FINAL REPORT

NUMERICAL SOLUTION FOR UNSTEADY SONIC FLOW OVER THIN WINGS

By

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FOREWORD

The work described herein was performed by The University of Tennessee Space Institute, Tullahoma, Tennessee, 37388, for The National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia 23365. The work was performed under Research Grant No. NGR-43-001-102 Supplement Number 1.

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SUMMARY

The unsteady transonic equation for oscillating thin wings is solved by a direct finite difference method in the case where the steady flow affects only one coefficient in the equation. Both concave and delta wing planforms are solved and the program may be used for relatively arbitrary planforms. Both pitching and plunging modes are calculated for a reduced frequency range from .2 to 1.0 .

The results are consistent with earlier asymptotic investigations and are in numeric agreement within the order of accuracy of these solutions. The thickness effect while small, increases as reduced frequency decreases to .2 .

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NOMENCLATURE

AR	Aspect ration $4\sigma^2/S$
b	Dimensional body length
C_p	Unsteady pressure coefficient
$f(x,y)$	Oscillation amplitude distribution
$g(x,y)$	Wing thickness distribution
k	Reduced frequency of oscillation
L_{ij}	Generalized force coefficient
M	Local Mach number
$s(x)$	Wing planform shape
S	Wing planform area
t	Non-dimensional time
$\bar{t}$	Dimensional time
U_o	Dimensional free stream velocity
w	$(f_x + ikf)$ - downwash
x,y,z	Non-dimensional Cartesian coordinates
$\bar{x},\bar{y},\bar{z}$	Dimensional Cartesian coordinates
γ	Ratio of specific heats
δ	Oscillation amplitude
ϵ	Thickness ratio of wing
σ	Semi-span-to-chord ratio
Γ	Parabolic constant
ω	Dimensional frequency
Φ	Transonic small perturbation potential
ϕ_1	Steady state potential

ϕ_2	Unsteady potential
φ	Complex amplitude of ϕ_2
Re( )	Real part of
Im( )	Imaginary part of
( )', ( )", etc	Derivatives with respect to x or ξ
$(\ )_x, (\ )_{xx}$, etc.	Partial derivatives
O( )	Of the order of
sgn z	Sign of z

1. INTRODUCTION

The prediction of unsteady flow requires taking into account the corresponding steady flow. Landahl's [1] solutions of steady flow completely separated the two effects thus omitting the effect of body thickness on the unsteady behavior of the body. Total inclusion of the terms omitted by Landahl on the other hand gives a difficult equation of mixed type similar to the non-linear steady transonic equation which must usually be solved by time consuming numeric methods. The only simplification is that the flow-type regions are identified in advance. But mixed type differencing is still required and is further complicated by the presence of in-phase and out-of-phase components of the solution. In addition, a multiplicity of solutions is needed for one body to account for several frequencies and for various modes of motion. Thus a method which is fast and yet provides some measure of the effect of the steady flow is badly needed.

The present report describes a numerical solution procedure of a simplified unsteady transonic equation which is very fast and reasonably accurate and which takes into account many of the effects of the steady flow field. The numeric solution of this equation is accurate within a few percent and can be accomplished on an IBM 360/65 computer in less than one minute per case (one frequency and one mode of oscillation). It is more flexible than the older transonic box method[2] in that it accommodates rather arbitrary planform shape and is easily capable of handling variable local Mach number effects from the steady flow.

2. PROBLEM FORMULATION

Consider a rigid pointed wing which performs harmonic pitching oscillations of small amplitude in a steady uniform transonic flow. It is required that the wing be smooth and sufficiently thin that the conditions for the validity of small perturbation theory are satisfied.

With the flow directed along the positive x axis and the wing oriented along the x axis, we may introduce non-dimensional variables $x = b\bar{x}$, $y = b\bar{y}$, $z = b\bar{z}$, and $t = (U_o/b)\bar{t}$ where b is the body length, U_o is the free stream speed and the bar coordinates are the physical coordinates. The steady state position of the wing, which for convenience is assumed to be symmetric, is then as shown in Figure 1. Denoting the thickness ratio by ϵ, the requirement that the wing be thin becomes $\epsilon \ll 1$.

Since the shock strength is or order ϵ^2, we may ignore the irrotationality effect of the shock effect in the solution up to order $\epsilon^2 \ln\epsilon$. Consequently, we may assume the existence of a velocity potential Φ such that the x, y and z components of the flow velocity are $(1 + \Phi_x)$, Φ_y, and Φ_z respectively. It has been shown (e.g. by Landahl[1], that the potential Φ must satisfy the following equation:

$$(1 - M^2)\,\Phi_{xx} + \Phi_{yy} + \Phi_{zz} - 2M^2\,\Phi_{xt} \tag{1}$$
$$- M^2\,\Phi_{tt} = (\gamma + 1)\,M^2\Phi_x\Phi_{xx}$$

Consistent with the requirement that the wing perform small oscillations of amplitude δ about its steady state position, we may write the equation for points on the wing as

$$z = \epsilon\, g(x,y)\ \mathrm{sgn}\ z + \delta\ \mathrm{Re}\ (e^{ikt}\, f(x,y)) \tag{2}$$

where $g(x,y)$ is the steady state wing shape of order 1 and $f(x,y)$, also of order 1, represents the change in shape due to oscillation. k is the reduced frequency of oscillation equal to $\omega b/U_o$ where ω is the physical frequency. We assume the oscillation to be a small disturbance to the steady state solution, so $\epsilon \gg \delta$.

The condition of tangential flow on the body becomes:

$$\Phi_z \Big]_{body} = \epsilon g_x \text{ sgn } z + \delta \text{ Re } ((f_x + ikf)e^{ikt}) + G(\epsilon,\delta) \tag{3}$$

where $G(\epsilon,\delta) = \Phi_y \Big]_{body} (\epsilon g_y \text{ sgn } z + \delta \text{ Re } (f_y e^{ikt}))$

Letting σ denote the semi-span to chord ratio, we let $\bar{y} = y/\sigma$ in the last term in (3).

$$G(\epsilon,\delta) = \Phi_y \Big]_{body} (\epsilon \sigma^{-1} g_{\bar{y}} \text{ sgn } z + \delta \sigma^{-1} \text{Re}(e^{ikt} f_{\bar{y}})) \tag{4}$$

Then, if the wing is assumed to be nearly planar so that $\sigma \gg \epsilon$, (4) is negligible and the boundary condition (3) becomes

$$\Phi_z \Big]_{body} = \text{Re} \left\{ \epsilon g_x \text{ sgn } z + \delta (f_x + ikf) e^{ikt} \right\} \tag{5}$$

Moreover, we assume that δ is sufficiently small that (5) may be evaluated on $z = \pm 0$ instead of (2).

The non-linear term in equation (1) is the fundamental difficulty in solving the above problem. Following the ideas of Lin, Reissner, and Tsien[3], Landahl showed that the non-linear term would be negligible for k sufficiently high, i.e. ($k \gg \sigma\epsilon \ln\sigma^{-1} \epsilon^{-1/3}$). This restricts the range of validity and leaves out completely the effect of body thickness as the equations for the unsteady part of the potential are then completely independent of the steady state solution. Teipel[4] extended an approach used by Oswatitsch and Keune[5] for steady sonic flow by approximating the Φ_{xx} term with a parabolic constant Γ transmitting the effect of the steady flow into the unsteady equation. This method has been exploited by Kimble, Liu, Ruo and Wu[6] to obtain asymptotic solutions for low aspect ratio pointed wings.

Instead of this approach Φ may be represented as the sum of a steady potential ϕ_1 and an unsteady potential ϕ_2 and assuming the unsteady flow is a small perturbation of the steady flow gives, as in [1],

$$(1 - M^2)\ \phi_{2xx} - M^2\ (\gamma + 1)\ (\phi_{1xx}\phi_{2x} + \phi_{1x}\phi_{2xx})$$

$$+ \phi_{2yy} + \phi_{2zz} - 2M^2\ \phi_{2xt} - M^2\ \phi_{2tt} = 0 \tag{6}$$

In sonic flow $1 - M = \mathcal{O}(\phi_{1x})$. It has become common practice to ignore the term $\phi_{1x}\phi_{2xx}$ in comparison to $\phi_{1xx}\phi_{2x}$. This reflects in part the vanishing of the receding wave as $M \to 1$. Kimble and Wu[7] carried out numerical solutions of two dimensional wings with and without the term $\phi_{1x}\phi_{2xx}$ using Spreiter's[8] steady state approximate solutions and found less than one-half percent difference in the solutions, well within the order of numeric approximation. This approximation requires further study to place it on a firmer physical foundation. The resulting governing equation for ϕ_2 is

$$[M^2\ (\gamma + 1)\ \phi_{1xx}]\ \phi_{2x} = \phi_{2yy} + \phi_{2zz} - 2M^2\phi_{2xt} - M^2\phi_{2tt} \tag{7}$$

Letting $\phi_2 = \delta\ \mathrm{Re}\ (\varphi\ e^{ikt})$ and ∇^2 denote the Laplacian with respect to y and z we have finally:

$$[M^2\ (\gamma + 1)\ \Phi_{1xx} + 2M^2 ki]\ \varphi_x = \nabla^2\ \varphi + k^2M^2\varphi \tag{8}$$

with boundary conditions

$$\varphi_z\ (x,y, +0) = (f_x + ikf), \quad |y| < s(x) \tag{9}$$

$$\varphi\ (x,y, +0) = 0 \quad , \ |y| \geq s(x)$$

$$\varphi\ (0,y,z) = 0, \quad z \geq 0$$

$$\lim_{y^2 + z^2 \to \infty} \varphi = 0, \quad x \geq 0.$$

3. SOLUTION METHOD

Ignoring the fact that the leading coefficient is complex, equation (8) resembles a heat equation with x playing the role of time and spatial variables y and z. (It is actually a biharmonic wave equation if the complex coefficient is eliminated by differentation). This suggests an adaptation of the Crank-Nicolson method for solution.

