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by James A. Martin

Langley Research Center
Hampton, Va. 23665

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James A. Martin

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SUMMARY

Results are presented from a general analytical treatment of a single-stage vehicle with multiple propulsion phases. This analytical treatment includes a closed-form solution for the cost and for the performance and a derivation of the optimal phasing of the propulsion. Linearized variations in the inert weight elements are included, and the function to be minimized can be selected. The derivation of optimal phasing results in a set of nonlinear algebraic equations for optimal fuel volumes, for which a solution method is outlined. Three specific example cases are analyzed: minimum gross lift-off weight, minimum inert weight, and a minimized general function for a two-phase vehicle. The results for the two-phase vehicle are applied to the dual-fuel rocket. Comparisons with single-fuel vehicles indicate that dual-fuel vehicles can have lower inert weight either by development of a dual-fuel engine or by parallel burning of separate engines from lift-off.

INTRODUCTION

Recent and projected advances in the state of the art of various aerospace vehicle technologies may make single-stage vehicles attractive for missions requiring acceleration through large velocity increments. Some of the missions of interest are Earth-to-orbit transportation, long-range Earth transportation, and orbit-to-orbit transportation. The development of the staged-combustion rocket engine (ref. 1) is an example of an advance in aerospace propulsion.

An advance in conceptual design may have been accomplished by the introduction of the "mixed-mode" propulsion principle by Robert Salkeld (ref. 2). This propulsion principle, simply stated, is the combination of one propulsion mode (a propulsion mode is defined as a method of using the propulsion system) which results in low vehicle structural mass with a propulsion mode which results in low vehicle fuel mass to achieve a vehicle which has certain advantages over vehicles designed with either propulsion mode alone. One application of the mixed-mode propulsion principle is the dual-fuel rocket vehicle (refs. 2 and 3). In this case, the first fuel has a high density impulse and the second fuel has a high specific impulse. Such a vehicle could be operated in several phases (a phase being a portion of the trajectory during which all vacuum propulsion characteristics remain constant): in the first phase, only the first fuel is burned; in the intermediate phases, both fuels are used; and in the last phase, only the second fuel is burned. (See ref. 3.)

In reference 2, Salkeld presented a preliminary analytical treatment of the mixed-mode propulsion principle and numerical point-design data for a dual-fuel rocket vehicle. The present report gives an expanded analytical treatment and selected numerical results from the analysis. The analytical treatment of this report is expanded relative to that of reference 2 in three ways: inert weight is calculated as a function of the volumes of the fuels rather than being assumed fixed; the function to be minimized is not restricted to total fuel volume, as in Salkeld's analysis; and the number of phases and propulsion modes is unlimited. Some numerical results have been reported previously (ref. 4) for dual-fuel rockets. The results discussed in the present report are matched to point-design data and include both series-burn (one fuel burned after the other) and parallel-burn (both fuels burned initially) concepts.

Results of similar previous work (refs. 5 to 7) have been published for multistage vehicles, but they differ from results for the single-stage vehicle considered herein in that the mass of early stages is jettisoned and does not have as great an effect on the performance. Much work has been done on single-stage vehicles with multiple propulsion modes in which air-breathing engines are used during some phase of the flight. (See, for example, refs. 8 and 9.) In general, such vehicles do not use the mixed-mode propulsion principle; propulsion mode changes are dictated by flight regime rather than by a trade-off of structural and fuel mass properties. Air-breathing propulsion systems have high specific impulse and should be used for the final portion of the flight according to the mixed-mode principle, but such systems are limited to a maximum operating velocity.

SYMBOLS

A	group of input terms relative to a phase
C	cost
C_E	cost per unit engine weight for a phase
C_F	cost per unit fuel weight for a phase

C_{in}	cost per unit injected weight
C_j, C_l	cost coefficient of ignition weight for phase j or l
C_{lo}	cost per unit lift-off weight
C_T	cost per unit tank weight for a phase
C_0	fixed cost
C_0'	fixed cost and other costs which do not vary with V_j
E	engine weight coefficient, representing engine weight per unit ignition weight for a phase
f	fraction of injected weight which varies linearly with total injected weight
g	acceleration due to gravity
H	modified cost function
I_s	vacuum specific impulse of a phase
T	tank weight coefficient, representing tank weight per unit fuel weight for a phase
V	fuel volume for a phase
W_{FX}	fixed weight
W_G	weight of glider vehicle with no propulsion
W_{ign}	ignition weight for a phase
W_{in}	injected weight (vehicle inert weight plus payload), $W_G + W_{PI}$
W_{lo}	lift-off weight
W_P	payload weight

W_{PI}	propulsion inert weight, part of injected weight required for propulsion
Δv	total performance (ideal velocity increment in this analysis)
Δv_j	performance from phase j
λ	Lagrangian multiplier
ρ	fuel density for a phase

Subscripts:

J	total number of phases
j	phase 1, 2, 3, . . .
k	phases before or after phase j
l	phase in which ignition weight is the variable of differentiation

STATEMENT OF THE PROBLEM

The problem considered is that of finding the phasing of a multiphase single-stage vehicle which minimizes a cost function for a given performance. The models assumed for the vehicle, cost function, and performance are now described.

Vehicle Model

The vehicle gross lift-off weight W_{lo} is defined in terms of the weight elements as follows:

$$W_{lo} = W_{in} + \sum_{j=1}^{J} \rho_j V_j \tag{1}$$

where ρ_j is the fuel density and V_j is the fuel volume for phase j. Each phase is denoted by j, and J is the total number of phases. The injected weight W_{in} is the burnout weight of the final phase and is determined by

$$W_{in} = W_P + W_{FX} + \sum_{j=1}^{J} \left(E_j W_{ign,j} + T_j \rho_j V_j\right) + fW_{in} \tag{2}$$

The first two terms are the payload weight and the fixed weight, which do not vary. The engine weight of each phase $E_j W_{ign,j}$ is assumed to be a linear function of the ignition weight of the phase $W_{ign,j}$, and the nonnegative coefficient E_j includes the effects of desired vehicle thrust-to-weight ratio at ignition, the engine thrust-to-weight ratio, and the thrust structure. Similarly, the tank weight of each phase $T_j \rho_j V_j$ is assumed to be a linear function of the fuel weight of the phase, and the nonnegative coefficient T_j includes the effects of tankage, body structure, and propellant lines. The last term fW_{in} (where $0 \leq f < 1$) is recovery weight and represents aerodynamic surfaces, maneuver and control propellants, and other weight elements which vary with injected weight itself. The ignition weight of each phase, which includes the injected weight and the fuel of the remaining phases indicated by the subscript k, is expressed as

$$W_{ign,j} = W_{in} + \sum_{k=j}^{J} \rho_k V_k \tag{3}$$

Note that $W_{ign,1} = W_{lo}$.

