N7514717

NASA CR-132563

STUDY OF DYNAMIC CHARACTERISTICS OF AEROELASTIC SYSTEMS UTILIZING RANDOMDEC SIGNATURES

By C.S. Chang

Prepared under Contract No. NAS1-12249 by
New Technology, Inc.
Huntsville, Alabama

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

ABSTRACT

Feasibility of utilizing the Random Decrement method in conjunction with a Signature analysis procedure to determine the dynamic characteristics of an aeroelastic system for the purpose of on-line prediction of potential on-set of flutter has been examined.

Digital computer programs were developed to simulate sampled response signals of a two-mode aeroelastic system. Simulated response data were used to test the Random Decrement method. A special curve-fit approach was developed for analyzing the resulting Signatures. A number of numerical "experiments" were conducted on the combined processess. The method was found to be capable of determining frequency and damping values accurately from Randomdec Signatures of carefully selected lengths.

FOREWORD

This report is prepared for the National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia by New Technology, Inc., under Contract NAS1-12249, " Study of Dynamic Characteristics of Aeroelastic Systems Utilizing Randomdec Signatures."

The Technical Representatives for NASA were Mr. J.T. Foughner, Jr., and Dr. R.M. Bennett, Aeroelasticity Branch, Structures and Dynamics Division, Langley Research Center. The principal NTI investigator was C.S. Chang. Computer programs were developed by G.O. Dennis and refined by G.A. Aramayo. C. Hsu provided technical consultation in all phases of work described in this report.

CONTENTS

LIST OF FIGURES

SUMMARY

An ensemble averaging method for determining the characteristic response function of an aeroelastic system from its turbulence-induced random vibrations was developed recently by Henry A. Cole, Jr. The most significant feature of this (the Random Decrement) method is that very little knowledge about the excitation is required to obtain useful results (Randomdec Signatures). Provided an automated numerical procedure can be developed to analyze Signatures, the Random Decrement method can be very useful for on-line prediction of the on-set of flutter during subcritical wind tunnel or flight testing.

The Signature analysis procedure selected for investigation under this project is that of a curve-fitting nature. A Randomdec Signature is approximated by the theoretical homogeneous solution of the mathematical model of the aeroelastic system under consideration. An error function between the Signature and the analytical expression is defined, determined and numerically minimized. Those coefficients in the theoretical solution which lead to a minimum error are said to be the best approximations of the dynamic properties of the aeroelastic system.

Digital simulation techniques were employed to carry out numerical experiments for investigating effects of variation of system, data acquisition, Randomdec and curve-fit parameters on the accuracy of the overall approach.

Section 1

INTRODUCTION

The need for an accurate, reliable and rapid means to forecast the onset of flutter during subcritical wind-tunnel and flight tests of aeroelastic structures is widely recognized. That this need exists today is evidenced by the number and variety of current and recently completed research and development programs in the United States and abroad (References 1, 2 and 3), as well as by the differences among methods used by various organizations (References 4, 5, 6 and 7).

Coupry (Reference 4) and Cole (References 5, 6, 8 and 9) both advocate the utilization and on-line processing of wind induced dynamic response signals from the test specimen for flutter prediction and failure detection. Their approaches have obvious advantages over other methods in which "controlled" excitations (impulsive, periodic and/or other kinds of forces) must be applied to induce responses in those vibrational modes which eventually flutter.

Coupry relies on rapid PSD analyses of the response signal and modal identification in the frequency domain. The accuracy of the method suffers when flutter modes are closely spaced on the frequency axis. The method also depends on the PSD of the excitation (in this case, the effects of turbulence) being reasonably flat over the frequency range in which the flutter modes reside.

Cole first introduced the concept of what was later named the Random Decrement method in Reference 5 while investigating applications of the correlation functions. He subsequently used it at NASA/Ames Research Center for detecting fatigue failures in a Space Shuttle wing flutter model (Reference 6). The method received full treatment by the inventor in References 8 and 9.

The Random Decrement method is essentially an ensemble averaging procedure which determines the characteristic response function (the Randomdec Signature) of an aeroelastic specimen under test from its turbulence-induced random vibrations.

To obtain a Randomdec Signature, one simply collects a number of segments of time series representing the random responses of a system, and ensemble-averages them. If the system is linear and the excitation random, the average time series converges towards the transient response of the system due to a set of initial conditions. The order of the system is arbitrary. The initial conditions to which the Signature corresponds can be manipulated almost at will by judicial "triggering" (selection of the starting point) of each ensemble member. For failure detection and for property identification of nonlinear structures, Cole favors triggering at a constant response level. The resulting Signature is an approximating time series of the characteristic response function* of the specimen in its natural environment at an amplitude which is equal to the trigger level. For flutter prediction in linear systems, constant-level triggering is not necessary, and various other methods become optionally available. Since the ensemble averaging procedure cannot be carried out indefinitely in practical situations, the Randomdec Signature will contain a certain amount of error which generally makes direct (visual) interpretation and determination of system dynamic characteristics difficult. The main objective of this study is to investigate the feasibility of utilizing the Random Decrement method in conjunction with a Signature analysis procedure to determine the dynamic characteristics of an aeroelastic system under test for the purpose of on-line prediction of potential on-set of flutter.

*The existence of a characteristic response function for a nonlinear system is an assumption.

The Signature analysis procedure selected for investigation under this project is that of a curve-fitting nature. A Randomdec Signature is approximated by the theoretical homogeneous solution for the mathematical model of the aeroelastic system. An error function between the Signature and the analytical expression is defined, determined and numerically minimized. Those coefficients in the theoretical solution which lead to a minimum error are said to be the best approximations of the dynamic properties of the aeroelastic system.

Digital computer programs were developed to simulate sampled response signals of a two-mode aeroelastic system. Simulated response data were used to test the Random Decrement method. A special curve-fit approach was developed for analyzing the resulting Signatures. A number of numerical "experiments" were conducted on the combined processes. Results of these experiments indicate definite feasibility of combination of the approaches.

Analyses of the Random Decrement method and the curve-fit procedures are presented in Sections 3 and 4 of this report, respectively. Section 5 deals with the simulation of response signals and the implementation of the Random Decrement procedures. Numerical results of the investigation are summarized in Section 6.

Computer programs developed for this study are described in Appendix A. A sample test case with typical input and output is included in Appendix B.

Section 2

SYMBOLS

All symbols used in the text portion of this report are defined when they are first introduced. The following is a cross-reference of these symbols arranged in alphabetical order.

SYMBOL	DEFINITION
a_i	Quasi-steady aerodynamic generalized force in the ith mode
$A_i(s)$	History dependent component of generalized force in the ith mode due to aerodynamics
b_i	See definition of $(-T_i/b_i)$
B_i	Amplitude in the ith mode of the anti-symmetric part of the periodic factor of a Randomdec Signature
$\underline{B}_i$	Best approximation of B_i determined by least-squares curve-fit
$BC_{i\kappa}$	"Box-car" components of the generalized force in the ith mode
c_i	Total modal damping coefficient in the ith mode
c_i'	Modal damping coefficient of the structure in still air
c_i''	Modal damping coefficient due to aerodynamics
c_k^ℓ	Intermediate variable used in the curve-fit process
d_m^ℓ	Correction factors calculated by the curve-fit procedure for the parameters α_m during the ℓth iteration
D_i	Amplitude in the ith mode of the symmetric part of the periodic factor of a Randomdec Signature
$\underline{D}_i$	Best approximation of D_i determined by least-squares curve-fit

$e(\omega)$	Infinite Fourier Transform of measured velocity response of an aeroelastic system, including effects of low-pass filters
E	Error function used to gauge the degree of success of a curve-fitting process
E^{ℓ}	Error function E after ℓ iterations of the curve-fit process
f_o	Bandwidth of the response simulation process, Hertz
$f_i(t)$	Inverse Laplace Transform of the factor in the transfer function on the ith mode which distinguishes an aeroelastic system from a purely mechanical system. For a purely mechanical system, $f_i(t)$ is a Dirac Delta function
$F_{j\kappa}$	Sequences of random numbers used to construct $Q_{i\kappa}$ in the simulation process
g	A dummy integer subscript used in intermediate steps in the curve-fit process
$h_i(t)$	Impulse response function of the ith mode due to an impulse applied at t=0
$h_i'(t)$	Convolution of $f_i(t)$ and $h_i''(t)$
$h_i''(t)$	Inverse Laplace Transform of the factor in the transfer function in the ith mode, which is responsible for flutter of an aeroelastic system when its poles move on to the imaginary axis
i	A subscript identifying modal parameters and variables, i=1,. . . . I
I	See definition of i.
j	Index used to identify those modes in which generalized forces are statistically related, j=1, ... J<I
J	See definition of j.
k	Sample counter, k=1,2, . . . K
k_i	Total generalized stiffness in the ith mode
k_i'	Generalized stiffness of the structure in still air

k''_i	Generalized stiffness due to aerodynamics
K	See defnintion of k
L { }	The Laplace Transform operator
m	An integer subscript used to associate intermediate variables in the curve-fit process with the various α_m's
m_i	Total generalized mass in the ith mode
m'_i	Generalized mass of the structure in still air
m''_i	Generalized mass due to aerodynamics
n	An integer counter used in script form for ensemble members in the Random Decrement process. It is also used in identifying the time associated with each ensemble observation.
N	See definition of n
p_{io}	Average initial velocity in the ith mode after N ensemble averages
p_{ino}	Velocity in the ith mode at $t=t_{no}$, which is also the initial modal velocity of the nth member used in the Random Decrement process
$P_{i\kappa}$	Simulated periodic components of the generalized forces in the ith mode
$q_i(t)$	Response (displacement) of the ith mode
$q_i(o)$	Initial displacement in the ith mode as a function of time
$\dot{q}_i(o)$	Initial velocity in the ith mode as a function of time
$\overline{q_i^2}$	Mean-squared value of response in the ith mode as a function of time
$\overline{q_i^2}(t)$	Expected squared value of response in the ith mode as a function of time
q_{io}	Average initial displacement in the ith mode after N ensemble averages
$q_{in}(t_n)$	Response in the ith mode collected by the Random Decrement process as the nth ensemble member, starting at $t_n=0$

q_{ino}	Displacement in the ith mode at $t=t_{no}$, which is also the initial modal displacement of the nth member used in the Random Decrement process
$Q_i(t)$	Generalized force in the ith mode
$Q_i'(t)$	Part of generalized force in the ith mode which is statistically independent of generalized forces in all other modes.
$Q_i''(t)$	Part of generalized force in the ith mode which is statistically related to a similar component in the jth mode
$\overline{Q_i^2}$	Mean-squared value of the generalized force in the ith mode
$\overline{Q_i'^2}$	Mean-squared value of Q_i'
$\overline{Q_i''^2}$	Mean-squared value of Q_i''.
$Q_{i\kappa}$	Simulated random components of generalized force for the ith mode. $Q_{i\kappa}$ is simulated at discrete time points $\kappa\delta$ only.
r_{gk}^{ℓ}	Intermediate variable used in the curve-fit process
R_g^{ℓ}	Intermediate variable used in the curve-fit process
R_{ij}	Coefficients relating generalized forces in different modes which are statistically dependent
s	Laplace Transform variable
$S_N(t')$	"Signal" part of a Randomdec Signature
$S_\infty(t')$	Ideal Randomdec Signature obtained by ensemble averaging infinitely many statistically independent response samples
t	Time
t'	Running variable for time for all members collected by the Random Decrement process. It is also used as the independent variable for the ensemble average of all members (the Randomdec Signature).
t_n	Running independent variable for the nth member collected in the Random Decrement process, $t_n=t-t_{no}$.

t_{no}	Time at which the nth ensemble member begins. It is also the nth triggering time in the Random Decrement process.
T	Length of a Randomdec Signature
T_b, T_c	Time constants of simulated low-pass filters used in data acquisition system
T_i	Time constant in the generalized force in the ith mode associated with $A_i(s)$
$(-T_i/b_i)$	Real pole of the transfer function in the ith mode of an aeroelastic system
$U(t)$	Measurable response of a dynamic system
$\overline{U^2}$	Mean-squared value of system response
$U_n(t')$	Total response function collected as the nth member of the ensemble by the Random Decrement process
$U_N(t')$	Ensemble average of $U_n(t')$
U_{Nk}	Value of the kth point in the Randomdec Signature
$y(t')$	Theoretical homogeneous solution of a purely mechanical system used to approximate an aeroelastic system
y_k	$y(t')\|_{t'=k\delta}$
$z(y_k)$	A weighting functional
α_1	Equivalent to B_1
α_2	Equivalent to $\zeta_1'\omega_1'$
α_3	Equivalent to $\overline{\omega_1'}$
α_4	Equivalent to D_1
α_5	Equivalnet to B_2
α_6	Equivalent to $\zeta_2'\omega_2'$
α_7	Equivalent to $\overline{\omega_2'}$

α_8 Equivalent to D_2

α_m General representation of α_1, α_2, ... α_8

β_m^ℓ Intermediate variable used in the curve-fit process

$\gamma_N(t')$ "Noise" part of a Randomdec Signature

γ_N^2 Mean-squared value of the noise term in a Randomdec Signature

$\gamma_N^2(t')$ Expected value of the noise term, $\gamma_N(t')$ in the Randomdec Signature

$\gamma_{iN}^2(t')$ Expected squared value of the noise term in a Randomdec Sigaature attributed to the ith modal response

δ Sampling period in a data acquisition process

Δf Frequency resolution of the response simulation process in units of Hertz

ε_m^ℓ Intermediate variable used in the curve-fit process

ζ Larger of ζ_1 and ζ_2

ζ_i Damping factor of the ith mode

ζ_i' Real parts of the complex poles of the transfer function in the ith mode of an aeroelastic system

$\underline{\zeta}_i'$ Best approximation of ζ_i' determined by least-squares curve-fit

η $(-1)^{\frac{1}{2}}$

κ An integer subscript used as a counter for simulated sampled response data

ℓ An integer superscript used as a counter in the iterative curve-fit process

ν See definition of ρ_ν

ρ_ν Multiplying scale factors for optimize step size in searching for least-squares curve fit, $\nu=1,2,3$ . . . $\rho_\nu=(0.5)^{(\nu-1)}$

τ	Integration (dummy) variable for t
ϕ^{ℓ}_{mgk}	Intermediate variable used in the curve-fit process
ψ^{ℓ}_{mg}	Intermediate variable used in the curve-fit process
ω_i	Natural frequency of the ith mode
ω'_i	Imaginary part of the complex poles of the transfer function in the ith mode of an aeroelastic system
$\bar{\omega}_i$	Damped natural frequency of the ith mode, $\bar{\omega}_i = (1 - \zeta_i^2)^{\frac{1}{2}}\omega_i$
$\bar{\omega}'_i$	Abbreviation for $(1 - \zeta_i'^2)^{\frac{1}{2}}\omega'_i$
$\underline{\omega}'_i$	Best approximation of ω'_i determined by least-squares curve-fit

Section 3

MATHEMATICAL FOUNDATIONS OF RANDOM DECREMENT

The mathematical foundation of the Random Decrement method is derived in the following for a linear time-invariant system.

Let t = time, the independent variable, $-\infty < t < \infty$;

$U(t)$ = measurable response of a system;

i = a subscript used to identify modal parameters and variables, $i=1, 2, \ldots I$, (the system is of order $2I$);

$q_i(t)$ = modal responses;

$Q_i(t)$ = generalized forces;

and $h_i(t)$ = modal response functions after an impulse applied at $t=0$.

The measured (total) response is the sum of all modal responses:

$$U(t) = \sum_{i=1}^{I} q_i(t).$$

For physically realizable (casual), second-order modes, we have

$$h_i(t) = 0, \quad 0 < t;$$

$$h_i(t) = e^{-\zeta_i \omega_i t} \sin[(1-\zeta_i^2)^{\frac{1}{2}} \omega_i t], \quad t > 0, \tag{1}$$

and $h_i(t)$ has a finite discontinuity at $t=0$. In the above expressions, ω_i is the undamped modal natural frequency and ζ_i the modal damping factor. For any point t_{no} ($n=1, 2, \ldots N$) on the time axis, let

$$q_{ino} = q_i(t)\big|_{t=t_{no}},$$

$$p_{ino} = \frac{dq_i(t)}{dt}\Big|_{t=t_{no}},$$

$$\text{and } \dot{h}_i(t) = \frac{dh_i(t)}{dt}, \quad t \neq 0.$$

The response function in each mode may be written, in general, as

$$q_i(t) = (p_{ino} + 2\zeta_i\omega_i q_{ino})h_i(t - t_{no}) + q_{ino}\dot{h}_i(t - t_{no})$$
$$+ \int_{t_{no}}^{t} Q_i(\tau)h_i(t - \tau)d\tau$$

where τ is an integration (dummy) variable. For each n, we define a new independent variable

$$t_n = t - t_{no},$$

and a new modal response function

$$q_{in}(t_n) = q_i(t_n + t_{no}),$$

which may be written for each n as

$$q_{in}(t_n) = (p_{ino} + 2\zeta_i\omega_i q_{ino})h_i(t_n) + q_{ino}\dot{h}_i(t_n)$$
$$+ \int_0^{t_n} Q_i(\tau - t_{no})h(t_n - \tau)d\tau.$$

Since t_n starts from zero for each n, it is unnecessary to distinguish them in the last equation. We may, therefore, use a common independent variable t' for all t_n and write

$$q_{in}(t') = (p_{ino} + 2\zeta_i\omega_i q_{ino})h_i(t') + q_{ino}\dot{h}_i(t') \qquad (2)$$
$$+ \int_0^{t'} Q_i(\tau - t_{no})h_i(t' - \tau)d\tau.$$

Summing over I modes, the response function for each observation beginning with $t=t_{no}$ is

$$U_n(t') = \sum_{i=1}^{I} (p_{ino} + 2\zeta_i\omega_i q_{ino})h_i(t')$$

$$+ \sum_{i=1}^{I} q_{ino}\dot{h}_i(t')$$

$$+ \sum_{i=1}^{I} \int_0^{t'} Q_i(\tau - t_{no})h_i(t' - \tau)d\tau.$$

The ensemble average of N such observed results is

$$U_N(t') = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{I} U_n(t')$$

$$= \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{I} (P_{ino} + 2\zeta_i\omega_i q_{ino})h_i(t')$$

$$+ \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{I} q_{ino}\dot{h}_i(t')$$

$$+ \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_0^{t'} Q_i(\tau - t_{no})h_i(t' - \tau)d\tau$$

$$= \sum_{i=1}^{I} h_i(t')[\frac{1}{N} \sum_{n=1}^{N} p_{ino}] \tag{3}$$

$$+ \sum_{i=1}^{I} 2\zeta_i\omega_i h_i(t')[\frac{1}{N} \sum_{n=1}^{N} q_{ino}]$$

$$+ \sum_{i=1}^{I} \dot{h}_i(t')[\frac{1}{N} \sum_{n=1}^{N} q_{ino}]$$

$$+ \frac{1}{N} \sum_{i=1}^{I} \sum_{n=1}^{N} \int_0^{t'} Q_i(\tau - t_{no})h_i(t' - \tau)d\tau.$$

The quantities $\frac{1}{N}\sum_{n=1}^{N} p_{ino}$ and $\frac{1}{N}\sum_{n=1}^{N} q_{ino}$ are the average (over N observations) initial modal velocity and displacement, p_{io} and q_{io}, respectively. We may, therefore, regard the ensemble average of the response functions as being of two parts. The first part has a deterministic functional form, and its magnitude is established by p_{io} and q_{io}. We shall call this part the "signal", and represent it by

$$S_N(t') = \sum_{i=1}^{I} [(p_{io} + 2\zeta_i\omega_i q_{io})h_i(t') + q_{io}\dot{h}_i(t')]. \tag{4}$$

The second part, the last term in Equation (3), is the system response to the generalized forces, and in our discussions below, it will be called the "noise", represented by $\gamma_N(t')$. It is the ensemble average of forced responses and its characteristics are dependent on the characteristics of the generalized forces $Q_i(t)$. By interchanging the order of summation and integration operations, the noise term in Equation (3) may be re-written in the following form:

$$\gamma_N(t') = \frac{1}{N}\sum_{i=1}^{I} \int_0^{t'} h_i(t' - \tau)[\sum_{n=1}^{N} Q_i(\tau - t_{no})]d\tau$$

Shorthand notations

$$Q_{iN}(\tau) = \sum_{n=1}^{N} Q_i(\tau - t_{no}), \text{ and}$$

$$\gamma_{iN}(t') = \int_0^{t'} h_i(t' - \tau)Q_{iN}(\tau)d\tau.$$

will be used.

For a given set of Q_i, the noise term may be evaluated. Since $Q_i(t)$ may be related to one another, it is necessary to split each generalized force into unrelated components $Q_i'(t)$ and $Q_{ij}''(t)$ as follows:

$$Q_i(t) = Q_i'(t) + \sum_{j=1}^{J} R_{ij}\, Q_j''(t), \text{ for each } i, \tag{5}$$

where coefficients R_{ij} control which and how the generalized forces are related.

We consider first the simple case defined by

$$R_{ij} = 0$$

for all combinations of i and j, and where each $Q_i'(t)$ is an independent and stationary random white noise. Then the expected squared value of the noise term as a function of t' in each mode may be evaluated by the technique outlined in Section 6.2, Reference 10, and is as follows:

$$\overline{\gamma_{iN}^2(t')} = N\, \overline{Q_i^2} \int_0^{t'} h_i^2(t' - \tau)d\tau,$$

where $\overline{Q_i^2}$ is the mean-squared value of the generalized force $Q_i(t)$. The expected value of $\gamma_N^2(t')$ is, therefore,

$$\overline{\gamma_N^2(t')} = \frac{1}{N} \sum_{i=1}^{I} \overline{Q_i^2} \int_0^{t'} h_i^2(t' - \tau)d\tau. \tag{6}$$

Cross products containing different indices do not appear because of the assumed statistical independences among the various generalized forces. Each integral on the right-hand side of Equation (6) can be evaluated:

$$\int_0^{t'} h_i^2(t' - \tau)d\tau = \int_0^{t'} e^{-2\zeta_i\omega_i(t' - \tau)} \sin^2[(\bar{\omega}_i(t' - \tau)]d\tau$$

$$= \frac{1}{4\zeta_i\omega_i} [1 - \zeta_i^2 - e^{-2\zeta_i\omega_i t'} (1 - \zeta_i^2 \cos 2\bar{\omega}_i t' + \zeta_i(1 - \zeta_i^2)^{\frac{1}{2}} \sin 2\bar{\omega}_i t')]$$

where $\bar{\omega}_i = (1 - \zeta_i^2)^{\frac{1}{2}}\omega_i$.

For small values of ζ_i, the above equation may be replaced by the approximation:

$$\int_0^{t'} h_i^2(t' - \tau)d\tau \approx \frac{1}{4\zeta_i\omega_i} (1 - e^{-2\zeta_i\omega_i t'}). \tag{7}$$

Using Expression (7) in Equation (6) we obtain

$$\overline{\gamma_N^2(t')} \approx \frac{1}{N} \sum_{i=1}^{I} \overline{Q_i^2}(1 - e^{-2\zeta_i\omega_i t'})/(4\zeta_i\omega_i).$$

The last equation may be cleared of the generalized forces explicitly by noting that the mean-squared response for each mode is

$$\overline{q_i^2} = \lim_{t\to\infty} \overline{q_i^2(t)} = \overline{Q_i^2}/4\zeta_i\omega_i.$$

Therefore,

$$\overline{\gamma_N^2(t')} \approx \frac{1}{N}\sum_{i=1}^{I} \overline{q_i^2}(1 - e^{-2\zeta_i\omega_i t'}), \tag{8}$$

and
$$\overline{\gamma_N^2} = \lim_{t\to\infty} \overline{\gamma_N^2(t')} \approx \frac{1}{N} \sum_{i=1}^{I} \overline{q_i^2}.$$

On account of the assumed statistical independency of the generalized forces, the modal response components are also independent of one another. The mean-squared value of the measured response is, consequently, the sum of the mean-squared modal responses:

$$\overline{U^2} = \sum_{i=1}^{I} \overline{q_i^2} \ .$$

So that

$$\overline{\gamma_N^2} = \frac{1}{N} \overline{U^2}. \tag{9}$$

If the generalized forces are not completely independent of one another as we have assumed in the first example, the general expression of Equation (5) should be used. Instead of Equation (6), a more complex expression will be obtained for $\overline{\gamma_N^2}(t')$. The added complexity, however, is only bookkeeping in nature and presents no fundamental difficulties. We will demonstrate, via a second example, that the same estimate on the noise term is still valid when the generalized forces are related to one another.

Let us restrict our attention to a two-mode system (i.e., I = 2), with generalized forces

$$Q_1 = Q_1'' + Q_2'',$$

and $$Q_2 = Q_2''.$$

In other words, we set

$$Q_1'(t) = Q_2'(t) = 0,$$

$$R_{11} = R_{12} = R_{22} = 1,$$

and $\quad R_{21} = 0,$

in Equation (5) for I = 2. The noise term in Equation (3) is in this case

$$\gamma_N(t') = \frac{1}{N}\int_0^{t'} h_1(t' - \tau)[\sum_{n=1}^{N} Q_i''(\tau - t_{no})]d\tau$$
$$+ \frac{1}{N}\int_0^{t'} [h_1(t' - \tau) + h_2(t' - \tau)][\sum_{n=1}^{N} Q_2''(\tau - t_{no})]d\tau.$$

Since $Q_1''(t)$ and $Q_2''(t)$ are statistically independent,

$$\overline{\gamma_N^2(t')} = \frac{1}{N}\overline{(Q_1'')^2}\int_0^{t'} h_i^2(t' - \tau)d\tau + \frac{1}{N}\overline{(Q_2'')^2}\int_0^{t'}[h_1(t' - \tau) + h_2(t' - \tau)]^2 d\tau$$
$$= \frac{1}{N}[\overline{(Q_1'')^2} + \overline{(Q_2'')^2}]\int_0^{t'} h_1^2(t' - \tau)d\tau$$
$$+ \frac{1}{N}\overline{(Q_2'')^2}\int_0^{t'} h_2^2(t' - \tau)d\tau \tag{10}$$
$$+ \frac{2}{N}\overline{(Q_2'')^2}\int_0^{t'} [h_1(t' - \tau)h_2(t' - \tau)d\tau.$$

where $\overline{(Q_1'')^2}$ and $\overline{(Q_2'')^2}$ are the mean-squared values of $Q_i''(t)$ and $Q_2''(t)$, respectively. We know from the first example that

$$[\overline{(Q_1'')^2} + \overline{(Q_2'')^2}]\int_0^{t'} h_1^2(t' - \tau)d\tau \approx \overline{q}_1^2(1 - e^{-2\zeta_1\omega_1 t'}).$$

and $$\overline{Q_2''2}\int_0^{t'} h_2^2(t' - \tau)d\tau \approx \overline{q}_2^2(1 - e^{-2\zeta_2\omega_2 t'}).$$

The last integral in Equation (10) is the only one which we have not seen until now. Upon carrying out the indicated integration, however, we find its magnitude to be of the order ζ, the greater of ζ_1 and ζ_2. For a lightly damped system, therefore, Equations (8) and (9) can still be used to estimate the mean-squared value of the noise term.

We now summarize results as follows: The ensemble average of N sample series of the total response function is the sum of a signal term, $S_N(t')$, and a noise term $\gamma_N(t')$:

$$U_N(t') = S_N(t') + \gamma_N(t'),$$

where the form of $S_N(t')$ is deterministic, and its magnitude is controlled by the average values of the modal responses and their derivatives at the the beginning of all samples ($t=t_{no}$, $n=1, 2, \ldots N$); and where $\gamma_N(t')$ has a mean-squared value which grows from a near-zero value at $t'=0$, to one Nth of the mean-squared value of the measured response at large t'. A pictorial representation of $U_N(t')$ is shown in Figure 1 for a one-mode system.

If the starting times t_{no} of all sample series are selected on the basis of the total response, its derivative, or a combination of both, in such a manner that $S_N(t')$ does not continually diminish with increasing N, i.e., if

$$\lim_{N\to\infty} U_N(t') = S_\infty(t') \neq 0 \text{ for all } t'.$$

Since $S_\infty(t')$ contains parameters required to specify the system, it will be called the Ideal Signature of the system.

A Randomdec Signature (of N averages), on the other hand, is the truncated function:

$$U_N(t') = S_N(t') + \gamma_N(t'), \; t' \leq T, \text{ a finite "signature length".}$$

where $S_N(t')$ is defined by Equation (4).

The selection of starting times t_{no} is called "triggering". A proper triggering method is required to guarantee the convergence of the ensemble average to a usable* Randomdec Signature in a reasonable** time. Two examples of triggering methods are shown in Figure 2. With the Level Triggering method, t_{no} = every time response signal crosses a preselected level, regardless of the sign of the slope. With the Zero-Crossing Triggering method, t_{no} = every time response crosses zero with a plus slope. A third method which is not shown in Figure 2 is the Every-Point Triggering method where t_{no} = every sampling time. Each triggering method will lead to a Randomdec Signature with a different apparent form. For a linear system, they all contain the same information so far as system dynamic properties are concerned. This is because

$$h_i(t') = e^{-\zeta_i \omega_i t'} \sin\bar{\omega}_i t', \text{ and}$$

$$\dot{h}_i(t') = - e^{-\zeta_i \omega_i t'} (\zeta_i \omega_i \sin\bar{\omega}_i t' - \bar{\omega}_i \cos\bar{\omega}_i t')$$

so that $U_N(t')$ may be written in the following general form for all Randomdec Signatures regardless of the trigger methods used, see Equations 3, 4 and 5:

$$U_N(t') = \sum_{i=1}^{I} e^{-\zeta_i \omega_i t'} (B_i \sin\bar{\omega}_i t' + D_i \cos\bar{\omega}_i t') + \Upsilon_N(t'). \qquad (12)$$

Different triggering methods merely give us different combinations of B_i and D_i.

* The usefulness of a Randomdec Signature depends on the method employed to analyze it, and is one of the main objectives of our study. More discussions will be found in Section 6.

**Our main concern is the minimum amount of data required to obtain a useful Randomdec Signature. Time is most appropriately measured in terms of the period of the lowest frequency content of interest.

Let us turn our attention to the forcing function on an aeroelastic structure at subcritical velocities. We will continue to assume that the structural properties are invariant with respect to time, and that the forcing function is stationary. In other words, the following analysis is applicable when the velocity is constant and below the flutter velocity. Let us consider the state of affairs in a typical mode. The generalized force may be split up into three basic parts: the first part, $Q_i(t)$, is independent of modal responses and is stationary, wideband and random; a second part which is dependent on the modal response, and can be further divided into aerodynamic inertia, damping and stiffness components; and a third part which is dependent on the response history. For the ith mode, let

m'_i = generalized mass of the structure in still air,

m''_i = generalized mass due to aerodynamics,

$m_i = m'_i + m''_i$,

c'_i = modal damping coefficient of the structure in still air,

c''_i = modal damping coefficient due to aerodynamics,

$c_i = c'_i + c''_i$,

k'_i = generalized stiffness of the structure in still air,

k''_i = generalized stiffness due to aerodynamics,

$k_i = k'_i + k''_i$,

$q_i(o)$ = modal initial displacement, and

$\dot{q}_i(o)$ = modal initial velocity.

The equation of motion for each mode of the aeroelastic systems can be expressed most readily with the help of the Laplace Transform and can be written as

$$[m_i s^2 + c_i s + k_i + A_i(s)]q_i(s)$$
$$= m_i Q_i(s) + m_i q_i(o)s + m_i \dot{q}_i(o) + c_i q_i(o) \quad (13)$$

where s is the transform variable, and

$$q_i(s) \equiv L\{q_i(t)\},^*$$

$$Q_i(s) \equiv L\{Q_i(t)\}.$$

The term $A_i(s)$ is a general representation of the history-dependent components of the unsteady aerodynamic forces. The simplest version of $A_i(s)$ is a first-order lag:

$$A_i(s) = \frac{a_i}{T_i s + 1}.$$

where a_i is the quasi-steady aerodynamic force and T_i is the associated time constant.

The following solution of the modal response function is obtained from Equation (13):

$$q_i(s) = \frac{[m_i Q_i(s) + m_i q_i(o)s + m_i \dot{q}_i(o) + c_i q_i(o)](T_i s + 1)}{(m_i s^2 + c_i s + k_i)(T_i s + 1) + a_i} \quad (14)$$

The denominator can be expanded, factored and put into the form

$$m_i(s^2 + 2\zeta_i' \omega_i' s + \omega_i'^2)(T_i s + b_i),$$

*The same symbol will be used for the variable and its Laplace Transform. The independent variables will always be used to distinguish them. However, when a symbol is used to represent an initial condition, the "o" in parentheses indicates that the quantity in question is a constant (the value of the associated variable at t=0 or t'=0) with b_i, ω_i' and ζ_i' related to the original set of coefficients by the following expressions:

$$b_i = 1 - 2(\zeta_i'\omega_i' - \zeta_i\omega_i)T_i, \tag{15}$$

$$\omega_i'^2 = \omega_i^2(1 + a_i/k_i)/b_i, \text{ and} \tag{16}$$

$$2\zeta_i'\omega_i' = [2\zeta_i\omega_i - (\omega_i'^2 - \omega_i^2)T_i]/b_i. \tag{17}$$

The modal response can now be written as

$$q_i(s) = \frac{Q_i(s) + q_i(o)s + \dot{q}_i(o) + 2\zeta_i\omega_i\, q_i(o)}{s^2 + 2\zeta_i'\omega_i's + \omega_i'^2} \times \frac{T_is + 1}{T_is + b_i}. \tag{18}$$

Let

$$L^{-1}\{\frac{1}{s^2 + 2\zeta_i'\omega_i's + \omega_i'^2}\} = h_i''(t), \tag{19}$$

$$L^{-1}\{\frac{T_is + 1}{T_is + b_i}\} = f_i(t), \text{ and} \tag{20}$$

$$h_i''(t) * f_i(t) = h_i'(t). \tag{21}$$

The time-domain solution for the stationary subcritical response function can then be written in terms of $h_i'(t)$, the initial conditions, and the motion-independent forcing function as follows:

$$q_i(t) = q_i(o)\dot{h}_i'(t) + [\dot{q}_i(o) + 2\zeta_i\omega_iq_i(o)]h_i'(t) + \int_0^t Q_i(\tau)h_i'(t - \tau)d\tau. \tag{22}$$

As (a_i/k_i) and $(T_i\omega_i')$ approach zero,

$$b_i \to 1,$$

$$\omega_i' \to \omega_i,$$

and $2\zeta_i'\omega_i' \to 2\zeta_i\omega_i$,

so that $h_i''(t) \to h_i(t)$ and $f_i(t)$ becomes a Dirac delta. The solution consequently approaches that for a purely mechanical system. As (a_i/k_i) and $(T_i\omega_i)$ deviate from zero, the impulse response function $h_i'(t)$ must be evaluated by the convolution between $h_i''(t)$ and $f_i(t)$. The corresponding Randomdec Signature is

$$U_N(t') = \sum_{i=1}^{I} [(p_{io} + 2\zeta_i\omega_i q_{io}] \, h_i'(t') + q_{io}\dot{h}_i'(t') \tag{23}$$

$$+ \gamma_N(t').$$

We see that for an aeroelastic system at a subcritical velocity, the Random Decrement procedure will produce a Signature which differs from that of a purely mechanical system, only in the difference between $h_i'(t)$ and $h_i(t)$. In representing the solution by Equations (15)-(22), the above mathematical differences between solutions of the two types of systems are more easily related to the physical differences.

In any case, we see that the addition of the lag term does not increase the number of independent initial conditions beyond two per mode.

