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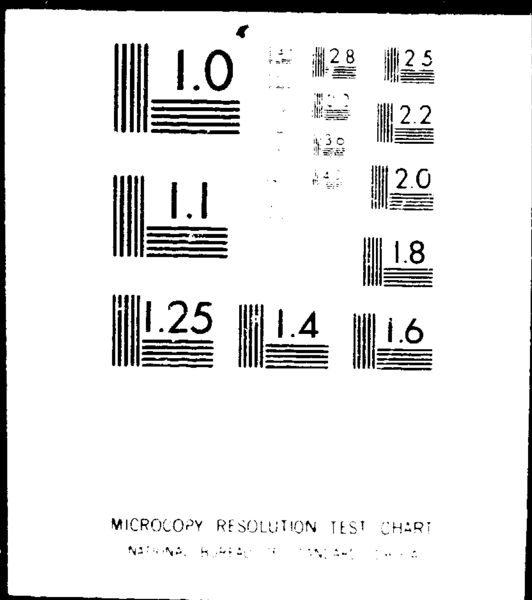

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# A THEORY FOR PREDICTING COMPOSITE LAMINATE WARPAGE RESULTING FROM FABRICATION

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# A THEORY FOR PREDICTING COMPOSITE LAMINATE WARPAGE RESULTING FROM FABRICATION


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## ABSTRACT

Linear laminate theory is used in conjunction with the moment-curvature relationship to derive equations for predicting end deflections due to warpage without solving the coupled fourth-order partial differential equations of the plate. Composite micro- and macromechanics are used in conjunction with laminate theory to assess the contribution of factors such as ply misorientation, fiber migration, and fiber and/or void volume ratio nonuniformity on the laminate warpage. Using these equations, it was found that a $1^{\circ}$ error in the orientation angle of one ply was sufficient to produce warpage end deflection equal to two laminate thicknesses in a 10 inch by 10 inch laminate made from 8 ply Mod-I/epoxy. Using a sensitivity analysis on the governing parameters, it was found that a $3^{\circ}$ fiber migration or a void volume ratio of three percent in some plies is sufficient to produce laminate warpage corner deflection equal to several laminate thicknesses. Tabular and graphical data are presented which can be used to identify possible errors contributing to laminate warpage and/or to obtain an a priori assessment when unavoidable errors during fabrication are anticipated.


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# INTRODUCTION

Flat composite laminates have been observed to warp upon removal from their fabrication mold. This warpage is the result of nonsymmetries and residual stresses that are present in the laminate. The warpage can be of a magnitude sufficient to render the laminate useless for its intended purpose. Thus, residual stresses are always present in angleplied laminates which are cured at elevated temperatures and then cooled down to room temperature. Bending nonsymmetries result from ply misorientations and fiber nonuniformities which occur inadvertently during the fabrication of the laminate and produce coupling modes in the laminate. When the laminate is subjected to either thermal or mechanical loads, these coupling modes produce laminate warpage. The various coupling modes resulting from combinations of nonsymmetries are discussed in reference 1. Exact determination of the surface of the warped laminate requires solution of coupled fourth order partial differential equations. In the literature, neither solutions to the coupled partial differential equations or approximate equations are available for predicting laminate warpage as a result of bending nonsymmetries and residual stresses. Therefore, the assessment of the effect of various factors that contribute to the warpage and the establishment of fabrication control procedures has not been possible in the past.

Equations, even of an approximate nature, for predicting laminate warpage could be of considerable practical value to both fabricators and designers. For example, the fabricator could use the equations to obtain an a priori assessment of the warpage resulting from factors which may be difficult to control accurately during laminate fabrication. The designer, on the other hand, could use the results to specify tolerances which would minimize warpage-producing nonsymmetries, or he could use the results to design the laminate with warpage-compensating nonsymmetries.

It is the objective of this report to show how linear laminate theory can be used to derive convenient equations without solving the coupled fourth-order partial differential equations as would normally be required. The use of these equations for predicting warpage is demonstrated.

## WARPAGE GEOMETRY AND ORIGIN

### Warpage Geometry

The laminate warpage geometry of interest in this investigation is depicted in figure 1. This type of warpage is typical of $(0_2,\pm\theta,\mp\theta_2,0_2)$ laminates which have the $0^{\circ}$ ply direction parallel to the x-axis. As can be seen in figure 1, the laminate is fixed along its x-edge (AB). Note that in this position, the laminate exhibits two primary modes of warpage. These are: (1) curvature (bending) along the y-direction and (2) a twist about an axis normal to and bisecting the plate x-edge. There is also a curvature along the x-direction. However, for the laminate configurations under consideration, the curvature along the x-direction is relatively small compared to that of the y-direction. The corner deflections C and D provide a quantitative measure of the amount of warpage. The twisting increases the bending corner deflection at C while it decreases the corner deflection at D. In an actual warped laminate, the corner deflections C and D can be readily measured by orienting the laminate as is shown in figure 1.

### Warpage Origin

Composite laminates are, in essence, nonuniform materials since they consist of several plies with different orientations through the thickness. Because of this nonuniformity and the lay-up procedure used during fabrication, some bending-stretching coupling in the laminate is the general rule rather than the exception. Bending-stretching coupling is present when a purely extensional deformation in the laminate produces, simultaneously, bending and stretching. Conversely, a purely bending or twisting deformation in the laminate can produce, simultaneously, an extensional deformation. Bending-twisting coupling is also possible in angleplied laminates. This is the case even if the angleplied laminate is balanced and symmetric.

Both ply misorientations and fiber nonuniformity will produce bending-stretching coupling which results in warpage of a flat laminate in the presence of residual stress. When these conditions are present in $(0_2,\pm\theta,\mp\theta,0_2)$ laminates, they will produce warpage having the geometry depicted in figure 1.

## THEORETICAL BACKGROUND AND GOVERNING EQUATIONS

In this section, the theoretical background leading to the governing equations for predicting laminate warpage corner deflections in the presence of residual stress are described. The equations presented for approximate deflection prediction are of convenient form and obtainable without solving the coupled fourth-order partial differential equations. A list of symbols used in the equations is contained in the Appendix of this report. Warpage corner deflections can be predicted more accurately by anisotropic thin-plate bending theory which accounts for all possible coupling responses present in such plates (Appendix, reference 1). Warpage corner deflections can also be predicted accurately by special finite elements accounting for the same coupling responses. However, neither anisotropic thin-plate bending theory nor finite element analysis which accounts for all coupling responses have been used to predict laminate warpage corner deflections in the presence of residual stress.

