AUBURN UNIVERSITY

THERMAL CONDUCTIVITY OF COMETS

Final Report
Contract NAS8-26579

by
R. I. Vachon, G. N. Kumar, M. S. Khader

School of Engineering
Department of Mechanical Engineering
Thermoscience Group

Under Contract With
George C. Marshall Space Flight Center
National Aeronautics & Space Administration

Administered Through
ENGINEERING EXPERIMENT STATION
AUBURN UNIVERSITY
Auburn, Alabama 36830

NOV 1974
RECEIVED
NASA STI FACILITY
INPUT BRANCH

September 1974

(NASA-CR-120524) THERMAL CONDUCTIVITY OF COMETS Final Report (Auburn Univ.) 26 p HC $3.75 CSCL 03B
N75-11805
Unclas
G3/89 02582

NOMENCLATURE

B	A constant in Eqs. (2) and (4)
C	A constant in Eqs. (2) and (4)
C'	Negative of C
K	Thermal conductivity (WATT/cm-^{o}k)
P	Phase volume fraction
P_1	Porosity
P_m	Mass fraction
R	Thermal resistance
T	Absolute temperature (^{o}k)
E	Modulus of Elasticity of solid particle
g	Gravitational constant
z	Depth below the surface
ρ	Density
$\bar{\rho}$	Average density
υ	Poisson's ratio

Subscripts

c	Continuous phase
d	Discontinuous phase
eff	Effective
S	Solid

INTRODUCTION

A comet is generally regarded to be composed of three primary regions: the nucleus or Kernal, the coma (a plasma region surrounding the nucleus), and the tail. Since not much is known about the composition and structure of the nucleus, it is thought that some useful information may be inferred by investigating the heat and mass transfer of the nucleus. Before conducting these investigations, it is necessary to study the material thermal properties of possible nucleus models.

The object of the present study program is to perform the following:

Part I: To recommend a value for the thermal conductivity of the frost layer and a value for the thermal conductivity of the water-ice solid debris mixture. The basis for the recommendation is a nucleus model consisting of water-ice mixed with basaltic and meteoritic materials in the form of dust and agglomerated particulates, all of which is surrounded by a layer of water-frost, assuming a constant thermal conductivity in each of the layers.

Part II: To recommend a value for the thermal conductivity of the porous structure of Part II as a function of depth only. The model for this part is the same model as in Part I with the following additional assumption. The assumption is that the frost layer and water-ice have been sublimating and evaporating, leaving a porous structure composed of the solid basaltic and meteoritic material with residual gases partially confined in the porous structure to varying amounts over some depth. This conductivity function should be regarded as a simplified approximation to be used for initial calculational estimates of heat transfer in the nucleus.

MODEL FOR PART I

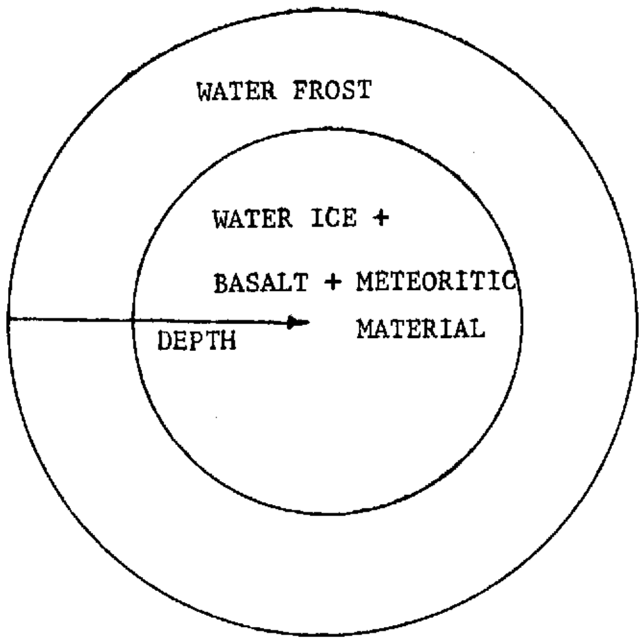

FIGURE 1(a).

MODEL FOR PART II

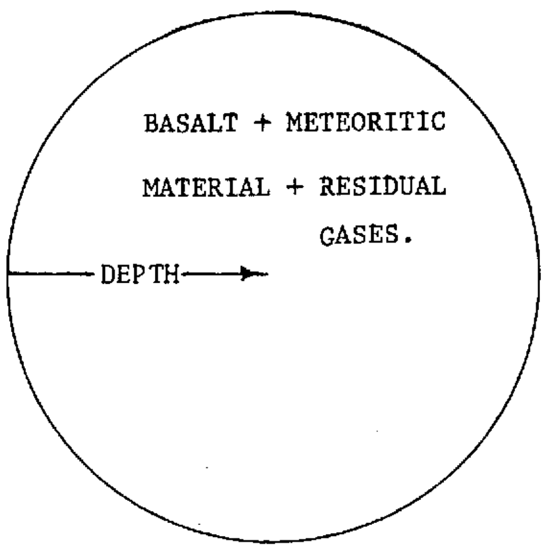

FIGURE 1(b)

COMETS CONSIDERED

i) Commet Encke: With a minimum temperature of 140°K, Radius: 4KM.

ii) Comet Halley: Min. Temp: 49°K, Radius: 15KM.

ANALYSIS FOR PART I

For the Water Ice-Solid Debris Mixture, the Cheng and Vachon Model [1] has been used to calculate the approximate effective thermal conductivity of this layer. The nucleus of a comet normally consists of 70% or more of ices by mass [2] and the rest, basaltic and meteoritic material. As a first approximation, in the absence of detailed information on the composition of the meteoritic material (other than that it might contain Fe, Ca, Mn, Mg, Cr, Si, Ni, Al, etc), the mixture of basaltic and meteoritic material has been approximated first with the properties of basalt and then with properties of basalt and iron which are mixed in different proportions.

(A): When the solid debris has been approximated with the properties of basalt, the above model is simplified to a mixture of water ice and basaltic material. Since the temperature of the nucleus varies with its distance from the Sun (Example: At Aphelion [3] temperature of nucleus of Encke = 142° K and that of Halley = 49°K), the thermal conductivities have been calculated for the various assumed temperatures of the nucleus. The calculations have also been performed with varying proportions of ice and basaltic material.

(B): When the solid debris was approximated by a mixture of basaltic material and iron, the model is simplified to a mixture of ice, basaltic material and iron. Again the calculations have been performed for different assumed temperatures of the nucleus (variation of thermal properties within the nucleus for a given average temperature have not been considered) and for varying proportions of water ice and solid debris. The calculations have also been performed with varying proportions of basalt and iron for a given ice, solid debris ratio.

A linear variation of thermal conductivity with temperature has been assumed for the water ice and using the values given in [4]. The following equations were obtained for the thermal conductivity and the desnsity variation of ice with temperature

$$\text{a)}\quad K_{ice} = [0.0481156 - 9.2444 \times 10^{-5}\ T]\ \text{Watt/cm-K}$$

$$\text{b)}\quad \rho_{ice} = [0.94323 - 9.2444 \times 10^{-5}\ T]\ \text{gm/cm}^3 \quad T \text{ in } {}^\circ K. \tag{1}$$

The following property values were used for the Basalt [5] and Iron [6].