Finally, we remark that by (1) setting $I=2$, $a_1=a_2$, $T_1=T_2$, and $Q_1(t) \propto Q_2(t)$, (2) linearly combining $q_1(t)$ and $q_2(t)$, and (3) disregarding all initial conditions, we will be able to simulate with Equation (14) the special case used by Houbolt (Equation (49), Reference 11).

Section 4

SYSTEM IDENTIFICATION VIA SIGNATURE CURVE FITTING

The Randomdec Signature is a truncated approximation of the characteristic response function (transient response function due to initial conditions) of the system in question. It contains information on the system characteristics. From subcritical flutter testing point of view, it is important to be able to determine the natural frequencies and damping ratios from the Signature. The method selected to accomplish this objective is described below.

Our efforts are concentrated on a two-mode aeroelastic system which is characterized by the ten parameters ω_1', ω_2', ζ_1', ζ_2', m_1, m_2, a_1, a_2, T_1 and T_2. The generalized masses m_1 and m_2 cannot be determined from Randomdec Signatures because they appear explicitly only in forced response solutions. Fortunately their determination is not needed in subcritical testing as they are only multiplying constants for the responses and do not directly relate to stability. To completely specify a Randomdec Signature for a two-mode system, four additional parameters corresponding to the initial amplitudes and velocities in the Signature as governed by the triggering method are required. The data acquisition equipment and process is assumed to introduce yet another three parameters: the sampling period, δ and two filter time constants, T_b and T_c. The objective of the curve-fit procedure is to determine the best estimates for the four most important system parameters ω_1', ω_2', ζ_1', and ζ_2' in the presence of all others, from an imperfect Signature in which not all effects of the forcing function have been averaged out. We will try to curve-fit the Signature, $U_N(t')$, by

$$y(t') = \sum_{i=1}^{I} e^{-\zeta_i'\omega_i't'} (B_i \sin\bar{\omega}_i t' + D_i \cos\bar{\omega}_i t') \tag{23}$$

with $\bar{\omega}_i' = (1 - \omega_i'^2)^{\frac{1}{2}}\omega_i'$

In approaching the problem this way, we are assuming that

(a) In a real test situation, the response transducer and signal conditioning equipment and process are reasonably good, so that the cut-off frequencies ω_b and ω_c corresponding to T_b and T_c, respectively. are very high in comparison with ω_1' and ω_2'.

(b) The effects of a_i and T_i are primarily on the natural frequencies and damping ratios, and by using ω_i' and ζ_i' in $h_i'(t)$, most of the differences between the impulse response functions of the aeroelastic system and a purely mechanical system has already been accounted for. Putting it in another way, the convolution indicated by Equation (21) is assumed to produce only small differences between $h_i''(t)$ and $h_i'(t)$. (The consequent error can be estimated by the difference between $f_i(t)$ and the Dirac Delta.)

In view of Equation (18), our approach is valid because flutter is reached when ζ_1' or ζ_2'(not when ζ_1 or ζ_2) become non-positive.

We begin the curve-fit procedure by defining an error-measuring function E. The most commonly used function for this purpose is the mean-squared error:

$$\frac{1}{T}\int_0^T [y(t') - U_N(t')]^2 dt'. \tag{24}$$

In the case where $U_N(t')$ is defined only at $t'=k\delta$, $k=1,2, \ldots .K$, (i.e., if sampled data are used), E is written as

$$E = \frac{1}{K}\sum_{k=1}^{K} (y_k - U_{Nk})^2,$$

where $y_k = y(k\delta)$,

and $U_{Nk} = U_N(k\delta)$.

A more general approach is to introduce weighting and use

$$E = \frac{1}{K}\sum_{k=1}^{K} [z(y_k) - z(U_{Nk})]^2. \tag{24a}$$

Parameter values $\underline{B}_i$, $\underline{D}_i$, $\underline{\omega}_i'$ and $\underline{\zeta}_i'$ which minimize E will be considered as the optimum approximations of the true parameters.*
The method of minimization is based on the Gauss-Newton method (Reference 12) modified by Roman (Reference 13) and further modified during the course of this project. An initial estimate of parameter values is made. This estimate is upgraded by information based on the slopes of the error function E with respect to variation of each of the parameters. For purpose of the following discussion, it will be convenient to rename the parameters in accordance with the following:

$$\begin{aligned}
&\alpha_1 = B_1 \\
&\alpha_2 = \zeta_1'\omega_1' \\
&\alpha_3 = \overline{\omega}_1' \\
&\alpha_4 = D_1 \qquad\qquad (25)\\
&\alpha_5 = B_2 \\
&\alpha_6 = \zeta_2'\omega_2' \\
&\alpha_7 = \overline{\omega}_2' \\
\text{and}\quad &\alpha_8 = D_2.
\end{aligned}$$

*The parameters B_i and D_i are not system properties but have to be determined in the curve-fit process.

Let α_m^o, m=1,2,3,4,5,6,7 and 8 be an inital set of estimated values of α_m, and let

$$C_k^\ell = (dz/dy)_k^\ell$$

be the indicated derivatives evaluated at $t'=k\delta$ during the ℓth iteration. In addition, we calculate

$$\beta_{mk}^\ell = C_k^\ell (\partial y/\partial \alpha_m)_k^\ell,$$

$$\phi_{mgk}^\ell = \beta_{mk}^\ell \beta_{gk}^\ell,$$

$$\psi_{mg}^\ell = \sum_{k=1}^{K} \phi_{mgk}^\ell,$$

$$r_{gk}^\ell = \beta_{gk}^\ell [z(y_k^\ell) - z(U_{Nk}^\ell)],$$

$$R_g^\ell = \sum_{k=1}^{K} r_{gk}^\ell.$$

$$[d_m^\ell] = -[\psi_{mg}^\ell]^{-1} [R_g^\ell],$$

and $$\varepsilon_m^\ell = d_m^\ell / \alpha_m^\ell.$$

In the Gauss-Newton method, if

$$|\varepsilon_m^\ell| > \varepsilon = \text{a predetermined band for all } m,$$

improved, $(\ell + 1)^{th}$ parameters are obtained by

$$\alpha_m^{(\ell+1)} = \alpha_m^\ell + d_m^\ell.$$

In our method, an additional optimum step size factor ρ_ν, is used to improve the rate of convergence and the last expression is modified as follows:

$$\alpha_m^{(\ell + 1), \nu} = \alpha_m^\ell + d_m^\ell \rho_\nu. \tag{26}$$

Initially, $\nu = 1$, and $\rho_1 = 1$ is used.

If the resulting error function

$$E^{(\ell + 1)} > E^\ell,$$

the step size is halfed, i.e.,

$$\rho_2 = 0.5,$$

and the error function is checked again. The process continues (ν = 3, 4, ...) until

$$E^{(\ell + 1)} < E^\ell.$$

The resulting set of new estimates $\alpha_m^{(\ell + 1)}$ will then be used to repeat the entire process until the condition

$$|\alpha_m^{(\ell + 1)} - \alpha_m^\ell| < \varepsilon \tag{27}$$

is satisfied for all m.

The curve-fit procedure is outlined in Figure 3. In addition to the above logic, the computer program is implemented to conduct both a four-parameter (one-mode) and an eight-parameter (two-mode) fit each time. For the one-mode case, the initial estimate for (a) the natural frequency is obtained by counting the number of samples per each "period" of the Signature; (b) the amplitude is the magnitude of maximum peak; (c) the damping is set to zero. For the two mode case, the above set of initial estimates are used for one of the modes, and the frequency and amplitude results of the one-mode curve-fit are used as initial estimates for the other mode. Damping is again set to zero.

Section 5

SUBCRITICAL AEROELASTIC RESPONSE SIMULATION

The simulation of desired response signals of an aeroelastic system may be achieved in the time domain by direct numerical integration of the governing differential equations,or by convolving, again in the time domain, the forcing function with the impulse response function of the system. The simulation of desired response signals may also be achieved by complex multiplication of the forcing function* with the frequency response function of the system in the frequency domain. The last approach was chosen in this study because it leads to the implementation of a more general computer program which can be easily altered to simulate different types of systems.

The velocity response of an aeroelastic structure in a subcritical condition will be simulated. Velocity signal is chosen for simulation because in practice it offers an optimum balance between low- and high-frequency signal magnitudes under normal test environments. From Equation (14) the modal response velocity is

$$p_i(s) = sq_i(s) - q_i(o)$$

$$= \frac{[sQ_i(s) + s\dot{q}_i(o) - \omega_i^2 q_i(o)](T_i s + 1) - a_i q_i(o)/m_i}{(s^2 + 2\zeta_i\omega_i s + \omega_i^2)(T_i s + 1) + a_i/m_i} \quad (28)$$

The Fourier transform for the modal velocity is obtained when s is replaced by $\eta\omega(\eta = \sqrt{-1}, -\infty < \omega < \infty)$ in Equation (28):

---

*Including initial conditions which may be considered as special forcing functions.

$$p_i(\omega) = \frac{[\eta\omega Q_i(\omega)/\omega_i^2 + \eta\omega\dot{q}_i(o)/\omega_i^2 - q_i(o)](\eta\omega T_i + 1) - a_i q_i(o)/(m_i\omega_i^2)}{(1 - \omega^2/\omega_i^2 + 2\eta\zeta_i\omega/\omega_i)(\eta\omega T_i + 1) - a_i/(m_i\omega_i^2)} \qquad (25)$$

The measured total velocity signal has a Fourier transform

$$e(\omega) = \frac{1}{(\eta\omega T_b + 1)(\eta\omega T_c + 1)} \sum_{i=1}^{I} p_i(\omega). \qquad (26)$$

Of course, when $e(\omega)$ is inverse transformed, the resulting time series simulates the velocity response signal.

The computer program developed during this investigation (Appendices A & B) uses an existing fast Fourier transform (FFT) subroutine to carry out all required forward and inverse transformations. The following restrictions are imposed on the simulation by the FFT*:

(a) Only equally spaced, sampled response data, in segments of finite lengths can be simulated, since the FFT is a finite discrete Fourier transform procedure.

(b) The transform of a sampled time series is computed for a finite number of equally spaced frequencies only. Both bandwidth and frequency resolution are limited.

(c) The number of data points in the time domain is the same as the number of complex frequency components computed in the frequency domain.

Based on procedural requirements of the Random Decrement method, on desired frequency resolution and bandwidth, and on computer core size limitations, simulated response signals are generated in 2,048-point

---

*Implications of, and solutions to overcome, these restrictions are discussed subsequently.

segments. Displacement and velocity components for each mode at the end of a segment are computed and used as initial conditions for the following segment.

The simulation procedure is outlined in the simplified flow diagram* of Figure 4. Generalized forces are created in the sampled time domain first. For each mode, the force can contain a random component and two deterministic components. Random components for both modes are generated simultaneously from two independent sequences of Gaussianly distributed random numbers, $F_{j\kappa}$, in accordance with the following mixing formula:

$$Q_{i\kappa} = Q_i(\kappa\delta) = \sum_{j=1}^{2} R_{ij} F_{j\kappa}$$

where i=1,2 and κ=1,2, . . . 2,048 for each segment of simulation, δ is the sampling period, and $\kappa\delta$ are the times at which the simulated sampling takes place. Values of the mixing coefficients, R_{ij}, are selected by the desired amount of "correlation" between the two generalized forces. Sequences $F_{1\kappa}$ and $F_{2\kappa}$ have zero means and unity standard deviations.

Deterministic generalized force components include a periodic part, $P_{i\kappa}$ and a "box-car" part $BC_{i\kappa}$. The amplitudes, frequencies and phase displacements of $P_{i\kappa}$, as well as the rise- and fall-times of $BC_{i\kappa}$ are user selected via input data. The total generalized force for each mode is

$$\overline{Q}_{i\kappa} = Q_{i\kappa} + P_{i\kappa} + BC_{i\kappa},\ i=1,2.$$

For each mode the above array is transformed into the discrete frequency domain by the FFT. The transform operates on an array of 4,096

*Detailed flow diagrams of all computer programs are found in Appendix A.

real points. The first half of the array contains $\overline{Q}_{iK}$; the second array are filled with zeroes*. After the transformation, only the first 2,048 complex points in the frequency domain are used. They are combined with terms containing initial conditions in accordance with Equation (29), and complex-multiplied with modal and filter frequency response functions. The real part of the product array of 2,048 complex points is extended symmetrically about the frequency origin, the imaginary part antisymmetrically. The resulting array of 4,096 complex points is then inverse transformed into the time domain. There will be 4,096 real points, of which only the first 2,048 are correct data simulating the sampled, filtered modal velocity response.

The process is repeated for the other mode. Modal responses are then summed point by point to obtain the desired total response signal. All three series are kept for subsequent Random Decrement processing.

The following are the two basic relationships among sampling period, δ (seconds), frequency resolution, Δf (Hertz), and bandwidth, f_o (Hertz):

$$\delta(\Delta f) = (1/4096)$$

$$f_o = 4096(\Delta f)$$

In addition, at a frequency 2,048(Δf), the number of samples per cycle is exactly two.

On account of the truncation of the frequency response function, the frequency resolutions should be set in such a manner that natural frequencies of the simulated system is less than $f_o/2$. Similarly, modal damping factors causing significant truncation errors in the time

*A standard preliminary procedure for subsequent convolution via frequency domain multiplication, see Reference 14.

domain (on the impulse response functions) should be avoided.

In the computer program, Random Decrement trigger points, t_{no}, are established by inspecting the total filtered response signals. The actual ensemble averaging processes are performed on individual modal components as well as the total response signal. This represents a luxury not enjoyed in real test conditions where modal components are not individually available. The added step is used in the simulation process to provide study data on how ensemble samples of individual responses converge.

Section 6

PARAMETRIC STUDIES

The digital computer programs were used to conduct a number of numerical experiments to establish the feasibility and performance characteristics of the Random Decrement, the Signature curve-fit and the combined processes.

6.1 FEASIBILITY OF RANDOM DECREMENT

The feasibility of applying the Random Decrement method to obtain an approximate characteristic response function was demonstrated for a two-mode system with and without aerodynamic lag forces. The two test cases were designed to accomplish the goal without relying on the Signature curve-fit procedure.

In the first test case, the aerodynamic lag forces were set to zero; the natural frequencies of the two modes are 5.908 Hz and 6.519 Hz (i.e., a difference of 9.836%); and the modal damping factors are both 0.020 (of critical). Referring to Equations (29) and (30), this test case is specified by the following set of parameters:

$\Delta f = 0.02546$ Hz

$\dot{q}_1(o) = \dot{q}_2(o) = q_2(o) = 0$

$q_1(o) = 1.0$ in.

$\omega_1 = 37.12$ rad/sec (5.098 Hz)

$\omega_2 = 40.96$ rad/sec (6.519 Hz)

$\zeta_1 = 0.020$

$\zeta_2 = 0.020$

$m_1 = 1.00 \quad lb\text{-}sec^2/in$

$m_2 = 1.00 \quad lb\text{-}sec^2/in$

$R_{11} = 1.00 \quad lb/(lb\text{-}sec^2/in)$

$R_{12} = R_{21} = 0$

$R_{22} = 1.00 \quad lb/(lb\text{-}sec^2/in$

$a_1 = a_2 = 0$

$T_1 = T_2 = T_b = T_c = 0$

Zero-crossing and every-point trigger methods were used. Results are summarized in the series of "log peak plots"* (LPP's) in Figures 5 (zero-crossing triggering) and 6 (every-point triggering). The first series of LPP's (Figures 5-a through 5-f) show the convergence of the Signature of the 5.908-Hz Model, the second series (Figures 5-g through 5-l) the 6.519-Hz mode, and the last series (Figures 5-m through 5-q) the total system. The number of ensembles used are indicated on the figures. These results readily demonstrate that triggering on the total response signal does indeed lead to a system Signature which corresponds to the sum of two individual Modal Signatures. They also show the left-to-right convergence trend characterizing the Random Decrement process, as hinted in earlier discussions. The ideal system Signature is shown in Figure 5-r.

Figures 6-a and 6-b are interim Modal Signatures obtained by the every-point trigger method. The amount of response data used to obtain these interim Signatures are the same as those used to obtain Signatures shown in Figures 5-d and 5-j, respectively. A comparison of

*The computer program finds both the positive and negative peaks in a Signature; it computes log(Positive Peak Values), and plots the results as Peak Nos. 0, 2, 4, . . . ; and it computes log(- Negative Peak Values), and plots the results on the same graph as Peak Nos. 1, 3, 5, . . Damping for a single mode can be determined easily from this plot of the Signature peaks by the slope of the curve. For reference, the straight line connection points (0,0) and (77,-42dB) represents a modal damping of 0.02. Figures 5 and 6 also illustrate the print-plot capability developed for the Univac 1108.

these results indicates that the every-point trigger method requires just as much data to achieve a given Signature accuracy. This method was, consequently, deleted from the computer program.

The following parameter set defines the second test case in which modal aerodynamic lag forces are present:

$\Delta f = 0.0200$ Hz

$q_1(o) = q_2(o) = \dot{q}_1(o) = \dot{q}_2(o) = 0$

$\omega_1 = 67.25$ rad/sec

$\omega_2 = 108.23$ rad/sec

$\zeta_1 = 0.03175$

$\zeta_2 = 0.00588$

$m_1 = 0.0260$ lb-sec^2/in

$m_2 = 0.0520$ lb-sec^2/in

$R_{11} = 1.920$ lb/(lb-sec^2/in)

$R_{12} = 0.385$ lb/(lb-sec^2/in

$R_{21} = 0.960$ lb/(lb-sec^2/in)

$R_{22} = 0.000$ lb/(lb-sec^2/in)

$a_1 = 24.69$ lb/in

$a_2 = -93.33$ lb/in

$T_1 = 3.721 \times 10^{-3}$ sec

$T_2 = 3.721 \times 10^{-3}$ sec

$T_b = T_c = 0.000.$

The case was designed to yield the following aeroelastic system frequencies and damping

$\omega_1' = 73.54$ rad/sec

$\omega_2' = 101.25$ rad/sec

$\zeta_1' = 0.00659$

$\zeta_2' = 0.0338.$

Simulated responses are shown for this test case in Figure 7. The zero-crossing trigger method was used to obtain the set of Signatures in Figure 8. Convergence of the Signatures indicated again the feasibility of the Random Decrement method.*

6.2 FEASIBILITY OF CURVE-FIT

The feasibility of the curve-fit procedure as a method to identify system dynamic characteristics was next established.

As pointed out earlier in Section 4, the selected study approach of this project is to apply a parameter identification technique which assumes that the Signature is the sum of a free response and a small-magnitude forced response of a two-degree-of-freedom system without aerodynamic forces. Therefore, checkout of the procedure can be logically divided into two steps. The first step is to verify that the procedure is accurate if the system to be identified is indeed a purely mechanical one. The second step is to determine whether force-fitting the Signature of a system with aerodynamic forces by free vibration solutions of a system without such terms would lead to useful answers for flutter prediction purposes.

Step 1 was accomplished by adding wideband random noises of various intensities to an ideal Signature to test the capability of the curve-fit procedure. The two components of the ideal Signature have equal initial amplitudes, and

$$\omega_2 = 2\,\omega_1$$

$$\zeta_1 = 0.018$$

$$\zeta_2 = 0.035$$

*Detailed analyses of results will be found in Section 6.3.

The noise component has a Gaussian distribution, zero mean, and RMS magnitudes of up to 18% of the initial amplitude of the Signature. One hundred sampled points were used, representing approximately 8 cycles of the Signature at the average frequency of the two modes. Near-perfect (within 1%) identification of all properties was accomplished. The number of iterations required to determine the least-squares curve-fit ranged from 7 to 20. The computer program was consequently modified to limit the number of iterations to prevent accidental high-cost computer runs. In its final form, the program will stop searching for the best fit if after 20 iterations, the convergence criterion of Equation (27) is still not satisfied.

It was also determined during this stage of checkout that as few as four sample points per cycle of the higher modal response frequency are sufficient for the curve-fit program to identify the correct system properties. This feature, together with its ability to determine individual modal damping values when the natural frequencies are very close*(and, consequently, with the total Signature displaying the characteristic "beating" of Figures 5-m through 5-r) are among the principal advantages of the procedure. In all subsequent investigations a sampling rate corresponding to approximately six points per cycle of the average frequency was used.

Figure 9 is a typical Simulated Randomdec Signature presented to the curve-fit routine. Figure 10 is the curve which fits the Signature in Figure 9 in the least-squares sense. Figure 11 shows the convergence paths of a typical parameter and the mean-squared error function.

*This ability was verified via test cases to be described later.

6.3 FEASIBILITY OF THE COMBINED PROCEDURE

Subsequently, the feasibility of combining Random Decrement and curve-fit procedures was demonstrated by curve fitting a Randomdec Signature of suitable length* obtained for the test case on page 39. Zero-velocity properties, i.e., those of the mechanical system, are

$$\omega_1 = 67.25 \quad \text{rad/sec}$$

$$\omega_2 = 108.23 \quad \text{rad/sec}$$

$$\zeta_1 = 0.03175$$

$$\zeta_2 = 0.00588$$

$$m_1 = 0.0260 \quad \text{lb-sec}^2\text{/in}$$

$$m_2 = 0.0520 \quad \text{lb-sec}^2\text{/in}$$

A lag time constant of 3.721×10^{-3} sec, or exactly one fourth of the natural period of the first mode, was assigned. The quasi-static aerodynamic forces are

$$a_1 = 24.69 \quad \text{lbs/in}$$

$$a_2 = -93.33 \quad \text{lbs/in}$$

corresponding to

$$a_1/k_1 = a_1/m_1\omega_1^2 = 0.2100$$

$$a_2/k_2 = a_2/m_2\omega_2^2 = 0.1422$$

The selected characteristics for the aerodynamic forces will simulate conditions for a velocity close to flutter**. The resulting values of b_1 and b_2 are 1.012 and 0.979, respectively. The factors $(T_i s + 1)/(T_i s + b_i)$ in Equation (18) will, consequently, not change the form of the impulse response function of the system. Based on Equations (15), (16) and (17), the calculated natural frequency for the low-frequency mode will increase

---

*The effect of Signature length on curve-fit accuracy will be investigated in Section 6.4

**So far as damping values are concerned. The natural frequencies were deliberately kept separated by a significant amount to minimize the cost of the computer run.

from a zero-velocity value of 63.25 rad/sec to 73.54 rad/sec, while that for the other mode will decrease from 108.23 rad/sec to 101.25 rad/sec. The corresponding changes in damping are:

$$(\zeta_1 = 0.03175) \rightarrow (\zeta_1' = 0.00659)$$

$$(\zeta_2 = 0.00588) \rightarrow (\zeta_2' = 0.0338)$$

The zero-crossing trigger method was used to obtain the Randomdec Signatures. A Signature length of 50 samples was used. The Signatures are shown in Figure 8. The following is a summary of numerical results from the test run:

No. of Ensembles Averaged	Numerical Results of Combined Process							
	Freq & Error, rad/sec				Damping and Error			
	ω_1'	Error	ω_2'	Error	ζ_1'	Error	ζ_2'	Error
234	73.48	+0.06	103.91	+2.66	.00806	+.00147	.0541	+.0203
461	73.54	0.00	102.17	+0.92	.00708	+.00049	.0373	+.0035
914	73.61	+0.07	101.42	+0.17	.00719	+.00060	.0386	+.0048
1806	73.53	-0.01	101.62	+0.37	.00590	-.00069	.0426	+.0088
3612	73.47	-0.07	101.24	-0.01	.00524	-.00135	.0351	+.0013
Exact	73.54		101.25		.00659		.0338	

The exceptionally good agreement between the exact and numerically obtained results indicate that the influence on the aerodynamic forces is, indeed, primarily on the frequencies and damping values and that the impulse response function of the aeroelastic system can be accurately approximated by the use of those for two second-order modes. The results also indicate that, when applied properly, the combined process is truly a useful tool for flutter prediction with subcritical test data.

6.4 SIGNATURE LENGTH AND AVERAGING TIME

Effects of Signature length on accuracy are demonstrated via results* of two series of tests described below. Problems to be expected in selecting

*Obtained with a CDC6600 version of the developed computer programs.

a suitable Signature length for curve-fitting for a system whose dynamic properties are totally unknown will be discussed in Section 6.5.

In the first test series, parameters are set up to specify a system which may be described as having separated natural frequencies (by $\pm$ 15% of the average frequency) and moderate damping (0.04 of critical for each mode). Case parameters are

$\Delta f = 0.020$ Hz

$q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0$

$\omega_1 = 73.91$ rad/sec

$\omega_2 = 99.55$ rad/sec

$\zeta_1 = 0.040$

$\zeta_2 = 0.040$

$m_1 = 0.0260$ lb-sec^2/in

$m_2 = 0.0520$ lb-sec^2/in

$R_{11} = 1.920$ lb/(lb-sec^2/in)

$R_{12} = 0.385$ lb/(lb-sec^2/in)

$R_{21} = 0.962$ lb/(lb-sec^2/in)

$R_{22} = 0.000$

$a_1 = a_2 = T_1 = T_2 = T_b = T_c = 0$

Signature lengths of 12, 25, 37, 50, 75 and 100 samples were curve-fitted. For reference, the number of samples per cycle at the higher natural frequency is approximately 5, and the period of beating between two sine waves with frequencies equal to ω_1 and ω_2 would be approximately 40 sample periods. Numerical results indicate that the natural frequencies are relatively easy to determine except when the Signature length was too short (the 12- or 25-point case where the curve-fit program did not have sufficient data to process), or when it was too long (the 100-point case, due to the excessive weighting of the tail section of the Signature which contains mostly random forced response signals). In all other cases, natural frequencies were accurately determined (by within 2.5% of the exact value) after the number of ensembles used for averaging reaches 1,000.

Damping determination, on the other hand, requires considerably more data--approximately 2,000 ensembles were required to determine damping to within 0.007 (out of 0.040), for the 37-, 50-, and 75-point cases. The corresponding average time is approximately 145 seconds. Damping accuracies are summarized in Figure 11. Based on these test cases, it may be said that Signatures shorter than a beat period are usable for the two mode case. On the other hand, a Signature length greater than 200% of the beat period requires too much averaging time to be considered practical. An optimum Signature length appears to be the 50-point case which is 125% of the beat period.

In the second test series, natural frequencies are spaced much closer (within $\pm$ 1.2% of the average frequency) and the damping values are 0.04 for one mode and 0.005 for the other. The case parameters are:

$\omega_1 = 84.72$ rad/sec

$\omega_2 = 86.73$ rad/sec

$\zeta_1 = 0.040$

$\zeta_2 = 0.005$

all other parameters are the same as the previous test case.
The beat period of two sine waves at frequencies equal to ω_1 and ω_2 is 512 sample periods. Signature lengths of 25, 50, 75, 100 and 512 were curve-fitted. Both the 25- and the 50-point Signatures failed to yield useful results after 16,836 averages. For the 75-point Signature, the natural frequency prediction did not yield useful results* until

*When the natural frequencies are close to each other, the accuracy criteria should be based on the measurement of the difference of the natural frequencies, rather the the natural frequencies themselves.

after 8,418 averages. For the 100-point case, 4,202 averages are required, for the 200- and 512-point case, 2,103 averages. The optimum Signature length for both natural frequency and damping predictions for this sample test series was found to be the 200-point Signature after 2,000 averages, or about 150 seconds at the 85 rad/sec average frequency. Damping measurement results are plotted against the number of averages for Signature lengths of 100, 200 and 512 in Figure 13.

6.5 APPLICATION NOTES

The practical usefulness of any on-line flutter prediction process greatly depends on the amount of data it has to use at near critical velocities. The ideal situation is when the process requires so little data that the need to hold constant velocity at various subcritical stages is eliminated. With respect to the overall approach adapted in this investigation, this consideration leads to the requirement of developing a method to select the proper Signature length so that the Random Decrement averaging time can be minimized.

The following factors must be considered:

(a) Based on discussions in Section 3, the rms value of the residual forced response term (the noise term $\gamma_N(t')$) starts from zero for $t'=0$, and increases with t' in accordance with Equation (8). The expected signal-to-noise ratio of the Randomdec Signature for a given number of averages is, therefore, maximum for small values of t'. Consequently, using a shorter length of the Signature would require less averaging and would yield answers with given expected accuracies in a shorter time.

(b) For a system known, a priori, to have only one degree of freedom, the Signature length and averaging time required to determine damping are derived in Reference (9).

(c) For systems with two degrees of freedom, the beating of the two Modal Signatures makes it necessary to select a Signature length which, in accordance with results described in Section 6.4, is somewhere between 50% to 125% of the beat period. Once the proper Signature length has been selected, the curve-fit procedure can be used with confidence.

This suggests the following approach in applying the techniques put together during this investigation. It is assumed that (a) on-line Random Decrement and curve-fitting programs (or hardware) have been implemented, (b) real-time display of the developing Randomdec Signature is available, and (c) it is possible to select the length and sample density of the Signature to be presented to the curve-fit program. Starting at a constant low velocity where the danger of explosive flutter is not present, obtain an accurate reference Randomdec Signature by using a large number of averages. A high sampling rate (about 16 points per cycle of the response signal) should be used so that the beat period can be determined from the visual display. Needless to say, the Signature should be long enough to cover more than one beat period. Select from this long and dense Signature the proper length (about one beat period) and sample density (about 4-6 points per cycle) and present it to the curve-fit program. With the properly selected Signature, we are assured of fairly accurate frequency and damping results. The minimum number of ensembles required to produce frequency and damping data to a given accuracy with respect to the reference Signature is then determined experimentally.

The velocity sweep can then begin. The real-time display of the current Randomdec Signature and results of the previous curve-fit analyses will be used to select the Signature length used for curve-fit, which will be actuated as soon as a sufficient number of ensemble averages have been reached.

It is, of course, possible to automate the above procedure and use preliminary curve-fit results to compute the proper Signature length for a second, more refined Signature analysis.

Section 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

We have demonstrated analytically in Section 3 that applying the Random Decrement data reduction procedure to the random responses of a multi-mode system will produce a Randomdec Signature which is an approximation of the characteristic response function of the system. The error contained in the Randomdec Signature is in the nature of a residual forced response whose mean-squared value decreases as one over the number of ensemble averages.

Using computer programs developed in this project, sampled random responses of a two mode system were simulated. The feasibility of the Random Decrement procedure was established via a number of numerical experiments on different simulated two-degrees-of-freedom systems, including both a purely mechanical and an aeroelastic system.

A parameter identification procedure using least-squares curve-fitting of the Randomdec Signature was adapted. The method was found to be capable of determining frequency and damping values accurately from Randomdec Signatures of carefully selected lengths.

For optimum results, a Signature length between 50% to 125% of the beat period created by the two frequencies of the Modal Signatures should be selected. The number of ensembles required to produce accurate damping results by the combined process is found to be approximately 2,000.

The study was limited to one- and two-mode systems.

7.2 RECOMMENDATIONS

Simulation studies in this project were limited in scope on account of computer costs. Since the programs for response simulation, Random Decrement and curve-fit analysis can all be implemented on less powerful computers, exhaustive statistical studies are recommended using such computers for reasons of economy.

While flutter usually occurs on account of interaction between two modes of an aeroelastic system, the presence of other non-flutter modes at nearby frequencies cannot be denied in practical situations. The Randomdec Signature will contain characteristic responses in all modes. The Signature analysis (curve-fit) program should, therefore, be extended for such situations. On-line flutter prediction programs or hardware can then be implemented by suitable modification of the application approach suggested in Section 6.5.

REFERENCES

1. Rosenbaum, Robert, "Survey of Aircraft Subcritical Flight Flutter Testing Methods," NASA CR-132 479, August 1974.

2. Baldock, J.C.A. and C.W. Skingle, "Flutter Technology in the United Kingdom - A Survey," AIAA Paper No. 73-330, Presented at the AIAA Dynamics Specialists Conference, Williamsburg, Virginia, March 19 and 20, 1973.

3. Dat, Rolland, "The Theoretical and Experimental Methods Used in France for Flutter Prediction," AIAA Paper No. 73-329, Presented at the AIAA Dynamics Specialists Conference, Williamsburg, Virginia, March 19 and 20, 1973.

4. Coupry, Gabriel, "Random Techniques for Flutter Testing in Wind-Tunnel and in Flight," Paper Presented at the Fifteenth Israël Annual Conference on Aviation and Astronautics, Tel-Aviv, Israël, March 15 and 16, 1973.

5. Cole, Henry A., Jr., "On-The-Line Analysis of Random Vibrations," AIAA/ASME 9th Structures, Structural Dynamics and Materials Conference, Palm Springs, California, April 1-3, 1968, Paper No. 68-288.

6. Cole, Henry A., Jr., "Failure Detection of a Space Shuttle Wing Flutter Model by Random Decrement," NASA TM X-62, 041, May 1971.

7. Baird, Eugene F., and W.B. Clark, "Recent Developments in Flight Flutter Testing in the United States," Presented at the 34th Meeting of the AGARD Structures and Materials Panel, Lyngby, Denmark, 9-14 April 1972.

8. Cole, Henry A., Jr., "Method and Apparatus for Measuring the Damping Characteristics of a Structure." United States Patent No. 3,620,069, Nov. 16, 1971.

9. Cole, H.A., Jr., "On-Line Failure Detection and Damping Measurement of Aerospace Structures by Random Decrement Signatures," NASA CR-2205, March 1973.

10. Laning, J.H. and R.H. Battin, "Random Processes in Automatic Control," McGraw-Hill Book Company, Inc., New York, 1956.

11. Houbolt, J.C., "Subcritical Flutter Testing and System Identification," NASA CR-132480, August 1974.

12. Box, M.J., D. Davies, and W.H. Swaan, "Nonlinear Optimization Techniques," I.C.I. Monograph No. 5, Olives and Boyd, Edinburg, 1969.

13. Roman, W.S. "Parameter Identification in Nuclear Reactor Systems," Argonne National Laboratories Report No. ANL-7967, September 1972.