## Theoretical Background

The derivation of the equations for predicting laminate warpage corner deflections is based on the force-deformation-temperature relationships derivable from linear thin-laminate bending theory (refs. 2 and 3). The two important assumptions in linear thin-laminate bending theory are: (1) the planform dimensions of the laminate are very large relative to its thickness and (2) the maximum deflection of the laminate is of the same order of magnitude as the laminate thickness. The second assumption is usually interpreted to mean accurate predictions for maximum deflections equal to plate thickness and good approximation for maximum deflections of 10-20 times the plate thickness. The 10-20 times factor is based on the maximum-deflection to plate-edge-dimensions ratio. If this ratio is such that the sine and tangent of small angles can be approximated by the angle itself (in radians) then linear thin-laminate bending theory would predict good approximations to maximum deflections.

To illustrate this point with an example, consider a $(0_2, \pm 45, \mp 45, 0_2)$ laminate with maximum tip deflection of 1.20 inches, plate edges of 10 inches, and plate thickness of .06 inches. The tangent of the angle subtended by the 1.20 inch dimension on a 10 inch base is 0.12. The corresponding angle in radians is 0.119 which is equal to the tangent when rounded off to two figures. Based on the aforementioned criteria, linear thin laminate bending theory would be applicable to these types of laminates even though they undergo corner deflections 20 times their laminate thickness.

The laminate force deformation relationships derivable from thin-laminate bending theory and of interest in this investigation are given in references 4 or 5. In matrix form and assuming forces due to residual stresses only, these equations become:

$$\begin{Bmatrix} \{\epsilon_{cox}\} \\ \{\kappa_{cx}\} \end{Bmatrix} = \begin{bmatrix} [A_{cx}] & [C_{cx}] \\ [C_{cx}] & [D_{cx}] \end{bmatrix}^{-1} \begin{Bmatrix} \{N_{c\Delta Tx}\} \\ \{M_{c\Delta Tx}\} \end{Bmatrix} \tag{1}$$

The notation in equation (1) is as follows: $\epsilon_{cox}$ denotes reference plane strains; $\varkappa_{cx}$ denotes the plate local curvatures; $A_{cx}$ represents a(3 x 3) array of axial (membrane or stretching) stiffness coefficients; $C_{cx}$ represents a (3 x 3) array of stretching-bending coupling stiffness coefficients; $D_{cx}$ represents a (3 x 3) array of bending (flexural) rigidities coefficients; $N_{c\Delta Tx}$ denotes thermal forces due to residual stresses and $M_{c\Delta Tx}$ are the corresponding thermal moments. Note the subscript x in equation (1) indicates that these relationships are referred to the laminates' structural axes (x,y,z, fig. 1). Note also the superscript -1 denotes the inverse of the matrix.

The equations used to generate the elements in the arrays $A_{cx}$, $C_{cx}$, and $D_{cx}$ are given in reference 4.The form of these equations, neglecting interply contribution, is:

$$[A_{cx}] = \sum_{i=1}^{N_l} \Delta T_{li}(z_{li+1} - z_{li})[R_{li}]^T[E_{li}]^{-1}[R_{li}] \tag{2}$$

$$[C_{cx}] = \frac{1}{2}\sum_{i=1}^{N_l} \Delta T_{li}(z^2_{li+1} - z^2_{li})[R_{li}]^T[E_{li}]^{-1}[R_{li}] \tag{3}$$

$$[D_{cx}] = \frac{1}{3}\sum_{i=1}^{N_l} \Delta T_{li}(z^3_{li+1} - z^3_{li})[R_{li}]^T[E_{li}]^{-1}[R_{li}] \tag{4}$$

The corresponding equations for the thermal forces and moments are:

$$\{N_{c\Delta Tx}\} = \sum_{i=1}^{N_l} \Delta T_{li}(z_{li+1} - z_{li})[R_{li}][E_{li}]^{-1} \{\alpha_{li}\} \tag{5}$$

$$\{M_{c\Delta Tx}\} = \frac{1}{2}\sum_{i=1}^{N_l} \Delta T_{li}(z^2_{li+1} - z^2_{li})[R_{li}]^T[E_{li}]^{-1} \{\alpha_{li}\} \tag{6}$$

The arrays $\{\alpha_{li}\}$, $[R_{li}]$ and $[E_{li}]$ in equations (2)-(6) are:

$$\{\alpha_{li}\} = [\alpha_{l11} \quad \alpha_{l22} \quad 0]^T_i \tag{7}$$

$$[R_{li}] = \begin{bmatrix} \cos^2\theta_l & \sin^2\theta_l & \frac{1}{2}\sin 2\theta_l \\ \sin^2\theta_l & \cos^2\theta_l & -\frac{1}{2}\sin 2\theta_l \\ -\sin 2\theta_l & \sin 2\theta_l & \cos 2\theta_l \end{bmatrix}_i \tag{8}$$

$$[E_{li}] = \begin{bmatrix} \frac{1}{E_{l11}} & -\frac{\nu_{l21}}{E_{l22}} & 0 \\ -\frac{\nu_{l12}}{E_{l11}} & \frac{1}{E_{l22}} & 0 \\ 0 & 0 & \frac{1}{G_{l12}} \end{bmatrix}_i \tag{9}$$

The notation in equations (2)-(9) is as follows. $N_l$ is the number of plies in the laminate; $\Delta T_{li}$ is the temperature of the i-th ply or the difference between cure and room temperatures in the present case; the difference $Z_{li+1} - Z_{li}$ locates the i-th ply relative to the reference plane (stacking sequence); $\alpha_{l11}$ is the ply thermal coefficient of expansion along the fiber direction and $\alpha_{l22}$ normal to it; $\theta_{li}$ is the angle locating the fiber direction in a ply relative to the laminate structural axes (ply orientation angle); $E_{l11}$ is the ply modulus of elasticity along the fiber direction and $E_{l22}$ normal to it; $\nu_{l12}$ is the major

Poisson's ratio and $\nu_{\ell 21}$ the minor; and $G_{\ell 12}$ is the inplane ply shear modulus. It is well known that the ply thermal coefficients of expansion, moduli and Poisson's ratios are related to constituent material properties using composite micromechanics when the types of constituents, the fiber volume ratio, and the void volume ratio are known. See reference 4.

The variables of interest in the present discussion are the laminate local curvatures ($\chi_{cx}$), defined in equation (1) which contribute to laminate warpage. It is seen by inspection from equations (1)-(9) that the laminate curvatures depend on: the number of plies in the laminate ($N_\ell$), the ply temperature difference ($\Delta T$), the ply stacking sequence ($Z_{\ell i+1} - Z_{\ell i}$), the ply orientation angle ($\theta_{\ell i}$), and through micromechanics on the constituent material properties (E, G, $\nu$, and $\alpha$), the fiber volume ratio, and the void volume ratio.

Equations (1)-(9) and the micromechanics equations for predicting ply properties from constituent properties have been programmed in the computer code described in references 4 and 5. This computer code is used herein to compute the laminate local curvature $\chi_{cx}$ explicitly as a function of the factors identified in the previous paragraph. Therefore, it becomes straightforward to compute perturbations of the factors contributing to warpage about the condition of bending symmetry. How the local curvatures will be used to predict laminate warpage corner deflections will be described in the second part of this section.

