Prediction of Pump Cavitation Performance

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A method for predicting pump cavitation performance with various liquids, liquid temperatures, and rotative speeds is presented. Use of the method requires that two sets of test data be available for the pump of interest. Good agreement between predicted and experimental results of cavitation performance was obtained for several pumps operated in liquids which exhibit a wide range of properties. Two cavitation parameters which qualitatively evaluate pump cavitation performance are also presented.

Cavitation in turbopumps is usually undesirable because it can produce vibrations, flow instabilities, and blade structural damage, as well as losses in head rise. However, in some cases, it is advantageous to operate with cavitation present. For example, the overall structural weight of rocket vehicles can generally be reduced if the turbopump is operated with a controlled degree of cavitation. This type of operation can permit greater pump speeds or lower propellant tank pressures; either can reduce vehicle weight, thereby allowing increased payload. These size and weight advantages offered by increased rotative speed are not limited to space applications.

Rocket engine pumps generally employ an inducer ahead of the main pump rotor. The inducer, by virtue of its design, can operate satisfactorily with a considerable amount of cavitation present on the suction surface of the blades. Because cavitation is a vaporization process which involves heat and mass transfer, the properties of the pumped liquid and its vapor and the flow conditions can affect the cavitation performance of the inducer. These combined effects of fluid properties, flow conditions, and heat transfer—termed thermodynamic effects of cavitation—can improve cavitation performance. For example, cavitation studies have shown that for certain liquids and liquid temperatures the net positive suction head (NPSH) requirements can be significantly less than that obtained in room temperature water (refs. 1 to 3). The accurate prediction of the thermo-

dynamic effects of cavitation and its relation to the NPSH requirements is therefore essential to an optimum design of a cavitating flow system.

This paper presents a method for predicting the cavitation performance of pumps and inducers. The method presented is based on, and is an extension of, the method discussed in references 4 and 5 for Venturis. Two parameters which qualitatively evaluate the cavitation performance are also presented.

ANALYSIS

The method for predicting pump cavitation performance presented here is detailed in reference 6. The method is based on Venturi cavitation results (refs. 5, 7, and 8). Use of the prediction method requires that two sets of appropriate test data be available for the pump and operating conditions of interest.

Assumptions

A prerequisite to the prediction method developed for Venturis is that similarity of the cavitating flow be maintained for all conditions to which the method is applied. For a given inducer design, similarity of cavitating flow is assumed to exist under the following conditions:

(1)   Constant flow coefficient. The flow is entering the blades at a constant incidence angle.

(2)   Constant head-rise coefficient ratio. The cavity on the suction surface of the blades and thus the vapor volume corresponding to a specific cavitating-to-noncavitating head-rise coefficient ratio is essentially constant irrespective of liquid, liquid temperature, or rotative speed.

(3)   The same or geometrically scaled inducers. The inducer should have similar velocity and pressure distribution along the blade surfaces.

Thermodynamic Effects of Cavitation

The differences in pump cavitating performance are generally attributed to the thermodynamic effects of cavitation which reflect the varying degrees of evaporative cooling that accompany vaporization of the particular liquid being pumped. In situations where evaporative cooling is significant, the cavity pressure and the vapor pressure of the liquid adjacent to the cavity are reduced by an amount corresponding to the reduced local temperature. This reduction in cavity pressure retards the rate of further vapor formation, thereby allowing satisfactory operation of the inducer at lower values of net positive suction head than would otherwise be possible.

The magnitude of thermodynamic effects of cavitation Δh_v can be estimated by setting up a heat balance between the heat required for

vaporization and the heat drawn from the liquid surrounding the cavity. The heat balance has been derived in references 4, 6, and 7. Thus, only the resulting equation is presented. All symbols are defined at the end of this paper.

$$\Delta h_v = \left(\frac{\rho_v}{\rho_l}\frac{L}{C_l}\frac{dh_v}{dT}\right)\left(\frac{\mho_v}{\mho_l}\right) \tag{1}$$

The vapor-to-liquid volume ratio $(\mho_v/\mho_l)$ in any real flow situation is not known nor can it be measured directly. However, from the Venturi cavitation studies (refs. 4 and 5), the following equation, which estimates the volume ratio relative to a reference value, was developed.

$$\left(\frac{\mho_v}{\mho_l}\right)=\left(\frac{\mho_v}{\mho_l}\right)_{ref}\left(\frac{\alpha_{ref}}{\alpha}\right)\left(\frac{V}{V_{ref}}\right)^{0.8} \tag{2}$$

For the experimental case where a value of Δh_v is known, the corresponding reference value of $\mho_v/\mho_l$ can be determined from equation (1). Then values of $\mho_v/\mho_l$ for other liquids, liquid temperatures, and flow velocities are estimated by equation (2). With the predicted value of $\mho_v/\mho_l$, determination of the corresponding value of Δh_v is possible using equation (1).

Similarity Relations

For the Venturi studies, a cavitation parameter $K_{c,min}$ was shown to be a constant for similarity of cavitating flow. In terms of pump parameters, the cavitation parameter can be expressed as

$$K_{c,min}+1=\frac{\text{NPSH}+\Delta h_v}{(V^2/2g)} \tag{3}$$

Because $K_{c,min}$ is constant for geometrically similar cavities in a given flow device, it follows that

$$\frac{\text{NPSH}+\Delta h_v}{\text{NPSH}_{ref}+(\Delta h_v)_{ref}}=\left(\frac{V}{V_{ref}}\right)^2 \tag{4}$$

where the subscript ref denotes a reference value which must be determined experimentally. Equation (4) may be used to predict the required NPSH for a particular pump operated at a fixed flow coefficient and head-rise coefficient ratio, but with changes in liquid, liquid temperature, and rotative speed.

