Multiple Pure Tone Noise Generated by Fans at Supersonic Tip Speeds[1]

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The existence of clusters of pure tones at integral multiples of shaft speed has been noted for supersonic-tip-speed operation of fans and compressors. A continuing program to explore this phenomenon, often called combination-tone noise, has been in effect at Pratt & Whitney Aircraft (P&WA) for several years. This paper reviews the research program, which involves a wide range of engines, compressor rigs, and special apparatus. Elements of the aerodynamics of the blade-associated shock waves are outlined and causes of blade-to-blade shock inequalities, responsible for the multiple tones, are described in some detail. Some of the data presented were obtained under sponsorship of the Federal Aviation Agency.

Multiple pure tone sound, frequently called combination-tone and "buzz-saw" noise, characterizes the spectra of turbofan inlet radiation fields at supersonic-tip-speed operation. Before discussing the high-speed regime, however, some relevant background information concerning subsonic-tip-speed fan noise will be mentioned.

Instances of discrete tones other than blade-passage harmonics have been noted in the inlet spectra of turbofan engines since the time of the first aircraft compressor noise studies. In those very early 1960's, when blade-passing frequency whine was the outstanding engine noise problem, such "extra frequencies" were called "subharmonics" and were dismissed from the mainstream of research activities. With continuing time and effort, a clear understanding of blade frequency noise was achieved. The growing evidence for the prevalence of extra frequencies could no longer be ignored, and they became the subject of increasing attention. A simple

[1] The authors acknowledge the cooperation of the Federal Aviation Agency, under whose sponsorship portions of the data presented here were obtained.

sorting test based on whether or not these frequencies remained fixed integral orders (multiples) of shaft rotation rate served a useful classifying purpose.

Examples of strong acoustic signals with nonintegral orders are aeolian tones and edge tones caused by oscillating flows over structural elements in the fan air passages. Such small-scale items as instrumentation probes, bosses, and sheet metal seams could excite substantial acoustic vibrations and in fact did so on more occasions than one would care to recall. Figure 1 shows some of the early P&WA laboratory models used for the study of aeolian and cavity tones in airflow duct sections. As has subsequently been pointed out (ref. 1), the duct modes can couple with the triggering mechanism to sustain intense fields. Despite such complexity, solution of many of these types of extra-frequency problems is a matter of careful attention to detecting and removing disturbances in and along the engine flow passages.

A more subtle category of extra-frequency compressor sounds includes cases of subsonic-tip-speed, two-stage fans with spectra having several integral order tones. The essential feature of these spectra, which clued discovery of the generating mechanism, is that the predominant orders are arranged in arithmetic sequence. The sequence may be broken occasionally and may contain a very few misfits, but the main sequence difference equals the difference in the number of rotating blades of the two fan stages. Figure 2 shows a pair of order diagrams for experimental engines run in 1961. By plotting narrow-band spectral components against engine shaft speed, frequencies that are related to speed by a factor or "order" are recognized by falling upon radial lines. Figure 2*a* pertains to a turbofan having 35 first-stage blades and 42 blades in the second stage, a difference of 7 blades. Lines of constant order have been labeled and the concentration of data points along lines separated by 7 orders is striking. In figure 2*b*, corresponding groupings for an experimental fan with 35 and 32 blades are seen along order lines separated by 3 units.

The sequence of important order lines in these and other instances of two-stage fans suggested that a common generating mechanism was responsible for many cases of extra-frequency spectra and that both rotors were mutually involved. From that point on it was quite straight-forward to show that the extra frequencies are generated when a rotor-stator interaction mode at some blade-passage harmonic of one rotor is scattered by the other rotor. This scattering, now called "secondary interactions" in multistage compressors, transforms acoustic energy in the frequency domain as well as effecting mode conversion. Mode conversion at fixed frequency is a result of stator scattering; frequency shifts require a rotating scatterer. The allowable orders in a two-stage fan are given by integral linear combinations of the blade numbers; i.e., $[k_1B_1+k_2B_2]$. Whether an allowable secondary interaction exists in significant strength (some are on

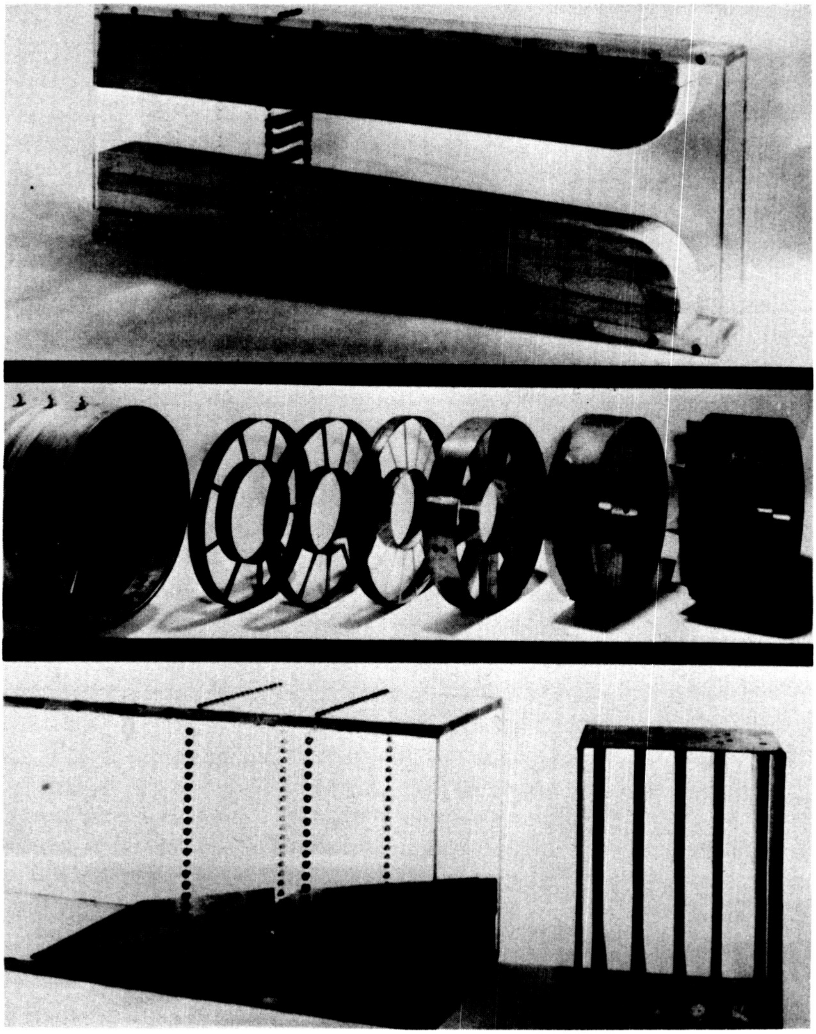


FIGURE 1.—*Apparatus used in studies of aeolian and cavity tones.*

occasion actually greater than blade-passage harmonics!) depends on many factors, including considerations of cutoff ratios of both the secondary interaction and the primary rotor-stator "generator" mode.