The three important points to be noted from the previous discussion are:

1. Laminate warpage, of interest in the present discussion, is within the realm of linear thin-laminate bending theory even when the laminates undergo large corner deflections compared to their thickness.

2. The factors contributing to laminate warpage are readily identified via the laminate force-deformation-temperature relations.
3. A computer program is available that can be used to assess the relative significance of the various factors contributing to laminate warpage.

## Governing Equations

The governing equations for predicting laminate warpage corner deflections are readily derived when the local curvatures are known and advantage is taken of the following observation. The laminate local curvatures ($\kappa_{cx}$, eq. (1)) resulting from uniform residual stress ( $\Delta T$ constant in eqs. (2)-(6)) are constant throughout the laminate. This observation is violated in the vicinity of the free edges within a distance approximately equal to the laminate thickness which is insignificant compared to laminate planform dimensions. Since the radius of curvature is the reciprocal of the curvature, constant curvatures yield constant radii of curvature, and therefore

$$R = 1/\kappa_c \qquad \text{is constant} \tag{10}$$

The governing equation for warpage corner deflection, for example, due to the curvature along the y-direction is obtained from the geometric relationships depicted in figure 2. Thus

$$R_y = 1/\kappa_{cyy} \tag{11}$$

$$\theta_y \approx \sin^{-1}(b/R_y) \tag{12}$$

$$\delta_y = R_y(1 - \cos \theta_y) \tag{13}$$

Similarly, for the warpage due to curvature along the x-direction:

$$R_x = 1/\kappa_{cxx} \tag{14}$$

$$\theta_x \approx \sin^{-1}(a/R_x) \tag{15}$$

$$\delta_x = R_x(1 - \cos \theta_x) \tag{16}$$

The corner deflection due to twisting of Point C, figure 1, is determined from recalling that twisting takes place about an axis through the laminate center:

$$R_{xy} = 1/\kappa_{cxy} \tag{17}$$

$$\theta_{xy} \approx \sin^{-1}(a/2R_{xy}) \tag{18}$$

$$\delta_{xy} = R_{xy}\,(1 - \cos\theta_{xy}) \qquad (19)$$

For twisting on the other edge, a is replaced by b in equation (18). The total corner deflection at point C, figure 1, is obtained by superposition since linear thin-laminate bending theory is used in deriving the curvatures. This procedure achieves the objective of obtaining an approximate solution for the corner deflections without having to solve the coupled fourth-order partial differential equations.

In addition to the approximations inherent in the linear thin-laminate bending theory, two other approximations were introduced in deriving equations (13), (16), and (19). These are:

1. The projected deformed laminate edge length is approximately equal to the undeformed edge length.
2. Warping due to twisting takes place about an axis through the laminate center.

Using these approximations, the laminate warpage corner deflections are readily derived when the local curvatures are known. The error introduced by the two approximations indicated above is not known.

## THEORETICAL WARPAGE CORNER DEFLECTIONS

In this section, theoretically calculated data are presented for warpage corner deflections resulting from small perturbations of the bending symmetry using the equations developed in the previous section. The predicted values were obtained by perturbing the ply orientation angles in the laminates $(0_2 \pm 30, \mp 30, 0_2)$ and $(0_2, \pm 45, \mp 45, 0_2)$, using the computer code (ref. 4) and the following procedure:

1. Perturb the ply orientation angles in the laminates $(0_2 \pm 30, \mp 30, 0_2)$ and $(0_2 \pm 45, \mp 45, 0_2)$.
2. Calculate the local curvatures $\varkappa_{cxx}$, $\varkappa_{cyy}$, and $\varkappa_{cxy}$ due to the perturbations using the computer code in reference 4.
3. Calculate the corresponding corner deflections using equations (13), (16), and (19).

The results obtained by this procedure are summarized in table 1. In this and subsequent tables, the laminate configuration is given in the first column. The next three columns are the local curvatures computed using the computer code (ref. 4). The corner deflection due to $\varkappa_{cyy}$ is computed using equation (13) and that due to $\varkappa_{cxy}$ using equation (19). The maximum warpage corner deflection is shown in the last column and is the algebraic sum of the two previous columns. The values for $x_{cxx}$ are negligible relative to $x_{cyy}$ and as a result are not included in the corner deflection calculations.

A better assessment is obtained by plotting tip deflections versus ply orientation perturbations. This is shown in figure 3 for the laminate $(0_2, \pm 30, \mp 30, 0_2)$. The following observations can be made:

1. The warpage corner deflections vary linearly with the perturbation angle.
2. A perturbation angle of $+1.35^{\circ}$, for example, in the $+30^{\circ}$ ply is sufficient to produce a warpage tip deflection of .22 inches which is approximately equal to four times the laminate thickness which, in this case, is 0.060 inches.

A corresponding plot for the $(0_2 \pm 45, \mp 45, 0_2)$ laminate is shown in figure 4. A corner deflection of 1.20 inches, for example, is obtained at a perturbation angle of $11.5^{\circ}$ of one of the $+45^{\circ}$ plies. This exercise shows that it is possible to obtain a corner deflection 20 times the laminate thickness with a 11.5° error in only one of the plies. Note the small nonlinearity in the curves in figure 4.

The effects on the corner deflection of replacing one of the -45° plies with a +θ ply are shown in figure 5. As can be seen in this figure, an angle θ of about 18° will produce a corner deflection of 1.20 inches. The effects on the corner deflection of the simultaneous perturbation in both the +45° and -45° plies are shown in figure 6. These types of perturbations produce negligible corner deflection due to twisting. However, they produce substantial corner deflection due to bending.

The important observations from the previous discussion are as follows:

1. The warpage corner deflections vary approximately linearly with small perturbations in only one of the plies.
2. Relatively small ply misorientations in only one ply can cause corner deflections several times the laminate thickness.
3. Combinations of perturbations may be selected to produce specific corner deflections.

## SENSITIVITY ANALYSIS

In this section, calculated results are presented which can be used to assess the significance of the following factors on laminate warpage:

1. Bending nonsymmetry (two ply equivalent laminate)
2. Ply stacking sequence
3. Fiber content nonuniformity (assuming voids in some plies)
4. Combinations of items 2 and 3 above

The results for the bending nonsymmetry case are summarized in table 2. It is seen from the results in this table that only the twisting component is present in this case. The graphical representation of the data in table 2 is shown in figure 7. Here it is seen that the corner deflection varies nonlinearly with ply angle. It increases rapidly with increasing ply angle in the range $0^{\circ} \leq \theta \leq 10^{\circ}$ and levels off at $\theta > 20^{\circ}$.