14. Hewlett-Packard Application Note 140-0, "Fourier Analyzer Training Manual, Section IV."

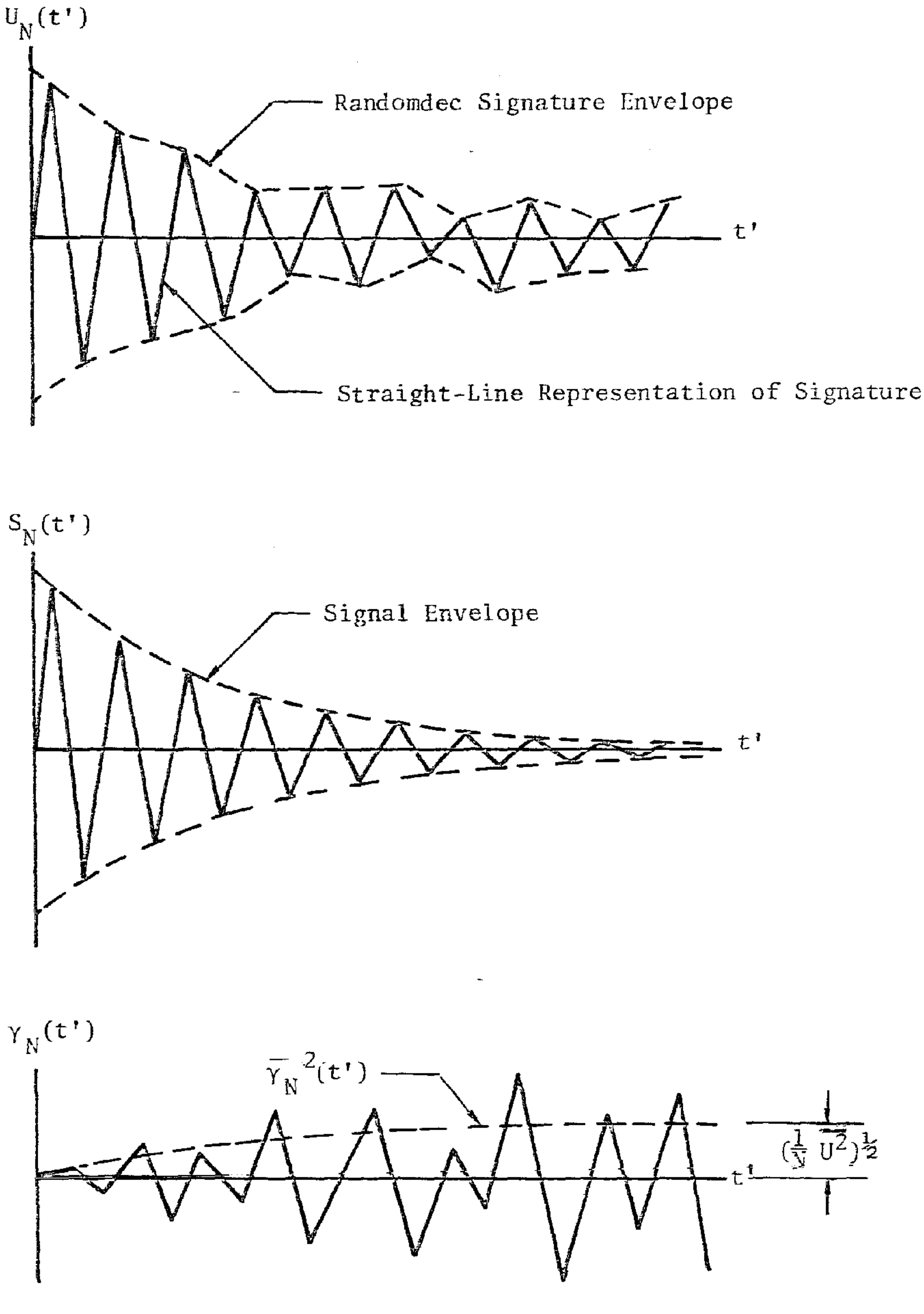


Figure 1 Composition of Randomdec Signature For a Single-Degree-of-Freedom System

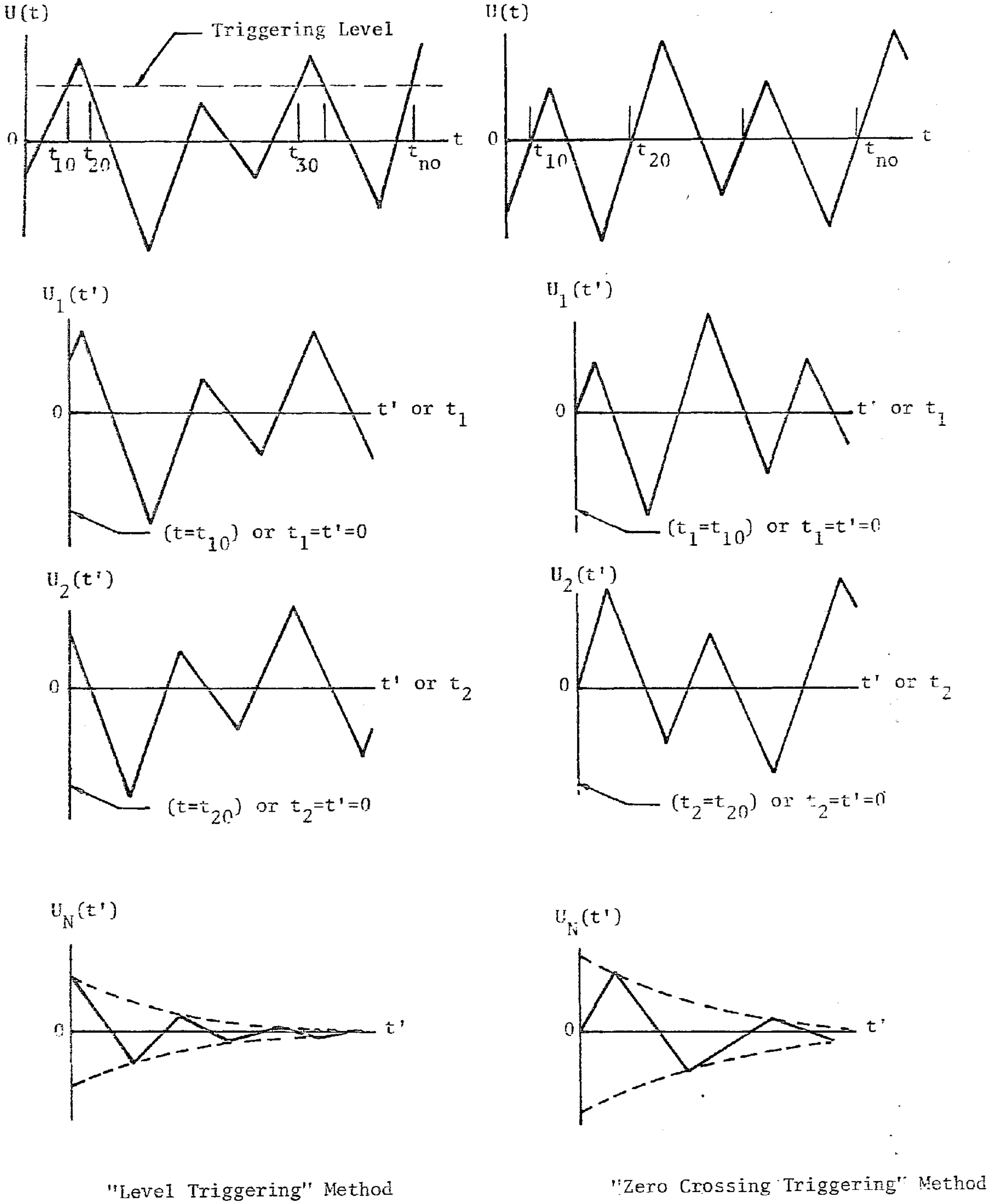


Figure 2 Examples of Random Decrement Triggering Methods

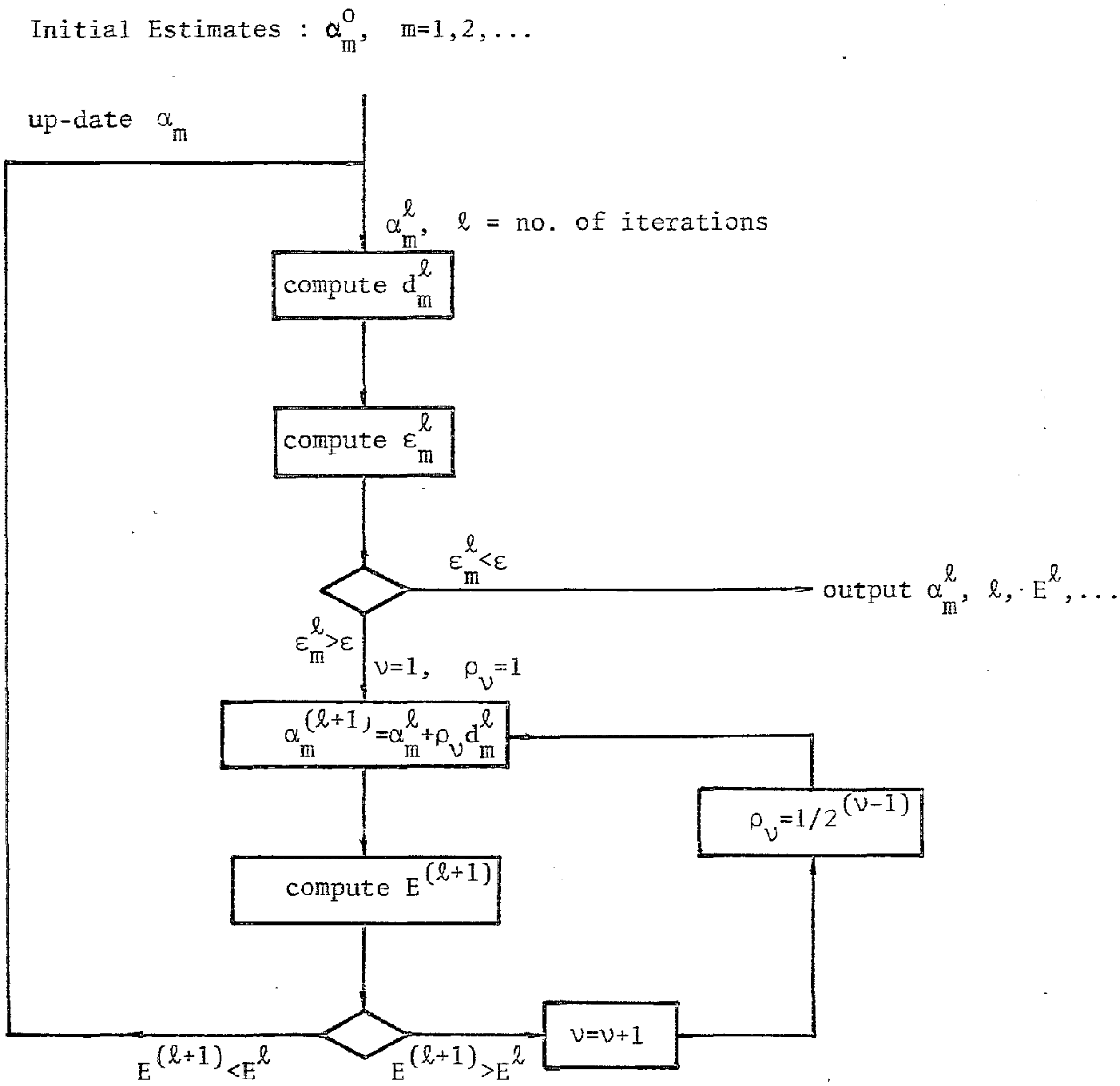


Figure 3 Simplified Flow Chart Showing Curve-Fitting Procedures

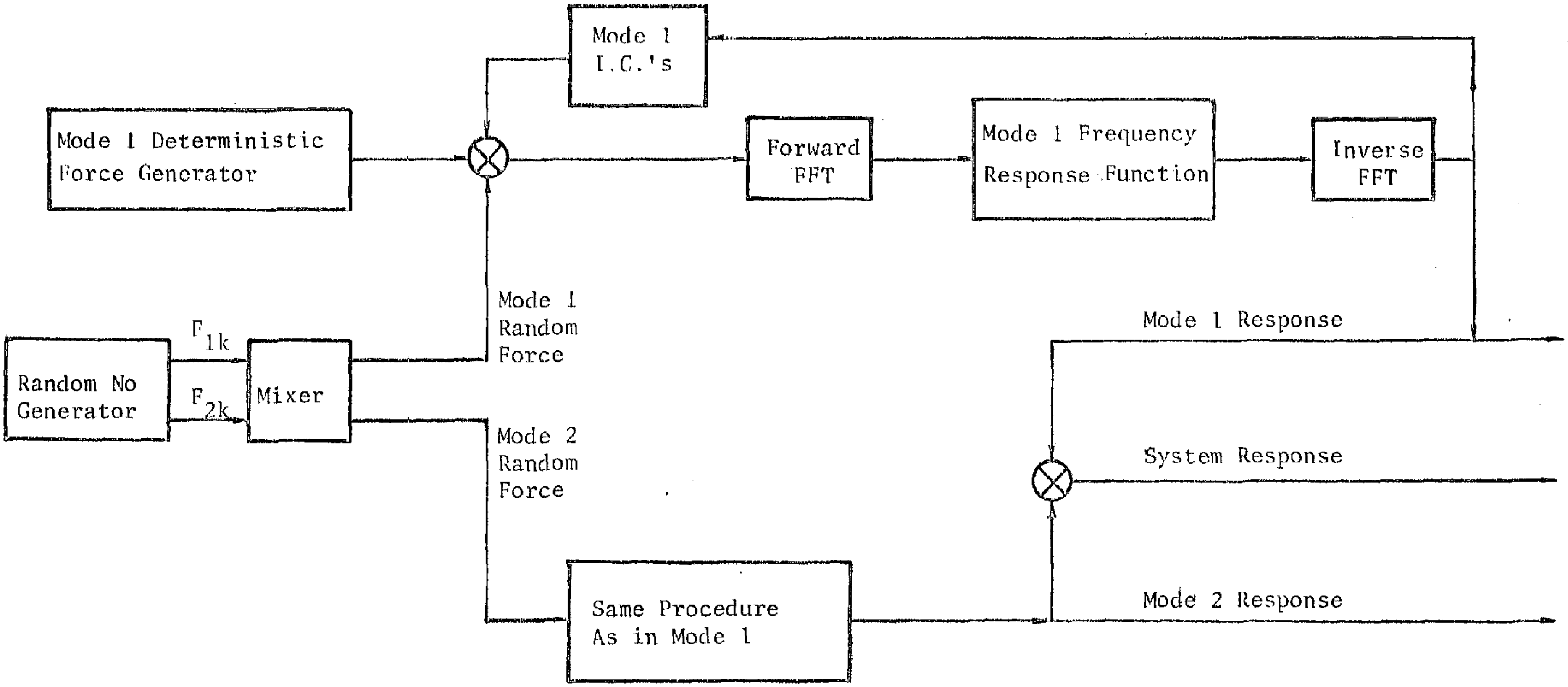


Figure 4   Simplified Block-Diagram Showing Response Simulation Process

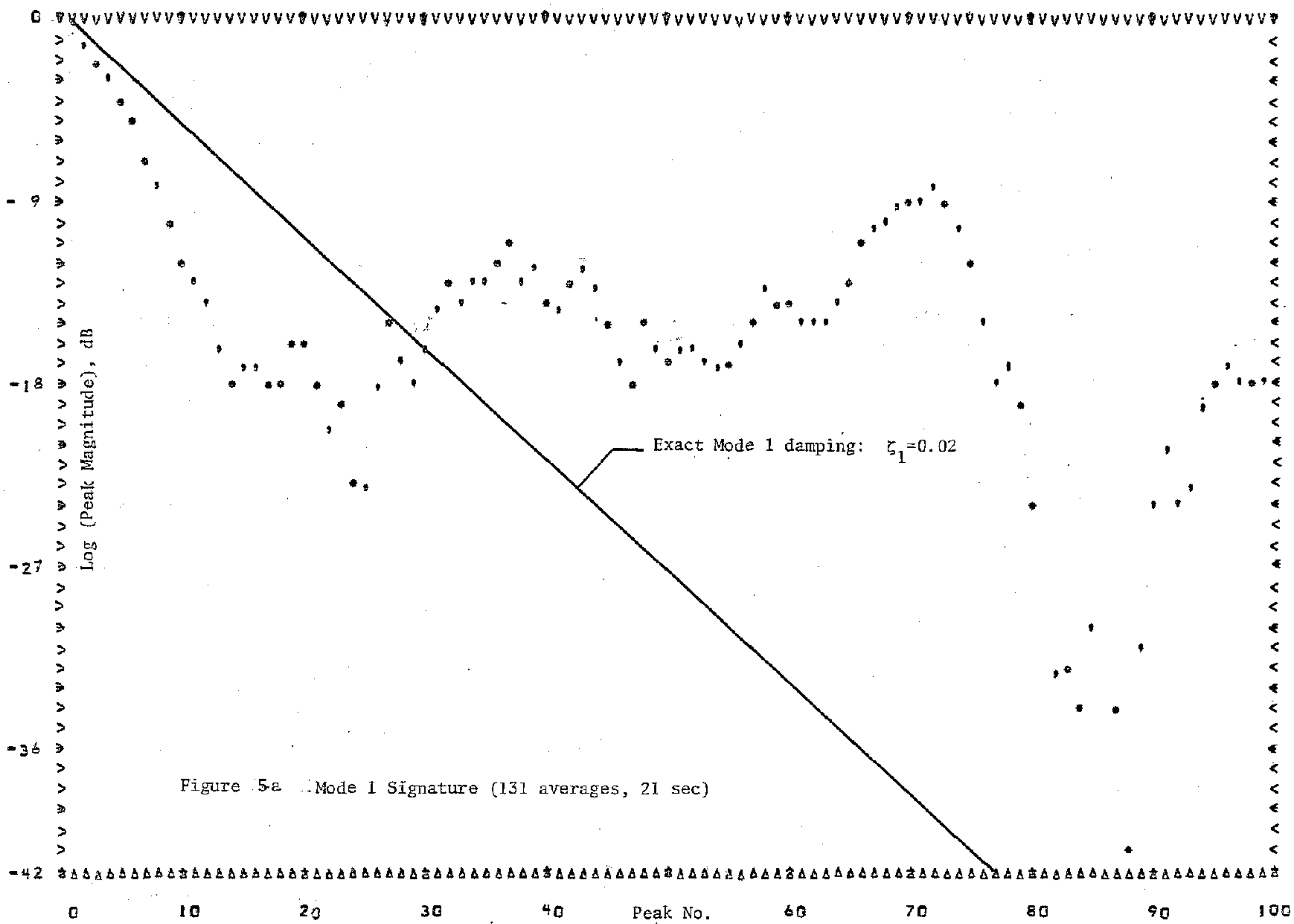


Figure 5-a Mode 1 Signature (131 averages, 21 sec)

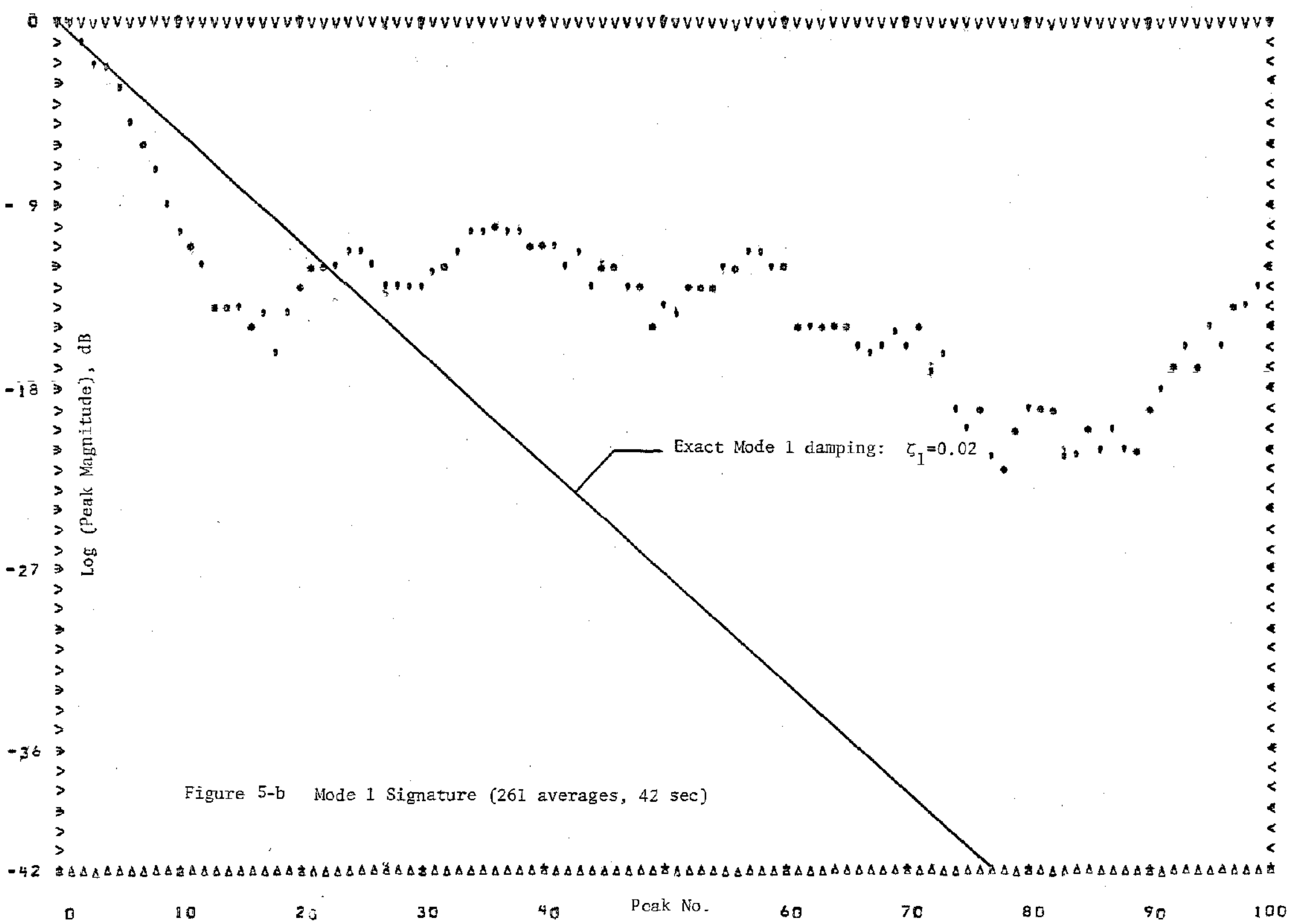


Figure 5-b Mode 1 Signature (261 averages, 42 sec)

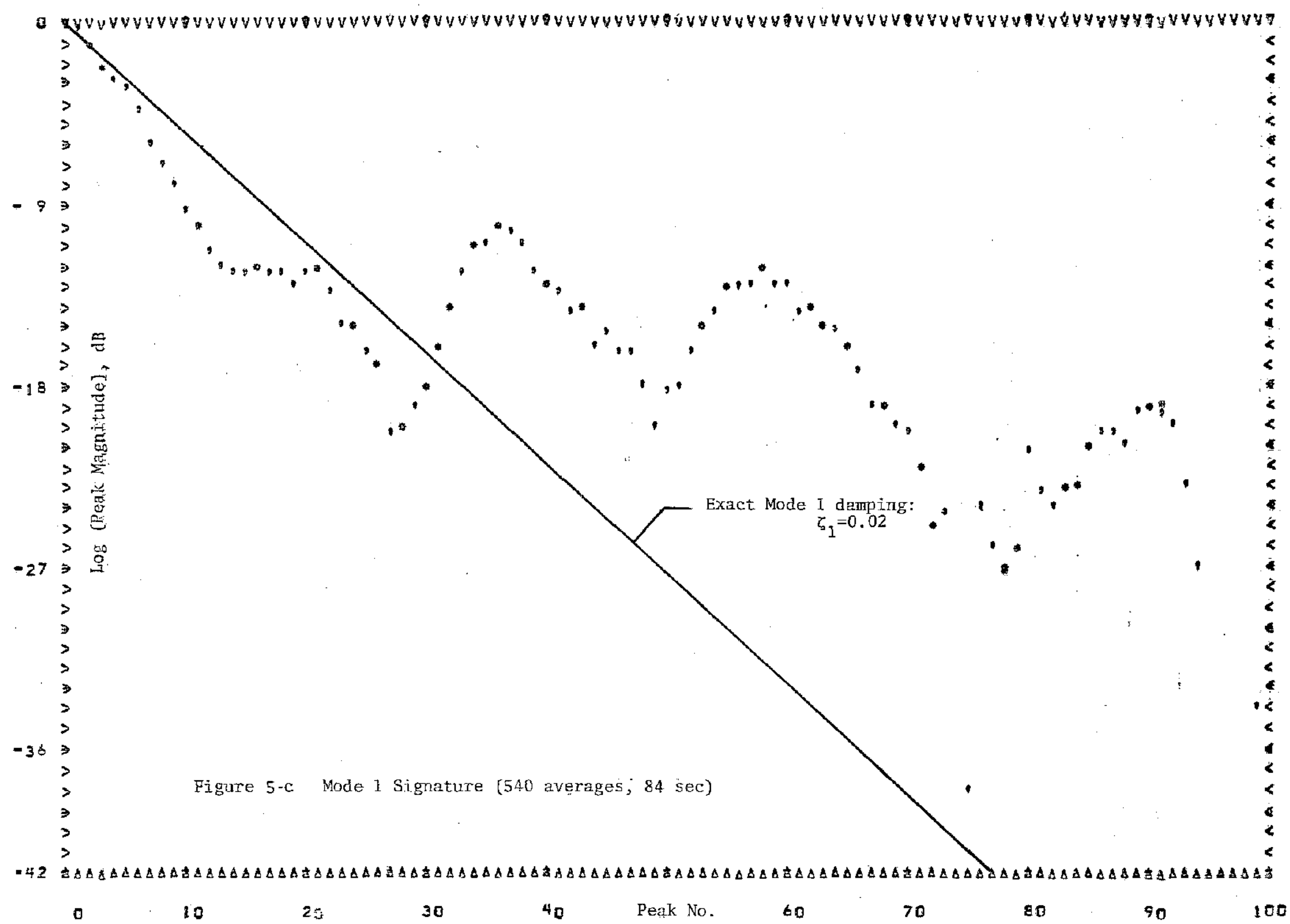


Figure 5-c Mode 1 Signature (540 averages, 84 sec)

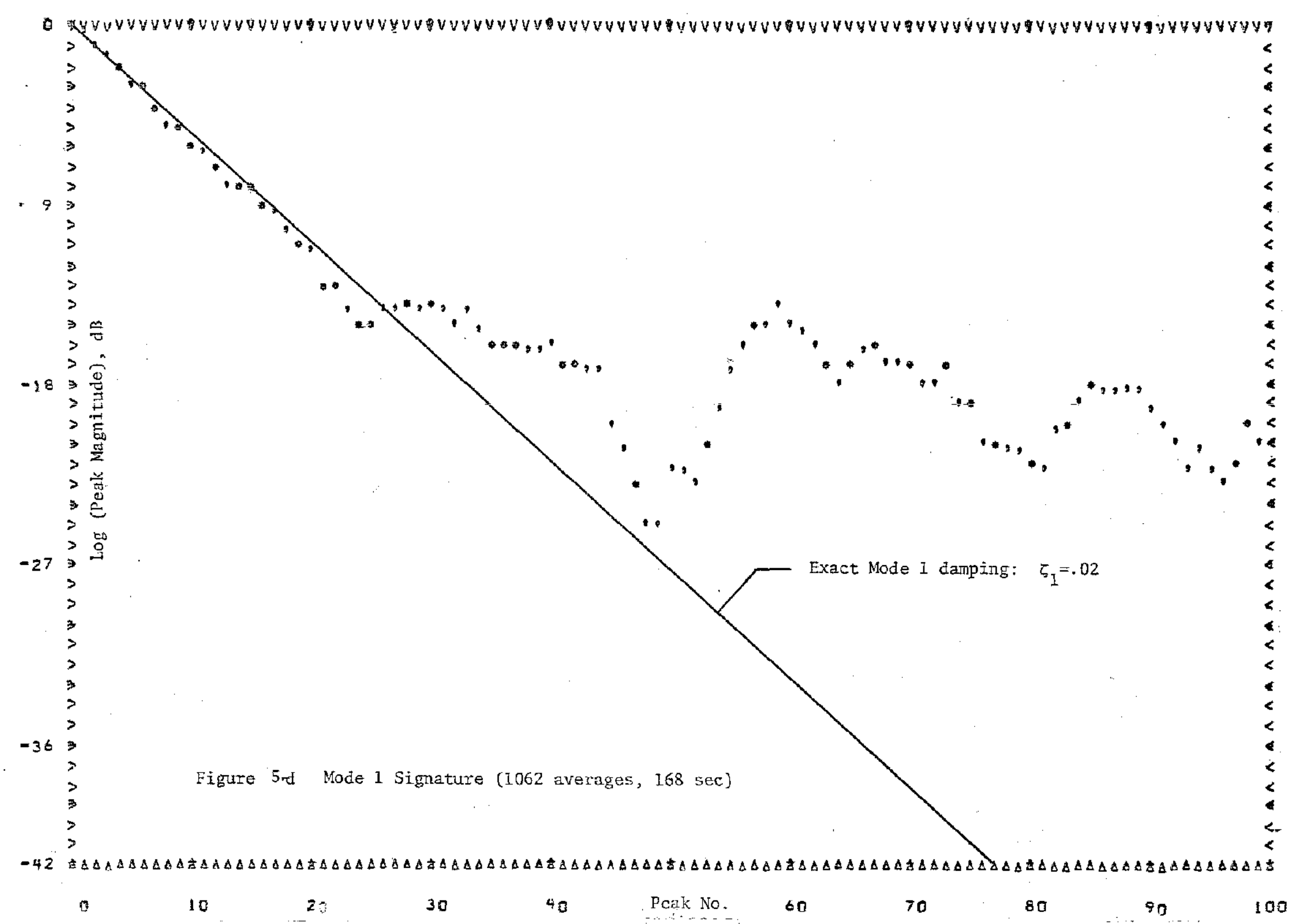


Figure 5-d Mode 1 Signature (1062 averages, 168 sec)

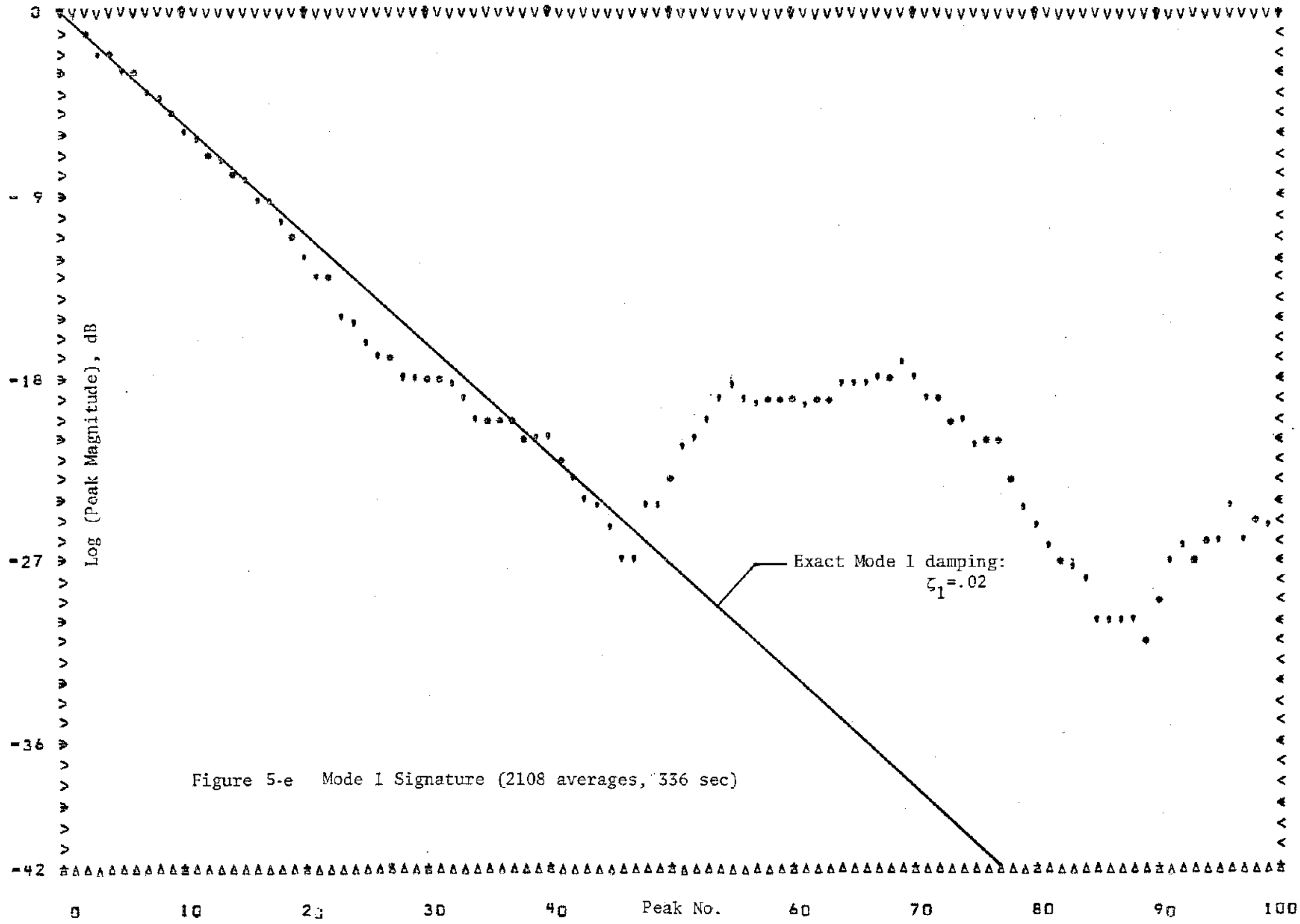


Figure 5-e Mode 1 Signature (2108 averages, 336 sec)

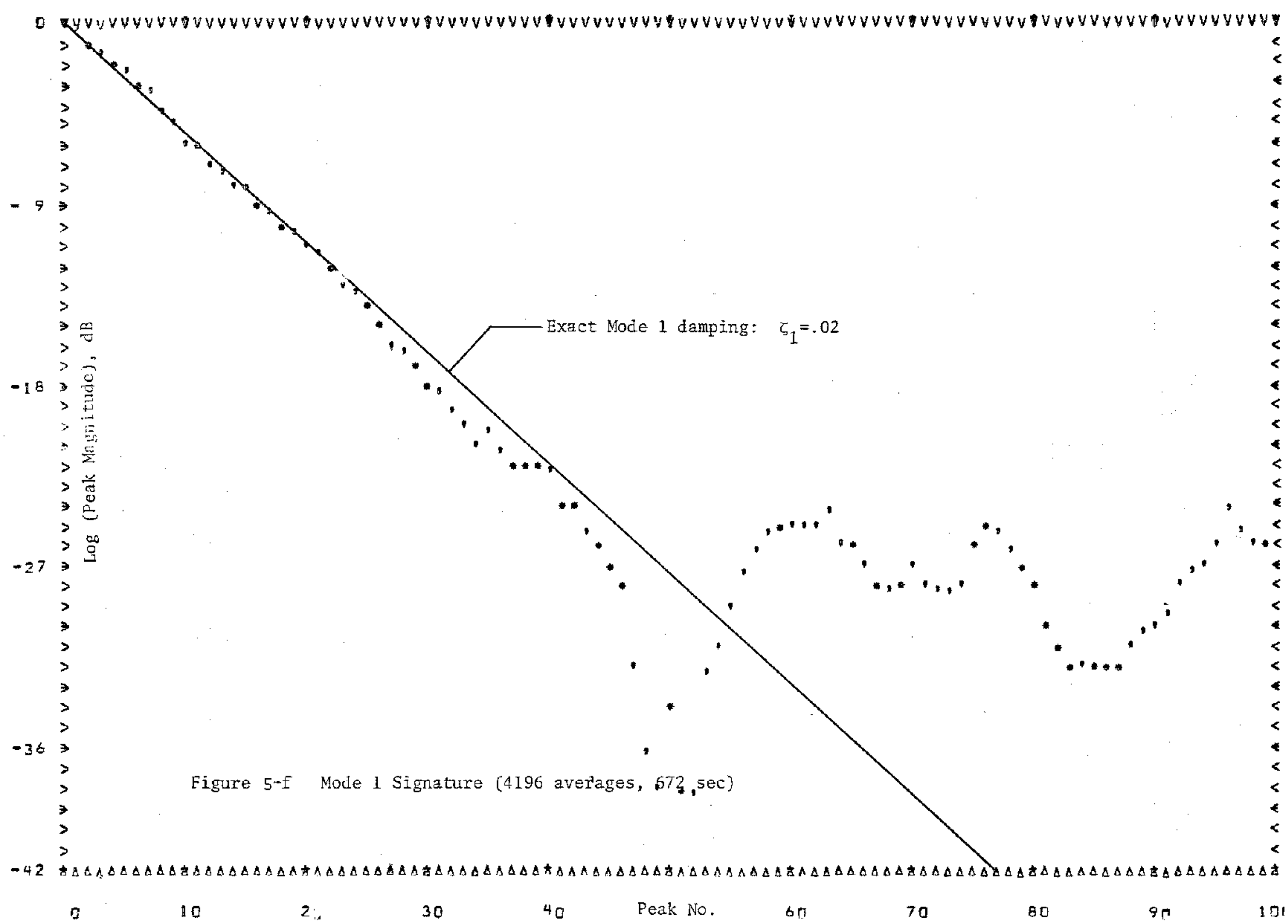


Figure 5-f Mode 1 Signature (4196 averages, 672 sec)