Cost Model

The cost function can be written as

$$C = \sum_{j=1}^{J} \left(C_{E,j} E_j W_{ign,j} + C_{T,j} T_j \rho_j V_j + C_{F,j} \rho_j V_j\right) + C_{lo} W_{lo} + C_{in} f W_{in} + C_0 \tag{4}$$

The first group of terms represent the cost of engines, tanks, and fuel for each of the phases. The remaining terms represent costs proportional to W_{lo}, the recovery weight, and a constant. This cost function is sufficiently versatile to allow the minimization of W_{lo}, inert weight, fuel costs, development costs, or any other linear function by choosing the coefficients appropriately. Although the W_{lo} term could be divided among the other terms, having W_{lo} as a separate cost item is convenient for costs proportional to W_{lo}.

Performance Model

The performance is measured by the total ideal velocity increment as follows:

$$\Delta v = \sum_{j=1}^{J} \Delta v_j = \sum_{j=1}^{J} g I_{s,j} \ln \frac{W_{ign,j}}{W_{ign,j} - \rho_j V_j} \tag{5}$$

For a single-stage vehicle, the burnout weight of a phase is the ignition weight of the next phase.

Since the form of the performance equation is the same as that of the Breguet range equation, the analysis is also applicable to the multiphase cruise problem if $gI_{s,j}$ is replaced by the Breguet factor. The Breguet factor may vary, but an average value can be used over short phases. Although this paper does not emphasize the cruise problem, the results indicate that multiple fuels can be used to advantage for cruise vehicles.

DERIVATION OF EQUATIONS

Expressions for Cost, Injected Weight, and Performance in Closed Form

The cost, the injected weight, and the performance are expressed in the following derivation in terms of a common set of independent variables in closed form. This derivation is useful in understanding the results and simplifies the optimization in subsequent sections.

The injected weight can be considered to consist of two terms, as follows:

$$W_{in} = W_G + W_{PI} \tag{6}$$

This relation comes from solving equation (2) for W_{in} and is illustrated in figure 1. The glider weight W_G represents the payload, fixed weight, and corresponding recovery weight and is given by

$$W_G = \frac{W_P + W_{FX}}{1 - f} \tag{7}$$

The glider would be the entire vehicle if no propulsion were required, and the glider weight does not vary for the purposes of this analysis. The propulsion inert weight W_{PI} includes the engines, tanks, and corresponding recovery weight and is given by

$$W_{PI} = \sum_{j=1}^{J} \left(\frac{E_j}{1 - f} W_{ign,j} + \frac{T_j}{1 - f} \rho_j V_j \right) \tag{8}$$

The effect of the recovery weight is now in the form of increases in the other inert weight elements.

The natural independent variables from a physical standpoint are the fuel volumes V_j, $j = 1, 2, \ldots, J$. Mathematically, however, the ignition weights of each phase $W_{ign,j}$ are more convenient. Equation (3) gives $W_{ign,j}$ in terms of V_j, and V_j can be found in terms of $W_{ign,j}$ from

$$W_{ign,j+1} = W_{ign,j} - \rho_j V_j \tag{9}$$

to be

$$V_j = \frac{W_{ign,j} - W_{ign,j+1}}{\rho_j} \qquad (j = 1, 2, \ldots, J - 1) \tag{10}$$

and

$$V_J = \frac{W_{ign,J} - W_{in}}{\rho_J} \tag{11}$$

Eliminating V_j in equation (8) by use of equations (10) and (11) gives

$$W_{PI} = \sum_{j=1}^{J} \frac{E_j}{1 - f} W_{ign,j} + \sum_{j=1}^{J} \frac{T_j}{1 - f} W_{ign,j} - \sum_{j=1}^{J-1} \frac{T_j}{1 - f} W_{ign,j+1} - \frac{T_J}{1 - f} W_{in} \tag{12}$$

If each index is reduced by 1 and the limits are increased by 1, the third summation can be rewritten in terms of $W_{ign,j}$ to give a term

$$- \sum_{j=2}^{J} \frac{T_{j-1}}{1 - f} W_{ign,j}$$

This summation term may be combined with the first two summations of equation (12), except for the $j = 1$ terms, and then equation (6) can be rewritten

$$W_{in} = W_G + \frac{E_1 + T_1}{1 - f} W_{lo} + \sum_{j=2}^{J} \frac{E_j + T_j - T_{j-1}}{1 - f} W_{ign,j} - \frac{T_J}{1 - f} W_{in} \tag{13}$$

Solving for W_{in} gives

$$W_{in} = \frac{W_G}{1 + \frac{T_J}{1 - f}} + A_1 W_{lo} + \sum_{j=2}^{J} A_j W_{ign,j} \tag{14}$$

where

$$A_1 = \frac{E_1 + T_1}{1 - f + T_J} \tag{15}$$

and

$$A_j = \frac{E_j + T_j - T_{j-1}}{1 - f + T_J} \qquad (j = 2, 3, \ldots, J) \tag{16}$$

Similar operations on the cost function (eq. (4)) lead to

$$C = C_0 + \left(C_{lo} + C_{E,1}E_1 + C_{T,1}T_1 + C_{F,1}\right)W_{lo} + \sum_{j=2}^{J} \left(C_{E,j}E_j + C_{T,j}T_j - C_{T,j-1}\right.$$

$$\left. + C_{F,j-1}\right)W_{ign,j} + \left(C_{in}f - C_{T,J}T_J - C_{F,J}\right)W_{in} \tag{17}$$

Substituting equation (14) into equation (17) and collecting all coefficients of the independent variable gives

$$C = C_0' + C_1 W_{lo} + \sum_{j=2}^{J} C_j W_{ign,j} \tag{18}$$

where

$$C_0' = C_0 + \left(C_{in}f - C_{T,J}T_J - C_{F,J}\right)\frac{W_G}{1 + \frac{T_J}{1 - f}} \tag{19}$$

$$C_1 = C_{lo} + C_{E,1}E_1 + C_{T,1}T_1 + C_{F,1} + \left(C_{in}f - C_{T,J}T_J - C_{F,J}\right)A_1 \tag{20}$$

and

$$C_j = C_{E,j}E_j + C_{T,j}T_j - C_{T,j-1}T_j + C_{F,j} - C_{F,j-1} + \left(C_{in}f\right.$$

$$\left. - C_{T,J}T_J - C_{F,J}\right)A_j \qquad (j = 2, 3, \ldots, J) \tag{21}$$

The expression for performance (eq. (5)) can be rewritten

$$\Delta v = \sum_{j=1}^{J} gI_{s,j} \ln W_{ign,j} - \sum_{j=1}^{J} gI_{s,j} \ln \left(W_{ign,j} - \rho_j V_j\right) \tag{22}$$

Eliminating V_j gives

$$\Delta v = gI_{s,1} \ln W_{lo} + \sum_{j=2}^{J} gI_{s,j} \ln W_{ign,j} - \sum_{j=1}^{J-1} gI_{s,j} \ln W_{ign,j+1} - gI_{s,J} \ln W_{in} \tag{23}$$

Rewriting the second summation term as a function of $W_{ign,j}$ and combining the two summation terms gives

$$\Delta v = gI_{s,1} \ln W_{lo} + \sum_{j=2}^{J} g\left(I_{s,j} - I_{s,j-1}\right) \ln W_{ign,j} - gI_{s,J} \ln W_{in} \tag{24}$$

Substituting for W_{in} from equation (14) gives Δv as a function of $W_{ign,j}$; however, equation (24) is a more convenient form for the optimization.

