Blade Selection for a Modern Axial-Flow Compressor

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In spite of the high-speed digital computer, no theoretical procedure for solving the real, three-dimensional, compressible flow, cascade problem yet appears imminent. However, under the pressure of engineering necessity, a practical, sophisticated combination theoretical and empirical design approach has evolved which utilizes a quasi-three-dimensional calculation procedure (the combination of essentially 2 two-dimensional procedures). The following is a satisfactory list of steps which have led to successful designs.

(1) Selection of the average pressure ratio and temperature rise or efficiency per stage (from past experience or literature)

(2) Definition of the span-wise variation of pressure ratio and loss or efficiency (from experience or literature)

(3) Under the assumption of axial symmetry, solution of the radial component of the equations of motion, yielding the blade row inlet and outlet velocity diagrams

(4) Selection or calculation of the two-dimensional blade sections to match the velocity diagrams determined in (3)

This paper is primarily concerned with (4): the methods used in the past, those presently in use, and some possible innovations which might extend the achievements of this approach to still higher performance levels.

A short summary of the published cascade data most used in this country is given along with some speculation on the type of design approach and blade section design concepts which might provide the next improvements in compressor technology.

The success of an axial-flow compressor design depends upon the proper execution of several steps. Assuming that the secondary flows, loss variations, and boundary layer growths are properly accounted for; that the ultimate loading limits are not exceeded; and that the axisymmetric calculation of blade row inlet and outlet velocity diagrams are correctly performed, the achievement of design point operation depends entirely on the proper selection of blade shapes. However, other factors such as Mach number and proximity of the selected design point to the ultimate loading limits may determine range of operation or off-design performance levels. Range of operation is, of course, important in high pressure ratio, multistage, engine compressors in that it may not be possible to operate the compressor through the midspeed regions to reach the design point without mechanical aids (e.g., bleed and/or variable vanes). Neglecting for the moment these considerations, which constitute more properly a study for off-design performance prediction, the process of blade shape selection is examined.

Three related approaches to cascade selection are theoretically possible.

(1) The purely experimental approach which relies entirely on the use of experimental results from identical cascades to satisfy the velocity diagrams calculated

(2) A purely analytical procedure whereby blade shapes are calculated from the theoretical cascade and viscous flow equations

(3) A semiempirical procedure which utilizes experimental data together with the theoretically derived functional relations to relate the cascade parameters (solidity, angle of attack, stagger angle, cascade turning, airfoil thickness, etc.)

Each approach has its advantages and disadvantages. The purely empirical approach (1) is most limited because it requires huge amounts of data to duplicate any design situation which might arise. However, if the proper data can be located (in literature or from in-house test programs) it provides a nerve-soothing assurance when expensive engine programs are contemplated. The purely theoretical approach (2) is ideal in all aspects except that of implementation. As of now, it is not possible to solve such real flow problems as those posed by today's practical turbomachinery designs. The final and almost universally used procedure (3) provides the optimum combination of assurance and range of application. Further, it allows encroachment into the more complex and comprehensive realm of the purely theoretical. The insertion of experimentally derived coefficients into the intractable theoretical relations provides a design capability which can, with continuing research, approach arbitrarily close to the generality offered by the purely theoretical.

CRITERIA FOR BLADE SECTION SELECTION

In the design of modern flight-type, axial-flow blade rows, size and weight are inevitably important. Assuming that a respectable efficiency could be maintained, it would then appear that only highly loaded, high Mach number sections are of current importance. However, there are many cases, such as the latter stages in multistage compressors, where

Mach numbers are moderate (due to the temperature rise) and incidence angle range requirements dictate moderate dimensionless loading as defined by D-factor and $\Delta P/q$. Moreover, the stator vanes in all current high-performance, flight-type compressors fit this description fairly closely. Consequently, blade sections which can operate with very low losses under these conditions are vitally important. Such sections exist in the empirical cascade domain. The NACA 65-series blower blade sections (or variations of these) have proved near optimum for this application (refs. 1, 2, and 3).

The NACA 65-series cascade data in its original form (ref. 1), in the form of carpet plots (ref. 2), or as replotted by Mellor's charts (ref. 3) constitute the most widely published cascade data available in America. Mellor's presentation, in particular, gives these experimental cascade results in a convenient form for designers.

It should be noted that the referenced NACA cascade data, whatever the presentation form, are two-dimensional data, while the designer is usually faced with a distinctly three-dimensional flow problem. Consequently, precautions should be taken to ensure that the cascade data are applied to sections which have the same effective circulation, considering both changes in radius between the section leading edge and trailing edge and inlet/exit annulus area ratio. Factors to be considered in adapting the two-dimensional cascade data to the three-dimensional compressor problem will be discussed in a later section.

PROCEDURES FOR VARIOUS BLADE SECTION DESIGNS

Depending on the type of cascade selected, the designer has various loosely established procedures for effecting his design. Table I summarizes the Mach number ranges in which the modern aerodynamic designer must work and the generally acceptable cascade types for each range.

Experience has shown that the simple expedient of selecting cascade sections according to these broad overlapping Mach number ranges essentially eliminates the possibility of designs with inherently improper section types.

65-Series Thickness Distribution on Circular Arc Meanline

Sections consisting of circular arc meanlines with 65-series thickness distributions are commonly and successfully used in the low-to-moderate Mach number range ($M \leq 0.78$). For these sections, empirical incidence and deviation angle rules usually replace the strict application of the NACA cascade data.

TABLE I.—*Mach Number Range—Blade Section Type Correlation*

Category number	Design point inlet Mach number	Recommended section types
1	$M \leq 0.78$	NACA 65-series blades, circular or parabolic arc meanlines
2	$0.70 \leq M \leq 1.20$	Double circular arc sections
3	$1.10 \leq M \leq 1.50$	Arbitrary straight leading edge sections, specially designed sections, multiple circular sections
4	$M > 1.50$	Special normal shock-free sections

Figure 1 shows the standard cascade parameters. Blade geometry is usually defined by the blade profile, the stagger or chord angle λ, camber angle φ, and the blade row solidity σ (see table of symbols). Also illustrated are the important aerodynamic parameters such as the cascade inlet velocity W_1, the flow inlet angle β_1', the outlet velocity W_2, the outlet flow angle β_2', and the blade deviation angle $\delta°$.

