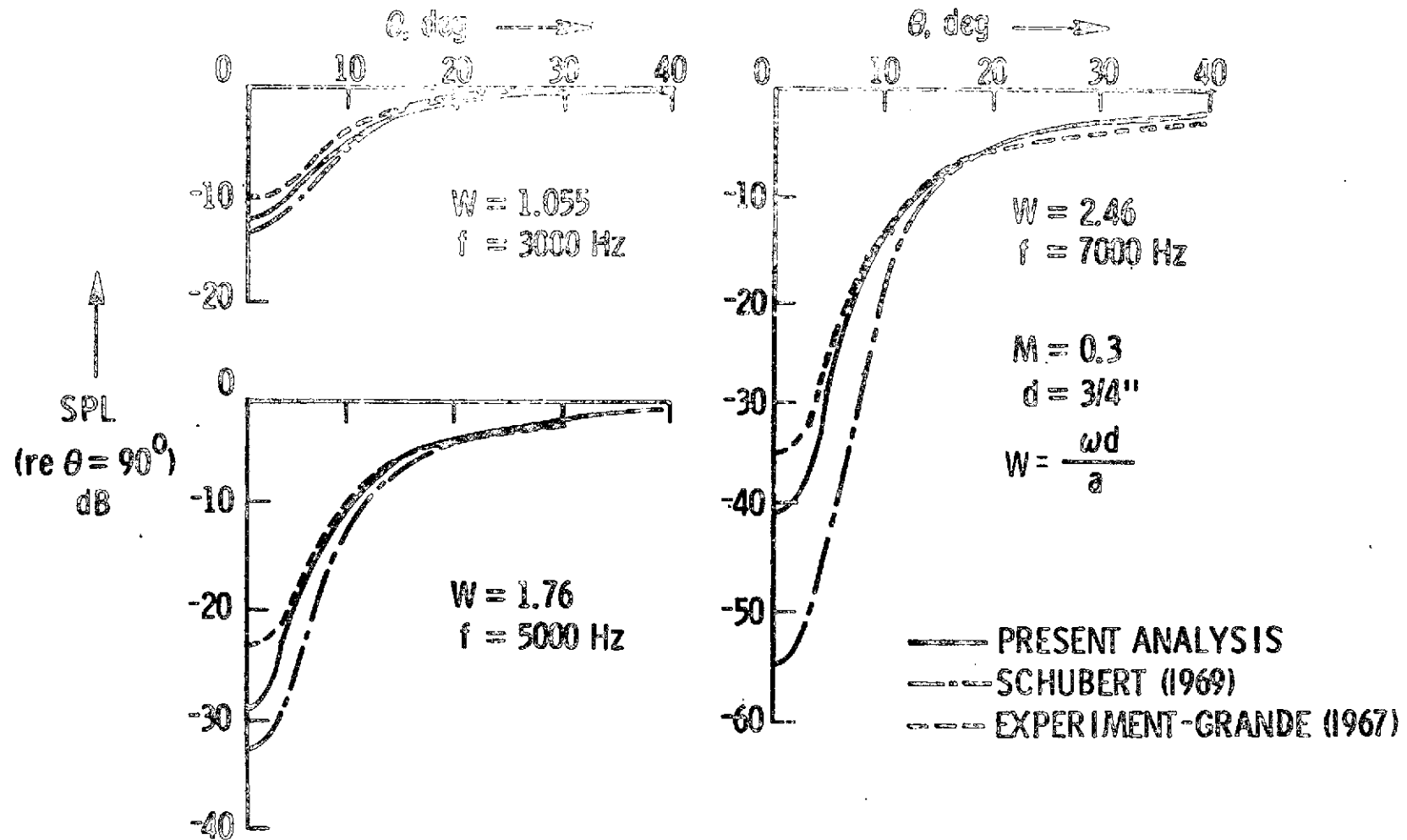


Figure 6. Sound pressure level at 100 d from the source for several frequencies. Source at 2 d.