Basalt:	ρ = 2830 Kg/m^3	K_s = 1.34 watt/m^{o}K
	E = 2.2 x 10'' N/m^2	ν = 0.2
Iron:	ρ = 7890 Kg/m^3	K_s= 63.7 watt/m°K.
	E = 2.07 x 10'' N/M^2	ν = 0.3

K_{ice} is of the order 3.8 watt/m°K at 100°K

I(A)

For this model, we see from the above that for the temperature range of 30-200°K, the ratio $\frac{K_{ice}}{K_{solid}}$ is in the range 1 to 4. Hence, from [1], for the case $K_c > K_d$ we have the effective or equivalent thermal resitance given by

$$R_{eff} = \frac{2}{\sqrt{(-c'(k_d - k_c)\ [K_c + B\ (K_d - K_c)]\ )}} \tan^{-1} \frac{B}{2} \sqrt{\{ \frac{-C'(K_d - K_c)}{K_c + B(K_d - K_c)}\}} + \frac{1-B}{K_c} \tag{2}$$

Where K_c = Thermal conductivity of continuous phase (water-ice in this case)

K_d = Thermal conductivity of discontinuous phase (basalt in this case)

P_d = Discontinuous phase volume fraction

$B = \sqrt{3P_{d/2}}$

$C = -4/B$

$C' = -C$

Then the approximate effective thermal conductivity is given by

$$K_{eff} = \frac{1}{R_{eff}} \tag{3}$$

Thus, the K_{eff} has been calculated for different temperatures of nucleus and for different phase volume fractions of the discontinuous phase (basalt in this model).

<u>I(B)</u>

In this model, we have a three phase mixture of water-ice, basalt and iron and we have to find the approximate thermal conductivity (K_{eff}) of this three-phase mixture. Cheng and Vachon [1] give the following method for calculating K_{eff}:

Since the major portion of the comet nucleus is made up of water-ice, we take water-ice to be the continuous phase in applying the equation developed by Cheng and Vachon for 3 phase mixtures.

Let K_c = Thermal conductivity of the continuous phase (i.e. of water ice)

K_{d_1} = Thermal conductivity of the first discontinuous phase (i.e. of Fe)

K_{d_2} = Thermal conductivity of the second discontinuous phase (that is of basalt)

Thus, from the given range of values for the above, we have

$$\frac{K_{d_1}}{K_c} > 10 \qquad\qquad \frac{K_{d_2}}{K_c} < 1$$

In order to determine the effective thermal conductivity (K_{eff}) of this 3 phase mixture, the 3 phase mixture is considered to be reduced to a two-phase mixture in which the continuous phase is composed of the original continuous phase of the three phase mixture and the second discontinuous phase. The discontinuous phase of this two phase mixture is the first discontinuous phase of the three-phase mixture.

For this two-phase mixture,

$$P_d = P_{d_1}$$

$$P_{ce} = P_c + P_{d_2}$$

$$K_d = K_{d_1}$$

K_{ce} = Thermal conductivity of the effective continuous phase

P_{ce} = Phase volume fraction of the effective continuous phase

K_{ce}, the effective thermal conductivity of the original continuous phase mixed with the second discontinuous phase of the 3 phase mixture, is determined as per the method given in Part I(A).

Now, since $\frac{K_{d_1}}{K_{ce}} > 10$, from ref [1], the effective resistance R_{eff} is given by

$$R_{eff} = \frac{1}{\sqrt{\{C'(K_{d_1} - K_{ce})\ [K_{ce} + B_1\ (K_{d_1} - K_{ce})]\}}}$$

$$\ln \frac{\sqrt{\{K_{ce} + B_1\ (K_{d_1} - K_{ce})\}} + \frac{B_1}{2}\sqrt{\{C'(K_{d_1} - K_{ce})\}}}{\sqrt{\{K_{ce} + B_1\ (K_{d_1} - K_{ce})\}} - \frac{B_1}{2}\sqrt{\{C'(K_{d_1} - K_{ce})\}}} + \frac{1 - B_1}{K_{ce}} \qquad (4)$$

where $B_1 = \sqrt{3P_{d_1}/2}$

$C' = -C = + 4/B_1$

and the effective thermal conductivity is then given by

$$K_{eff} = \frac{1}{R_{eff}} \tag{5}$$

This calculation has been performed for different assumed temperatures of the nucleus (Here also, the thermal property variation inside the nucleus has not been considered) and for different phase volume fractions of the continuous phase. For a given value of phase volume fraction of water ice, calculations have been performed for varying proportions of basalt and iron.

ANALYSIS FOR PART II

For the model in Part I, we now assume that the frost layer and the water ice has been sublimating and evaporating which occurs as the comet nears the sun (distance less than 2 AU), thus leaving a porous structure.

A simplified approximation for the effective thermal conductivity as a function of depth is obtained using the theoretical model developed by Khader and Vachon [5] for heterogeneous mixtures. In the application of the above model to our case, it has been assumed that since the gas is at low pressure the thermal conductivity of the residual gases is quite small compared to the thermal conductivity of basalt and meteoritic material. Hence, the thermal conductivity through the void can be neglected. Also, since the temperatures involved are of the order of 100°K, radiation through the void space (i.e. through the gases at very low pressures) could be neglected as being very small in comparison to conduction through the solid.

II(A)

Thus, with these simplifications, from Ref [5] we get the following expression for the effective thermal conductivity when the solid is approximated with the properties of basalt.

$$K_{eff} = \frac{3(1 - P_t)(1 - C_1^2\, Z^{2/3})\, K_s}{[\frac{\pi}{2}(\frac{3}{2\, c_1 Z^{1/3}} - 1) + (4 - 1.2\, C_1^2\, Z\, 2/3)]} \qquad (6)$$

where $C_1 = \left[\frac{2 \quad g\, \bar{\rho}\pi(1-\nu^2)}{16\, E\, (1 - P_t)}\right]^{1/3}$

$\bar{\rho}$ = Average Density of the Solid.

P_t = Porosity of the solid material.

The calculations have been performed for different porosities and for different densities up to a depth of 2500 meters.

II(B)

When the solid material is approximated by a heterogeneous mixture of basalt and Fe, the problem is solved in 2 stages. First, an effective value of thermal conductivity is obtained for the heterogeneous mixture of basalt and Fe using the Cheng and Vachon model [1].

Then the model for the second stage consists of the solid material whose thermal conductivity was determined in Stage 1 and whose voids are assumed to be filled with low pressure residual gases. The effective thermal conductivity of this stage is determined using the Khader and Vachon model as explained for II(A).

RESULTS AND DISCUSSION: PART I

Part I(A):

Fig. 2 shows the variation of effective thermal conductivity with the average temperature when the solid debris is approximated with the properties of basalt for different values of the density of the discontinuous phase at a given value of mass fraction of the discontinuous phase [For example: The above graph is for the case when there is 80% of water-ice and 20% of basalt by mass]. This figure shows that the effective thermal conductivity of this layer is of the same order as that of water-ice and that its variation with temperature is also linear. The reason for this is that the major portion of the nucleus (about 70-80%) is made up of water-ice and the thermal conductivity of basalt is also of the same order of magnitude as that of water-ice. The figure also shows that at a given temperature there is not much variation in K_{eff} with change in density for this case, especially near temperatures of the order of 200°K.

