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FINAL REPORT
CONVOLUTIONAL CODING TECHNIQUES
FOR DATA PROTECTION

NASA Grant NGL 15-004-026

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RESEARCH PERIOD REPORTED: September 15, 1967 to July 31, 1974
(Grant was inactive during the period
Sept. 16, 1971 to Oct. 15, 1972)

I. Introduction

Under this grant, NASA has supported for six years the work of the Principal Investigator and his associates centering on the use of convolutional codes in data communications. The breadth of this work can be seen from a perusal of the list in Section IV of the publications issuing under this grant.

Because the research in the first five years of this grant has previously been described in reports to NASA, we give a description in Section II only of the research performed in the final year of the grant. In Section III, we list the personnel who have been involved in the research under this grant since its inception and, as mentioned previously, we give a complete listing of the grant publications in the final Section IV.

II. Summary of Research in Final Year of Grant (October 16, 1973 to July 31, 1974)

It has been the policy of the Principal Investigator to communicate research results under this grant to NASA as timely as possible through appropriate technical reports and/or preprints of submitted journal articles. Most of the research performed during the final year of this grant has already been communicated in this manner to NASA and hence this section will be devoted primarily to citing of the pertinent reports together with a description of certain fragmentary results that have not been previously reported. The following four sub-sections contain this description of the research as divided into topical areas.

A. Convolutional Coding Fundamentals

The nearly six years of extensive research into convolutional coding techniques conducted under this grant has given the Principal Investigator an unparalleled opportunity to explore the advantages of "non-block" coding for

communications purposes. In an effort to share the fruits of this research with the communications engineering community, the Principal Investigator has recently prepared the following self-contained and extensive treatment of non-block coding:

J. L. Massey, "Error Bounds for Tree Codes, Trellis Codes and Convolutional Codes with Encoding and Decoding Procedures," Lectures presented at the Summer School on "Coding and Complexity," Centre International des Sciences Mechanique (C.I.S.M.), Udine, Italy, July 15-26, 1974, to appear in a future volume in the C.I.S.M. Lecture Series.

The above paper, which runs to 78 typewritten pages, while written in a tutorial style is also repository for a number of new results including:

(1) A useful definition of the general classes of tree and trellis codes with proofs of their error-correcting power,

(2) A quantitative demonstration of the advisability of choosing the "memory" greater than the "tail length" for convolutional codes used with sequential decoding, and

(3) A description of the smallest class of convolutional codes for which the random coding error bounds apply.

Motivated by (1) and (2) above, Mr. Rolf Johannesson, research assistant under this grant, conducted extensive simulations to verify the required excess of memory over tail length for minimal error probability and also generalized the bounds for tree and trellis codes. These results are reported in:

R. Johannesson, "On the Error Probability of General Tree and Trellis Codes with Applications to Sequential Decoding," Tech. Rpt. No. EE-7316, Dept. of Electrical Engineering, University of Notre Dame, Decmeber 1974 (in review for publication by the IEEE Trans. on Communications.)

Motivated further by the differing importance of the parameters "row distance," "column distance," and "free distance" of convolutional codes [all of which parameters were first identified as useful by previous research under this grant] dependent on the type of decoder used [sequential decoder, Viterbi decoder, or algebraic decoder], Johannesson sought to find codes which would

be simultaneously optimal or near-optimal for all three criteria. This work was successfully concluded and is described in:

R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional Codes," Tech. Rpt. No. EE-7403, Department of Electrical Engineering, University of Notre Dame, July 3, 1974 (to appear in IEEE Trans. on Information Theory.)

and was in part presented orally as

R. Johannesson, "Some Rate 1/2 Binary Convolutional Codes Which Are Optimal for Various Criteria," Presented at IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

These codes are expected to be prime candidates for adoption by NASA in future coding standards for deep-space missions.

