139124

Computations of

M_2 and K_1 Ocean Tidal Perturbations

in Satellite Elements

by

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16. Abstract

Semi-analytic perturbation equations for the influence of M_2 and K_1 ocean tidal constituents on satellite motion are expanded into multi-dimensional Fourier series and calculations made for the BE-C satellite. Perturbation in the orbital elements are compared to those of the long period solid earth tides.

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FOREWORD

This report describes the work performed by Business and Technological Systems, Inc. under Contract No. NAS 5-20045 with the NASA/Goddard Space Flight Center. The work deals with the development of a computer program to determine the ocean tidal perturbations in the elements of artificial earth satellites.

1. INTRODUCTION

Tidal forces have special significance in geophysics since their study can give insight into the properties of the earth's interior and its elastic responses and ocean dynamics. Until recently the analysis of tidal perturbations of artificial satellites due to the *combined* effect of earth and ocean tides has been applied to the determination of values for the characteristic parameters (Love numbers) which describe the elastic response of the earth with only the influence of solid earth tides taken into consideration. These results yield a value for the Love number k_2 somewhat smaller than anticipated and show an apparent dependence of k_2 on the orbital inclination. The perturbations due to ocean tides could be the primary cause of this disagreement.

As the ocean tide, which manifests itself directly through gravitational attraction as well as indirectly through effects such as ocean loading is observable in satellite data, a basic understanding of the mechanism involved and an accurate ocean tidal theory is required for precise analysis of satellite perturbations based on the models of the principal ocean tidal components. Existing models of tidal component amplitudes and phase lags can be used to give a general estimate of oceanic effects on satellites. However, the comprehensive treatment of the probelm requires the solution of the Laplace Dynamical Tidal Equation with realistic boundary conditions, friction and ocean loading to obtain more accurate tidal component models.

In the present work we follow the semi-analytical treatment of Musen [5] for the oceanic perturbation equations and develop computer programs to calculate the influence of ocean tides on the satellite orbital elements in terms of the tidal amplitudes and lags of existing models for the M_2 and K_1 ocean tidal components. It is with pleasure that the author acknowledges the direction and support of Dr. Peter Musen of NASA/Goddard Space Flight Center in this effort.

2. TIDAL CONSTITUENTS

The sum of the gravitational potentials of the moon and sun at any point on the surface of the earth are periodic functions of time. In 1921 Doodson [2] presented a harmonic expansion based on the Brown lunar theory which is the basis of modern tidal analysis. In particular, Doodson represented the tidal potential as a function of six basic astronomical variables in the form

$$\sum_{a_1,\ldots,a_2} A(a_1,\ldots,a_6)\exp i[a_1\tau + a_2 s + a_3 h + a_4 p + a_5 N' + a_6 p_s] \tag{1}$$

where

- τ is mean lunar time
- s is the mean longitude of the moon
- h is the mean longitude of the sun
- p is the longitude of the lunar perigee
- N' is the negative of the longitude of the ascending lunar node
- p_s is the longitude of perihelion

and where the sum is taken over all positive and negative integer values of the a_i. Each term in the expansion displaces the geoid a distance

$$\delta h = \frac{A(a_i)}{g}$$

where g is the value of the local gravity, and these displacements are called tides or constituents.

The tides fall into three major species denoted by $a_1 = 0$ (long period), $a_1 = 1$ (diurnal) and $a_1 = 2$ (semidiurnal), and are usually denoted by the Doodson number , defined as

$$(a_1,\ a_2+5,\ a_3+5,\ a_4+5,\ a_5+5,\ a_6+5).$$

Certain major constituents are labeled by letters, with a subscript denoting the integer a_1. For example M_2, P_1, K_1, O_1 denote (2,5,5,5,5,5), (1,6,3,5,5,5), (1,6,5,5,5,5) and (1,4,5,5,5,5) respectively.

The astronomical variables are very nearly linear functions of time and are expressed in mean solar days (T) from January 0, 1900 mean noon of Greenwich as

$$s = 270^\circ.43659 + 13^\circ.1763967\ T$$
$$h = 279^\circ.69668 + 0^\circ.9856473\ T$$
$$p = 334^\circ.32956 + 0^\circ.11140408\ T$$
$$N' = 100^\circ.81672 + 0.^\circ 0529539 T$$
$$p_s = 281^\circ.22083 + 4^\circ.70642026 \times 10^{-5}\ T.$$

Mean Lunar time (τ) and mean solar time (t) are related to sidereal time (θ) through the relations

$$\theta = t + h$$
$$\theta = \tau + s$$

so that

$$\tau = t + h - s.$$

and the variables ℓ, ℓ', F, D, Γ of the lunar theory are related to the Doodson variables such that

$$\ell = s - p$$
$$\ell' = h - p_s$$
$$F = s + N'$$
$$D = s - h$$
$$\Gamma = p_s$$

or

$$s = \ell' + D + \Gamma$$
$$h = \ell + \Gamma$$
$$p = -\ell + \ell' + D + \Gamma$$
$$N' = -\ell' + F - D - \Gamma$$
$$p_s = \Gamma\ .$$

For example, the M_2 tide (2,5,5,5,5,5) has an argument

$$2\tau + 0 \cdot s + 0 \cdot h + 0 \cdot p + 0 \cdot N' + 0 \cdot p_s = 2t + 2h - 2t - 2D$$

and the K_1 tide (1,6,5,5,5,5) has

$$\tau + s = \theta = t + h.$$

Typically the solutions for a tidal constituent $a_1=m$ is represented in the form

$$F_m(x,\phi)\ \cos(\sigma t+v+S(x,\phi)) \tag{2}$$

where the phase angle $S(x,\phi)$ is the retardation of the high water at (x,ϕ) relative to high water at Greenwich and t is the number of mean solar days from epoch t_0. Tidal chart data is generally in the form of co-range lines (contours of equal amplitude) and co-tidal lines (isochrones of the retardation) [7] with the time origin at lunar transit at Greenwich.

3. OCEAN TIDAL PERTURBATIONS

As developed by Musen[5], the satellite perturbations due to ocean tides are expanded into Fourier series with the orbital elements Ω and ϖ of the satellite and ℓ, ℓ', F, D and Γ of the lunar theory as arguments. The coefficients of the expansion are numerical and depend upon the satellite elements. For the sake of completeness we present a brief description of the Musen development.

The disturbing function V can be written

$$V = 3\frac{\kappa}{\kappa_0}\frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{a}\right)^{n+1}(1+k_n')\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}N_{nm}P_{nm}(\sin\delta)$$

$$\times\frac{1}{4\pi}\iint\xi(x',\phi',t)\,P_{nm}(\sin\phi')\cos(m[\alpha-\theta-x'])d\sigma' \tag{3}$$

where G is the universal gravitational constant, M is the mass of the earth, r is the geocentric distance of the satellite, a is the semi-major axis of the satellite orbit, R is the radius of the mean level surface of the ocean, $R\xi$ is the height of the water tide, κ_0 and κ are the densities of the earth (mean) and sea water respectively, k_n' are Love numbers associated with the effects of loading of the tidal water mass, α and δ are the right ascension and declination of the satellite, θ is the sidereal time at Greenwich, P_{nm} are the associated Legendre polynomials and

$$N_{nm} = \frac{2(n-m)!}{(n+m)!} - \delta_{m,0} \tag{4}$$

where δ_{ij} is the familiar Kronecker delta.