Results for some additional ply stacking sequence cases (not examined previously) are summarized in table 3. As can be seen from the results in this table, ply stacking sequences can be selected to yield corner deflections 40 times the laminate thickness. Note also that laminates can be made with ply stacking sequences which have (1) approximately the same bending and twisting curvatures and (2) identical curvatures in the x and y directions.

The results from the fiber content nonuniformity sensitivity analysis are summarized in table 4. For this case only, the laminate $(0_2 \pm 45, \mp 45, 0_2)$ was used and the fiber content nonuniformity was introduced via voids. As can be seen from the results in table 4, fiber content nonuniformity in the form of about five percent voids in only two plies produces corner deflection approximately equal to the laminate thickness. Note that fiber content nonuniformity of the type investigated here causes relatively small curvature in the x-direction as compared with the other two. The graphical representation of the results in table 4 are shown in figure 8 when the fiber content nonuniformity is expressed as a void volume ratio. As can be seen from the curves in this figure, the corner deflections vary nonlinearly with fiber content nonuniformity (void volume ratio) and increases more rapidly as the amount of nonuniformity becomes large. Note that at void volume ratios of .10 or less, the curves in figure 8 are approximately linear.

Results of sensitivity analysis for the combined case of small ply misorientations and fiber content nonuniformity are summarized in table 5. As can be seen from the data in this table, the combinations selected, which on the surface appear to be minor variations from case to case, produce substantially different corner deflections. Comparing corner deflections from the individual cases in tables 1 and 4 with the corresponding values in table 5, it is seen that some combinations are compensatory. The tabulated curvatures in tables 1-5 can be used to predict laminate warpage corner deflections in laminates of similar configuration but different planform dimensions. For these cases, the edge dimensions are the only variables that change in equations (11)-(19).

It is noted here that the same procedure used for the corner deflections can be used to calculate warpage deflection at any point (x, y; fig. 1) in the laminate. For these calculations, the sign of the subtended angle is determined from the following equations:

$$\theta_y = \sin^{-1}(y/R_y) \tag{20}$$

$$\theta_x = \sin^{-1}(x/R_x) \tag{21}$$

$$\theta_{xy} = \sin^{-1}(x/2R_{xy}) \tag{22}$$

$$\theta_{yx} - \sin^{-1}(y/2R_{yx}) \tag{23}$$

where the physical problem will dictate whether equation (22) or (23), or some combination thereof, should be used.

The important points from the previous discussion are:

1. Relatively small perturbations in various ply stacking sequences and fiber content nonuniformities produce considerable deflections.
2. At small values of the perturbation, the corner deflection varies approximately linearly with the perturbation.
3. Combined perturbations can produce compensatory effects on the corner deflection compared to that produced by the individual cases.

## APPLICATION OF EQUATIONS FOR CALCULATING WARPAGE

The approximate equations for warpage corner deflections derived previously can be used to suggest possible combinations of nonsymmetries that could explain the warpage measured in two actual laminates.

### Description of Laminates

Laminates were fabricated to have symmetry with respect to bending and supplied to NASA-Lewis Research Center by an outside vendor. These laminates were 12 inch square plates, eight plies thick (0.060 inches) of $(0_2, \pm\theta, \mp\theta, 0_2)$ ply stacking sequence, and were made from MOD-I/ERLA 4617 with about 50 percent fiber volume ratio and cured at about 370°F. Two of these laminates, $(0_2, \pm 30, \mp 30, 0_2)$ and $(0_2 \pm 45, \mp 45, 0_2)$, exhibited warpage with measurable corner deflections as depicted in figure 1. After curing, the laminates were reheated in an unrestrained condition to about 300°F and then cooled to room temperature.

## Corner Deflection Measurements

The warpage corner deflections of the two laminates $(0_2,\pm45,\mp45,0_2)$ and $(0_2,\pm30,\mp30,\pm45)$ were measured at points C and D as depicted in figure 1. The measured values are summarized in table 6. Note that the last two columns in table 6 separate the corner deflections into two components: bending and twisting. The bending component equals the algebraic average of the two corner deflections and the twisting component equals their algebraic difference. As is observed from the values shown in table 6, the maximum corner deflections can be substantial: 1.20 inches for the $(0_2,\pm45,\mp45,0_2)$ laminate and 0.22 inches for the $(0_2,\pm30, \pm30,0_2)$ laminate. Compared to 0.06 inches for the laminate thickness, these deflections are 20 times the laminate thickness for the $(0_2,\pm45,\mp45,0_2)$ laminate and about four times for the $(0_2,\pm30,\mp30,0_2)$ laminate.

## Some Possible Combinations of Nonsymmetries

The following procedure was used to identify a few of a large number of possible combinations of nonsymmetries that could have contributed to the warpage corner deflections that were measured in the actual laminates.

For the $(0_2,\pm30,\mp30,0_2)$ laminate, the data in table 1 were used as a guide because the 12-inch edge of the actual laminate is only 20 percent longer than the 10-inch edge which was used to generate the curves in figure 3. Comparing corresponding values from table 6 and table 1, it is seen that the measured value of the maximum corner deflection lies between those calculated in the laminates $(0_2,31,-30,\mp30, 0_2)$ and $(0_2,32,-30\mp30,0_2)$. Using a=b=12 in equations (11) to (13) and (17) to (19) and plotting the data as shown in figure 3 yields a perturbation nonsymmetry of 0.9°. The components of the maximum warpage deflection produced by this perturbation are:

0.174 inches due to bending (.17 inches measured)

0.046 inches due to twisting (.05 inches measured)

It is of interest that the measured values are quite similar to those calculated for the arbitrarily assumed perturbation.

If the ply orientations in the $(0_2,\pm45,\mp45,0_2)$ laminate were $(0_2,\pm45,\mp39.5,0_2)$, the calculated maximum warpage corner deflection would be 1.20 inches which is equal to that measured.

The corresponding components of the deflection are:

1.175 inches due to bending (1.00 inches measured)

0.025 inches due to twisting (0.20 inches measured)

Again, it is of interest to note that the measured values are quite similar to those calculated for the arbitrarily assumed perturbation and that the perturbations required to produce the warpage corner deflections are relatively small.

## GENERAL REMARKS

The equations derived herein together with the results presented should be of considerable practical value to both designers and fabricators of fiber composites. For example, the fabricator can use the equations to obtain an a priori assessment of the warpage resulting from factors which may be difficult to control accurately during laminate fabrication. The designer may use the results to specify tolerances which will minimize warpage-producing nonsymmetries; or he may use the results to design the laminate with warpage-compensating nonsymmetries.