The blade resulting from the application of the 65-series thickness distribution (see table I, ref. 1) to a circular arc meanline is a hybrid. Consequently, some means of determining the blade setting and blade camber is required. In order to permit some use of the cascade data from reference 1, an equivalence has been established between the NACA ($a=1$) (uniformly loaded) meanline and the circular arc meanline. This process is described in reference 4 and the actual equivalence is presented in figure 2 along with some other useful geometrical relations. It is now possible to obtain reasonable setting angles for the hybrid section from the experimental 65-series cascade data given in references 1 through 3.

For practical compressor blade (or vane) designs using this type section, it may be more prudent to use the store of single stage blade element performance data of the type published in many NACA references; e.g., references 4 through 6. These reports present, for a range of geometrical conditions, experimental blade element data which relate element loss coefficient and deviation angle as functions of blade element incidence angle. These are precisely the parameters which must be set by the designer to implement his design. These data have been used for blade setting with excellent results in spite of the differences in thickness distribution between the hybrid and the biconvex sections. The acceptability of the procedures just described is implied by the results shown in reference 7.

Reference 7 presents a comparison of the performance of the British NGTE circular arc meanline (10C4/30C50) sections with the equivalent

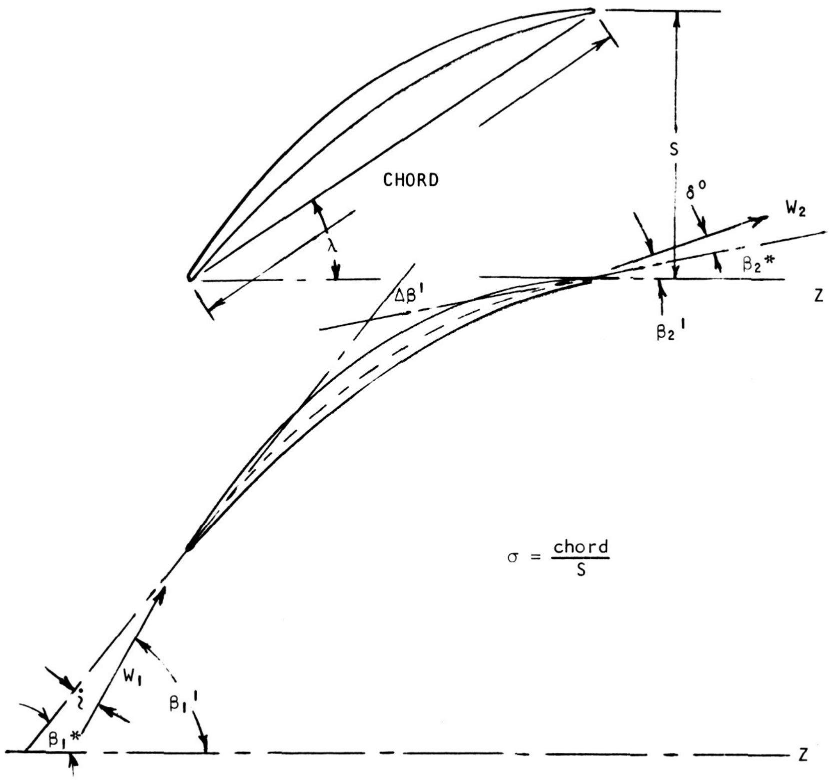


FIGURE 1.—*Standard cascade parameters.*

NACA 65 ($12A_{10}$) 10 section. The relatively small turning angle differences (approximately 3° maximum) between the NGTE and NACA tabulations appear to result from the fact that porous cascade tunnel walls were used for the NACA section while the NGTE used solid walls. Loss variation, surface velocities, and turning angles are shown to be very similar when tested under the same conditions. The surface pressure distributions indicate a slightly higher critical Mach number for the NACA sections.

The deviation angles for the hybrid sections have been shown to correlate well with Carter's deviation angle rule (ref. 8). Carter's rule gives the deviation angle as a function of turning angle, cascade solidity, and a parameter m in the following relation.

$$\delta^\circ = m \frac{\varphi}{\sqrt{\sigma}} \tag{1}$$

The parameter m is given in figure 3 and may be expressed closely as a function of the blade stagger angle λ by the following expression:

$$m = .216 + .000875\lambda + .00002625\lambda^2 \tag{2}$$

where λ is in degrees.

The hybrid section is not recommended for design Mach numbers above 0.78 and hence choking considerations are not too vital in general. However, because the operating conditions in many multistage units may