The tidal oscillations of the sea at a given point (x', ϕ') on the mean sea surface is represented as a sum of periodic components of the form

$$\xi(x',\phi,t) = \sum_m f_m = \sum_m F_m(x',\phi')\cos[m(\theta+x')+q_m(x',\phi')+\psi_m] \tag{5}$$

where ψ_m is expressed as a linear combination of the lunar elements ℓ, ℓ', F, D and Γ, and m denotes the distinct tidal component (K_1, O_1, P_1, M_2, etc.). This is to be compared with Eqn. (2). The local sidereal time $\theta + x'$ is averaged out when only the long period effects are considered and the slowly changing part $q_m(x',\phi') + \psi_m$ remains.

The disturbing function may then be written as a sum of components

$$V = \sum_m V_m \tag{6}$$

where for a given constituent

$$V_m = 3/2 \ \frac{GM}{R} \ \frac{\kappa}{\kappa_0} \sum_{n=m}^{\infty} V_{nm} \tag{7}$$

and

$$V_{nm} = \left(\frac{R}{a}\right)^{n+1} (1+k_n') A_{nm} N_{nm} \left(\frac{a}{r}\right)^{n+1} P_{nm}(\sin\,\delta) \cos(m\alpha + \psi_{nm}) \tag{8}$$

$$\psi_{nm} = \psi_m + \varepsilon_{nm} \tag{9}$$

$$\begin{aligned} A_{nm}\cos\,\varepsilon_{nm} &= \frac{1}{4\pi} \iint F_m(x',\phi') P_{nm}(\sin\,\phi') \cos\,q_m(x',\phi')\,d\sigma' \\ A_{nm}\sin\,\varepsilon_{nm} &= \frac{1}{4\pi} \iint F_m(x',\phi') P_{nm}(\sin\,\phi') \sin\,q_m(x',\phi')\,d\sigma' \ . \end{aligned} \tag{10}$$

The long period effects are obtained from perturbation differential equations in which the short period terms are averaged over the instantaneous satellite orbit. Denoting

$$Y_{nm} = P_{nm}(\sin\,\delta)\ \cos(m\alpha + \psi_{nm}) \tag{11}$$

$$Z_{nm} = [\sin\,i\,\sin\,\Omega\ \frac{\partial}{\partial\lambda} - \sin\,i\,\cos\,\Omega\ \frac{\partial}{\partial\mu} + \cos\,i\ \frac{\partial}{\partial\nu}] Y_{nm} \tag{12}$$

where λ, μ, ν are the equatorial components of the geocentric unit vector in the direction of the satellite, we have

$$\frac{d\delta e_{nm}}{dt} = -\frac{n\sqrt{1-e^2}}{e} B_{nm} \cdot \frac{1}{2\pi}\int_0^{2\pi} \left(\frac{a}{r}\right)^{n+1} \frac{\partial Y_{nm}}{\partial \varpi}\, dM \tag{13}$$

$$\frac{d\delta i_{nm}}{dt} = \frac{n}{\sqrt{1-e^2}} B_{nm} \cdot \frac{1}{2\pi}\int_0^{2\pi} \left(\frac{a}{r}\right)^{n+1} Z_{nm} \cos(f+\varpi-\Omega)\, dM \tag{14}$$

$$\frac{d\delta \Omega_{nm}}{dt} = \frac{n}{\sqrt{1-e^2}\sin i} B_{nm} \cdot \frac{1}{2\pi}\int_0^{2\pi} \left(\frac{a}{r}\right)^{n+1} Z_{nm} \sin(f+\varpi-\Omega)\, dM \tag{15}$$

$$\frac{d\delta \varpi_{nm}}{dt} = \frac{n}{e\sqrt{1-e^2}} B_{nm} \cdot \frac{1}{2\pi}\int_0^{2\pi} \left(\frac{a}{r}\right)^{n+1} \left\{ (n+1)\left(\frac{1}{2}e + \cos f + \frac{1}{2}e\cos 2f\right) Y_{nm} \right.$$

$$\left. + \left(2\sin f + \frac{1}{2}e\sin 2f\right)\frac{\partial Y_{nm}}{\partial f} \right\} dM + 2\sin^2 i/2\, \frac{d\delta\Omega_{nm}}{dt} \tag{16}$$

$$\frac{d\delta M_{nm}}{dt} = 2nB_{nm}(n+1) \cdot \frac{1}{2\pi}\int_0^{2\pi} \left(\frac{a}{r}\right)^{n+1} Y_{nm}\, dM - \sqrt{1-e^2}\, \frac{d\delta\varpi_{nm}}{dt}$$

$$+ 2\sqrt{1-e^2}\sin^2 i/2\, \frac{d\delta\Omega_{nm}}{dt} \tag{17}$$

where

$$B_{nm} = \frac{3}{2}\frac{\kappa}{\kappa_0}\left(\frac{R}{a}\right)^n (1+k_n')A_{nm}N_{nm} \tag{18}$$

and where f is the true anomaly, e, ϖ, Ω, a and i are mean elliptic elements of the satellite, n is the mean motion and M is the mean anomaly.

The closed form integration of the perturbation equations (13) - (17) is facilitated by use of special purpose multi-dimensional Fourier series manipulation subroutines. For a specified tidal component m, Y_{nm} and Z_{nm} are expanded into Fourier series with arguments f, ϖ, Ω and ψ_{nm} to arbitrary values of n by recursion on functions of lower order and λ, μ, ν through the formulas

$$Y_{n+1,m} = \frac{1}{n-m+1}\left\{(2n+1)\nu Y_{nm} - Y_{n-1,m}\right\} \tag{19}$$

$$Z_{n+1,m} = \frac{1}{n-m+1}\left\{(2n+1)\nu Z_{nm} + (2n+1)\cos i\; Y_{nm} - (n+m)Z_{n-1,m}\right\} \tag{20}$$

where

$$\begin{aligned}
\lambda &= \cos^2 i/2 \cos(f+\varpi) + \sin^2 i/2 \cos(f+\varpi-2\Omega) \\
\mu &= \cos^2 i/2 \sin(f+\varpi) - \sin^2 i/2 \sin(f+\varpi-2\Omega) \\
\nu &= \sin i \sin(f+\varpi-\Omega) \; .
\end{aligned} \tag{21}$$

The coefficients in the expansions are then numerical and dependent on the satellite inclination.

The recursion is initiated by formally calculating

$$\begin{aligned}
Y_{11} &= \lambda \cos\psi - \mu \sin\psi \\
Z_{11} &= \sin i \sin(\Omega+\psi)
\end{aligned} \tag{22}$$

for m=1 and

$$Y_{22} = 3(\lambda^2 - \mu^2)\cos\psi - 6\lambda\mu \sin\psi$$

$$Z_{22} = 6\lambda \sin i \sin(\Omega+\psi) + 6\mu \sin i \cos(\Omega+\psi) \tag{23}$$

for $m=2$ while noting that Y_{jk}, Z_{jk} are zero for $j < k$ in Eqn. (19). After all expansions are calculated ψ is replaced by the proper value of ψ_{nm}.