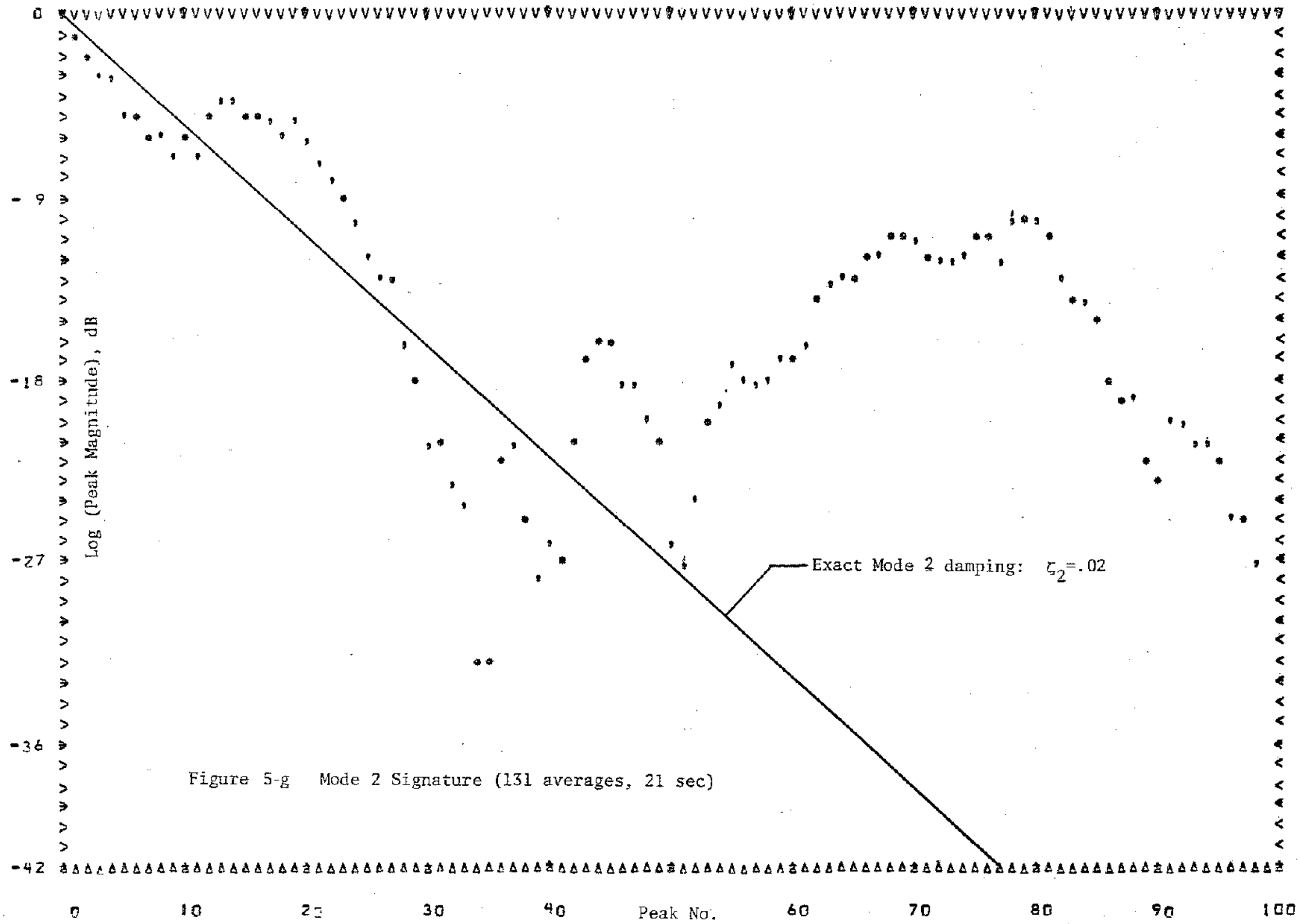


Figure 5-g Mode 2 Signature (131 averages, 21 sec)

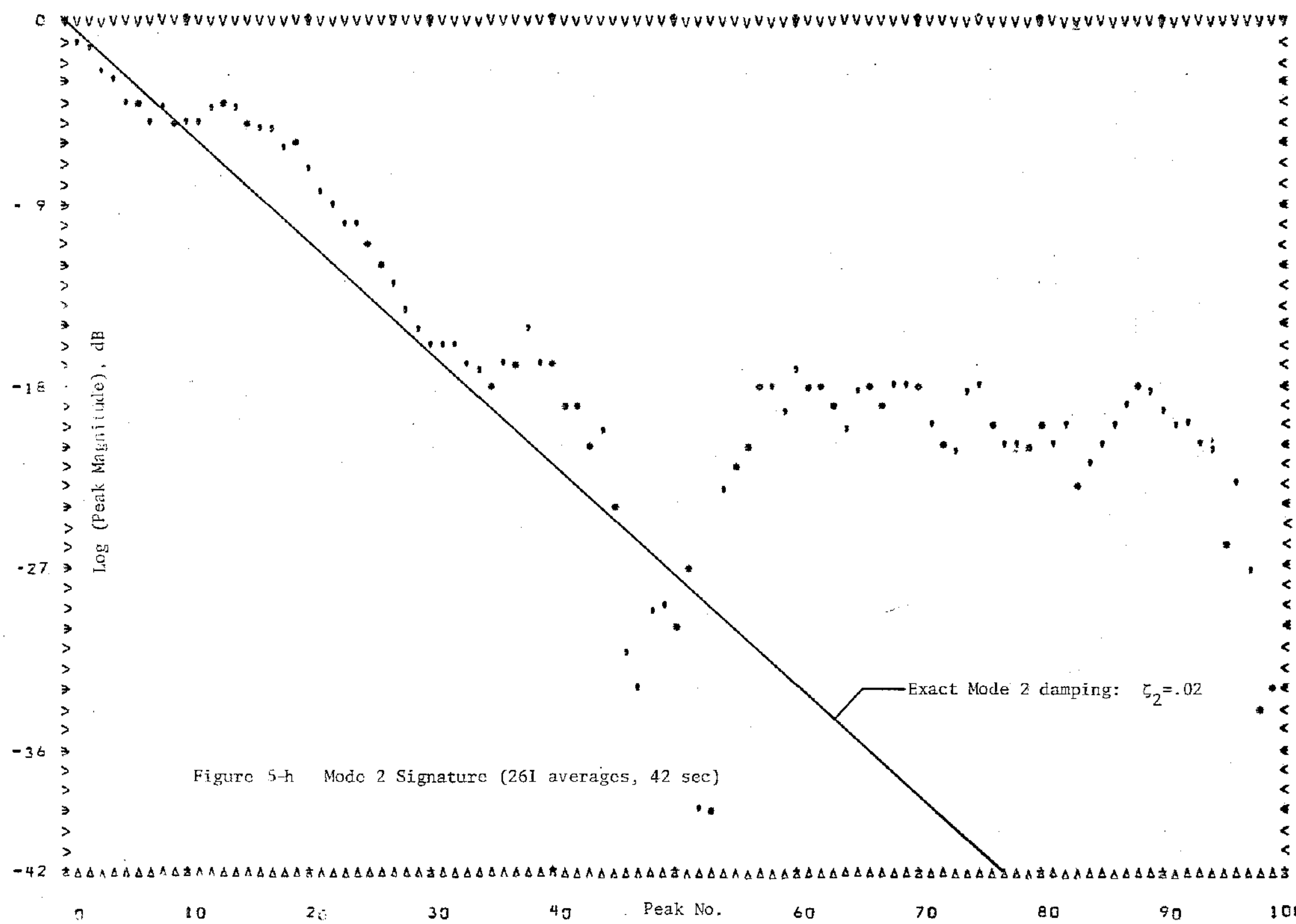


Figure 5-h Mode 2 Signature (261 averages, 42 sec)

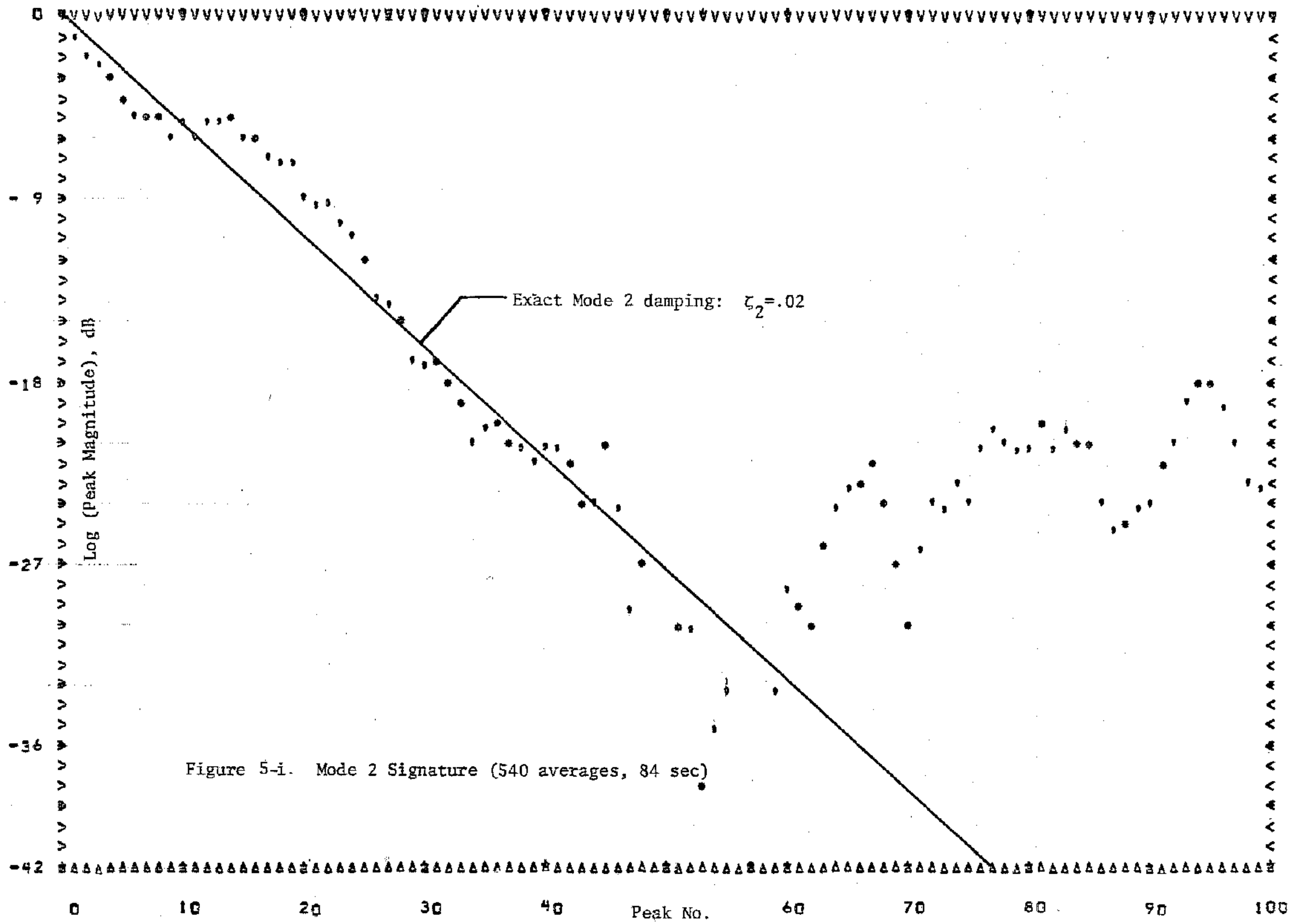


Figure 5-i. Mode 2 Signature (540 averages, 84 sec)

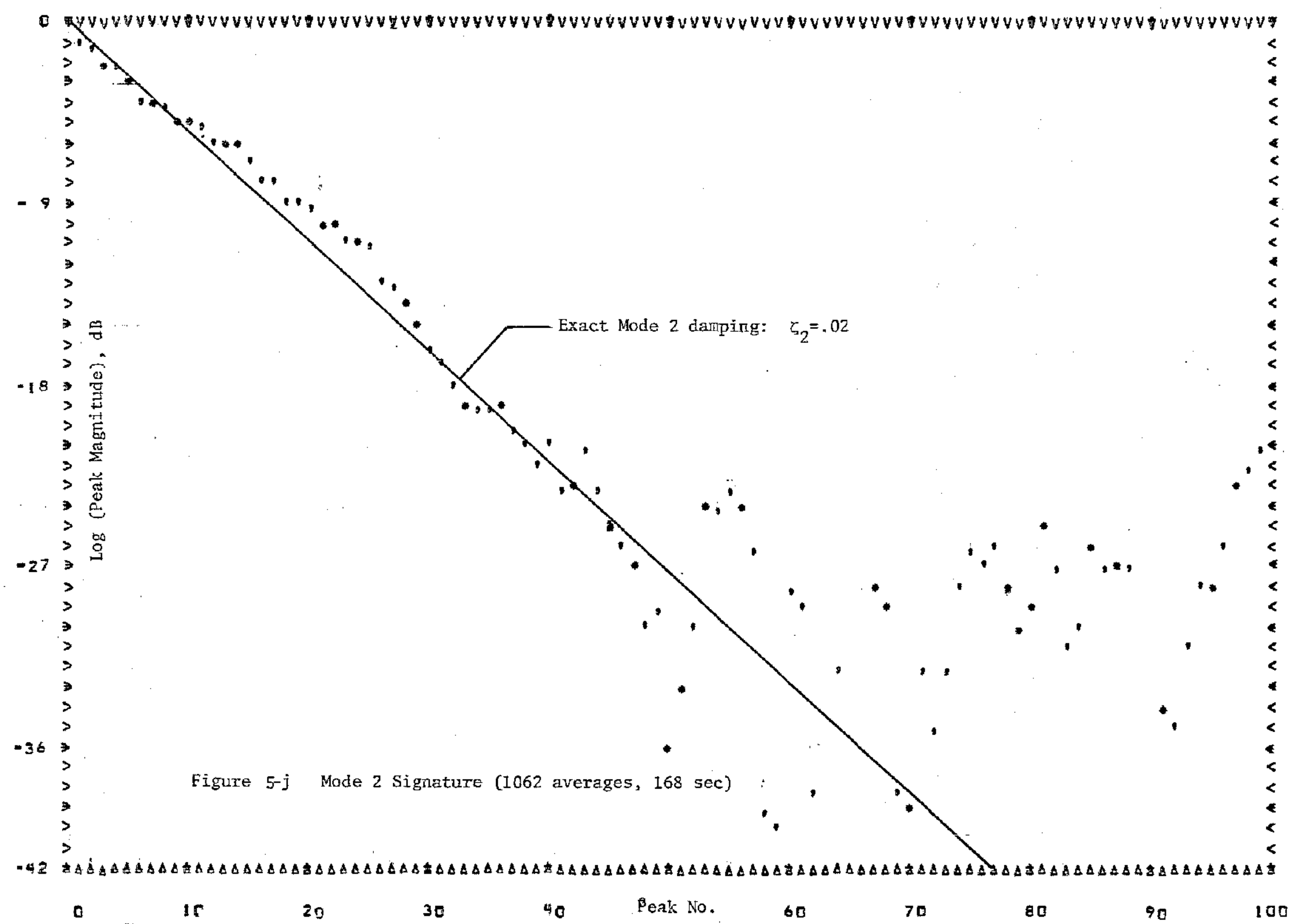


Figure 5-j Mode 2 Signature (1062 averages, 168 sec)

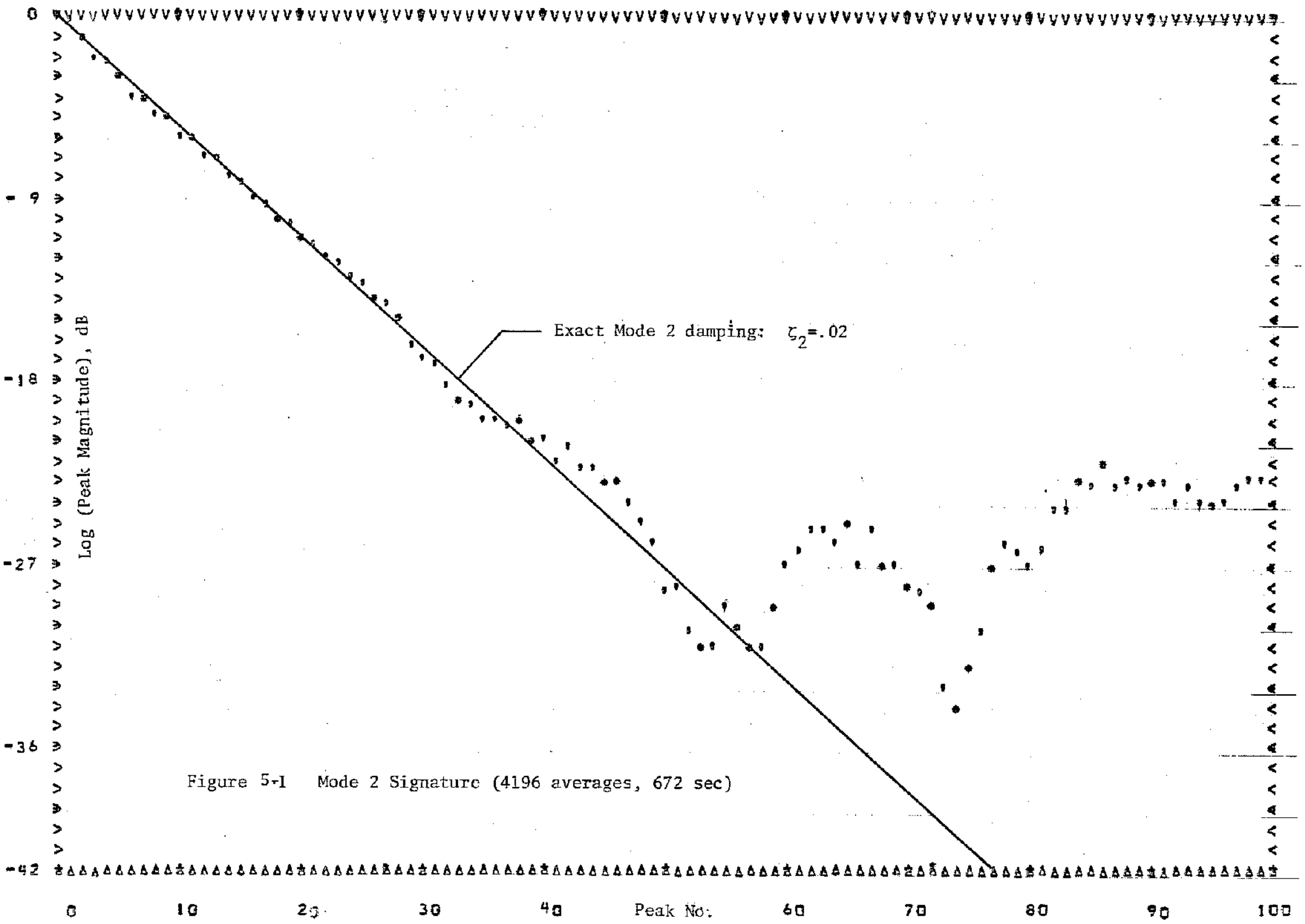


Figure 5-1 Mode 2 Signature (4196 averages, 672 sec)

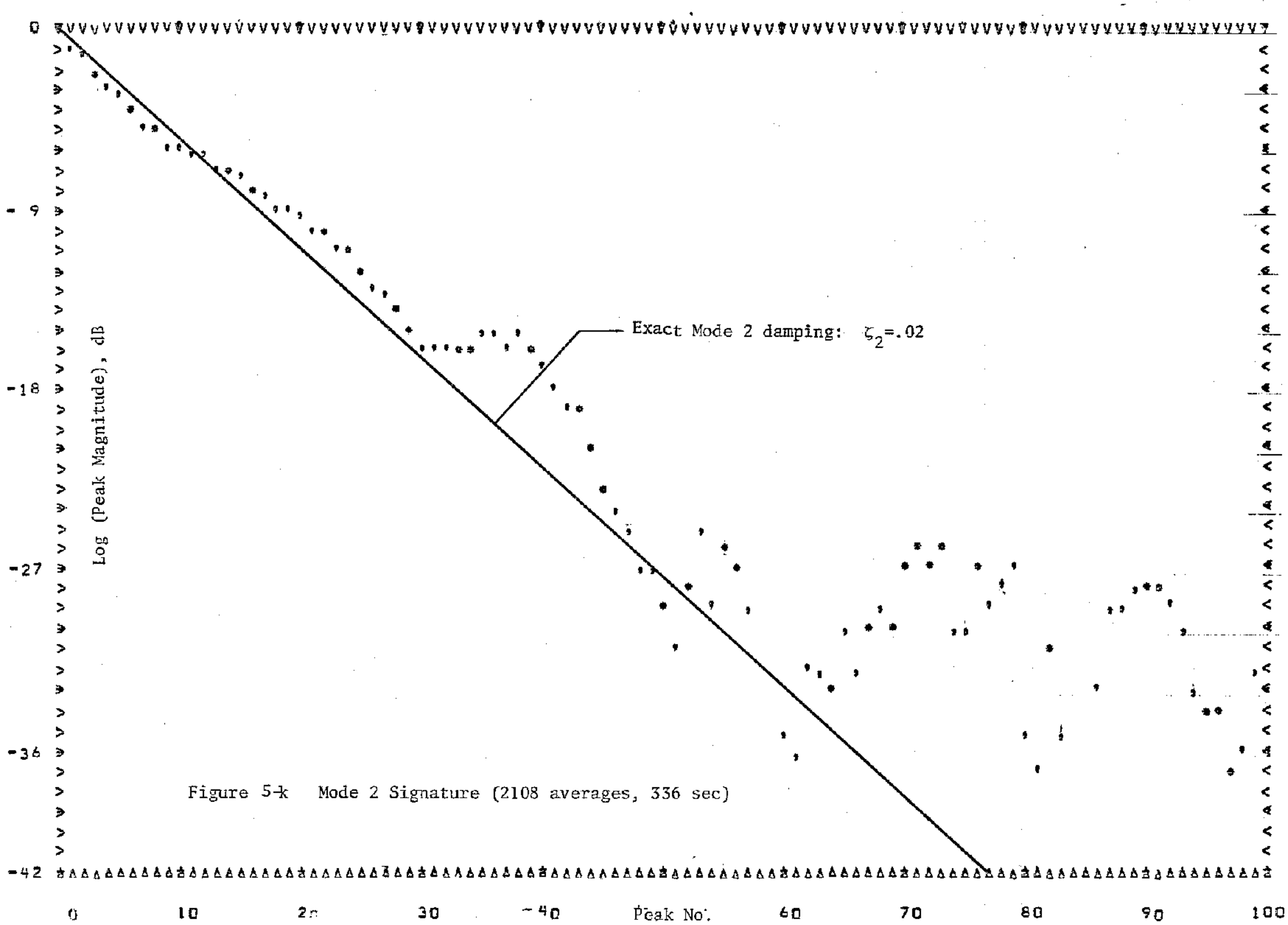


Figure 5-k Mode 2 Signature (2108 averages, 336 sec)

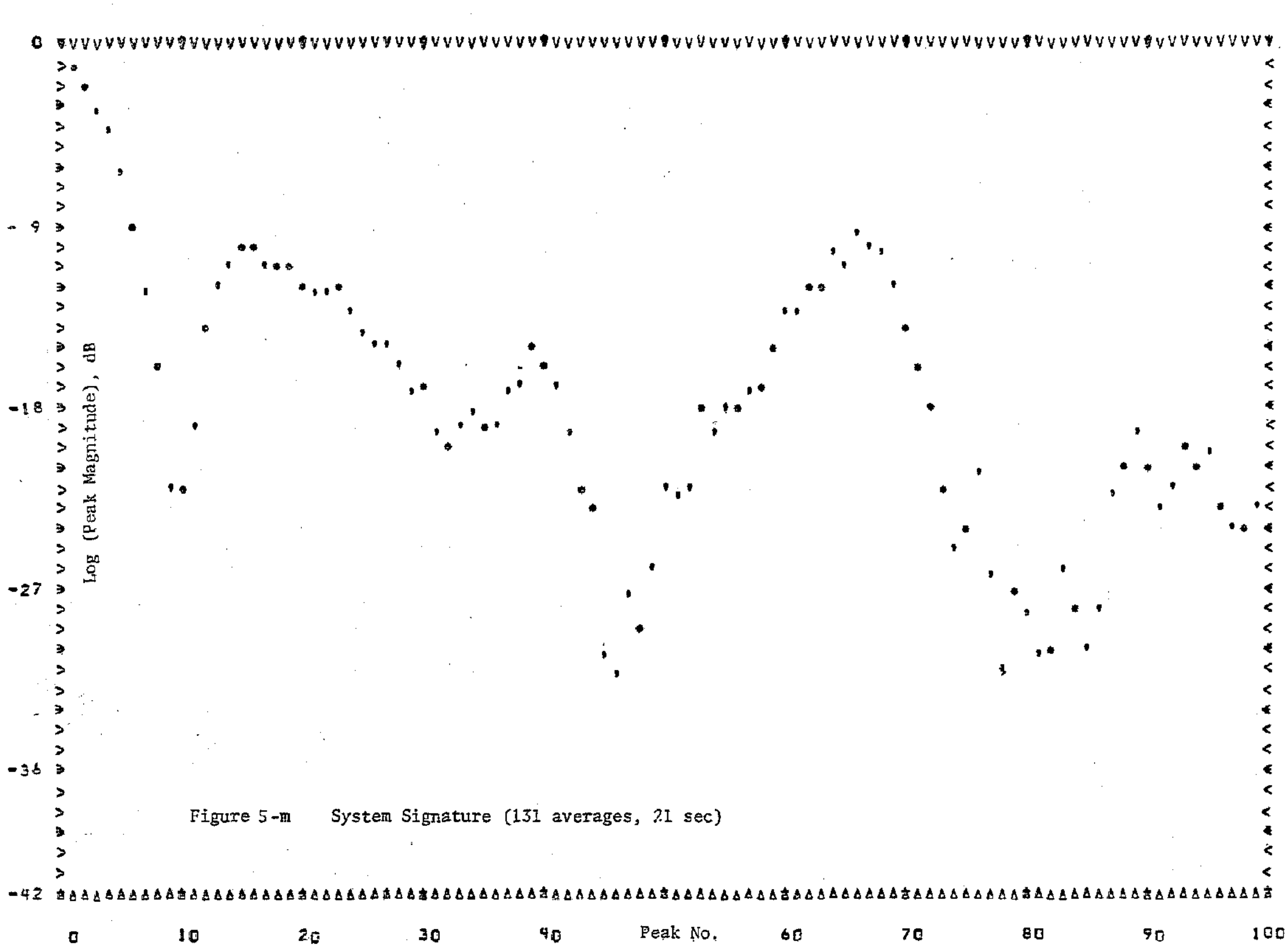


Figure 5-m System Signature (131 averages, 21 sec)

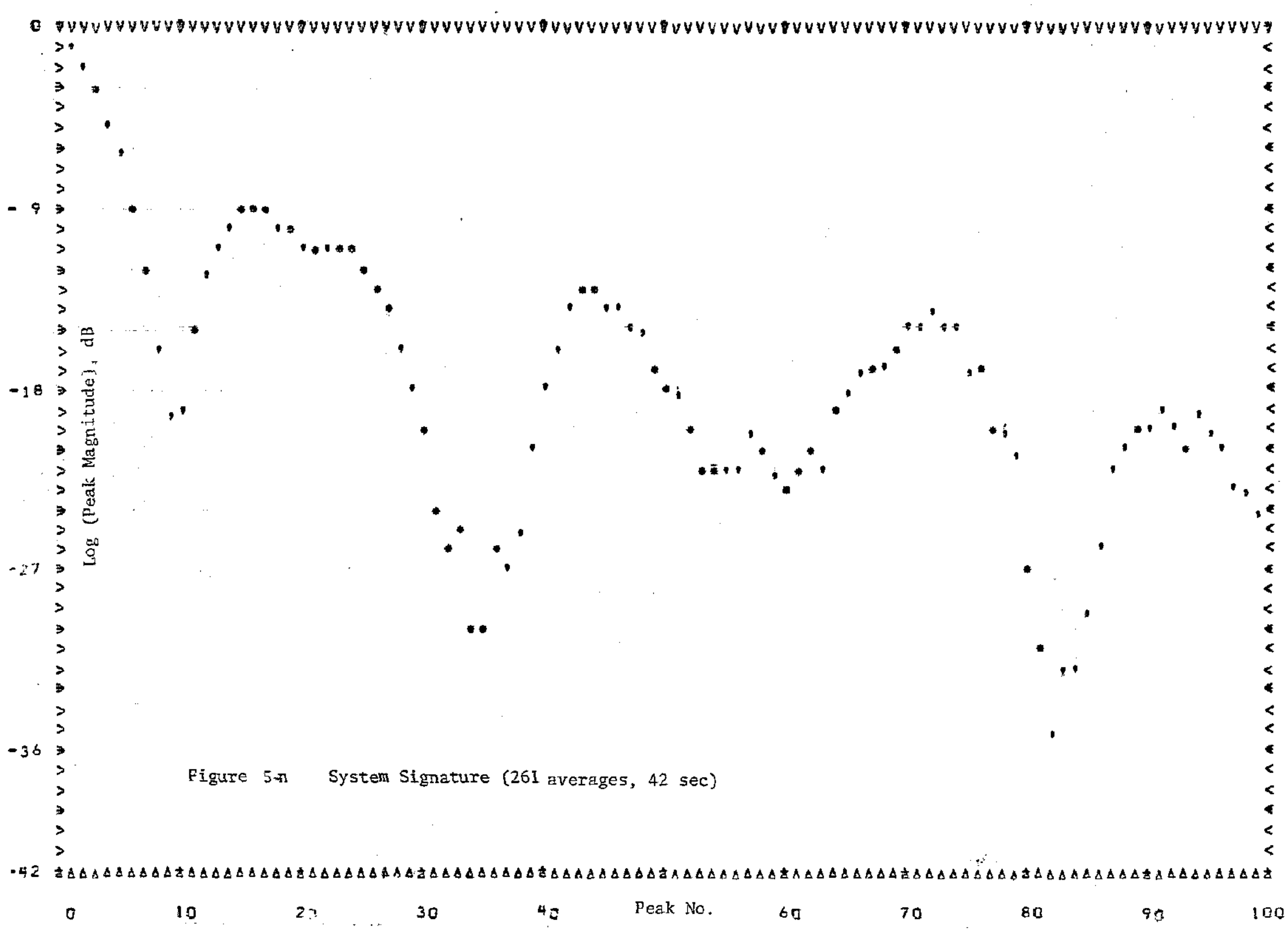


Figure 5-n System Signature (261 averages, 42 sec)

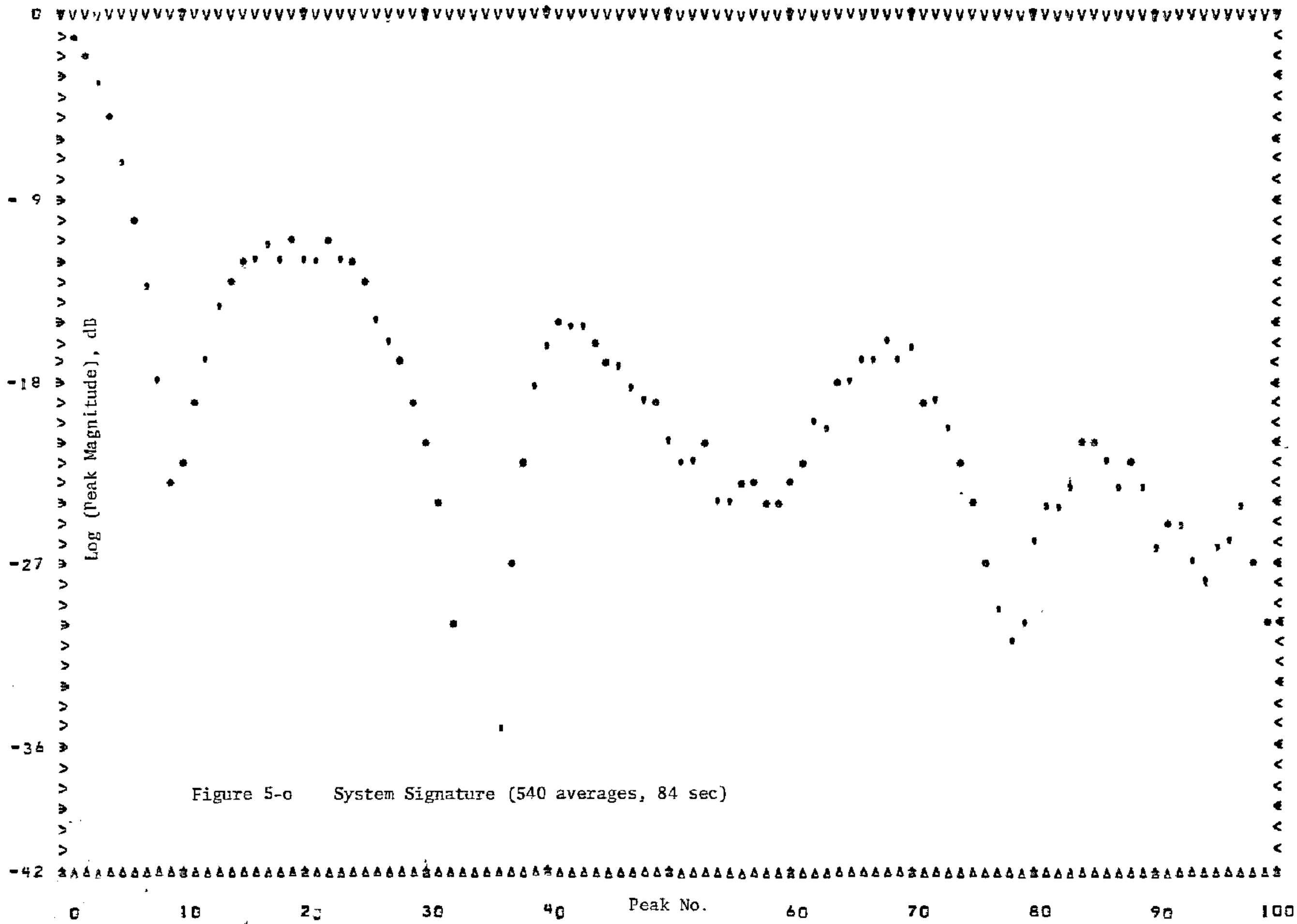


Figure 5-o System Signature (540 averages, 84 sec)

