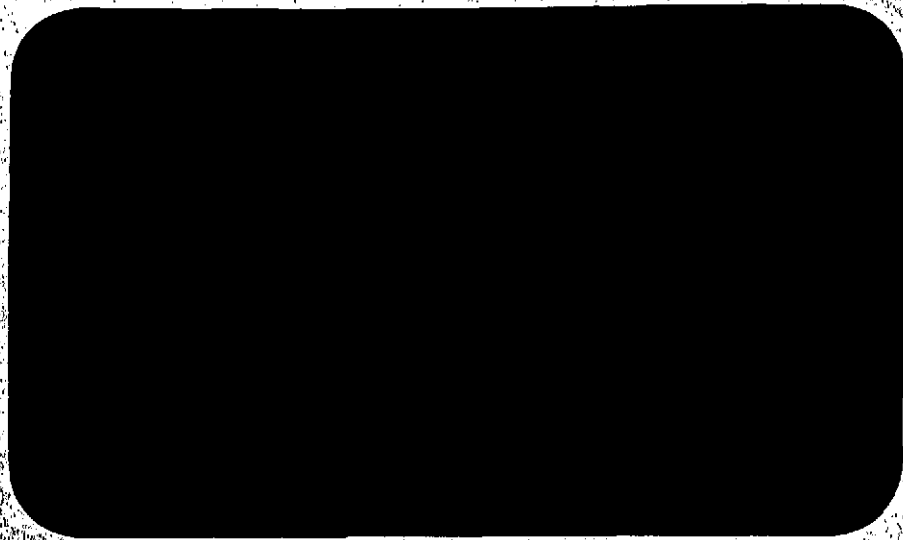


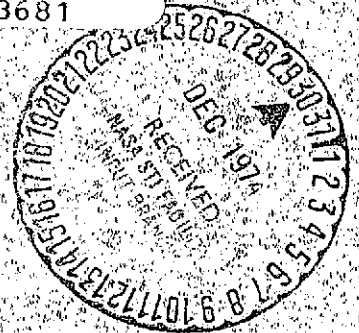
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ACOUSTICS OF THE PIEZO-ELECTRIC
PRESSURE PROBE

G. S. Dutt

Report 1179

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August 1974

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I. INTRODUCTION

This report relates some acoustical properties of a piezoelectric device for measuring the pressure in the plasma flow from an MPD arc. Such probes have been used to measure pressures with a fast risetime of a few μsec corresponding to pressure changes from less than 0.001 atmospheres to 0.1 atmospheres.

Further development of the piezoelectric pressure probe is not being continued in this laboratory at the present time. The purpose of this report is therefore to record and to summarize the progress in our understanding of the probe performance and its acoustics. In addition, an extended list of useful references with some brief comments is included. It is hoped that this report may serve as a starting point for further use and development of piezoelectric probes.

The report is subdivided into the following sections:

- I) Description of the elements of the probe and their acoustical behavior in a probe.
- II) Analysis of the acoustical phenomena taking place in a piezoelectric probe with a description of some dispersion, impedance matching and damping. This section is largely qualitative.
- III) Experimental procedure of verification of impedance matching and wave absorption by resonant discs fitted onto the probe rod in support of the acoustical analysis of Section II. The experimental results are presented in a set of oscillographic records.
- IV) A list of references and some comments.

II. DESCRIPTION OF THE PROBE

The probe is intended to measure the quasi-steady pressure at different locations in the MPD accelerator. The MPD arc discharge to be studied has a duration of up to 1 msec. During much of the latter part of this interval the flow conditions are constant (quasi-steady). The probe is required to measure the pressure during this period.

The probe described is similar to the one used by Cory¹ insofar as the acoustics are concerned. The active transducing element is a thin disk of a piezoelectric crystal, PZT-5, manufactured by Clevite². This disk is bonded between cylindrical Plexiglas rods 5 and 90 cm long of the same diameter as the disk (0.95 cm). The pressure pulse to be measured acts on the free end of the shorter rod, the front end of the probe. The pulse is transmitted as a longitudinal stress wave along the rod to the crystal through which it passes into the longer backing rod. The acoustic wave in the backing rod is reflected from its remote end and returns to the crystal. The length of the rod is so chosen as to prevent the reflected wave from reaching the crystal until after the time of interest.

A piezoelectric material responds to a change in stress applied to its surface by generating a proportionate electrical charge over certain crystal faces. The charge developed may be amplified and monitored on an oscilloscope and the state of stress on the material determined.

In this probe, the plane faces of the crystal are stressed by the longitudinal wave passing through it. The charge developed on the same faces is monitored through thin copper foils attached on these faces and wired to a Kistler charge amplifier³.

Since the magnitude of the stress in the wave does not remain constant, in general, the stresses on either face of the disk are unequal. The difference in forces on the disk gives rise to an acceleration of the disk. When the stress difference is small, it may be assumed that the charge developed is proportional to the mean stress. Since the probe is designed to measure a pressure that is steady for a large part of 1 msec, during which period the stress in the wave is constant, no limitation is imposed. For harmonic waves, the stress difference across the crystal is small when the wavelength is large compared to the thickness of the crystal disk.