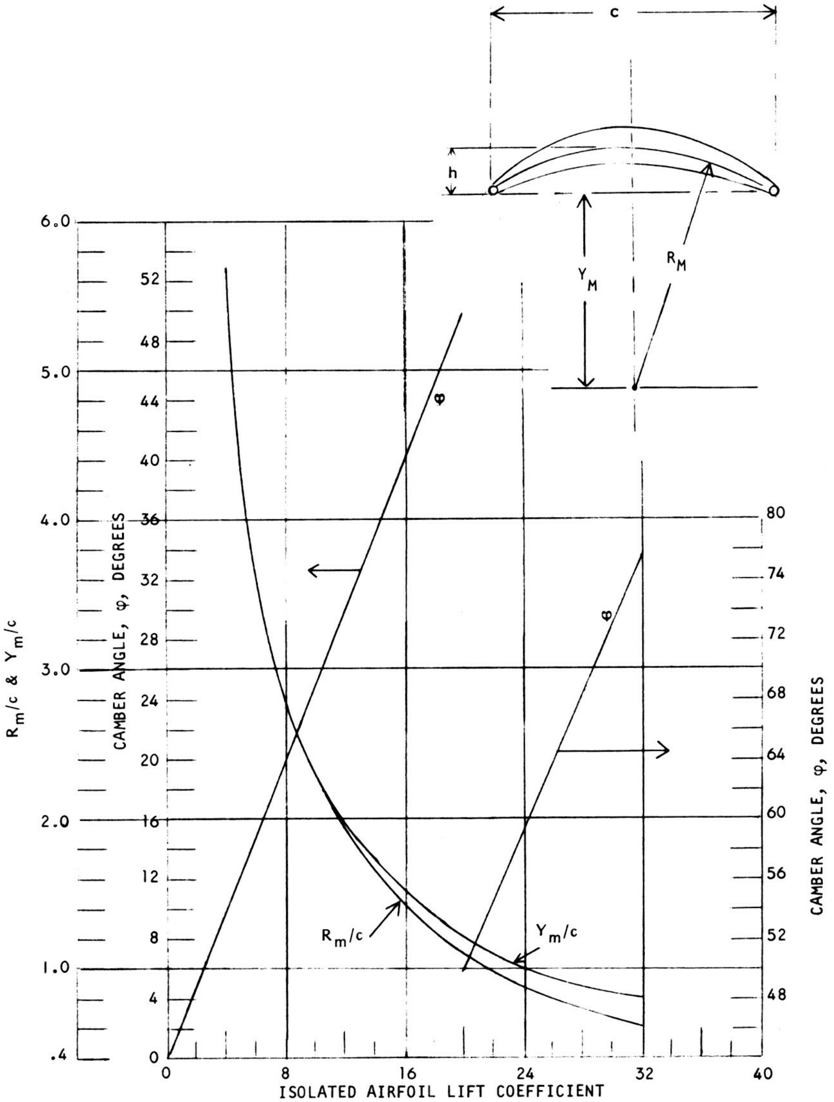


FIGURE 2.—*Equivalence between circular arc meanline camber and the NACA* (a = *1*) *lift coefficient.*

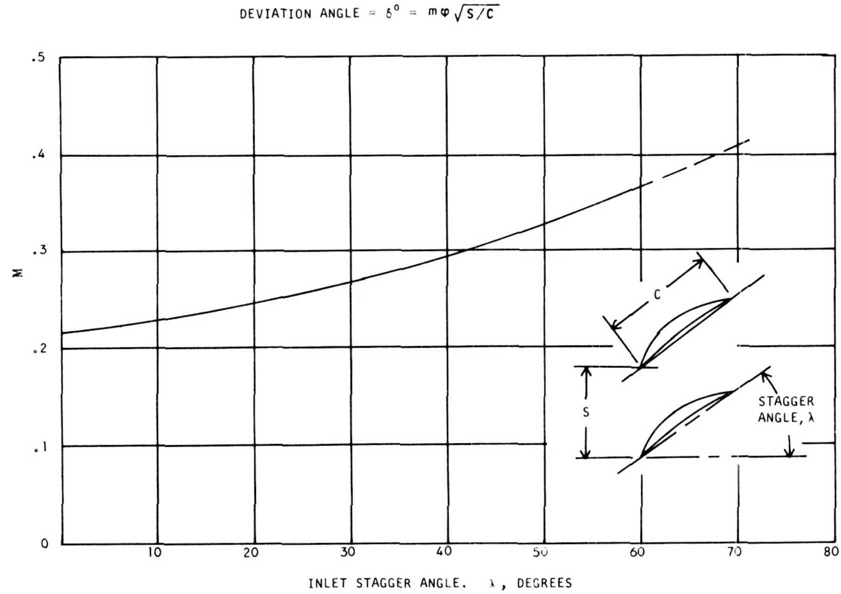


FIGURE 3.—*Carter's rule for deviation angle at optimum incidence for compressor cascades using circular-arc camber lines (ref. 8).*

undergo large swings from design, the critical Mach number (Mach number where the suction surface velocity first reaches unity) and the cascade choking inlet incidence Mach number should be determined.

Standard 65-($a=1$ Meanline) Section

The 65-series blades of reference 1 were derived to be used directly in the design of cascades in category 1, Table I. These data have been used with contemporary success in the design of both single and multistage compressors (refs. 9 and 10).

The cascade summary of either of the 65-series data formulations (refs. 1 through 3) could be used in the design procedure for the standard sections. The cascade data of reference 1, which was obtained by holding the flow angle and varying the cascade angle, has been cross plotted in reference 3 to permit a more convenient section selection procedure. Referring to figure 4, the inlet and outlet flow angles β_1 and β_2 constitute the abscissa and ordinate, respectively. Lines of constant stagger angle λ and angle of attack α are also shown. The inlet and exit flow angles may now be taken from the velocity diagrams resulting from the radial equilibrium calculations and located on a plot of the type of figure 4 for an acceptable solidity and lift coefficient. The criterion for acceptance of these parameter values is a combination of the desired location of the

β_1,β_2 intersection in the experimentally defined useful range between the positive (stalling) angle of attack and the negative (choking) value on the desired solidity figure. The blade sections to match the stagger, solidity, angle of attack, camber and percent thickness may then be defined as shown in reference 1.

The results of reference 9 provide a clear indication that the pure 65-series ($a=1$ meanline) sections may still be appropriately used in category 1 ($M \leq 0.78$). It is primarily for simplicity that the circular arc meanline has largely replaced the original ($a=1$ meanline) calculated for uniform chordwise loading.

It is currently common practice to use the 65-series thickness distribution for stator sections whether ($a=1$) or circular arc meanlines are

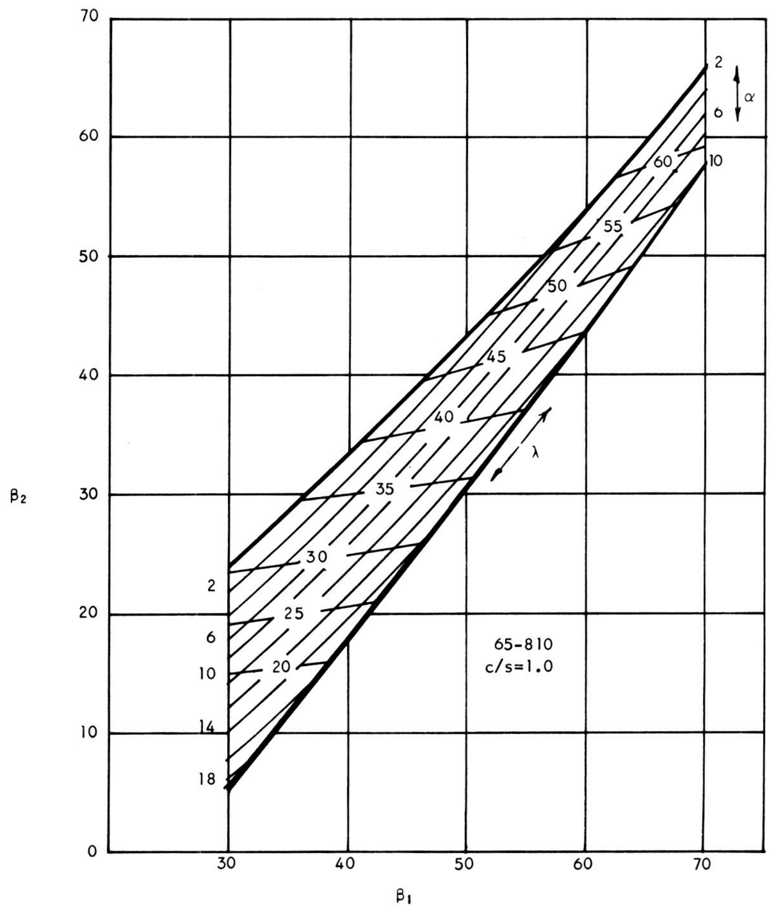


FIGURE 4.—*Typical representation of the Mellor (ref. 3) display of the NACA 65-series cascade data.*