The factors multiplying $(\frac{a}{r})^{n+1}$ in the integrands of Equations (13) - (17) are now finite Fourier series with arguments f, ϖ, Ω and ψ. They are formally integrated term by term through the relation

$$\frac{1}{2\pi}\int (\frac{a}{r})^{n+1} \left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\} (n_f f + n_\varpi \varpi + n_\Omega \Omega + n_\psi \psi)\, dM$$

$$= (1-e^2)^{-n+\frac{1}{2}} \; a_{n_f}^{n-1}(e) \cdot \left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\} (n_\varpi \varpi + n_\Omega \Omega + n_\psi \psi) \tag{24}$$

where

$$a_k^m(e) = \sum_{\ell=0}^{[\frac{m-k}{2}]} \binom{m}{k+2} e^{k+2\ell} \frac{(k+2\ell)!}{(2\ell)!!(2k+2\ell)!!} \;. \tag{25}$$

The resultant differential equations may now be integrated in closed form to obtain the perturbations, since the arguments ϖ, Ω and ψ_{nm} of the averaged Fourier series are very nearly linear functions of time where

$$\Omega = \Omega_0 + \dot{\Omega}(t - t_0)$$

$$\varpi = \varpi_0 + \dot{\varpi}(t - t_0)$$

and (Brouwer, 1959)

$$\dot{\Omega} = -\frac{3}{2} n \frac{J_2 R^2 \cos i}{a^2 (1-e^2)^2}$$

$$\dot{\varpi} = -\frac{3}{4} n \frac{J_2 R^2}{a^2 (1-e^2)^2} [1+2 \cos i - 5 \cos^2 i] \;. \tag{26}$$

4. PROGRAM FORMULATION

A set of PL/I programs have been developed for the manipulation of the multi-dimensional Fourier series required for the perturbation equations. The storing, processing, combining, multiplying, integrating, averaging and evaluating of the series has been subroutinized and the main program performs the numerical calculations by a sequence of calls to these routines. The user has control of the program core size through variably dimensional arrays representing the trigonometric series. The user specifies the minimum value of the series coefficients to be retained and NMAX, the maximum value for n, where

$$n=m,\ m+1,\ \ldots\ \mathrm{NMAX}$$

denotes the number of terms calculated in the expansion of the disturbing function, Eqn. (7).

The scheme for storing the trigonometric series is as follows: Let γ_{nm} denote a trigonometric series associated with the calculation of perturbations for the n^{th} term in the expansion of the disturbing function for constituent m such that

$$\gamma_{nm} = \sum_{\ell=1}^{\ell_{MAX}} B_\ell \left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\} (n_1 f + n_2 \varpi + n_3 \Omega + n_4 \psi).$$

Then an integer array dimensioned $(\ell_{MAX}, 6)$ stores the trigonometric function and arguments while an array of dimension ℓ_{MAX} stores the coefficients. For a particular value of ℓ, the first element of the integer array stores 0 for a cosine term and 1 for a sine term, while elements 2 through 6 store n_1, n_2, n_3, n_4 and n respectively. The number of terms in the series, ℓ_{MAX}, is stored separately as an integer variable.

The program structure consists of a main program TIDES in which the user specifies the input for a particular satellite and for the tidal constituent desired, and the following subprograms:

EXCUT: Calculates the ocean tidal perturbation on the satellite by recursively calculating Y_{nm} and Z_{nm}, forming the perturbation differential equations, symbolically averaging the differential equations over the satellite orbit, integrating the equations in in closed form, and calculating for perturbations in elements over a given time interval.

PAF: Computes the partial derivative of a trigonometric series with respect to f, ϖ or Ω.

ANALIN: Calculates the average of a trigonometric series $(\frac{a}{r})^{n+1} \gamma_{nm}$ over the instantaneous orbit of the satellite.

CØMBAL: Computes the combinatorial coefficient $\binom{m}{k}$.

SERMUL: Multiplies two trigonometric series together.

SERSUM: Adds two trigonometric series together.

HANSEN: Calculates the Hansen coefficient $a_k^m(e)$ by recursion.

EVAL1: Prints a trigonometric series and the period of each term.

LUNARG: Computes the arguments ℓ, ℓ', F, D, Γ at time t.

EVALDS: Evaluates a series at time t.

COMP: Compresses a trigonometric series by deleting terms whose coefficients are less than a specified tolerance.

ENTSER: Integrates a series in closed form with respect to time.

PASSER: A FØRTRAN subroutine which generates plots of the perturbations in elements.

The parameters A_{nm}, ε_{nm} of Equation (10) have been obtained from cotidal and corange data for M_2 [7] and K_1 [8] from charts and included in the program. These charts are presented in Figures I and II. The chart contour data was obtained on magnetic tape from an electronic digitizing machine and then interpolated onto a 3°×3° global grid. The function $q_m(x, \phi)$ was calculated at the grid points from the cotidal data and the quadrature of Equation (10) over the global ocean was performed using a recursive two-dimensional integration scheme and a world function map which is zero on land masses and unity on the oceans.

5. PROGRAM OPERATION

The PL/I program, as currently developed, operates for tidal constituents M_2 and K_1. The investigation of other tides would required the input of the parameters A_{nm} and ε_{nm} for those constituents in the program EXCUT. User input to the program is through the main program TIDES, where the initial conditions

SI = i (inclination in radians);

ECC = e (eccentricity),

PIEO = ϖ (longitude of perigee in radians);

ØMEGO = Ω (longitude of ascending node in radians);

LØNG = $L = \varpi + M$ (mean longitude in radians);

SN = n (mean motion in radians/day);

P = $\frac{1}{a}$ (reciprocal semi-major axis in earth radii);

TJD = Julian date at epoch;

are specified for a particular satellite. The tidal constituent is defined by

$$M = \begin{Bmatrix} 2 \text{ for } M_2 \\ 1 \text{ for } K_1 \end{Bmatrix} ;$$

while the constituent argument $\psi_m = n_1\ell + n_2\ell' + n_3F + n_4D + n_5\Gamma$ is given by the integer array NPSI(5) where

$$NPSI(i) = n_i \ ;$$

and $\psi_m = -2\ell' - 2D - 2\Gamma$ for M_2 and $\psi_m = 0$ for K_1.

The recursion for the terms of the disturbing function is continued from

$$n = m, m+1, \ldots, NMAX$$

and terms in the perturbation series are deleted for values of the coefficients less than TØLER. Both NMAX and TØLER are user specified. Additionally, the integer MAXDM defines the dimensionality of the arrays storing the trigonometric series and is a function of NMAX and TØLER. As NMAX or TØLER (or both) become larger, more terms will appear in the perturbation series so that MAXDM must be increased.

The variables ASTART, AFIN and AINC specify the interval in days about the epoch TJD for which perturbations will be calculated. This interval is (TJD + ASTART, TJD + AFIN) in increments of AINC days. Note that these variables are not necessarily integers and may be positive or negative.