General Optimum Phasing

The cost may be minimized subject to the constraint of fixed Δv by writing

$$H = C + \lambda\left[gI_{s,1} \ln W_{1o} + \sum_{j=2}^{J} g\left(I_{s,j} - I_{s,j-1}\right) \ln W_{ign,j} - gI_{s,J} \ln W_{in} - \Delta v\right] \tag{25}$$

with the understanding that W_{in} is defined by equation (14). The optimum is then found by taking the derivative of H with respect to each independent variable and setting each derivative equal to zero. The J equations are, with l denoting the variable of differentiation,

$$\frac{\partial H}{\partial W_{1o}} = 0 = C_1 + \lambda\left[gI_{s,1} \frac{1}{W_{1o}} - gI_{s,J} \frac{1}{W_{in}} A_1\right] \tag{26}$$

$$\frac{\partial H}{\partial W_{ign,l}} = 0 = C_l + \lambda\left[g\left(I_{s,l} - I_{s,l-1}\right) \frac{1}{W_{ign,l}} - gI_{s,J} \frac{1}{W_{in}} A_l\right] \qquad (l = 2, 3, \ldots, J) \tag{27}$$

Solving equation (26) for λ and substituting the resulting expression into equation (27) gives, upon canceling g and returning to the subscript j,

$$0 = C_j - \left(\frac{C_1}{\dfrac{I_{s,1}}{W_{1o}} - \dfrac{I_{s,J}A_1}{W_{in}}}\right)\left(\frac{I_{s,j} - I_{s,j-1}}{W_{ign,j}} - \frac{I_{s,J}A_j}{W_{in}}\right) \qquad (j = 2, 3, \ldots, J) \tag{28}$$

These $J - 1$ equations, with equation (14) and either equation (24) or equation (5), determine the $J + 1$ unknowns W_{1o}, W_{in}, and $W_{ign,j}$ for $j = 2, 3, \ldots, J$. The equations are nonlinear and no general explicit solution is available, although explicit solutions for specific cases are shown in subsequent sections. Rewriting equation (28) reveals that $W_{ign,j}/W_{in}$ is determined explicitly for $j = 2, 3, \ldots, J$ by the parameter W_{1o}/W_{in}:

$$\frac{W_{ign,j}}{W_{in}} = \frac{I_{s,j} - I_{s,j-1}}{\left(I_{s,1} \dfrac{W_{in}}{W_{1o}} - I_{s,J}A_1\right) \dfrac{C_j}{C_1} + I_{s,J}A_j} \qquad (j = 2, 3, \ldots, J) \tag{29}$$

If W_{in}/W_{1o} is assumed, $W_{ign,j}/W_{in}$ can be calculated from equation (29). The Δv corresponding to the assumed W_{in}/W_{1o} can then be calculated from equation (5), since

$$\frac{W_{ign,j}}{W_{ign,j} - \rho_j V_j} = \frac{W_{lo}/W_{in}}{W_{ign,2}/W_{in}} \qquad (j = 1) \qquad (30a)$$

$$\frac{W_{ign,j}}{W_{ign,j} - \rho_j V_j} = \frac{W_{ign,j}/W_{in}}{W_{ign,j+1}/W_{in}} \qquad (j = 2, 3, \ldots, J-1). \qquad (30b)$$

$$\frac{W_{ign,j}}{W_{ign,j} - \rho_j V_j} = \frac{W_{ign,J}}{W_{in}} \qquad (j = J) \qquad (30c)$$

A one-dimensional search is required to find the W_{in}/W_{lo} which produces the desired Δv. Once the correct W_{in}/W_{lo} and the corresponding $W_{ign,j}/W_{in}$ are found, equation (14) can be used to determine the actual value of W_{in} (and therefore $W_{ign,j}$ and W_{lo}) for the given glider weight. The resulting minimum cost can then be calculated from equation (18), and V_j can be calculated from equations (10) and (11) for $j = 1, 2, \ldots, J$.

An alternate solution method is graphical, and this graphical method is used in the section "Application to Dual-Fuel Rocket Vehicle." The graphical method requires the following steps:

(1) Assume a value of W_{in}/W_{lo}.

(2) Calculate $W_{ign,j}/W_{lo}$ from equation (29).

(3) Calculate Δv from equation (5).

(4) Calculate W_{in} from equation (14).

(5) Calculate the cost from equation (18) or calculate other desired quantities.

(6) Plot the quantities of interest as a function of Δv.

(7) Increment W_{in}/W_{lo} and repeat.

When the cost or other quantities have been plotted as a function of Δv, the value at the required Δv can be read. Also, this method provides an insight into the variations of the solution with variations in the Δv requirement.

Minimum Lift-Off Weight

For minimum W_{lo}, choose

$$C_{lo} = 1 \qquad (31)$$

and

$$C_{E,j} = C_{T,j} = C_{F,j} = C_{in} = C_0 = 0 \tag{32}$$

which results in the cost function (eq. (4)) becoming

$$C = W_{lo} \tag{33}$$

Then, from equations (20) and (21), $C_1 \neq 0$ and

$$C_j = 0 \qquad (j = 2, 3, \ldots, J) \tag{34}$$

In this case equation (27) can be solved explicitly to give

$$\frac{W_{ign,l}}{W_{in}} = \frac{I_{s,l} - I_{s,l-1}}{I_{s,J}A_l} \qquad (l = 2, 3, \ldots, J) \tag{35}$$

Then $W_{lo}/W_{ign,2}$ can be found from the Δv constraint (eq. (5)). The interesting conclusion shown by this solution is that the ideal velocity performance increments Δv_j of all but the first phase are invariant with total ideal velocity. Only the first-phase ideal velocity changes to meet the Δv constraint.

Minimum Inert Weight

Since the payload weight is fixed, minimum inert weight corresponds to minimum injected weight W_{in}. The cost function represents injected weight if

$$C_{in} = \frac{1}{f} \tag{36}$$

and

$$C_{E,j} = C_{T,j} = C_{F,j} = C_{lo} = C_0 = 0 \tag{37}$$

which results in the cost function

$$C = W_{in} \tag{38}$$

From equations (20) and (21), this cost function gives

$$C_1 = A_1 \tag{39}$$

and

$$C_j = A_j \qquad (j = 2, 3, \ldots, J) \tag{40}$$

In this case the constant terms in the denominator of equation (29) cancel, and W_{in} can then be canceled to give

$$\frac{W_{ign,l}}{W_{lo}} = \frac{I_{s,l} - I_{s,l-1}}{I_{s,1}\frac{A_l}{A_1}} \qquad (l = 2, 3, \ldots, J) \tag{41}$$

Then $W_{in}/W_{ign,J}$ can be found from the Δv constraint. The interesting conclusion shown by this solution is that the ideal velocity performance increments Δv_j of all but the last phase are invariant with total ideal velocity. Only the last-phase ideal velocity changes to meet the Δv constraint.