For pumps, direct measurement of $(\Delta h_v)_{ref}$ needed for use in equation (4) is not feasible. However, it is possible to estimate Δh_v without measuring cavity pressure directly. This requires that two sets of experimental data be available at the flow coefficient and head-rise coefficient ratio of

interest. Although these experimental data do not have to be for the same liquid, liquid temperature, or flow velocity, one set of data must yield measurable thermodynamic effects of cavitation. Measured values of NPSH and V from one of the test conditions are arbitrarily chosen as the reference value. Since both Δh_v and $(\Delta h_v)_{ref}$ are unknown, an iterative solution of equations (1), (2), and (4) is required to determine $(\Delta h_v)_{ref}$. With the value of $(\Delta h_v)_{ref}$ once determined, the required NPSH can be predicted for other liquids, liquid temperatures, and flow velocities.

RESULTS AND DISCUSSION

The prediction method has been applied to several axial-flow inducers and centrifugal pump impellers. These rotors were operated in several different liquids at various liquid temperatures. The rotors were tested at various rotative speeds and flow coefficients.

Liquid Temperature Effects

The cavitation performance for an 84° research inducer (fig. 1) is presented in figure 2. The data are for liquid hydrogen at temperatures from 27.5° R to 36.6° R (15.3 K to 20.3 K) and for a constant rotative speed of 20 000 rpm (ref. 3). The flow coefficient is also constant. The data are plotted in terms of head-rise coefficient ratio ψ/ψ_{NC} as a function of NPSH. For a given value of ψ/ψ_{NC}, the required NPSH decreased rapidly with increasing liquid temperature. The performance results obtained at 27.5° R and 31.7° R (15.3 K and 17.6 K) were arbitrarily chosen as the reference data to predict the NPSH requirements for
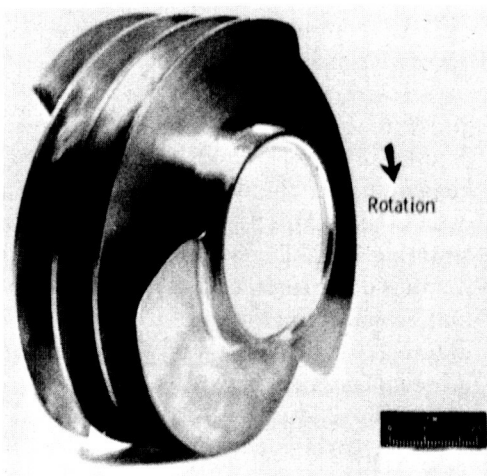

FIGURE 1.—*84° flat-plate helical inducer. Tip diameter, 4.986 inches (12.66 cm); hub diameter, 2.478 inches (6.29 cm); number of blades, 3.*
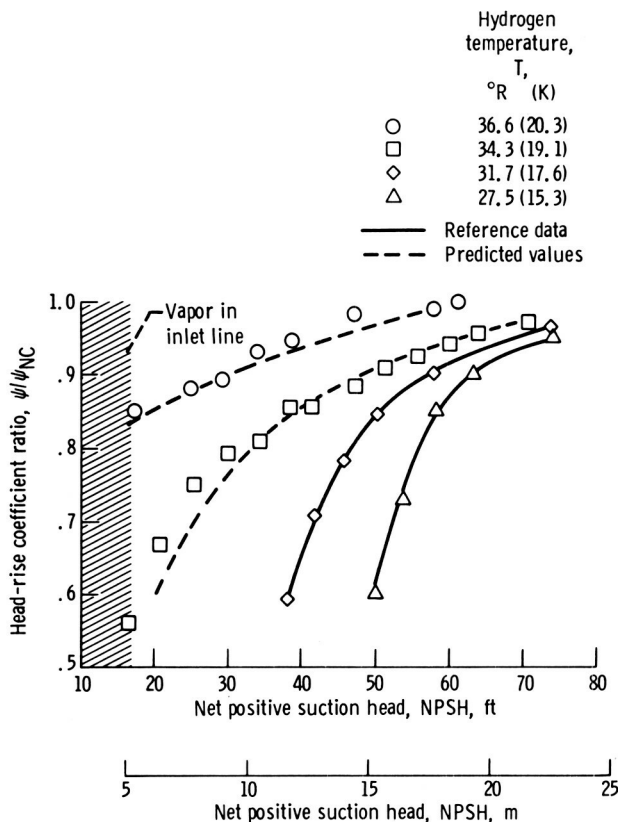
FIGURE 2.—*Effect of liquid temperature on cavitation performance of 84° inducer. Rotative speed, 20 000 rpm.*

hydrogen at 34.3° R and 36.6° R (19.1 K and 20.3 K). The predicted NPSH requirements (dashed lines of fig. 2) agree reasonably well with the experimental results.

The static pressure in the inlet line of a pump is less than the inlet total pressure as a result of the flow velocity. However, the inlet total pressure is used in defining NPSH. Thus, at some relatively low value of NPSH, depending on the design and mode of operation, a condition is reached where the static pressure at the pump inlet equals the pressure of the entering fluid. The fluid in the inlet line is therefore in a boiling state. Any further decrease in NPSH will cause more vapor to be generated; and this vapor will be ingested by the pump. For the inducer performance data of figure 2, this condition of boiling in the inlet line occurs at an NPSH of about 17 feet (5.2 m), as noted by the vertical line. The data for 36.6° R (20.3 K) show that the test inducer was capable of pumping boiling hydrogen while it maintained 0.85 of the noncavitating head rise.

The present prediction method applies to two-phase flow (cavitation) within the blade passages of a pump or inducer. However, when a boiling liquid is pumped, two-phase flow also exists in the inlet line ahead of the pump (shaded area of fig. 2, for example). The condition of a vapor-liquid mixture in the inlet line presents an added complication to the prediction of cavitation performance; the present prediction method does not apply to this condition.

The magnitude of the thermodynamic effects of cavitation realized for this inducer at a rotative speed of 20 000 rpm and ψ/ψ_{NC} of 0.7 is shown in figure 3. The Δh_v values increased rapidly with increasing temperature.