Secondary interactions are mentioned here because they provide one of the first cases where the phrase "combination tone" was applied to a powerplant noise. This expression was also chosen to refer to multiple pure tone supersonic noise, in preference to "buzz saw," on the basis of

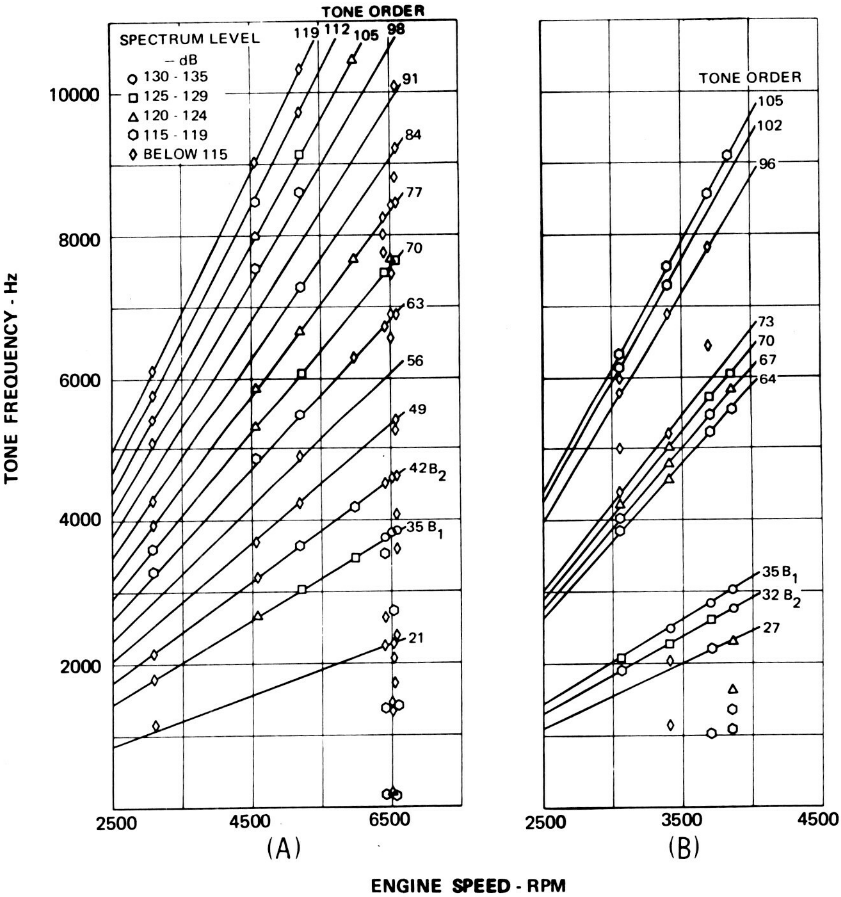


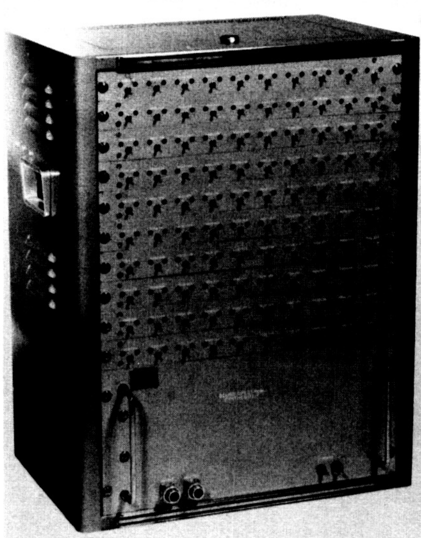
FIGURE 2.—*Combination tones at subsonic rotor speeds due to secondary interactions.*

psychoacoustic considerations that will be discussed presently. A further purpose of including some of the investigations into extra-frequency engine noise is to show the rather wide range of phenomena that can be made understandable by application of linear acoustic theory. Despite these instances of success, linear acoustic theory proved unable to account for the characteristics of supersonic-tip-speed combination tones. Detailed experimental studies have revealed the fundamental nonlinear nature of the supersonic case.

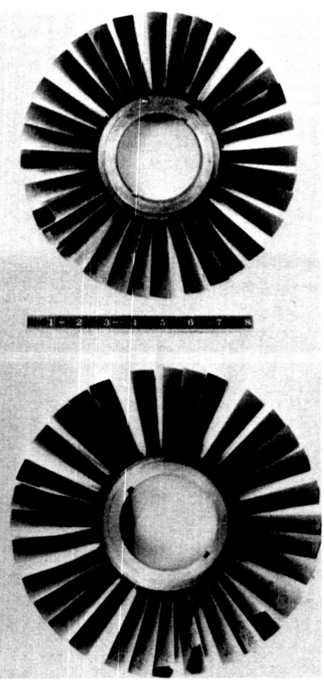
PSYCHOACOUSTIC FACTORS

An unusual feature of combination-tone noise is that its spectrum is, as a matter of natural course, much less annoying to the listener than would be the case if the transformation of energy from blade-passage harmonics failed to occur. Probably the most common examples of reducing annoyance by spreading line spectra are the irregularly spaced blade arrangements used in automotive cooling fans and the irregular tread patterns used for snow tires. This spectral control feature resurfaces every so often in the literature (refs. 2, 3, 4), but the example involved here with supersonic fan multiple-tone noise is the first known reported instance of its occurrence in machinery as a natural outcome rather than as a deliberate design feature.

At the time when blade-vane interaction noise at subsonic rotor speed was the central power plant noise problem, two psychoacoustically oriented programs were instituted to explore the possibilities for reduction of compressor-whine annoyance. One program was based on using controlled off-synchronization of the speeds of the four engines so that the airplane far-field spectra contained blade-passage components, combining in a typical sequence as, for example, 2000, 2100, 2200, and 2300 Hz and harmonics of these. The nonlinear characteristics of the human ear are



a.—*Sound-spectrum synthesizer.*



b.—*10-inch-diameter asymmetric rotors.*

FIGURE 3.—*Equipment for tone-spectrum evaluation.*

such as to supply perceptions of many extra tones that are linear combinations of these basic frequencies. Stevens (ref. 5) discusses these phenomena, employing the apt term "combination tone." Where several signals separated by a common difference (e.g., 100 Hz) are presented to the ear, reinforcement of the low-frequency first-order difference tones (100, 200, 300 Hz, etc.) is especially pronounced, giving the impression of a much lower pitched sound than is actually implied by the physical spectrum. Psychoacoustic tests were conducted using a spectrum synthesizer with 100 controllable frequency outputs, as shown in figure 3*a*. This program was suspended when analysis of flight operational considerations contraindicated use of the complex engine speed control required.

A more straightforward scheme for spectral spreading was explored in model form using unequally spaced blade arrangements (ref. 10), as illustrated in figure 3*b*. This program was also dropped because of other conflicting engine requirements, although, as noted earlier, the concept has many working applications in current use.

There are indications that the spectra of combination-tone noise in supersonic rotors are subject to some measure of control. For this reason, as well as the need to predict "perceived noisiness" more reliably than current rating systems can accomplish, extensive psychoacoustic studies are required. A facility for testing multiple subjects simultaneously in exceptionally uniform sound fields has been recently developed and is shown in figure 4.

FIGURE 4.—*Psychoacoustic test facilities.*

HISTORICAL INFORMATION

As far as we have determined, the first reported incident of supersonic combination-tone noise appeared more than a decade ago in an internal report on an experimental engine (ref. 6). Conclusion number six states, "Engine noise had an odd quality . . . which was best described as sounding like a buzz saw. It was generally considered to be a less annoying sound than the compressor blade passing noise at lower thrust settings." At the time of these remarks, when no theory of compressor noise existed and when a "fix" for the acute blade-passage noise of the JT3D turbofan then entering service was being urgently sought, it is somewhat ironic to have uncovered a glimpse of a phenomenon that would assume practical significance at a later date.