III. ANALYSIS OF ACOUSTICAL PHENOMENA

The stress wave in the rod suffers from geometric and viscous dispersion⁴. Geometric dispersion occurs if the lateral dimension of the rod is significant compared to the dominant wavelengths in the wave. Viscous dispersion is caused by a departure from perfectly elastic behavior and is a property of the medium. Only the dispersion that occurs in the short Plexiglas rod is of interest since only the dispersion occurring between the start of the stress wave and its arrival at the crystal distorts the signal registered. Neither form of dispersion has any noticeable magnitude in the 5 cm length of Plexiglas. The principal wavelength in a pulse of 1 msec in a wave traveling at about 2000 msec is 2m. Higher Fourier components have correspondingly smaller wavelengths associated with them. In order to determine the circumstances under which geometric dispersion is important, the frequency corresponding to a wavelength of 1 cm, i.e. the diameter of the rod, is calculated and is found to be 2×10^5 Hz. So, when the frequency of the wave is small compared to 2×10^5 Hz, effects of geometric dispersion may be neglected.

Impedance Matching

Ideally, in the 1 dimensional approximation, a small compressive stress σ applied at the end of a solid rod produces a compression that will be transmitted along the rod with velocity $a_1 = (E_1/\rho_1)^{1/2}$ inducing a particle velocity

$$v_1 = a_1 \sigma E_1^{-1} = \sigma (E_1 \rho_1)^{-1/2}$$

E_1, ρ_1 are the Young's modulus of elasticity and the density of the medium. The variable $(E_1 \rho_1)^{1/2} = Z$ has been termed the acoustic impedance. Such a stress wave propagating from medium 1 to medium 2 must satisfy the boundary conditions at the interface that $\sigma_1 = \sigma_2$ and $v_1 = v_2$. Accordingly, it can be shown that the ratio of the reflected to the transmitted stress can be expressed⁵

$$\frac{\sigma_{1R}}{\sigma_{1T}} = \frac{\gamma-1}{\gamma+1} \quad \text{where} \quad \gamma = \frac{Z_2}{Z_1}$$

The stress wave travelling in our probe encounters two interfaces - one on either side of the crystal. For most accurate measurements it is necessary that the acoustic impedance of the crystal is the same as that of the rods.⁶ This is not so in our case and indeed some reflection takes place. This feature and its experimental investigation will be described later.

The circumstances under which the wave travels through the crystal are further complicated because it is in the form of a thin disk. The disk has numerous bending modes that are easily excitable, in addition to the mode of longitudinal oscillation that lies within the domain of our one dimension approximation. It should, however, be noted that in order to excite the bending modes of the disk, the stress wave needs to be non-symmetric about the axis of the Plexiglas rods.

When the stress wave first encounters the crystal, a part is reflected. When this reflected part reaches the probe tip, it is reflected in the opposite sense. This wave reaches the crystal an appropriate time interval after the original wave; again a part is transmitted and the rest reflected. In each succeeding reflected wave the amplitude is decreased, the decrease being substantial if the impedance mismatch at the interface is small. In such a case, the stress wave remaining in the shorter Plexiglas rod is damped out within a few reflections. The wave reflected from the other crystal-Plexiglas interface which is transmitted into the short Plexiglas rod follows the wave reflected from the front interface into the same rod after an integral multiple of the time it takes the wave to go back and forth within

the crystal. Similarly, the wave that is transmitted into the backing rod after a single pass through the crystal is followed by waves that have suffered reflections in the crystal and reflections from the front end of the short rod. The exact description of the reflection pattern for a given imposed stress cannot be obtained by a theoretical analysis since there are numerous unknowns associated with the construction of the probe - the effect of the copper foils and silicone grease between the Plexiglas rods and the crystal, and that of the 'Eccosil' potting compound that surrounds the crystal-rod interface - to name but a few. Whether or not the probe response is adequate for the purpose it is to serve must be determined experimentally. The above analysis serves to indicate which are the frequencies that one would expect as a result of the existing interactions. The experiments, described later, then indicate the relative importance of the various effects.

The stress wave that passes into the backing rod travels along it until the end. If the end is free then this wave is reflected in the opposite sense, i.e. a compression wave is reflected as a tension wave. The backing rod then acts as an acoustic delay line which keeps this reflected wave from interfering with the crystal response until after the time of interest.

The backing rod needs to be about 1m long for a delay of 1 msec using this 1-D wave propagation mode. The existing probe has an overall length of 1m and is 0.95 cm in diameter. It is suspended within a tube of diameter 7 cm. The vacuum tank within which the accelerator is mounted is 90 cm in internal diameter and 180 cm long. The relatively large size of the probe limits its maneuverability within the tank particularly when rotation of the probe with respect to the

tank axis is desired. Extensive spatially resolved measurements are thus not possible and reduction in the probe size would be very useful. Since the principal contributor to the size is the 90 cm length of backing rod, an alternative to this is first sought.

The effect of varying the conditions at the free end of the rod on the reflected stress wave was investigated. The shape of the rod at its end is of no significance. The wavelength of the principal wave components are around a meter while the disturbances caused by varying the end shape are localized to within 1 cm, the lateral dimension of the Plexiglas rod. Since this is much smaller than the relevant wavelengths the waves cannot "see" these disturbances and consequently are not affected by them. This has been experimentally verified.