B. Modulation and Coding Interaction

The concern about the effect of the bit tracking loop memory on sequential decoding computation that arose in the HELIOS program motivated the principal investigator to pay increased attention to the interaction of the coding scheme with the modulation scheme in a communication system. Such considerations led to a "new" communications theory adequate to treat simultaneously coding and modulation and first reported in:

J. L. Massey, "Coding and Modulation in Digital Communications," Proc. International Zurich Seminar on Digital Communications, pp. E2(1)-E2(4), Zurich, Switzerland, March 12-15, 1974.

This paper gives (1) the first analytical approach to optimal "soft-decision" demodulation for binary modulation and (2) a simple method to evaluate the quality of a modulation signal set. The approach taken in this paper is that the function of the modulation system is not to minimize bit error probability, as has been traditional, but to create the best channel for coding. "Best" here means having the largest cut-off rate R_o (the "two-codeword random coding exponent" or, equivalently, the rate above which the average computation for sequential decoding becomes infinite.) As an application of the approach,

it is shown that the simplex signal set is optimal for the R_o criterion for the additive white Gaussian noise channel.

Johannesson has continued the investigation of the R_o approach to channels with memory and some fragmentary results were obtained before the expiration of the grant period. This work was continued under a new grant (NASA Grant NSG 5025 "Coordinated Design of Coding and Modulation Systems") and a technical report now under preparation under the new grant will contain these earlier fragmentary results as well as more extensive further results. Johannesson also began a study for the analytical calculation of the computational distribution for sequential decoding and obtained some results under this grant which will be reported in another technical report, under the new grant, now in preparation.

To illustrate the importance of soft-decision modulation, particularly for channels with memory, if coding is to be profitably employed, the Principal Investigator applied the experience gained under this grant to the problem of communication against ionospheric scintillations as has recently been encountered in communication satellite systems. This research was reported in:

J. L. Massey, "Methods of Alleviation of Ionospheric Scintillation Effects on Digital Communications," Paper 74-55, 12th AIAA Aerospace Sciences Meeting, Washington, D.C., January 30-February 1, 1974.

C. Concatenated Coding Systems

Mr. Lin-nan Lee, research assistant under this grant, has been investigating the use of convolutional codes as the inner codes in concatenated coding systems. An early realization from this study was of the need for extending Massey's optimal soft-decision theory for binary modulation to the non-binary case because the digits in the outer code of a concatenated coding system are non-binary. The results of this work were reported in:

L. Lee, "Optimal Soft-Decision Demodulation for Non-Binary Signalling," Presented at the IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

Lee has continued this research under the new grant mentioned above (NASA Grant NSG 5025) and is preparing a technical report thereunder which will include the detailed derivation of the results in the above presentation. The object of his research is to produce simple, yet powerful, concatenated coding systems in which the inner decoder will be a "soft-decision Viterbi decoder."

D. Data Compression with Convolutional Codes

Mr. Teofilo C. Ancheta, Jr., research assistant under this grant, has been continuing his investigation of the "syndrome-source-coding" technique by which error-correcting codes are used for data compression in a novel manner. His work under this grant is reported in:

T. C. Ancheta, Jr., "Syndrome-Source-Coding for Data Compression," Tech. Rpt. No. EE-7312, Dept. of Electrical Engineering, University of Notre Dame, July 25, 1973 and Revised July 1974 (in review for publication by IEEE Trans. on Information Theory), also presented at IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

To elucidate the difference and the potential advantage of "syndrome-source-coding" over the conventional or "codeword method" of using error-correcting codes for data compression, the Principal Investigator prepared the following tutorial paper:

J. L. Massey, "The Codeword and Syndrome Methods for Data Compression with Error-Correcting Codes," to appear in Proceedings NATO Advanced Study Institute on New Directions in Signal Processing and Control, Darlington, England, August 15-17, 1974.