The boundary condition on φ_z would behave as if one were heating a plate at the edge over an expanding region. This causes severe difficulties in the use of a difference method mesh which is uniform in x. The mesh would not match the wing planform edges exactly (Figure 2a). Nonetheless, this approach was completely implemented and failed badly. Severe oscillations were present in the numeric solution on the wing surface. These oscillations which are normally a sign of instability in the numeric method did not extend into the field. In addition, a Von-Neumann stability analysis showed the field equations were stable. A similar difficulty was experienced with the transonic box method[2] and was partially repaired by an edge correction. Various corrections were tried but a difference scheme can only recognize a change of the boundary to an accuracy the same as mesh size. This approach was therefore abandoned.

A transformation used by Landahl[1] to transform a delta wing problem gave the fundamental idea for a means to transform the problem in such a way that the mesh aligned itself with the planform edges thus eliminating the boundary instability problem.

Let
$$y = \bar{y}s(x) \qquad (10)$$
$$z = \bar{z}s(x)$$

Then the equation (8) becomes)

$$\alpha\,[s^2\,\varphi_x - s's\,(\bar{y}\,\varphi_{\bar{y}} + \bar{z}\,\varphi_{\bar{z}})] + \beta\,s^2\,\varphi = \bar{\nabla}^2\,\varphi \qquad (11)$$

where $\alpha = [M^2 (\gamma + 1) \phi_{1xx} + 2M^2 ki]$ and $\beta = k^2 M^2$ while

$w = f_x + ikf$

The boundary conditions (8) become

$$\varphi_{\overline{z}} (x, \overline{y}, +0) = w \cdot s \qquad |\overline{y}| < 1 \tag{12}$$

$$\varphi (x, \overline{y}, +0) = 0 \qquad |\overline{y}| \geq 1$$

and the other boundary conditions are unchanged. See figure 2b.

This problem was programmed using a relatively straight forward adaptation of the Crank-Nicolson method for variable coefficients and complex φ. The stability was excellent and the previous difficulty completely overcome . Unfortunately, an iterative method of solving the equations at each x step was used. The amplification factors found by the Von-Neumann analysis approached one for x near the trailing edge and $\phi_{1xx} = 0$. The iteration procedure took more and more iterations to converge as x approached 1.0. Under relaxation lead to eventual convergence but only after consuming a large amount of computer time. The effects were not as severe with $\phi_{1xx} \neq 0$ but efficiency still suffered.

Since reprogramming was again necessary to use a direct elimination procedure, an additional transformation was also tried to reduce storage requirements. Many mesh points must be used off the wing in the lateral as well as the vertical direction to take account of the field effects. The transformation

$$\overline{y} = \sin \overline{\overline{y}} \cosh \overline{\overline{z}} \tag{13}$$

$$z = \cos \overline{\overline{y}} \sinh \overline{\overline{z}}$$

eliminates the need for mesh points displaced laterally off the wing and in addition smoothes the edge singularity in the derivatives of φ giving even better numeric accuracy there. See figure 2c.

The equation and boundary conditions (with the double bars dropped) become

$$\alpha\ [(\cos^2 y + \sinh^2 z)\ s^2\ \varphi_x - ss'\ (\sin\ (2y)\ \varphi_y/2 + \sinh\ (2z)\ \varphi_z/2)]$$

$$+\ \beta\ (\cos^2 y + \sinh^2 z)\ s^2\ \varphi = \nabla^2\ \varphi \tag{14}$$

$$\varphi_z\ (x,y,0) = s\ w\ \cos\ y, \quad |y| < \frac{\pi}{2} \tag{15}$$

$$\varphi\ (x, \pm\frac{\pi}{2}, z) = 0\ , \quad z \geq 0$$

$$\varphi\ (0,y,z) = 0\ , \quad z \geq 0, \quad |y| \leq \frac{\pi}{2}$$

$$\lim_{z\to\infty} \varphi = 0, \quad x \geq 0, \quad |y| \leq \frac{\pi}{2}$$

This method gave good stability and accuracy of the in-phase part of φ. However, the out-of-phase part of φ is one orders of magnitude smaller than the in-phase part. Without fine mesh the out-of-phase part was lost in noise. To correct this φ was split into two parts, one a known function related to the slender body solution, and the other, ψ, of comparable magnitudes in both in and out-of-phase parts.

$$\varphi = \psi + s\ w\ \cos(y) e^{-z} \tag{16}$$

This was substituted into (14) and (15) and only ψ was computed numerically resulting in very accurate solutions for both in and out-of-phase parts of φ.

4. PRESSURE COEFFICIENT AND GENERALIZED FORCES

The pressure coefficient is computed using central differences to approximate the derivatives of φ from

$$C_p = -2\ (\varphi_x + ik\varphi) \tag{17}$$

We define the generalized aerodynamic force coefficients by

$$L_{ij} = \frac{4}{S\,\delta_i} \iint_S \left[\varphi_{i,x} + ik\varphi_i \right]_{z=0} f_j \, dxdy \tag{18}$$

where S is the wing planform area, the displacement distribution function for harmonic oscillations in mode j is given by $f_j(x,y) \cos kt$ (f_j is of order 1), and φ_i is the unsteady potential due to mode i. The integrals were evaluated (after integration by parts to eliminate derivatives) by Simpson's rule.

5. RESULTS

5.1 Comparison With Transonic Box Method

The program was tested for a case solved by Rodemich and Andrew[2] by the transonic box method. The planform was a delta wing of aspect ratio 1.5 oscillating in plunge at a reduced frequency k of .5 . Results are plotted in Figure 3 and show agreement to within about 3%. The finite difference solution uses 16 points in the x direction and 8 in the y direction. In a sense this is comparable to 128 boxes in the transonic box method. The transonic box solution shown, however, required approximately 300 boxes on the wing.

The far field was approximated by setting the correction potential to 0 at z_∞ approximately 4. This means that at $z_\infty = 4$ the potential is forced to agree with the slender body solution. A value of $z_\infty = 8$ caused only about one percent change in the solutions whereas $z_\infty = 2.0$ caused much more severe variation.

Similar tests were made with varying mesh sizes. Doubling the number of mesh points in any direction again caused only about one percent variation in the solutions.

5.2 The Delta Wing With Thickness Effect.

The generalized moments L_{11} and L_{22} were computed for a delta wing of aspect ratio 1.5 in pitching and plunging modes at reduced frequencies .2, .5, .8 and 1.0 with and without a thickness effect.

The thickness effect chosen for this study was based on earlier work of Liu and Ruo[9] which extended the earlier work of Teipel mentioned above. It was assumed that $\epsilon = .0683$, $\sigma = .375$ and that the wing was a biconvex with simple wedge chord sections. This implies that the equivalent body is a cone of base area .0683. According to [9] this gives a parabolic constant $\Gamma = .55$ which serves as an approximation to the steady state term $(\gamma + 1)\ M\ \phi_{1xx}$. The flow was assumed to be sonic (M = 1) everywhere for this report. There is no limitation on the program (or its efficiency) to constant values of $(\gamma + 1)\ M\ \phi_{1xx}$, however. Better data can be incorporated in future studies.

The results are shown in Figures 4 and 5. Note the expanded scales. The effect of thickness is seen to increase as reduced frequency decreases which is in agreement with past asymptotic analysis of thickness effect[9], [6]. The near agreement with and without thickness effect at k = 1.0 is rather coincidental although as frequency increases, thickness effect will vanish. The accuracy suffers somewhat near k = 1.0 because of the relatively large mesh size.

The phase angle is affected only very little by thickness. Note that L_{22} seems to show very little thickness effect. This can be traced to the fact that φ is not normalized by k as it is in the plunging case. We suggest using the downwash w = ikx and normalizing φ to accurately account for thickness effect in this case.

5.3 The Concave Wing With Thickness Effect

To illustrate the capabilities of the method to handle general planforms, a concave wing with planform given by

$$s(x) = \sigma \ (x + x^2)/2$$

and $\sigma = .4167$ corresponding to aspect ratio 2.0 was chosen. The results show a slower onset of thickness effect with decreasing frequency and a slower decay as frequency increases. The first tendency is again consistent with qualitative trends noted in the earlier asymptotic investigation at low frequency. The second tendency has not been seen before and requires further investigation. Again phase angle is not greatly affected and the pitching results are largely swamped due to lack of normalization.

5.4 Pressure Coefficients

Pressure Coefficients were calculated for both wings oscillating in the plunging mode at $k = .2$. The amplitudes of pressure show little change when thickness effect is included. Thickness effect is largely reflected in rather significant changes in the phase angles. This emphasizes the importance of calculating the out-of-phase part of φ correctly since this part almost totally determines the phase shift due to thickness effect.

6. CONCLUDING REMARKS AND RECOMMENDATIONS.

An efficient and accurate method has been developed for solving a simplified version of the unsteady transonic equation. The equation may be modified to include the effect of ϕ_{1x} wherever this is not negative without interfering with efficiency or accuracy. Such an approach is reasonable in sonic flow when the receding wave effects can be ignored. In consequence wake influence is ignored and shocks must be weak. Nevertheless, when the sonic pocket extends over most of the wing, these assumptions are very nearly fulfilled. The present method can easily account for variable local Mach number and rather arbitrary planform so long as the basic assumptions are fulfilled.

Additional convergence studies in the case of curved planforms would be valuable since accuracy is somewhat affected by such changes. The present program may be modified to use approximately 70% less storage and about the same proportion less time to make such studies feasible.

It is recommended that pitching results be compared using a downwash function $w = ikx$ rather than the more conventional $w = 1 + ikx$ since the present procedure gives solutions largely dominated by the plunge-like case $w = 1$. Comparison of results calculated with $w = ikx$ will allow normalization by frequency k and will more accurately reflect differerences due to thickness, planform, and local Mach number.