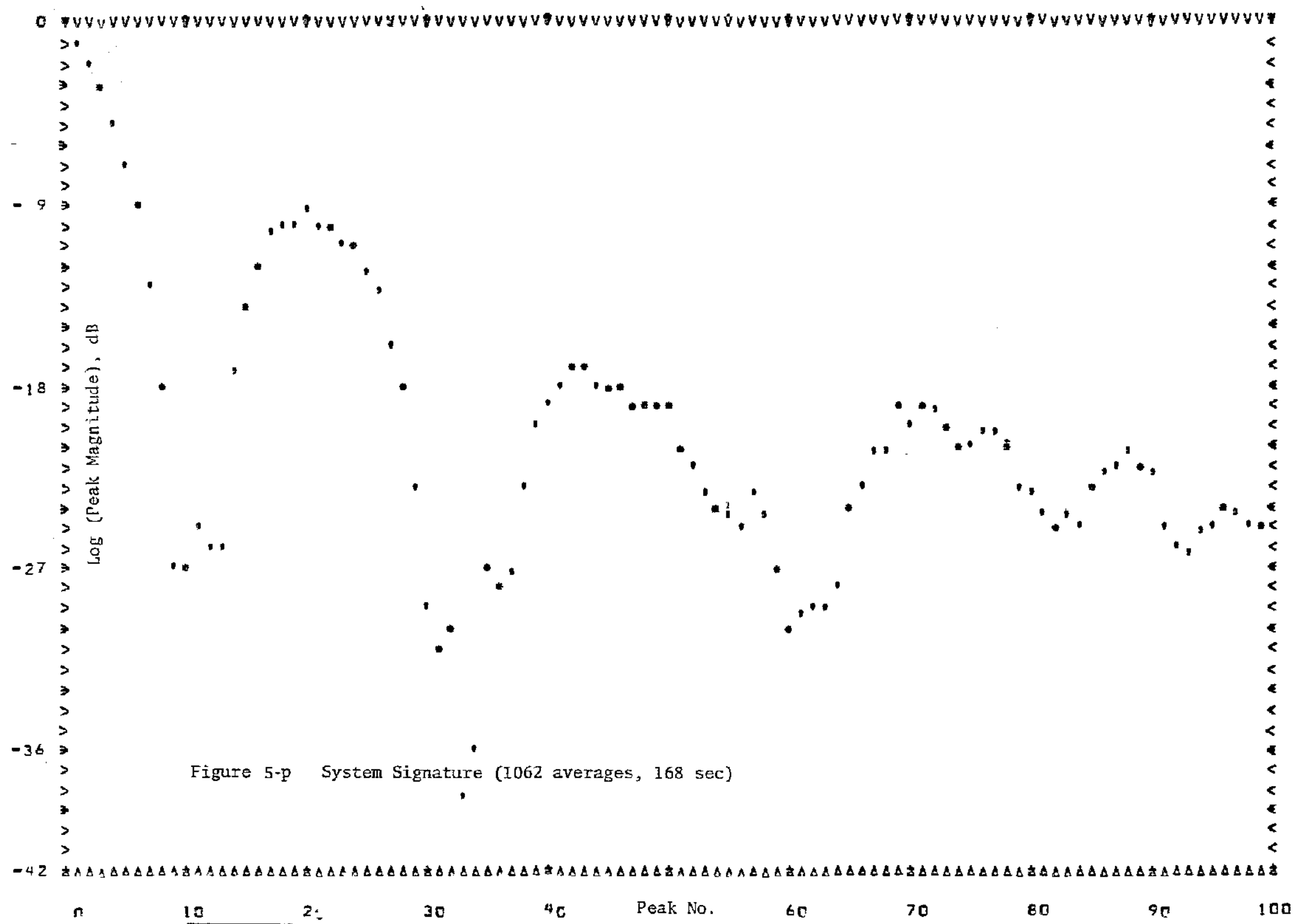


Figure 5-p System Signature (1062 averages, 168 sec)

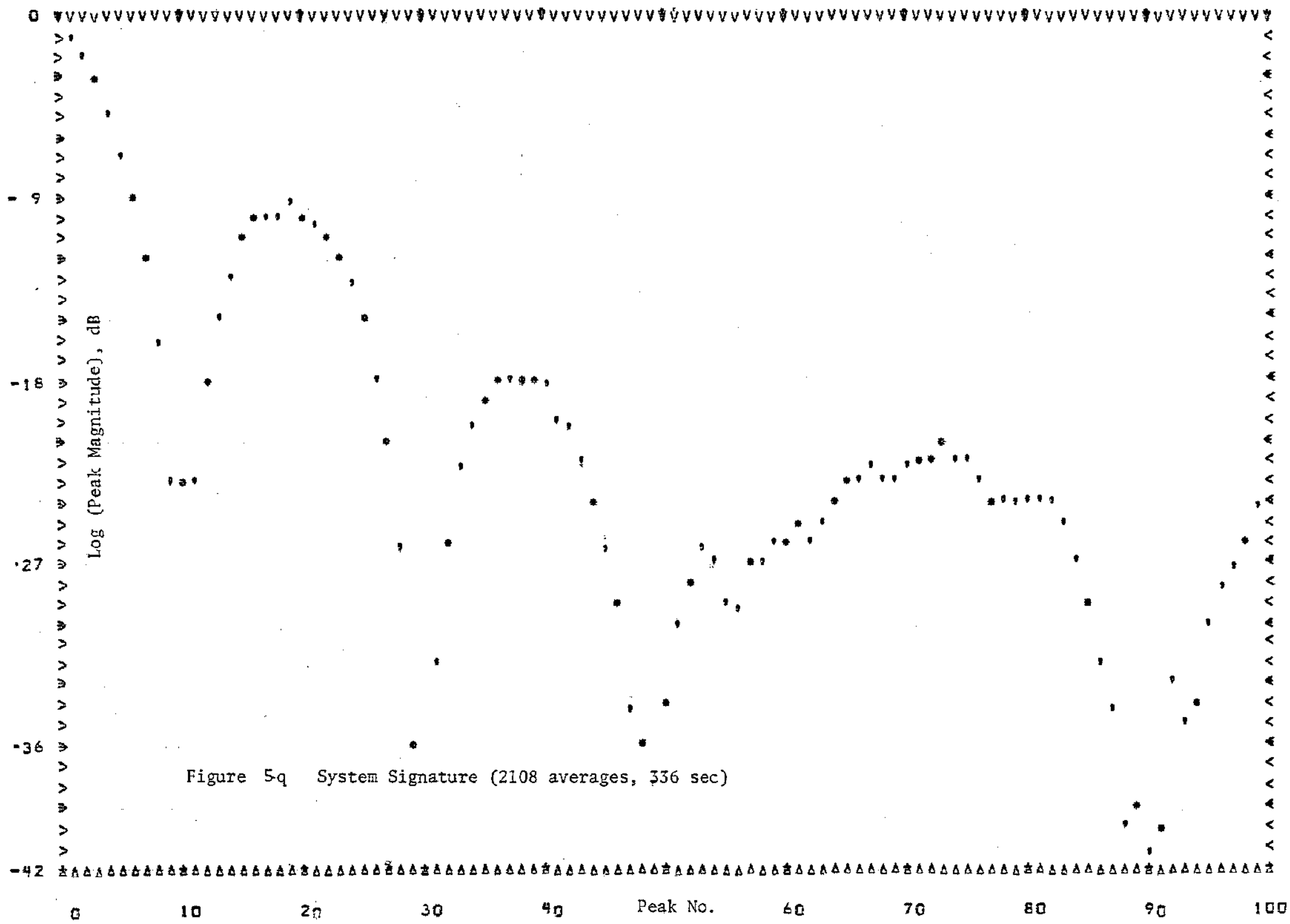


Figure 5-q System Signature (2108 averages, 336 sec)

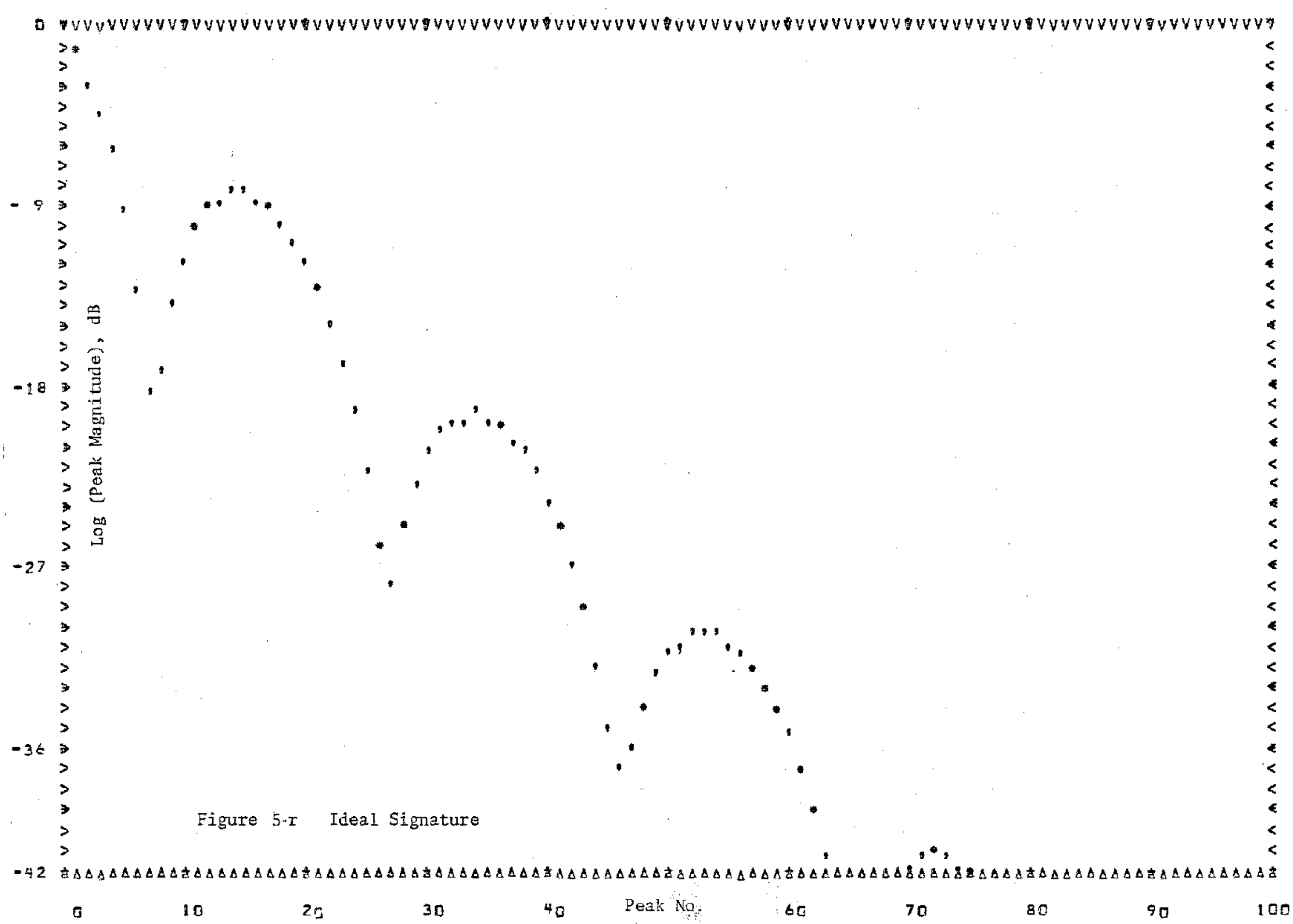


Figure 5-r Ideal Signature

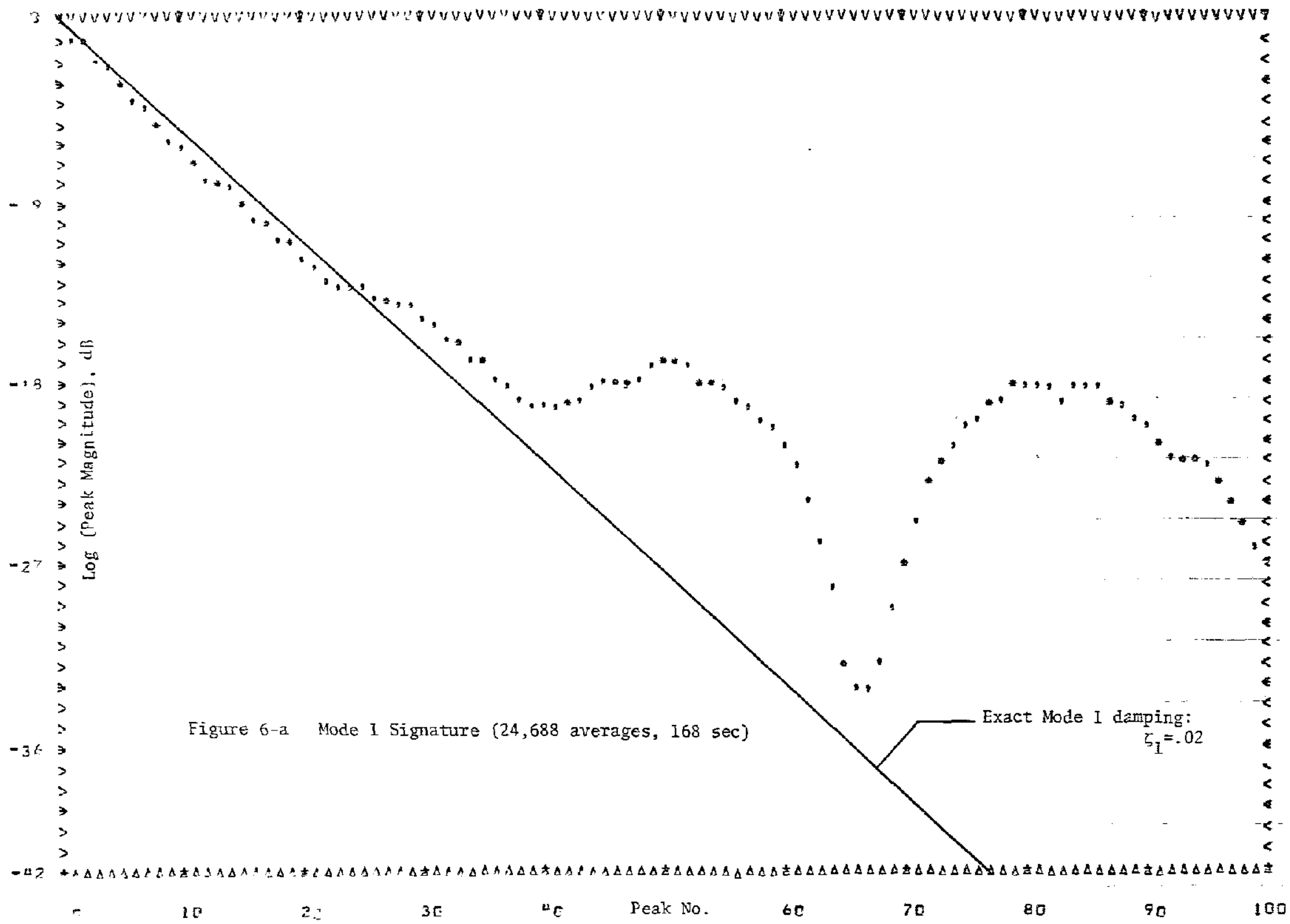


Figure 6-a Mode 1 Signature (24,688 averages, 168 sec)

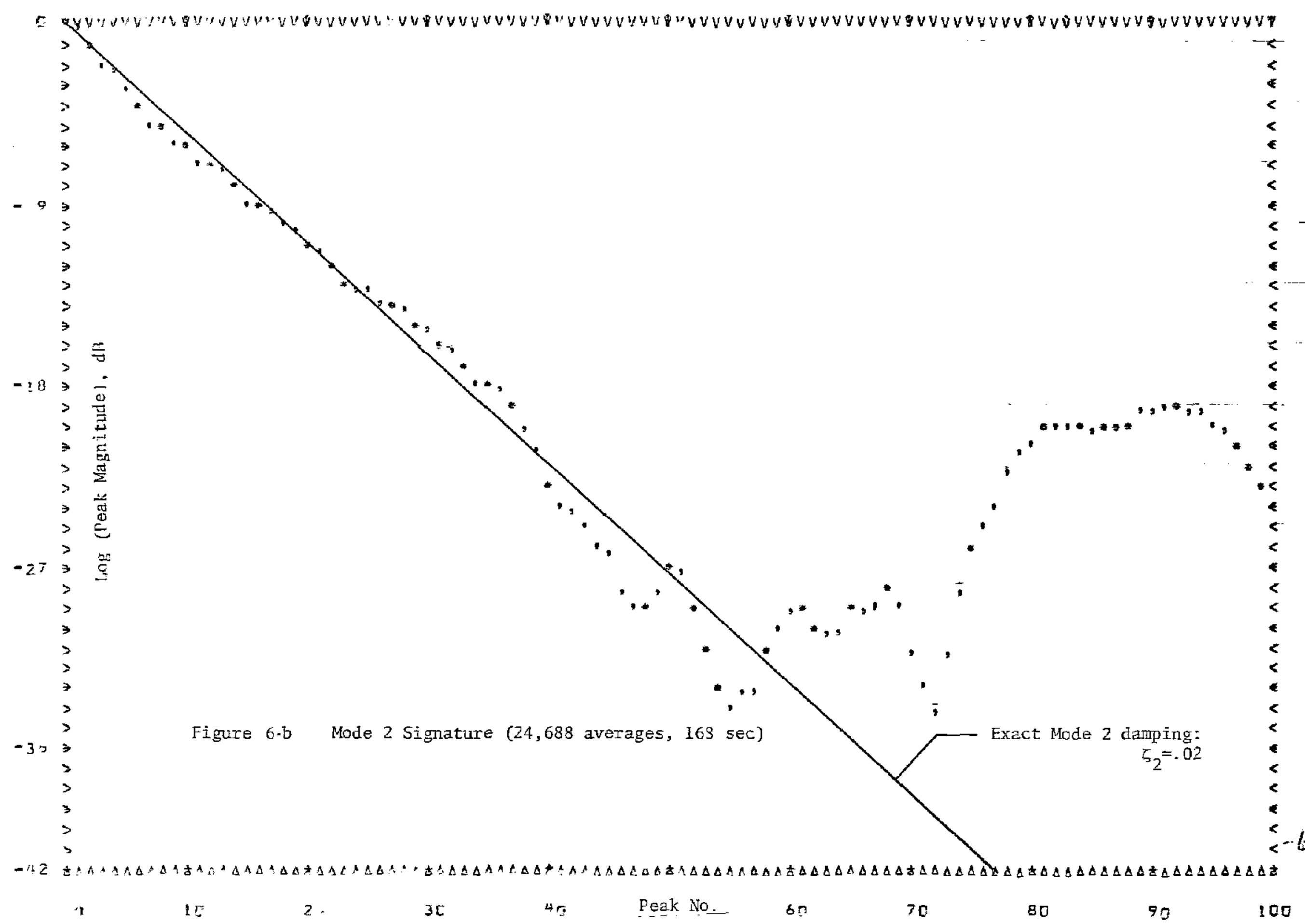
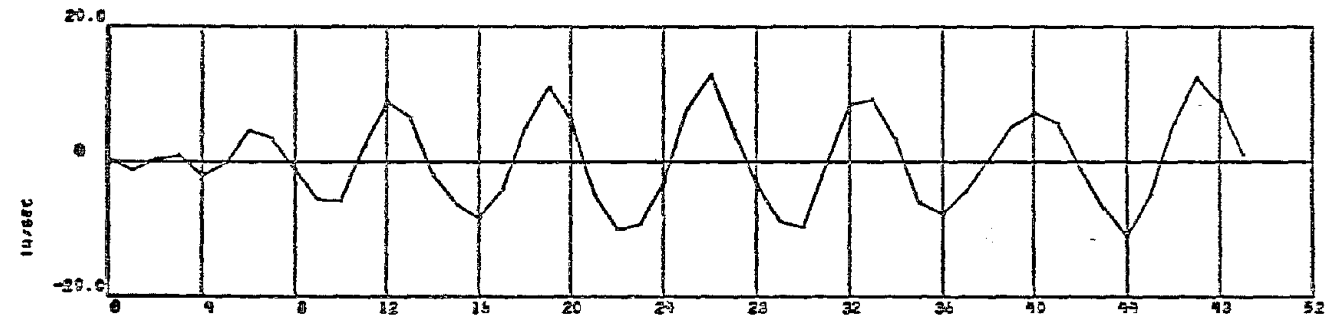


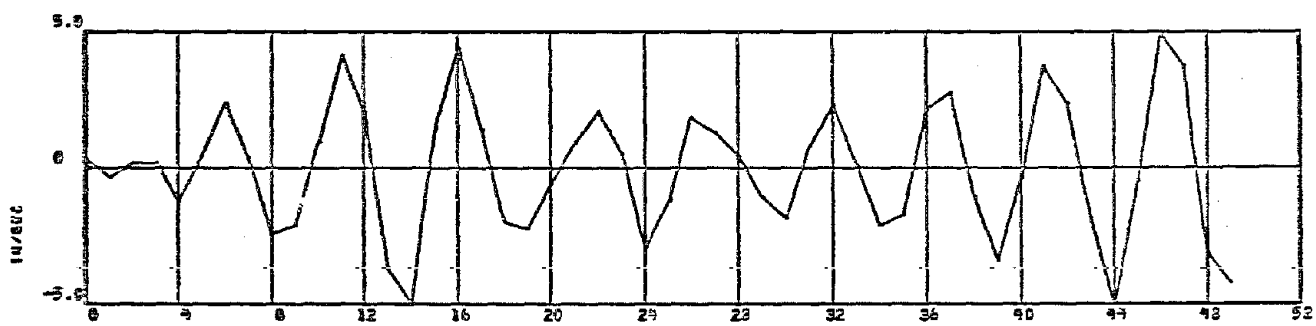
Figure 6-b Mode 2 Signature (24,688 averages, 163 sec)

SIMULATED RESPONSE SIGNAL (MODE 1)



N (t = .0003N sec)

SIMULATED RESPONSE SIGNAL (MODE 2)



N (t = .0003N sec)

Figure 7 Simulated Modal Response Signals

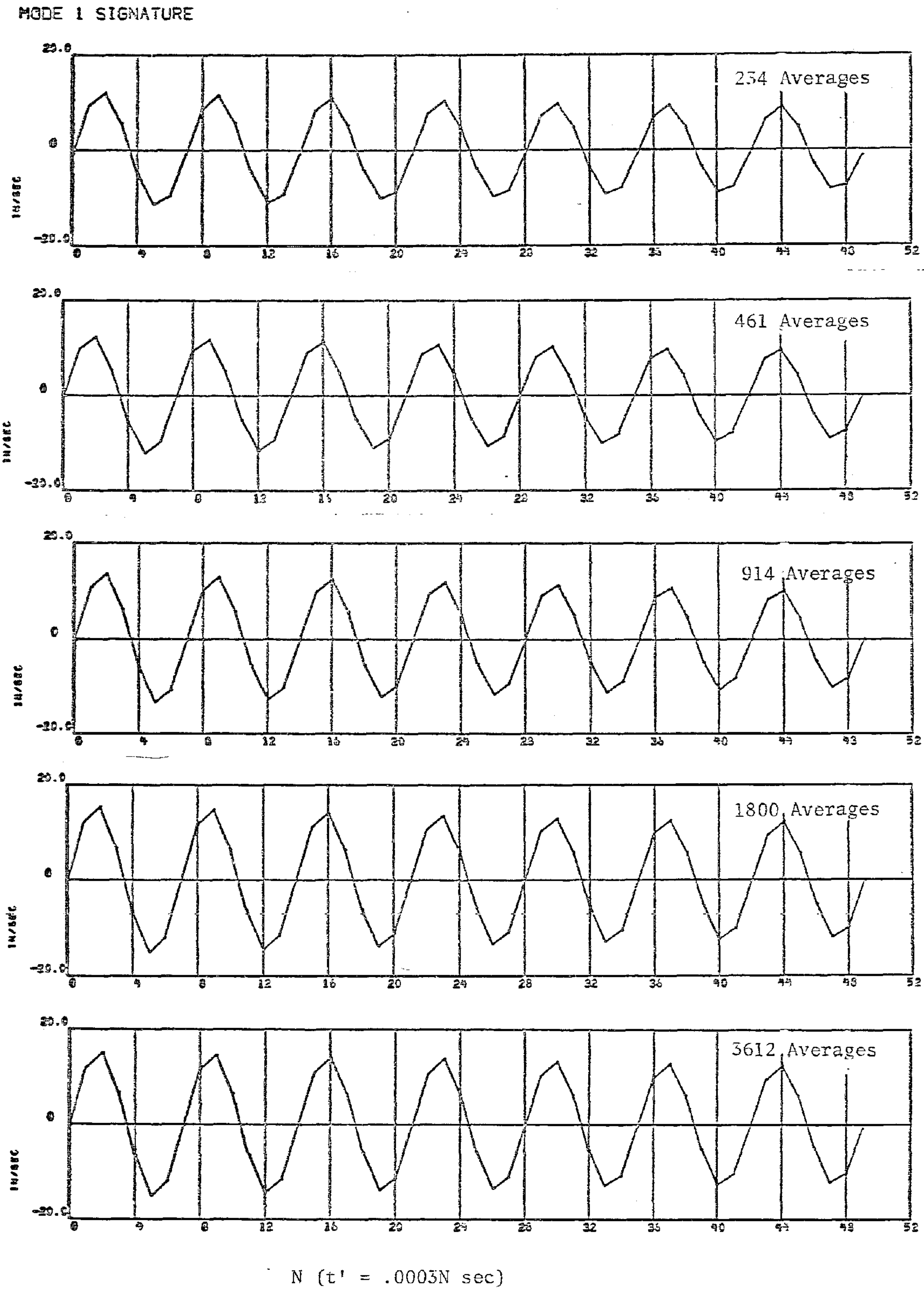


Figure 8-a Mode 1 Signatures

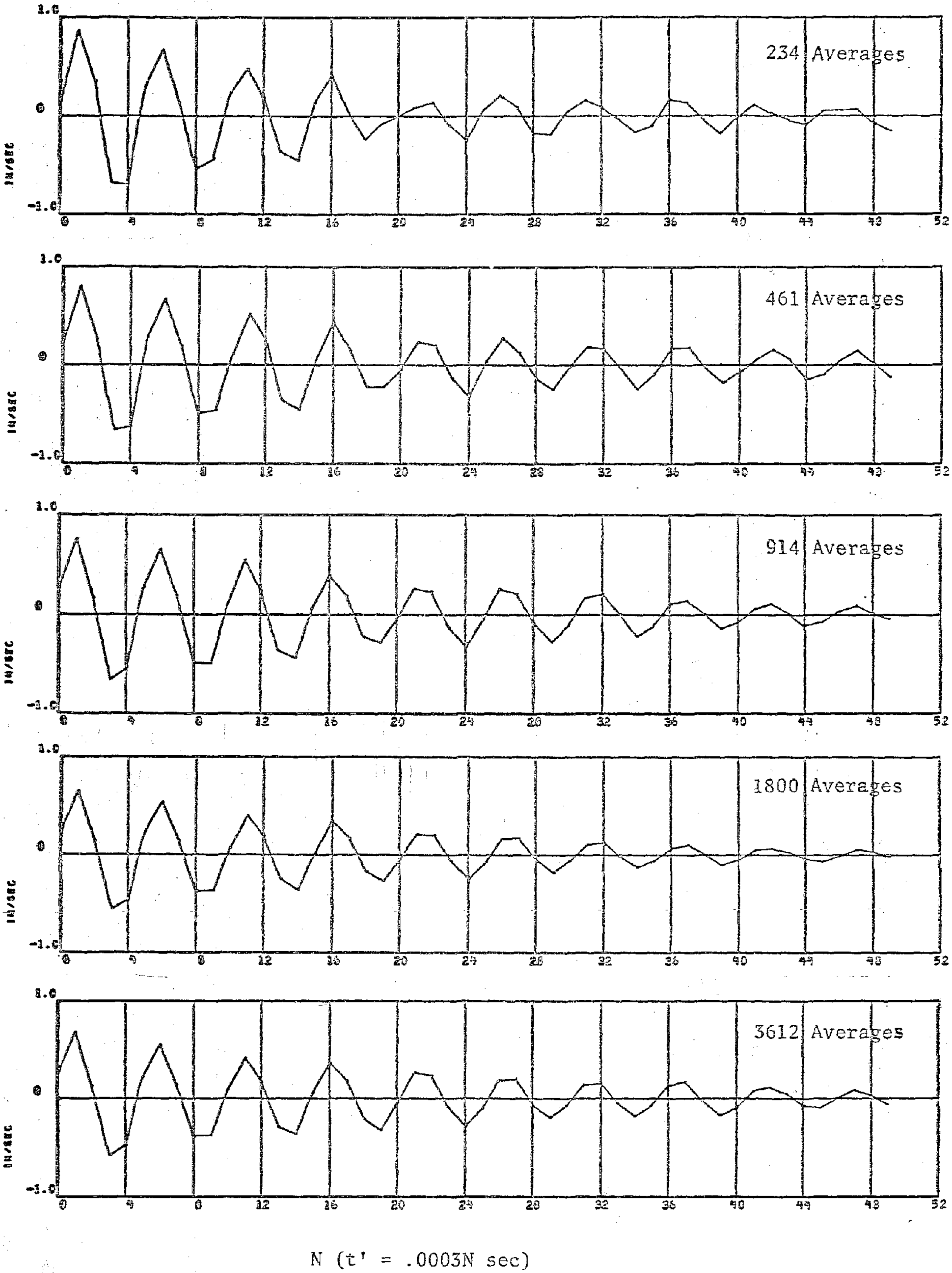
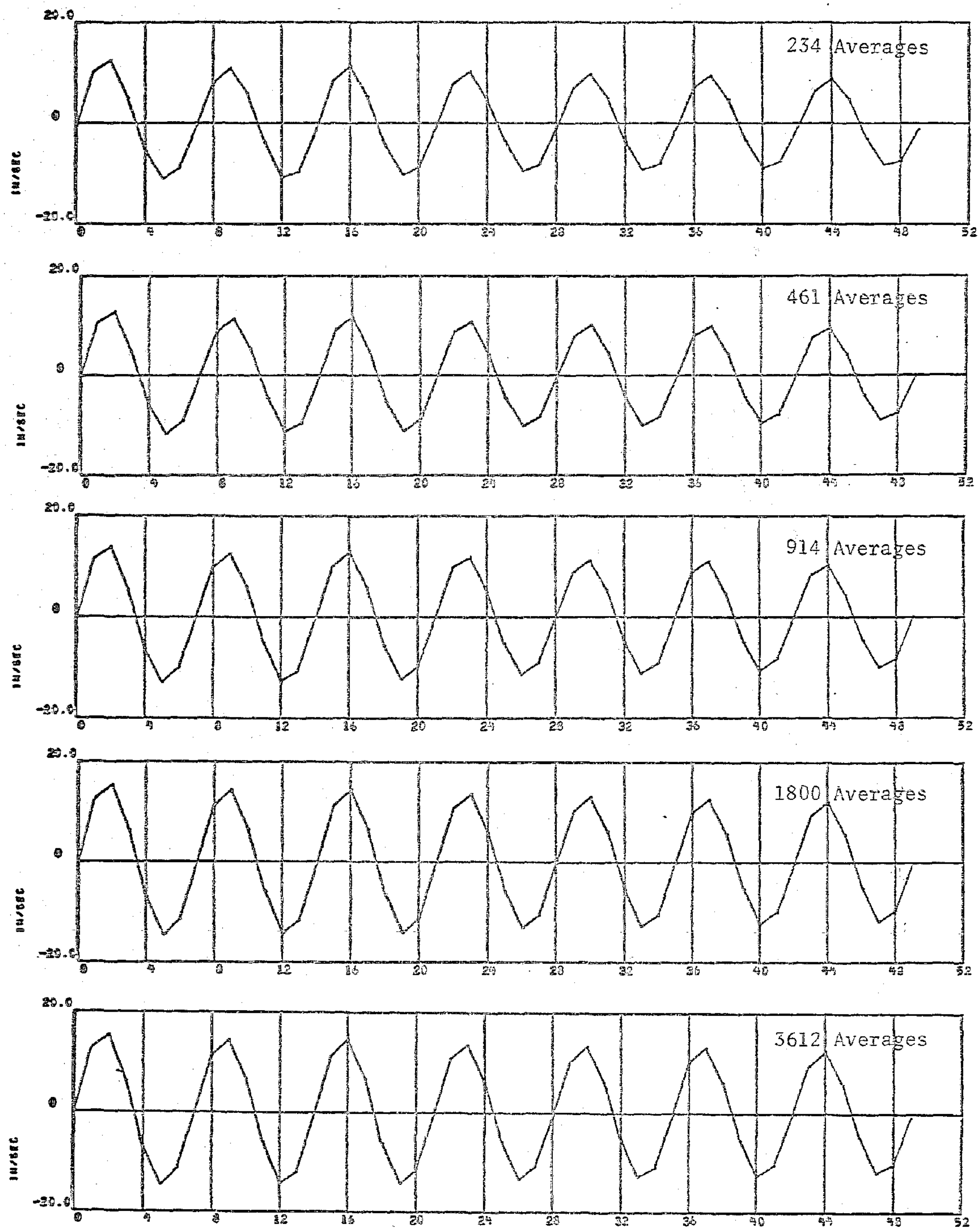


Figure 8-b Mode 2 Signatures



N (t' = .0003N sec)