used. There is some indication that sections using a 65-series thickness distribution, particularly in the section nose region can tolerate a wider range of incidence regardless of the meanline to which it is applied. Accordingly, the procedure for defining the leading edge nose radius illustrated in figure 5 has been used for sections which have symmetrical thickness about the meanline. The smaller radius represents the lower limit on the leading edge dimensional tolerance. The radius of the larger circle represents the upper tolerance limit on leading edge diameter. A smooth fairing from the smaller circle to the basic contour at four large diameters downstream of the leading edge completes the blade section. Although the fairing illustrated using the small circle is preferred, the difficulty of maintaining accurate dimensions on these very small leading edge dimensions make anything between and including the two contours illustrated acceptable.

Double Circular Arc and Other Circular Arc Meanline Sections

In the early 1950's, the NACA set out to provide experimental blade element design data for transonic blade sections via single stage compressor tests (ref. 11). These are the sections designated in table I as most satisfactory for category 2. Early NACA transonic, single stage tests (e.g., ref. 12) had demonstrated good performance with double circular arc sections (sections composed of two circular arcs of different radii intersecting at two points). Moreover, these sections were simple, easy to manufacture, and possessed the characteristics which appeared analytically desirable for transonic blades. Hence the NACA program of transonic compressor element design data accumulation utilized these sections.

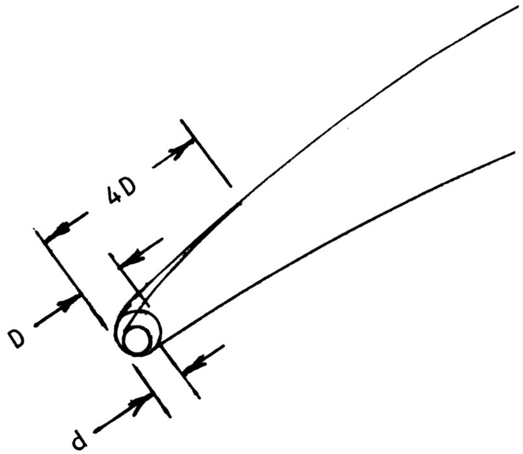


FIGURE 5.—*Application of blade leading edge tolerance.*

In a sense, this program constituted a departure from the original concept of a quasi-three-dimensional design procedure wherein 2 two-dimensional calculations (in the radial-axial plane and the tangential plane) were combined to define the entire flow field. Actually, compressor element performance data, as tabulated by NASA, reflect many phenomena which do not originate in the two-dimensional flow field. Therefore, in an inadvertent way, such effects as secondary flow, boundary layer centrifugation, tip clearance vorticity, wall boundary layer, and annulus convergence are confounded in the single stage element data with the true cascade variations. Under these conditions the true relations between the cascade parameters are not easily isolated. Even so, blade rows of a sufficient range of radius ratios, Mach numbers, stagger angles, camber angles, solidities, and thickness were tested to provide the designer with a suitable model for almost any configuration he contemplated designing. (See refs. 4, 5, 6, 11, 12, 13, and 14.) In addition, several analyses and correlations (refs. 15 through 18) provided aid in interpreting and adapting the cascade data substitutes obtained from the single stage tests to actual design problems. In spite of the obvious confounding of two- and three-dimensional effects, the results of using these data in industrial transonic compressor design must be considered highly successful.

In England, Andrews (ref. 19) investigated the circular arc sections in a 2-dimensional cascade tunnel. These results demonstrated clearly that the basic double circular arc sections could provide highly satisfactory transonic blades. The designer was now able to cross check the single stage elements data (obtained from single stage tests) with that from circular arc sections in 2-dimensional cascades.

The double circular arc has been closely associated with the transonic compressor from its inception. This fact is clearly illustrated by reference 20 in which the highlights of the transonic compressor development are summarized. R. O. Bullock discusses the various NACA investigations designed to illuminate the phenomena involved in transonic compression and make more general the cascade design data obtained.

With the advent of the transonic stages and the single stage element data the presentation format was altered from, for example, the cascade results of reference 1. A typical plot of element data for a single section taken from reference 5 is shown in figure 6. Here the design parameters loss coefficient $\bar{\omega}$, deviation angle $\delta°$, D-factor, blade row inlet relative Mach number M_1', element efficiency η, work coefficient $\Delta H/U_t^2$ and axial velocity ratio C_{z_2}/C_{z_1} are given as functions of the independent variable incidence angle i. Such data are extremely useful in predicting the actual performance to be expected in a real design once certain similarity criteria (e.g., loading, solidity, stagger angle, and Mach number) have been satisfied.

The work of Lieblein (ref. 21) gave a significant tool to the transonic compressor field in its formulation of a loss criterion. Although the correlation of the large store of available loss data against a diffusion parameter D showed a high degree of scatter, a rational and useful basis for loss projection was created.

Chapter VII of reference 22, on blade element flow in annular cascades, provides some basic information on the correction and use of available blade element data on a generalized basis. In particular, it provides losses and curves for conversion of two-dimensional incidence and deviation angle data for use in annular cascades, considering both Mach number and radial position in the annulus of the element under consideration.