Liquid Effects

The measured and predicted NPSH requirements for three small commercial pumps that were operated in water, methyl alcohol, butane, and Freon-11 at various temperatures are compared in table I. The measured NPSH values listed are taken from previously reported results (refs. 1 and 2). All values are for a cavitating-to-noncavitating head rise of 0.97 and for a speed of about 3550 rpm. The two reference values of NPSH listed for each pump (denoted by a) were arbitrarily chosen as the
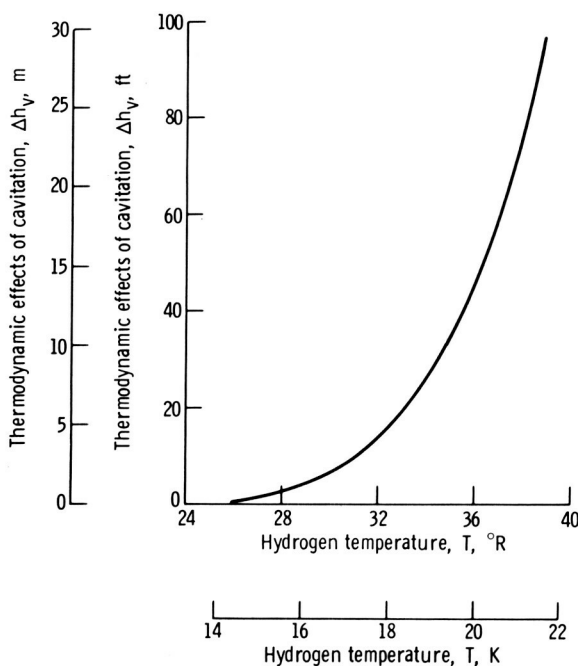

FIGURE 3.—*Thermodynamic effects of cavitation as function of hydrogen temperature for 84° inducer. Rotative speed, 20 000 rpm; head-rise coefficient ratio, 0.70.*

reference test data for use in the prediction method. The predicted NPSH values are in good agreement with experimental results. In fact, except for the 756° R (420 K) water data for pump 1 the predicted values are within the experimental accuracy of measured NPSH values. For pump 3 operated in 870° R (483 K) water, the predicted NPSH value is zero, which from previous discussion implies that this particular pump should have been capable of satisfactory operation (3-percent loss) at an NPSH value sufficiently low to cause vapor to form in the inlet line. As stated in reference 1, there were indications that flashing in the inlet line did occur at rated flow; consequently, the pump was operated at a less-than-design flow rate and the resulting measured NPSH was extrapolated to design conditions. This last set of data further illustrates that, because of the thermodynamic effects associated with cavitation, it may have been possible to pump this liquid under boiling conditions satisfactorily.

The magnitudes of the thermodynamic effects of cavitation realized with these various commercial pumps are also listed in table I. For each liquid, the Δh_v values increase with increasing temperature. It should also be noted that the Δh_v values for the same temperature liquid vary with the various pumps.

Rotative Speed Effects

Liquid hydrogen cavitation data were also obtained for a small research centrifugal pump impeller (fig. 4) operated at various rotative speeds (ref. 6). The experimental and predicted cavitation performance are compared in figure 5. The data were obtained for four rotative speeds at a constant temperature and flow coefficient. The performance curves
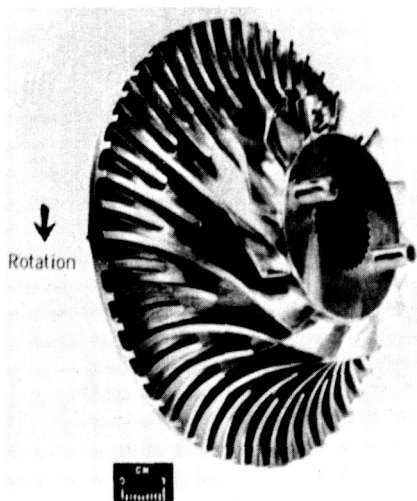

Rotation

FIGURE 4.—*Centrifugal pump impeller. Overall diameter, 4.56 inches (11.58 cm); number of blades, 12; number of splitter vanes, 36; inlet tip blade angle, 71.3° (from axial direction).*

TABLE I.—*Effect of Liquid and Liquid Temperature on Pump Cavitation Performance*

[Nominal rotative speed, 3550 rpm; head-rise coefficient ratio, 0.97.]

Pump	Liquid	Temperature, T		Net positive suction head, NPSH				Thermodynamic effects of cavitation, Δh_v	
				ft		m			
		°R	K	Measured (±0.5 ft)	Predicted	Measured (±0.2 m)	Predicted	ft	m
I (ref. 2)	Water	534	297	17.5[1]	----	5.3[1]	----	0	0
		600	333	17.5	17.5	5.3	5.3	0	0
		670	372	17.5	17.4	5.3	5.3	0.1	0
		710	394	17.0[1]	----	5.2[1]	----	0.5	0.1
		735	408	16.0	16.5	4.9	5.0	1.0	0.3
		756	420	13.3	15.6	4.1	4.8	1.9	0.5
	Methyl alcohol	560	311	17.5	17.4	5.3	5.3	0.1	0
		611	339	16.7	16.8	5.1	5.1	0.7	0.2
		641	356	15.5	15.3	4.7	4.7	2.2	0.6
		663	368	13.5	13.3	4.1	4.1	4.2	1.2

II (ref. 1)	Water	530	294	12.3	11.8	3.7	3.7	0	0
		710	394	11.0¹	--	3.4¹	--	0.8	0.3
		760	422	8.6	8.7	2.6	2.7	3.1	1.1
	Butane	495	275	9.8	10.2	3.0	3.1	1.6	0.5
		515	286	8.8¹	--	2.7¹	--	3.0	0.9
		540	300	5.1	5.6	1.6	1.7	6.2	1.9
		550	306	3.5	3.4	1.1	1.0	8.4	2.6
	Freon-11	545	303	10.2	10.5	3.1	3.2	1.3	0.4
		580	322	8.4	8.1	2.6	3.7	3.7	1.1
III (ref. 1)	Water	530	294	12.0¹	--	3.7¹	--	0	0
		750	417	9.5¹	--	2.9¹	--	2.5	0.8
		785	436	6.0	6.0	1.8	1.8	6.0	1.9
		870	483	2.0²	0	0.6	0	12.0	3.7