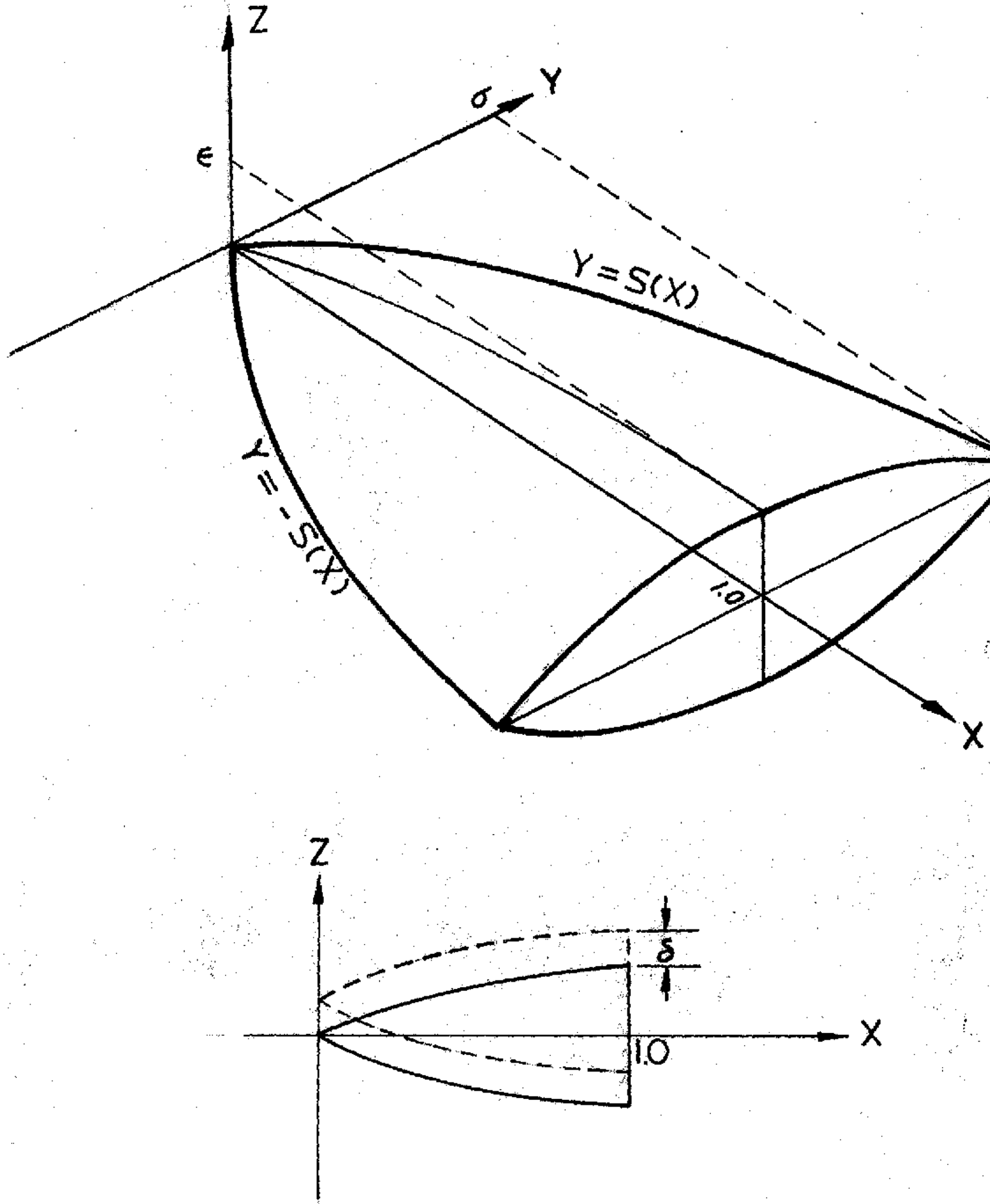

Figure 1. Nomenclature

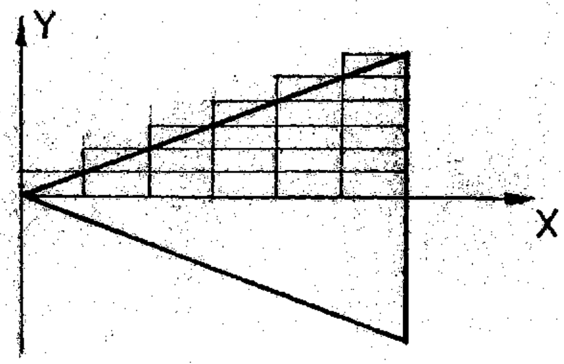

Figure 2a. Mesh Does Not Match Wing Edge.

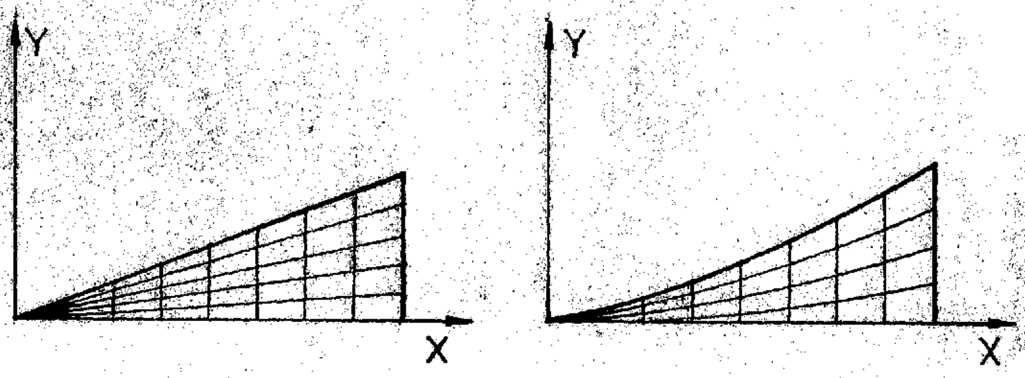

Figure 2b. Effect Of Stretching Transformation On Mesh.

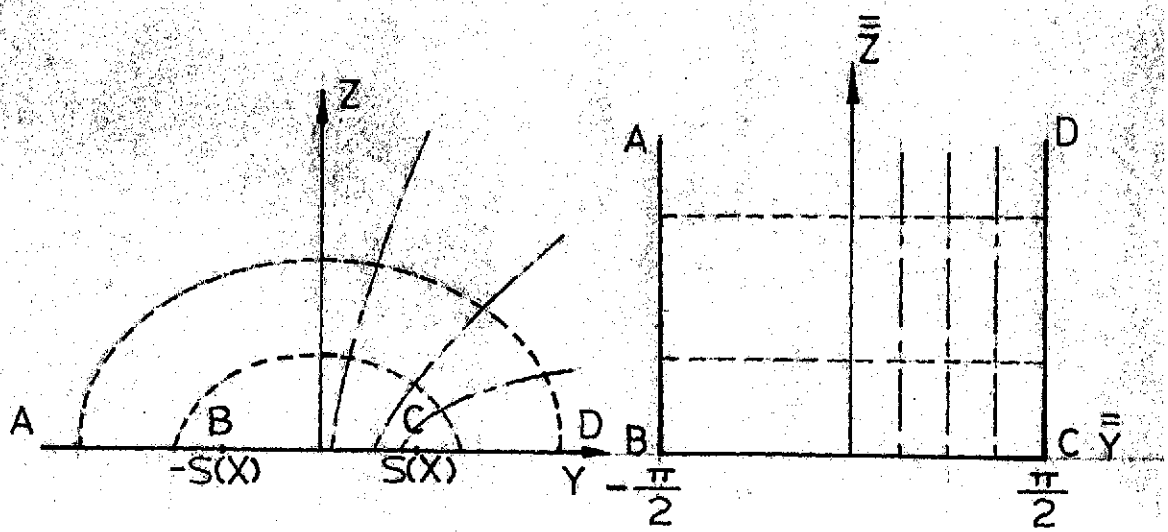

Figure 2c. Effect Of Total Transformation On Mesh.