The results of this investigation also illustrate that suitable combinations of warpage-producing factors are compensatory and can be used to alleviate some fabrication problems. It is also noted that because a laminate is flat, or meets some flatness specification tolerances, it may not be free of warpage-producing nonsymmetries. It is possible that warpage will result if the laminate is reheated and cooled again. This type of warpage may cause difficulties in designs where the laminate is part of a structure restrained along its edges and is subjected to thermal cyclic environment. In this case, alternating stresses due to warpage would not have been taken into account in designing the laminate.

Although the data presented were not obtained using hybrid composites, such composites will exhibit similar warpage in the presence of bending nonsymmetries and residual stresses. The approximate equations deriged herein are applicable to these composites as well.

## SUMMARY OF RESULTS

The major results of this investigation are summarized below:

1. A convenient set of equations has been derived which can be used to approximate laminate warpage corner deflections resulting from small errors during fabrication. This approach does not require solutions of coupled fourth-order partial differential equations which would normally be the case.

2. Using the derived equations, it was found that an error of only one degree in one ply orientation was sufficient to produce a warpage corner deflection of a magnitude twice the laminate thickness. This ply mis-orientation, among a large number of other possible ones, could have caused the warpage corner deflection that was measured in a 10 inch x

10 inch x .06 inch $(\theta_2, \pm 30, \mp 30, \theta_2)$ laminate made from MOD-I/ERLA-4617.

3. Using the derived equations in a sensitivity analysis of the various factors contributing to laminate warpage, it was found that a $3^{\circ}$ fiber migration or a five percent void volume ratio in some plies is sufficient to produce laminate warpage corner deflections several times the ply thickness. It was also found that errors in ply stacking sequences can produce laminate warpage corner deflections as much as 40 times the laminate thickness.

4. Combinations of errors or tolerances can produce compensating effects on the warpage and initially flat laminates may warp as a result of unrestrained reheating and cooling.

5. The warpage corner deflection varies linearly with errors up to $10^{\circ}$ in ply misorientations and up to 10 percent in fiber content nonuniformity.

# APPENDIX

## SYMBOLS

| | |
|---|---|
| $A_{cx}$ | Array of composite axial stiffnesses referred to composite structural axes |
| $a$ | Laminate edge dimension along x |
| $b$ | Laminate edge dimension along y |
| $C_{cx}$ | Array of composite coupling stiffnesses referred to composite structural axes |
| $D_{cx}$ | Array of composite bending (flexural) stiffnesses referred to composite structural axes |
| $E_{\ell i}$ | Array of strain-stress relations (elastic constants for the $i^{th}$ ply) |
| $E_{\ell 11}$ | Ply longitudinal modulus |
| $E_{\ell 22}$ | Ply transverse modulus |
| $G_{\ell 12}$ | Ply shear modulus |
| $k_v$ | Fiber and void volume ratios, respectively |
| $M_{c\Delta Tx}$ | Vector of unbalanced thermal moments referred to composite structural axes |
| $N_{c\Delta Tx}$ | Vector of unbalanced thermal forces referred to composite structural axes |
| $R$ | Radius of curvature (subscripts denote direction) |
| $R_{\ell i}$ | Array of transformation coefficients for the $i^{th}$ ply |
| $\Delta T_{\ell i}$ | Temperature difference for the $i^{th}$ ply |
| $x,y,z$ | Structural axes |
| $z_{\ell i}$ | Location of the $i^{th}$ ply relative to the reference plane |
| 1,2,3 | Material axes |

| | |
|---|---|
| $\alpha_{\ell i}$ | Vector of thermal coefficients of expansion of the $i^{th}$ ply |
| $\alpha_{\ell 11}$ | Ply longitudinal thermal coefficient of expansion |
| $\alpha_{\ell 22}$ | Ply transverse thermal coefficient of expansion |
| | Corner deflection (subscript denotes component due to corresponding curvature) |
| $\epsilon_{cox}$ | Vector of composite strains referred to composite structural axes at the reference plane |
| $\theta_{\ell}$ | Ply angle measured from the composite structural axes to the ply material axes |
| $\varkappa_{cx}$ | Vector of composite local curvatures referred to composite structural axes |
| $\nu_{\ell 12}$ | Ply major Poisson's ratio |
| $\nu_{\ell 21}$ | Ply minor Poisson's ratio |
| $\varkappa_{cxx}$ | Local curvature, x-direction |
| $\varkappa_{cyy}$ | Local curvature, y-direction |
| $\varkappa_{cxy}$ | Local curvature, twisting x-y plane |

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TABLE 1 - THEORETICALLY CALCULATED EFFECTS OF SMALL PLY MISORIENTATIONS ON THE RESIDUAL STRESS WARPAGE OF FLAT LAMINATES[a]

| Laminate and perturbation | Curvature, 1/in. | | | Maximum corner deflection (in.) due to | | |
|---|---|---|---|---|---|---|
| | x-direction $\kappa_{xx}$ | y-direction $\kappa_{yy}$ | twist $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy} \pm \kappa_{xy}$ |
| $[0_2, \pm 30]_{sym}$ (control) | 0 | 0 | 0 | 0 | 0 | 0 |
| $[0_2, 31, -30, \mp 30, 0_2]$ | .00002 | -.00264 | -.00278 | -.132 | ±.035 | - .167 |
| $[0_2, 32, -30 \mp 30, 0_2]$ | .00004 | -.00531 | -.00541 | -.265 | ±.068 | - .333 |
| $[0_2, 35, -30, 0_2]$ | .00017 | -.01337 | -.01237 | -.660 | .154 | - .814 |
| $[0_2, \pm 45]_{sym}$ (control) | 0 | 0 | 0 | 0 | 0 | 0 |
| $[0_2, \pm 45, \pm 45, 0_2]$ | 0 | 0 | -.05126 | 0 | ±.611 | .611 |
| $[0_2, \pm 45, \mp 40, 0_2]$ | .00052 | -.01517 | .00128 | -.746 | ±.016 | - .762 |
| $[0_2, \pm 45, \mp 35, 0_2]$ | .00091 | -.03314 | .00019 | -1.532 | 0 | -1.532 |
| $[0_2, \pm 45, \mp 50, 0_2]$ | -.00055 | .01176 | -.00290 | .582 | ±.036 | + .618 |
| $[0_2, \pm 45, \mp 55, 0_2]$ | -.00107 | .02038 | -.00679 | 1.030 | ±.085 | +1.115 |

[a]Here and in subsequent tables unless otherwise noted:

1. The composite material is MODMOR-1/ERLA 4617 at .5 fiber volume ratio and cured at 370° F yielding a $\Delta T$ of -300° F.
2. The laminate dimension are a = b = 10 in.: h = .06 in. (fig. 1).