Two-Phase Vehicle

Most examinations of multiphase vehicles would start with a two-phase vehicle. In this case the relationship (29), which represents $J - 1$ equations, results in the single equation

$$\frac{W_{ign,2}}{W_{in}} = \frac{I_{s,2} - I_{s,1}}{\left(I_{s,1}\frac{W_{in}}{W_{ign,2}}\frac{W_{ign,2}}{W_{lo}} - I_{s,2}A_1\right)\frac{C_2}{C_1} + I_{s,2}A_2} \tag{42}$$

Also, equation (5) becomes

$$\Delta v = gI_{s,1}\ln\frac{W_{lo}}{W_{ign,2}} + gI_{s,2}\ln\frac{W_{ign,2}}{W_{in}} \tag{43}$$

These two equations determine the two unknowns $W_{lo}/W_{ign,2}$ and $W_{ign,2}/W_{in}$. An iterative process in which one of these ratios is assumed or a graphical method can be used to solve for the two weight ratios. The procedure is the same as outlined for the multiphase case. When values are found for these two weight ratios, the actual weights can be found from equation (14) which becomes

$$W_{in} = \frac{W_G}{1 + \frac{T_2}{1 - f}} + A_1W_{lo} + A_2W_{ign,2} \tag{44}$$

Solving for W_{in} in terms of these two weight ratios yields

$$W_{in} = \frac{W_G}{\left(1 + \frac{T_2}{1 - f}\right)\left(1 - A_1\frac{W_{lo}}{W_{ign,2}}\frac{W_{ign,2}}{W_{in}} - A_2\frac{W_{ign,2}}{W_{in}}\right)} \tag{45}$$

Thus, for a fixed value of W_G, W_{in} is determined, which in turn determines W_{lo} and $W_{ign,2}$. The cost can be calculated when the actual weights are known from equation (18), which in this case becomes

$$C = C_0 + C_1 W_{1o} + C_2 W_{ign,2} \tag{46}$$

The groups of input terms for the two-phase case, from equations (15), (16), (19), (20), and (21), become

$$A_1 = \frac{E_1 + T_1}{1 - f + T_2} \tag{47}$$

$$A_2 = \frac{E_2 + T_2 - T_1}{1 - f + T_2} \tag{48}$$

$$C_0' = C_0 + \left(C_{in} f - C_{T,2} T_2 - C_{F,2}\right) \frac{W_G}{1 + \frac{T_2}{1 - f}} \tag{49}$$

$$C_1 = C_{1o} + C_{E,1} E_1 + C_{T,1} T_1 + C_{F,1} + \left(C_{in} f - C_{T,2} T_2 - C_{F,2}\right) A_1 \tag{50}$$

and

$$C_2 = C_{E,2} E_2 + C_{T,2} T_2 - C_{T,1} T_2 + C_{F,2} - C_{F,1} + \left(C_{in} f - C_{T,2} T_2 - C_{F,2}\right) A_2 \tag{51}$$

Results of Derivation

An analysis of a single-stage vehicle with multiphase propulsion has been performed with a general cost function. Linearized variations of the injected weight with the fuel loading of each phase were assumed. As a consequence of the linearization, a closed-form solution was found for the ideal velocity performance and the cost function as a function of any given set of values for either the fuel loading of each phase or the ignition weight of each phase.

The cost function was then minimized subject to a constraint on the ideal velocity. The result was a set of nonlinear equations which have not been solved explicitly, but an implicit method of solving the equations has been found; the implicit method requires either a one-dimensional iteration or a graphical method.

Explicit minimum-cost solutions were found for the specific cost functions of gross weight and inert weight. For these cost functions, the ideal velocity increment of all but one phase is invariant with the total ideal velocity requirement. For minimum gross weight, only the first-phase ideal velocity increment changes to meet the total ideal velocity requirement. For minimum inert weight, only the last-phase ideal velocity increment changes to meet the total ideal velocity requirement.

The results of the general optimum-phasing analysis were also written specifically for the two-phase vehicle. For the two-phase vehicle, the total ideal velocity requirement and one equation governing the condition for optimality must be solved for two unknowns which are essentially the ideal velocity increments of both phases.

APPLICATION TO DUAL-FUEL ROCKET VEHICLE

The results of the optimum-phasing analysis for a two-phase vehicle with a general cost function have been programed for a Wang 720C programable calculator and 702 plotting output writer. The programs have been used to calculate and to plot weight and fuel-cost parameters for vehicles with single-fuel and dual-fuel rocket propulsion.

The fuels which were considered are hydrogen and RJ-5, a cyclical synthetic hydrocarbon similar to kerosene; both fuels are burned with liquid oxygen. The coefficients which were used are summarized in table I. The values of the coefficients were chosen to match the results of unpublished point-design data for single-fuel and dual-fuel vehicles and to minimize inert weight or fuel cost.

Minimum Inert Weight

Figure 2 shows a comparison of a dual-fuel rocket vehicle with similar single-fuel rocket vehicles using two fuels separately. (The design Δv calculations in the data figures were made in U.S. Customary Units.)

The glider weight is representative of the payload, and W_G/W_{in} is greatest for minimum inert weight. This ratio is plotted as a function of the ideal velocity increment for which the vehicle is to be designed. The vehicle that uses only RJ-5 fuel has the highest (best) value of W_G/W_{in} at low design ideal velocity increments, but the value drops rapidly as design Δv increases. The hydrogen-fueled vehicle has a better value of W_G/W_{in} than the RJ-5 vehicle at high design velocities. The dual-fuel vehicle essentially combines the best of both fuels by using RJ-5 at first and then switching to hydrogen at the design Δv indicated as the beginning of dual fuel. This method of using the fuels could result in a curve which has the shape of the hydrogen curve and an initial point on the RJ-5 curve, and such a curve would represent the potential of a dual-fuel vehicle. The dual-fuel vehicle does not achieve its full potential because it must carry not only the RJ-5 burning engines to use at lift-off but also some provision to burn the hydrogen later in the trajectory. The dual-fuel vehicle in figure 2 utilizes a dual-fuel engine which allows hydrogen to be burned in some of the RJ-5 engines (ref. 3). The hardware required for the hydrogen-burning capability is a penalty that is shown as the difference between the initial point of the dual-fuel curve and the RJ-5 curve at that design velocity. The dual-fuel vehicle has a better value of W_G/W_{in} at moderate and high design velocities than either single-fuel vehicle.

Figure 3 shows the weight breakdown into components of the dual-fuel vehicle as a function of Δv. As noted in the section "Minimum Inert Weight," the first-phase ratio of fuel weight to lift-off weight $\left(W_{lox-RJ-5}/W_{lo}\right)$ is not a function of design Δv. This fact can be understood by referring to figure 2 and noting that the slope of the hydrogen curve was always better (i.e., less negative) than the slope of the RJ-5 curve at design velocities greater than that denoted by the point as the beginning of dual fuel. Any velocity above the denoted point should, therefore, be gained with hydrogen. Figure 3 also shows that no dual-fuel optimum solution exists below the denoted point; the vehicle using RJ-5 is better than any dual-fuel vehicle in that regime.