6. NEW TECHNOLOGY

The effort under this contract consisted of the development and programming of techniques to calculate the influence of ocean tides on satellite orbital elements in terms of the tidal amplitudes and lags of existing constituent models. High order expansion of the oceanic tidal potential was obtained by recursion with multi-dimensional Fourier series and the perturbation differential equations averaged over the satellite orbit and analytically integrated. The parameters needed for the M_2 and K_1 tides were obtained from global integration of tidal data taken from existing charts. The M_2 and K_1 perturbations acting on the BE-C satellite were evaluated over a one hundred day interval and compared with the perturbations due to solid earth tides.

Frequent reviews and a final survey for new technology were performed. It is believed that the mathematical and programming techniques and algorithms developed do not represent "reportable items," or patentable items, within the meaning of the New Technology Clause. Our reviews and final survey found no other items which could be considered reportable items under the New Technology Clause.

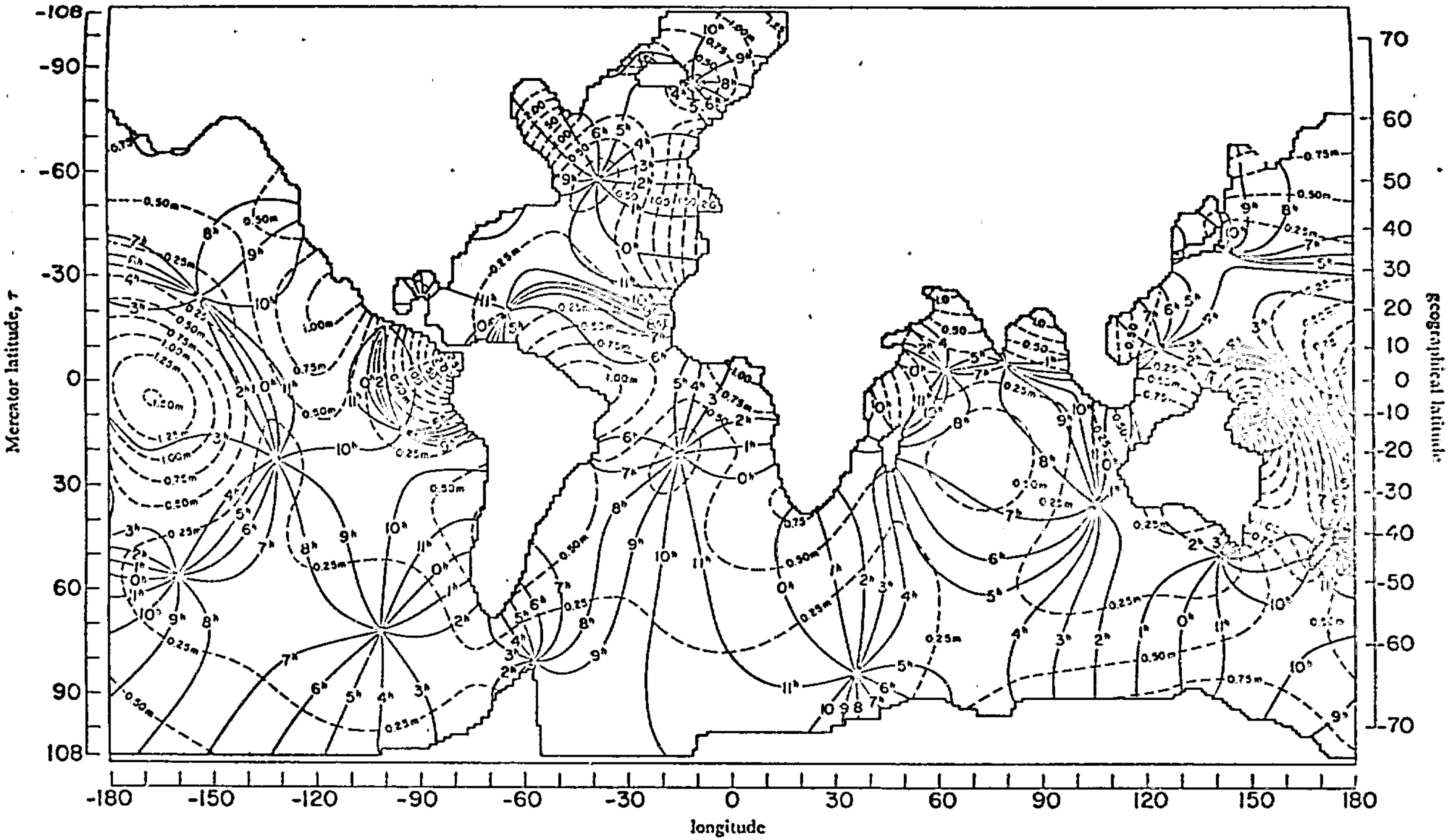

FIGURE 10. Cotidal lines (——) and corange lines (metres) (- - -) for the M_2 tide; map 1° A; computation grid, $k = 1°$; $f = \sigma_2/2\omega = 0.9635$; $F_B/\rho\sigma_2 h_0 = \alpha(h_0/h)^{n-1} U$ ($\alpha = 0.5$, $h_0 = 1$ km, $n = 2$, F_B = frictional force/unit area).