By 1964, uprated engine modifications and advanced experimental prototypes were appearing with higher fan tip speeds. Reports of unusual inlet noises began to grow. Finally, full-scale inlet noise measurements on a run of several engines, together with high-operating-speed data on a single-stage compressor rig, provided the necessary information. Narrow-band spectral analysis clearly and consistently showed a plurality of tones at multiples of shaft rotational frequency and linked their appearance with the onset of supersonic rotor tip speed. Compressor noise theory, by now reasonably developed, indicated that these integral order tones were linked with rotor blade inequalities. Since the phenomena occurred even with rotors having new, closely uniform blades, it was also clear that some sort of aerodynamic amplification was involved, whereby small geometric blade-to-blade variations were transformed into very substantial nonuniformities in the pressure fields rotating with the blades.

In 1965 design work was progressing on a new generation powerplant, the JT9D, which incorporated all known fan noise reduction features in the basic configuration (ref. 7). Interaction noise had been designed out of the powerplant, but at supersonic tip speeds, where no method other than absorbing lining was available to attenuate the propagating pressure field of the rotor, it had originally been feared by some that the direct rotor field would generate an objectionable siren-like noise. However, the test information related above, supplemented by noise data on a related larger turbofan, indicated that combination-tone noise rather than blade-passage noise would be radiated from the inlet at high powers. A significant body of information thus indicated that we could reasonably expect to avoid this piercing blade-passage whine problem. Early in 1966, tests of a prototype JT9D confirmed this fortunate state; the concept behind asymmetric fans had finally reached practicable embodiment with the natural, small blade-to-blade nonuniformities of aircraft rotors operating at supersonic tip speeds.

Disclosure of combination-tone noise as an inherent characteristic of supersonic-tip-speed operation was made in a pair of 1966 survey papers (refs. 8, 9). Studies to advance the understanding of this noise and to explore methods for its control were pursued. Extensive use was made of the 28-in.-diameter single-stage compressor rig shown in figure 5. Internal traversing of the inlet of this rig revealed the blade-tip shock patterns and gave definitive evidence of the nonlinear behavior involved in supersonic combination-tone noise. Many of the detailed noise characteristics learned in these programs were described by J. D. Kester in 1969 (ref. 10) and were also disclosed in a short motion picture released in 1968 (ref. 11).

Several compressor rigs and many engines have been used to establish the current state of understanding. The wide-scale range of equipment used is highlighted in figure 6a. Posed in the inlet of an experimental full-scale flight-test JT9D engine is a wind-tunnel-model-powered nacelle designed for installation aerodynamic programs. Its 4-in.-diameter fan, capable of spinning at 80 000 rpm design speed, develops a tip Mach number of 1.3. Noise data were obtained with a high-frequency microphone system, recorded at high speed on magnetic tape, and played back at ¼ speed through a narrow-band wave analyzer. A sample inlet noise spectrum is shown in figure 6b. The prevalence of multiple pure tones may be noted and compared with the full-scale JT9D inlet noise spectrum of figure 6c. This model program provided convincing evidence for the reliability of scaling to blade Mach number, a principle of acoustic testing

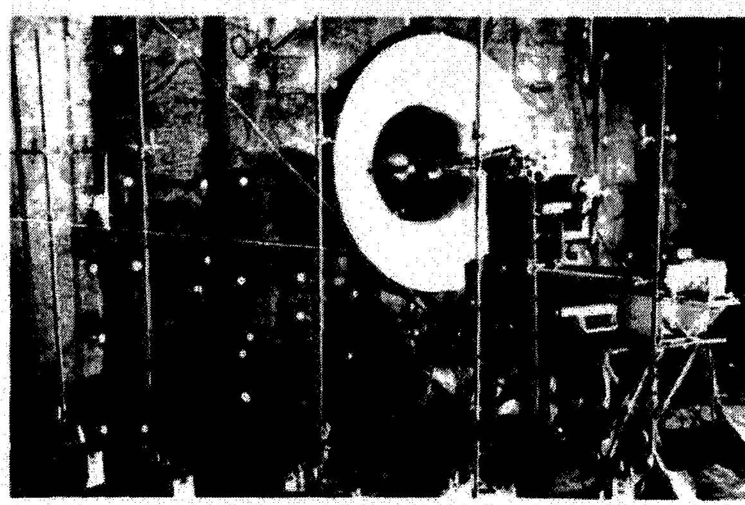
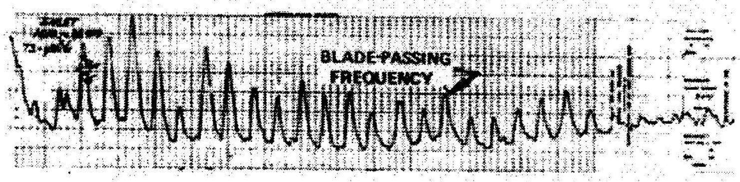
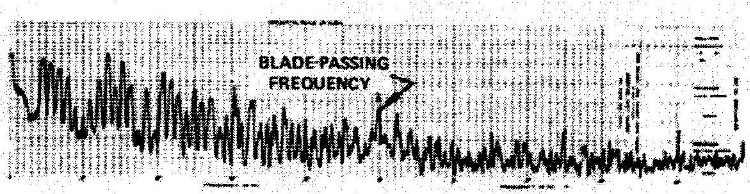


FIGURE 5.—*28-inch-diameter compressor rig.*

a.—*JT9D and scale-model powered nacelle.*



b.—*Scale-model spectrum—[illegible]-inch diameter.*

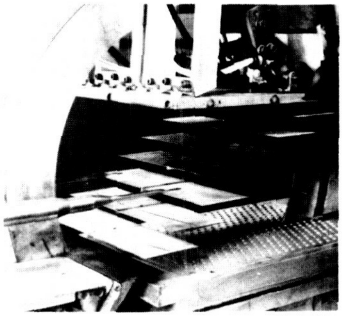


c.—*JT9D spectrum—92-inch diameter*

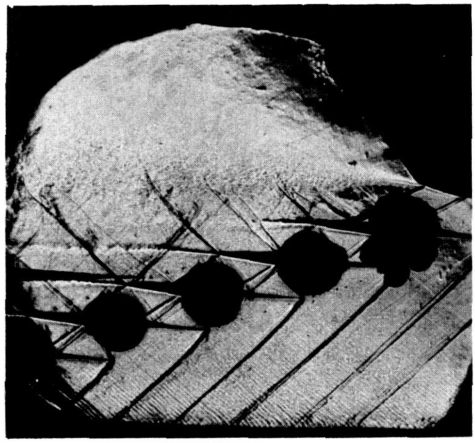
FIGURE 6.—*Effect of fan diameter on combination-tone noise.*

that had been stated explicitly for subsonic compressor operation by Wells and McGrew (ref. 12).

A supersonic cascade wind tunnel was used in a program to explore the consequences of various blade nonuniformities upon cascade shock-wave variations (refs. 13, 14). Figure 7 gives a view of the installation and a Schlieren photograph of the shock structure taken during operation with a uniform reference configuration.



a.—*Cascade arrangement.*



b.—*Schlieren photograph.*

FIGURE 7.—*Supersonic cascade tunnel.*

To supplement full-scale engine studies of combination-tone noise with a variety of blade designs and flexible test programs, a new single-stage fan rig has been developed (ref. 15). As seen in figure 8, the rig is installed in an outdoor stand where reliable far-field noise surveys are made and where inlet traversing equipment can be emplaced quickly for very near field data acquisition.