The data shown so far have been generated by assuming the use of a dual-fuel engine which burns both fuels in the same engine sequentially. Development of such a dual-fuel engine could lead to difficulties and development costs. Therefore, dual-fuel vehicles which do not have dual-fuel engines should be considered. Without dual-fuel engines, complete separate engines must be carried to burn the second fuel. If the engines are burned in series, the penalty for the second set of engines is severe. Figure 4 presents results for separate engines and series burn compared with the results for the dual-fuel engine. The region of no dual-fuel optimal solution increases if separate engines are used, but the penalty for the second set of engines is still so great that the single-fuel hydrogen vehicle provides a better value of W_G/W_{in}. Using separate engines in parallel is discussed in a subsequent section.

Minimum Fuel Cost

Figure 5 shows the weight breakdown into components of the dual-fuel vehicle as a function of Δv for minimum fuel cost. The fuel and oxygen cost per unit weight for the second phase was assumed to be 1.7 times that for the first phase, which is representative if the first fuel is a hydrocarbon such as kerosene. If the first fuel is a synthetic such as RJ-5, the cost would be much greater. Reference 4 includes comparisons of fuels with widely varying costs. The only difference between figure 5 and figure 3 is that the cost function was changed from inert weight to fuel cost. The result of this change was that the first-phase ratio of fuel weight to lift-off weight $\left(W_{lox-RJ-5}/W_{lo}\right)$ was greater in figure 5 at low design Δv. At high design Δv, the phase split approaches and finally equals the minimum-inert-weight split.

PARALLEL BURN

A distinctly different mode of operation for a dual-fuel vehicle, parallel burn, was also investigated. For parallel-burn operation, separate engines are used, rather than dual-fuel engines, and all engines are ignited at lift-off. The optimization procedure of this report is not applicable to the parallel-burn mode of operation; therefore, a parametric optimization was used in conjunction with the closed-form solution. The parametric optimization consisted of selecting several values for $W_{ign,2}/W_{lo}$ (where the subscript 2 implies the phase after the hydrocarbon engines are shut down) and plotting the resulting values of W_G/W_{in} as a function of Δv. The results of this optimization showed that the optimum phasing required a value of approximately 0.6 for $W_{ign,2}/W_{lo}$. The curve for parallel burn with $W_{ign,2}/W_{lo} = 0.6$ is compared with the dual-fuel-engine curve in figure 6. The figure shows that there is little difference between the two modes of operation. For example, at $\Delta v = 8992$ m/sec (29 500 ft/sec), the value of W_G/W_{in} was 0.3305 for the vehicle with dual-fuel engines and 0.3310 for the vehicle with parallel burn. The closeness of these values of W_G/W_{in} indicates that dual-fuel rocket vehicles with parallel-burn propulsion systems should be considered in more depth.

SUMMARY OF RESULTS AND CONCLUSIONS

The ideal velocity increment (total performance) of a single-stage vehicle with multiphase propulsion was analyzed with a general cost function. Linearized variations of the injected weight with the fuel loading of each phase were assumed. As a consequence, a closed-form solution for the cost and for the performance as a function of any given fuel loading was obtained.

The fuel loading of each phase was then optimized, and the result was a set of simultaneous, nonlinear, algebraic equations. An implicit solution method was outlined. When either gross weight or inert weight was minimized, the solution was found explicitly. The derivation results were written in some detail for the vehicle limited to two phases while the generality of the cost function was maintained. The two-phase vehicle results were programed and applied to the dual-fuel rocket vehicle.

The results of the analysis suggest the following specific conclusions:

1. When multiphase vehicles are designed for minimum gross lift-off weight, the ideal velocity increment of all but the first phase is invariant with total ideal velocity increment.

2. When multiphase vehicles are designed for minimum inert weight, the ideal velocity increment of all but the last phase is invariant with total ideal velocity increment.

The results of the application to dual-fuel rocket vehicles and of the parallel-burn analysis are summarized as follows:

1. Vehicles with dual-fuel rocket propulsion can have lower inert weight than similar vehicles with single-fuel propulsion at moderate and high design velocity increments.

2. The requirement for the second set of engines significantly penalizes the series-burn dual-fuel vehicle if a dual-fuel engine is not used.

3. The parallel-burn dual-fuel vehicle appears attractive and does not need a dual-fuel engine.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., October 7, 1974.

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TABLE I. - COEFFICIENT INPUTS[a]

Propulsion system	E	T	I_s, sec
Single fuel:			
RJ-5	0.0225	0.0224	348
Hydrogen	0.0326	0.0419	465
Dual fuel:			
RJ-5	0.0225	0.0224	341
Hydrogen, dual-fuel engine/separate engine	0.0061/0.0326	0.0397	462
Parallel-burn phase	[b]0.0264	0.0290	387

[a]The following coefficients, which did not vary with propulsion system, were also used in this analysis:

$$f = 0.328$$

Minimum inert weight:

$$C_{in} = \frac{1}{f}; \quad \text{All other cost coefficients} = 0$$

Minimum fuel cost:

$$C_{F,1} = 1.0; \quad C_{F,2} = 1.7; \quad \text{All other cost coefficients} = 0$$

[b]No additional engine weight is added for hydrogen-alone burn.