Figure 8-c System Signatures

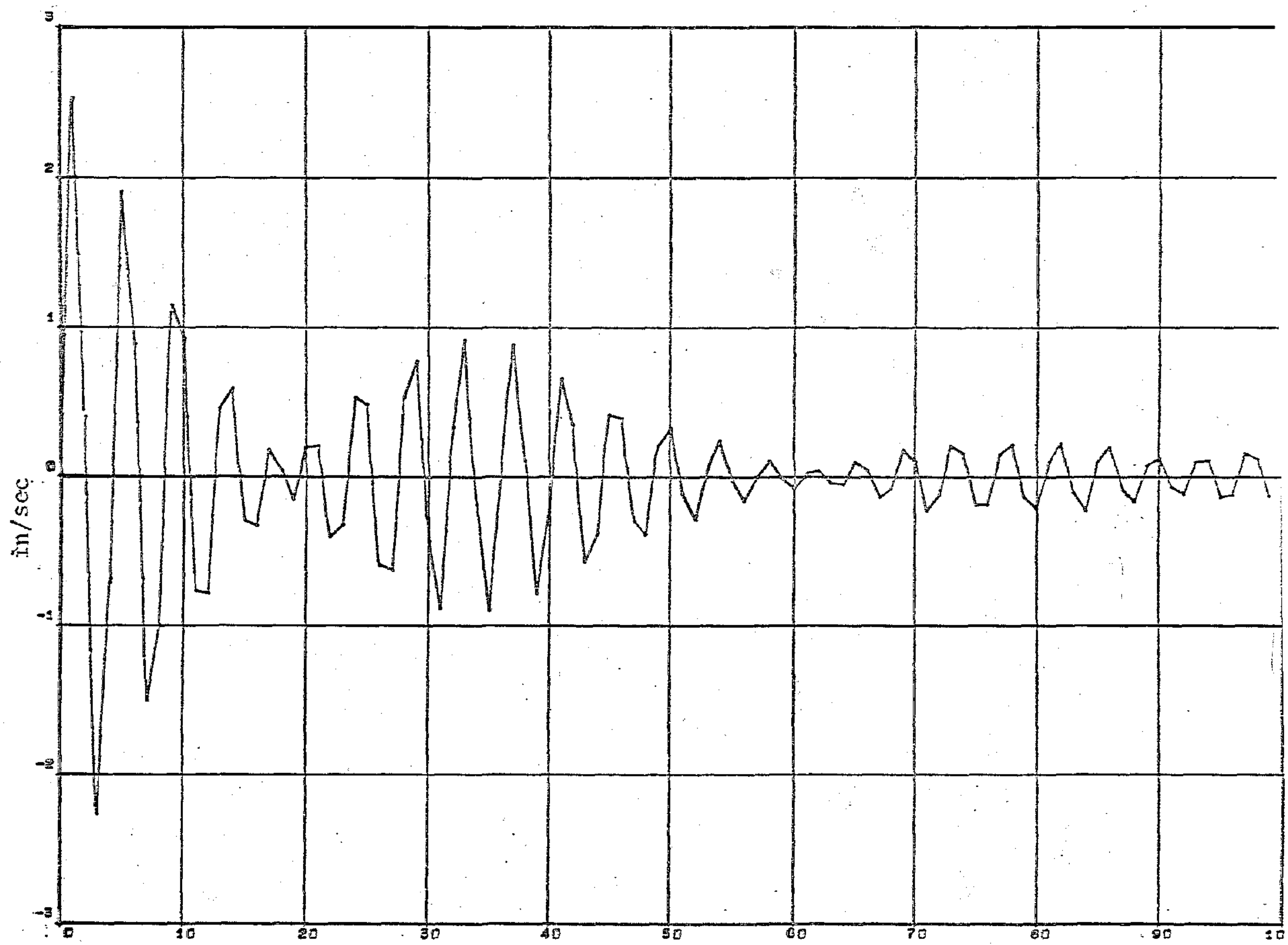


Figure 9 Sample System Signature Used for Curve-Fitting

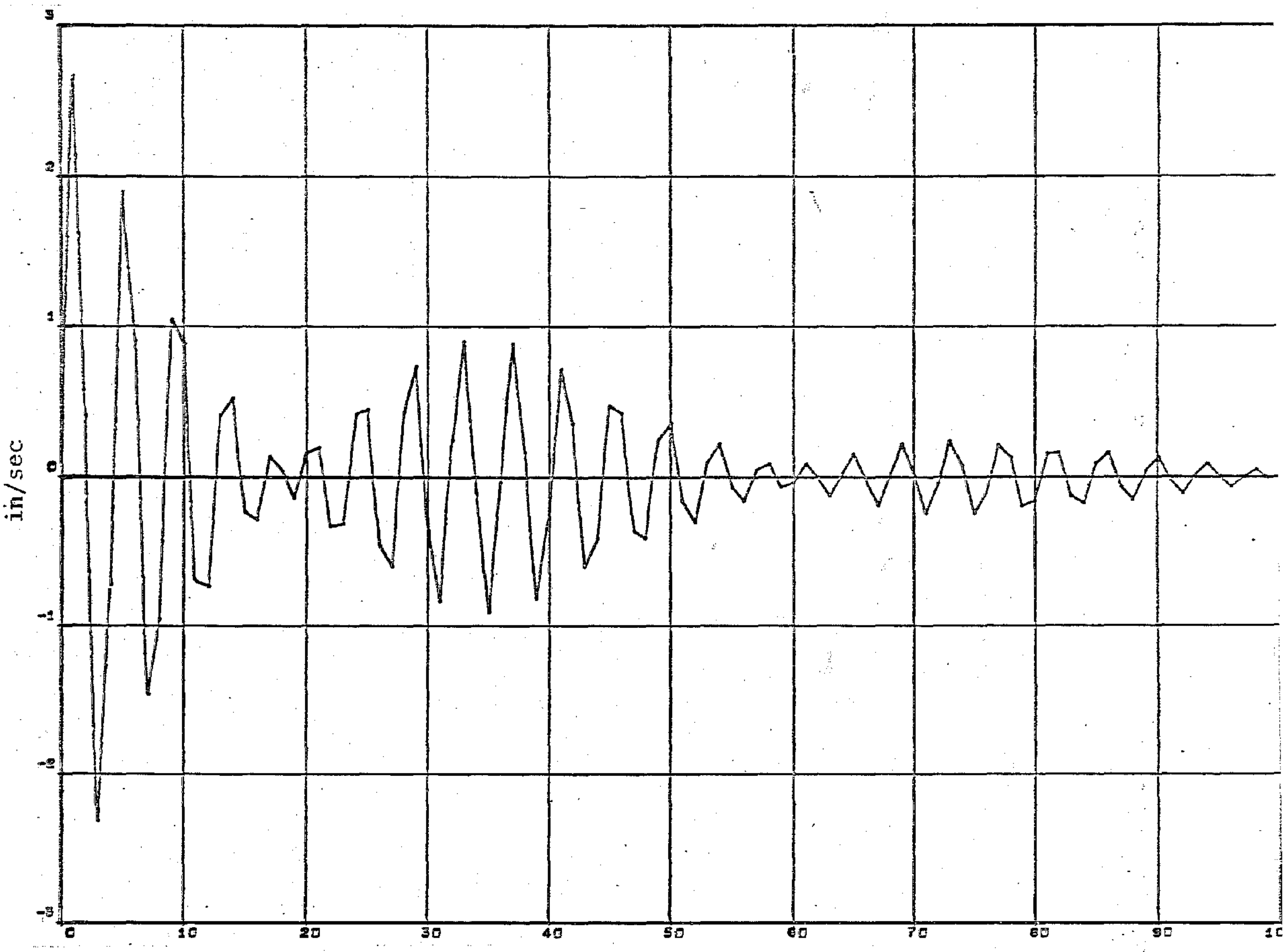


Figure 10 Least-Squares Curve-Fit Result for Signature of Figure 9

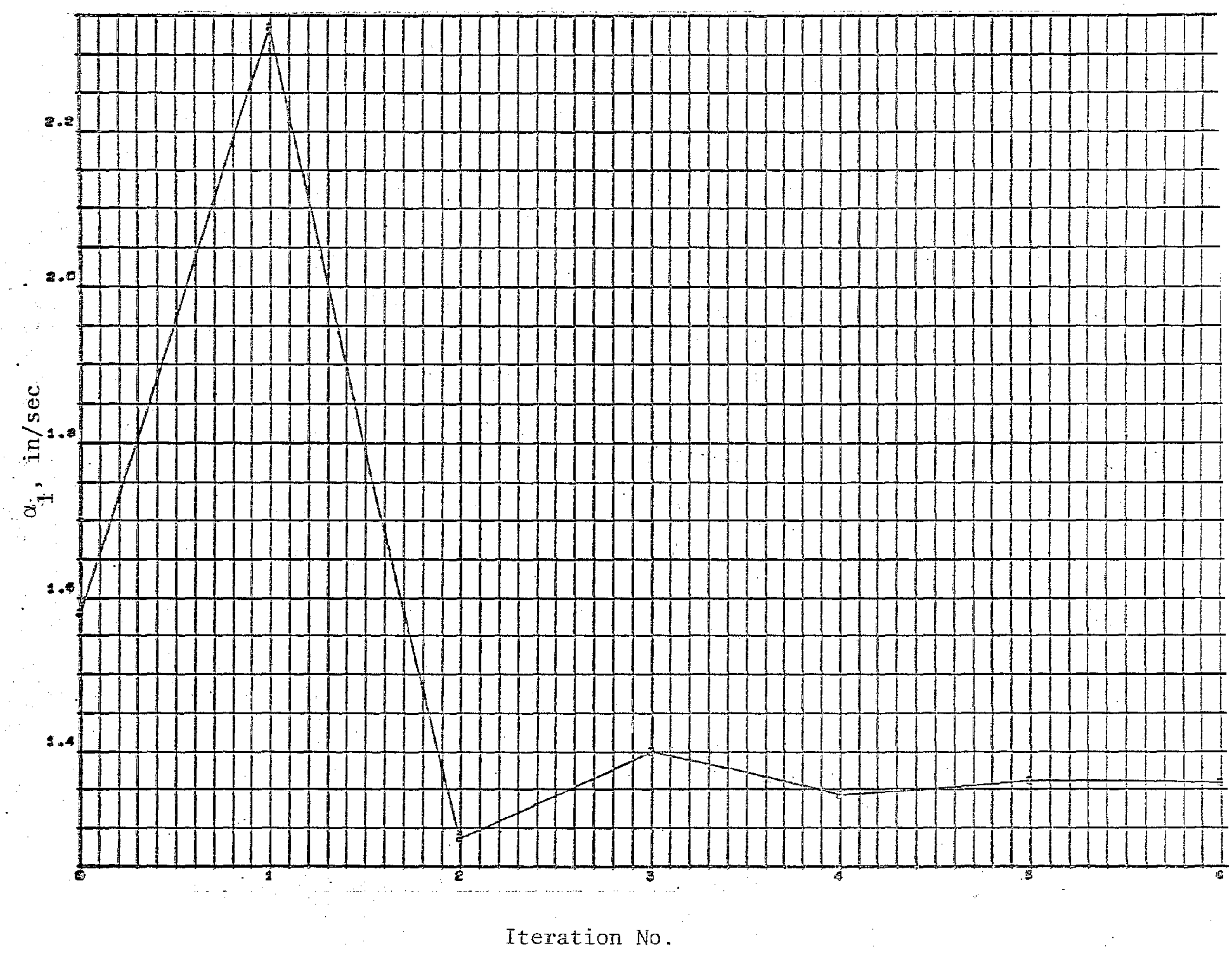


Figure 11-a Convergence Path of α_1

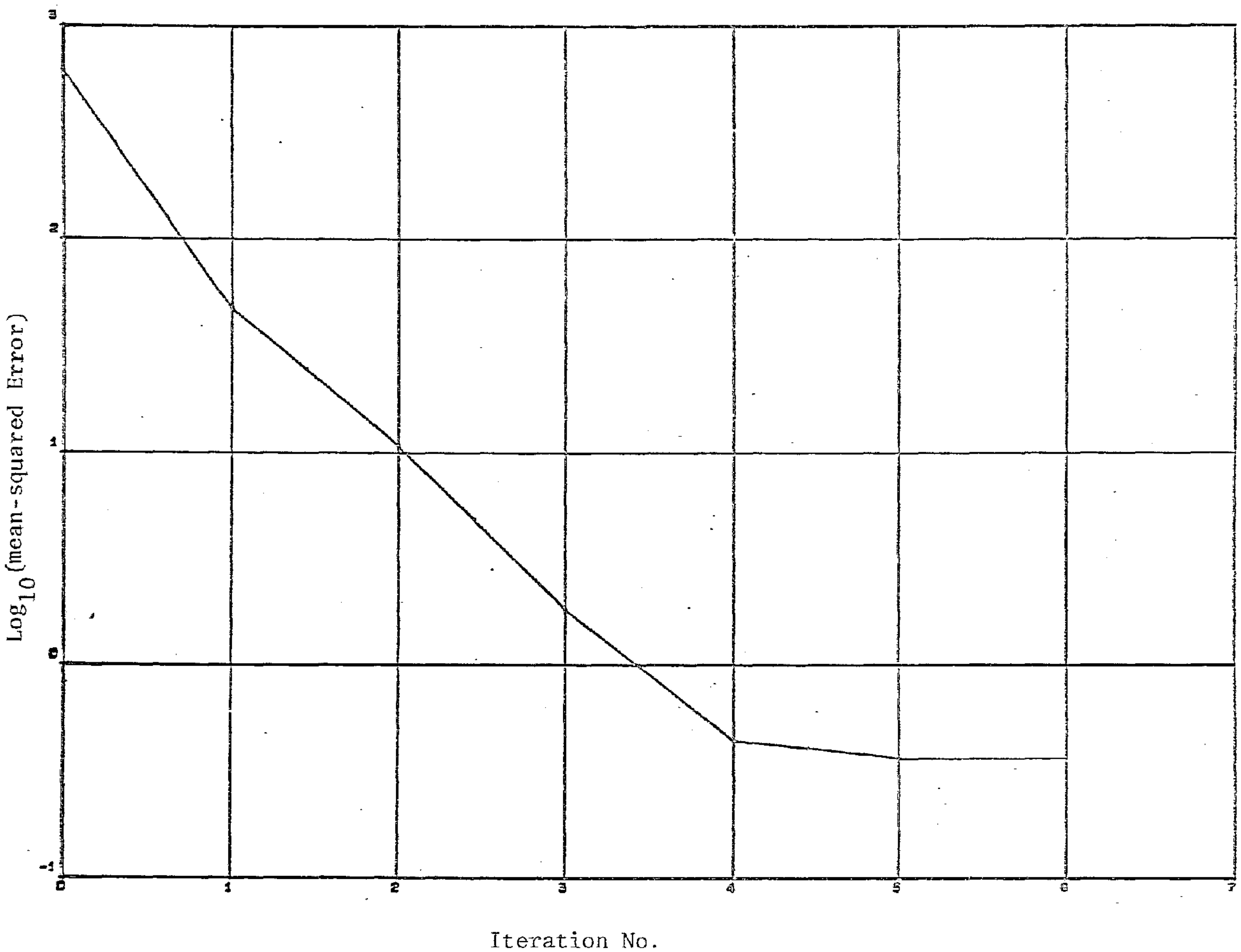


Figure 11-b Convergence Path of Mean-Squared Error

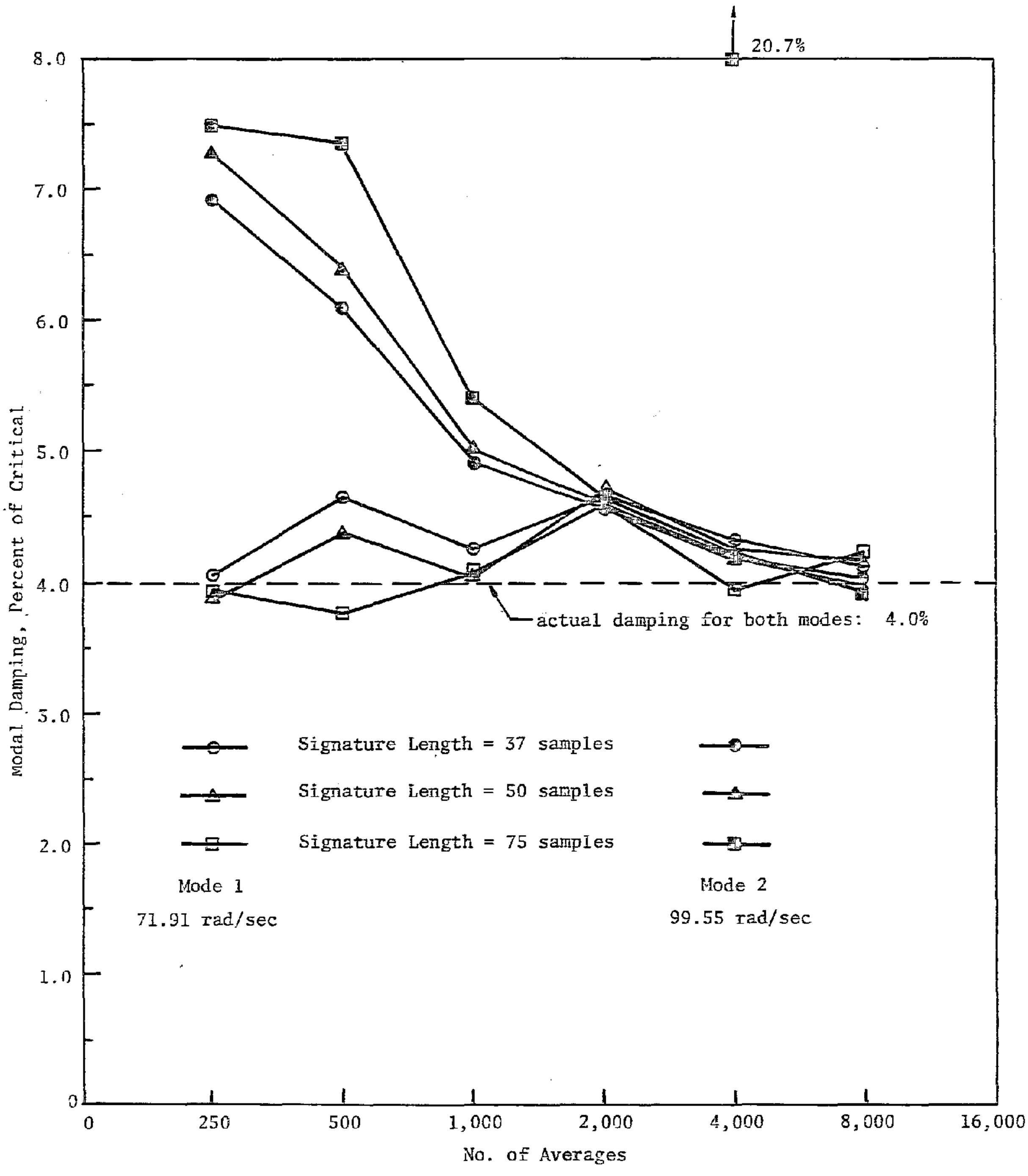


Figure 12-a Accuracy of Damping Determination vs. No. of Averages (Signature Lengths of 37, 50 and 75 samples)

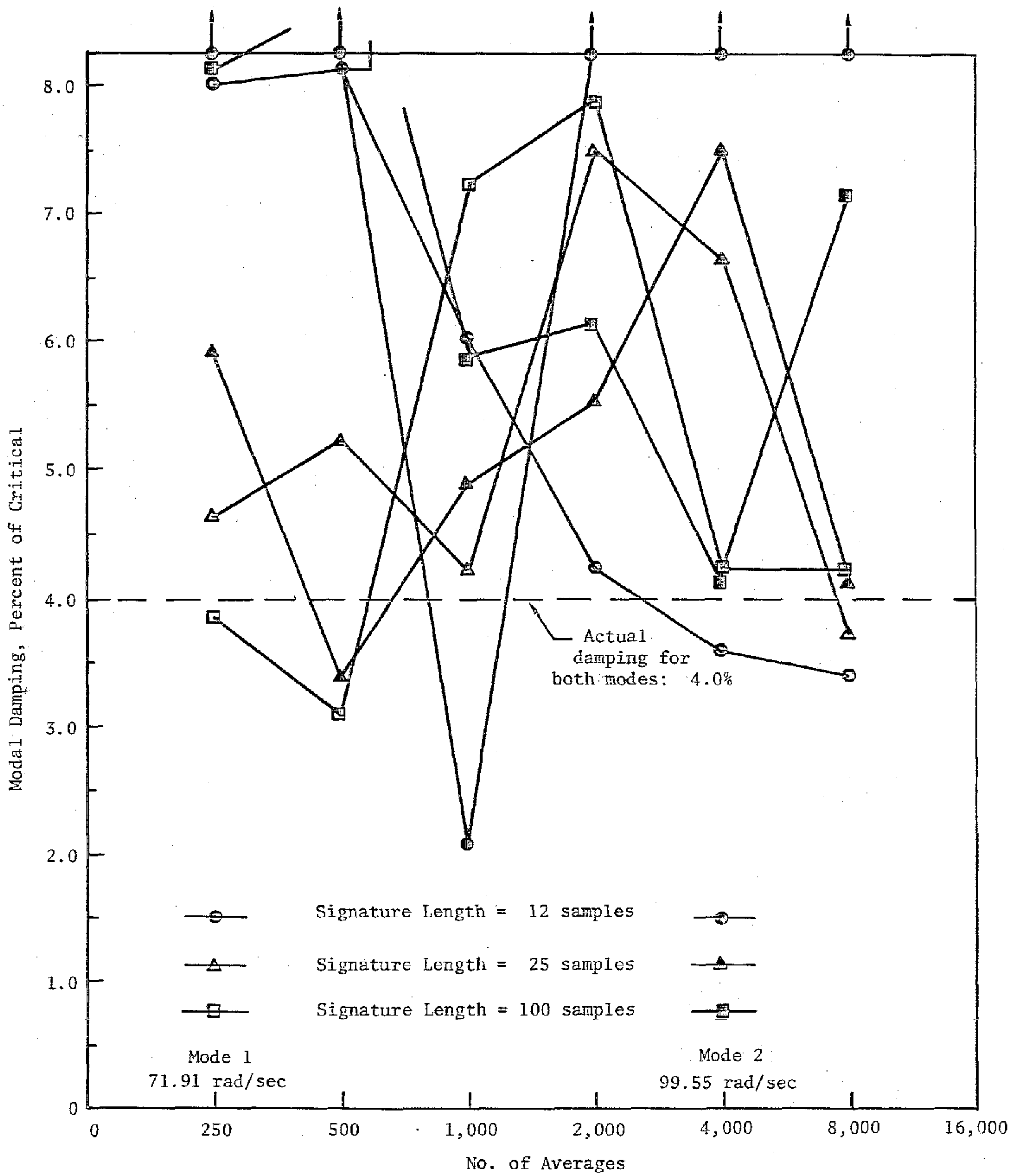


Figure 12-b Accuracy of Damping Determination vs. No. of Averages (Signature Lengths of 12, 25 and 100 samples)

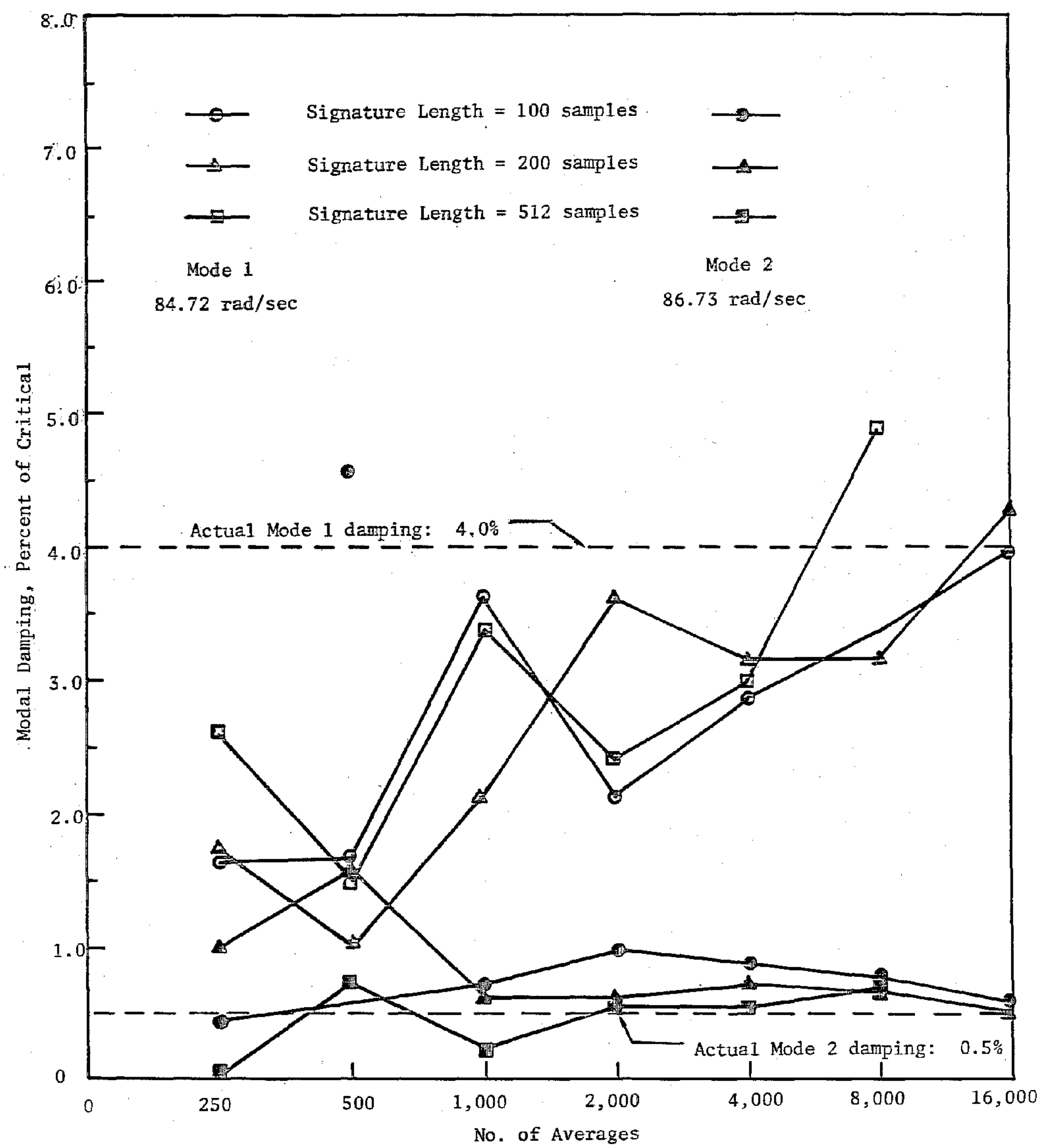


Figure 13 Accuracy of Damping Determination vs. No. of Averages
(Closely Spaced Modes)

Appendix A

PROGRAM RADCUF
USER DOCUMENTATION

1 SUMMARY

Program RADCUF synthesizes response data of an aeroelastic structure, generates Randomdec Signatures and determines the system dynamic characteristics (frequency and damping for each of the two modes of the system) via curve-fit procedures. The program is used to conduct parametric studies, and to verify the practicality of both Random Decrement and curve-fit procedures via numerical experiments.

The program is written in Fortran IV and is compatible with the UNIVAC 1108 Exec 8 version compiler. The overlay structure of this program as implemented in the UNIVAC computer is shown in Figure A-1.

Data-flow during execution can be summarized as follows: The main program, DRVR, resides in the primary link and directs the logical flow of four subprograms of the secondary link. The first subprogram called by DRVR is IDENT which provides the identification (name of the program, bin number, run ID, job number, date and time) of the job on output plots. The second subprogram called is FLTR which generates random responses of a simulated aeroelastic system and obtains Randomdec Signatures from the data. The third subprogram called is CURVFT which curve-fits the Signature generated by FLTR. The last subprogram called is ENDJOB which writes an END OF JOB on the last page of the output plots.

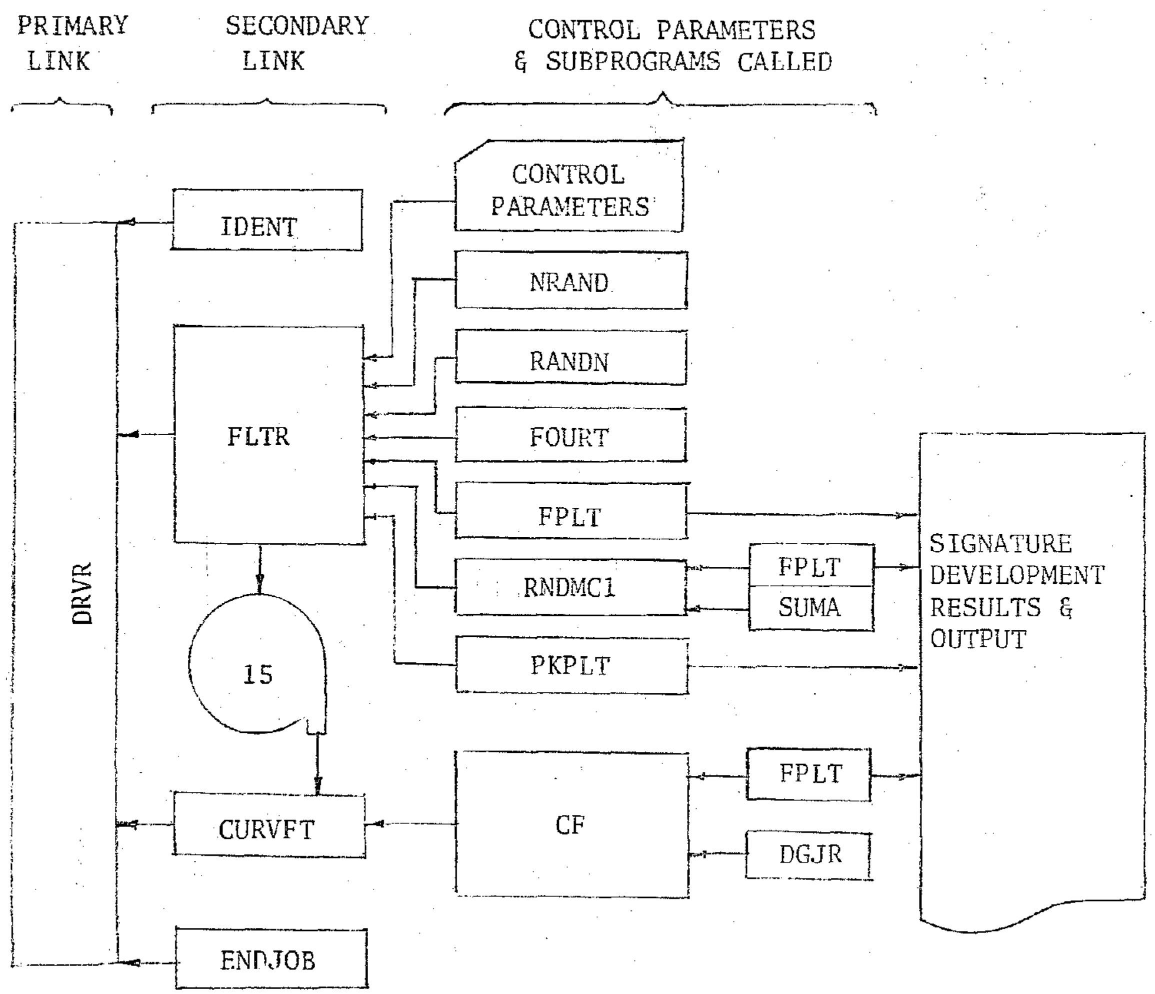


FIGURE A-1 OVERLAY STRUCTURE-PROGRAM RADCUF

2 DESCRIPTION OF PROGRAM

2-1 Primary Link

The primary link is named DRVR. The primary function of this program is procedural only, and it does not contain executable statements.

2-2 Secondary Link

Two major programs reside in the secondary link. They are FLTR and CURVFT. Their executions are mutually exclusive, and each of these programs can be executed independently from the other. Although program CURVFT normally requires input data generated by program FLTR, this data could be written on Tape Unit 15 by any other program, so long as the format is compatible.

2-2-1 FLTR

FLTR is a program that resides in the secondary link of the system. It creates an environment for the input of general control parameters, generates response data of a two-mode aeroelastic system and computes Randomdec Signatures. Basic functions of this program are:

(a) Accepts input data.

(b) Generates response data.

(c) Calls subroutine FPLT to plot intermediate or final results.

(d) Calls subroutine PKPLT to (1) determine and plot the peaks of Randomdec Signature, (2) determine initial estimates for subsequent curve-fitting and (3) establish initial conditions for subsequent response data.

(e) Calls subprogram RNDMC1 to generate Randomdec Signatures.

(f) Writes final Randomdec Signatures, initial parameter estimates, and all system and Random Decrement parameters on Tape Unit 15.

2-2-2 CURVFT

CURVFT is a curve-fitting algorithm that operates on Randomdec Signatures created by FLTR. The following is the procedure:

(a) Reads Tape Unit 15 for Randomdec Signatures and all other data written by FLTR.

(b) Sets up an environment to curve-fit the Randomdec Signature with a four-parameter expression corresponding to a one-mode approximation of the Signature.

(c) Calls subroutine CF which accomplishes the actual curve-fitting procedure.

(d) Calls subroutine FPLT to plot the Randomdec Signature being curve-fitted, the analytical expression after convergence of the procedure and convergence paths of all parameters, and the error function. All problem oriented parameters are tabulated by FPLT also.

(e) Repeats steps (b), (c), and (d) above for an eight-parameter curve-fit.

(f) Calls subroutine CF to provide the final output of the program under simple English text headings.

3 PROGRAM INPUT

Control parameters input to program RADCUF are made via cards utilizing Logical Unit 5 in the UNIVAC 1108 system. Four cards are used to define one typical case. The following logic is designed in the program for multi-case execution.

In the first execution, the program will read the first set of four data cards and execute until the generation of the Randomdec Signature for this case is completed. Then the flow of execution will be directed to read another card corresponding to Card Form 4 or to find an EOF (End of File). If a Card Form 4 is encountered, the program will execute utilizing the new parameters given by this card and previous input parameters given by Card Forms 1, 2 and 3. If an EOF is encountered, the program then will expect to find either another EOF or a completely new set of four data cards. If the second EOF is encountered, the program then proceeds to curve-fit the generated data.

The following is the card stream for typical multi-case type execution:

CARD FORM 1
CARD FORM 2
CARD FORM 3
CARD FORM 4
CARD FORM 4
CARD FORM 4
CARD FORM 4
EOF
CARD FORM 1
CARD FORM 2
CARD FORM 3
CARD FORM 4
EOF
EOF

It should be noted that Randomdec Signatures for all cases to be investigated are recorded on Tape Unit 15 before curve-fitting begins.

Input Card Form 1

Parameters: DF, MU(1), MU(2), QQ(1), QQ(2), P(1), P(2), A(1), A(2), LD(1), LW(1), LD(2), LW(2)

Format: 9F6.0, 4I6

Input Card Form 2

Parameters: KREF, NREF, NPRT, METHR, NRR, METHS, NRS, NUT, STN, RQ, AV(1), AV(2), SD(1), SD(2)

Format: F8.0, 2I4, 2(I2, I3), 5X, I1, 6F8.0

Input Card Form 3

Parameters: NA(1), NA(2), NB, NC, ISRS, IERS, ISC, IEC, ISS, IES, ISF, IEF, ISP, IEP, NSF, ISYM, NPF

Format: 4F7.1, 13I4

Input Card Form 4

Parameters: NU(1), NU(2), Z(1), Z(2), R(1,1), R(1,2), R(2,1), R(2,2) C(1), C(2), NP(1), NP(2), TH(1), TH(2)

Format: 2F4.0, 12F6.0

All input parameters are defined in Table A-1.