As is true in the linear cases of compressor noise phenomena, the three-dimensional nature of the inlet geometry presents complications in the nonlinear domain of supersonic fan noise. Radial or spanwise variations in the aerodynamic characteristics of the flow very near the rotor face, as well as acoustic field variations symptomatic of higher radial mode structure farther out the inlet, raise significant obstacles to theoretical and experimental studies alike. A simplified basis for further study would be established if it could be demonstrated in a narrow annulus inlet that the power in predominantly blade-passage harmonics in the very near shock field of the rotor were transformed into multiple tones at shaft rotation harmonics, as occurs in conventional fan inlets.

A short program has been conducted recently under Federal Aviation Agency sponsorship to determine if combination-tone noise is developed in a narrow annular inlet region (ref. 16). By inserting a sleeve spaced concentrically in the inlet of the 28-in.-diameter rig (figure 9), a narrow annulus was formed within which the outboard blade shock field could travel. Confined between the closely spaced (1-$\frac{1}{2}$ in.) walls, shock structure in the very near field of the rotor evolved into the characteristic form of combination-tone noise as the pattern spiraled forward. Figure 9*b* shows typical combination-tone results obtained several inches from the rotor in the annulus.

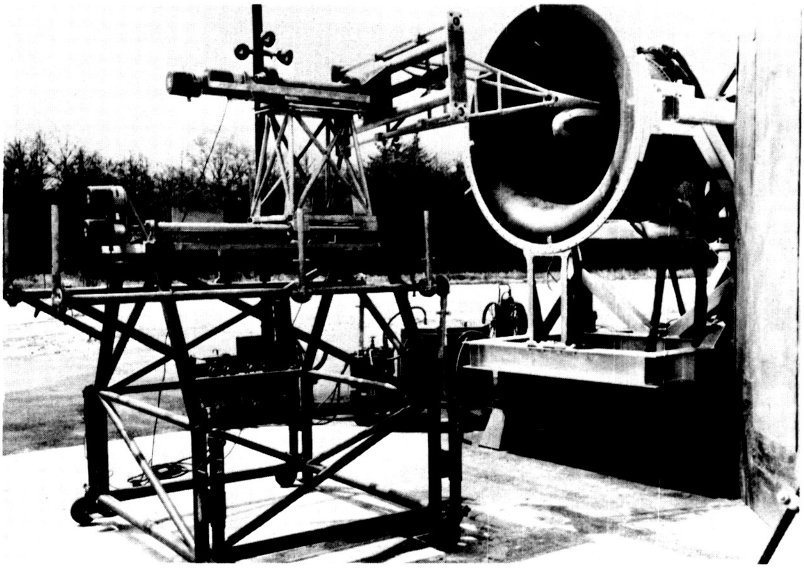
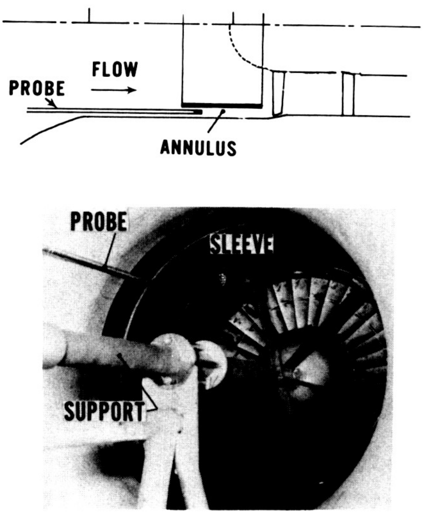
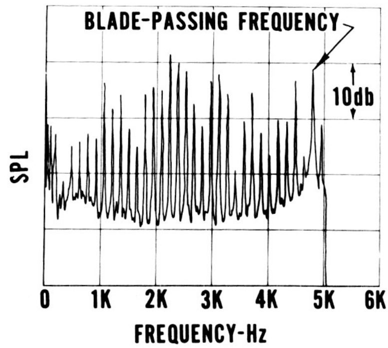


FIGURE 8.—*52-inch-diameter fan rig.*



a.—*Annulus and probe.*



b.—*Spectrum inside annulus.*

FIGURE 9.—*Combination-tone development in narrow-annulus inlet.*

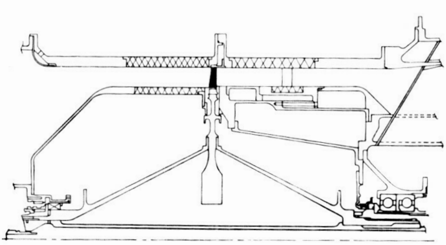
These tests confirmed that combination-tone studies could indeed be made on a two-dimensional basis and that no essential features are lost by ignoring radial complications. Accordingly, consideration is currently being given to possible fabrication and use of a special narrow annulus rig or rotating annular cascade (figure 10) in future studies.

COMBINATION-TONE CHARACTERISTICS

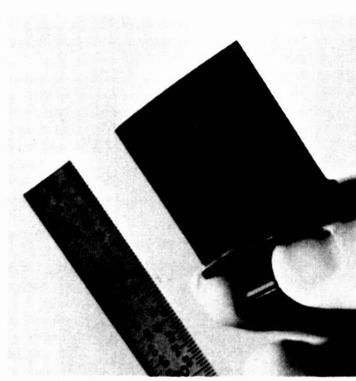
The essential features of combination-tone noise, based on the work reported by J. D. Kester (ref. 10) and on additional programs, are summarized below, prior to an aerodynamic explanation of the shock-wave evolution responsible for this noise.

At supersonic tip speeds, shock waves form at the rotor blade tips. Because of small blade-to-blade irregularities, the shocks display slight nonuniformity of amplitude and spacing. Probe microphone measurements taken very close to the leading edge plane of the rotor reveal that these irregularities are small compared to their average values. Typical data for the very near field (figure 11*a*) show the essential uniformity of pressure signature, repetitive at blade-passage frequency. The corresponding spectrum confirms the visual impression, extra harmonics of shaft rotational rate being at least 20 dB below blade-passage levels. Parenthetically, these small traces of extra orders are also present in the subsonic spectra of actual fans.

Only when the pressure sensor is drawn farther away from the rotor do the characteristics of combination-tone phenomena appear. The rotor signature exhibits relatively more blade-to-blade irregularity. At this distance the shock pattern, locked to and spinning with the rotor, assumes