¹ Reference test data.
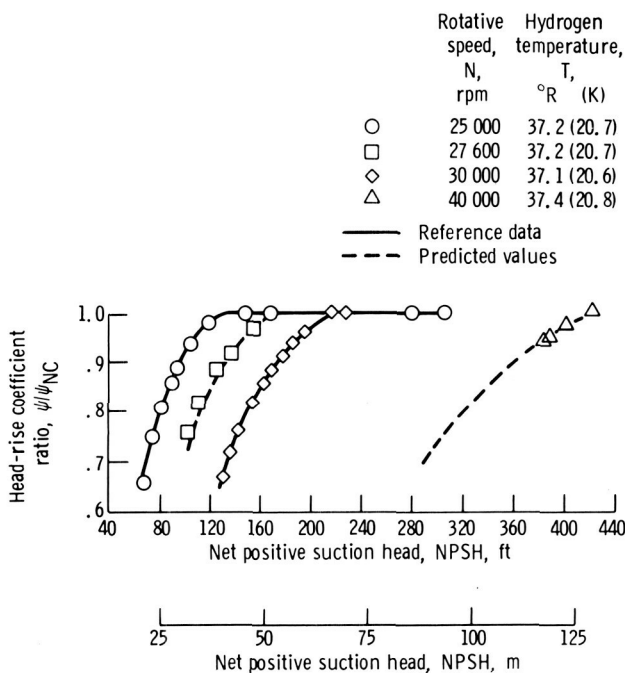² Estimated accuracy of 2.0 ft (0.6 m).

FIGURE 5.—*Effect of rotative speed on cavitation performance of centrifugal pump impeller.*

obtained at 25 000 rpm and 30 000 rpm (solid curves) were used as reference test data to predict the NPSH requirements for 27 600 rpm and 40 000 rpm (dashed lines). The predicted and experimental results are in good agreement. The predicted magnitude of the thermodynamic effects of cavitation for this pump operated in 37.2° R (20.7 K) liquid hydrogen is shown in figure 6. The Δh_v values increase with increasing rotative speed. For the data of figure 5, the pump NPSH was sufficiently large that the condition of boiling in the inlet line was not encountered.

Flow Coefficient Effects

The prediction method was also used to predict the cavitation performance of a 80.6° inducer (fig. 7) operated in liquid hydrogen at a rotative speed of 30 000 rpm (refs. 9 to 12). The required NPSH for a ψ/ψ_{NC} of 0.7 is plotted as a function of flow coefficient in figure 8. For each liquid temperature, the required NPSH increases with increasing flow coefficient. The performance curves for 31.0° R and 34.1° R (17.2 K and 18.9 K) hydrogen were used as the reference data. The predicted and experimental values are in good agreement. The predicted magnitude of the thermodynamic effects of cavitation is shown in figure 9. For each
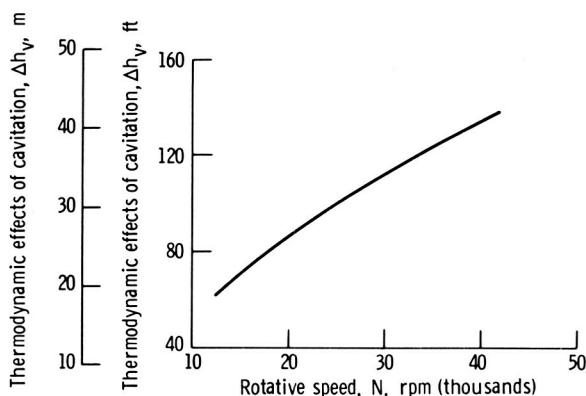
FIGURE 6.—*Thermodynamic effects of cavitation as function of rotative speed for centrifugal pump impeller. Hydrogen temperature, 37.2° R (20.7 K); head-rise coefficient ratio, 0.70.*
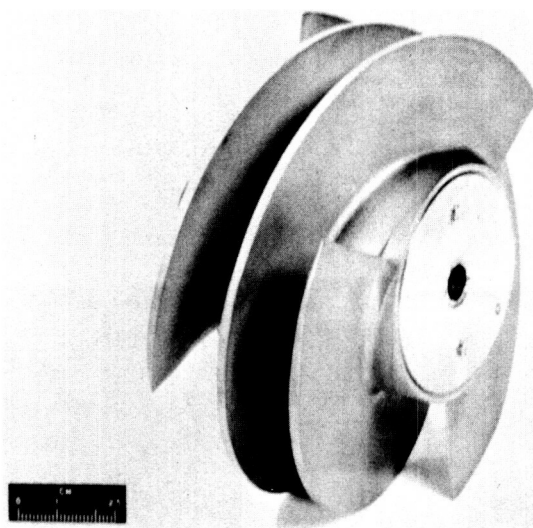

FIGURE 7.—*80.6° flat-plate helical inducer (Inducer A). Tip diameter, 4.980 inches (12.65 cm); hub diameter, 2.478 inches (6.29 cm); number of blades, 3.*

temperature above the triple point temperature, Δh_v decreased with increasing flow coefficient. As was the case with the other inducer and pumps, Δh_v increased with increasing liquid temperature.

Cavitation Parameters

As shown here and in references 3, 6, and 9 to 14, both the required NPSH and the magnitude of the thermodynamic effects of cavitation vary with liquid, liquid temperature, rotative speed, flow coefficient and inducer

or pump design. Thus, when values of NPSH and Δh_v are given, they must be accompanied by each of these variables.