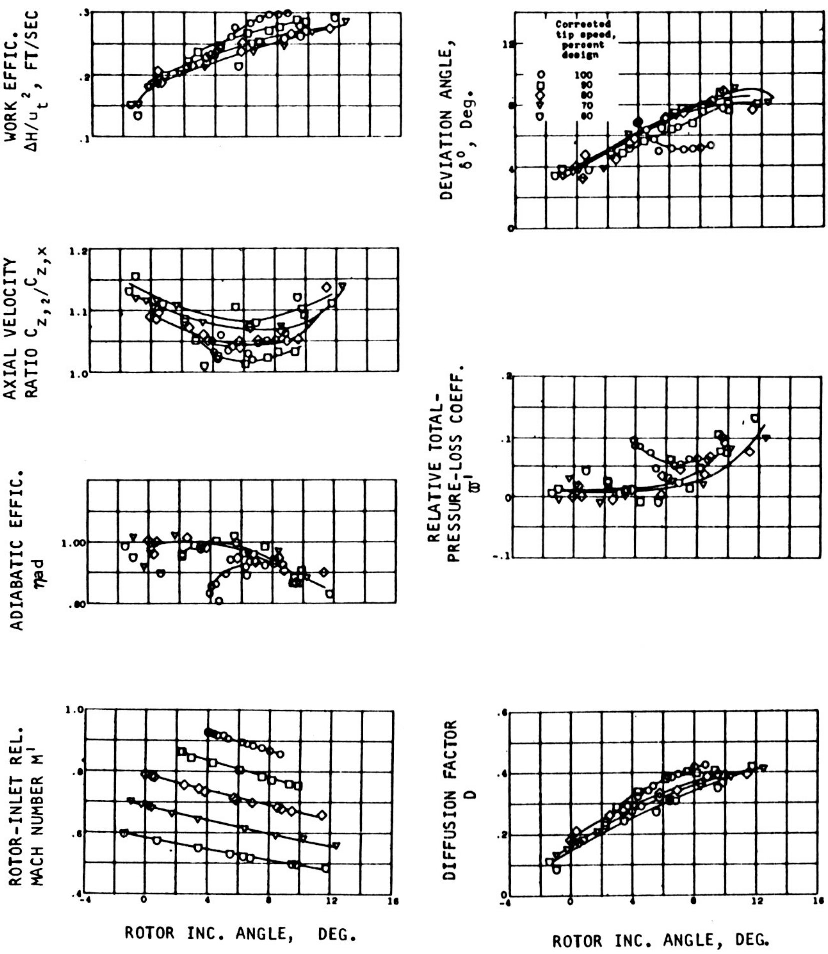


FIGURE 6.—*Typical presentation format for single-stage element data (ref. 5).*

One of the early multistage transonic compressors was the 1954 NACA 5-stage for which design and experimental evaluation are recounted in references 23 through 26. This compressor, which utilized an engine as a source of drive power, had all circular arc meanline sections and each of its five stages operated with transonic inlet relative velocity. A basic objective of this program was demonstration of the practicality of staging high pressure ratio transonic stages. The results from this program provided a large amount of information on the interaction of transonic stages and essentially verified the plausibility of the blade element or annular cascade approach as well as the utility of double circular arc and circular arc meanline sections.

With the velocity diagrams known and a section type selected, the empirical element data such as loss and deviation angle versus incidence angle data just discussed (e.g., ref. 22), permit the straightforward design of compressors in Mach number category 1.

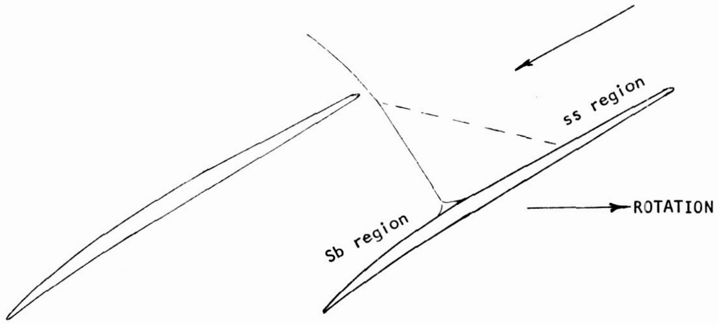
Arbitrary or Multiple Circular Arc Sections

It has been demonstrated that cascade losses go up as Mach number exceeds the section critical Mach number. However, for rotors these losses are largely compensated for by the increased work generated by the higher wheel speed. Hence, for Mach numbers up to approximately 1.10 to 1.20, little, if any, penalty in rotor efficiency is experienced. At blade relative Mach numbers somewhere above 1.1 for the current state of the art, efficiency begins to drop in spite of the increase in useful work with the double circular arc or circular arc meanline sections. Detail study of the passage flow phenomena indicated that the Mach numbers at which the tip passage shocks occurred were high enough to cause, for the first time, nonnegligible shock losses as well as secondary losses resulting from shock separation of the boundary layer.

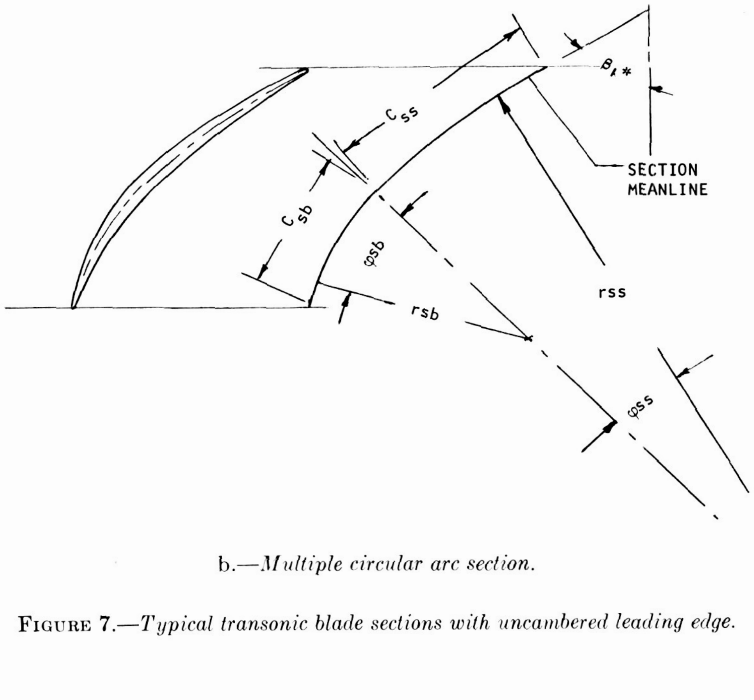
Some experimental studies utilizing high-frequency response probes showed clearly the mechanism of the rotor tip shock structures (Ref. 15). From this understanding evolved a better picture of the type of blade which would minimize these shock losses at a given Mach number and increase the tip speed and hence Mach numbers and pressure ratio before the efficiency would begin to drop. Generally, it became clear that curvature of the suction surface of the blades between the leading edge and the final, strong shock generated Prandtl-Meyer expansions, increasing the preshock inlet Mach numbers appreciably. Apparently, reduction in this curvature or leading edge camber would lower the preshock Mach number and reduce losses. The rotor of reference 27 demonstrated that this did in fact occur. A major effort was made in this rotor to balance the dimensionless static pressure rise between the rotor and stator. However, perhaps a more important feature was the design procedure which yielded a very small blade leading edge camber. The resulting rotor demonstrated a very creditable peak design speed efficiency of 0.89 at a pressure ratio of 2.12 in spite of a design tip relative Mach number of 1.48.