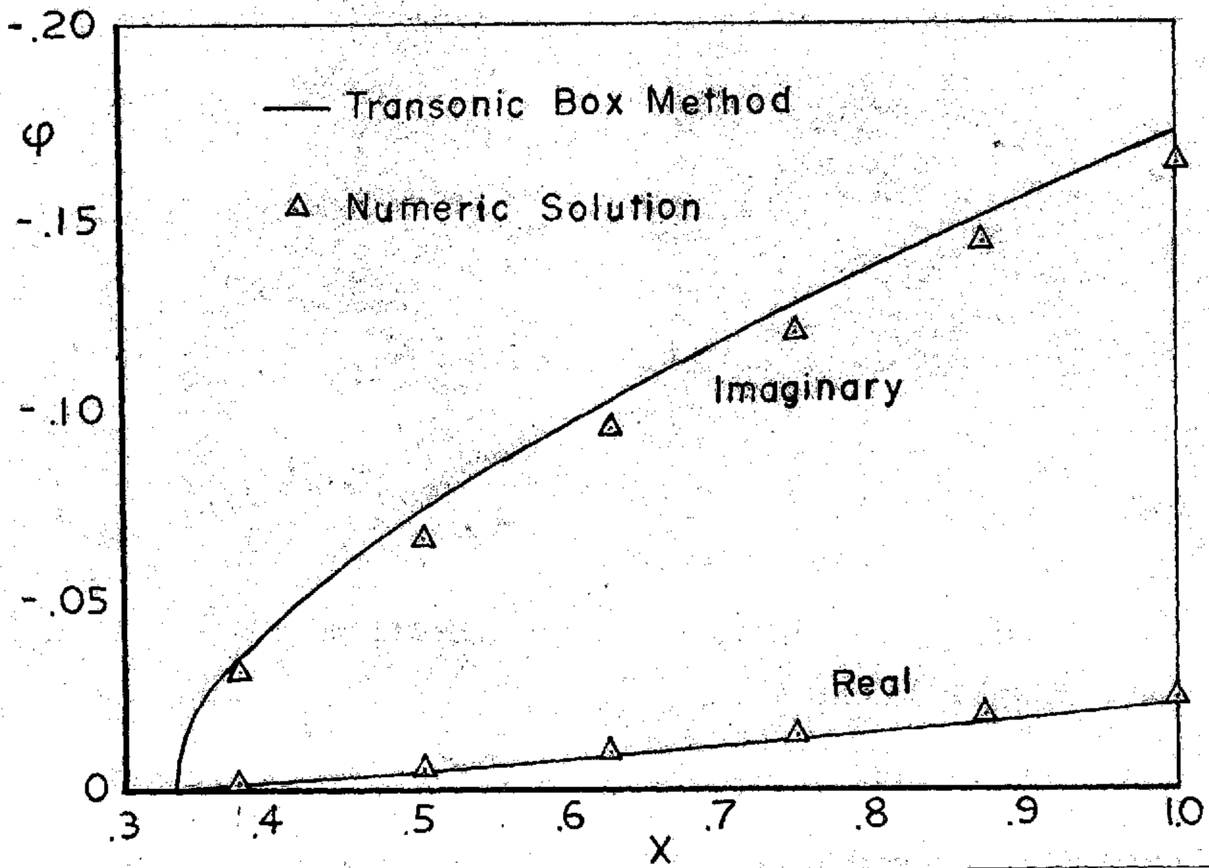

Figure 3. Real And Imaginary Parts Of the Unsteady Potential φ In The Plunging Mode For An Aspect Ratio 1.5 Delta Wing At $k = 0.5$; Chordwise Distribution For $y = 0.125$.

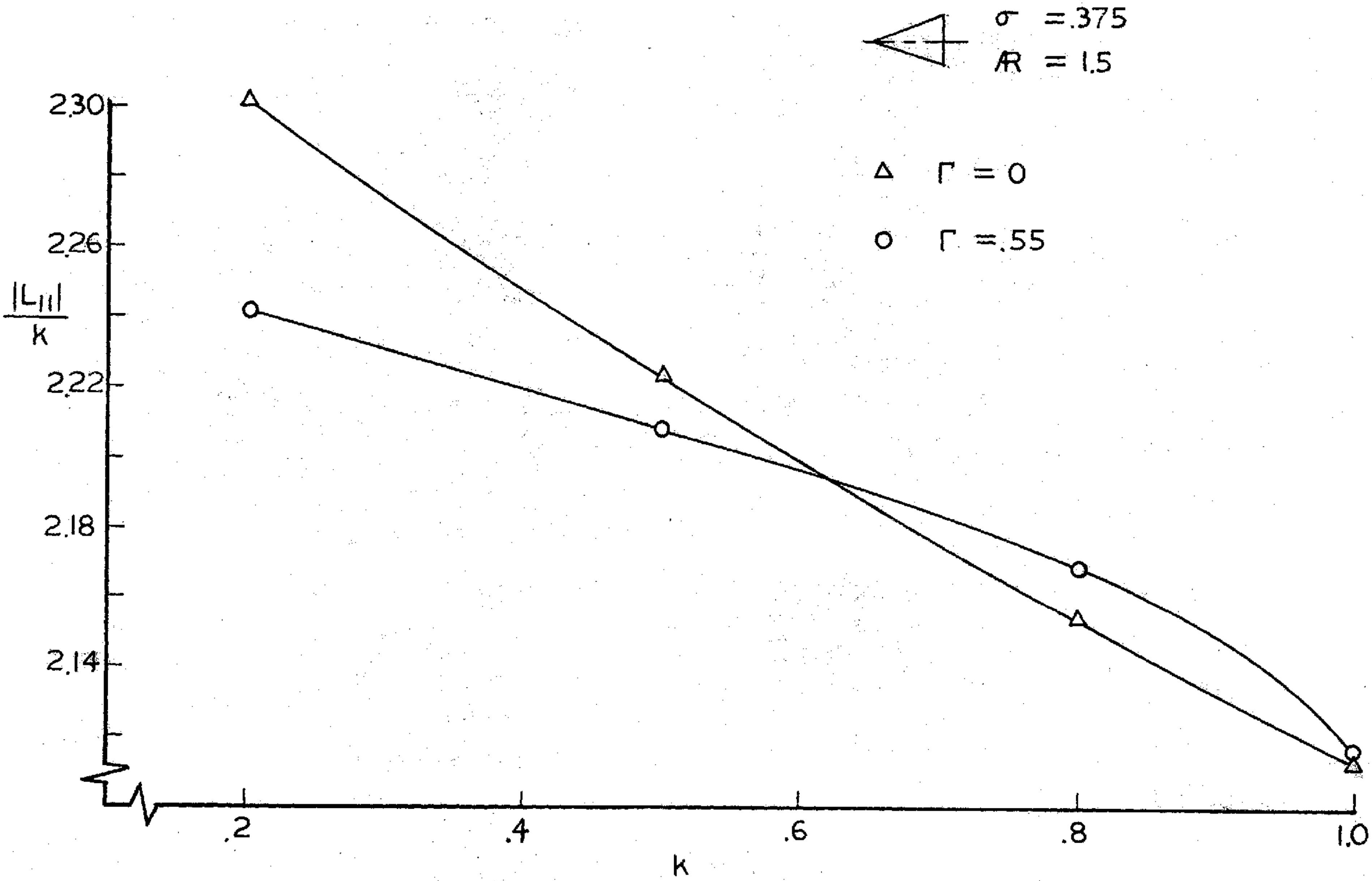

Figure 4. Amplitude Of Coefficient Of Total Lift Due To Translation For A Delta Wing.

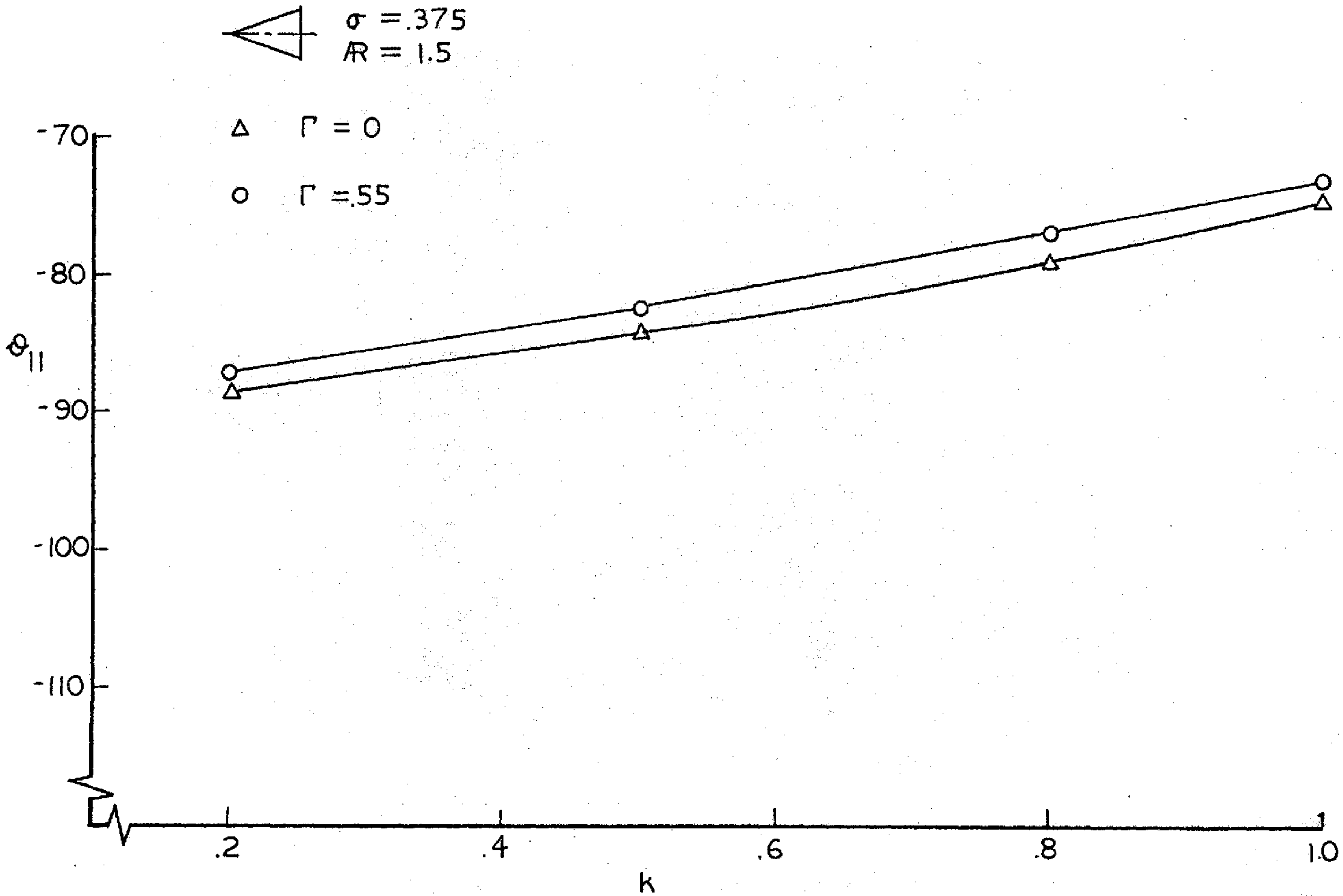

Figure 5. Phase Of Total Lift Due To Translation For A Delta Wing.