Fig. 3 shows the variation of K_{eff} with mass fraction of solid debris (i.e., discontinuous phase approximated by basalt in this case). It shows that the variation is not appreciable in the range of mass fraction predicted for the nucleus of a comet and that there is a slight decrease in the thermal conductivity as mass fraction of solid debris is increased. It also shows that the variation is more pronounced at temperatures near 40^{o}K than near 200^{o}K where it is almost flat.

Part I(B):

For the model of I(B), variation of thermal conductivity with the total mass fraction of basalt and iron is shown in Fig. 4 for different proportions of Fe and basalt. As can be seen from this figure, though the curves start at almost the same value of K_{eff} for very low values of mass fraction (the reason for which is that practically the whole nucleus in this case will be made up of water-ice), the slope of the curve changes from being negative for the model with only basalt, to positive for the model with only Fe as the solid debris material. The reason for the positive slope in the latter case is that since the thermal conductivity of iron is much greater than that of water-ice, the thermal conductivity of the mixture increases as the percentage of Fe increases. When Fe and basalt are in the ratio 1:1, even though the slope is positive, the increase is not marked.

PART II

Figs. 5, 6, 7 show the plots of variation of the effective thermal conductivity with depth for different values of porosity, each figure for a particular proportion of Fe and basalt. They all exhibit the same general trend of increase of K_{eff} with depth, the increase being sharper during the earlier part and then becoming gradual. The reason for this is that as the depth increases the contact area between the particles increases due to the increase in loading and consequently smaller thermal contact resistance and in turn thermal conductivity increases. As the porosity increases, the amount of solid material per unit volume decreases and hence thermal conductivity decreases with porosity.

A comparative study of Figs. 5, 6, 7 is made in Fig. 8. Comparing curves 1 and 5 we see that the thermal conductivity increases by a factor of about 75 when basalt is replaced by Fe. This marked increase in K_{eff} with increase in percentage of Fe compared to basalt is because of the very high value of thermal conductivity for Fe compared to basalt. This figure also shows that the curve of K_{eff} sharply rises for the first few hundred meters and then becomes very gradual (almost flat). Thus after about 500 meters, the effect of depth on thermal conductivity is not pronounced. This is because after this depth, no appreciable increase in contact area between the particles occurs with depth.

Fig. 9 shows the variation of effective thermal conductivity with depth for the case II(A) [i.e. with only Basalt as solid material] for the first 200 meters from the surface. This shows that even though the graph still follows the trend of Figs. 5, 6, 7, actually the major portion of the increase in the value of K_{eff} takes place within a few meters from the surface and afterwards the increase is very gradual. Since the same pattern was followed for the other cases (for varying proportions of basalt and Fe) they have not been plotted.

Fig.10 illustrates the variation of K_{eff} with density at different depths when the solid material was approximated with the properties of basalt. The increase in thermal conductivity with density is very gradual and the reason for the increase in K_{eff} with density is the increased contact area between the particles which in turn is due to increase in loading due to heavier mass/unit volume at a given porosity. Fig. 11 also illustrates the same.

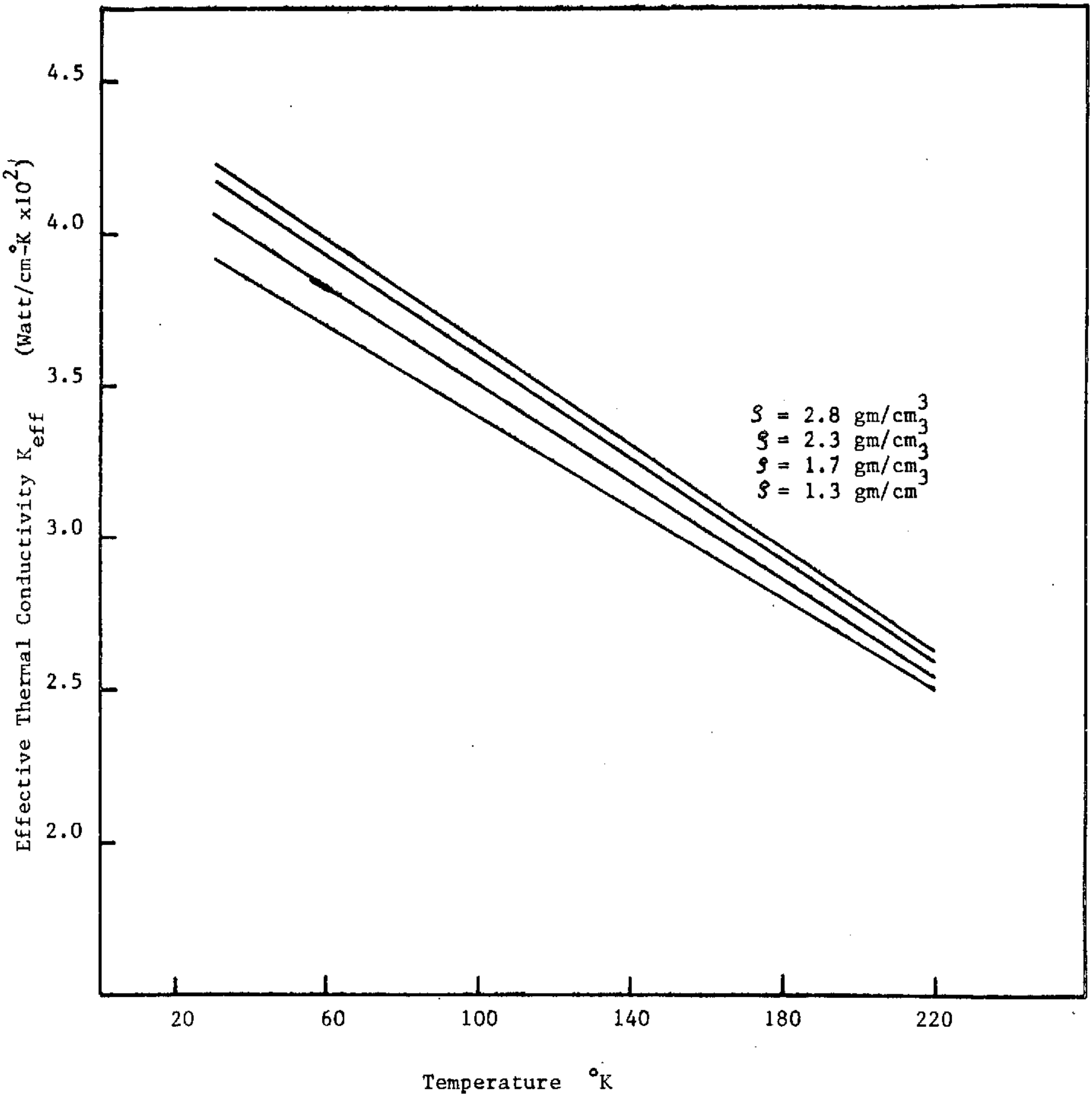

FIGURE 2: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF TEMPERATURE AT CONSTANT DENSITY FOR PART I(A).