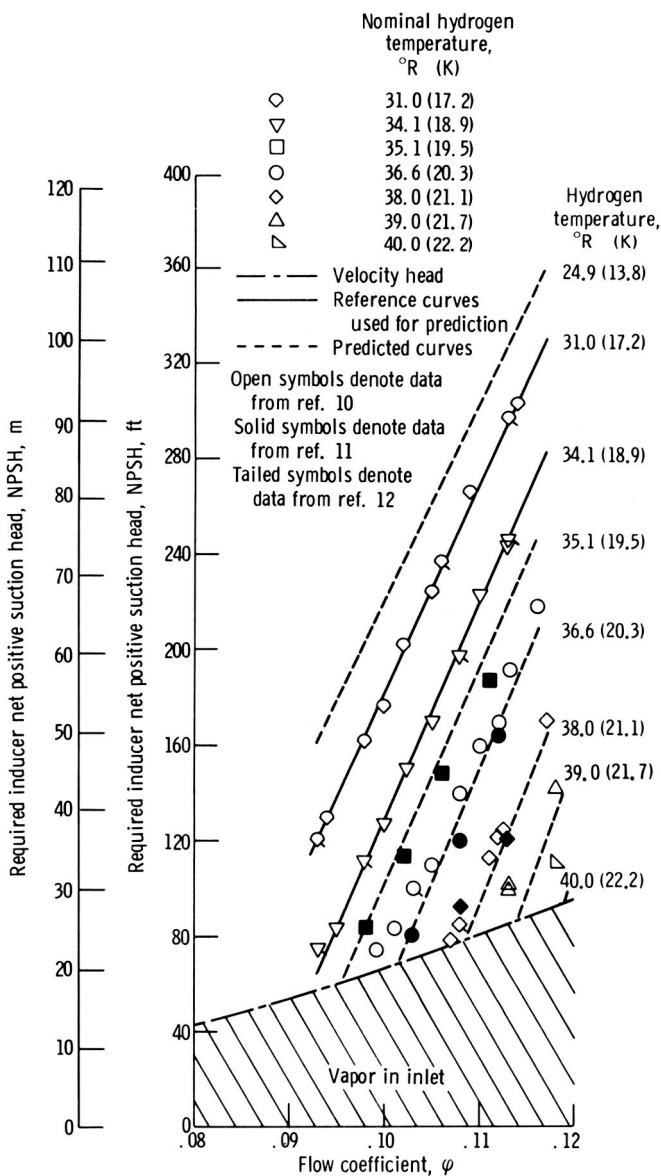A more convenient way to express the cavitation performance is to express equation (3) as

FIGURE 8.—*Effect of flow coefficient on cavitation performance of 80.6° inducer (Inducer A). Rotative speed, 30 000 rpm; head-rise coefficient ratio, 0.70.*

$$K\text{-factor} = K_{c,\min} + 1 = \frac{\text{NPSH} + \Delta h_v}{(V^2/2g)} \tag{5}$$

With the K-factor known, the cavitation performance of an inducer operated in a liquid without thermodynamic effects can be evaluated. The greater K-factor will result in the greater required NPSH.

Another way to evaluate the thermodynamic effects of cavitation is to express the reference values in equation (2) as

$$M\text{-factor} = \left(\frac{\mho_v}{\mho_l}\right)_{\text{ref}} (\alpha_{\text{ref}}) \left(\frac{1}{V_{\text{ref}}}\right)^{0.8} \tag{6}$$

With the substitution of equation (6), equation (2) will become

$$\left(\frac{\mho_v}{\mho_l}\right) = M\text{-factor} \left(\frac{1}{\alpha}\right) (V)^{0.8} \tag{7}$$
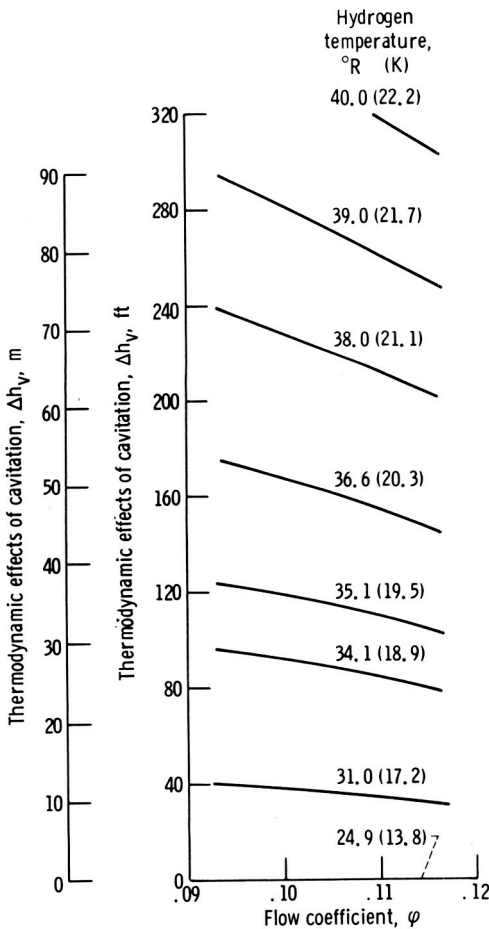

FIGURE 9.—*Thermodynamic effects of cavitation as function of flow coefficient for several hydrogen temperatures for 80.6° helical inducer (Inducer A). Rotative speed, 30 000 rpm; head-rise coefficient ratio, 0.70.*

Equation (7) can then be used in conjunction with equation (1) to determine the value of Δh_v for the condition of interest. Thus only the M-factor has to be known to qualitatively evaluate which inducer (or pump) will have the most thermodynamic effects of cavitation. When the same temperature liquid is being pumped at a particular speed, the inducer with the greater M-factor will have the greater thermodynamic effects.