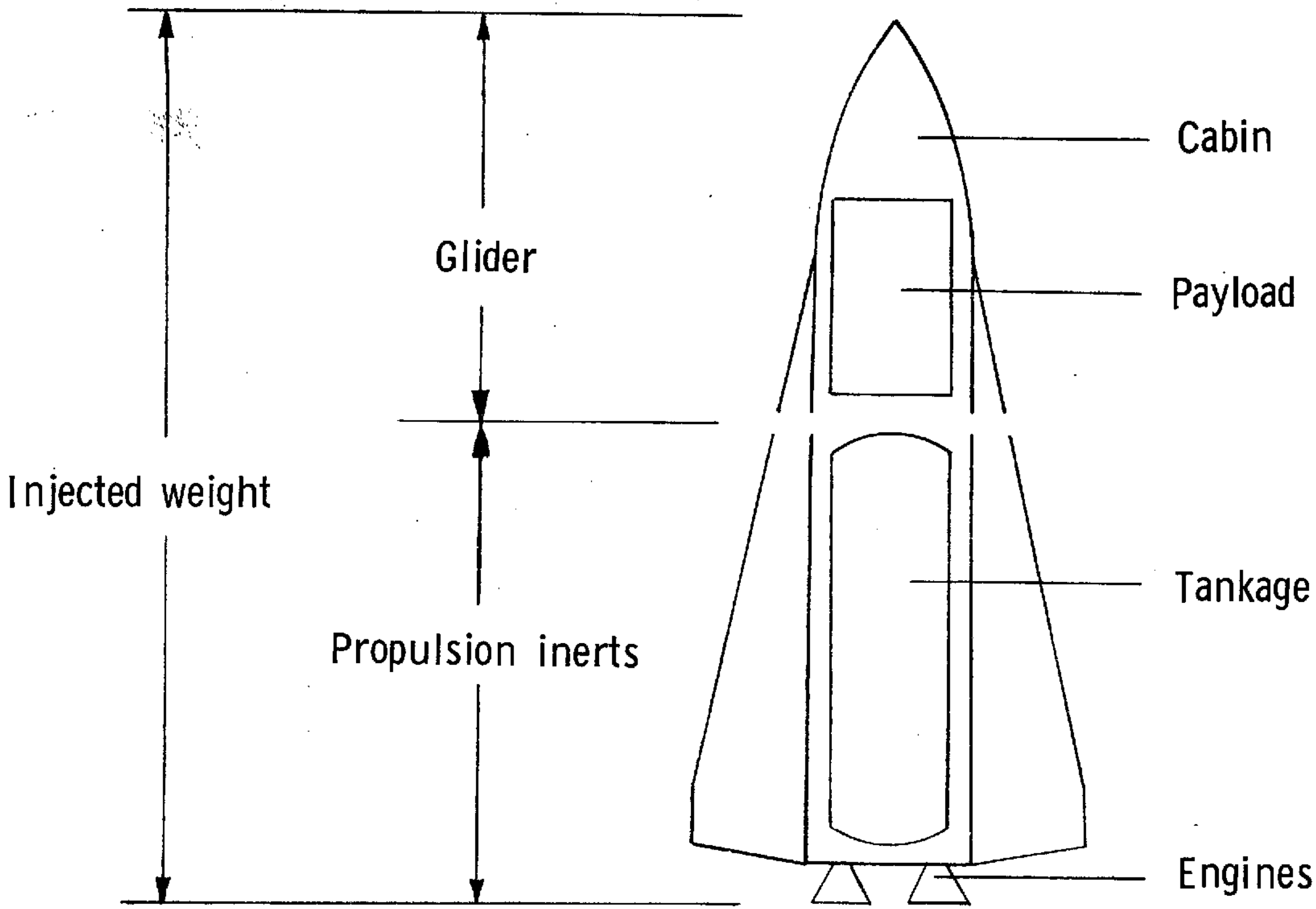

Figure 1.- Sketch of division of injected weight.

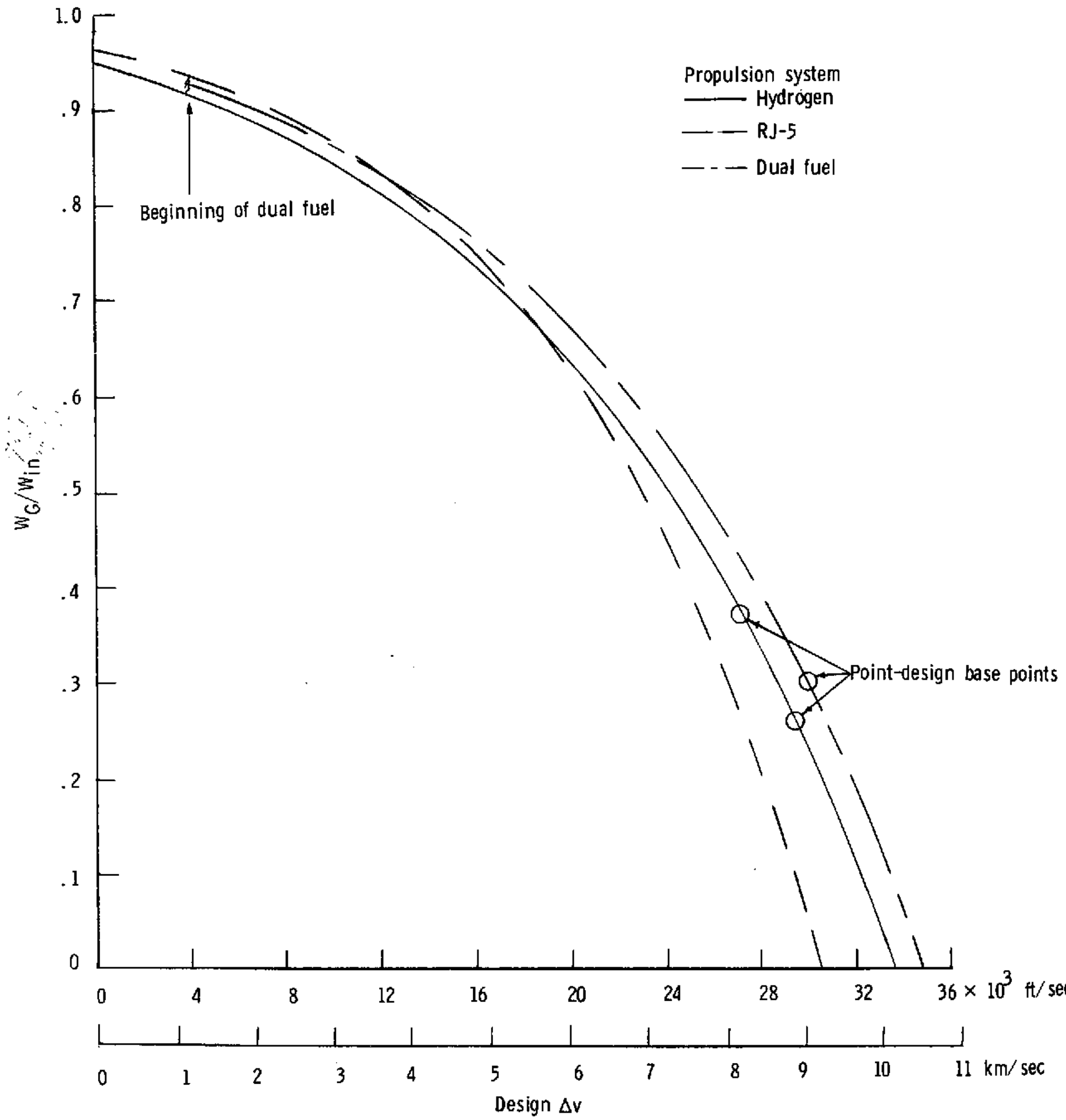

Figure 2.- Comparison of single-fuel and dual-fuel propulsion systems in single-stage vehicle.