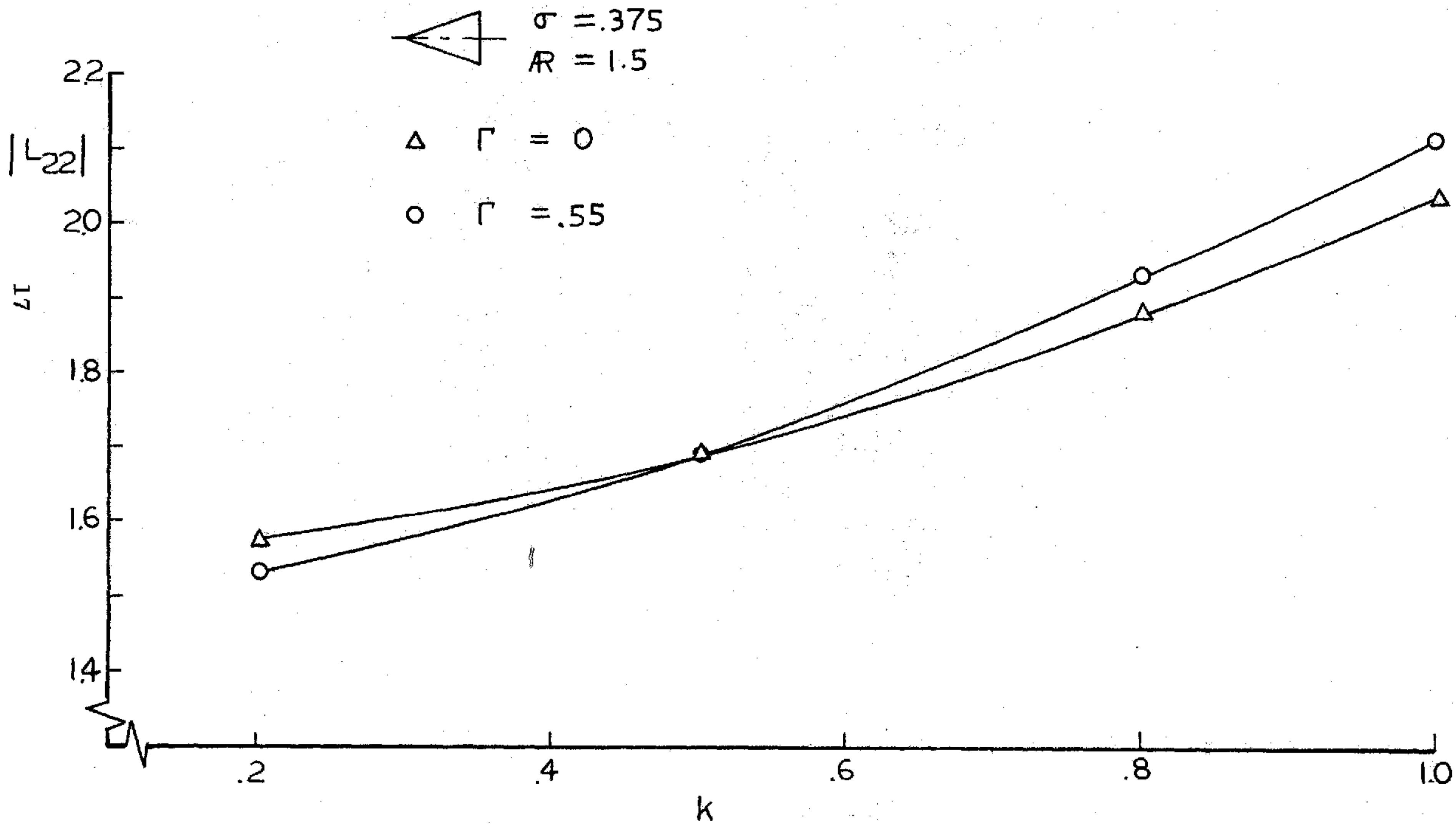

Figure 6. Amplitude Of Total Lift Due To Pitch For A Delta Wing.

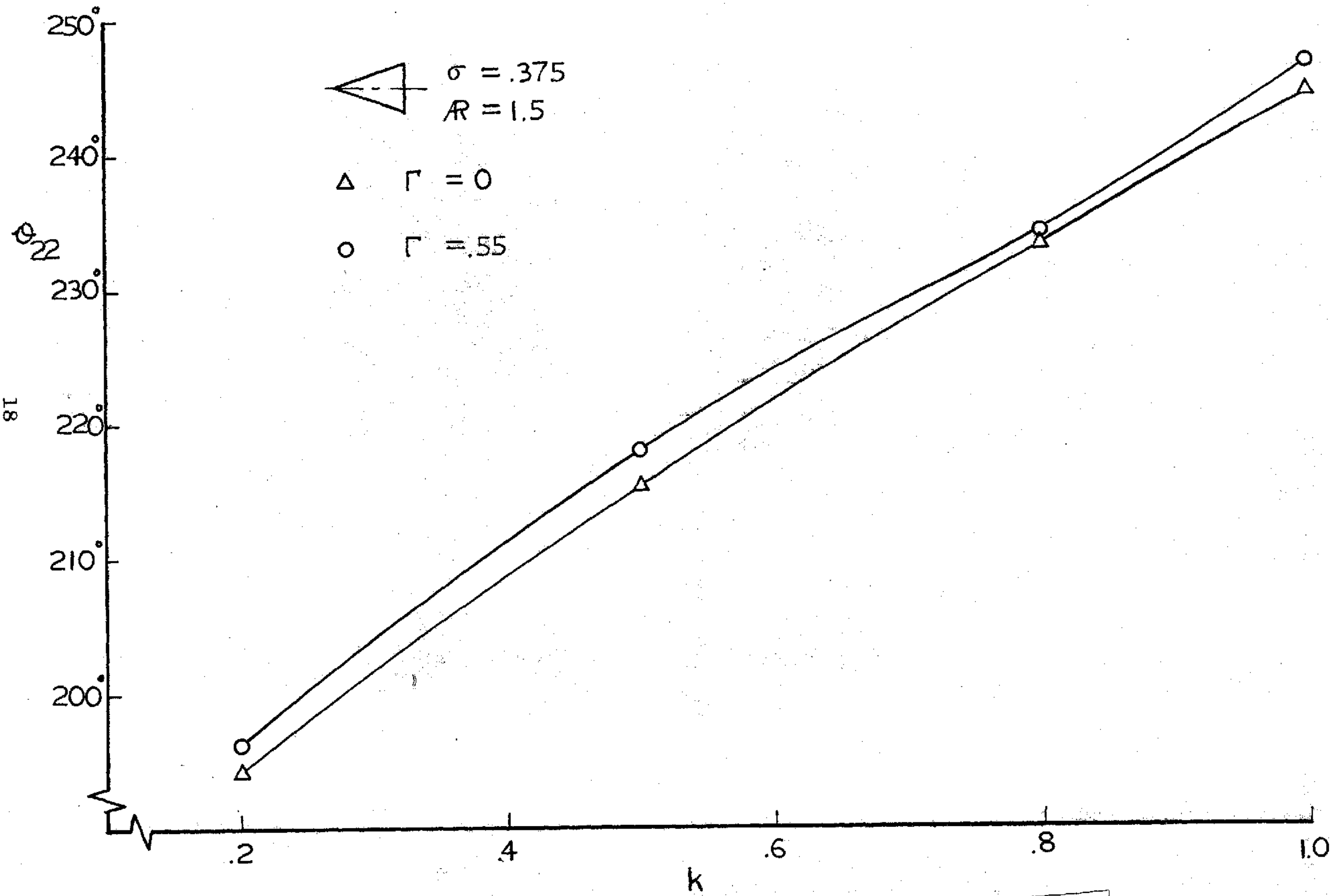

Figure 7. Phase Of Total Lift Due To Pitch For A Delta Wing.