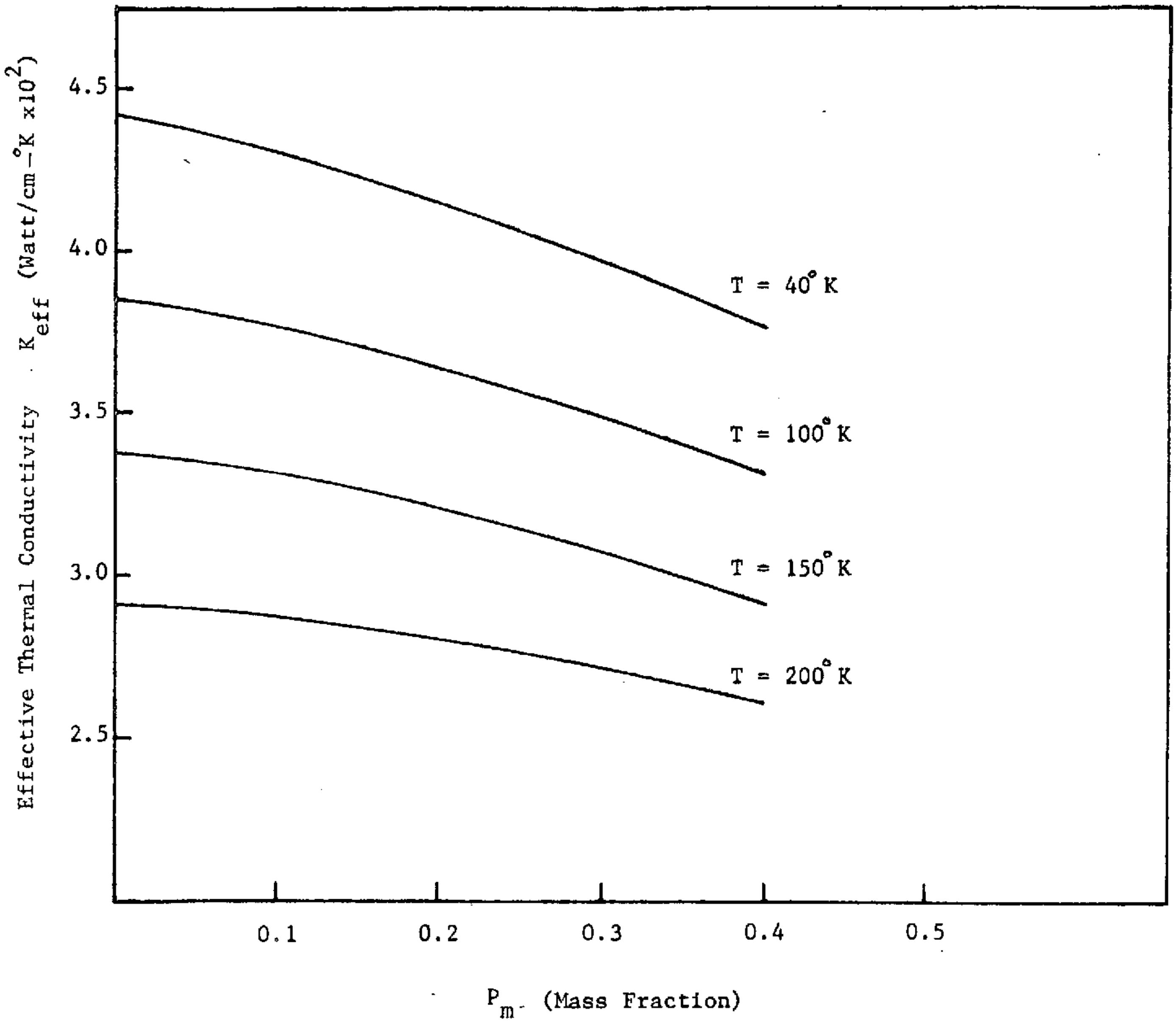

FIGURE 3: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF MASS FRACTION OF SOLID DEBRIS(BASALT + METEORITIC MATERIAL) AT CONSTANT TEMPERATURE FOR PART I(A).

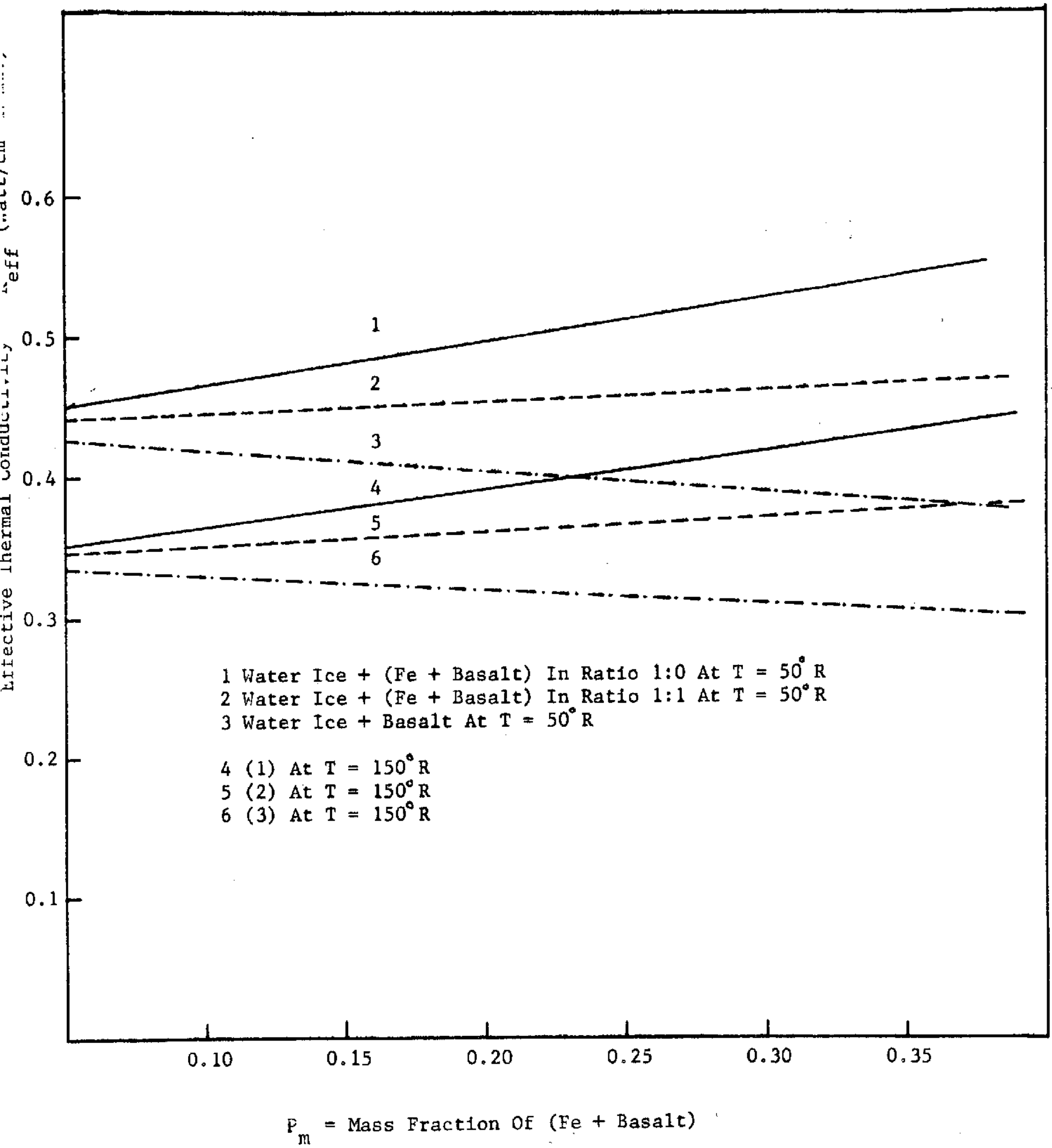

FIGURE 4: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF MASS FRACTION OF SOLID DEBRIS (BASALT + METEORITIC MATERIAL) AT CONSTANT TEMPERATURE FOR PART I(B).