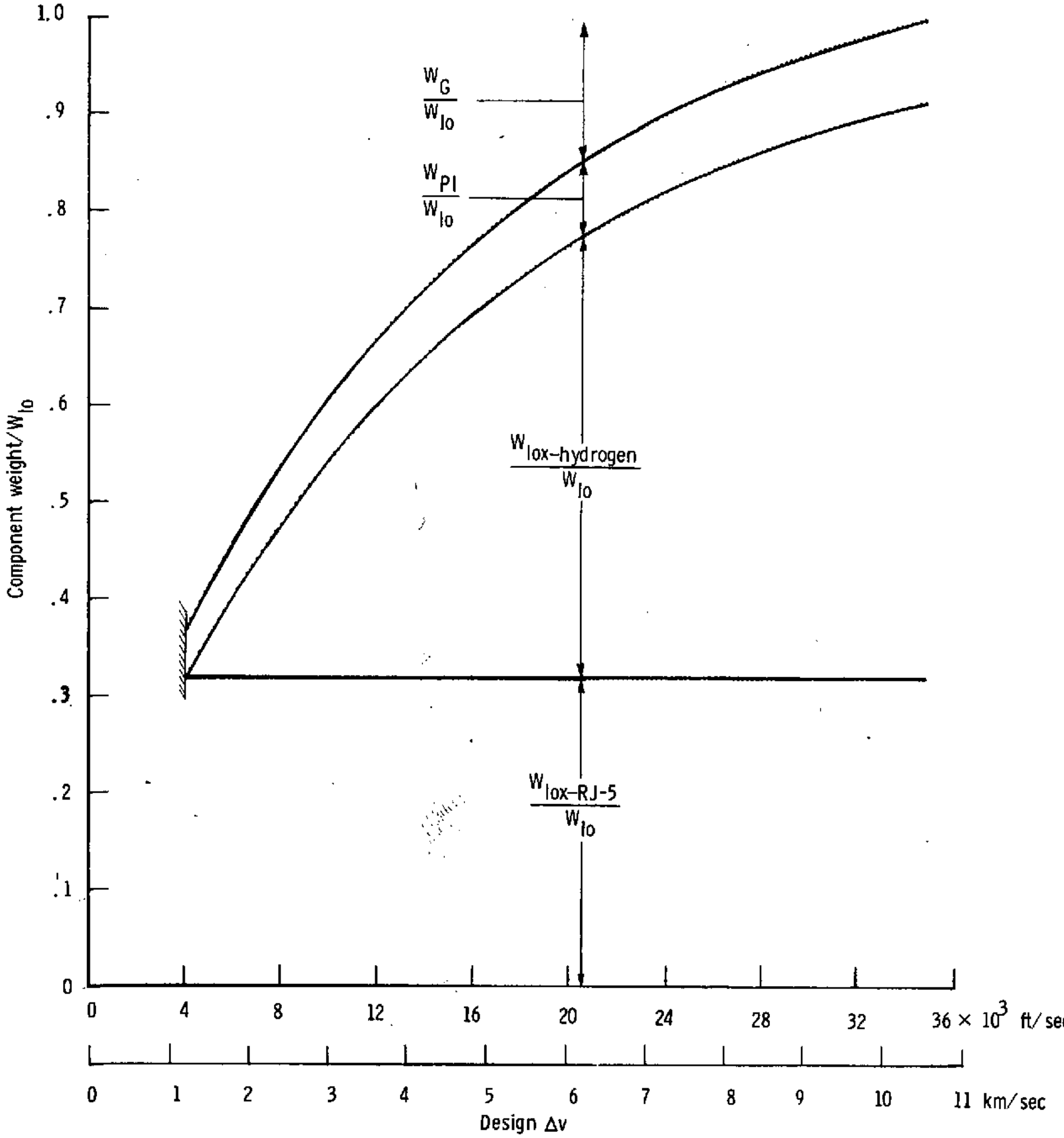

Figure 3.- Dual-fuel weight breakdown for minimum inert weight.

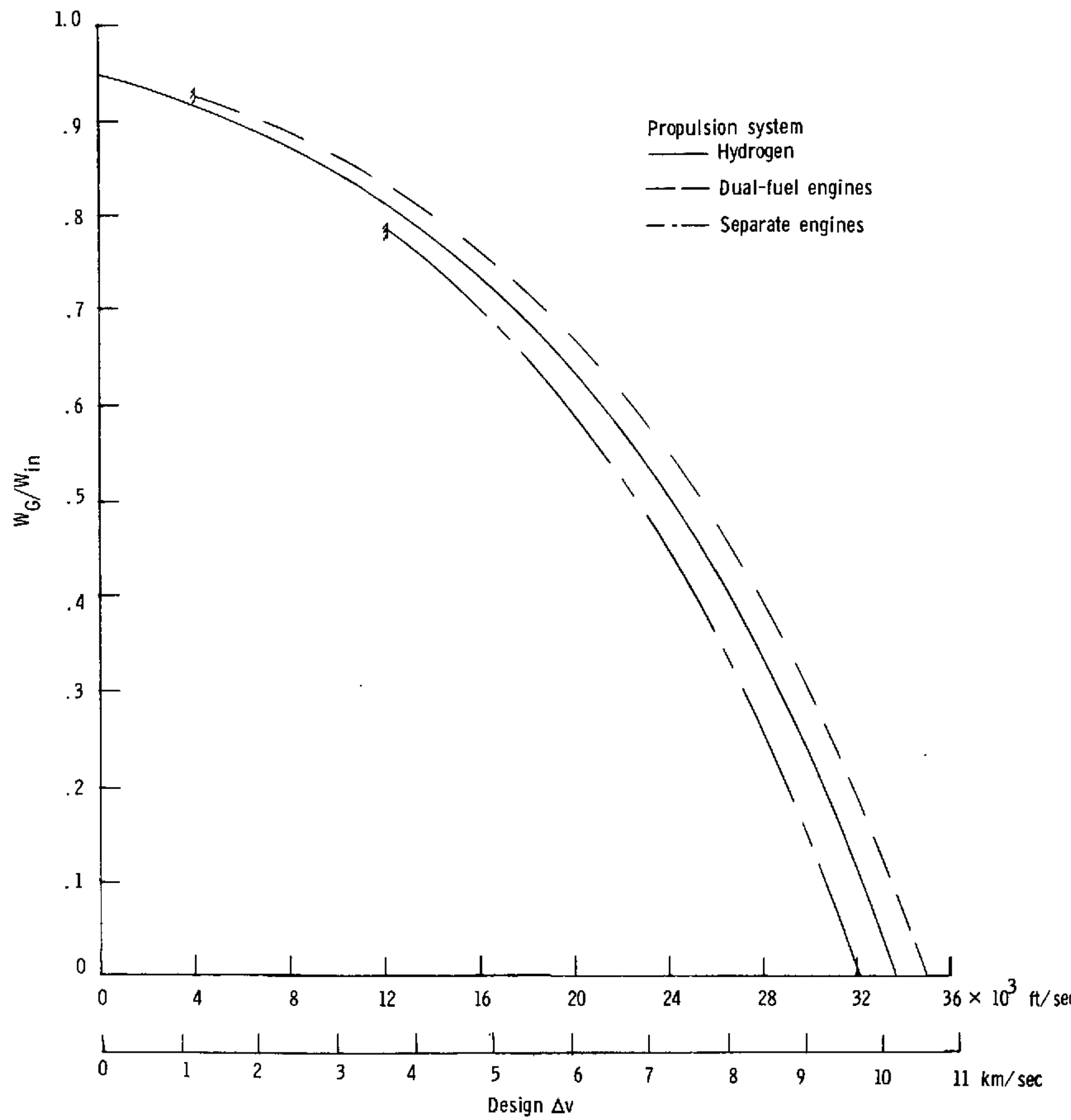

Figure 4.- Comparison of dual-fuel engine and separate engines with series burn.