TABLE 1. - Continued. THEORETICALLY CALCULATED EFFECTS OF SMALL PLY MISORIENTATIONS ON THE RESIDUAL STRESS WARPAGE OF FLAT LAMINATES

| Laminate and perturbation | Curvature 1/in. | | | Maximum corner deflection (in.) due to | | |
|---|---|---|---|---|---|---|
| | x-direction $\kappa_{xx}$ | y-direction $\kappa_{yy}$ | twist $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy} \pm \kappa_{xy}$ |
| $[0_2, \pm 45, \mp 45, 0_2]$ (control) | 0 | 0 | 0 | 0 | 0 | 0 |
| $[0_2, \pm 45, -43, 45, 0_2]$ | .00004 | -.00183 | .00151 | - .094 | ±.019 | - .113 |
| $[0_2, \pm 45, -40, 45, 0_2]$ | .00009 | -.00468 | .00408 | - .234 | ±.051 | - .255 |
| $[0_2, \pm 45, -45, 43, 0_2]$ | .00018 | .00369 | .00076 | .184 | ±.010 | + .194 |
| $[0_2, \pm 45, -45, 40, 0_2]$ | .00044 | .00945 | .00271 | .469 | ±.034 | + .503 |
| $[0_2, \pm 45, \mp 45, -1_2]$ | ~0 | .00035 | .00098 | .018 | ±.012 | + .030 |
| $[0_2, \pm 45, \mp 45, -2_2]$ | .00001 | .00072 | -.00196 | .036 | ±.024 | + .060 |
| $[0_2, \pm 45, \mp 45, 1_2]$ | .00001 | .00032 | .00097 | .016 | ±.012 | + .028 |
| $[0_2, \pm 45, \mp 45, 2_2]$ | .00003 | .00062 | .00194 | .031 | ±.024 | + .054 |
| $[0_2, \pm 45, -45, 35, 0_2]$ | .00085 | -.01961 | -.00834 | - .990 | ±.104 | -1.094 |
| $[0_2, \pm 45, -25, 45, 0_2]$ | .00004 | -.01991 | .02040 | -1.006 | ±.256 | -1.262 |
| $[0_2 \pm 45, -30, 45, 0_2]$ | .00011 | -.01493 | .01461 | - .751 | ±.183 | - .934 |
| $[0_2 \pm 45, -35, 45, 0_2]$ | .00013 | -.00974 | .00903 | - .488 | ±.113 | - .601 |

TABLE 2. - THEORETICALLY CALCULATED EFFECTS OF BENDING NONSYMMETRY ON THE RESIDUAL STRESS WARPAGE OF "2-PLY" FLAT LAMINATES

| Laminate and perturbation | Curvature 1/in. | | | Maximum corner deflection (in.) due to | | |
|---|---|---|---|---|---|---|
| | x-direction | y-direction | twist | | | |
| | $\kappa_{xx}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy}+\kappa_{xy}$ |
| $[0_8]$ (control) | 0 | 0 | 0 | 0 | 0 | 0 |
| $[.1_4, -.1_4]$ | ~0 | ~0 | -.00130 | ~0 | ± .016 | + .016 |
| $[.5_4, -.5_4]$ | --- | --- | -.00651 | --- | ± .018 | + .018 |
| $[1_4, -1_4]$ | --- | --- | -.01300 | --- | ± .162 | + .162 |
| $[3_4, -3_4]$ | --- | --- | -.03803 | --- | ± .463 | + .463 |
| $[5_4, -5_4]$ | --- | --- | -.06045 | --- | ± .708 | + .708 |
| $[7.5_4, -7.5_4]$ | --- | --- | -.08334 | --- | ± .923 | + .923 |
| $[10_4, -10_4]$ | --- | --- | -.10020 | --- | ±1.057 | +1.057 |
| $[20_4, -20_4]$ | --- | --- | -.12690 | --- | ±1.226 | +1.226 |
| $[30_4, -30_4]$ | --- | --- | -.13170 | --- | ±1.251 | +1.251 |
| $[45_4, -45_4]$ | ~0 | ~0 | -.13440 | ~0 | ±1.265 | +1.265 |

TABLE 3. - THEORETICALLY CALCULATED EFFECTS OF PLY STACKING SEQUENCE NONSYMMETRY ON THE RESIDUAL STRESS WARPAGE OF FLAT LAMINATES

| Laminate and perturbation | Curvature 1/in. | | | Maximum tip corner deflection (in.) due to | | |
|---|---|---|---|---|---|---|
| | x-direction $\kappa_{xx}$ | y-direction $\kappa_{yy}$ | twist $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy} \pm \kappa_{xy}$ |
| $[0_2, \pm 45, \mp 45, 0_2]$ (control) | 0 | 0 | 0 | 0 | 0 | 0 |
| $[0, 45_2, 90_2, -45_2, 0]$ | ~0 | ~0 | -.07740 | ~0 | ±.871 | - .871 |
| $[45, 0, 90_2, 0, -45_2]$ | ~0 | ~0 | -.06683 | ~0 | ±.771 | - .771 |
| $[45, 0, 45, 90_2 -45, 0, -45]$ | ~0 | ~0 | -.06265 | ~0 | ±.730 | - .730 |
| $[45, 0, 90, 45, -45, 90, 0, -45]$ | ~0 | ~0 | -.04934 | ~0 | ±.590 | - .590 |
| $[0_2, 90_2, 45_2, -45_2]$ | -.03204 | .03204 | -.06408 | 1.503 | ±.744 | +2.247 |
| $[0_4, 90_4]$ | -.06720 | .06720 | ~0 | 2.530 | ~0 | 2.530 |

TABLE 4. - THEORETICALLY CALCULATED EFFECTS OF VOID NONUNIFORMITY ON THE RESIDUAL STRESS WARPAGE OF FLAT LAMINATES

| Laminate and perturbation | Curvature 1/in. | | | Maximum tip corner deflection (in.) due to | | |
|---|---|---|---|---|---|---|
| | x-direction $\kappa_{xx}$ | y-direction $\kappa_{yy}$ | twist $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy} \pm \kappa_{xy}$ |
| [0, 0, 45, -45, -45, 45, 0, 0] ply<br>[.0, .0, .0, .0, .0, .0, .0, .0] voids | 0 | 0 | 0 | 0 | 0 | 0 |
| [.0, .0, .0, .0, .05, .05, .0, .0] | ~0 | -.00089 | -.00075 | -.044 | ±.009 | -.053 |
| [.0, .0, .0, .0, .10, .10, .0, .0] | ~0 | -.00185 | -.00156 | -.092 | ±.020 | -.112 |
| [.0, .0, .0, .0, .0, .05, .05, .0] | .00003 | .00082 | -.00208 | .041 | ±.026 | +.067 |
| [.0, .0, .0, .0, .0, .10, .10, .0] | .00006 | .00163 | -.00427 | .018 | ±.053 | +.134 |
| [.0, .0, .0, .0, .0, .0, .05, .05] | .00006 | .00274 | -.00123 | .167 | ±.015 | +.182 |
| [.0, .0, .0, .0, .02, .02, .02, .02] | .00002 | .00074 | .00078 | .037 | ±.010 | +.047 |
| [.0, .0, .0, .0, .05, .05, .05, .05] | .00006 | .00189 | .00199 | .094 | ±.025 | +.199 |
| [.0, .0, .0, .0, .10, .10, .10, .10] | .00018 | .00389 | .00410 | .194 | ±.051 | +.245 |
| [.0, .0, .0, .0, .20, .20, .20, .20] | .00025 | .00837 | .00780 | .416 | ±.097 | +.513 |
| [.0, .0, .0, .0, .30, .30, .30, .30] | .00041 | .01362 | .01426 | .672 | ±.178 | +.850 |