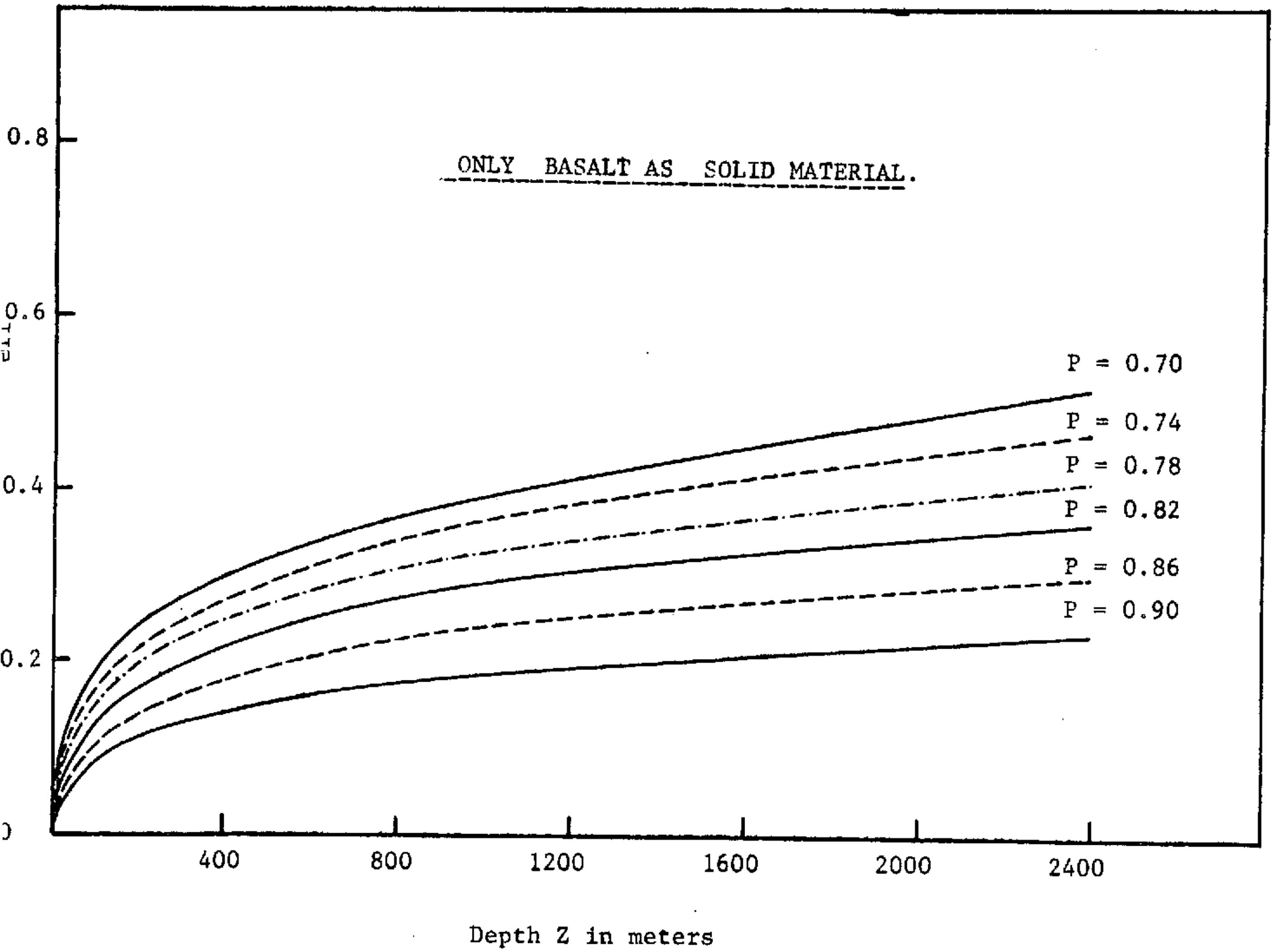

FIGURE 5: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DEPTH AT CONSTANT POROSITY FOR PART II(A).

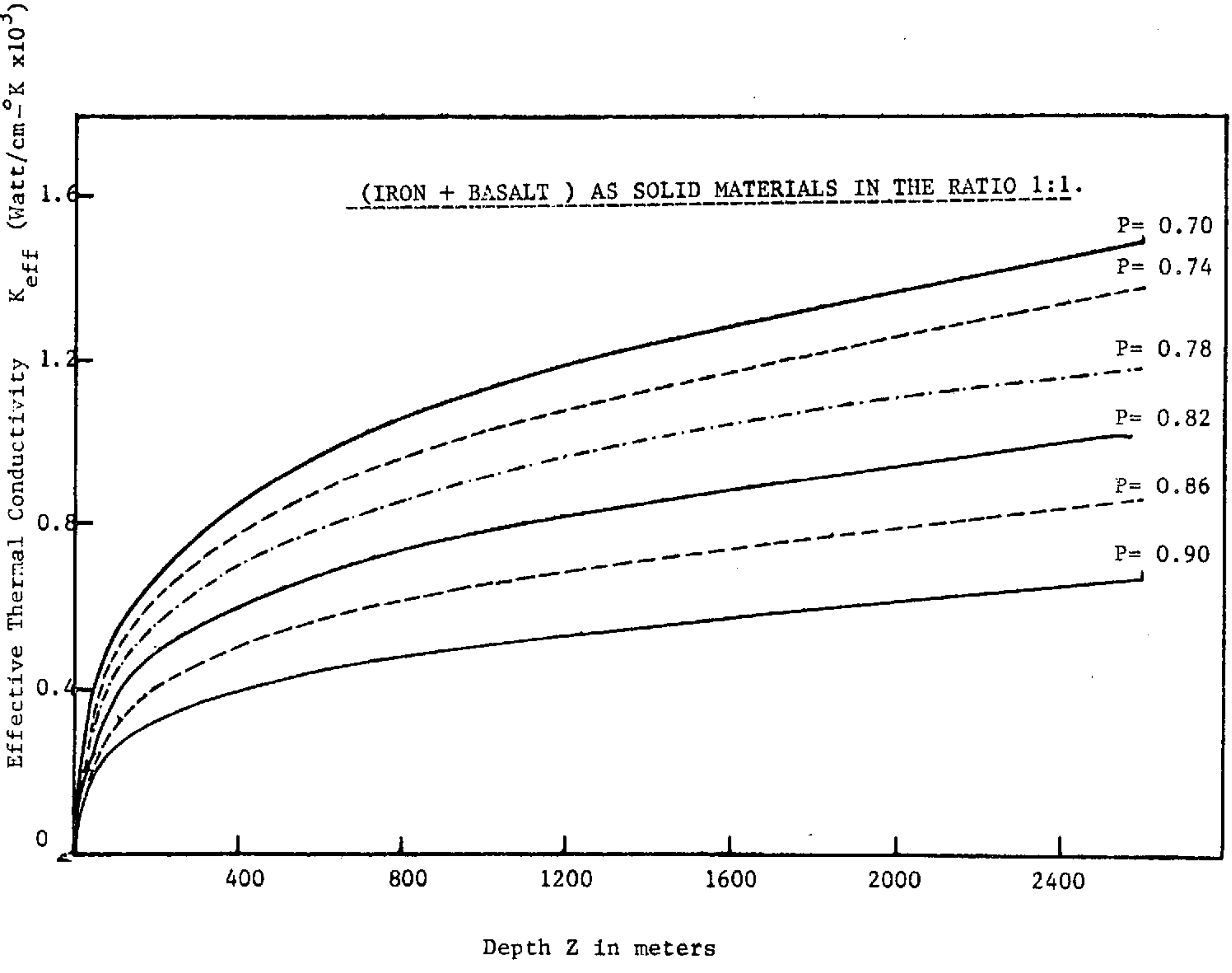

FIGURE 6: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DEPTH AT CONSTANT POROSITY FOR PART II(B).