We have considered the effect of the end of the rod when it is free. Changing the boundary conditions at the end appears promising. A compression wave is reflected as a tension wave at a free end and as a compression wave at a perfectly rigid end. The possibility of combining these two effects in order to have no net reflection at all presents itself. This could be achieved in a number of ways: a) by placing the end of the rod against a bar of some material more rigid than Plexiglas and then varying the pressure between the two and hence the degree of contact in order to achieve what is necessary for no reflection, b) by drilling a hole of half the cross-sectional area of the rod at its end and fastening the end to a rigid support so that the compressive and tensional reflections cancel each other. It should be noted, however, that alternative (a) can be achieved by adding an extra length of Plexiglas to the rod end which would then transmit the entire stress wave through

the interface with no reflection. This serves no purpose since it does not decrease the size of the probe. If this method is to succeed it is imperative that the wave is transmitted into the rigid bar where it excites some mode of vibration other than the one-dimensional longitudinal mode which it is intended to replace.

Even if a suitable mechanism for utilizing either alternative can be devised, this method will present several disadvantages which must be considered. Since the probe will be connected to a rigid support it will experience a considerable amount of mechanical vibration. If this is ameliorated by improving the vibration isolator for the probe assembly, then the rigidity of the support is lost. Whether or not this method can be used to prevent reflections is not known at present. A few simple experiments were performed but did not yield conclusive results.

Other ways of avoiding the reflected signal from interfacing with the probe response would be to damp it out by transmission through a viscous medium, or to use some other, more compact, elastic delay line or, indeed, a combination of these.

Any simple solution along either route is severely limited by the requirement of an impedance match on either side of the crystal, e.g. if we used a rubber backing rod in order to take advantage of its viscosity or low velocity of wave propagation then the large impedance mismatch at the crystal rubber interface would cause most of the wave to be reflected there. This would give rise to long transients as the wave gets damped out over numerous reflections in the front rod, resulting in complex boundary conditions at the crystal faces and no effective delay.

Experiments conducted to investigate possibilities of reduction of probe size retained the basic concept of one-dimensional wave propagation in a slender rod and attempted various additions to them.

The technique that proved to be most promising was based on the principle of the resonator. A disk may be made to fit around the backing rod. The dimensions of the disk may be chosen to resonate with certain frequencies and thus eliminate these frequencies from the propagating stress wave. The question immediately arises as to which frequencies should be chosen - frequencies based on the first n Fourier components of the 1 msec stress pulse or some persistent frequencies associated with the probe, e.g. the dominant frequency in the reflected stress pulse.

IV. EXPERIMENTAL PROCEDURE AND RESULTS

Most of the data on the different acoustic properties of the probe were obtained from very similar experiments. The main experiments performed involved the impact of a small sphere on the probe, and the principal artifact was the use of an extension rod for the probe. These will be described first before we look at the data obtained for distinct acoustic properties of the probe.

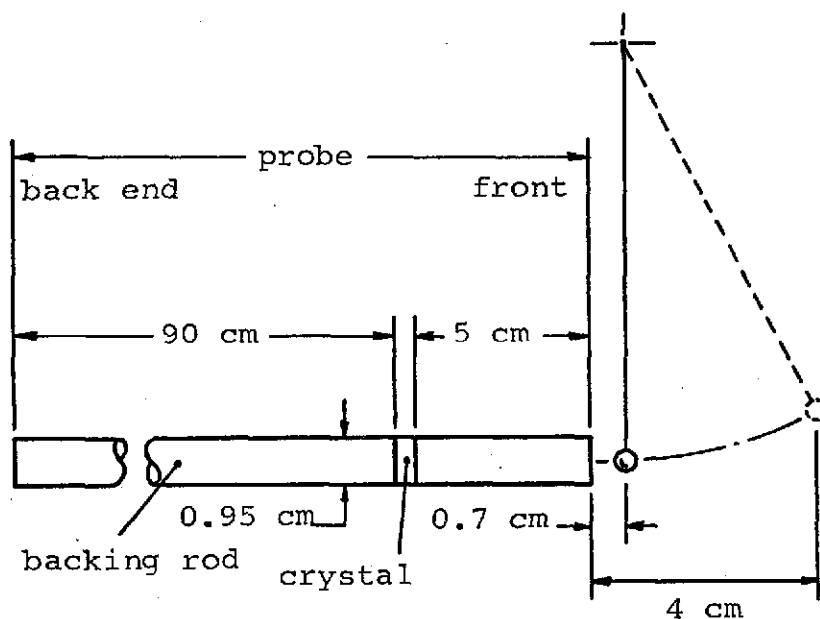


Fig. 1 Impact of steel ball on the center of the probe tip

Relevant information:

Ball diameter (nom 0.318 cm)	=	3.2 mm
Mass	=	0.129 g
Length of string from point of support	=	19.5 cm
Mass of string (19.5 cm)	=	.0069 g (negligible)

The probe is suspended horizontally by a pair of rubber bands. The impact is as described above. The crystal in the probe is connected through a charge amplifier to an oscilloscope. The oscilloscope was set to trigger when the measured signal itself exceeded a certain value.