Blade Leading Edge Thickness Effects

To evaluate the effect of blade leading edge thickness on the cavitation performance of an inducer, the 80.6° inducer (fig. 7) was tested with three different fairings. The leading edges of the blades were faired as shown in figure 10. The results from those tests (refs. 9, 13, and 14) are summarized in figure 11 where the cavitation parameters, M-factor and K-factor, are
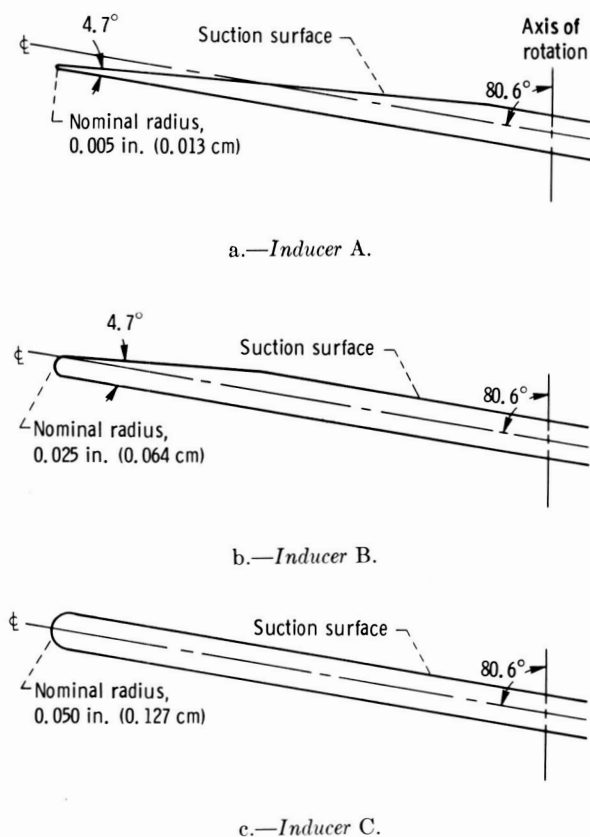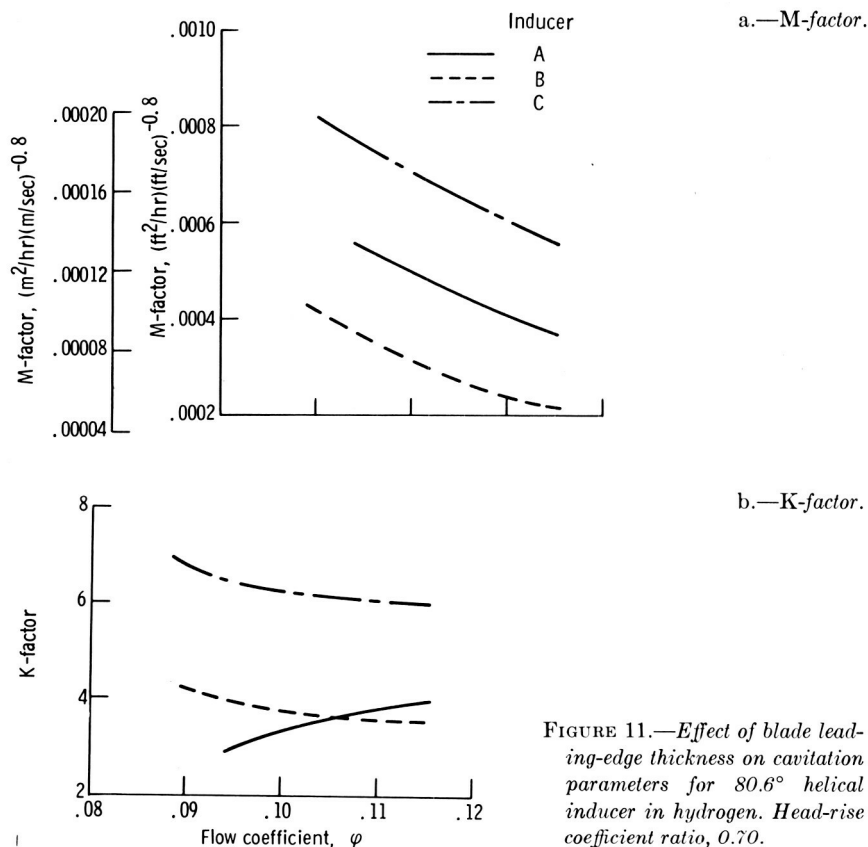

a.—*Inducer A.*

b.—*Inducer B.*

c.—*Inducer C.*

FIGURE 10.—*Blade leading edges for three 80.6° helical inducers.*

a.—M-*factor.*

b.—K-*factor.*

FIGURE 11.—*Effect of blade leading-edge thickness on cavitation parameters for 80.6° helical inducer in hydrogen. Head-rise coefficient ratio, 0.70.*

plotted as a function of flow coefficient. Inducer A is the same as that shown in figure 7.

Although Inducer C has the greatest thermodynamic effects of cavitation (indicated by the greatest M-factor), that inducer does not necessarily have the best cavitation performance (indicated by the highest K-factor). Depending on liquid temperature, the increase in Δh_v gained by blunting the blades may not be large enough to overcome the increase in NPSH requirements. Thus, compromises in blade fairing must be made in order to optimize the inducer design over a temperature range.

The values of Δh_v and NPSH can be obtained for each inducer from the cavitation parameters, M-factor (eq. (6)) and K-factor (eq. (5)). For example, the M-factor for Inducer A will yield the Δh_v-values shown on the curves of figure 9. The K-factor for Inducer A can be used to calculate the required values of NPSH shown in figure 8 for liquid hydrogen at 24.9° R (13.8 K). The values of Δh_v obtained from the M-factor are

then subtracted from the required NPSH at 24.9° R (13.8 K) to obtain the required NPSH at other liquid temperatures (fig. 8).

CONCLUDING REMARKS

A method for predicting the cavitation performance of pumps was developed. The prediction method, formulated from theoretical and supporting experimental studies, provided good agreement between predicted and experimental results for several inducers and pumps handling liquids of widely diverse physical properties. The use of the method requires that two appropriate sets of test data, not necessarily for the same liquid, liquid temperature, or rotative speed, be available for the pump of interest. From these reference tests, the cavitation performance for the pump can then be predicted for any liquid, temperature, or rotative speed provided that flow similarity is maintained. The method also has useful application in the generalization of experimental data obtained with liquids that exhibit substantial thermodynamic effects of cavitation. For example, pump cavitation studies that use a particular liquid need not be conducted at predetermined precise values of temperature and rotative speed since, through the use of the cavitation parameters, the data can be normalized.