In general, it can be stated that the "codeword method" is more suited to data sources that exhibit much symmetry and little memory whereas the "syndrome method" matches better to sources with little symmetry and much memory. Since the latter type of source seems to approximate real sources better than the former, we expect syndrome-source-coding to find practical

applications in data compression systems. In order to study source models with memory, it was found necessary to enlarge the present understanding of Markov source models which seem to be the most realistic and convenient models for sources with memory. Our results in this area were reported in:

J. L. Massey, "Markov Information Sources," to appear in Proceedings NATO Advanced Study Institute on New Directions in Signal Processing and Control, Darlington, England, August 15-17, 1974.

III. Summary of Personnel Supported by Grant

One of the valuable aspects of a research grant to a University is the resultant educational value to the graduate students who perform research to meet grant objectives. Through Grant NGL 15-004-026, NASA has provided such support to ten graduate students, six of whom have already received the Ph.D. degree from the University of Notre Dame for dissertations based on their grant research and two more of whom are expected to do so in the near future. A ninth graduate student, Rolf Johannesson, spent the period September 1, 1973 to October 31, 1974 on leave from the University of Lund, Sweden, to perform research under the Principal Investigator which he will now submit as his doctoral research at Lund. In this respect, NASA has through this grant played a remarkably extensive and vital role in support of graduate education. Several of these past graduate students have already won considerable recognition for their research under this grant. The following is a table of all graduate students who performed research under this grant under the direction of the Principal Investigator

Name	Period of Research Support under Grant	Remarks
Teofilo C. Ancheta, Jr.	10/16/72 to 7/31/74	Ph.D. expected August 1975.
Jian K. Chang	6/16/68 to 8/15/71	S.M.E.E. received, June 1971.
Daniel J. Costello, Jr.	9/16/68 to 9/16/69	Ph.D. received, Sept. 1969.
John M. Geist	9/16/69 to 9/15/70	Ph.D. received, Sept. 1970.
William F. Hartman	(unsupported)	Ph.D. requirements completed for June 1975.
Rolf Johannesson	(unsupported)	Ph.D. expected, Univ. of Lund, Sweden, June 1975.
Lin-nan Lee	10/16/72 to 7/31/74	Ph.D. expected, June 1975.
Thomas N. Morrissey, Jr.	(NASA traineeship 9/16/67 to 9/15/68)	Ph.D. received, Sept. 1968.

Raymond R. Olson	9/16/67 to 8/15/68	Ph.D. received, Sept. 1970.
Gerald E. Seguin	9/16/69 to 8/15/71	Ph.D. received, Jan. 1972.

Besides the above graduate students, research support under this grant was furnished during the period 6/16/68 to 8/15/68 to Dr. Michael K. Sain and to Dr. Kasivanathan Vairavan, the former of whom acted as a co-investigator during this period and the latter of whom served as a postdoctoral research assistant. Dr. James L. Massey has been the Principal Investigator throughout the grant period.

IV. Summary of Grant Publications

Research is generally of little value, even to its sponsor, unless it is communicated rapidly and clearly to the general scientific world and contributes thereby to the advance of its technical field. The research from this grant has resulted in 20 papers published in archival journals, 21 conference presentations and/or conference publications and 21 technical reports. These publications are listed in sub-sections A, B and C below. (Two further papers are presently in process of review for possible journal publication as mentioned in Section II.) Preprints and/or reprints of all these publications were distributed previously to NASA in accordance with the established grant procedures and contained acknowledgment of NASA support.

A. Journal Articles

J. L. Massey, "Some Algebraic and Distance Properties of Convolutional Codes," appearing in Error Correcting Codes (Editor: H. B. Mann), John Wiley & Sons, Inc., New York, pp. 89-109, 1968.

M. K. Sain and J. L. Massey, "Invertibility of Linear Time-Invariant Dynamical Systems," IEEE Transactions on Automatic Control, AC-14, pp. 141-149, April 1969.