Probe Extension Rod

During the course of these experiments, many variations on the backing rod were called for. In order to obviate the need for constructing a new probe each time, the method used was to extend the backing rod by various, easily removable, extension rods of the same diameter. The desired variability was limited to the extension rod. The form of the extension is illustrated below:

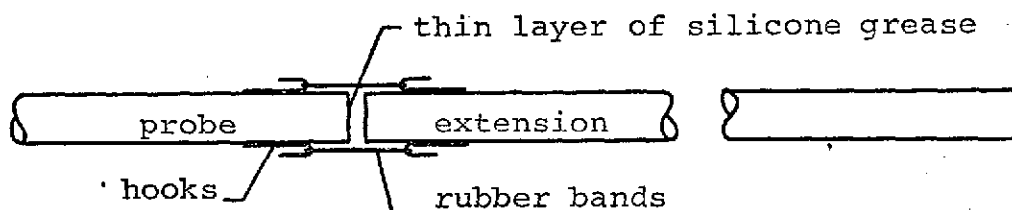


Fig. 2 Probe extension

A pair of hooks were taped on diametrically opposite edges of both the probe rod and the extension. A thin layer of silicone grease was applied to both ends. The joint was compressed in order to squeeze out excess grease. The function of the grease is to establish uniform contact between the 2 surfaces without acting as an intermediate layer. The connecting rubber bands were then attached. Ideally, we require

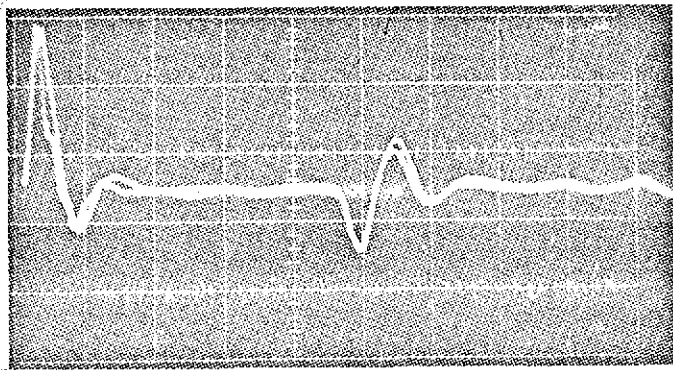
the joint to be so good as to make the extended probe acoustically equivalent to a single probe of the same length. This is impossible to achieve.

To test how good the joint is, the following experiment may be performed. The probe is suspended from 2 rubber bands in a horizontal position. An oscillogram trace of the impact test is taken at 0.2 ms/div as described above. The reflection from the back end returns to the crystal 0.86 ms after the original signal. Then the extension rod is attached to the probe. This rod must be supported by another rubber band to ensure that the entire assembly is straight and that the joint has uniform pressure. The impact test is then repeated. This time there are two reflected waves - one which is reflected at the interface between the probe and the extension rods, and the other reflected from the remote end. If the signal reflected from the interface is small compared to that reflected from the free end of the extension rod, then the joint is good. Traces corresponding to no extension rod and those with good and bad joints are shown on Figs. 3a-b-c.

Information regarding the impedance matching at the crystal-Plexiglas interfaces can be obtained from impact traces taken on extended time scales. Figure 3d shows such a trace taken at 38 μ sec/div. Initially, the trace is part of a bell shaped curve typical of elastic impacts. After a lapse of 57 μ sec, this curve gets distorted. The distortion is caused by that part of the stress pulse that upon arriving at the short Plexiglas rod-crystal interface is reflected and again reaches the interface after travelling back and forth in the short rod. Thus the period after the start of the impulse may be subdivided into intervals of 57 μ sec in order to investigate the effect of successive reflections.