TABLE 5. - THEORETICALLY CALCULATED EFFECTS OF SMALL PLY MISORIENTATIONS AND VOID NONUNIFORMITY ON THE RESIDUAL STRESS WARPAGE OF FLAT LAMINATES

| Laminate and perturbation | Curvature 1/in. | | | Maximum tip corner deflection (in.) due to | | |
| --- | --- | --- | --- | --- | --- | --- |
| | x-direction $\kappa_{xx}$ | y-direction $\kappa_{yy}$ | twist $\kappa_{xy}$ | $\kappa_{yy}$ | $\kappa_{xy}$ | $\kappa_{yy} \pm \kappa_{xy}$ |
| [0, 0, 45, -45, 45, 0, 0] ply<br>[.0, .0, .0, .0, .0, .0, .0, .0] voids | 0 | 0 | 0 | 0 | 0 | 0 |
| [0, 0, 45, -45, -43, 45, 0, 0]<br>[.0, .0, .0, .0, .10, .10, .10, .10] | .00017 | .00205 | .00259 | .102 | ±.032 | +.134 |
| [.0, .0, .0, .0, -45, 43, 0, 0]<br>[.0, .0, .0, .0, .10, .10, .10, .10] | .00031 | .00026 | .00488 | .013 | ±.061 | +.074 |
| [.0, .0, .0, .0, -45, 45, 1.0, 1.0]<br>[.0, .0, .0, .0, .10, .10, .10, .10] | .00014 | .00359 | .00318 | .179 | ±.040 | ÷.219 |
| [.0, .0, .0, .0, -45, 45, -1.0, -1.0]<br>[.0, .0, .0, .0, .10, .10, .10, .10] | .00009 | .00423 | .00502 | .211 | ±.063 | +.274 |

TABLE 6. - MEASURED PLATE WARPAGE CORNER DEFLECTIONS (REF. FIGURE 1, a=b=12 IN., THICKNESS = .06 IN. GRAPHITE FIBER MOD-I/EPOXY ERLA 4617)

| Laminate | Corner deflections, in. | | | |
|---|---|---|---|---|
| | Maximum point C | Minimum point D | Due to bending[a] | Due to twisting[b] |
| $[0_2, \pm 45, \mp 45, 0_2]$ | 1.20 | .80 | 1.00 | ±.20 |
| $[0_2, \pm 30, \mp 30, 0_2]$ | .22 | .12 | .17 | ∓.05 |

[a]Bending component = 1/2 (deflection at C + deflection at D)

[b]Twisting component = 1/2 (deflection at C - deflection at D)

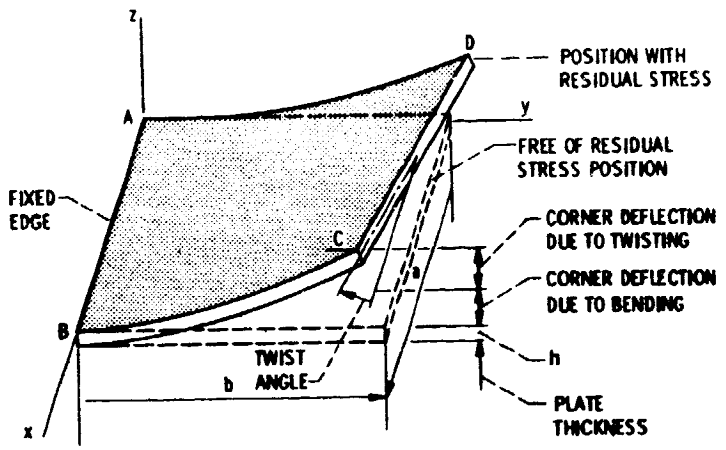


Figure 1. - Schematic of deformed nonsymmetric angleplied laminate with residual stresses.

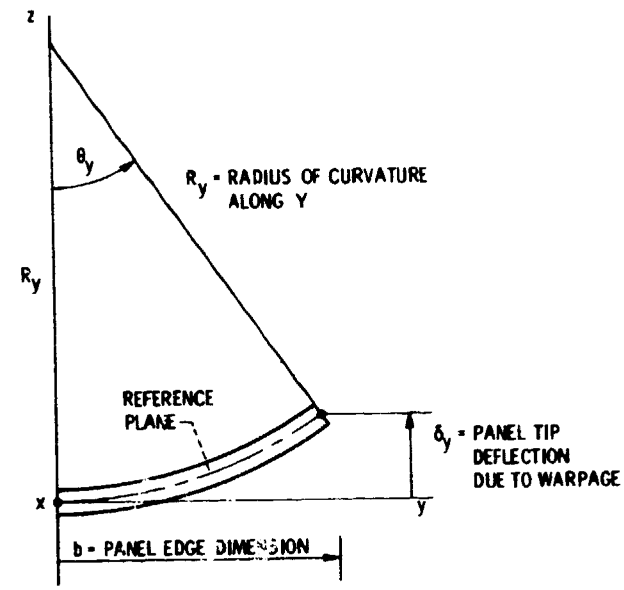


Figure 2. - Schematic depicting geometric relationships between corner deflection, panel edge dimension and radius of curvature.

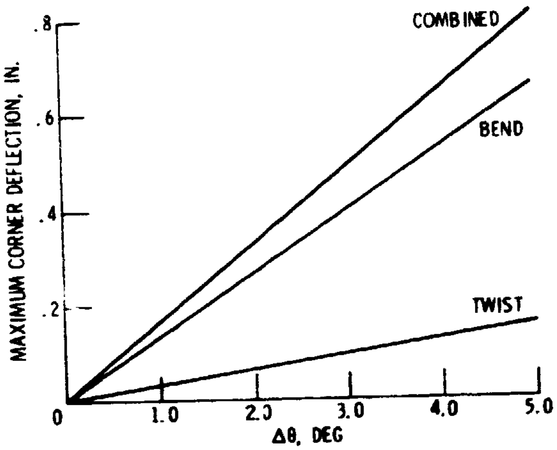


Figure 3. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stress in a $[0_2, +30+\Delta\theta, -30, \mp 30, 0_2]$ laminate from MOD-I/ERLA-4617 composite at 0.5 fiber volume ratio and $\Delta T = -300^\circ$ F.