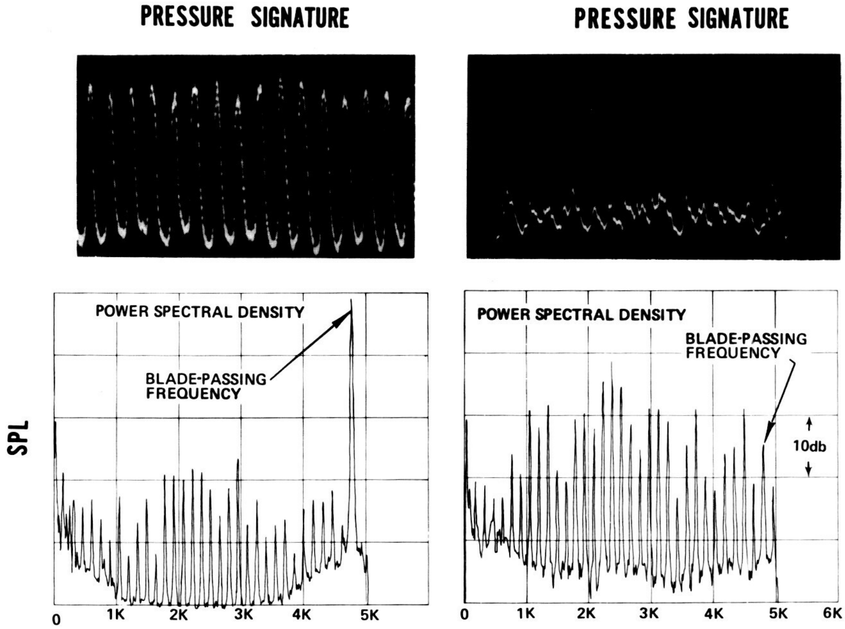


a.—*Sketch of rotating annular cascade.*



b.—*Blade for rotating annular cascade.*

FIGURE 10.—*Rotating annular cascade rig.*



a.—*Uniform very near field.* b.—*Combination-tone field.*

FIGURE 11.—*Waveforms and spectra in combination-tone development.*

different features around its perimeter. Correspondingly, the extra frequencies present in the spectrum have levels that are no longer negligible relative to blade-passage harmonics. At stations still farther from the rotor plane (figure 11*b*), the rotating pressure field has lost most if not all of its original repetitive blade imprint, and its spectrum is overwhelmingly multiple pure tone. This sequence portrays the complete evolution of the combination-tone pattern; further probe travel discloses no essential novelty.

The earliest studies of combination-tone structure inside a fan inlet drew heavily on instrumentation systems and concepts that had previously proved successful in explaining the nature of compressor noise at subsonic rotor speeds. In that field it is expedient to work with such concepts of linear analysis as spectral parameters (e.g., amplitude and phase of blade-passage frequency) and duct characteristic functions (circumferential and radial modes). These techniques have limited value for examining combination-tone behavior. Examination of the *unfiltered* pressure signature in the course of axial traversing shows the complex evolving progression of the train of unequal shocks. Filtering techniques in frequency and resolution into modal components actually obscure the clear information in the pressure-time signature. No amount of spectral information can substitute for witnessing the overtaking and merging of shocks when seeking understanding of combination-tone generation.

In supersonic rotors having subsonic exit flow relative to the blades, the shock structure producing combination-tone noise is restricted to the regions ahead of the rotor. The effects of inlet guide vanes noted to date are largely predictable: Preswirl lowers the resultant flow speed entering the fan and defers the onset of shock behavior. Guide vanes may affect the transmission of the combination-tone wave field as they do blade-passage noise, in a manner described by Mani, Horvay, and others (ref. 17). However, guide vanes reintroduce the prime source of blade-vane interaction noise.

Combination-tone noise has been successfully reduced by sound-absorbing liners (figure 12). Because the shock-wave field is confined to the vicinity of the inlet wall rather than distributed radially inward to the rotor hub, a short treatment length is more effective for the attenuation of combination-tone noise than it would be for interaction noise.

AERODYNAMIC THEORY—IDEAL, UNIFORM FANS

Having illustrated the essential features of combination-tone noise with examples from several rigs and engines, we will investigate in more detail that region ahead of the rotor called the transition zone, where sound power is transferred from blade-passing frequency into other harmonics of engine rotation frequency. We shall first consider the shock-wave behavior ahead of an ideal, symmetrical fan and then extend the results to actual fans.

A theoretical investigation using shock-wave theory is somewhat complicated by the low supersonic regime in which most fans operate and the three-dimensional nature of the flow. It is possible, however, using the methods of finite-amplitude wave theory as developed by Whitham (ref. 18) and Lighthill (ref. 19), and extended by Blackstock (ref. 20), Morfey (ref. 21), and Fink (ref. 22), to obtain the asymptotic behavior of two-dimensional steady shock waves from cascades.

Applying finite-amplitude wave theory to an isolated airfoil in supersonic flow, the leading-edge shock wave decays eventually as the inverse square root of distance from the airfoil and is of parabolic locus (ref. 19 sec. 9). The distance from the airfoil for which this solution is valid varies with the free-stream Mach number and the leading-edge thickness for the flow regime considered. If this airfoil is now assembled in a two-dimensional cascade, it can be seen from figure 13 that the shock wave from that airfoil will be intersected by or will eventually meet the expansion-wave region of the preceding blade. The fields are illustrated for straight- and curved-blade entrance profiles in figures 13*a* and 13*b*.

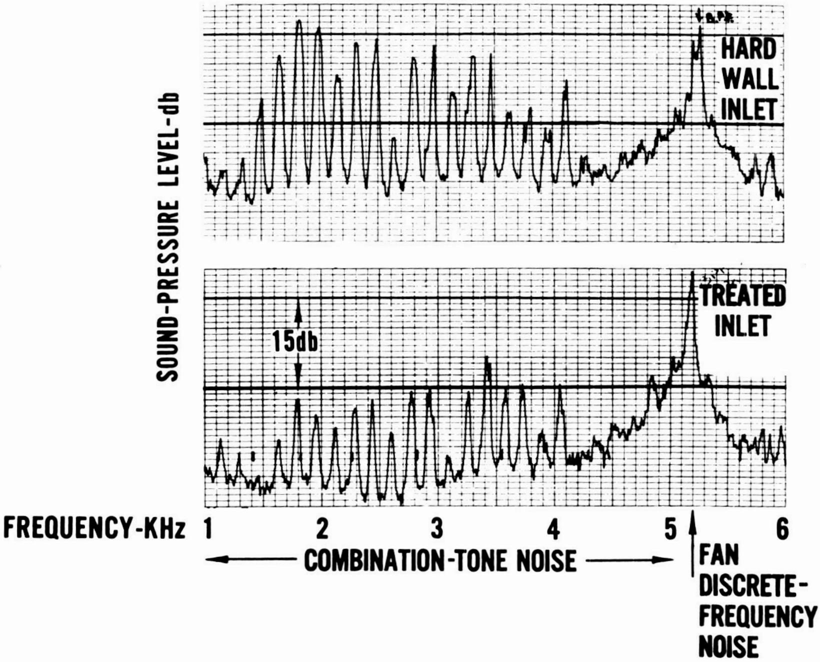
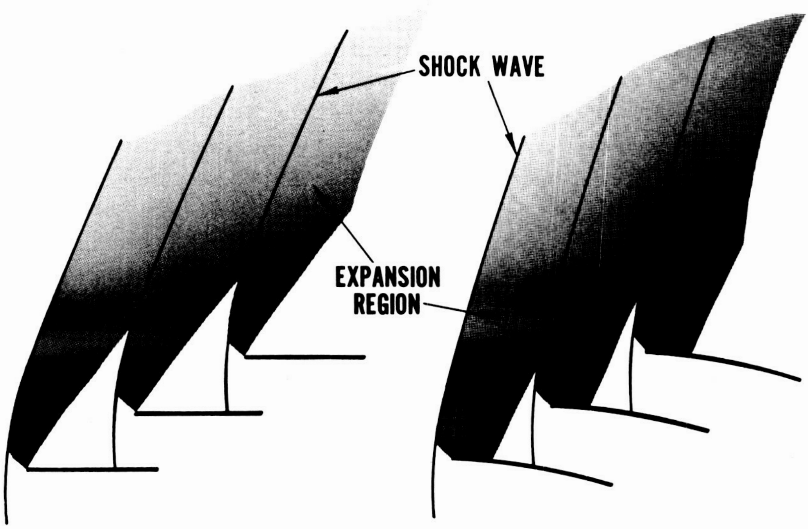