0.2 msec

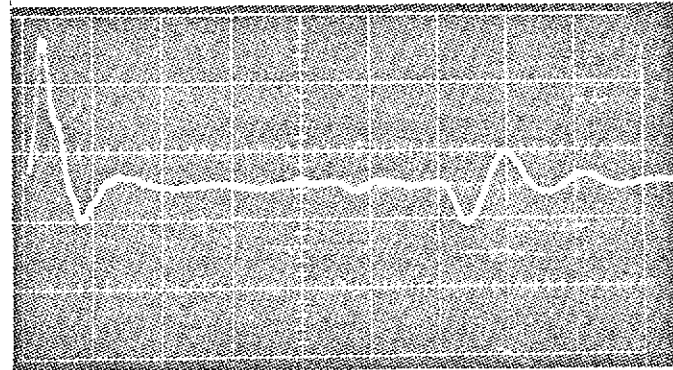
0629/11



0.2 V/DIV

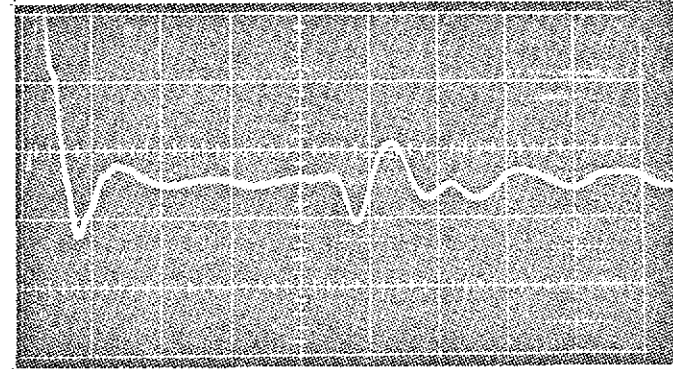
a) NO EXTENSION

0629/14



b) EXTENDED ROD: GOOD JOINT

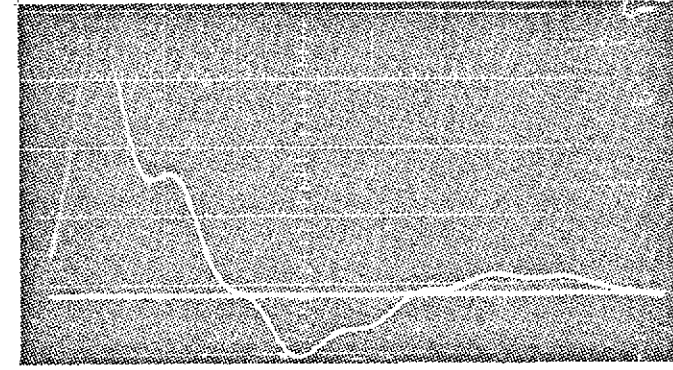
0629/13



c) EXTENDED ROD: IMPROPER JOINT

38 μsec

1106/8



0.1 V/DIV

d) EXTENDED SCALE

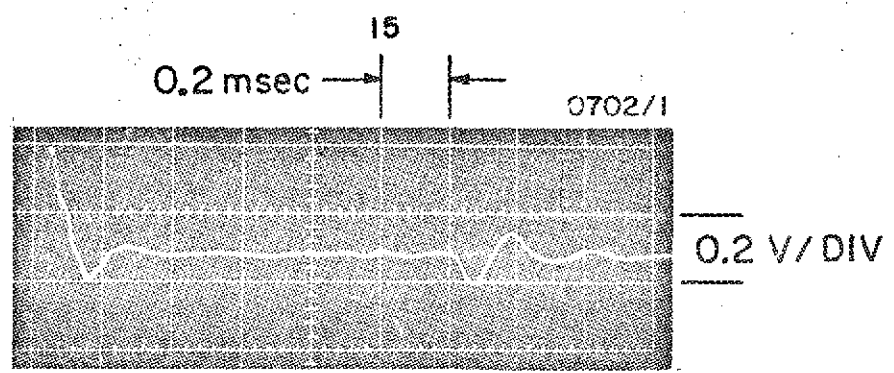
IMPACT TEST
FIGURE 3

If there had been a perfect match of impedance on either side of the crystal, then these reflections would be absent. The probe response would be just the bell shaped impact curve followed by no signal until the reflection from the end of the probe returns to the crystal. The reflections cause transients which last about 0.6 ms from the start of the pulse. The correct parameter to measure the response time to a steady pressure is the time elapsed between the initial stress pulse going to zero at the end of impact and the instant when the probe response indicates a zero signal. This figure can only be estimated since we cannot observe the end of the initial stress pulse because of the reflections which start before the pulse ends. An estimate of 0.45 ms may be obtained by guessing the terminal point of the initial pulse.

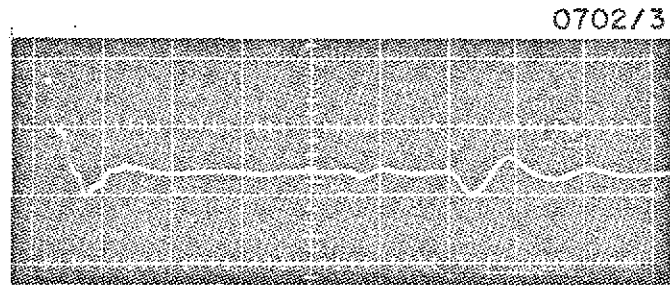
Mention was made of multiple reflections within the length of the crystal itself and its consequent effect on the wave propagation. The transit time of waves through the crystal is 1 μ sec. The probe response does not indicate that such a time period exists in the probe response; reflection of waves within the crystal do not seem to have a significant effect on the probe response.

The effect of the backing rod in delaying the reflected signal was observed in connection with the use of the extension rod. In that instance the free end of the extension rod was plane and normal to the rod axis. Traces using extension rods with their free end plane and at various angles to the axis are shown on Figs. 4a, 4b, 4c and 4d. These confirm that changing the end shape does not alter the acoustic delay in the rod.

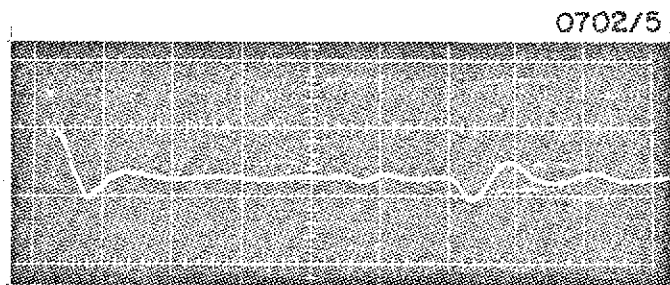
Attempts to alter the boundary conditions at the end of the rod were not extensive since, as mentioned earlier, this method would considerably increase the problems of