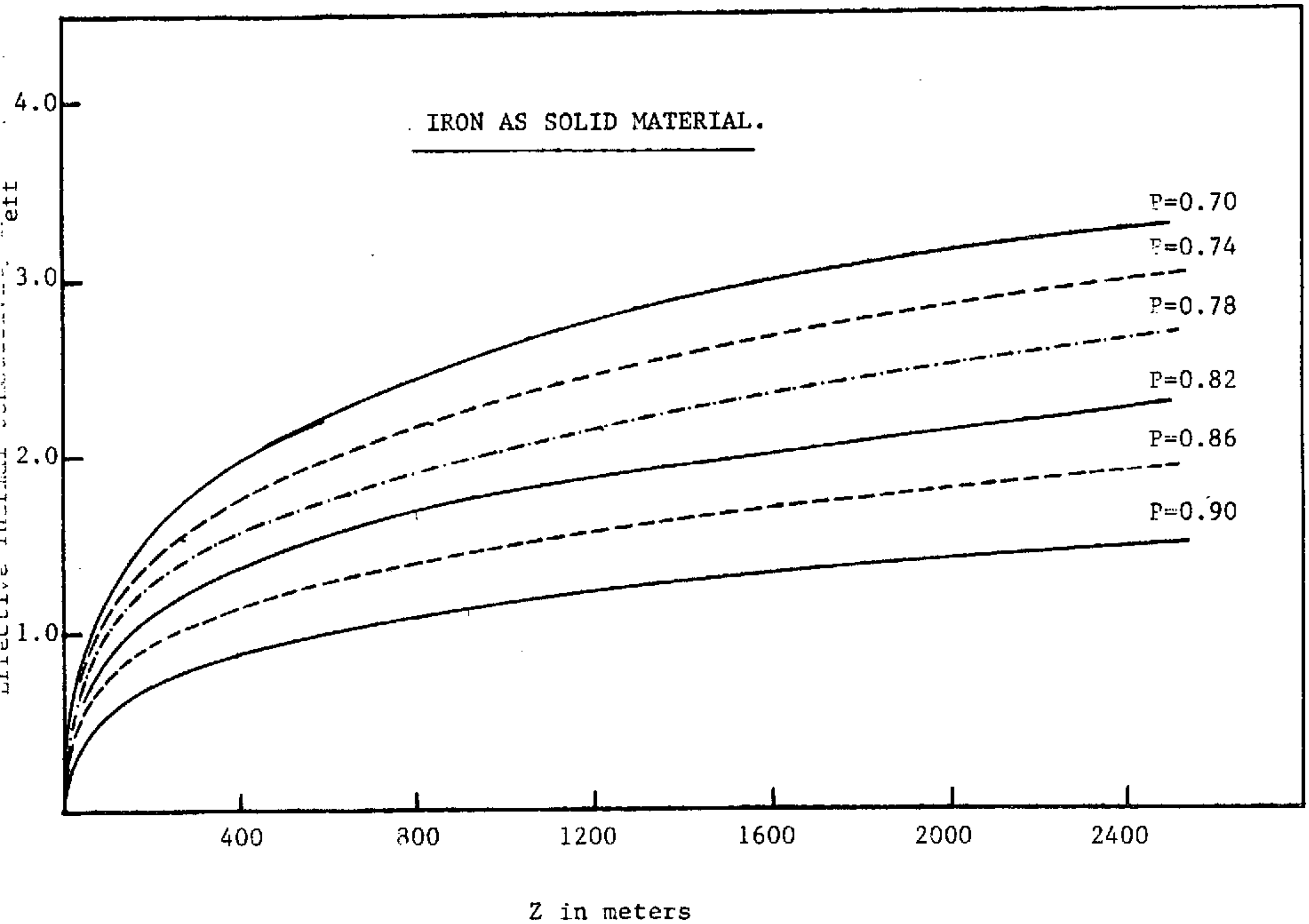

FIGURE 7: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DEPTH AT CONSTANT POROSITY FOR PART II(B).