Figure I. The M_2 tide of Pekeris and Accad

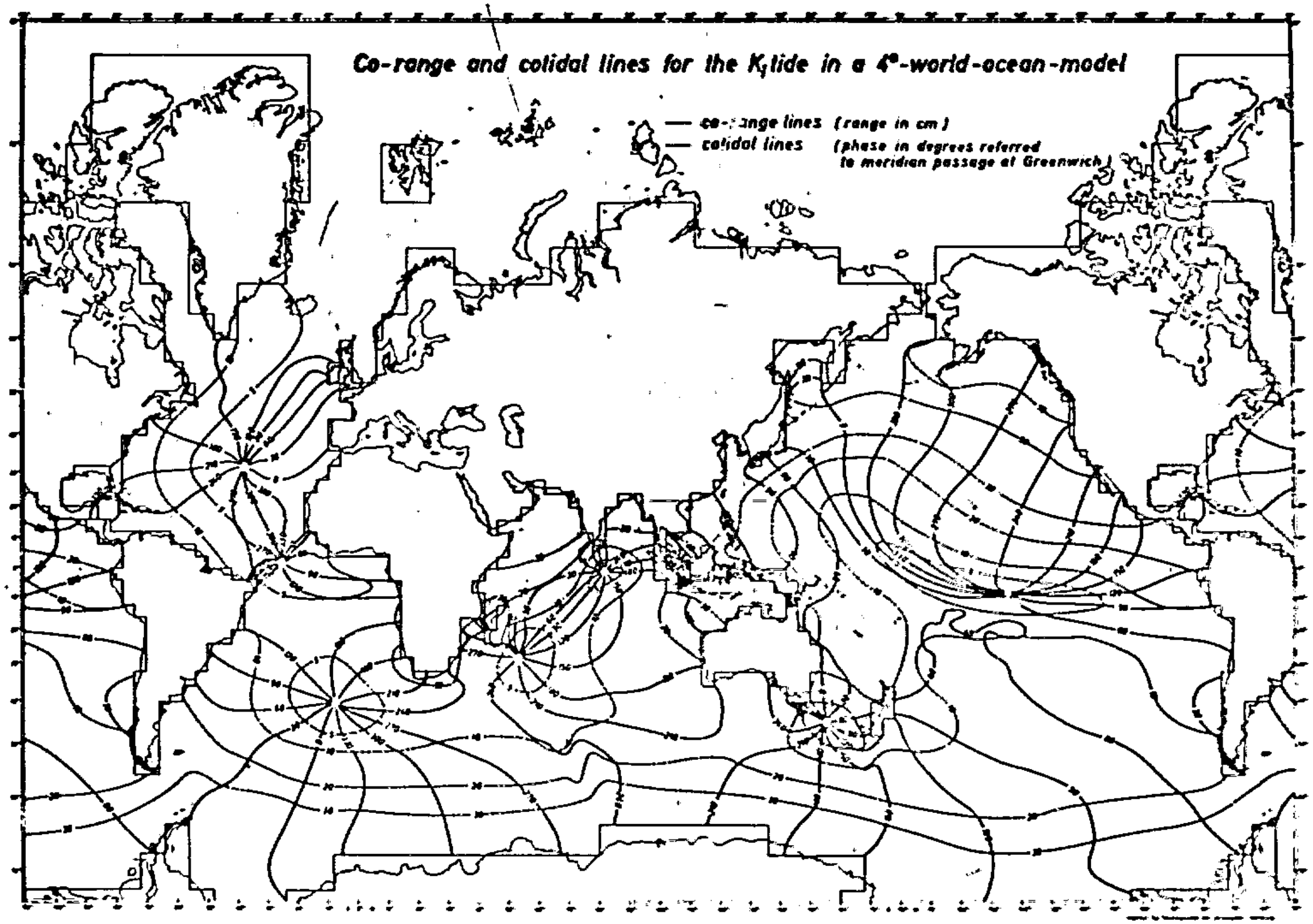

Figure II. The K_1 tide of Zahel

7. NUMERICAL COMPUTATIONS

The M_2 and K_1 tidal perturbation equations were expanded to order $n=20$ for the BE-C satellite with the following initial conitions:

$$e = .025037$$

$$i = 41^\circ.1929$$

$$a = 1.177 \quad \text{(earth radii)}$$

$$\Omega = 301^\circ.712$$

$$\varpi = 272^\circ.139$$

$$M = 353^\circ.164$$

$$n = 83.8567 \text{ (radians per day)}$$

$$T_0 = 2440812.5 \text{ (Julian days).}$$

These expansions produced no terms in the expressions for the perturbed elements for n greater than 14 whose coefficients were within two orders of magnitude of the principal terms of the perturbation. Consequently, for the results presented in this report the expansions were truncated at $n=15$. The principal terms for the perturbed elements are listed in Tables II and III, while Figures III - VII display the M_2 and K_1 perturbations of the BE-C satellite orbit evaluated over a one hundred day interval compared to the perturbations due to solid earth tides $(k_2 = .29)$ [6].

The M_2 and K_1 tides produce an appreciable affect on the orbital perturbations. In the inclination these perturbations can be approximately 10 per cent of those due to solid earth tides. Although the M_2 tide has a larger amplitude than K_1 over portions of the world ocean, K_1 has a greater influence on the orbit of BE-C. Computations for the BE-C orbit has shown that the indirect effect of the M_2 and K_1 ocean tides with J_2 has a negligible influence, in contrast to the solid earth tides.[6]

It must be noted that these results are of a preliminary nature and are dependent on the perturbation parameters A_{nm} and ε_{nm} calculated from existing chart data. These calculations require the co-range and co-tidal values at global grid points and thus demand extrapolation in some regions of sparse chart contour data. In addition, to more fully understand the *total* effects of ocean perturbations acting on satellites, other influential constituents such as O_1, P_1, S_2, etc., must be investigated. This suggests the need for a new accurate numerical integration of the Laplace Tidal Equations with realistic force models for the important tides on a uniform basis. The knowledge of the ocean perturbations due to the superposition of the major tidal constituents will then lead to improved values of the earth's elastic parameters (Love numbers) from satellite observations.

	M_2		K_1	
n	$A_{nm} \times R_E$ (centimeters)	ϵ_{nm} (radians)	$A_{nm} \times R_E$ (centimeters)	ϵ_{nm} (radians)
1			.7894	1.069
2	25.629	-1.89	2.583	0.008
3	11.556	-0.042	2.724	2.357
4	55.309	1.308	2.509	2.609
5	55.497	-2.282	1.580	-1.099
6	13.856	-3.05	0.760	-0.567
7	45.265	-1.649	2.164	-1.015
8	22.731	1.849	0.145	2.573
9	29.368	-0.615	0.847	3.113
10	17.565	1.148	0.476	2.563

TABLE I. Perturbation parameters obtained from the M_2 (Pekeris and Accad) and K_1 (Zahel) tides.

Perturbation	Terms	Period (days)
δe	$-1.836\times10^{-8}\ \sin(\varpi+\Omega+\psi_{32})$	12.13
	$+2.439\times10^{-8}\ \sin(\varpi-3\Omega-\psi_{52})$	8.99
	$+1.093\times10^{-8}\ \sin(\varpi+\Omega+\psi_{72})$	12.13
δi	$1.2734\times10^{-7}\ \cos(2\Omega+\psi_{22})$	10.33
	$-\ .9254\times10^{-7}\ \cos(2\Omega+\psi_{42})$	10.33
$\delta\Omega$	$-1.455\times10^{-7}\ \sin(2\Omega+\psi_{22})$	10.33
	$+\ .1489\times10^{-7}\ \sin(2\Omega+\psi_{62})$	10.33
$\delta\varpi$	$-7.339\times10^{-7}\ \cos(\varpi+\Omega+\psi_{32})$	12.13
	$-1.992\times10^{-7}\ \cos(\varpi-3\Omega-\psi_{32})$	8.99
	$+9.760\times10^{-7}\ \cos(\varpi-3\Omega-\psi_{52})$	8.99
	$+4.382\times10^{-7}\ \cos(\varpi+\Omega+\psi_{72})$	12.13
	$-2.761\times10^{-7}\ \cos(\varpi-3\Omega-\psi_{72})$	8.99
	$-1.252\times10^{-7}\ \cos(\varpi+\Omega+\psi_{11,2})$	12.13
	$+1.027\times10^{-7}\ \cos(\varpi-3\Omega-\psi_{11,2})$	8.99
$\delta L=\delta\varpi+\delta M$	$-2.875\times10^{-7}\ \sin(2\Omega+\psi_{22})$	10.33
	$+3.040\times10^{-7}\ \sin(2\Omega+\psi_{42})$	10.33

Table II. Principal terms from the expansion for perturbed elements due to the M_2 tide for the BE-C satellite.