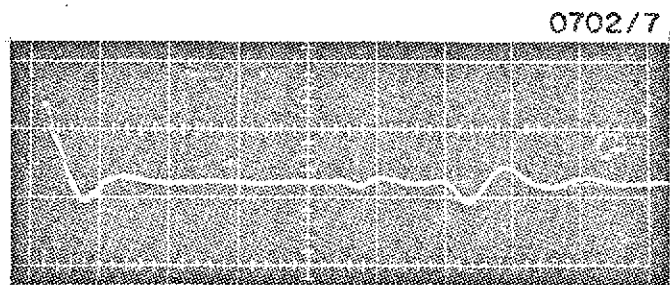
a) 27°



b) 30°



c) 44°



d) 63°

EXTENSION ROD END AT VARIOUS ANGLES

FIGURE 4

mechanical isolation of the probe. Figures 5a-b-c show the result of impact tests carried out with the probe in a vertical position with the remote end of the probe placed on various surfaces, as noted. The results are interesting and may lead to solutions if pursued further.

This brings us to the last and most promising means of delaying reflected waves - by using resonant absorption of vibration.

As mentioned in the discussion earlier, we have to choose a characteristic dominant frequency in the stress wave. We use Fig. 6 and consider the frequency equivalent to the bell shaped curve representing the impact. The frequency is estimated to be within the range 3.7×10^4 to 4.6×10^4 rad./sec.

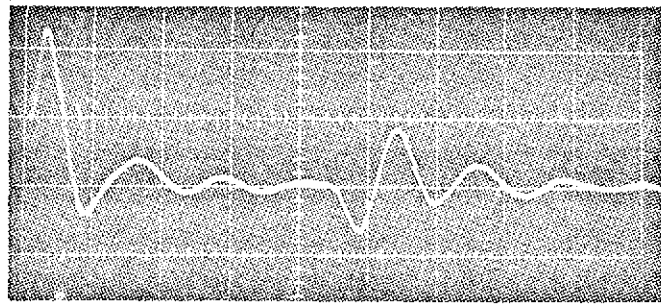
The vibration absorber used was a circular disc of Plexiglas with a central hole of 0.9 cm diameter. For the computation of its modes of vibration the disc was considered to be a solid circular plate with a free edge. The dimensions of the disc were so chosen as to resonate at the dominant frequency of the stress waves which we are attempting to remove. The requisite theory is summarized in Ref. 7. The mode of vibration is characterized by the tabulated nondimensional eigenvalues λ^2 and the integral number n of diametral nodes, and the integral number s of axially symmetric nodes.

λ^2 is related to the frequency and the dimensions of the disk by the expression

$$\lambda^2 = \omega a^2 \sqrt{\rho/D} \quad (1)$$

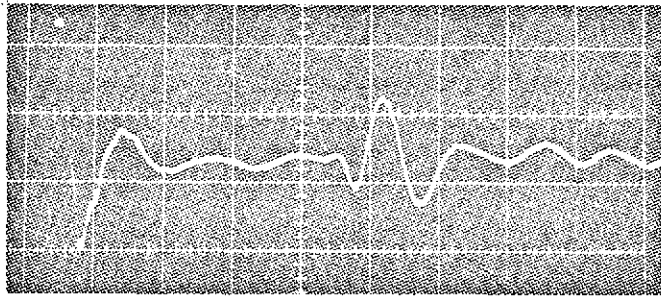
where: ω = resonant frequency
 a = radius of the disk
 ρ = mass density, per unit plate area

0.2 msec → | ←

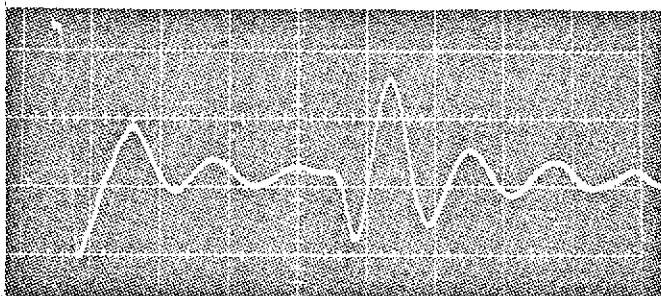


0.1 V/DIV

a) HARD RUBBER



b) PLEXIGLAS PLATE



c) PLEXIGLAS ON HARD RUBBER

PROBE SUPPORT

FIGURE 5

$$D = \frac{Eh^3}{12(1-\nu^2)}, \text{ where}$$

E = Young's modulus of elasticity

ν = Poisson's ratio, and

h = thickness of the plate.

The smallest value of λ^2 corresponding to a Poisson's ratio of $\nu = 0.33$ for Plexiglas was chosen since this leads to the smallest, most convenient disc size. This value of λ^2 is 5.235 and occurs for $n = 2, s = 0$.

The expression (1) for the eigenvalue can now be used to determine the diameter and the thickness of the disk for its $n = 2, s = 0$ mode of vibration. In the experiments all discs were cut from 5.19 cm diameter Plexiglas stock.