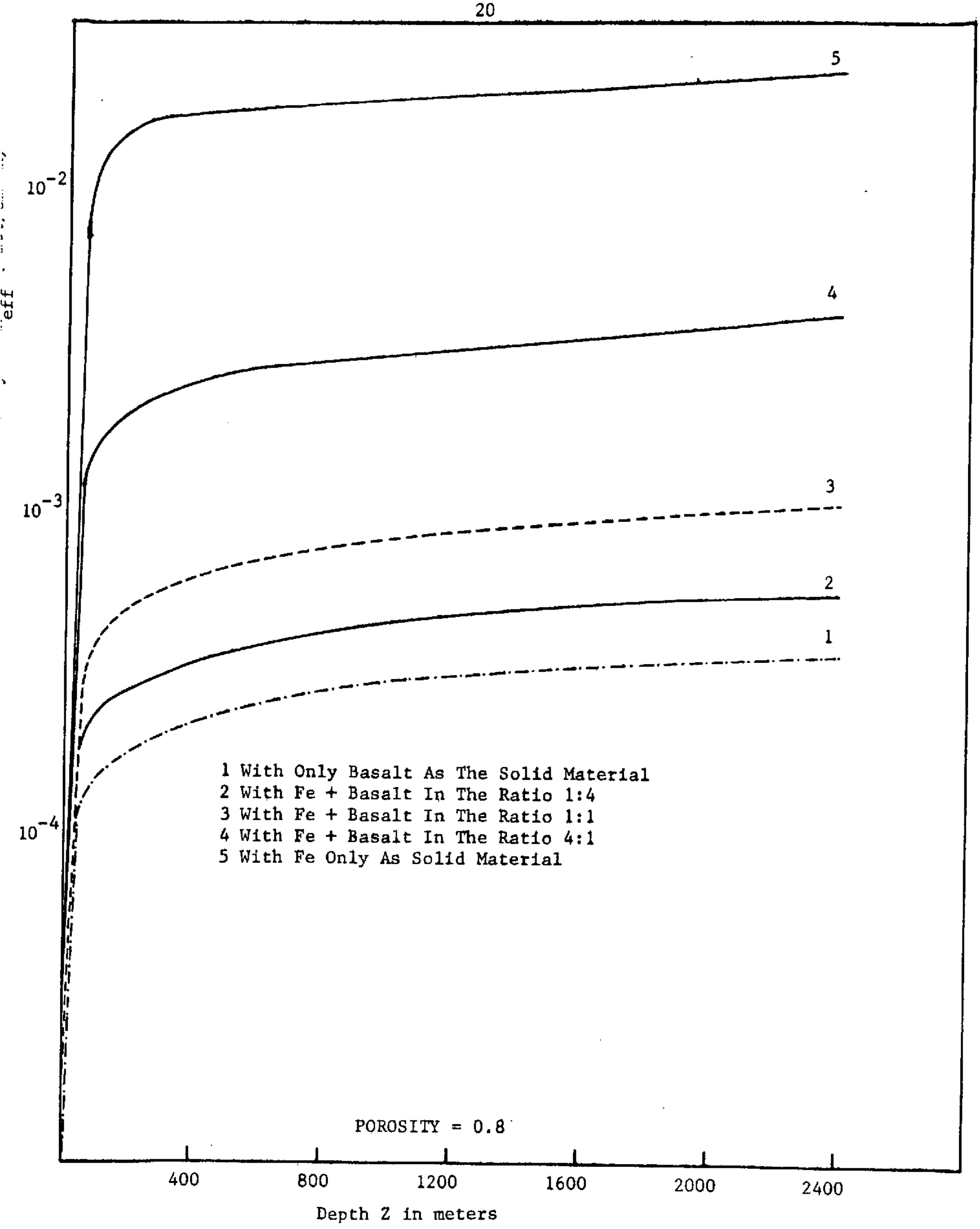

FIGURE 8: COMPARISION OF EFFECTIVE THERMAL CONDUCTIVITY FOR DIFFERENT PROPORTIONS OF BASALT AND IRON AS A FUNCTION OF DEPTH AT CONSTANT POROSITY FOR PART II(B).

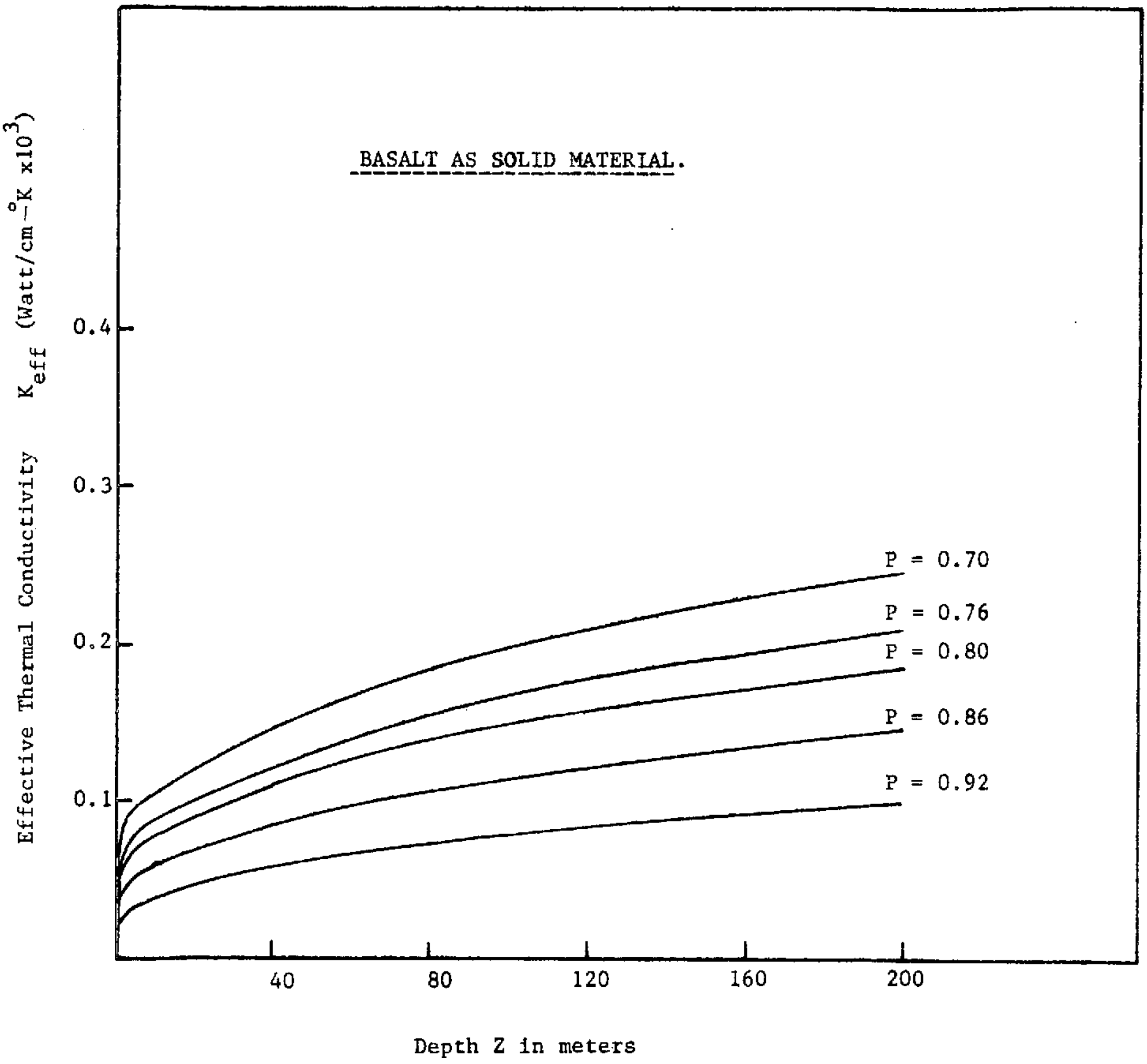

FIGURE 9: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DEPTH AT CONSTANT POROSITY FOR PART II(A).