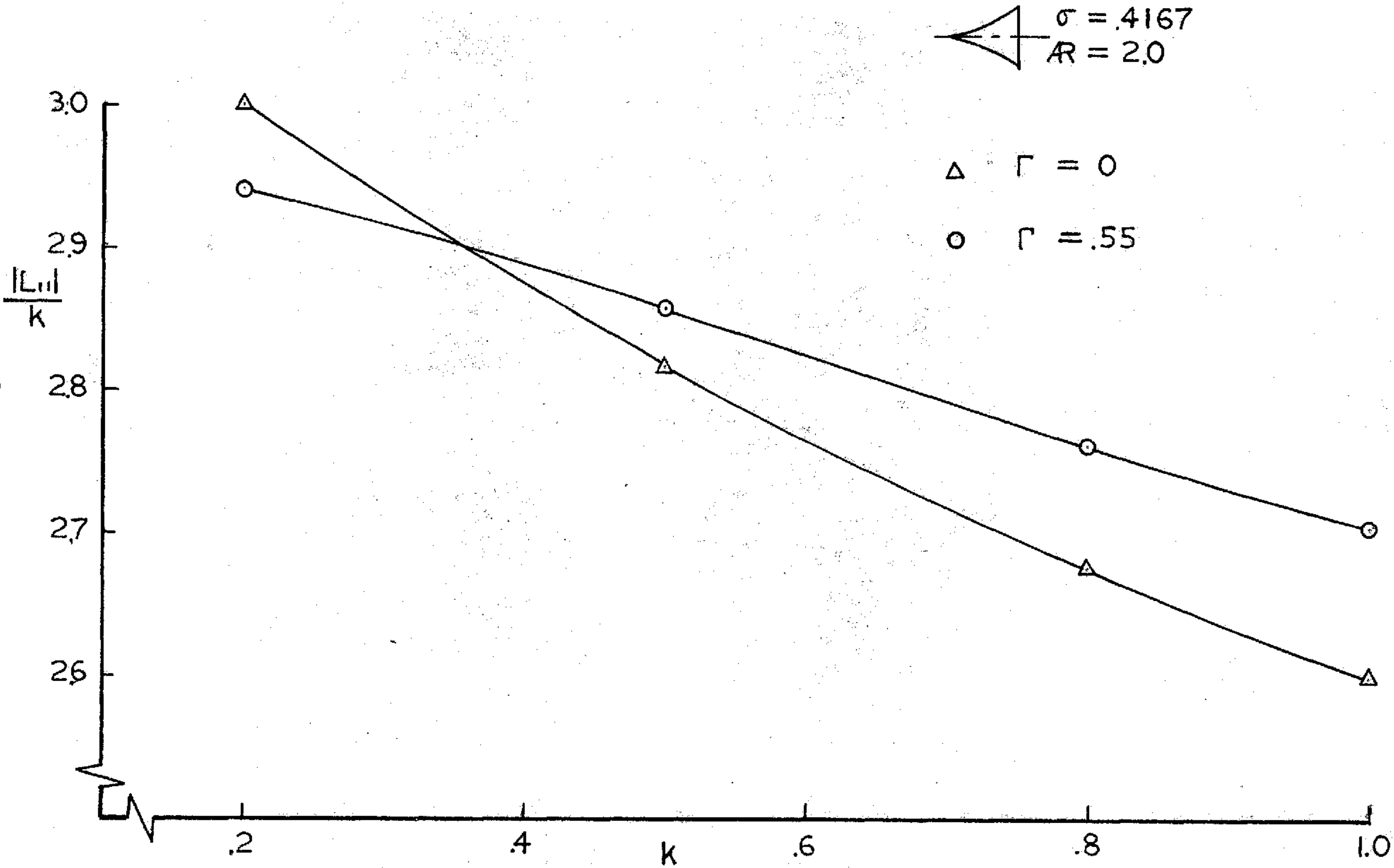

Figure 8. Amplitude Of Total Lift Due To Translation For A Concave Wing.

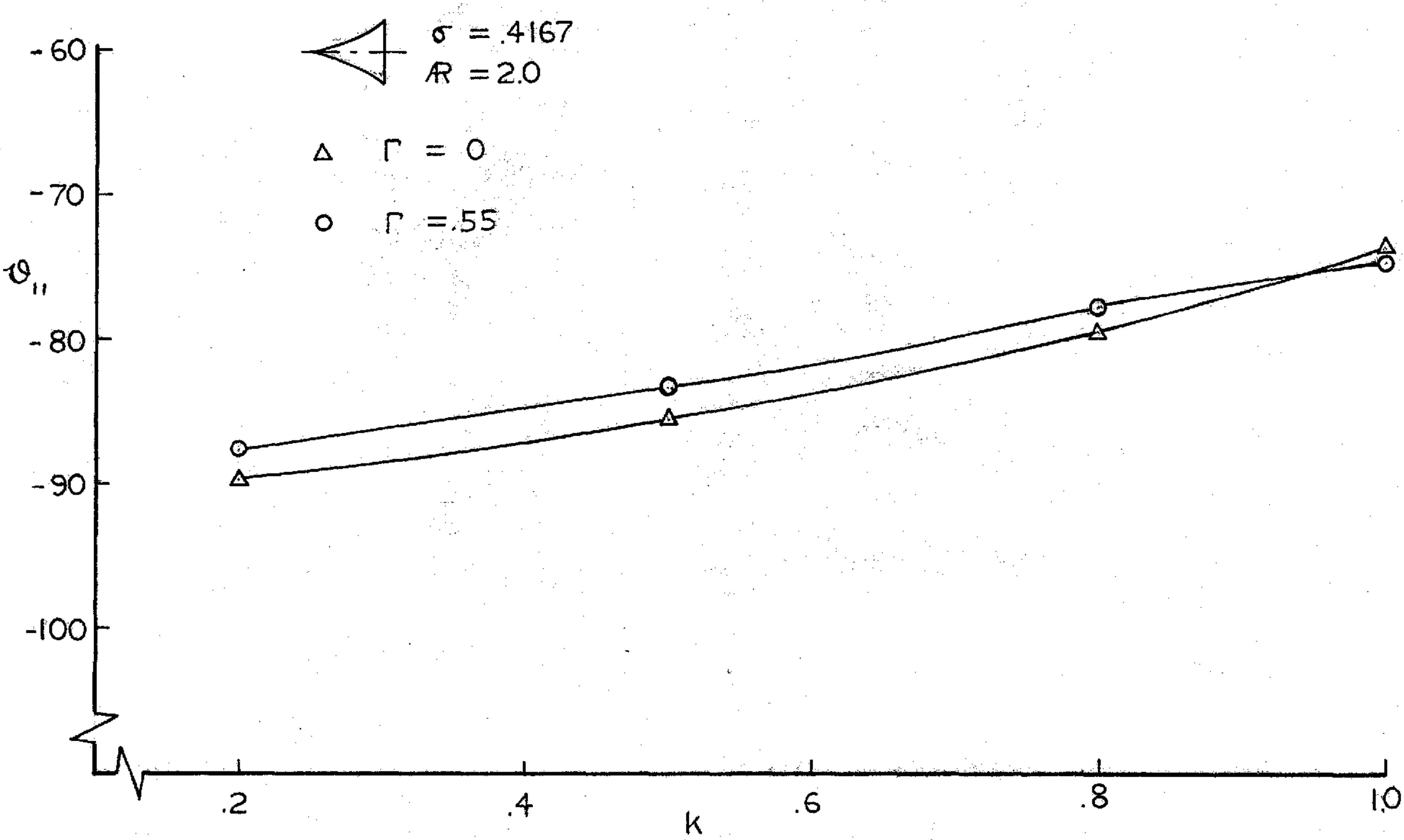

Figure 9. Phase Of Total Lift Due To Translation For A Concave Wing.

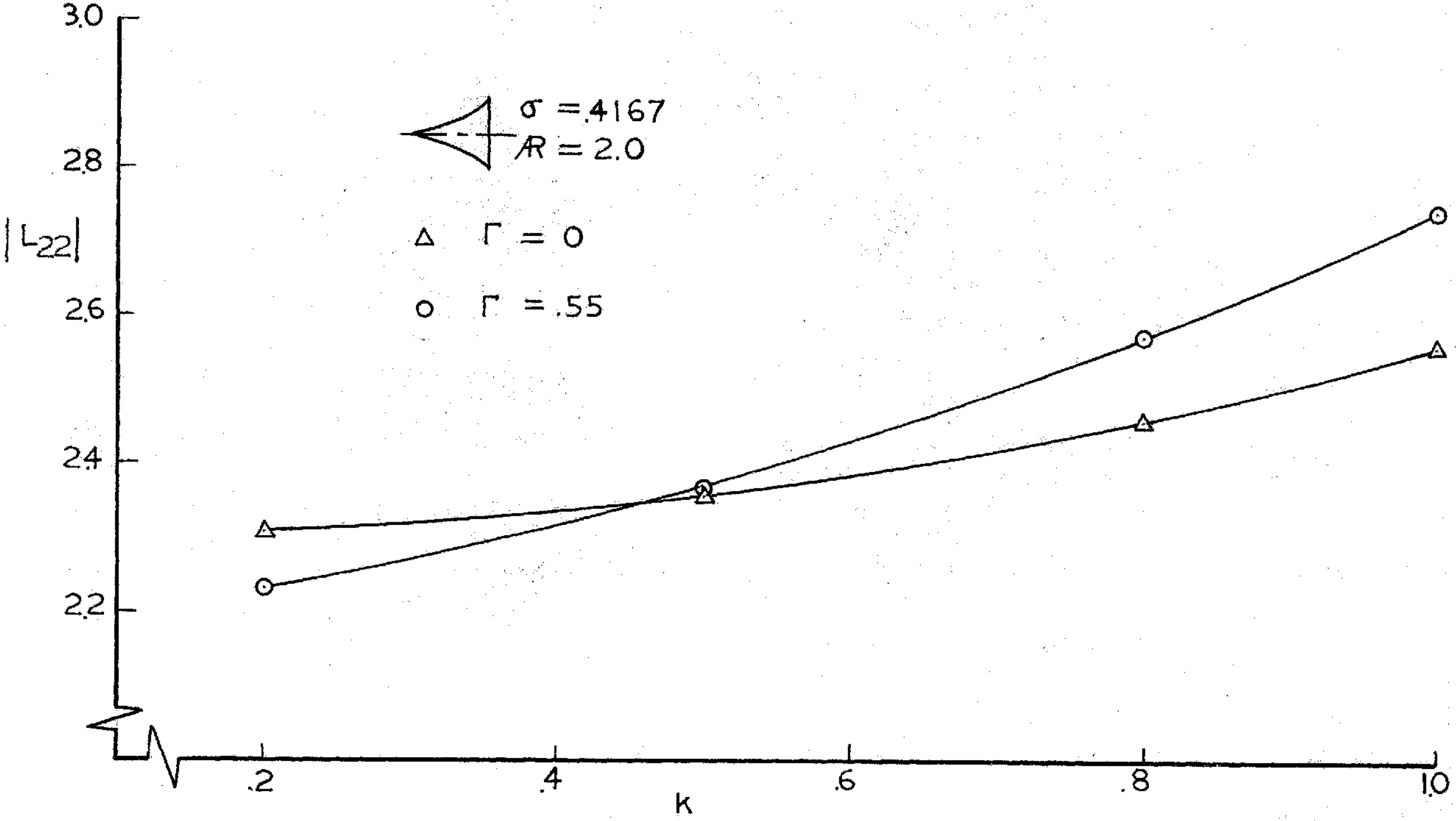

Figure 10. Amplitude Of Total Lift Due To Pitch For A Concave Wing.