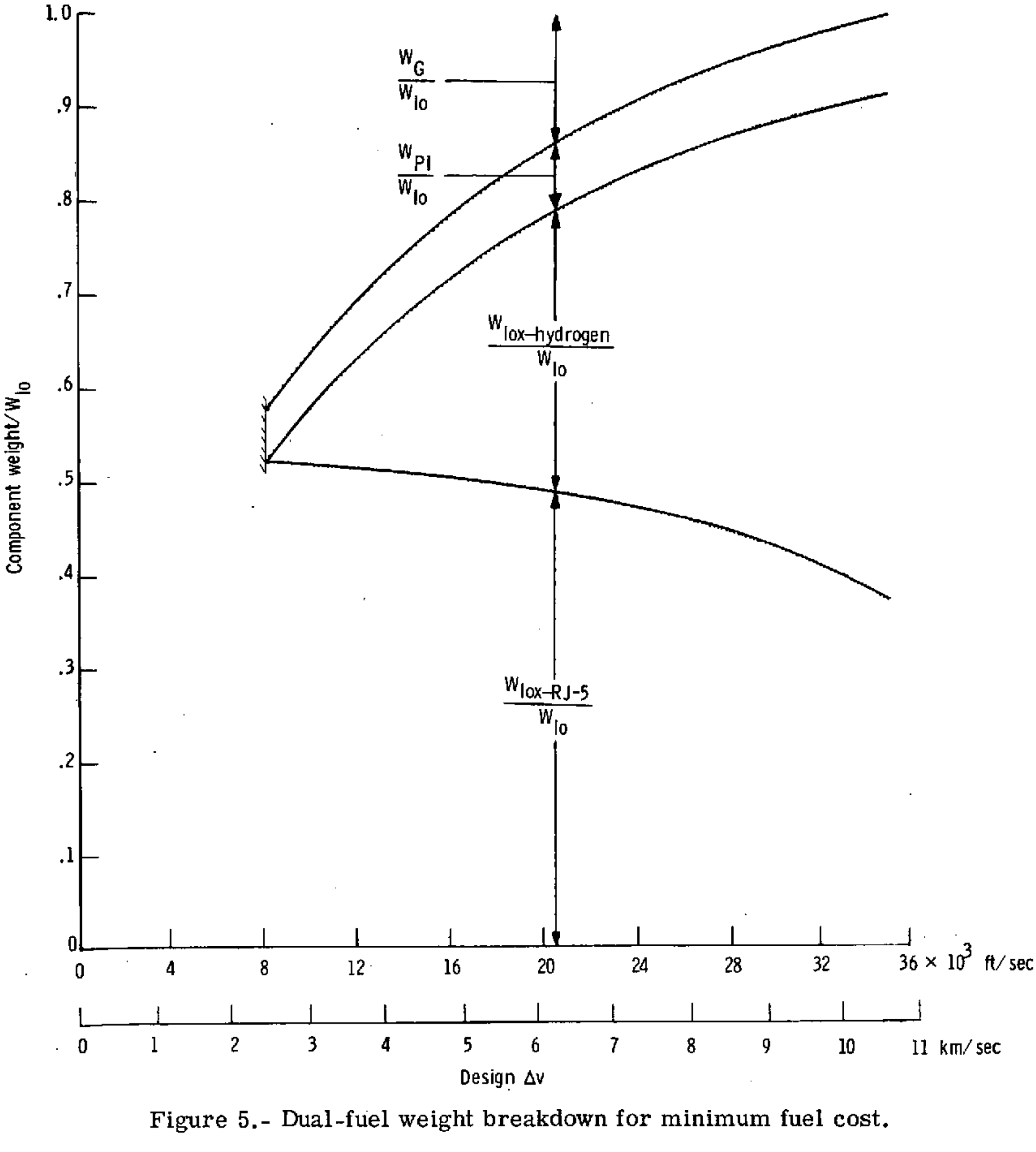

Figure 5.- Dual-fuel weight breakdown for minimum fuel cost.

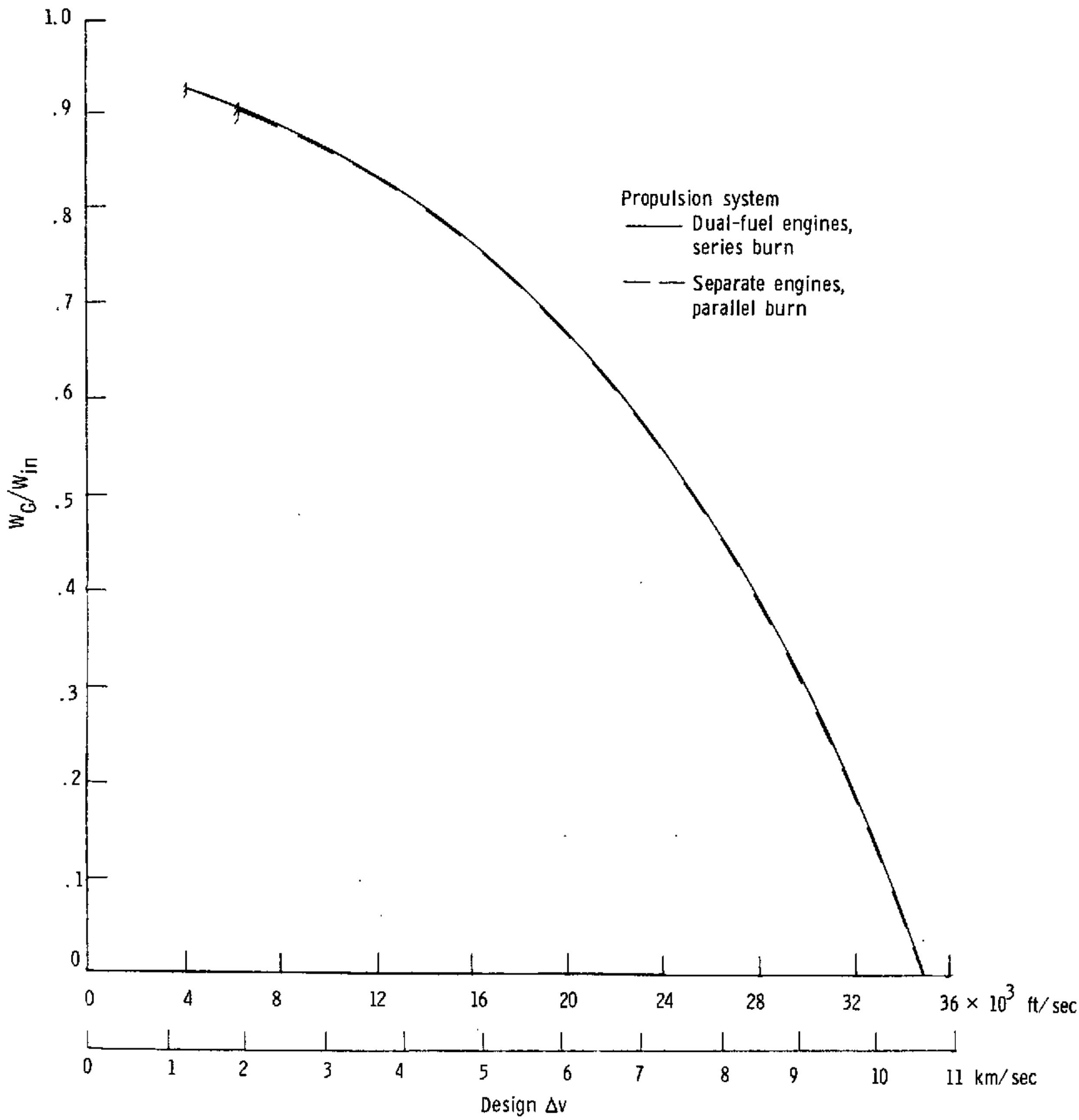

Figure 6.- Comparison of propulsion systems with series burn and parallel burn.

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