Because of the large thermodynamic effects of cavitation for liquid hydrogen, it is possible to pump hydrogen in a boiling state with relatively small losses in pump performance. The net positive suction head requirements will depend on the combination of fluid properties and on the flow conditions.

LIST OF SYMBOLS

C_l Specific heat of liquid, Btu/(lbm)(°R); J/(kg)(K)

dhv/dT Slope of vapor pressure head to temperature curve, ft of liquid/ °R; m of liquid/K

g Acceleration due to gravity, ft/sec²; m/sec²

ΔH Pump head rise based on inlet density, ft of liquid; m of liquid

Δh_v Decrease in vapor pressure because of vaporization (magnitude of thermodynamic effect of cavitation), ft of liquid; m of liquid

$K_{c,min}$ Developed cavitation parameter based on minimum cavity pressure

k Liquid thermal conductivity, Btu/(hr)(ft)(°R); J/(hr)(m)(K)

L Latent heat of vaporization, Btu/lbm; J/kg

N Rotative speed, rpm

NPSH   Net positive suction head, ft of liquid; m of liquid

U_t Blade tip speed, ft/sec; m/sec

V	Average axial velocity at inducer inlet, ft/sec; m/sec
$\mathcal{V}_l$	Volume of liquid involved in cavitation process, in.^{3}; cm^3
$\mathcal{V}_v$	Volume of vapor, in.^{3}; cm^3
α	Thermal diffusivity of liquid, $k/\rho_l C_l$, ft^2/hr; m^2/hr
ρ_l	Density of liquid, lbm/ft^3; kg/m^3
ρ_v	Density of vapor, lbm/ft^3; kg/m^3
φ	Flow coefficient, V/U_t
ψ	Head-rise coefficient, $g\Delta H/U_t^2$
ψ/ψ_{NC}	Cavitating-to-noncavitating head-rise-coefficient ratio

Subscripts

| NC | Noncavitating |
| ref | Reference value obtained from experimental tests |

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DISCUSSION

P. COOPER (TRW Inc. and Case Western Reserve University): Mr. Moore presents conclusive evidence in support of a theory that is the result of careful application to pumps of fundamental discoveries made at NASA with regard to cavitating flows. Basic to the theory presented is the discovery of the relationship between the NPSH and other relevant quantities that exists for similarly appearing flow configurations. That this relationship furnishes a pump cavitation performance map is emphasized if one approaches the problem of predicting such performance by using the classical methods of dimensional reasoning.

In outlining the problem in this way, we must distinguish between the *dynamic similarity* to which dimensional reasoning is applied and the *flow configuration similarity* that is basic to the theory presented by Mr. Moore. For the former, we first write, in terms of the quantities evidently involved, the physical equation for pump performance:

$$\Delta H = f\left(V, \Omega, r, \rho_l, \rho_v, \frac{dt}{dh_v}, C_l, L, \alpha, g\mathrm{NPSH}, m, \{G_i\}\right) \quad \text{(D–1)}$$

where Ω is the angular speed; r, the pump inlet tip radius; $\{G_i\}$, the set of geometrical lengths that describe the pump shape; m, another independent quantity or combination of quantities implied as being involved by the discoveries made; and the other symbols are those defined by the author. Recognizing the dimensions of these quantities and the combinations of them that appear in the theory, we obtain a dimensionless functional equation for pump performance:

$$\frac{\Delta H}{\Omega^2 r^2} = f_1\left(\frac{V}{\Omega r}, \frac{g\mathrm{NPSH}}{(V^2/2)}, \frac{dt}{dh_v}\frac{C_l}{L}\frac{\rho_l}{\rho_v}\frac{V^2}{2}, \frac{rV}{\alpha}, \frac{m}{r^{4/5}V^{1/5}}, \left\{\frac{G_i}{r}\right\}\right) \quad \text{(D–2)}$$

or, abbreviating these dimensionless groups,

$$\psi = f_1(\phi, k, \theta, P_e, \hat{m}, \{\hat{G}_i\}) \quad \text{(D–3)}$$

Here, ψ can be replaced by ψ/ψ_{NC}, since ψ_{NC} is a constant of the problem. (ϕ is fixed, and noncavitating Reynolds number effects are apparently ignored.) ϕ is the flow coefficient; k is a cavitation number; θ is essentially the parameter presented in an earlier NASA-sponsored inducer study (ref. D–1)[D–1]; and P_e is a Peclet number.

[D–1] There the parameter is developed as $H \simeq dt/dh_v \, C_l/L \, \rho_l/\rho_v \, \Omega^2 r^2 = 2\theta/\phi^2$.

Now, Mr. Moore's theory embodies the method of solution of equation (D-3), and it is in that solution that we see the independent parameter $\hat{m}$ involved. The relationships given by him in his equations (1), (2), and (3) (with his equation (2) modified by the addition of the size effect D/D_{ref} from equation (11) of NASA TN D-5292 (ref. 6) may be combined and written as follows:

$$k + \frac{\hat{m}P_e}{\theta} - 1 = K_{c,\min} \qquad (D-4)$$

$K_{c,\min}$ and m, where

$$m = \hat{m}r^{4/5}V^{1/5} = \frac{\mathcal{v}_v}{\mathcal{v}_l}\frac{r^{4/5}V^{1/5}}{P_e} = \frac{\mathcal{v}_v}{\mathcal{v}_l}\frac{\alpha}{V^{4/5}r^{1/5}}$$

are constants for fixed values of the dependent parameter ψ/ψ_{NC}, ϕ, and the shape $\{\hat{G}_i\}$.

From the standpoint of the rationalist who must evolve equation (D-1), it is unfortunate that the constant dimensional quantity m is unknown. It must be some combination of the known quantities that are involved, such as surface tension, viscosity, etc., just as

$$\frac{dt}{dh_v}\frac{C_l}{L}\frac{\rho_l}{\rho_v}$$

is another such combination. This is to be expected because, among other possibilities, no Reynolds number appears explicitly in the analysis.