Subsequently, many investigators devised design procedures which guaranteed a blade of zero or near zero camber in the supersonic region of the blade (see fig. 7). The design techniques included the solution of the radial equilibrium equations inside the blade row and "tailoring" the blade to the relative flow direction throughout the blade row (see ref. 28).

The benefits of blade sections with uncambered supersonic regions are now generally accepted and substantial effort has been devoted to



a.—*Arbitrary sections with uncambered leading edge.*



b.—*Multiple circular arc section.*

FIGURE 7.—*Typical transonic blade sections with uncambered leading edge.*

systematizing such sections (see ref. 29). The selected sections of reference 29 are defined by multiple circular arcs. The meanlines of these sections are made up of two circular arcs with mutual tangents at their juncture (see fig. 7*b*). The larger radius arc forms the leading edge (supersonic) portion of the blade while the smaller arc constitutes the rearward (subsonic) portion. It is concluded in reference 29 that the optimum blade achieves the best balance between the supersonic shock losses and the subsonic diffusion losses. The ratio of the leading edge (supersonic region) camber to the total blade camber is called the camber ratio. This ratio provides a parameter to which systematic changes may be made to achieve the optimum loss balance.

Reference 3 reports the experimental performance from four multiple circular arc rotors designed and investigated by General Electric under NASA contract. The results appear significant, varying in such a manner as to verify the loss mechanism hypothesized for transonic blades. It can, therefore, be concluded that the multiple circular arc section shape format does provide a satisfactory cascade base for systematic variation of the geometric and hence aerodynamic properties.

Moderate Supersonic Mach Number Sections

Supersonic compressors have been seriously considered in America since the early 1940's. As conceived at that time, the supersonic rotor would consist of a row of blades comprising a series of rotating supersonic diffusers. The blade relative inlet velocity was supersonic along the entire blade span and a strong shock was stabilized in each passage (refs. 31 and 32). That this somewhat idealized concept did not yield an immediately practical engine compressor is probably primarily due to the shock boundary layer interaction, the radially nonconstant angles and velocity at the rotor inlet, and the difficulty in satisfying simultaneously radial equilibrium and the shock equations as well as other real flow effects.

Most blade sections were developed using a two-dimensional method of characteristics which did not rely at all on cascade data. The principal outputs to date from the U.S. supersonic compressor programs have been, first, the feasibility demonstration, and, second, provision of detail insight into the flow phenomena associated with transonic compressors.

Today there is a renewed interest in supersonic compressors for various applications. However, the concepts are more mature and the requirements more demanding. Considerable interest is focussed on the potential of rotors which utilize high tip speeds and weak oblique shocks which do not diffuse through Mach 1.0 in the tip regions where shock losses could be large. Possible advantages of this type rotor include the possibility of high pressure ratio, uniformly loaded rotors with low radius ratios, and good efficiency along the entire blade span. Also, current high bypass ratio fan engines are usually restricted to low rpm by fan tip speed while the smaller diameter fan turbine has many stages to compensate for its low tip speed.

The type of blade sections required for this category 4 type of supersonic compressor are not yet generally defined. These sections will probably require a more intimate mingling of the two- and three-dimensional flow calculations than previous compressor types utilized. In order to avoid the strong shocks at the design point, the passage through flow areas must be carefully controlled, and the old procedures for section layouts (probably not on constant radius surfaces) utilizing a supersonic method of characteristics will probably be required. As always, the stream surfaces will reflect the radial equilibrium equations.

OTHER CASCADE CONSIDERATIONS

In the application of either two-dimensional design rules or cascade data to the design of real compressor blades, most often the stream surface intersects the blade leading edge at one radius and intersects the trailing edge at another. As previously stated, blade sections defined on such surfaces should be related to true two-dimensional sections on the basis of equivalent circulation. The following expressions define $\beta'_{2,e}$ and $\beta_{2,e}$ the equivalent (two-dimensional cascade) trailing edge angles for a rotating and a stationary blade section, respectively.

$$\left.\begin{aligned} \tan\beta'_{2,e} &= \frac{r_2 C_{z_2}}{r_1 C_{z_1}} \tan\beta_2 + \frac{U_1}{C_{z_1}}\left[1-\left(\frac{r_2}{r_1}\right)^2\right] \\ \tan\beta_{2,e} &= \frac{r_2 C_{z_2}}{r_1 C_{z_1}} \tan\beta_2 \end{aligned}\right\} \tag{3}$$

These angles represent the three-dimensional blade (or vane) discharge angles which would give the same circulation as the actual section along the actual stream surface but with constant radius and axial velocity. The use of these discharge angles permits the direct application of two-dimensionally derived incidence or deviation angles or cascade data.

The angles β and β' are defined as $\tan^{-1} C_u/C_z$ and $\tan^{-1} W_u/C_z$, respectively, and neglect the radial components of velocity. It therefore became necessary to make geometrical corrections to the blade angles β when sections are defined on cylindrical surfaces and the blade elements are neither radial nor normal to the axis (see Appendix B of ref. 36), and radial velocity components exist.

The final expression for the corrected blade angle on the cylindrical surface is given as

$$\tan \beta(\text{corrected}) = \frac{\tan \varphi \tan \epsilon + \dfrac{\tan \beta}{\cos \varphi}}{1 - \tan L \tan \epsilon} \tag{4}$$

In the case where the element is normal to the axis (sweep angle $L=0$) the considerably simpler expression which follows may be derived for the corrected β.

$$\tan \beta(\text{corrected}) = \tan \beta - \tan \varphi \tan \epsilon \tag{5}$$

It can be seen that for large hub slopes and lean angles a substantial correction is required for the blade tangent angle on the cylindrical surface. This complication may be eliminated by laying the blade out on a stream surface. The stream surface can usually be assumed to be a conical surface with little error. A procedure and the computer program for defining double and multiple circular arc blades on conical surfaces are given in reference 33. The procedure is designed to preserve the rate of change of surface angle with distance on the specified conical surface. From these calculated sections, new sections outlined by the intersections of plane surfaces with the blade which results from the conical surface profiles are defined. The area, center of area, and moments of inertia are computed for use in the blade stress analysis. The program also can be made to give any degree of tilt (tangential lean) desired for stress reasons.