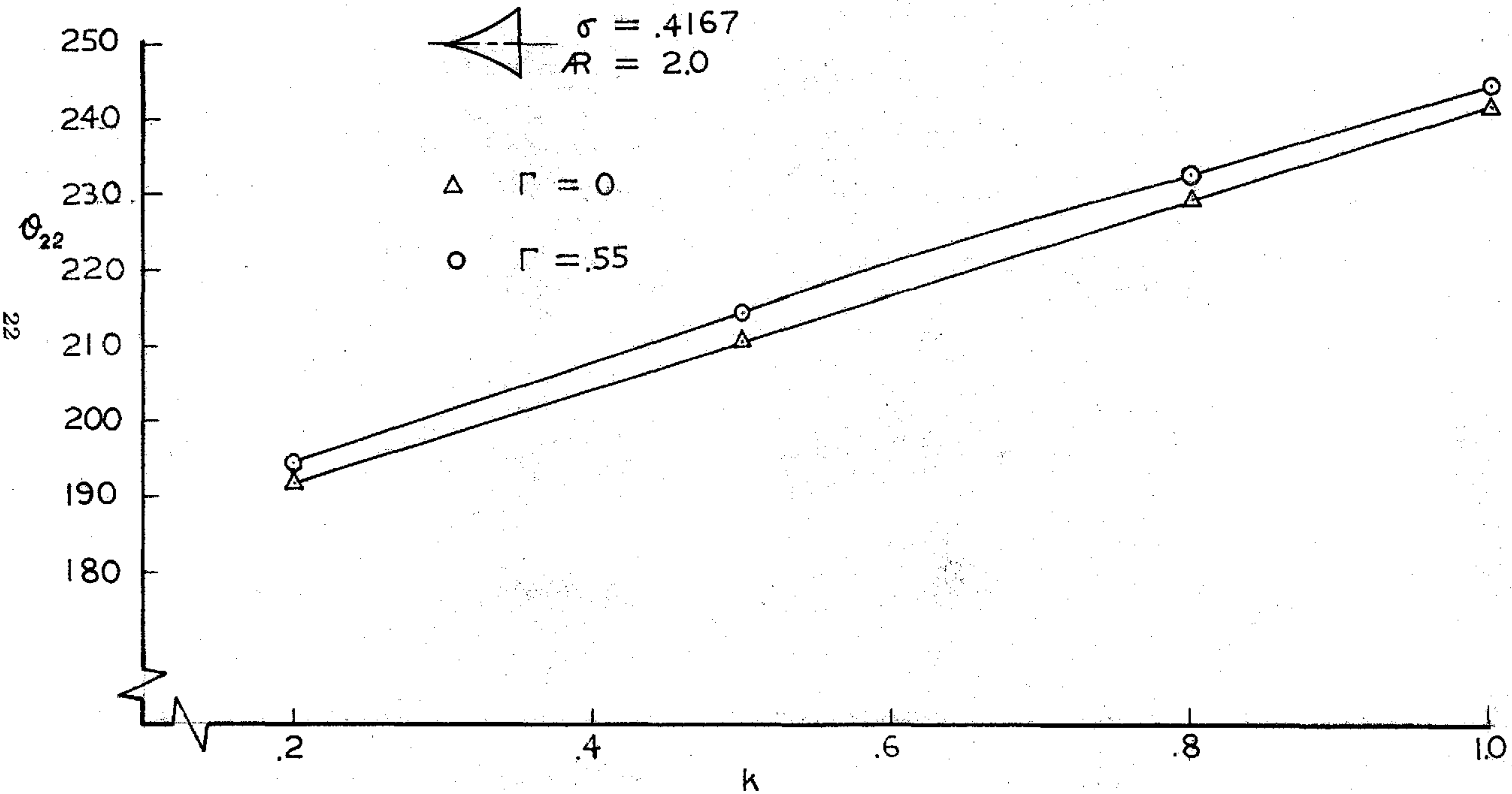

Figure 11. Phase Of Total Lift Due To Pitch For A Concave Wing.

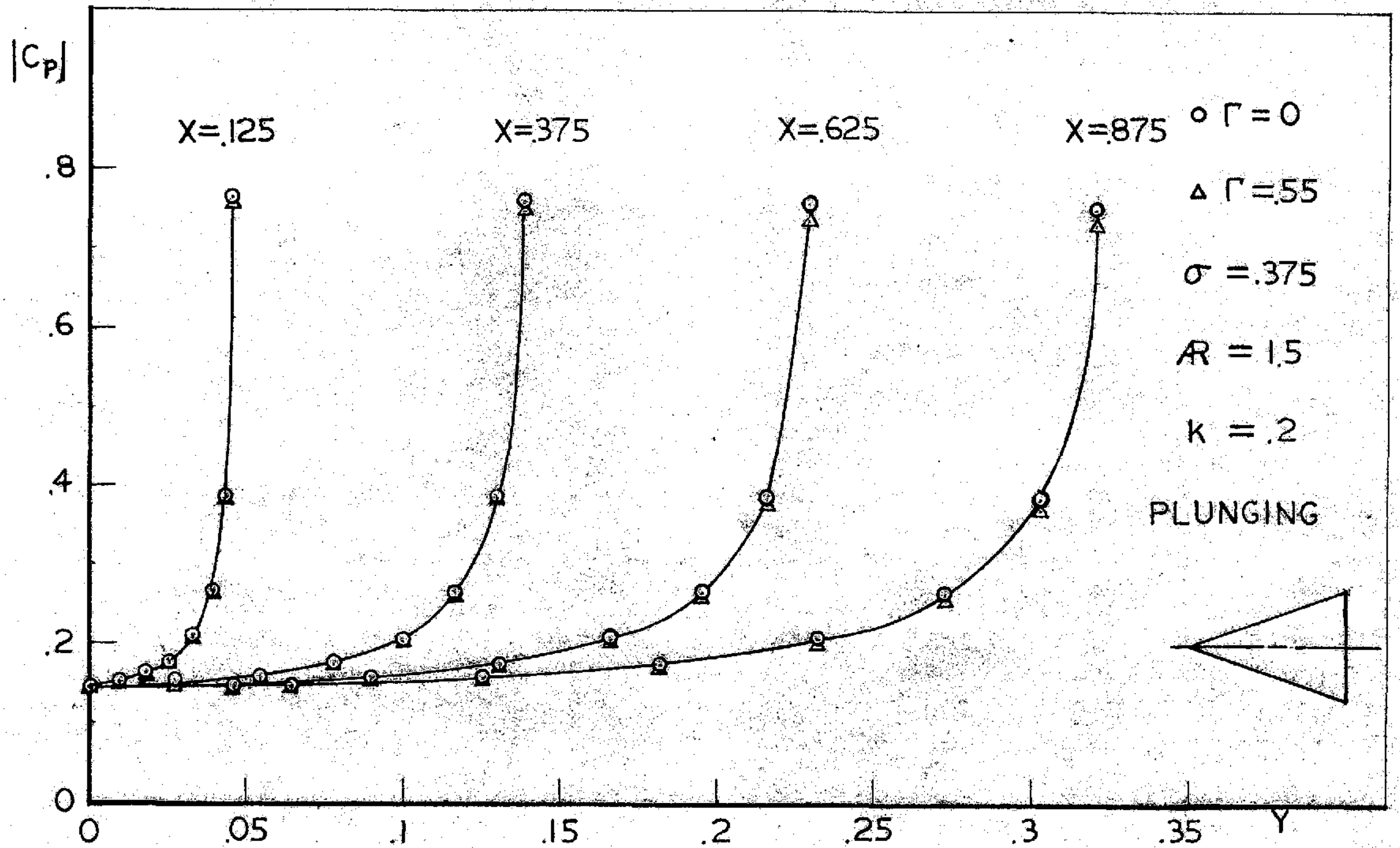

Figure 12. Amplitude Of Pressure Coefficient For A Plunging Delta Wing.