J. L. Massey, "Shift-Register Synthesis and BCH Decoding," IEEE Trans. Info. Th., Vol. IT-15, pp. 122-127, January 1969.

D. J. Costello, Jr., "A Construction Technique for Random-Error-Correcting Convolutional Codes," IEEE Transactions on Information Theory, IT-15, pp. 631-636, September 1969.

T. N. Morrissey, Jr., "Analysis of Decoders for Convolutional Codes by Stochastic Sequential Machine Methods," IEEE Trans. Info. Th., Vol. IT-16, pp. 460-469, July 1970.

R. R. Olson, "Note on Feedforward Inverses for Linear Sequential Circuits," IEEE Transactions Electronic Computers, Vol. C-19, pp. 1216-1221, December 1970.

D. J. Costello, Jr., and T. N. Morrissey, Jr., "Strengthened Lower Bound on Definite Decoding Minimum Distance for Periodic Convolutional Codes," IEEE Trans. Info. Th., Vol. IT-17, pp. 212-214, March 1971.

J. M. Geist, "An Empirical Comparison of Two Sequential Decoding Algorithms," IEEE Trans. Communication Technology, Vol. COM-19, No. 4, pp. 415-419, August 1971.

J. L. Massey and D. J. Costello, Jr., "Nonsystematic Convolutional Codes for Sequential Decoding in Space Applications," IEEE Trans. Comm. Tech., COM-19, pp. 806-813, October 1971.

J. L. Massey and O. N. Garcia, "Error-Correcting Codes in Computer Arithmetic," Chapter 5 in Advances in Information Systems Science, Vol. 4 (Ed. J. Tou), Plenum Press, New York, pp. 273-326, 1972.

J. L. Massey, "Variable-Length Codes and the Fano Metric," IEEE Trans. Info. Theory, IT-18, pp. 196-198, January 1972.

J. L. Massey, "Optimum Frame Synchronization," IEEE Trans. Communications, COM-20, pp. 115-119, April 1972.

J. L. Massey, M. K. Sain and J. M. Geist, "Certain Infinite Markov Chains and Sequential Decoding," Discrete Mathematics, Vol. 3, pp. 163-175, September 1972.

W. F. Hartman, "Modular Arithmetic Weight and Cyclic Shifting," Information and Control, Vol. 21, No. 4, pp. 339-343, November 1972.

J. L. Massey, D. J. Costello, Jr., and J. Justesen, "Polynomial Weights and Code Constructions," IEEE Trans. Information Theory, Vol. IT-19, No. 1, pp. 101-110, January 1973.

J. M. Geist, "Search Properties of Some Sequential Decoding Algorithms," IEEE Trans. Information Theory, IT-19, pp. 519-526, July 1973.

J. L. Massey, "On the Fractional Weight of Distinct Binary n-Tuples," IEEE Trans. Information Theory, Vol. IT-20, p. 131, January 1974.

D. J. Costello, Jr., "Free Distance Bounds for Convolutional Codes," IEEE Trans. Info. Th., Vol. IT-20, pp. 356-365, May 1974.

J. L. Massey, "Error Bounds for Tree Codes, Trellis Codes, and Convolutional Codes with Encoding and Decoding Procedures," Lectures presented at the Summer School on "Coding and Complexity," Centre International des Sciences Mechanique (C.I.S.M.), Udine, Italy, July 15-26, 1974, to appear in a future volume in the C.I.S.M. Lecture Series.

R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional Codes," to appear in IEEE Trans. Info. Th., 1975.

B. Conference Presentations and Articles Published in Conference Proceedings

J. L. Massey, "Catastrophic Error-Propagation in Convolutional Codes," Proceedings of 11th Midwest Circuit Theory Symposium, University of Notre Dame, Notre Dame, Indiana, pp. 583-587, May 13-14, 1968.

J. L. Massey and M. K. Sain, "Trunk and Tree Searching Properties of the Fano Sequential Decoding Algorithm," Proc. Sixth Allerton Conf. on Circuit and System Th., pp. 153-160, Monticello, Ill., Oct. 2-4, 1968.