Once the blade sections are completely defined and stacked, the section layouts together with the stream surface traces in the radial-axial plane permit the determination of the cascade throat areas. With these data the blade choking incidence angles may be determined. It is important to compare the section choking incidences with the selected design values and ascertain that some margin (e.g., 2° to 6°, depending on the design) exists between choke and design.

Some other miscellaneous considerations include the importance of blade surface finish and trailing edge thickness. These topics are covered in references 34 and 35. Indications are that for a trailing edge thickness up to 30 percent of the maximum thickness, no increase in loss with thickness has been observed for compressor cascades. The case for surface finish is less conclusive. There have been indications from time to time that, under some conditions of low Reynolds number and/or high Mach number operation, some surface roughness is beneficial. The conclusions in reference 35 are that nothing is gained by using finishes better than 40 microinches root-mean-square for conventional compressors.

Multistage Example

The range of section types used in current jet engine turbomachinery is illustrated by a typical multistage mix in figure 8. A high performance,

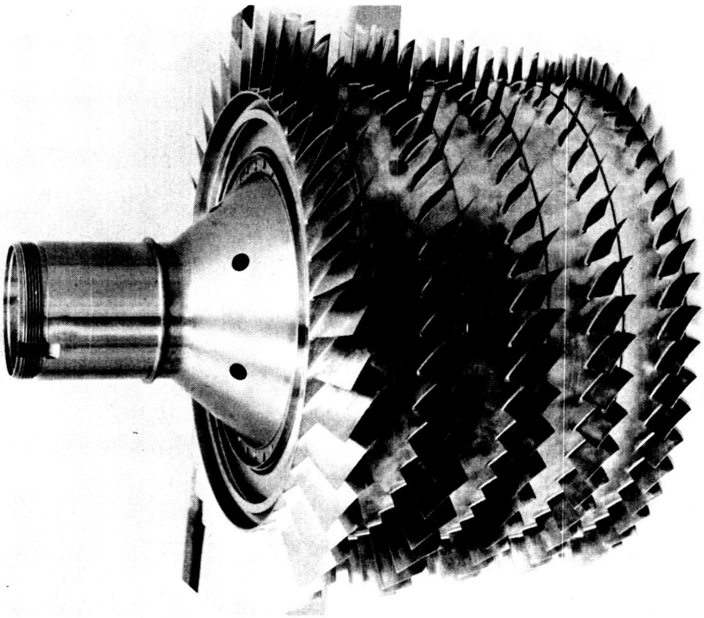


FIGURE 8a.—*High-performance five-stage compressor rotor.* (*AiResearch Manufacturing Division, Garrett Corporation*)

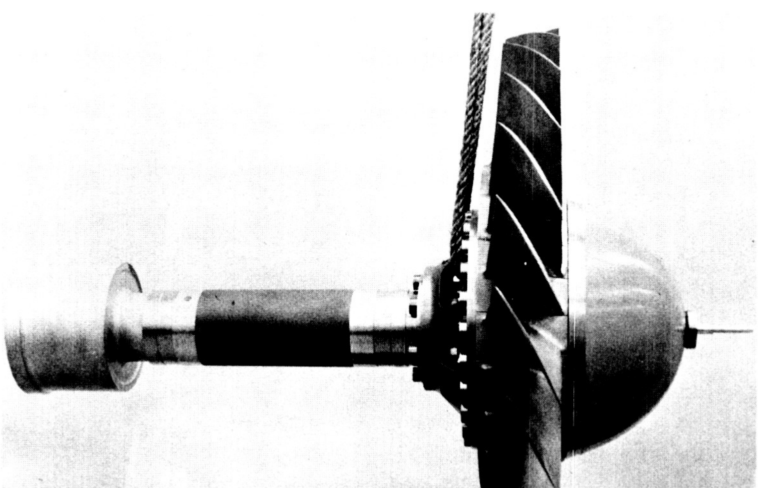


FIGURE 8b.—*Single-stage fan.* (*AiResearch Manufacturing Division, Garrett Corporation*)

5-stage compressor rotor is shown in figure 8*a*. Rotor 1 has special or arbitrary sections; all stators are circular arc meanlines with 65-series thicknesses. Rotors 2 through 5 utilize double circular arc sections. The single stage fan shown in figure 8*b* gave efficiencies in the high eighties at a design speed pressure ratio of 1.6. The sections varied from the arbitrary, uncambered leading edge region at the tip to a near circular arc meanline hub.

CONCLUDING REMARKS

The preceding remarks constitute a highly condensed and heavily referenced summary of the methods of cascade selection and design available today to the practical compressor aerodynamicist. Throughout the entire field, the emphasis has been, and still is, on the semiempirical approach. For the designer who dedicates himself to understanding the fundamental phenomena involved in cascade flow, the methods described can be used quite successfully. Beyond this, possibilities for future improvement in cascade performance appear abundant, and it is largely from this source that large gains in future turbine engines will probably come.

LIST OF SYMBOLS

C	Absolute velocity, ft/sec
c	Blade chord
D	Dimensionless loading parameter, $\left(1-\frac{W_2}{W_1}\right)+\frac{R_2C_{u2}-R_1C_{u1}}{2R\sigma W_1}$
ΔH	Enthalpy rise, ft-lb/slug
h	Blade camber
i	Incidence angle, $\beta_1'-\beta_1^*$, degrees (see fig. 1)
L	Blade sweep angle in plane of blade leading edge, degrees
M	Mach number
$\Delta P/q$	Dimensionless static pressure rise coefficient
R_m	Blade mean camber-line radius, inches
R_m/c	Blade mean camber-line radius normalized by blade chord
U	Wheel speed, ft/sec
W	Velocity relative to rotor, ft/sec
Y_m	Distance from center of meanline circle to chord
Y_m/c	Distance from center of meanline circle to chord normalized by blade chord
α	Angle between the flow direction and the blade chord, degrees

β	Angle measured from axial, $\tan \frac{C_u}{C_z}$, degrees
$\delta°$	Deviation angle $\beta_2' - \beta_2^*$, degrees (see figure 1)
ϵ	Blade lean angle, degrees
σ	Solidity, blade chord/blade spacing
φ	Camber angle, $\beta_2^* - \beta_1^*$, degrees
$\bar{\omega}$	Total pressure loss coefficient

Superscripts

*	Geometrical
′	Relative to the rotor

Subscripts

e	Equivalent
m	Mean
sb	Subsonic
ss	Supersonic
t	Tip
u	Tangential component
z	Axial component
1	Blade row inlet
2	Blade row outlet

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DISCUSSION

L. H. KING AND A. C. BRYANS (General Electric Co): The author has presented a comprehensive summary of various design approaches used to select blading for axial-flow compressors. This paper will be a valuable reference for the compressor designer.