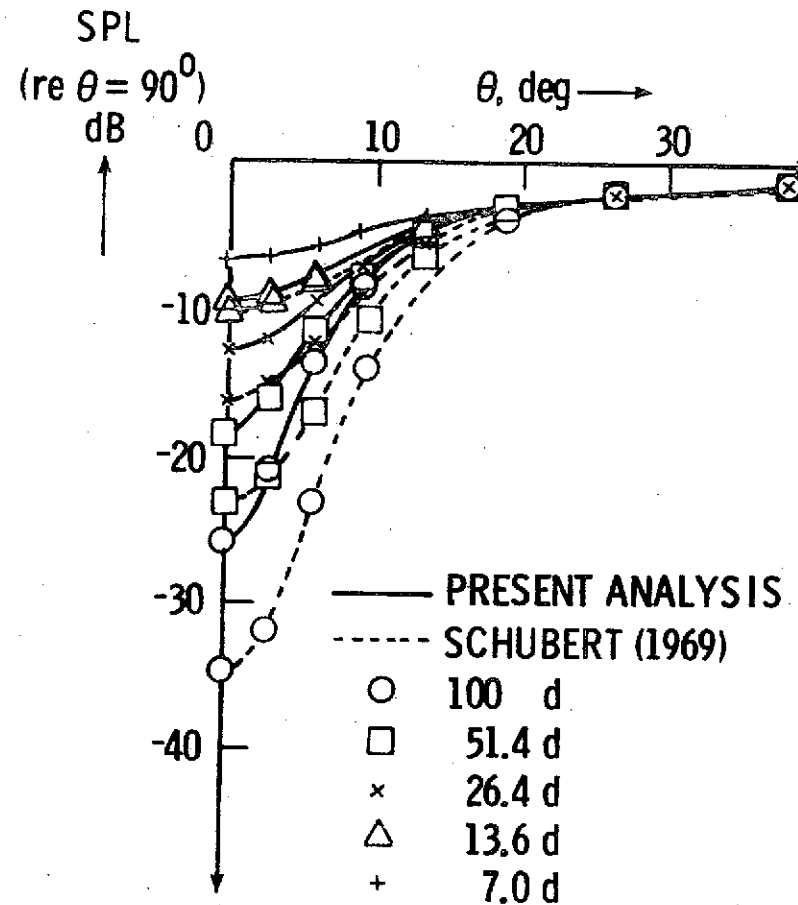


Figure 7. Sound pressure level at various distances from the source.  $M=0.7$ ,  $W=1.055$  (3000 Hz for a 0.75" jet), source at 2 d.

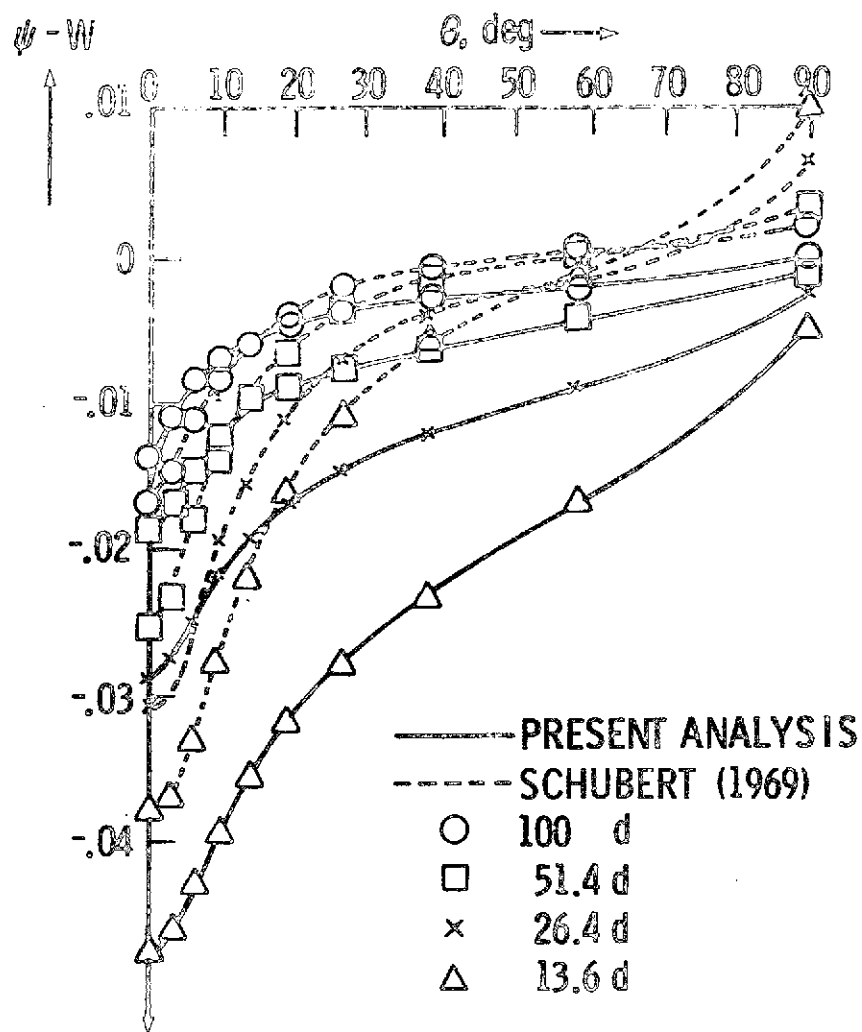


Figure 8. Phase at various distances from the source.  
 $M=0.7$ ,  $W=1.055$  (3000 Hz for a 0.75" jet), source at 2 d.