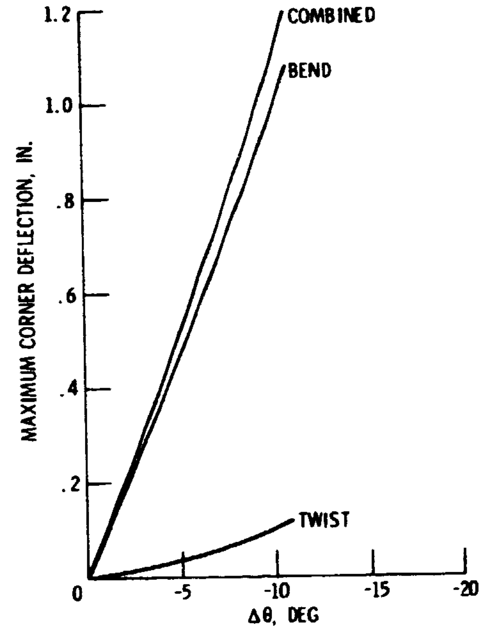


Figure 4. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stress in a $[0_2, \pm 45, -45, +45+\Delta\theta, 0_2]$ laminate from MOD-I/ERLA-4617 composite at 0.5 fiber volume ratio and $\Delta T = -300^\circ$ F.

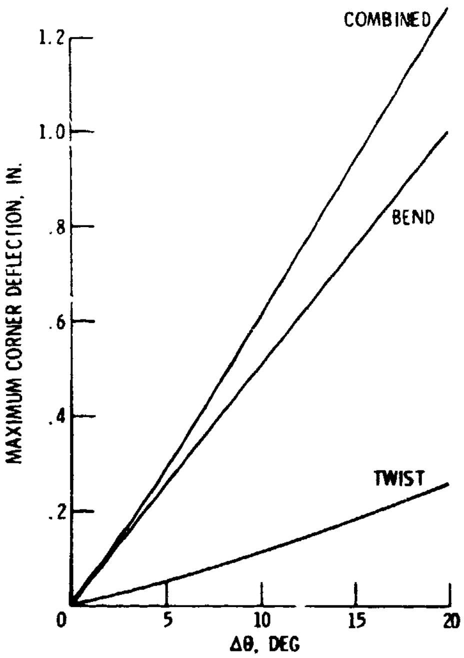


Figure 5. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stress in a $[0_2, \pm 45, -45+\Delta\theta, +45, 0_2]$ laminate from MOD-I/ERLA-4617 composite at 0.5 fiber volume ratio and $\Delta T = -300^{\circ}$ F.

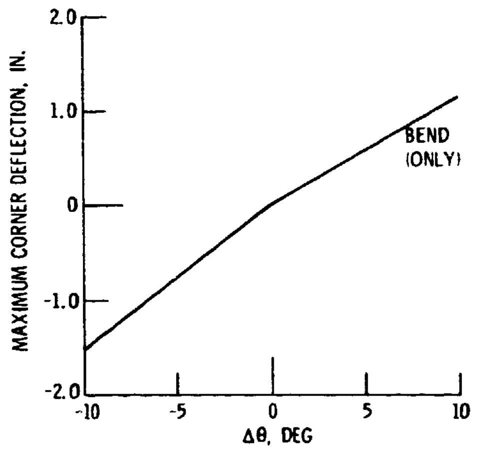


Figure 6. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stress in a $[0_2, \pm 45, \mp 45 \mp \Delta\theta, 0_2]$ laminate from MOD-I/ERLA 4617 composite at 0.5 fiber volume ratio and $\Delta T = -300^{\circ}$ F.

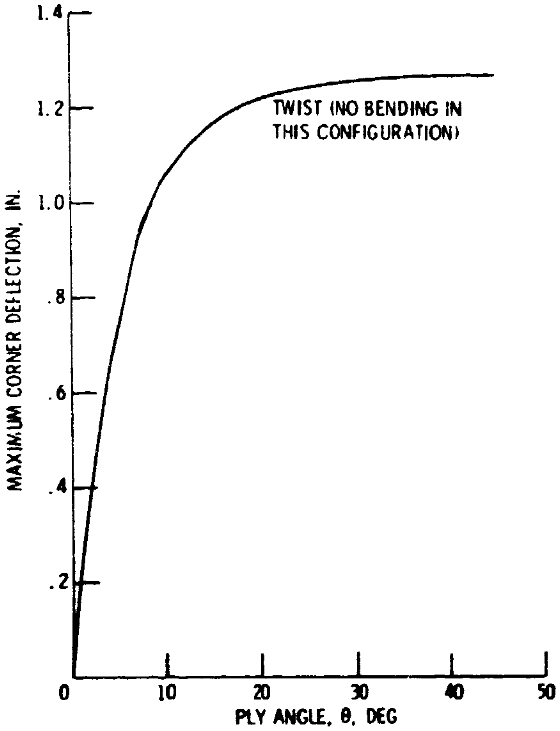


Figure 7. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stresses in a $[+\theta_4, -\theta_4]$ unsymmetric laminate made from MOD-I/ERLA 4617 at 0.5 fiber volume ratio and $\Delta T$ = -300° F.

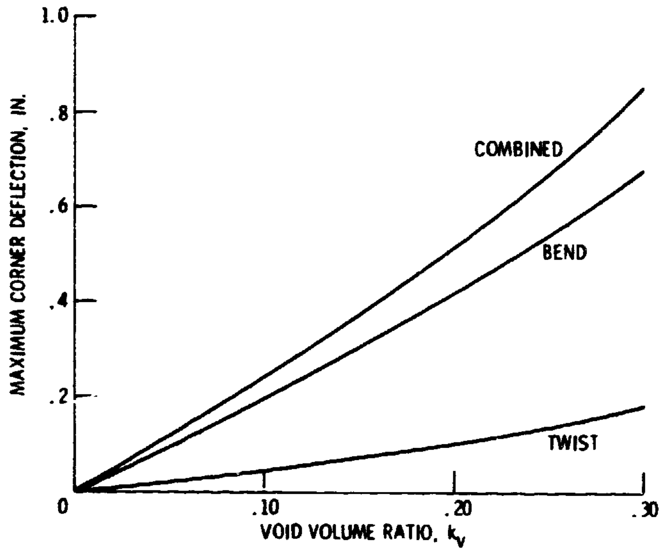


Figure 8. - Predicted maximum warpage corner deflection (point C, fig. 1, a = b = 10 in.) due to residual stress in a $[0_2, \pm45, \mp45, 0_2]$ laminate with void volume ratio $k_v$ [4(0.0), 4($k_v$)] made from MOD-I/ERLA-4617 composite at 0.5 fiber volume ratio and $\Delta T$ = -300° F.

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