Perturbation	Terms	Period (days)
δe	$3.497\times10^{-8}\ \cos(\omega - 2\Omega - \psi_{31})$	38.27
	$-1.395\times10^{-8}\ \cos(\omega - 2\Omega - \psi_{51})$	38.27
	$-2.144\times10^{-8}\ \cos(\omega + \psi_{71})$	391.38
	$-2.803\times10^{-8}\ \cos(\omega + \psi_{91})$	391.38
	$+1.113\times10^{-8}\ \cos(\Omega + \psi_{11,1})$	391.38
δi	$-2.408\times10^{-7}\ \sin(\Omega + \psi_{21})$	84.84
	$3.843\times10^{-8}\ \sin(\Omega + \psi_{41})$	84.84
$\delta\Omega$	$-6.439\times10^{-8}\ \cos(\Omega + \psi_{21})$	84.84
	$-2.664\times10^{-7}\ \cos(\Omega + \psi_{41})$	84.84
$\delta\varpi$	$-1.398\times10^{-6}\ \sin(\omega - 2\Omega - \psi_{31})$	38.27
	$-6.060\times10^{-6}\ \sin(\omega + \psi_{51})$	391.38
	$+8.594\times10^{-7}\ \sin(\omega + \psi_{71})$	391.38
	$+1.126\times10^{-6}\ \sin(\delta + \psi_{91})$	391.38
$\delta L = \delta\varpi + \delta M$	$-9.674\times10^{-7}\ \cos(\Omega + \psi_{21})$	84.84

Table III. Principal terms from the expansion for perturbed elements due to the K_1 tide for the BE-C satellite.

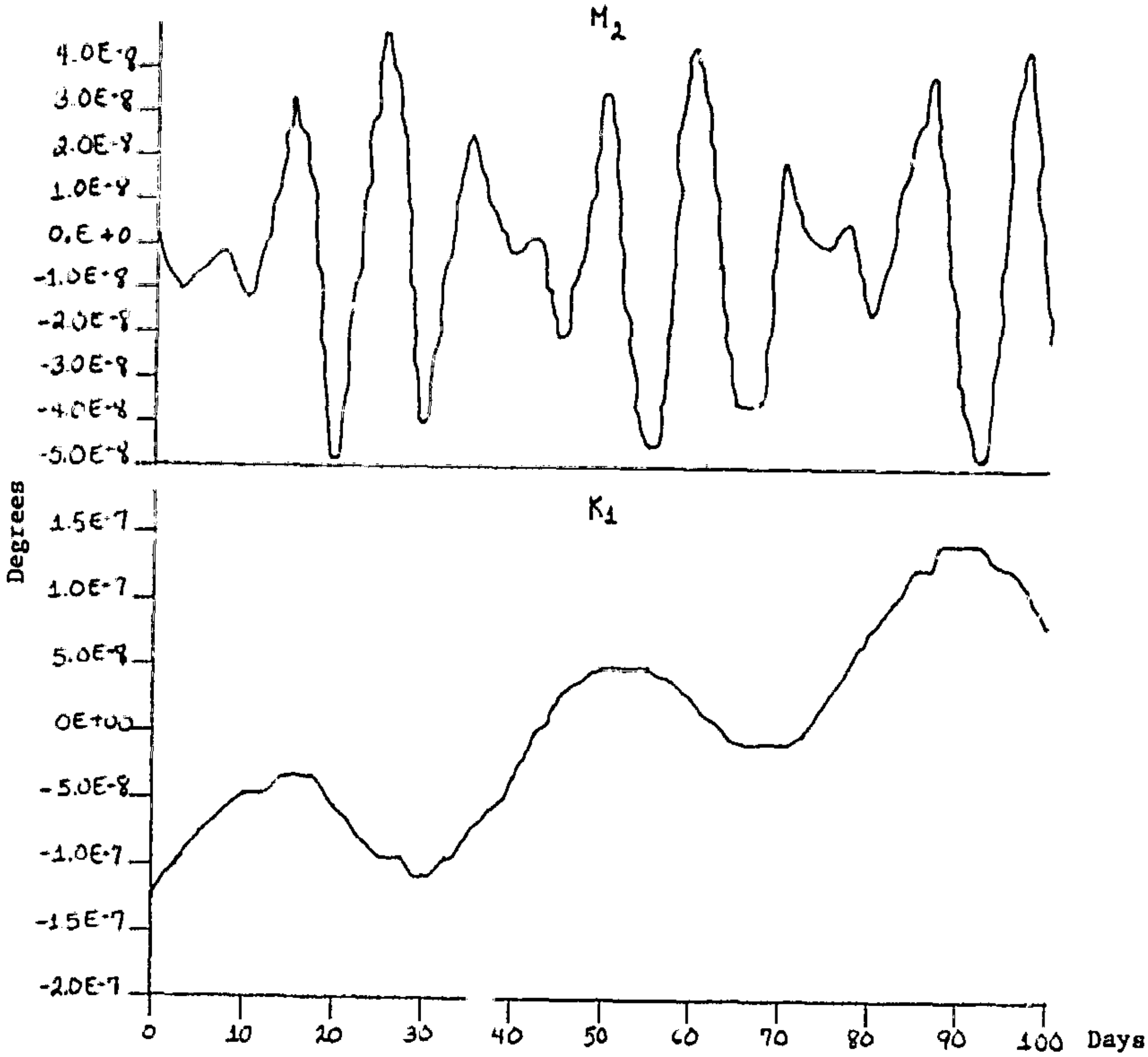

Figure III. Perturbations in eccentricity, δe, for BE-C due to the M_2 and K_1 tides.