In our case,

$$\lambda^2 = 5.253$$

$$a = 2.59 \text{ cm}$$

$$E = 6.2 \times 10^9 (\pm 16\%) \text{ N/m}^2, \text{ Handbook of Chemistry and Physics}$$

The range of h extends from

$$\begin{aligned} &6.43 \text{ mm for } 3.66 \times 10^4 \text{ rad/sec} \\ &\text{to } 8.15 \text{ mm for } 4.64 \times 10^4 \text{ rad sec,} \end{aligned}$$

using the mean value of E . To take into account possible variability of $\pm 16\%$ in the value of E and the effects of using boundary conditions that were only approximate and use of a theory which assumed the thickness of the plate small compared to its diameter, we extended the range of thicknesses.

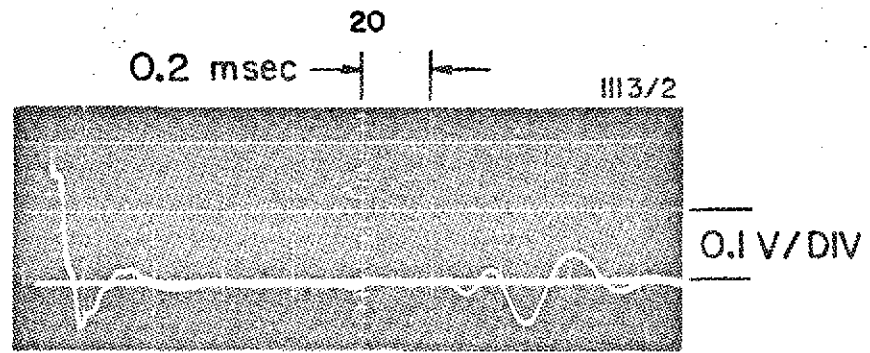
A set of disks of the following dimensions were machined initially (all of Plexiglas stock 5.19 cm in diameter).

Disk No.	Thickness (cm)
1, 2	0.554
3	0.500
4	0.595
5	0.714
6	0.780
7	0.844
8	1.037
9	0.650
10	0.965

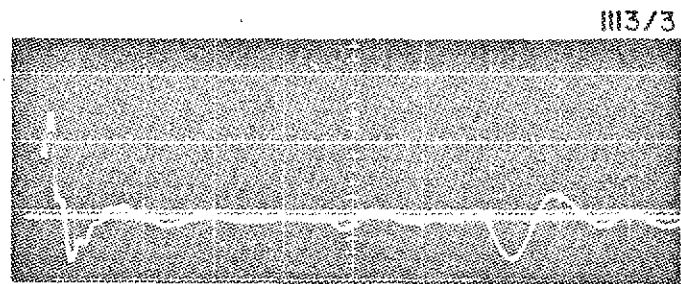
The disks were fitted one by one, on the end of the extension rod. The responses of the extended probes to the impact tests are shown on Figs. 6a - b - c - d - e corresponding to disks 3, 4, 5, 6 and 8. It was observed that the reflected signal was significantly altered by the addition of the disks. In particular, it was noted with interest that most of the reflection was delayed by a period of about 0.15 msec when disks 3, 4 and 5 in Fig. 6a, 6b and 6c were used. During this period, there was some reflection, however, but the amplitude of the reflected wave during the first 0.15 msec was small compared to the amplitude of the reflection that is delayed. The relative amplitudes of the 2 reflections are noted below

Disk No.	Primary reflection (cm of signal trace)	Main reflection
3	- 0.159	0.571, 0.620
4	- 0.053	0.604, 0.762
5	+ 0.159	0.555, 0.650

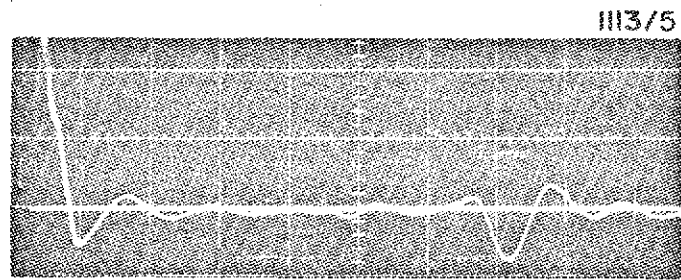
The sign of the primary reflection refers to the sign of the first reflected wave. This is of interest since there is a reversal in the sign, and to obtain optimum delay, the resonant disk must be so chosen as to minimize the amplitude of the primary reflection. On the basis of the above table, it is apparent that the optimum size of the disk lies between