TABLE A-1 (Part 1)

TEXT SYMBOL	FORTRAN NAME	DEFINITION	UNIT	RANGE
δ		sampling period	sec	$2\pi/(4096(DF)$
Δf		frequency resolution of response simulation process	Hertz	
$2\pi(\Delta f)$	DF	frequency resolution of response simulation process	radians/sec	
m_1	MU(1)	generalized mass of Mode 1 (including aerodynamic effects)	mass	
m_2	MU(2)	generalized mass of Mode 2 (including aerodynamic effects)	mass	
$q_1(o)$	QQ(1)	initial displacement of Mode 1	length	
$q_2(o)$	QQ(2)	initial displacement of Mode 2	length	
$\dot{q}_1(o)$	P(1)	initial displacement of Mode 1	length/sec	
$\dot{q}_2(o)$	P(2)	initial displacement of Mode 2	length/sec	
$a_1/(2\pi\Delta f)^2$	A(1)	magnitude of aerodynamic lag force per unit displacement in Mode 1	mass/length	
$a_2/(2\pi\Delta f)^2$	A(2)	magnitude of aerodynamic lag force per unit displacement in Mode 2	mass/length	
	LD(1)	rise-point no. of Box Car part of generalized force in Mode 1 rise point = $(LD(1))(\delta)$ seconds after initiation of simulation	δ	1-2047
	LW(1)	fall-point no. of Box Car part of generalized force in Mode 1	δ	2-2048
	LD(2)	rise-point no. of Box Car part of generalized force in Mode 2	δ	1-2047
	LW(2)	fall-point no. of Box Car part of generalized force in Mode 2	δ	2-2048
	KREF	(Random Decrement trigger level)/(rms response)	non-dimensional	
	NREF	number of points in each segment of simulated response data to be used for Random Drecrement processing	δ	2-1536
	NPRT	number of points required in Random Signature	δ	1-512

TABLE A-1 (Part 2)

TEXT SYMBOL	FORTRAN NAME	DEFINITION	UNIT	RANGE
	METHR	trigger method (METHR > 0) = level trigger method; (METHR = 0) = zero-crossing-with-positive slope trigger method; (METHR < 0) = modified zero-crossing trigger method	non-dimensional	
	NRR	number of response points to be simulated/2048	non-dimensional	
	METHS	not used in final version of program		
	NRS	not used in final version of program	non-dimensional	
	STN	signal-to-noise ratio (maximum response)/(std. dev. of noise)	non-dimensional	
	RQ	external key for random number generator	non-dimensional	
	AV(1)	mean value of random number sequence used to generate random component of generalized forces	non-dimensional	
	AV(2)	mean value of random number sequence used to generate random component of generalized forces	non-dimensional	
	SD(1)	standard deviation of random number sequence used to generate random component of generalized forces	non-dimensional	
	SD(2)	standard deviation of random number sequence used to generate random component of generalized forces	non-dimensional	
$1/(2\pi T_1(\Delta F))$	NA(1)	cut-off frequency of generalized force in Mode 1 corresponding to aerodynamic lag forces	$2\pi(\Delta f)$	1-4096
$1/(2\pi T_2(\Delta f))$	NA(2)	cut-off frequency of generalized force in Mode 2 corresponding to aerodynamic lag force	$2\pi(\Delta f)$	1-4096
$1/(2\pi T_b(\Delta f))$	NB	first-order low-pass filter cut-off frequency	$2\pi(\Delta f)$	1-4096
$1/(2\pi T_c(\Delta f))$	NC	first-order low-pass filter cut-off frequency	$2\pi(\Delta f)$	1-4096
	ISRS	starting point number of transform of forcing function to be plotted	$2\pi(\Delta f)$	≤IERS
	IERS	ending point number of transform of forcing function to be plotted	$2\pi(\Delta f)$	≤NPRT

TABLE A-1 (Part 3)

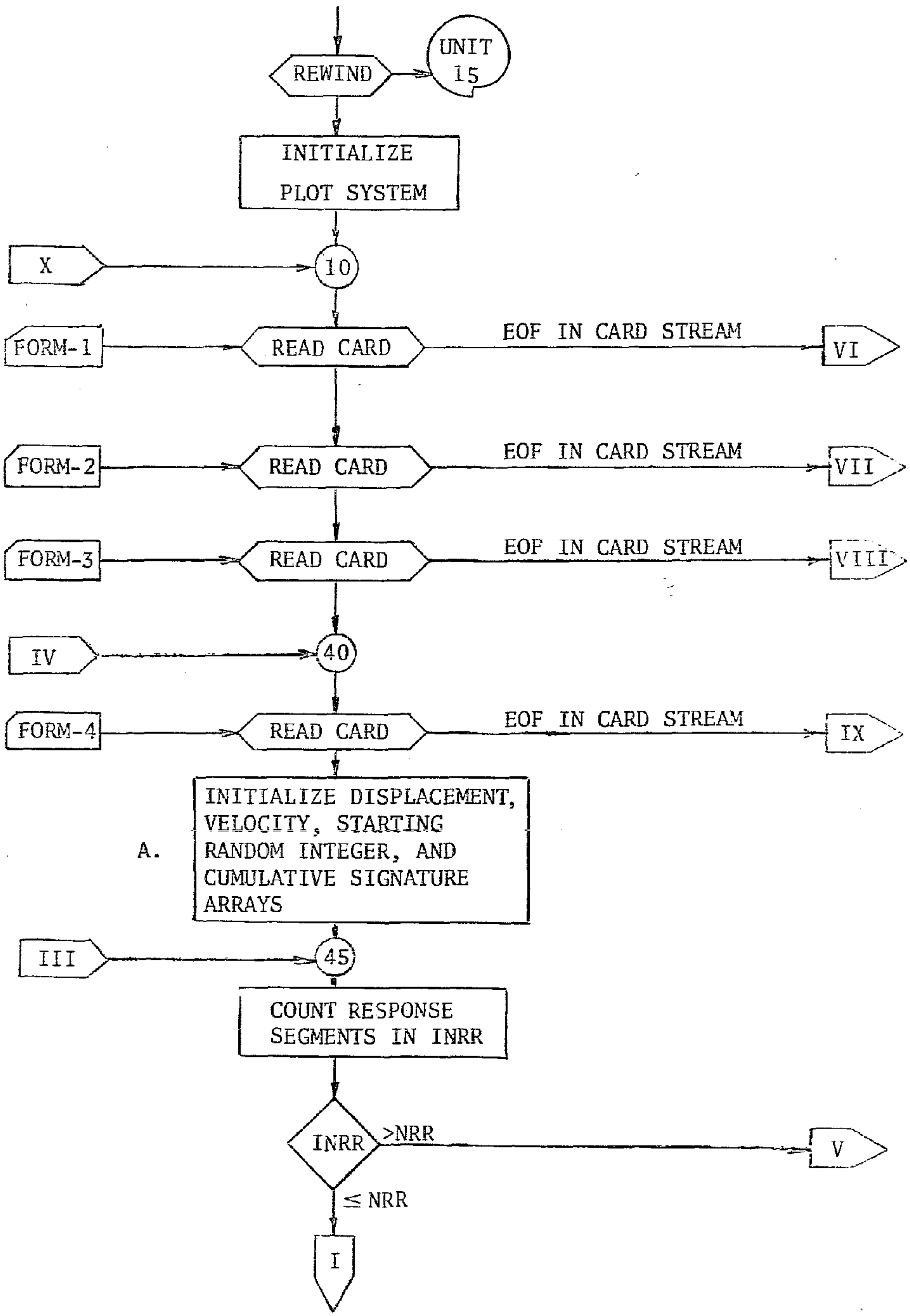
TEXT SYMBOL	FORTRAN NAME	DEFINITION	UNIT	RANGE
	ISC	starting time (point no.) of response data to be plotted	δ	$\leq$IEC
	IEC	ending time (point no.) of response data to be plotted	δ	$\leq$NPRT
	ISS	starting time (point no.) of Signatures to be plotted	δ	$\leq$IES
	IES	ending time (point no.) of Signatures to be plotted	δ	$\leq$NPRT
	IEF	if set >0, interim Signatures will be plotted	(plot controller)	0 or positive
	ISF	not used in final version of the program		
	ISP	not used in final version of the program		
	IEP	not used in final version of the program		
	NSF	number of subframes to be plotted per page		1,2 or 3
	ISYM	plot controller, set to zero for print plots, to 35 for SC4020 plots	(plot controller)	0 or 35
	NPF	number of horizontal divisions per subframe	1 + function plotted	
$\omega_1/(2\pi(\Delta f))$	NU(1)	natural frequency of Mode 1	$2\pi(\Delta f)$	1-2048
$\omega_2/(2\pi(\Delta f))$	NU(2)	natural frequency of Mode 2	$2\pi(\Delta f)$	1-2048
ζ_1	Z(1)	damping factor of Mode 1 (fraction of critical modal damping)	non-dimensional	
ζ_2	Z(2)	damping factor of Mode 2 (fraction of critical modal damping)	non-dimensional	
$R_{11}m_1$	R(1,1)	magnitude and mixing coefficients of random components of generalized forces	mass-length/sec^2	
$R_{12}m_1$	R(1,2)	magnitude and mixing coefficients of random components of generalized forces	mass-length/sec^2	

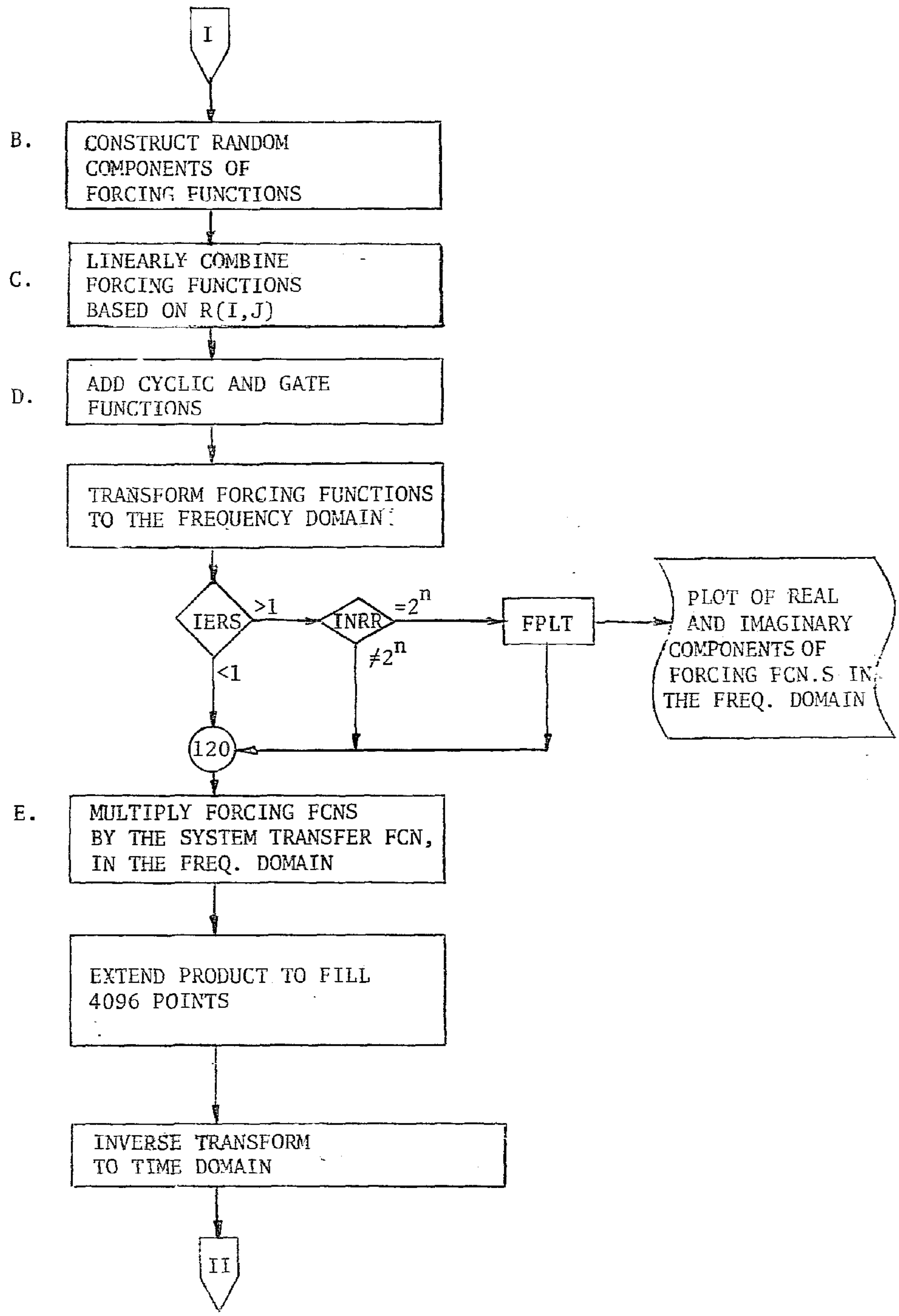
TABLE A-1 (Part 4)

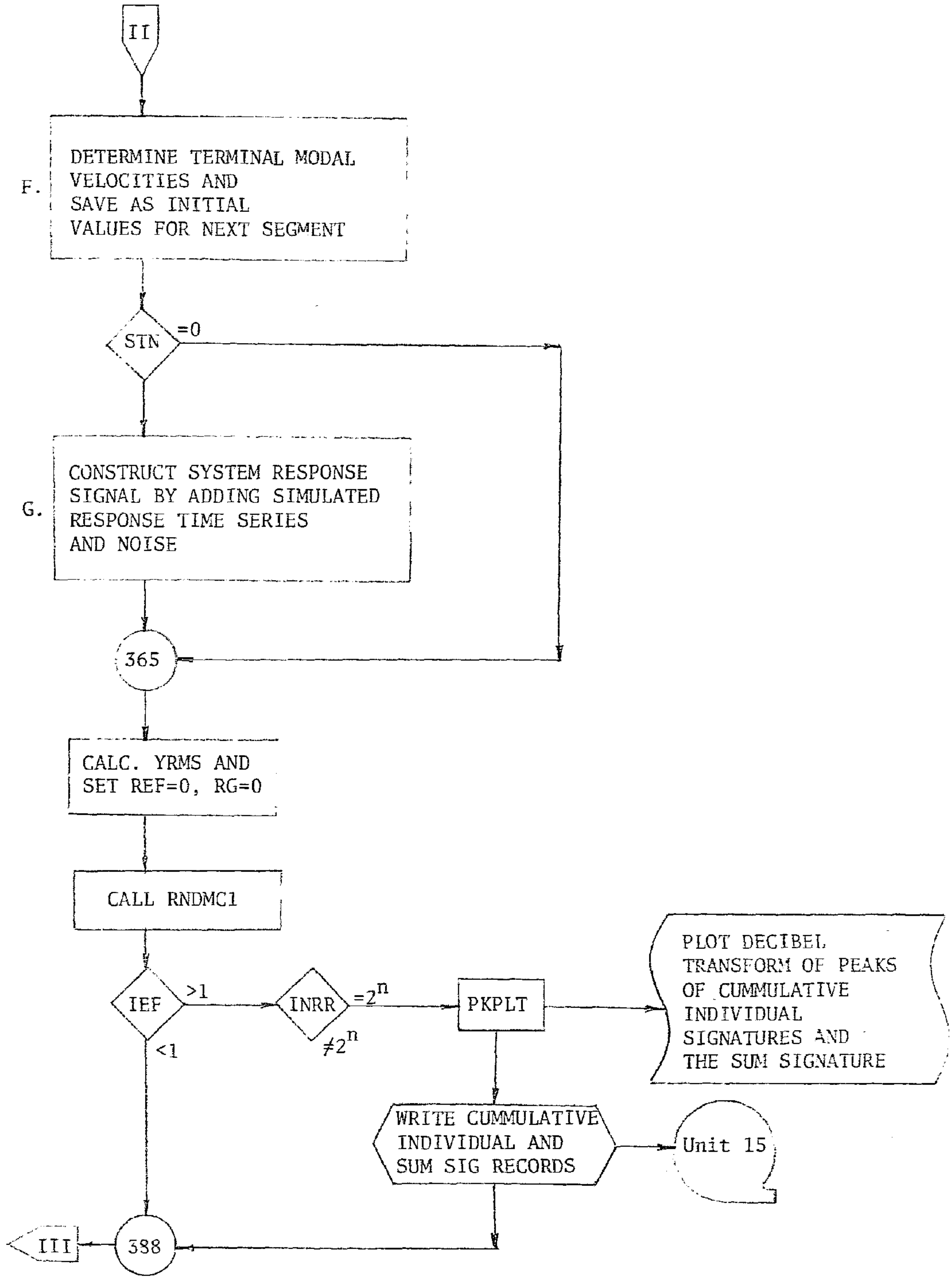
TEXT SYMBOL	FORTRAN NAME	DEFINITION	UNIT	RANGE
$R_{21}m_2$	R(2,1)	magnitude and mixing coefficients of random components of generalized forces	mass-length/sec^2	
$R_{22}m_2$	R(2,2)	magnitude and mixing coefficients of random components of generalized forces	mass-length/sec^2	
	C(1)	amplitude of sinusoidal component of generalized force in Mode 1	mass-length/sec^2	
	C(2)	amplitude of sinusoidal component of generalized force in Mode 2	mass-length/sec^2	
	NP(1)	frequency of sinusoidal component of generalized force in Mode 1	mass-length/sec^2	
	NP(2)	frequency of sinusoidal component of generalized force in Mode 2	mass-length/sec^2	
	TH(1)	phase displacement of sinusoidal component of generalized force in Mode 1	radians	
	TH(2)	phase displacement of sinusoidal component of generalized force in Mode 2	radians	
	NUT	Units to be written on Signature plots (NUT = 1) = in/sec; (NUT = 2) = cm/sec; (NUT = 3) = ft/sec; (NUT = 4) = m/sec; (NUT = 5) = Rad/sec		

4-1 FLTR Procedure

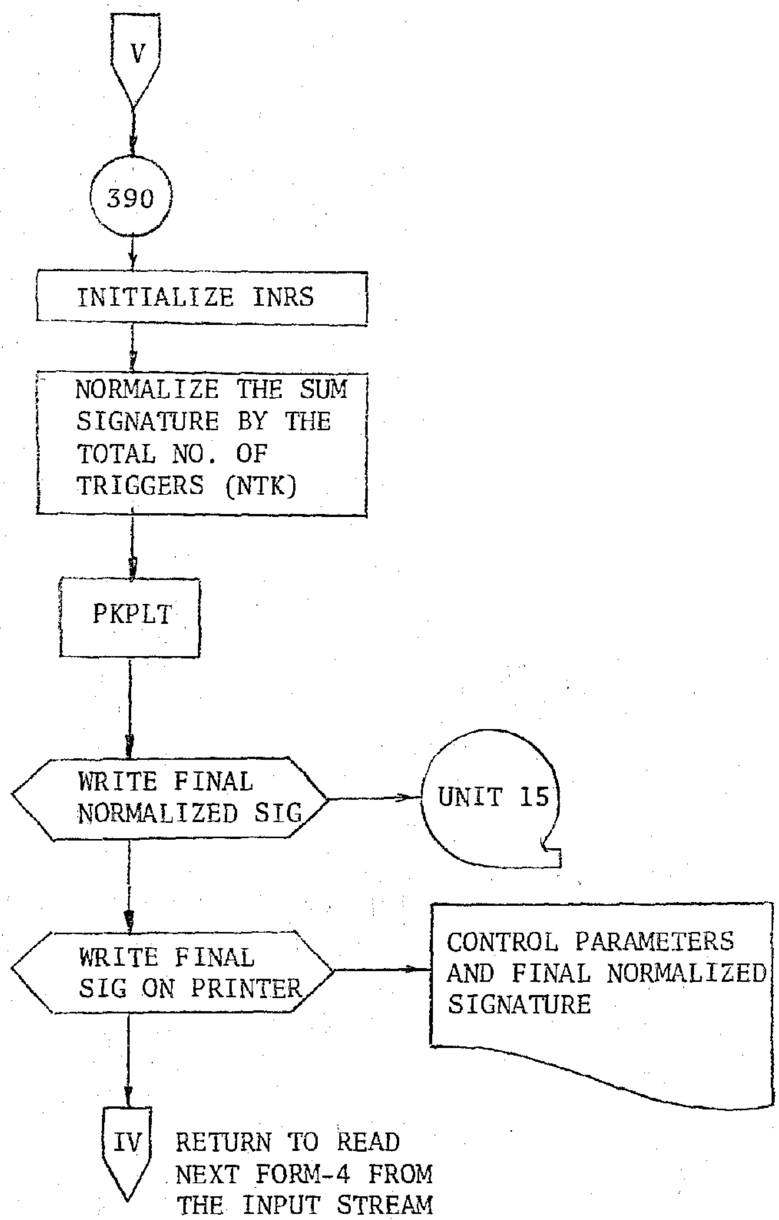
The diagram below shows the order of execution in the response simulation program, FLTR. Contents in the lettered blocks in the diagram are described in the correspondingly lettered descriptive paragraphs following these flow charts.

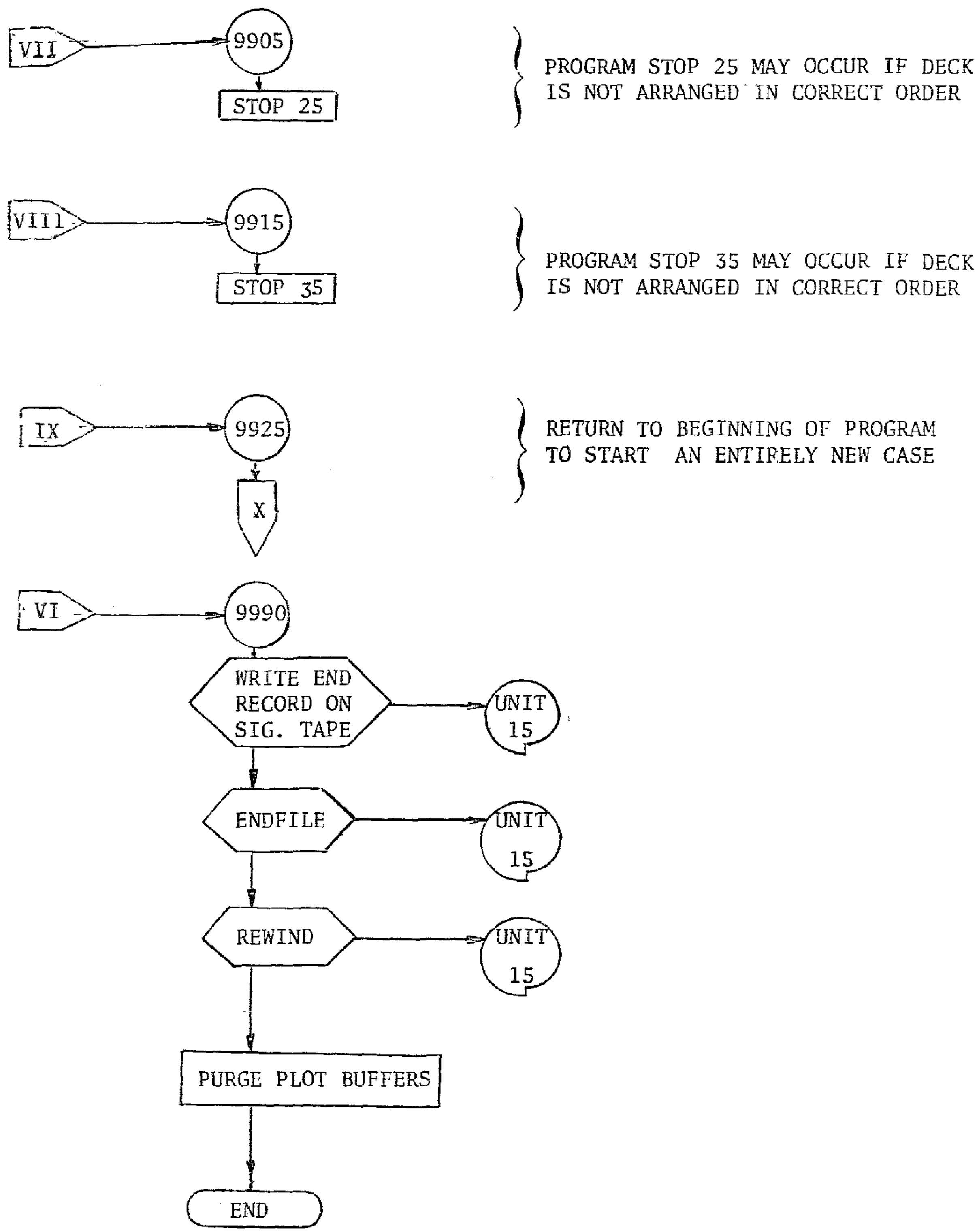



I
B.
CONSTRUCT RANDOM COMPONENTS OF FORCING FUNCTIONS
C.
LINEARLY COMBINE FORCING FUNCTIONS BASED ON R(I,J)
D.
ADD CYCLIC AND GATE FUNCTIONS
TRANSFORM FORCING FUNCTIONS TO THE FREQUENCY DOMAIN
IERS
>1
<1
INRR
=2^n
≠2^n
FPLT
PLOT OF REAL AND IMAGINARY COMPONENTS OF FORCING FCN.S IN THE FREQ. DOMAIN
120
E.
MULTIPLY FORCING FCNS BY THE SYSTEM TRANSFER FCN, IN THE FREQ. DOMAIN
EXTEND PRODUCT TO FILL 4096 POINTS
INVERSE TRANSFORM TO TIME DOMAIN
II


II
F.
DETERMINE TERMINAL MODAL VELOCITIES AND SAVE AS INITIAL VALUES FOR NEXT SEGMENT
STN
=0
G.
CONSTRUCT SYSTEM RESPONSE SIGNAL BY ADDING SIMULATED RESPONSE TIME SERIES AND NOISE
365
CALC. YRMS AND SET REF=0, RG=0
CALL RNDMC1
IEF
>1
<1
INRR
=2^n
≠2^n
PKPLT
PLOT DECIBEL TRANSFORM OF PEAKS OF CUMMULATIVE INDIVIDUAL SIGNATURES AND THE SUM SIGNATURE
WRITE CUMMULATIVE INDIVIDUAL AND SUM SIG RECORDS
Unit 15
388
III

C-2


V
390
INITIALIZE INRS
NORMALIZE THE SUM SIGNATURE BY THE TOTAL NO. OF TRIGGERS (NTK)
PKPLT
WRITE FINAL NORMALIZED SIG
UNIT 15
WRITE FINAL SIG ON PRINTER
CONTROL PARAMETERS AND FINAL NORMALIZED SIGNATURE
IV
RETURN TO READ NEXT FORM-4 FROM THE INPUT STREAM


VII
9905
STOP 25
PROGRAM STOP 25 MAY OCCUR IF DECK IS NOT ARRANGED IN CORRECT ORDER
VIII
9915
STOP 35
PROGRAM STOP 35 MAY OCCUR IF DECK IS NOT ARRANGED IN CORRECT ORDER
IX
9925
X
RETURN TO BEGINNING OF PROGRAM TO START AN ENTIRELY NEW CASE
VI
9990
WRITE END RECORD ON SIG. TAPE
UNIT 15
ENDFILE
UNIT 15
REWIND
UNIT 15
PURGE PLOT BUFFERS
END

Block A. JRQ is an internal variable, which contains the key for the generation of the next random integer by NRAND. It is initialized to the user specified input value, RQ, each time a new case is started with respect to reading a Card Form 4.

INRR is an integer program counter which indicates the number of segments of response data which have been processed for a given case. When it exceeds the required number of segments given by the user as NRR, "End-of-Case" procedures are initiated.

The variable NTM is 0-origin integer sample point counter in the time domain. The first point of the second response segment is considered point No. NTM = 4096 and etc.

NTK is an integer to be assigned as the total number of summation of input signature.

AK, BK and CK are constants to be assigned for the generation of complex function.

Fortran arrays SRF(L), SRG(L), S(L) for L=1, 4096, accumulate Signatures (triggered responses) of Mode 1, Mode 2 and the sum of Modes 1 and 2, respectively.

The variables P(1) and P(2) are the initial values of velocities of Modes 1 and 2. respectively for the first segment of response simulation.

The variables QQ(1) and QQ(2) are the initial displacements of Modes 1 and 2, respectively, used by the program in generating the first segment of the simulated response data.

The initial velocities and displacements of subsequent segments of simulation are obtained according to explanation for Block F below. Simulation will be started from initial input values each time a new case is started after reading a Card Form 4.

Block B. The arrays (QR1(L) and QR2(L), L=1, 4096, are used for storage of pseudo-random number subsets used for the generation of forcing functions.

The Fortran function NRAND is used to generate the next pseudo-random integer from the previous one. This integer, converted to floating form and stored in the first location of the mode array, is used as a trigger to generate the pseudo-random number subset for each modal force. The means and standard deviations of these pseudo-random subsets are specified by the user as AV(1), and SD(1), and AV(2) and SD(2), respectively.

This logic assures that the same pseudo-random number subset will be used for each case with respect to Card Form 4 parameter variation.

Block C. After the construction of the random sequence component, functions are mixed. The equations of mixing are given by:

$$\left.\begin{array}{l} F(L) = R(1,1) * F(L) + R(1,2) * G(L), \\ G(L) = R(2,1) * F(L) + R(2,2) * G(L), \end{array}\right. \quad L = 1,2049,$$

where F is the forcing function for the 1st mode and G for the 2nd mode. Obviously, no mixing occurs if

R(1,2) = R(2,1) = 0.0 in the control parameters.

The real and imaginary components of the resulting functions may be plotted against frequency for each segment number which is an integer power of 2. These plots are specified values of ISRS and IERS on control Card Form 3 upon the user's request. The subset of each segment from point No. ISRS to IERS will be plotted.

Block D. A cyclic component is added to the pseudo-random subset contained in QR1(L) and QR2(L) as follows:

$$QR1(L) = QR1(L) + C(1) * COS\ [\frac{2\pi}{4096} * NP(1) * NTM + TH(1)]$$

for L=1, 2049 and similarly for QR2(L). NP(1) and NP(2) are the frequencies in cycles/rad. TH(1) and TH(2) are the associated phase displacements. These parameters are specified by the user on Card Form 4.

In addition to adding cyclic components to the forcing functions, positive gate functions can also be added. The gate function generator, called DST in the program, is a function of NTM. The first 'rise' points of the functions are given by LD(1) for Mode 1, and LD(2) for Mode 2, on Card Form 1. These point No.s refer to the value of NTM, not the segment index. The number of 'rise' points is given by LW(1) and LW(2). The height of each gate is unity.

Block E. The forcing function and initial conditions are complex-multiplied by the modal frequency response functions.

The real parts are extended symmetrically about CF(2049) or CG(2049) respectively. The imaginary parts are also extended but antisymmetrically. IM(CF(2049)) and IM(CF(2049)) are set to zero. The resulting complex functions, of length 4096 each, are inverse transformed into the time domain. The imaginary components of the resulting time series are zero-valued. The real valued time series are modal velocity data.

Block F. Initial conditions for the second and subsequent segments of simulation are determined by the procedure below.

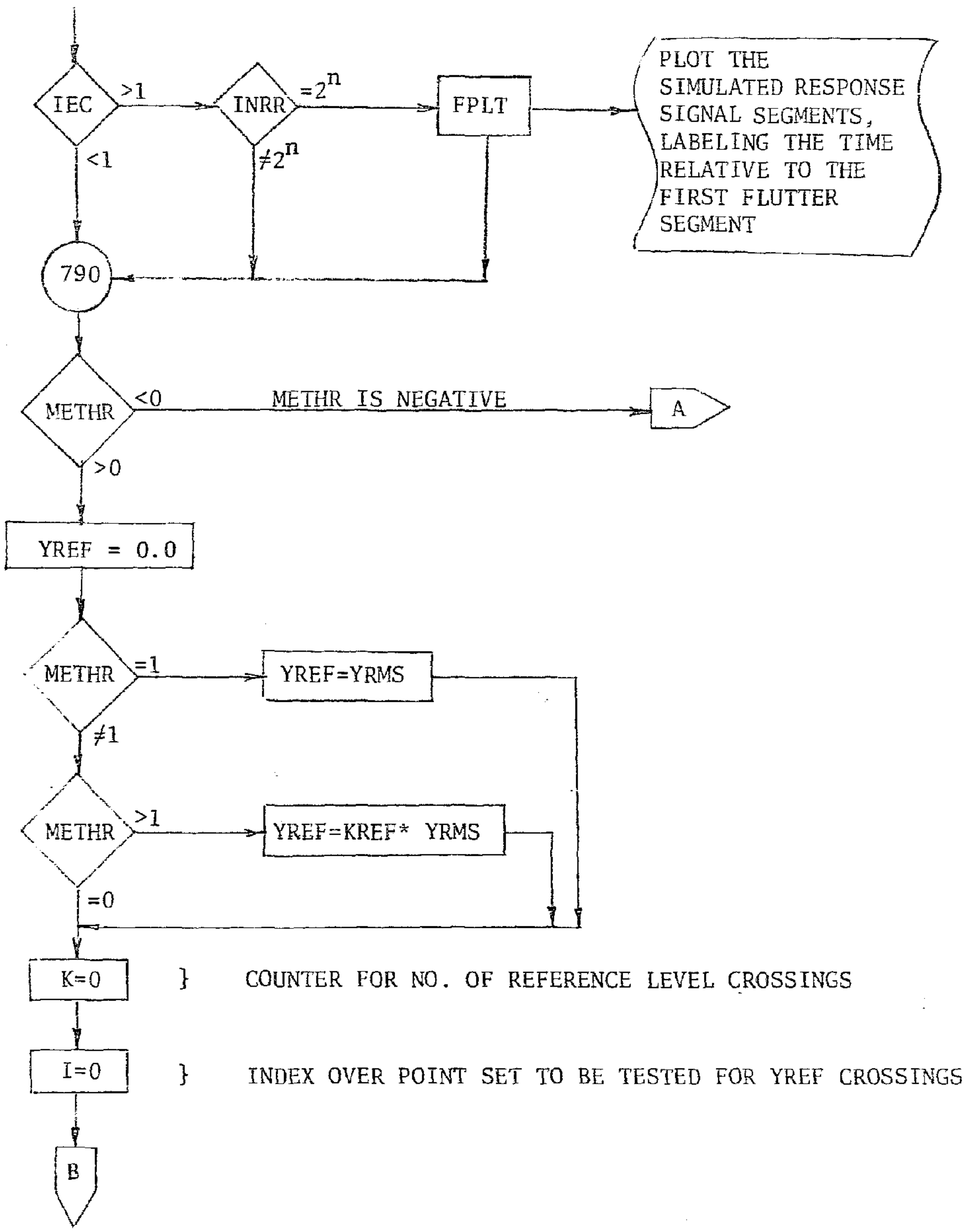
The PKPLT subroutine is called to find the final peak in the modal velocity signals of each segment. These peak velocities are the respective initial velocities of the following segment.

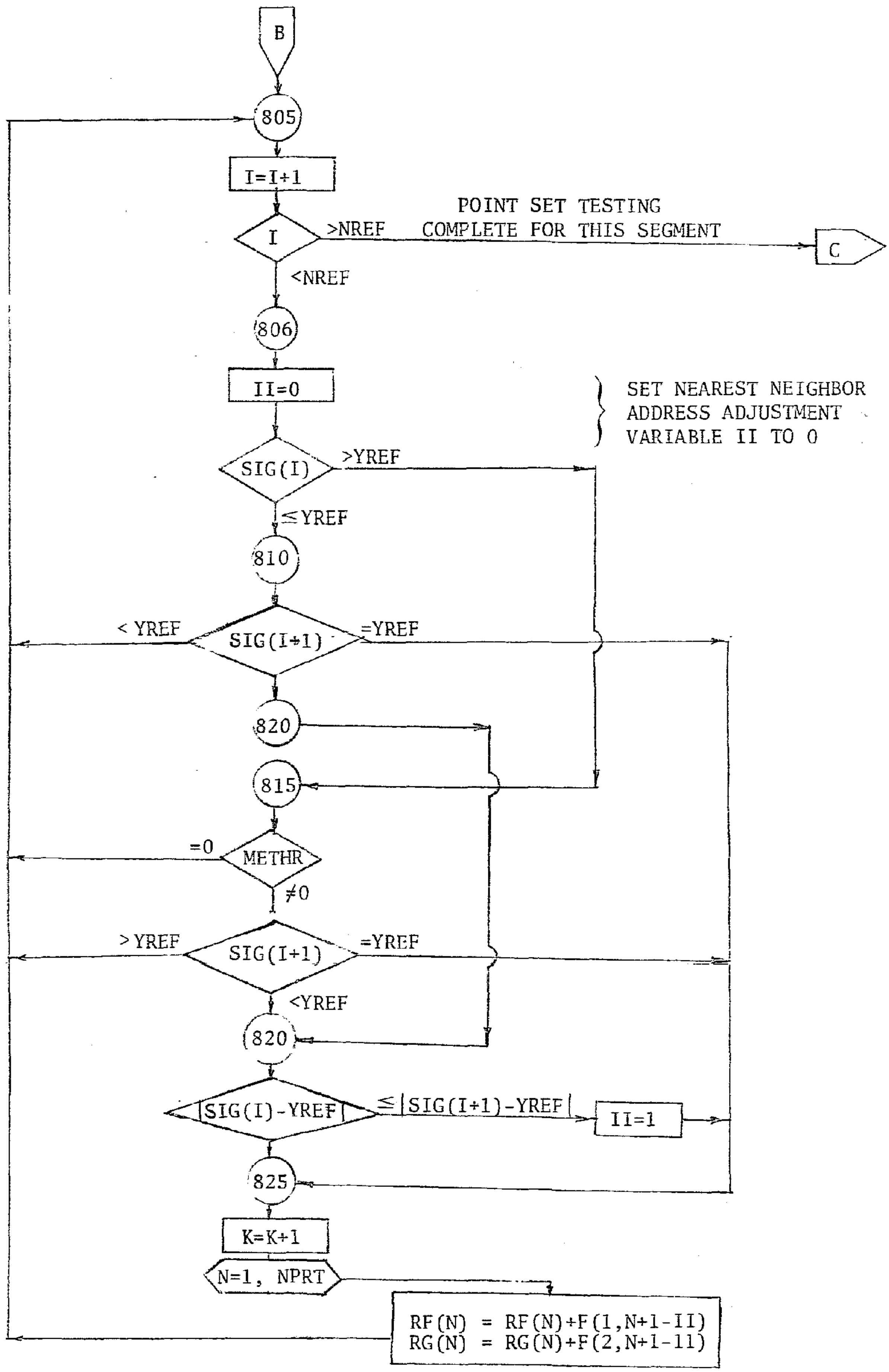
The modal displacements are zero when the modal velocities peak. The initital displacements for the second and all subsequent segments are, therefore, zero.