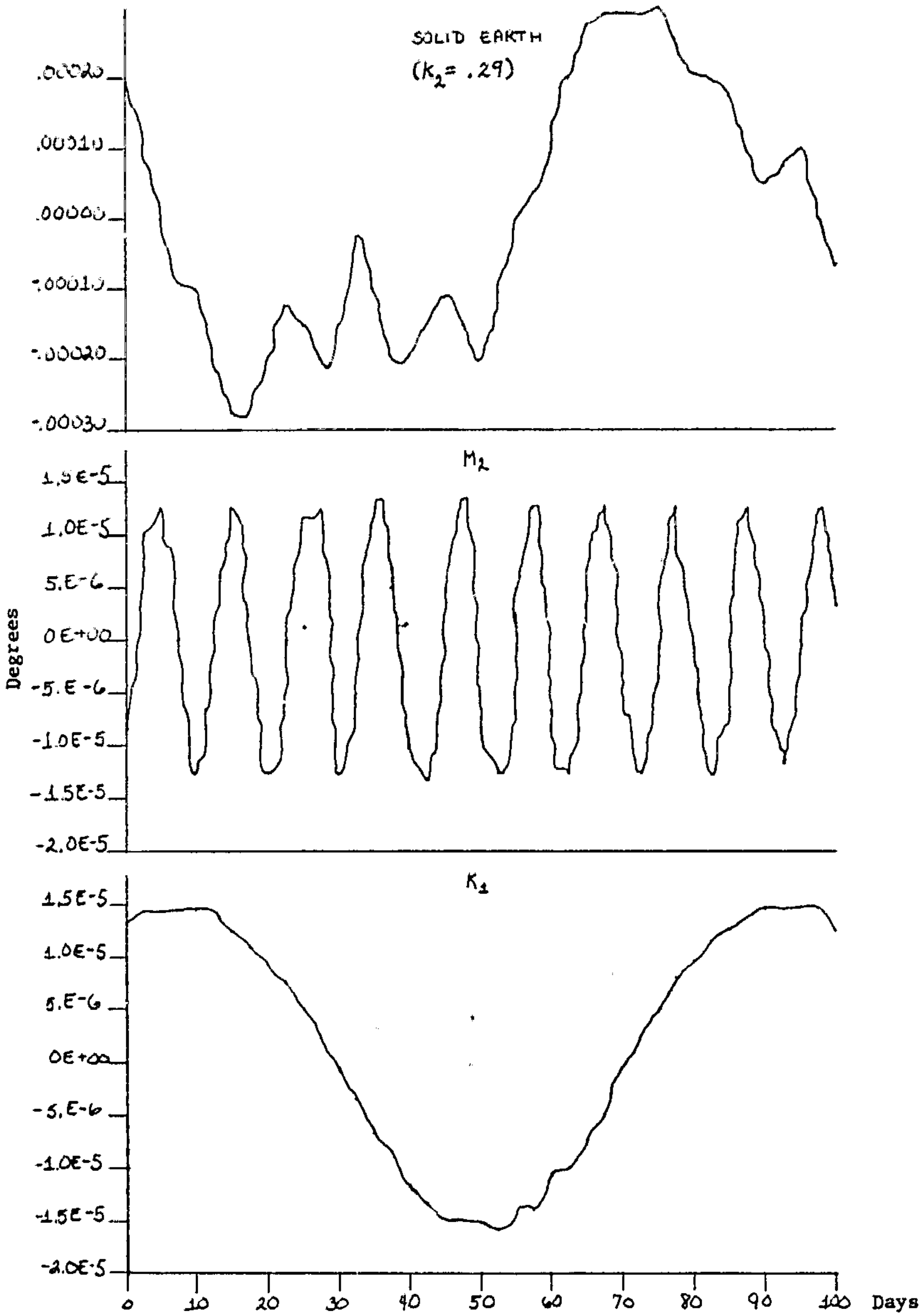

Figure IV. Perturbations in inclination, δi, for the BE-C due to tides.

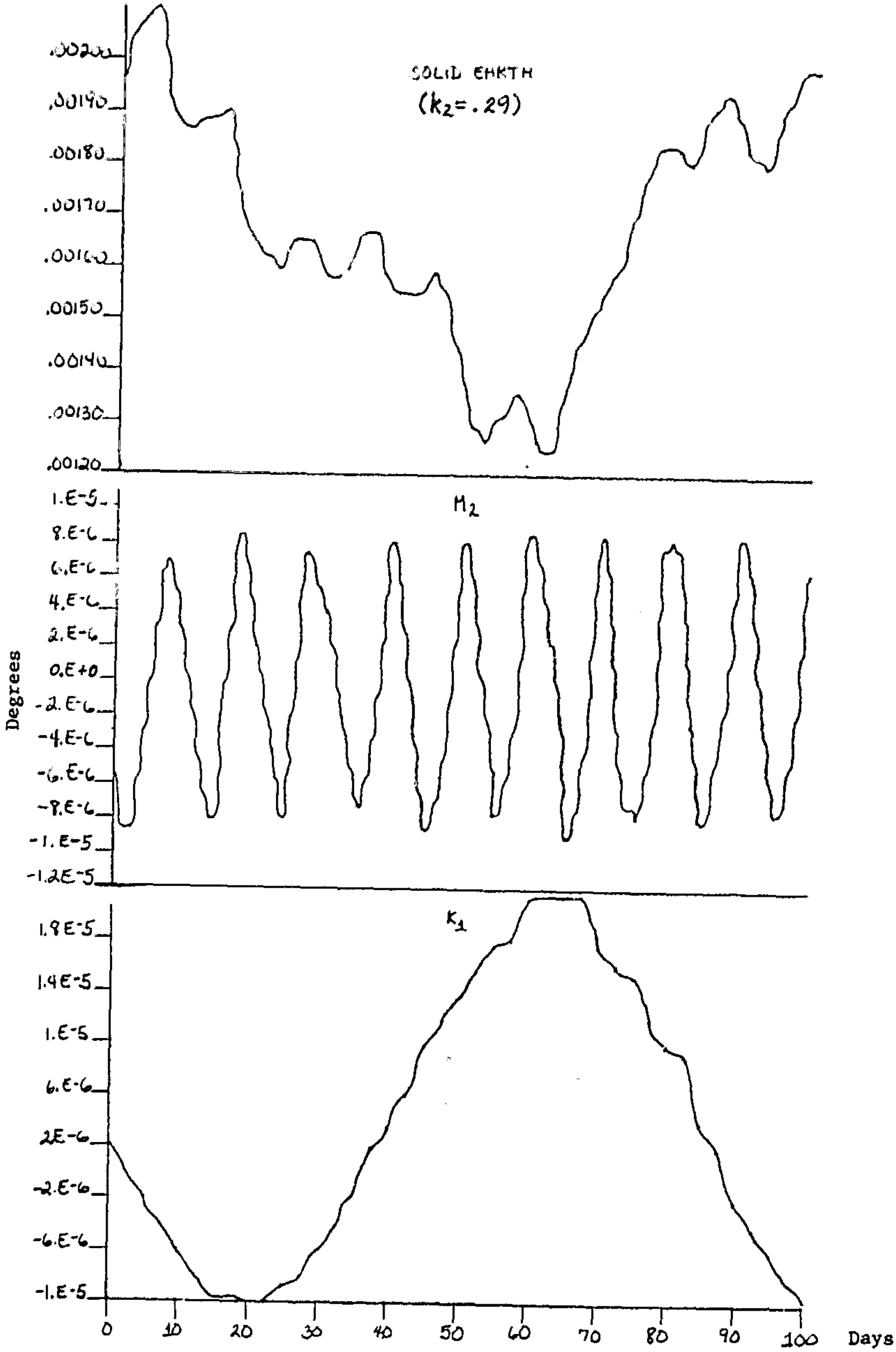

Figure V. Perturbations in the node, δΩ, for the BE-C due to tides.