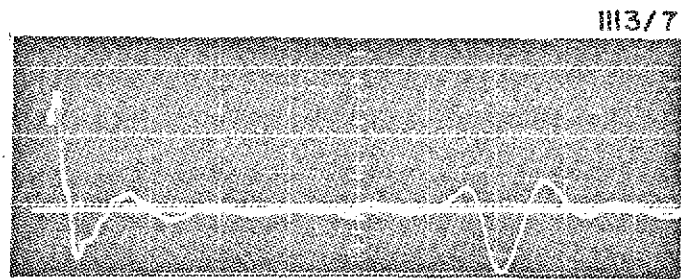
a) DISC # 3 ; 0.50 cm



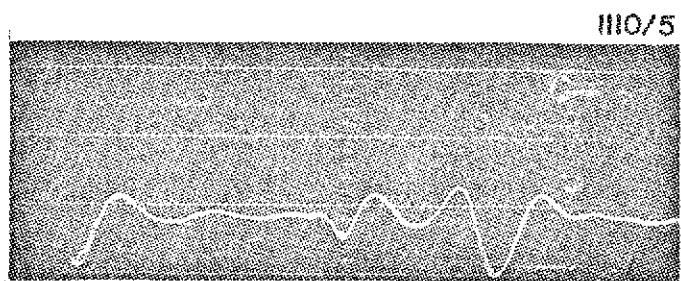
b) DISC # 4 ; 0.595 cm



c) DISC # 5 ; 0.714 cm



d) DISC # 6 ; 0.780 cm



e) DISC # 8 ; 1.037 cm

PROBE WITH DISC ON EXTENSION ROD

FIGURE 6

disks 4 and 5, and from a linear interpolation on the basis of the amplitudes of the reflected waves, we arrive at an optimum disk thickness of 0.622 cm.

New disks were machined to the thicknesses listed. The material for the disks 11, 12, 14, 15 and 16 was from a different stock than used for the earlier disks 1 - 10, and 13. The elastic properties could be slightly different.

Disk No.	Thickness (cm)
11	0.610
12	0.625
13	0.633
14	0.640
15	0.599
16	0.586

Typical responses using disks 11, 12, and 13 are shown in Figs. 7a, 7b and 7c.

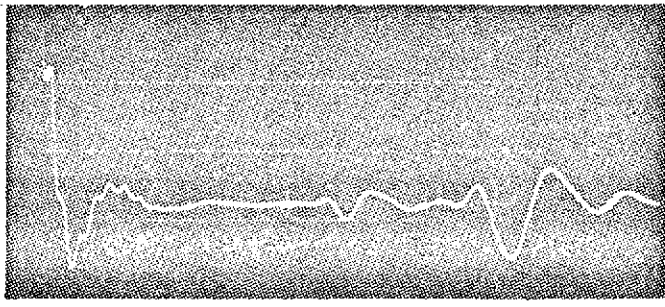
Disk 12 appeared to provide the most effective delay for 0.15 msec. With such a disk it is possible to have a backing rod of 75 cm instead of 90 cm. Some attempts were made to obtain additive delays but were not successful.

It is instructive to note the nature of this delay. When the stress wave travelling along the rod reaches the base of the disk it sets this in motion. Provided a suitable impedance match exists, the disk is set into vibration; the stress wave at the base of the disk travels outwards in the disk, and returns to the disk base after a certain time period. This explains why the delay of approximately 150 μ sec is about the same as the time period of the wave for which we are trying to obtain resonance. The primary reflection is due to the impedance mismatch at the base of the disk, i.e. when the resonant frequency of the disk is different from the frequency of the stress wave.

The results of the experiments described above are not conclusive with regard to the effectiveness of the delay since all the experiments used a particular impact which

0.2 msec →

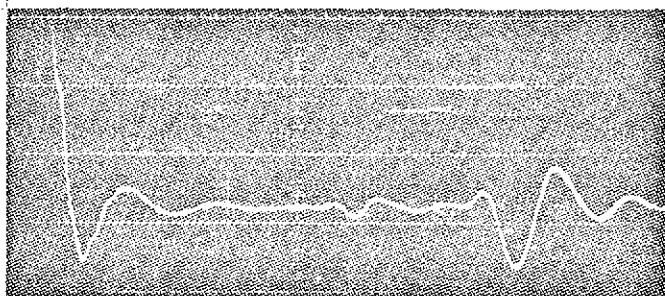
0524/2



0.1 V / DIV

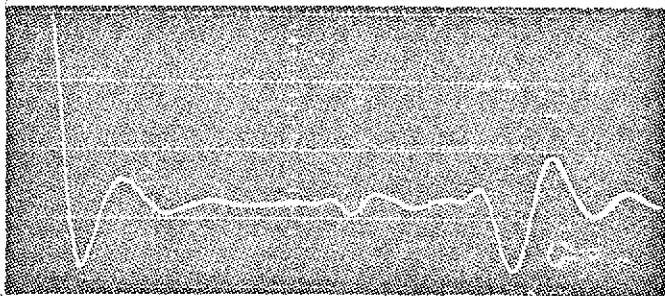
a) DISC #11 ; 0.610 cm

0524/4



b) DISC #12 ; 0.625 cm

0524/6



c) DISC #13 ; 0.633 cm

PROBE WITH DISC ON EXTENSION ROD

FIGURE 7

has associated with it a very distinct frequency. More extensive experiments need to be carried out with other impacts, e.g. obtained by varying the material and size of the impact sphere. This would yield data over a wider range of frequencies associated with the impact. The acid test must come when a suitably tailored probe is used in the tank to measure the pressure.

In passing, a few more ideas are worthy of mention. Attempts may be made to damp out the vibration of the resonant disk by using a visco-elastic material such as polyisobutylene rubber constrained to vibrate with the disk by a thin layer of a stronger substance. Reference may be made to the thesis of Dr. Yan⁸ which is related to the problem of constrained layer damping of vibrating plates.

The properties of visco-elastic materials may be used in other ways, e.g. to damp out the waves in the rod while maintaining an acoustic impedance match at the crystal interface. The textbook by Ferry⁹ is a good introduction to this extensive field.

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