FIGURE 12.—*Effects of sound-absorbing inlet liner on combination-tone noise.*



a.—*Straight entrance-region blades.* b.—*Curved entrance-region blades.*

FIGURE 13.—*Shock patterns for uniform blade cascades.*

Shock waves that are both preceded and followed by expansion waves were first investigated by Whitham (ref. 18); he found that the shock strength at large distances varied as the inverse first power of distance, converging to a line parallel with the free-stream Mach lines. Subsequent investigation by Blackstock (ref. 20) and Morfey (ref. 21) confirmed these results. Specifically, the shock strength at large distances, x, was shown to be

$$\frac{\Delta P}{P_0}=\frac{2\gamma}{\gamma+1}\frac{\lambda}{x}(\sin\varphi-M_x) \tag{1}$$

where λ is the gap between shocks, P_0 is the ambient pressure, M_x is the axial Mach number, φ is the angle between the free-stream Mach wave and the axial direction, and γ is the ratio of specific heats. Notice that equation (1) is independent of the initial amplitude of the shocks and that it does not contain any dissipative terms. This latter consideration is a restriction of the validity of equation (1) for very large distances when the shock strength is so small as to make dissipative terms comparable with nonlinear, steepening effects.

It can be concluded that the shock waves ahead of an ideal fan will initially advance on the expansion regions of preceding blades and their strength will decay as the inverse square root of distance from the fan. At distances of the order of one blade gap, where the expansions preceding and following the shocks are comparable,[2] the shocks will no longer advance on the expansion regions of the preceding blades, and their amplitude will decay more like the inverse first power of distance.

The conclusions were validated in the two-dimensional field of the narrow-annulus test mentioned earlier in the paper. The annulus was probed with a $\frac{1}{8}$-in. B&K microphone so that the strength of the average[3] shock wave could be measured as a function of distance ahead of the fan. The results for three operating points of the compressor are presented in figure 14. The pressure rise across the average shock is normalized by its value at the first measuring point ahead of the rotor, and lines with slopes of $-\frac{1}{2}$ and -1, corresponding to inverse square-root and inverse first-power decay respectively, are superimposed upon the measured data with evident fit. The change from $1/\sqrt{x}$ to $1/x$ decay is seen to occur around 2.5 in., which is slightly less than one blade gap.

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[2] That is to say, the areas preceding and following the shock on a pressure-time curve are comparable. Lighthill (ref. 19) illustrates similar situations in his treatment of plane shock-wave formation (sec. 7) that can be extended to include the development of nonplane shock waves (sec. 9).

[3] The data, taken on an actual fan, naturally reflect blade-to-blade inequalities. It was necessary to process these data so that the behavior of the corresponding uniform, ideal case could be inferred.

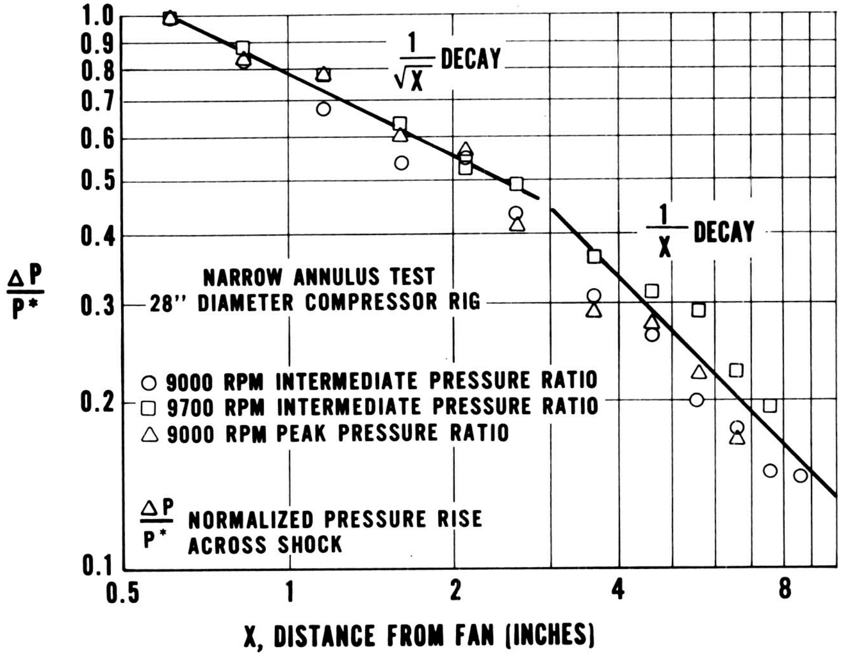

FIGURE 14.—*Changing rate of shock-wave decay.*

Figure 15 shows the development of the waveform from an ideal, symmetrical fan at distances of $\frac{1}{4}$, $\frac{1}{2}$, and 1-$\frac{1}{2}$ tip chords ahead of the rotor. At the $\frac{1}{4}$- and $\frac{1}{2}$-chord locations, it can be seen that the shock fronts have not caught up with the expansion regions of the preceding blades and thus still decay as $1/\sqrt{x}$. At the 1-$\frac{1}{2}$-chord position, the expansion regions preceding and following the shocks are comparable, and subsequent shock decay varies as $1/x$.

It is now possible to infer some of the details of the evolution of the shocks ahead of real fans containing nonuniformities and to explain how the sound power at blade-passing frequencies is transformed into harmonics of engine rotation frequency.

AERODYNAMIC THEORY—REAL FANS

In all real fans, small nonuniformities within manufacturing tolerances are present or are created by routine wear in service. These blade-to-blade nonuniformities are reflected in supersonic operation by variations in shock strengths and associated expansion regions. Very close to the blades, however, the nature of the nonuniformity closely preserves regularity of

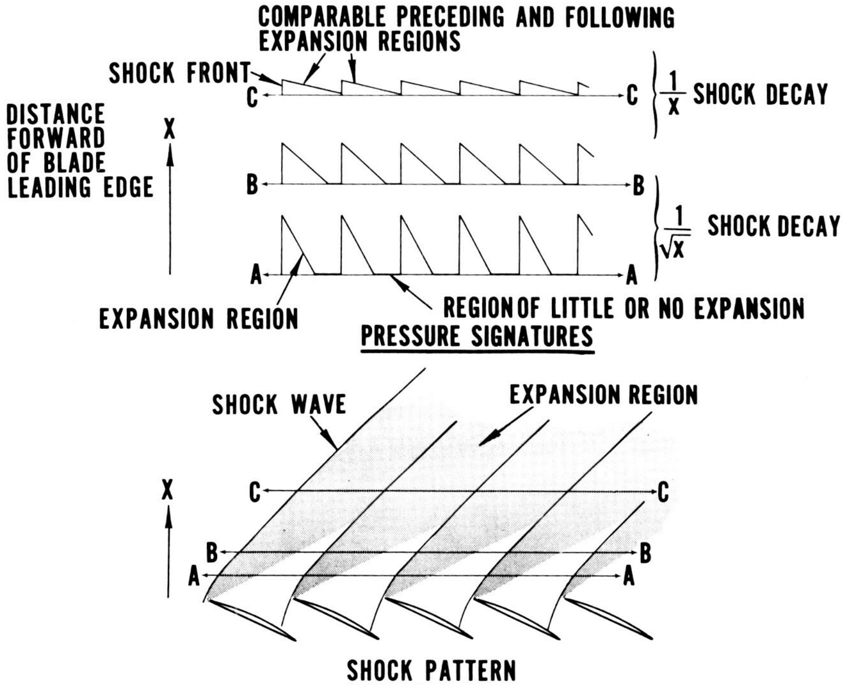

FIGURE 15.—*Shock-wave pattern from a uniform fan.*

interval between shocks. A Fourier analysis of waveforms produced by such rotating shock-wave patterns has been investigated as a particular case of a general analysis of time- and amplitude-modulated waves (ref. 22). An important result of this investigation is that the expected power spectrum depends critically upon variations of the intervals between shocks and only slightly upon variations in shock strength. Therefore, since the variance in interval between shocks is small close to the fan, most of the sound power will be concentrated at blade-passing frequency even if there are appreciable variations in shock strength. Probe data taken very close to a variety of fans confirms this spectral characteristic, the power at blade-passing frequency being as much as 20 dB greater than the power in other engine harmonics.