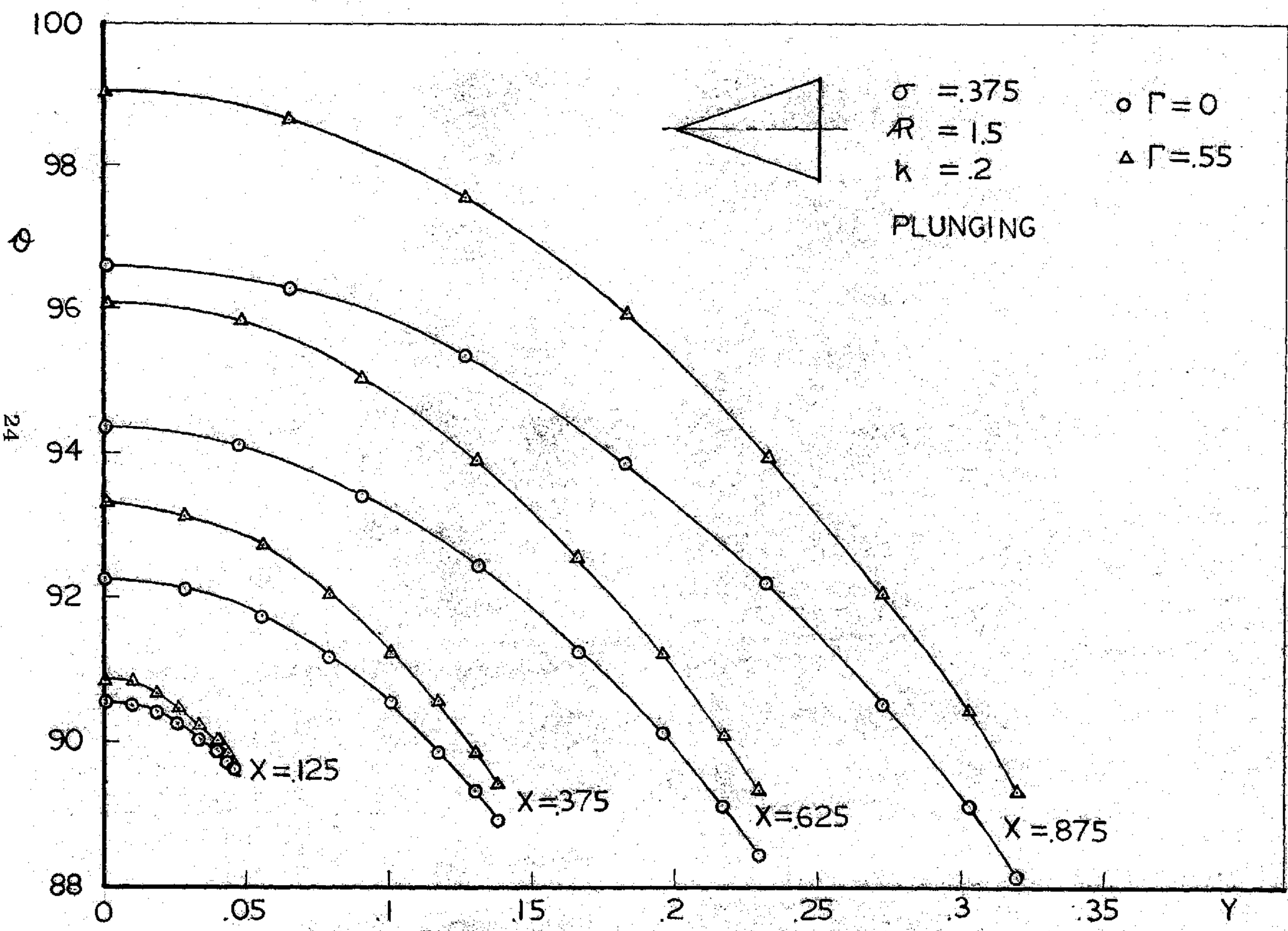

Figure 13. Phase Of Pressure Coefficient For A Plunging Delta Wing.

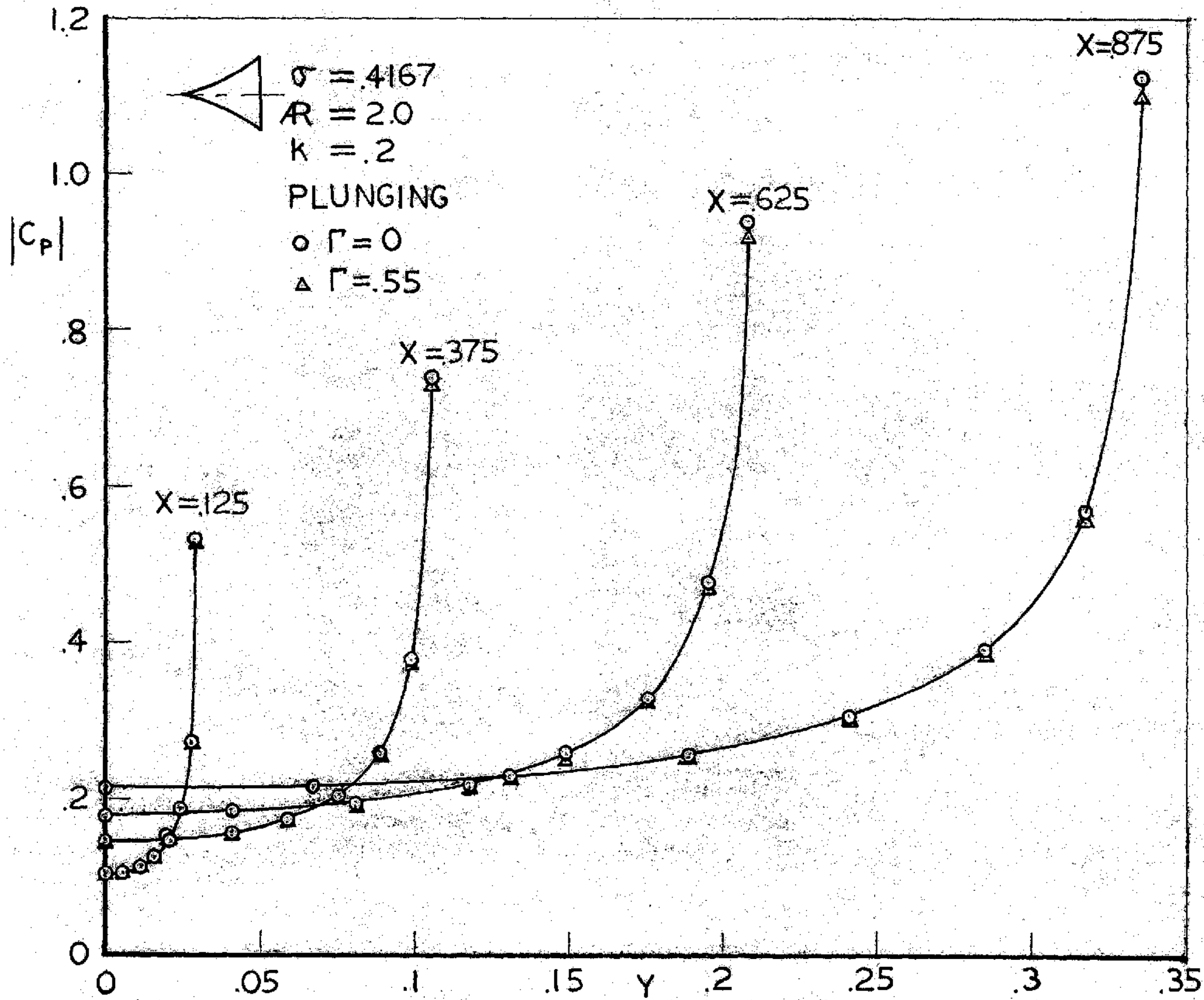

Figure 14. Amplitude Of Pressure Coefficient For A Plunging Concave Wing.

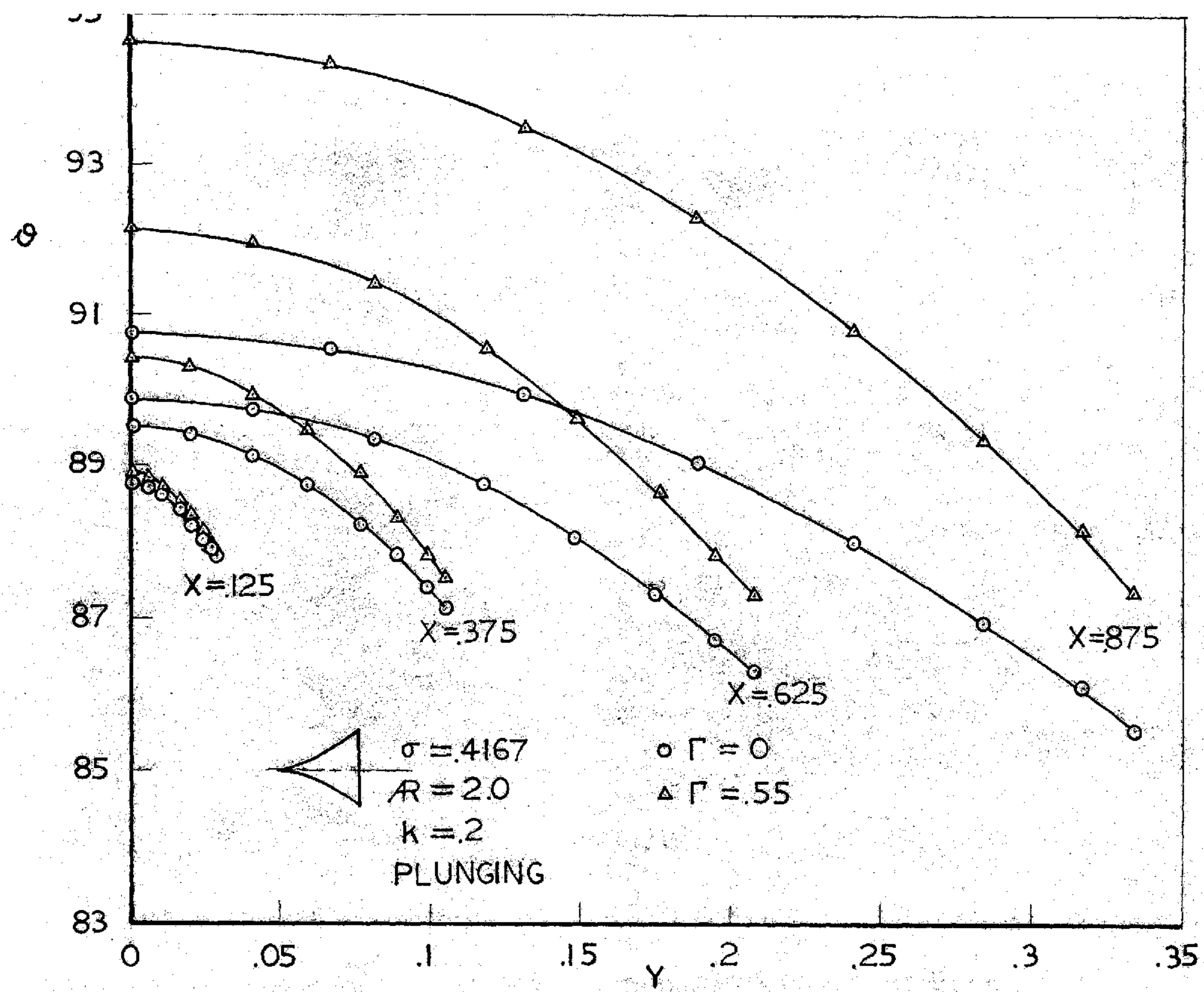

Figure 15. Phase Of Pressure Coefficient For A Plunging Concave Wing.

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