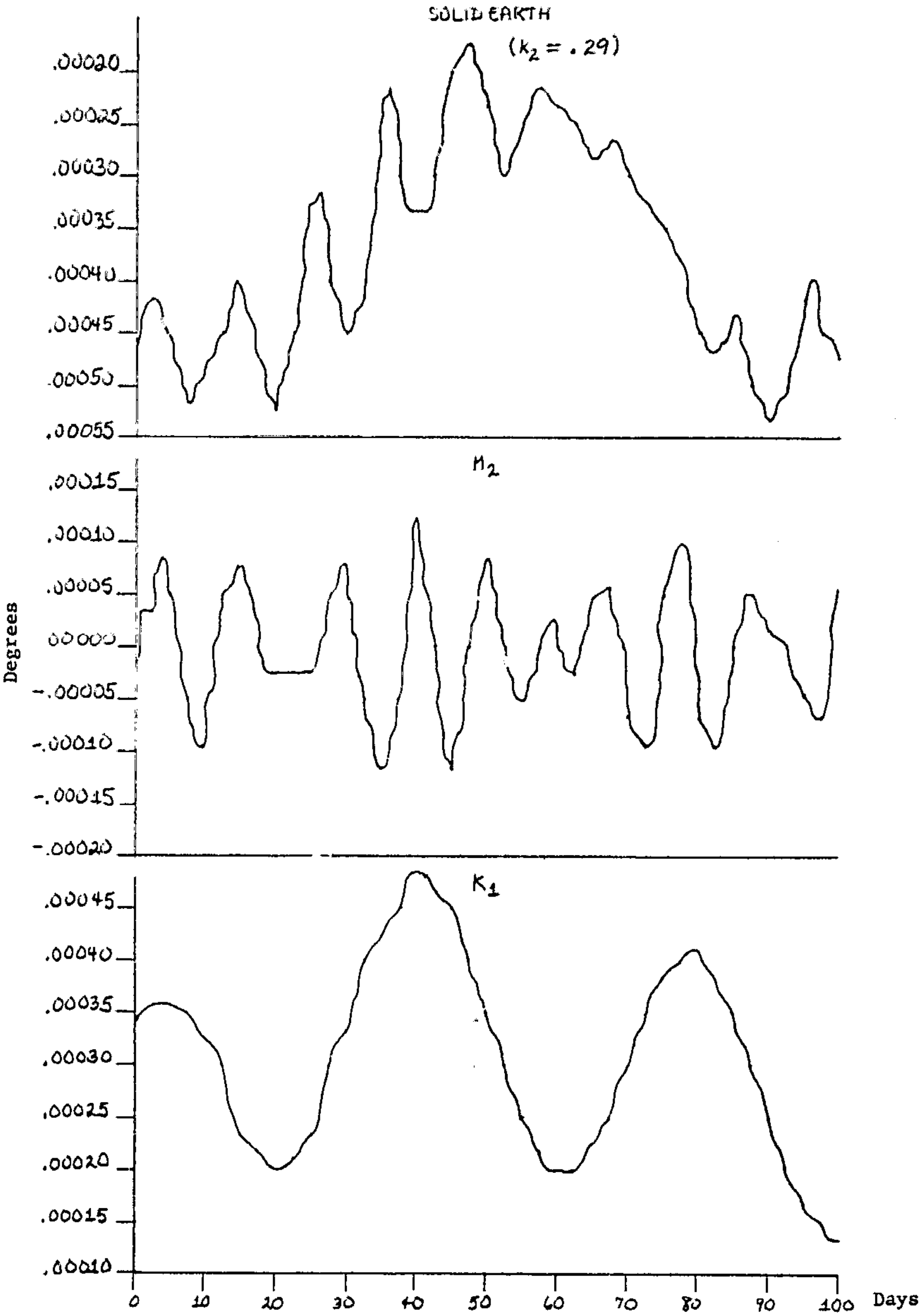

Figure VI. Perturbations in the longitude of perigee, $\delta\varpi$, for BE-C due to tides.

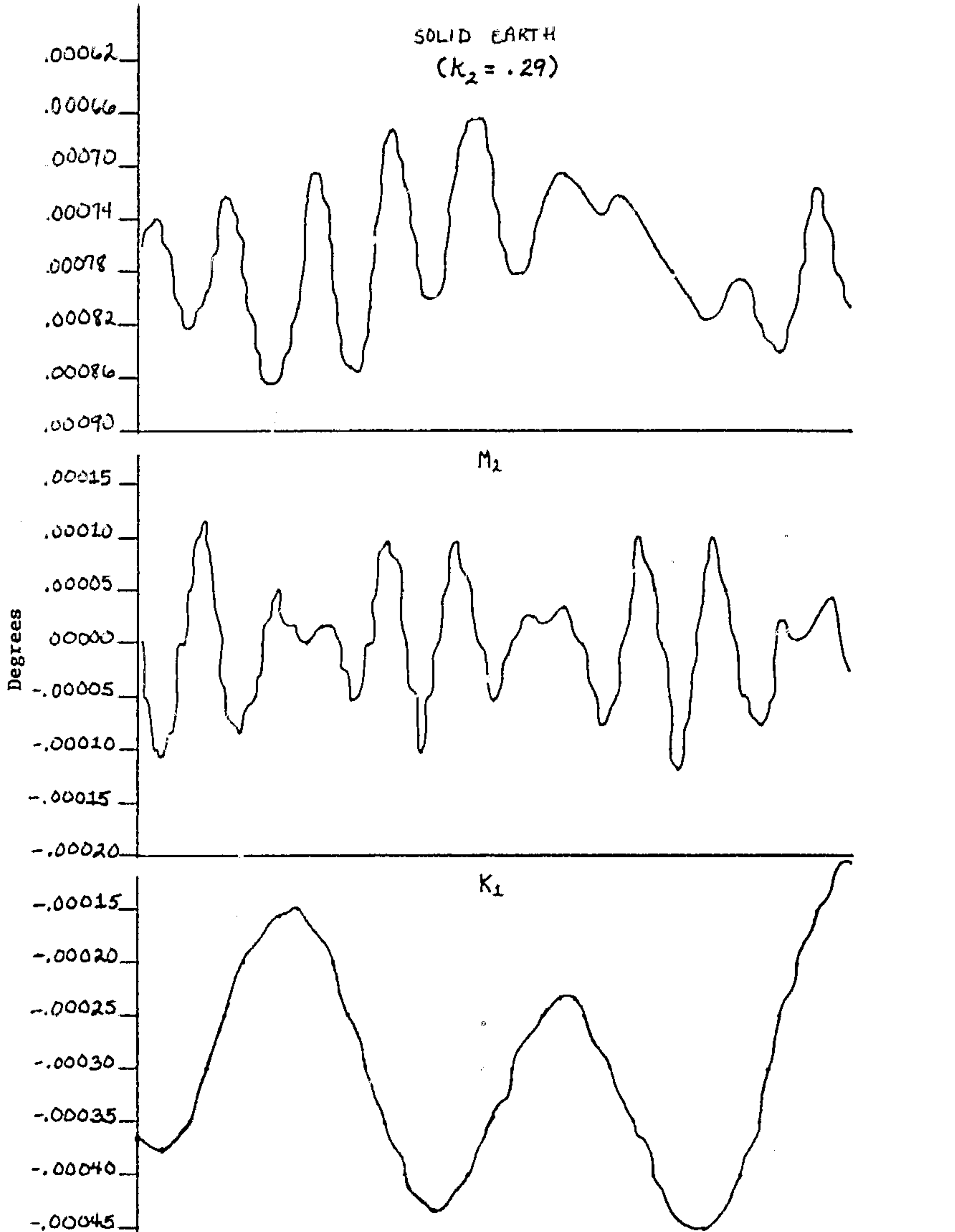

Figure VII. Perturbations in the measured anomaly, δM, for BE-C due to tides.

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