J. L. Massey and M. K. Sain, "Distribution of the Minimum Cumulative Metric for Sequential Decoding," Presented at IEEE International Symposium on Info. Theory, Ellenville, N.Y., January 28-31, 1969.

M. K. Sain and J. L. Massey, "A Modified Inverse for Linear Dynamical Systems," Proceedings of IEEE 8th Adaptive Processes Symposium, Penn. State Univ., pp. 5-a-1--5-a-3, November, 1969.

T. N. Morrissey, Jr., "A Markovian Analysis of Viterbi Decoders for Convolutional Codes," Proc. of the National Electronics Conference, Chicago, Ill., Vol. 25, pp. 303-307, December, 1969.

D. J. Costello, Jr., "A Strong Lower Bound on Free Distance for Periodic Convolutional Codes," "Long Paper" presented at IEEE International Symposium on Information Theory, Noordwijk, The Netherlands, June 15-19, 1970.

G. Seguin, "Binary Sequences with a Relaxed Barker Criterion," Proc. National Electronics Conference, Vol. 26, Chicago, Illinois, pp. 456-457, December 1970.

J. L. Massey, "Non-Systematic Convolutional Codes for Sequential Decoding," Presented at NASA-Wide Coded Communications Conference, JPL-Pasadena, Calif., February 26-27, 1970.

J. M. Geist and J. L. Massey, "A Comparison of the Fano and Jelinek Sequential Decoding Algorithms,"Presented at NASA-Wide Coded Communications Conference, JPL-Pasadena, Calif., February 26-27, 1970.

J. L. Massey and J. J. Uhran, Jr., "Analysis of Satellite Communications in a Multipath Environment," Proc. IEEE International Communications Conf., pp. 10-20--10-24, San Francisco, Calif., June 8-10, 1970.

J. L. Massey and J. J. Uhran, Jr., "Sub-Baud Coding and Cyclic Codes," Presented at IEEE International Symposium on Information Theory, Noordwijk, Netherlands, June 15-19, 1970.

D. J. Costello, Jr., "Using Cyclic Codes to Generate Good Convolutional Codes," Proceedings of the Mervin J. Kelly Communications Conference, U. of Missouri, Rolla, Missouri, October 5-7, 1970.

D. J. Costello, Jr. and J. L. Massey, "Constructing Good Convolutional Codes from Cyclic Block Codes," Presented at IEEE International Symposium on Info. Theory, Asilomar, Calif., January 1972.

J. L. Massey, "An Error Bound for Random Tree Codes," Presented at IEEE International Symposium on Information Theory, Ashkelon, Israel, June 1973.

J. L. Massey, "Methods of Alleviation of Ionospheric Scintillation Effects on Digital Communications," Paper 74-55, 12th AIAA Aerospace Sciences Meeting, Washington, D.C., January 30-February 1, 1974.

J. L. Massey, "Coding and Modulation in Digital Communications," Proc. International Zurich Seminar on Digital Communications, pp. E2(1)-E2(4), Zurich, Switzerland, March 12-15, 1974.

J. L. Massey, "Markov Information Sources," to appear in Proc. NATO Advanced Study Institute on New Directions in Signal Processing and Control, Darlington, England, August 15-17, 1974.

J. L. Massey, "The Codeword and Syndrome Methods for Data Compression with Error-Correcting Codes," to appear in Proceedings NATO Advanced Study Institute on New Directions in Signal Processing and Control, Darlington, England, August 15-17, 1974.

L. Lee, "Optimal Soft-Decision Demodulation for Non-Binary Signalling," Presented at IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

T. C. Ancheta, Jr., "Syndrome-Source-Coding for Data Compression," Presented at IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

R. Johannesson, "Some Rate 1/2 Binary Convolutional Codes Which Are Optimal for Various Criteria," Presented at IEEE International Symposium on Information Theory, Notre Dame, Indiana, Oct. 27-31, 1974.