The author offered one practical comment on 65-series thickness distribution blading that should be qualified. The statement that the leading edge shape cannot be obtained consistently in manufacturing and therefore would justify allowing a tolerance band is valid. However, the magnitude of the variation between the lower and upper tolerance limit must be specified and then types of acceptable nose profiles defined within these limits. Since the paper does not deal with the problem of blading tolerance limits, it seems inappropriate to introduce this unqualified comment.

The writers' experience with small axial compressor blading has shown that tolerance specification and manufacturing control are important aspects of the designer's role in obtaining the desired performance levels.

The tip region of transonic rotors still requires a great deal of study before we can adequately determine the flow field. The present paper makes reference to the procedure of designing at axial stations within the blade row. L. H. Smith's paper (ref. D–1) develops rigorously such a procedure, dealing therein with the effects arising from blade-to-blade variations which do not exist in a true axisymmetric field. Dr. Smith indicates that these perturbation effects are analogous to Reynolds stresses in turbulence theory. We are concerned that current practice still continues to use a linear distribution to represent the blade-to-blade variation in the force field when we know that shocks must result in some radical departure from this idealized situation. The normal concept that a blade row is equivalent to a force field and this force field, in turn, is equivalent to a vorticity distribution requires that the correct vorticity be used to define the blade. We would suggest some research into the significance of the rate of change of vorticity as it affects the flow field in the supersonic regime. We would further suggest that the mathematical model used to represent the shock structure in this region be checked for radial equilibrium.

D. M. SANDERCOCK (NASA Lewis Research Center): The author has presented a very useful summary of blade sections currently used in blade designs and their general areas of applicability.

There is one additional point of interest regarding the multiple circular arc blade shape. The use of low turning in the forward portion of the blade section does indeed tend to lower the shock inlet Mach number and presumably the shock losses. However, the amount of fluid turning upstream of the shock also affects the throat area of the blade passage, which must be large enough to pass the required mass flow. Thus, the throat area requirement can place some restrictions on the amount of turning that can be assigned to the inlet portion of a blade section. Also, there is a line of reasoning that indicates that close control of the throat area along the blade span may have a more significant effect on overall blade losses and stable operating range than that realized from minimizing blade passage shock loss. Mr. Benser, in his paper "Transonic Compressor Technology Advancements," discusses the effects of throat area observed from experimental data from transonic blade rows over a range of blade speeds.

For the cases of high solidity stator blade sections, added turning is sometimes introduced in the forward portion of the blade section in order to attain required throat areas to pass the mass flow.

R. M. HEARSEY (Ohio State University Research Foundation): Have you considered selecting cascade solidity by means of the total pressure loss parameter/diffusion factor relationship presented in Figure 203 of NASA SP 36?

WRIGHT (author): It is true, as Mr. Sandercock states, that the leading edge camber of transonic, multiple-circular-arc sections has a pronounced effect on blade throat area. Of course, so do the blade thickness and the convergence of the annular stream tubes. It would be interesting to compare the performance of blades designed for the same inlet velocity diagram and with the same throat area obtained via different combinations of the three variables: camber, thickness, and annular convergence. It is my unverified opinion that the best performance (and the maximum choking flow) will result for the configuration yielding the most uniform throat flow. I don't believe that this would be the blade with the more cambered leading edge because its integrated ρV would include some supersonic suction surface values and some subsonic pressure surface values and the variations between.

It is my opinion, further, that a straight leading edge blade with a higher design incidence will also yield a more uniform throat than a curved inlet blade and hence a lower suction surface Mach number at the normal shock for peak efficiency operation. The existence of a stronger expansion around the suction surface of the leading edge following the sonic velocity point, which in turn follows the leading edge stagnation point, is believed to be preferable to the later suction surface expansions of the curved section for the same reason (throat uniformity). Hence, until further proof is offered, I believe that the throat area requirements must be satisfied, but with minimum blade suction surface curvature.

On the leading edge tolerance discussed by Messrs. Bryans and King, I simply offer this for what it is worth to the reader. We at AiResearch have noticed what I take to be an advantage, particularly with respect to stalling incidence, of the asymmetric leading edge contour. And, inasmuch as some manufacturing tolerances are required, this approach offers the potential of converting a problem area into an advantage.

With regard to the rotor internal calculations, it is well known that the force field created by the blade sections is not rigorously represented by linear cross channel gradients. Further, the rigorous calculation of the rotor internal flow configurations requires complete departure from the quasi-three-dimensional procedures (a radial-axial plane flow based on axial symmetry plus the 2-*D* blade flow) which have traditionally, and perhaps surprisingly, yielded satisfactory performance and reasonable checks of rotor outlet distributions for moderate supersonic Mach number levels. Even neglecting the formation of the secondary flow vortices which in real life are extremely difficult to avoid, the stream surfaces between the hub and tip shrouds will not be axisymmetric surfaces. A qualitative approach to minimizing these perturbations from axial symmetry would be the use of very high solidity. The rationale would be similar to that by which the assumption of infinite solidity was used in early cascade theory in order to drop derivatives in the tangential direction.

I agree with Mr. King and Mr. Bryans that some research aimed at a closer approach to the rigorous representation of the supersonic flow field in a high Mach number rotor would be a worthwhile undertaking. The most practical fallout from this research would probably be the derivation of some analytical expressions for the radial accelerations caused by the shocks and the resulting warped, equilibrium shock.

Relating to Mr. Hearsey's question, and as commented by Mr. Benser, my experience is that the NASA SP36 figure 203 gives solidities somewhat lower than the optimum for transonic blades at optimum incidence. We have used this approach on stators in particular where the inlet Mach numbers are low.

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