Now, this picture changes as the shock waves spiral upstream in the inlet. Because there are variations in shock strength, the stronger shocks advance on the weaker ones and the intervals between them become noticeably irregular, causing the combination tones to emerge. In this region of the duct, the shocks from each blade will behave more like isolated shocks, being influenced only slightly by the other blades, and will decay in amplitude as the inverse square root of distance. The irregularities are enhanced when each shock encroaches on the expansion region of the preceding blade, and begins to decay more nearly as the inverse first power of distance. Because of the existing irregularities, this change in decay will occur at different points in the inlet for each shock. Thus, until all the shocks are decaying as the inverse first power, there is a region in the inlet where essentially two different types of decay mechanism are present. The intervals between the shocks will change appreciably in this region since some shocks will be advancing upon the expansion regions of the preceding blades while others will already be converging toward their respective asymptotes. As all the shocks approach their asymptotes, there will be no further changes in shock intervals. When dissipative forces have produced acoustic waves, noise radiates to the far field with no further exchange of power among the tones. Figure 16 illustrates, for a cascade, the various stages of shock-wave evolution. It may be seen how the essentially uniform pattern very close to the rotor is transformed into one containing marked irregularities.

Figures 17, 18, and 19, showing the inlet-duct pressure signatures of three different fans, illustrate various aspects of shock-wave decay considered in this section. High-frequency-response pressure transducers, flush-mounted in the inlet of an experimental JT9D engine, produced the waveforms in figure 17. It is clear that the shock waves close to the fan are not preceded by expansions until about the $\frac{3}{4}$-tip-chord position forward of the rotor. At this point, an irregular sawtooth waveform has developed.

Figure 18 displays waveforms obtained by probing the inlet of a 52-in.-diameter fan rig. An arrow draws attention to the evolution of a particular shock at various axial positions. It is both preceded and followed by stronger shocks, the following one overtaking the reference shock 8 in. ahead of the fan. The combined shock is then indicated by the arrow and its subsequent decay proceeds as the inverse first power of distance. This latter decay is consistent with theory since at 8 in. the combined shock has comparable preceding and following expansion regions.

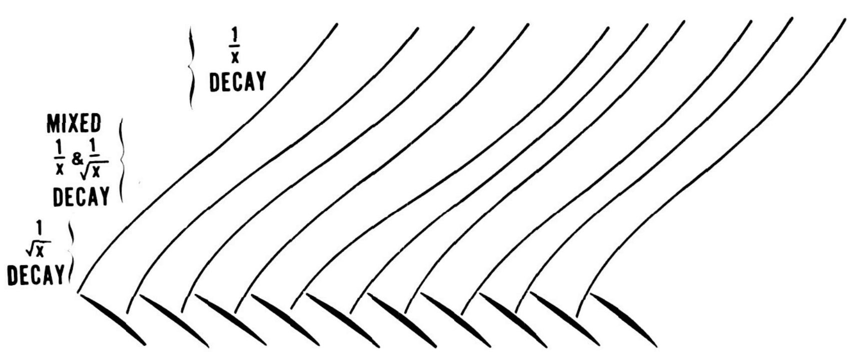

FIGURE 16.—*Shock-wave pattern from an actual fan.*

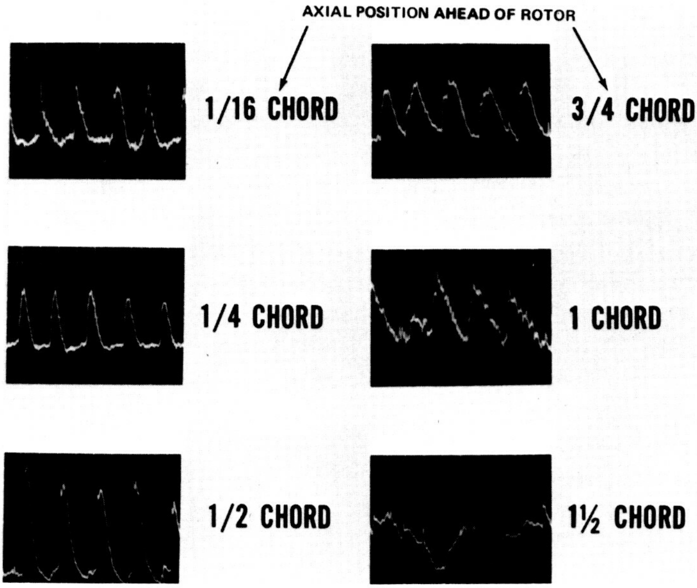

FIGURE 17.—*Blade-pressure signatures—experimental JT9D.*

The development of the three shocks ahead of the arrow is of interest also because they combine after 16 in. to produce a single shock that is seen to be approaching an acoustic waveform after 28 in. At 32 in., a typical shock-free, acoustic waveform can be seen, and it may be assumed that little or no subsequent transfer of sound power between the combination tones will take place.

Extensive data have been acquired on the 28-in.-diameter rig, which has been referred to frequently here. Sample waveform patterns for a blade configuration in this rig are shown in figure 19. One region of waveform is called to attention by an arrow, showing the behavior of a weaker shock decaying between two stronger shocks. Here again may be seen examples of how the nonlinear aerodynamic behavior of shock waves accounts for combination-tone noise.

SUMMARY

It has been shown theoretically and experimentally how finite-amplitude wave theory can account for the transfer of sound power from blade-passage harmonics into harmonics of engine rotation frequency. This transfer, occurring in the inlet forward of a supersonic rotor, produces noise with a multiple-tone spectrum and a deep, distinctive sound, characteristic of combination-tone noise.