C. Technical Reports

R. R. Olson, "Note on Feedforward Inverses for Linear Sequential Circuits," Tech. Rpt. No. EE-684, University of Notre Dame, April 1, 1968.

T. N. Morrissey, Jr., "Analysis of Decoders for Convolutional Codes by Stochastic Sequential Machine Methods," Tech. Rpt. No. EE-682, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Indiana, May 1, 1968.

D. J. Costello, Jr., "A Construction Technique for Random-Error Correcting Convolutional Codes," Tech. Rpt. No. EE-685, U. of Notre Dame, Notre Dame, Indiana, May 1, 1968.

J. L. Massey, "Shift Register Synthesis and BCH Decoding," Elec. Engr. Memo No. EE-684, University of Notre Dame, June 18, 1968.

M. K. Sain and J. L. Massey, "Invertibility of Linear Time-Invariant Dynamical Systems," Memo EE-687, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., August 8, 1968

J. L. Massey and M. K. Sain, "Trunk and Tree Searching Properties of the Fano Sequential Decoding Algorithm," Elec. Engr. Memo. No. EE-6817, Univ. of Notre Dame, Notre Dame, Ind., October 1, 1968.

T. N. Morrissey, Jr., "A Unified Markovian Analysis of Decoders for Convolutional Codes," Tech. Rpt. No. EE-687, Dept. of Elect. Engr., U. of Notre Dame, Notre Dame, Ind., October 24, 1968.

D. J. Costello, Jr., "A Construction Technique for Random Error Correcting Convolutional Codes," Memo EE-6822, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., November 14, 1968.

D. J. Costello, Jr., "Construction of Convolutional Codes for Sequential Decoding," Tech. Rpt. EE-692, Dept. of Elec. Engr., U. of Notre Dame, August 1969.

J. J. Uhran, Jr. and J. L. Massey, "Analysis of Satellite Communications in a Multipath Environment," Elec. Engr. Memo EE-701, Univ. of Notre Dame, Notre Dame, Indiana, January 1970.

J. M. Geist, "A Comparison of the Fano and Jelinek Sequential Decoding Algorithms," Tech. Rpt. No. EE-701, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., January 31, 1970.

R. R. Olson, "On the Invertibility of Finite State Machines," Tech. Rpt. No. EE-703, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Indiana, July 23, 1970.

G. Seguin, "Binary Sequences with a Relaxed Barker Criterion," Tech. Rpt. No. EE-704, Dept. of Elec. Engr., Univ. of Notre Dame, August 10, 1970.

J. M. Geist, "Algorithmic Aspects of Sequential Decoding," Tech. Rpt. No. EE-702, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Indiana, August 1970.

W. F. Hartman, "Note on Arithmetic Codes and Arithmetic Distance," Tech. Report EE-705, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., November 1970.

G. Seguin, "Cyclotomic Sequences," Tech. Rpt. No. EE-7102, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Ind., July 26, 1971. (Submitted to IEEE Transactions on Information Theory.)

G. Seguin, "Binary Sequences with Specified Correlation Properties," Tech. Rpt. No. EE-7103, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., August 1971.

L. Lee, "Real-Time Minimal-Bit-Error-Probability Decoding of Convolutional Codes," Tech. Rpt. No. EE-7312, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., July 25, 1973.

T. C. Ancheta, Jr., "Syndrome-Source-Coding for Data Compression," Tech. Rpt. No. EE-7313, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., July 25, 1973 (Revised July 1974.)

R. Johannesson, "On the Error Probability of General Tree and Trellis Codes with Applications to Sequential Decoding," Tech. Rpt. No. EE-7316, Dept. of Elec. Engr., Univ. of Notre Dame, December 1973.

R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional Codes," Tech. Rpt. No. EE-7403, Dept. of Elec. Engr., Univ. of Notre Dame, July 3, 1974.