Mr. Moore's method has made it unnecessary for us to know m from the beginning; he merely asks that two tests be run to determine it, together with $K_{c,\min}$. This points up another attribute of m, at least for the cavity-type flow configuration: It is independent of the fluid used and depends only on the quantities ψ/ψ_{NC}, ϕ, and $\{\hat{G}_i\}$. Apparently m is some compensating combination of physical properties and some of the other independent quantities listed in equation (D-1) such that it can have this constancy character.

The quantity $K_{c,\min}$ is not an independent parameter of the functional performance equation (D-3), but rather is a function of the independent dimensionless parameters that turns out to be constant under the given conditions for similarly appearing flow configurations of the type assumed to exist in cavitating pumps. Since m is also constant under these conditions, the parameter $\hat{m}$ could be a combination of more independent parameters that might have different influences in a basically different flow structure.

The flow of vapor and liquid in an inlet line is an example of a situation with which is associated a flow configuration fundamentally different from the cavity type basic to the prediction method presented by Mr.

Moore. If, as was done in the earlier inducer study already noted, one assumes such a flow to be fog-like (i.e., a homogeneous variable-density continuum consisting of many small bubbles in equilibrium with the surrounding liquid), $\dot{m}$ and P_e are eliminated from equation (D–3). Furthermore, the solution of the entire resulting compressible flow field can be sought using an equation of state for the mixture density ρ:

$$\rho = \frac{\rho_l}{1 + \Theta C_{pv}} \qquad \text{(D-5)}$$

This was applied wherever the pressure depression coefficient $C_{pv} = (p_v - p)g/(\rho_l \Omega^2 r^2)$ exceeded zero, p_v being the initial vapor pressure of the fluid and $p_v - p$ being the vapor pressure depression (since p is then the local vapor pressure).

This fog-flow approach was applied to inducers. The assumption of a cavity on the suction side of each blade is well substantiated by Mr. Moore for the operating conditions investigated. However, at complete head breakdown, or near such a condition with vapor in the inlet line, the blades are unloaded, and the fog-flow approach might more closely describe the flow, especially in the inlet region. Such an analysis applied to an 83.8° inducer yielded dimensionless NPSH results for this limiting situation over an Θ-range of 10 to 1000 that agree qualitatively with those indicated in figure 2 for the 84° inducer of figure 1 of Mr. Moore's paper. His range of the parameter Θ was from 46 to 1260 for the corresponding LH_2 temperature range of 36.6° R to 27.5° R at 20 000 rpm—the lower values of Θ corresponding to the greater thermodynamic effects of cavitation. Of particular interest is the fog-flow performance calculation, which indicated no head breakdown for the 83.8° inducer at $\Theta = 10$. There the NPSH was zero, and there was vapor in the inlet line.

Attention has been called to these fog-flow calculations as an indicator of the ideal for which the cavitation performance predictor strives; namely, the analytical description of cavitating flow fields under all possible conditions. Obviously a successful description, complete with losses, is a long way off. Thus there seems no doubt that the procedure presented by Mr. Moore will serve as the prediction method for a long time. His empirical determination of pump cavitation performance from two sets of test results shows excellent correlation of experiment and theory for all practical pump flows.

H. G. PAUL: The method for predicting thermodynamic effects of cavitation described by Mr. Moore has been used at the Marshall Space Flight Center (MSFC) for the past several years to predict the suction performance of cryogenic propellant pumps. It is our experience that the effect of pump speed and fluid temperature on suction performance can be predicted accurately. However, attempts to predict cavitation per-

formance of a pump when several fluids were involved have not always been completely satisfactory. The following data for the J-2 rocket engine fuel pump are cited as an example of the problem encountered. In this assessment, it is assumed that water has negligible vapor pressure depression and all NPSH deviations from the similarity laws in cryogenic liquids are due to thermodynamic effects. The data are

Fluid	LH_2	LN_2[D-2]	H_2O[D-3]
Speed, rpm	27 100	7590	6320
Temperature, °R	38	140	530
NPSH @ ref head loss, ft	80	12.5	12.2

If the water and hydrogen data are used to predict the vapor pressure decrease in LN_2 for the above conditions, a value of 2.5 ft is calculated compared to a value of 5.0 ft obtained from the nitrogen and water data. Similarly, if the water and nitrogen data are used to predict the vapor pressure depression in LH_2 at the above conditions, a value of 245 ft is calculated compared to a value of 145 ft obtained directly from the water and hydrogen data. From this result, it is concluded that some caution should be exercised when using these methods to predict cavitation performance of a pump in a fluid other than that fluid for which test data exist.

An additional point which merits comment is the capability to satisfactorily pump two-phase fluids under certain circumstances as discussed by Mr. Moore. This capability is of considerable interest as an operational mode for space vehicle pumping systems. Cryogenic liquids used in these applications are stored at close to saturation conditions under zero "g" conditions. In order to supply NPSH for pump operation, a vent and pressurization cycle is required. This complicates the system operation and is costly from a weight standpoint because of the requirement for stored pressurization gas. "Zero tank NPSH" operation, which implies two-phase operation at the pump inlet, has the potential for eliminating the vent and pressurization cycle and is therefore viewed as a highly desirable capability.

MOORE (author): I would like to thank Mr. Cooper and Mr. Paul for their comments. Mr. Cooper has presented an interesting discussion of the theory. Dimensionless parameters involving, V, α, and r might have been found had a model different from the conduction model been chosen. I agree that any and all theories should be examined in an effort to develop an analytical description which can predict cavitation performance under all conditions.

[D-2] Reference D-2.
[D-3] Reference D-3.

It is realized that large errors can occur when the predicted speed is several times greater than the reference value as is the case with Mr. Paul's example for hydrogen. Examination of the similarity equation (eq. 4) will show that any measurement errors occurring at the lower speed will increase by the speed squared at the higher speed. It is felt that prediction for various speeds should be limited to about a 2-to-1 change in speeds. A study made by C. F. Braun & Company, under NASA contract, indicated that pump similarity parameters were reasonably accurate to only a 2-to-1 change in speed.

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