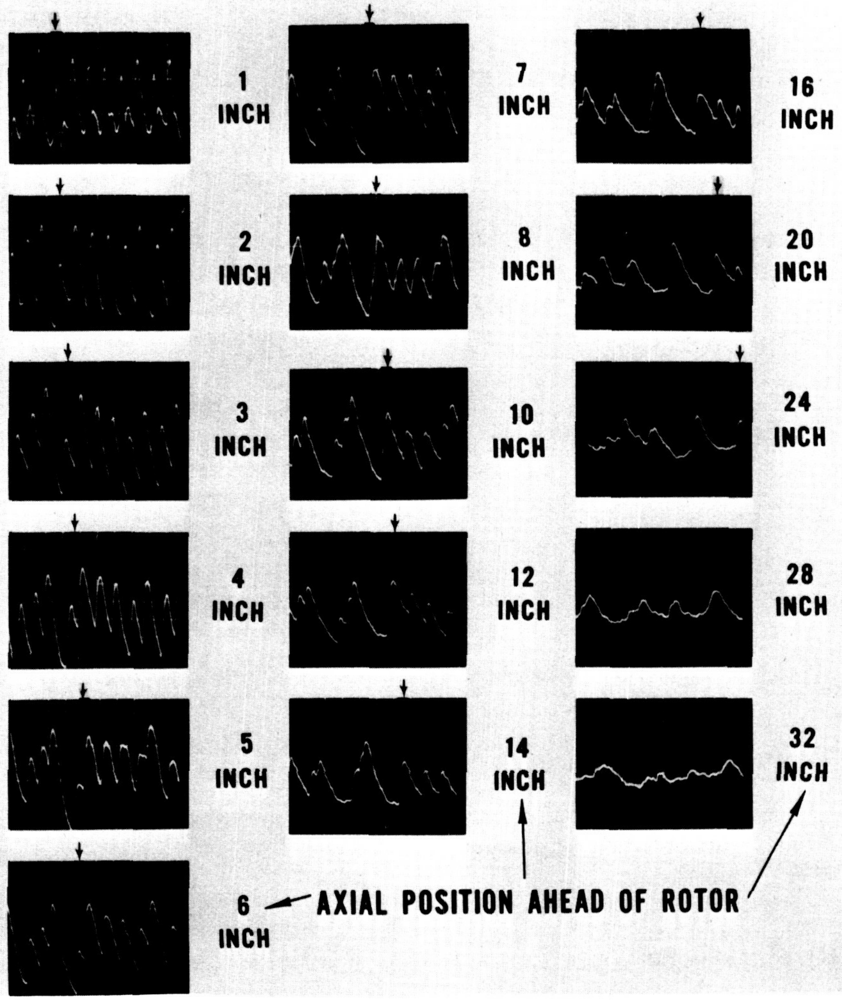

FIGURE 18.—*Blade-pressure signatures—52-inch-diameter fan.*

The work reported here and recent appearances of other accounts in the literature (refs. 24, 25) provide interesting examples of significant progress in aircraft noise reduction. Combination-tone noise is neither a new phenomenon nor has it suddenly become more severe. However, a new generation of high-bypass-ratio engines is appearing with important improvements in jet exhaust roar and fan interaction discrete noise. These reduced levels of previously dominant engine sources have unmasked combination-tone noise as a target of renewed noise reduction efforts.

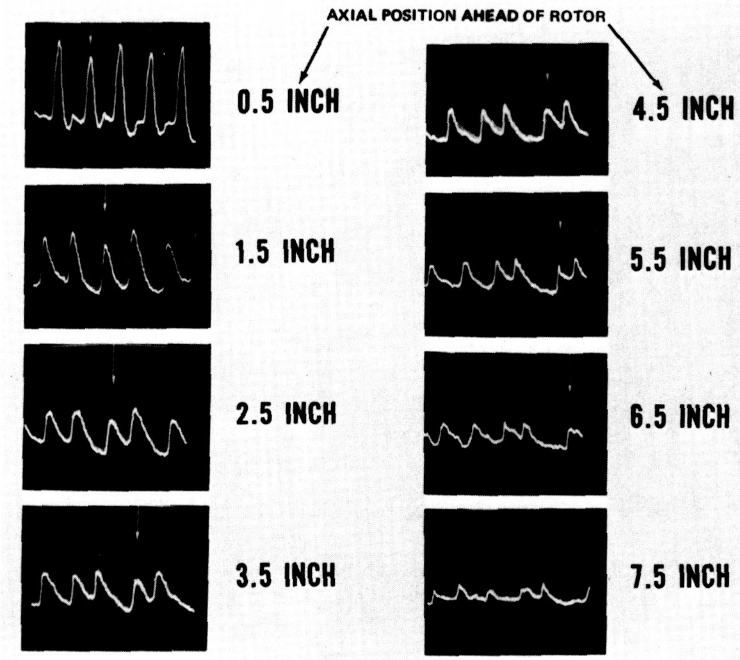

FIGURE 19.—*Blade-pressure signatures—28-inch-diameter fan.*

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DISCUSSION

J. E. FFOWCS WILLIAMS (Imperial College, London, England): I completely agree that the shock formation and steepening is a nonlinear process and I find your model completely consistent. However, there's one point that I would like to make and that is that I would argue that it is possible to get buzz saw even if you were to devise a supersonic fan with very small perturbations in pressure across the blades. And I would just take you up on the point you made at the beginning of your talk where you said, "We surely don't make fans that chaotic in spacing." But what is the measure of whether the error is large or small? It isn't on any absolute geometric scale.

The point I am stressing is that for a small error in blade positioning, it wouldn't matter at all, as long as the pressure signature was slowly varying on that scale. But if the signature is varying rapidly on that scale it would matter. In this case, if you've got supersonic flow, the pressure would rise abruptly. There will be a sort of a step wave, and, if you would misposition that step by any error, however small, the result would inevitably give rise to some buzz-saw noise.

I agree that your situation is entirely nonlinear. So for you to quote your experiments as evidence on my hypothetical linear supersonic fan problem isn't really to the point.

C. L. MORFEY (Southampton University): Referring to figure 11 where you show the very near field compared with the far field after the shocks are developed, the first question is whether the base lines are the same on these two figures. Second, I wonder if you could give us any guidance as to why it is we often see a peak in the envelope of these plots at roughly half the blade-passing frequency? This seems to be a common feature of some of your spectrographs. If the baseline is the same in figure 11*a* and figure 11*b*, then it shows your levels at about one-half blade-passing frequency have increased.

R. D. WELLIVER (The Boeing Co.): In all this discussion, there's just one point that I think gets lost a little bit that I would like to make. We've done a number of fan experiments in fans that run at supersonic tip speeds and when we think of the buzz saw being created by the non-uniformity of the blade or things of this nature, one thing comes to mind that's become very clear in many of our experiments, and that is that the magnitude of the buzz-saw levels as combination tones varies considerably with the blade loading. If we consider a fan running at a constant speed across the fan operating map from the maximum air flow to the surge line for a given speed, the Mach number really doesn't change too much since the inlet relative Mach number of the fan varies very little. If you look at the vector diagrams, obviously the absolute Mach number as well as the inlet relative Mach number for a high-speed fan doesn't vary too much, and yet if we measure the noise and look at the spectrum, across the speed line, we find that the buzz-saw noise could vary as much as 20 dB for a high-speed fan. It sometimes goes from disappearing perhaps at some lower loading to be very dominant. Now, I didn't change the speed, I didn't really change the Mach number, and I didn't change the blading obviously as I moved across the speed line, and so this is a question which I've certainly had and I think some explanation of this phenomenon is required.

SOFRIN AND PICKETT (authors): In reply to Professor Ffowcs Williams, we agree that combination-tone noise would be generated for the hypothetical linear supersonic fan problem. It would differ, however, from combination-tone noise produced from actual fans in two important ways: Firstly, the power at the harmonics of engine rotation frequency would not be comparable to blade passage harmonics and also there would be no transfer of power among the harmonics as a function of distance forward of the fan. These characteristics of combination-tone-noise generation can be explained only by using a nonlinear model.

With reference to Dr. Morfey's questions, figure 11 shows the differences in spectrum shapes at two axial locations in the inlet and was not prepared to compare the spectra at the two locations on a quantitative basis. He observes correctly that the envelope of the tones seems to peak at roughly half of blade-passing frequency. The reasons for the typical envelope shapes are discussed in reference 23.

Concerning the question of effect of blade loading on combination-tone noise as raised by Mr. Welliver, a reexamination of our results has confirmed that, over the operating range available on the rigs and engines reported in our paper, the spectra shape can often change, but the total energy contained in the tones remains effectively the same.