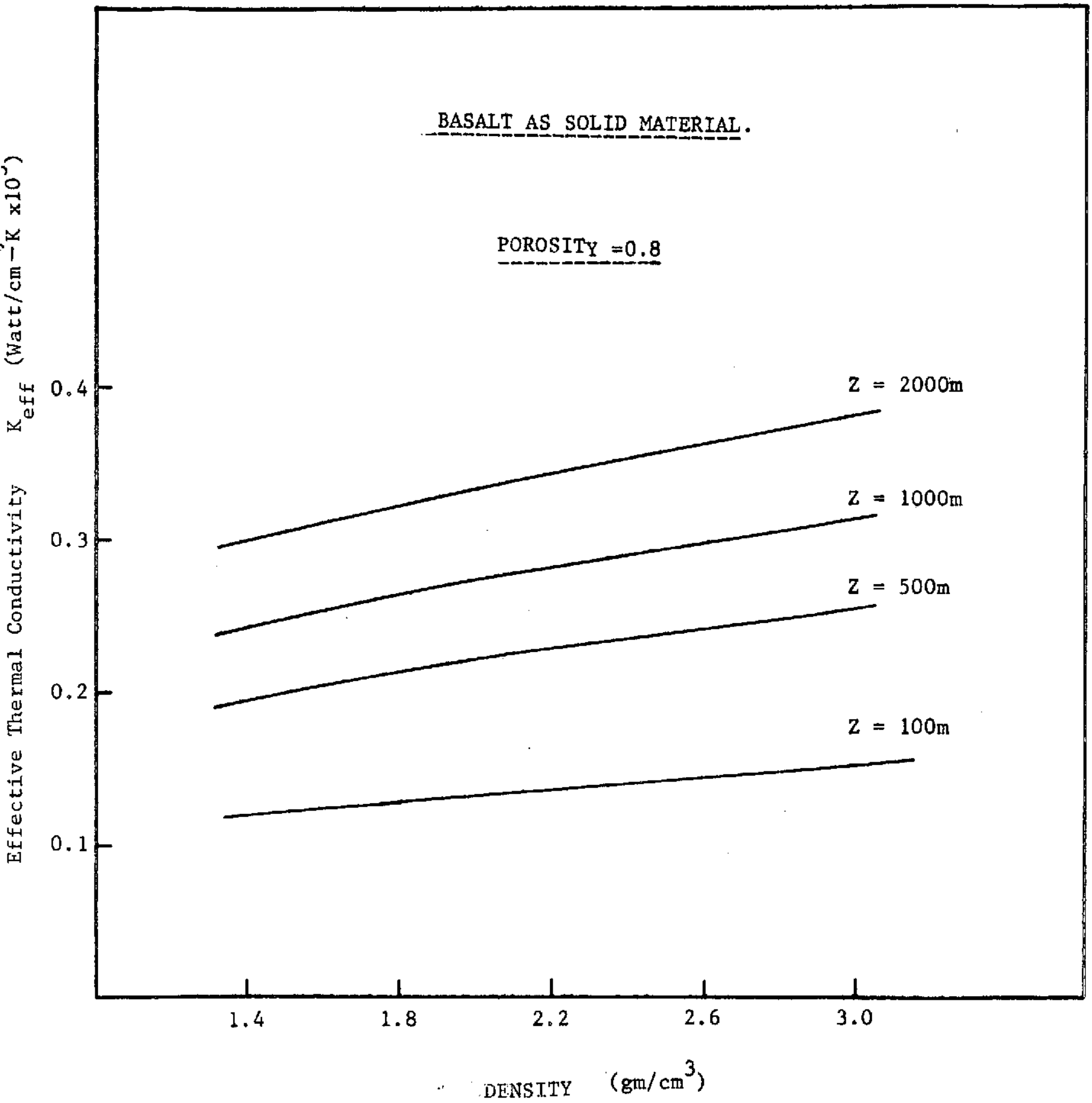

FIGURE 10: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DENSITY AT CONSTANT DEPTH FOR PART II(A).

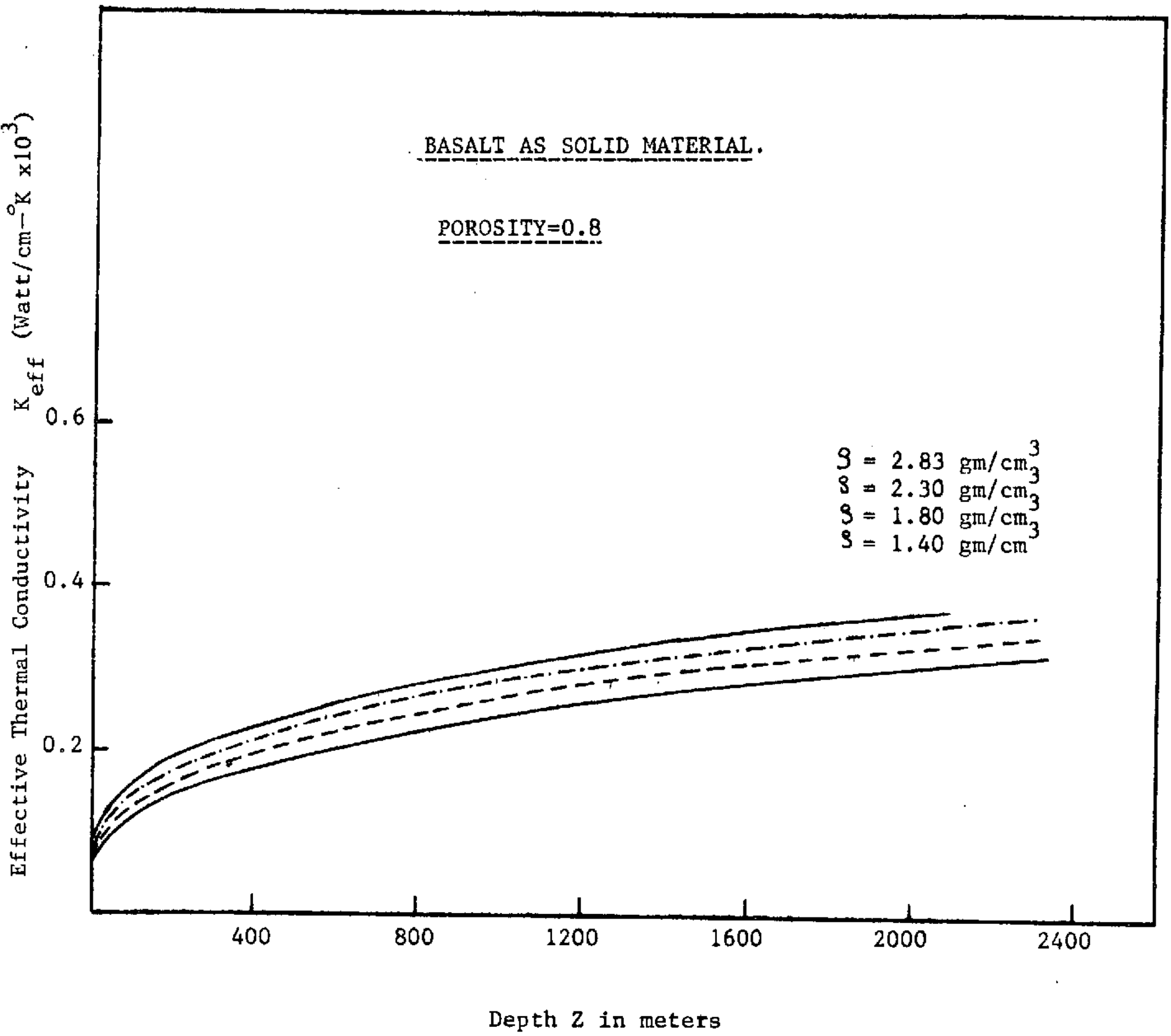

FIGURE 11: EFFECTIVE THERMAL CONDUCTIVITY AS A FUNCTION OF DEPTH AT CONSTANT DENSITY FOR PART II(A).

REFERENCES

(1) Cheng, S. C. and Vachon, R. I., "The Prediction of the Thermal Conductivity of Two and Three Phase Solid Heterogeneous Mixtures" Int. J. Heat Mass Transfer, Vol. 12, pp 249-264, (1969).

(2) F. L. Whipple, "A Comet Model II - Physical Relations for Comets and Meteors" Astrophysical Journal, Vol. 113, pp 464-474, (1951)

(3) M.G.J. Minnaret, "On the Temperature of the Cometary Nucleii" Proceeding of Academy of Sciences, A. M. Sterdam, #50, pp 826, (1947).

(4) Lentz, C. P. "Thermal Conductivity of Meats, Fats, Gelatine, Gels and Ice. Food Technology , 15, pp 243-244, 1961.

(5) M. S. Khader and R. I. Vachon, "A Theoretical Model for Lunar Surface Material Thermal Conductivity". ASME Journal of Heat Transfer, HT-35-1973.

(6) Eckert and Drake: Analysis of Heat & Mass Transfer. McGraw-Hill, 1972.