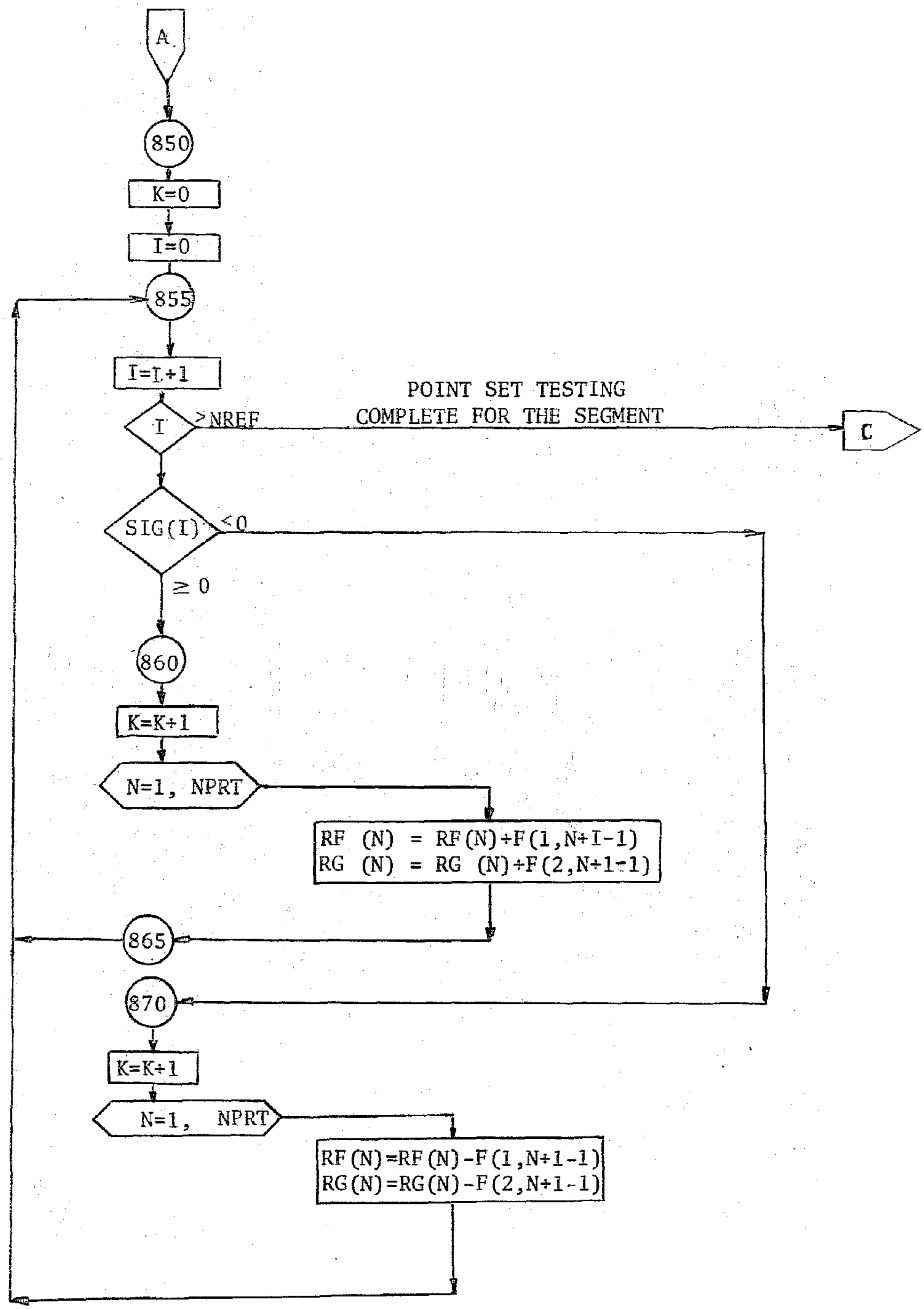
Block G. STN is the value of signal to noise ratio which is an option to include the random noise to the total system response. The noise signals can be obtained by RANDN subroutine if STN is not equal to zero. The system response is, then, simply the summation of simulated response signals and the noise. The standard deviation of the noise component is equal to the absolute maximum value of the response divided by STN which is established by input.

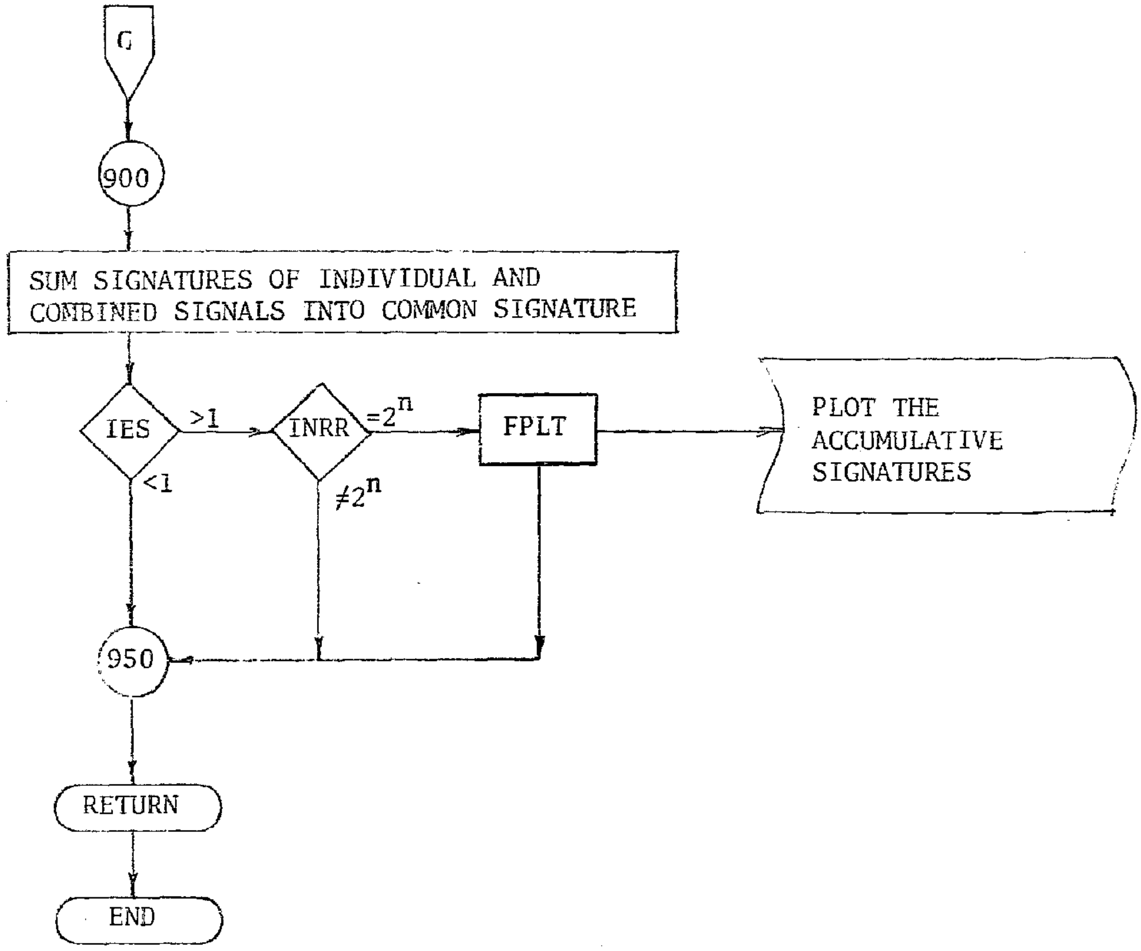
The resulting data segments are input data for the RNDMC1 algorithm (Randomdec). The Fortran array SIG(L), L=1, 4096 is the sum of the two modal response segments and is used by RNDMC1 to establish trigger points, i.e., points which become origins of the ensemble summing operation which generates the Signature. The Fortran arrays RF(L), and RG(L), L=1, 4096 are used to hold the components of the Signature which arise within the current segment of Modes 1 and 2 respectively. On completion of RNDMC1 algorithm, RF(L) is summed into SRF(L) and RG(L) is summed into SRG(L), the array S(L), L=1, 4096, is the cumulative sum of the two modal Signatures. YRMS is the RMS value of SIG(L), L=1, 4096.

The following diagrams show the logic of the RNDMC1 algorithm and the application of its related control parameters, from Card Form 2.



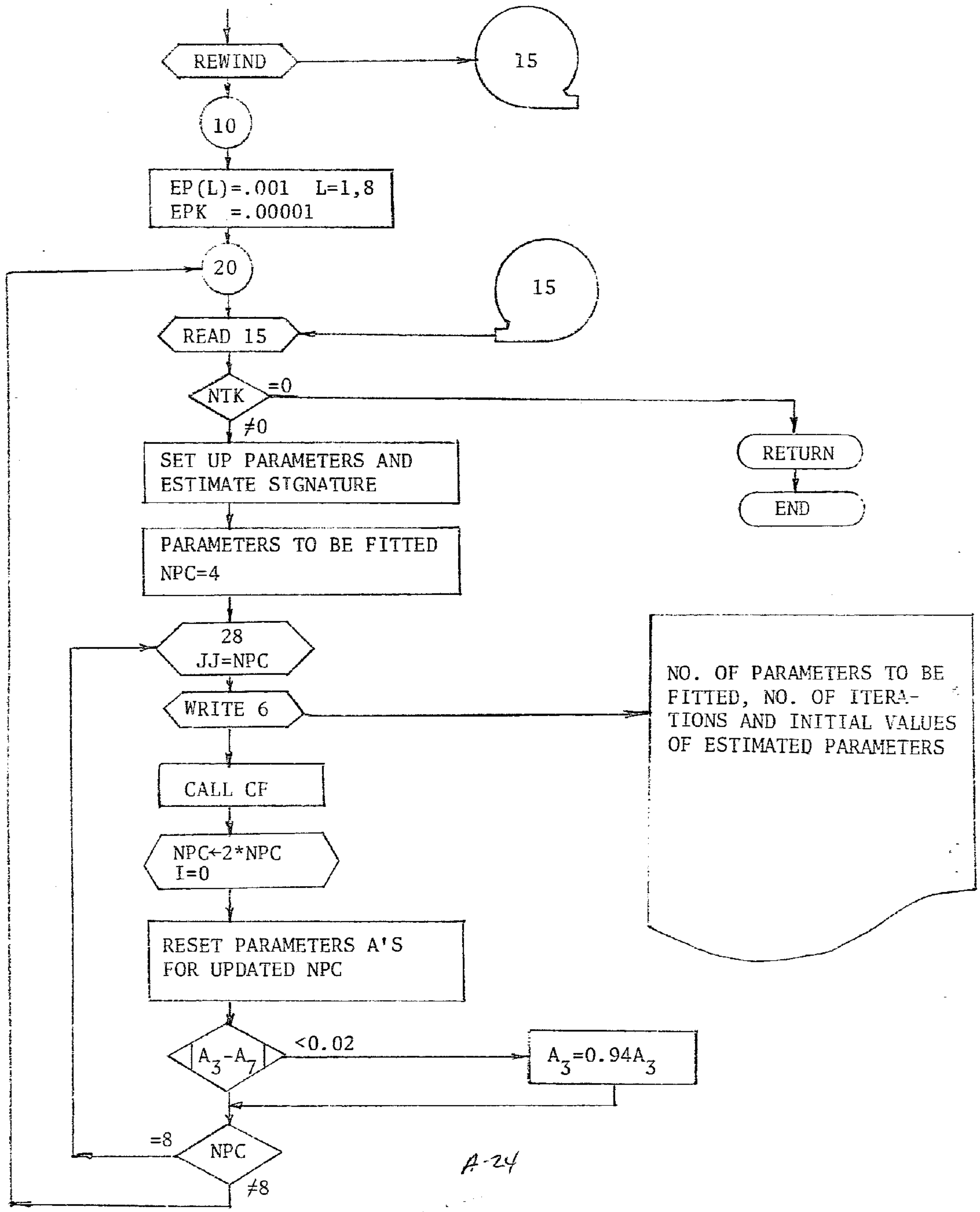

B
805
I=I+1
I
>NREF
POINT SET TESTING
COMPLETE FOR THIS SEGMENT
C
<NREF
806
II=0
SET NEAREST NEIGHBOR
ADDRESS ADJUSTMENT
VARIABLE II TO 0
SIG(I)
>YREF
≤YREF
810
< YREF
SIG(I+1)
=YREF
820
815
=0
METHR
≠0
> YREF
SIG(I+1)
=YREF
<YREF
820
|SIG(I)-YREF|
≤ |SIG(I+1)-YREF|
II=1
825
K=K+1
N=1, NPRT
RF(N) = RF(N)+F(1,N+1-II)
RG(N) = RG(N)+F(2,N+1-11)


A
850
K=0
I=0
855
I=I+1
I
>NREF
POINT SET TESTING
COMPLETE FOR THE SEGMENT
C
SIG(I)
<0
≥ 0
860
K=K+1
N=1, NPRT
RF (N) = RF(N)+F(1,N+I-1)
RG (N) = RG (N)+F(2,N+1-1)
865
870
K=K+1
N=1, NPRT
RF(N)=RF(N)-F(1,N+1-1)
RG(N)=RG(N)-F(2,N+1-1)


G
900
SUM SIGNATURES OF INDIVIDUAL AND COMBINED SIGNALS INTO COMMON SIGNATURE
IES
>1
<1
INRR
$=2^n$
$\neq 2^n$
FPLT
PLOT THE ACCUMULATIVE SIGNATURES
950
RETURN
END

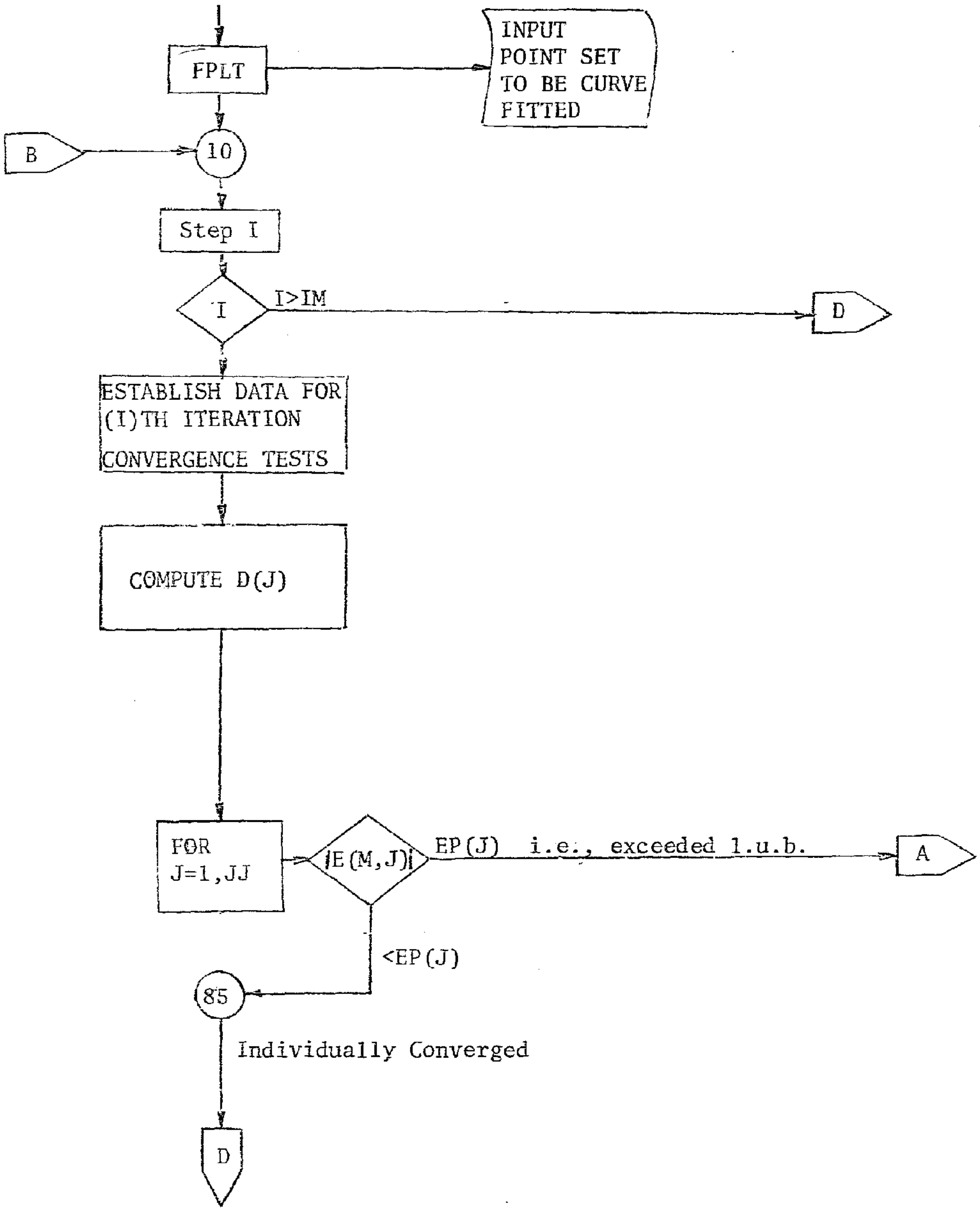
4.2 Curvft Procedure

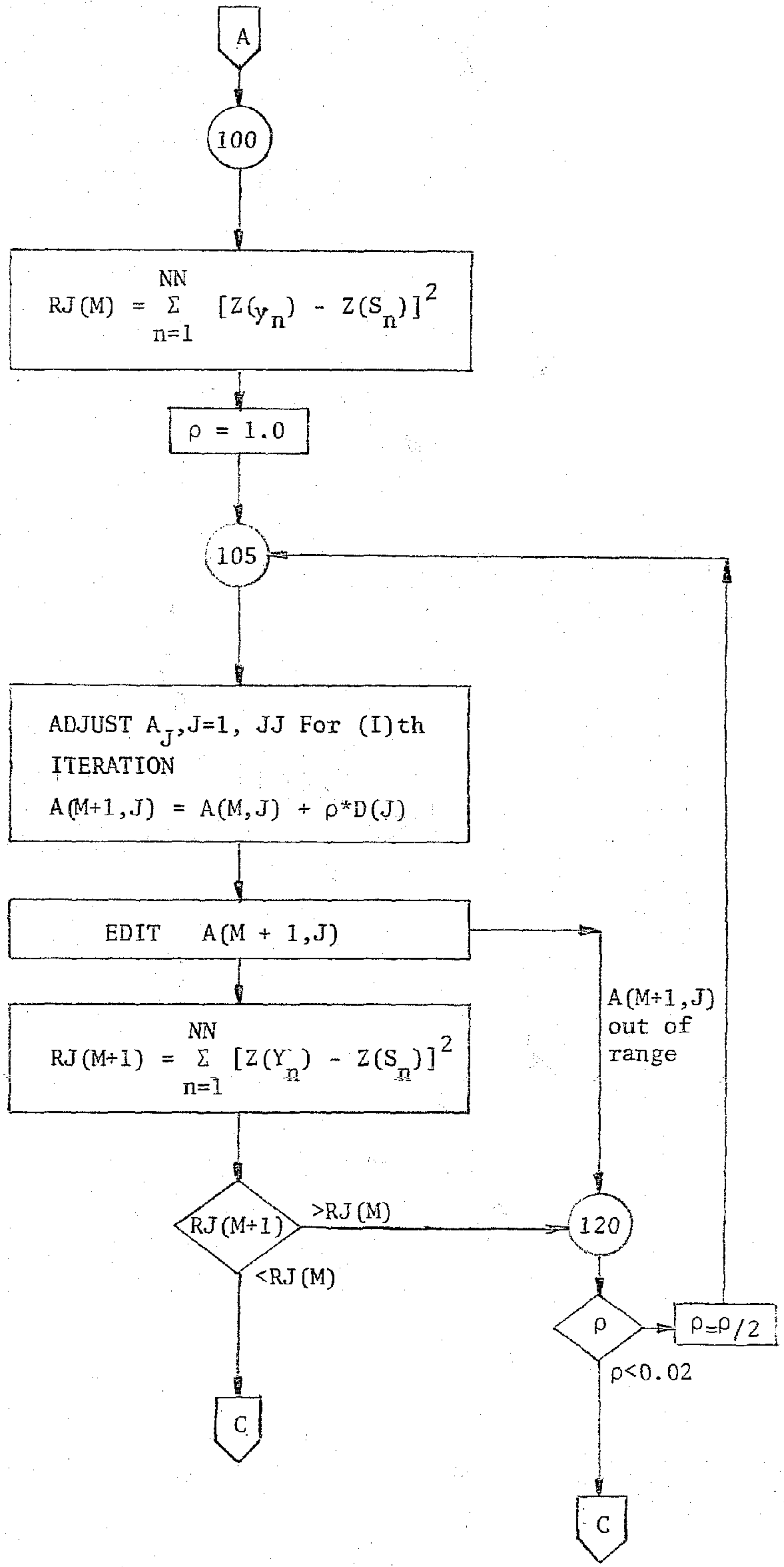
The following diagrams show the Curve-Fit algorithm.

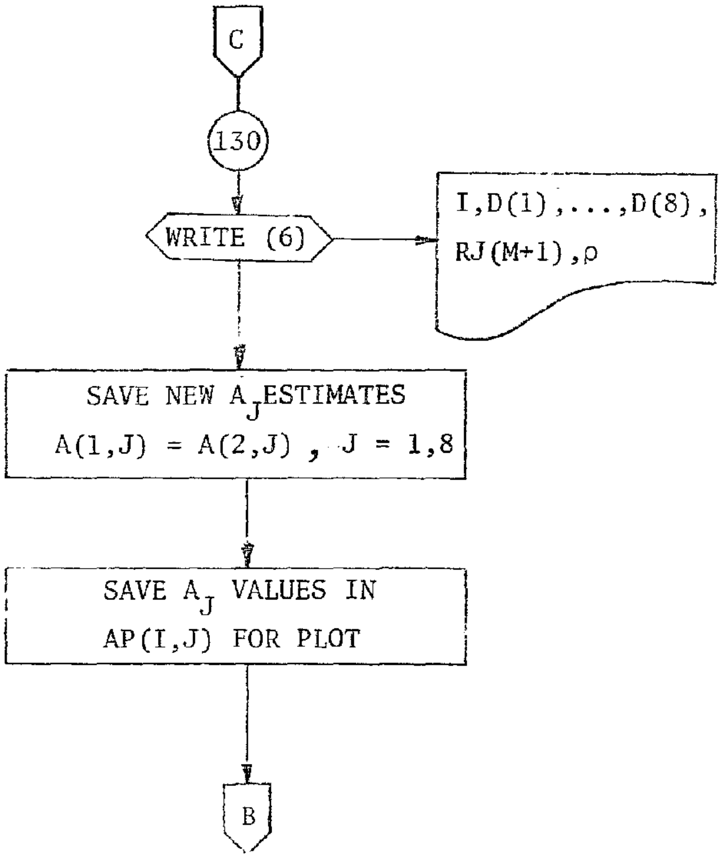


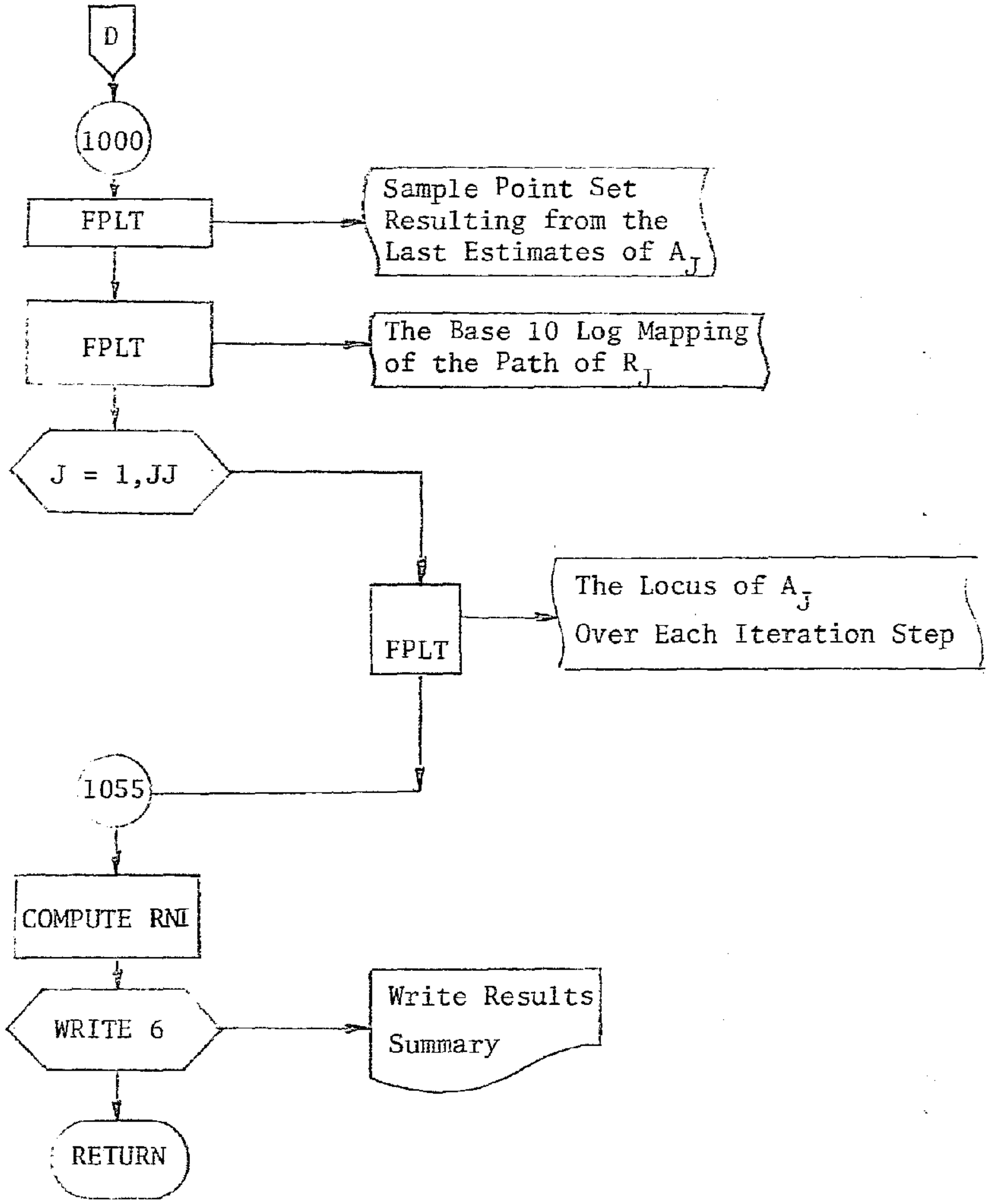
CF Procedures

The execution path of the procedure is shown in the flow diagram below. Contents in lettered blocks in the flow diagram are further described in the correspondingly lettered paragraphs following the flow diagram.




A
100
RJ(M) = Σ_{n=1}^{NN} [Z(y_n) - Z(S_n)]^2
ρ = 1.0
105
ADJUST A_J, J=1, JJ For (I)th ITERATION
A(M+1,J) = A(M,J) + ρ*D(J)
EDIT A(M + 1,J)
A(M+1,J) out of range
RJ(M+1) = Σ_{n=1}^{NN} [Z(Y_n) - Z(S_n)]^2
RJ(M+1)
>RJ(M)
<RJ(M)
120
ρ
ρ=ρ/2
ρ<0.02
C
C


C
130
WRITE (6)
I,D(1),...,D(8),
RJ(M+1),ρ
SAVE NEW A_J ESTIMATES
A(1,J) = A(2,J) , J = 1,8
SAVE A_J VALUES IN
AP(I,J) FOR PLOT
B


D
1000
FPLT
Sample Point Set Resulting from the Last Estimates of A_J
FPLT
The Base 10 Log Mapping of the Path of R_J
J = 1,JJ
FPLT
The Locus of A_J Over Each Iteration Step
1055
COMPUTE RNI
WRITE 6
Write Results Summary
RETURN

Appendix B

SAMPLE TEST CASE

PROGRAM RADCUF

INPUT DATA

(Refer to Pages A-5 to A-10)

```
@ELT,ILD  ,DATA42
ELT006-RL1867-10  01/11-17:10:16
CYCLE (001
000001      000  .12570.026  .052   0.0   0.6   0.0   0.024.693-93.33999999    999099          INPUT CARD NO. 1
000002      000       0.01536  50   16          1    0.0     7.0     0.0     0.0     1.0     1.0       2
000003      000  2140.0  2140.0   0.0   0.0   0   0   1  50   1  50   0  11   0   0   3  35  50        3
000004      000  535.861..03175.00588  0.05  0.01  0.05  0.00  0.0  0.0  0.1  0.1  0.0  0.0          4
000005      000  @EOF
000006      000  @EOF

@XQT   ADRVR
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

OUTPUT DATA

CASE PARAMETERS

OMEGA= .2570	M(1) = .20000-01	M(2) = .52000-01	WW(1) = .00000	WW(2) = .00000	P(1) = .00000
P(2) = .00000	A(1) = 24.693	A(2) = -93.330	NA(1) = .2140+04	NA(2) = .2140+04	NB = .0000
NC = .0000	NU(1) = .5350+03	NU(2) = .8610+03	Z(1) = .3175-01	Z(2) = .5880-02	A(1,1)= .50000-01
A(1,2)= .10000-01	A(2,1)= .50000-01	A(2,2)= .00000	KREF = .00000	NREF = 1536	NPRT = 50
METHR = 0	NCASE = 16	STN = .0000			

RANDOM NOEUE FOR GENERALIZED NOISE = 7.0000
MEAN FOR F AND G RESPECTIVELY .00000 .00000
STANDARD DEV OF F AND G 1.0000 1.0000

OUTPUT BELOW IS THE FINAL RANDOMDEC SIGNATURE AFTER 3612 TRIGGERS
TIMES TRIGGERED=3612 YRMS= 14.4049 YREF= .000000 SIGNATURE PERIOD= 7.00000 MAX PEAK= 55541.2

N	RSIG(N+0)	RSIG(N+1)	RSIG(N+2)	RSIG(N+3)	RSIG(N+4)	RSIG(N+5)	RSIG(N+6)	RSIG(N+7)
1	-21.7791	46551.6597	55541.1694	22372.0183	-25657.7126	-53059.4424	-41323.6167	-32.6575
9	41085.0069	51763.7393	24266.3987	-21260.9001	-51303.6265	-43056.3384	-2095.4823	40815.6924
17	52613.0352	24874.1938	-22336.6698	-51089.5342	-41025.3291	-373.7287	39870.3315	49403.6836
25	22107.2864	-20669.0955	-47670.8184	-39014.6001	-1805.5617	36791.9497	47875.7793	23181.6245
33	-18919.9766	-46618.2114	-39394.0840	-2279.9561	36481.8921	47369.3359	22278.2449	-19222.5732
41	-45582.4272	-37435.7324	-1912.6465	34868.1968	45170.8799	21583.4048	-17695.4495	-43318.1357
49	-34712.7524	-2749.0374						

```
CALL CF FOR MODE 4 FIT OF  4 PARAMETERS AFTER    1 SEGMENTS OF SIMULATED RESPONSE
INITIAL PARAMETER ESTIMATES ARE GIVEN AS FOLLOWS
A(1,1) =        12.5354
A(1,2) =        .000000
A(1,3) =        .897696
A(1,4) =        12.5354
A(1,5) =        .000000
A(1,6) =        .000000
A(1,7) =        .000000
A(1,8) =        .000000

NO. OF TRIGGERS ON SIGNATURE =        234
CF ENTERED
```

GRADIENT MATRIX COMPUTED ON PASS 1 Q(J,K), J= 4 , K= 4

.245000+002	.740191+004	-.764683+004	-.201228-005	.740191+004	.620178+007	.194939+006	.770902+004
-.764683+004	.194939+006	.656266+007	.795394+004	-.201228-005	.770902+004	.795394+004	.255000+002

PRODUCT OF W*W(I) ON PASS 1 QQ(J,K), J= 4 , K= 4

.100000+001	-.269229-014	.436211-014	.130184-017	.157736-019	.100000+001	-.121431-016	-.135525-019
.530734-020	-.173472-017	.100000+001	-.677626-020	-.533951-017	.532907-014	.355271-014	.100000+001

I.	A(J) J=1 , 4				RJ(D), AND RHO ARE LISTED IN BELOW AS FUNCTION OF ITERATION NO.	
1	-.2980	-.3554-02	.2521-02	-12.91	64.51	1.000
2	.1152	-.3776-02	-.3599-02	-.1689-01	6.741	1.000
3	.7127-02	.3937-04	-.6727-03	.6441-01	6.239	1.000
4	.3417-04	.3188-05	.5333-05	-.1178-02	6.239	1.000

THERE IS CONVERGENCE IN THE LOCAL SENSE FOR I= 4

INPUT SIGNATURE LENGTH = 50
NO. OF ITNS = 4
NORMALIZED STD. DEVIATION = .482652-01

RESULTING PARAMETERS
(FREQUENCY IN RAD/SEC)
(DAMPING IS RATIO TO CRITICAL)

A(1)= .1236+02 E(1)= -.1146-05 A(2)= -.7287+02 E(2)= -.7292-05 A(3)= .8959+00 E(3)= -.7024-08
A(4)= -.3256+00 E(4)= -.1300-04 A(

MODE	NATURAL FREQ.	DAMPING
1	73.4118	-.81336D-02

```
 CALL CF FOR MODE 4 FIT OF  8 PARAMETERS AFTER    1 SEGMENTS OF SIMULATED RESPONSE
INITIAL PARAMETER ESTIMATES ARE GIVEN AS FOLLOWS
 A(1,1) =        12.3698
 A(1,2) =        .000006
 A(1,3) =        .895852
 A(1,4) =       -.325486
 A(1,5) =        12.5354
 A(1,6) =        .000000
 A(1,7) =        1.13849
 A(1,8) =        12.5754

NO. OF TRIGGERS ON SIGNATURE =        234
CF ENTERED
```

GRADIENT MATRIX COMPUTED ON PASS 1 W(J,K), J= 8 , K= 8

.245377+002	.744570+004	.504042+002	-.396409-001	-.128880+001	.871985+002	.173077+004	-.554044+000
.744570+004	.301373+007	-.926515+005	-.348407+003	-.777217+003	.250744+006	.965656+006	.916985+003
.504042+002	-.926515+005	.316609+007	.769509+004	-.127640+004	-.117611+007	.486329+006	-.763490+003
-.396409-001	-.348407+003	.769509+004	.254623+002	.277064+000	-.207116+004	.474608+003	-.747602+000
-.128880+001	-.777217+003	-.127640+004	.277064+000	.249021+002	.763047+004	-.791657+004	-.113866+000
.871985+002	.250744+006	-.117611+007	-.207116+004	.763047+004	.617415+007	-.115429+006	.743928+004
.173077+004	.965656+006	.486329+006	.474608+003	-.791657+004	-.115429+006	.653029+007	.772538+004
-.554044+000	.916985+003	-.763490+003	-.747602+000	-.113866+000	.743928+004	.772538+004	.250979+002

PRODUCT OF A*Q(I) ON PASS 1 QQ(J,K), J= 8 , K= 8

.100000+001	-.316414-014	.249800-014	.444523-017	-.211284-016	-.754952-014	.249249-013	-.104083-016
.590217-020	.100000+001	-.314419-017	-.831161-020	-.147569-019	.333934-016	.390313-017	-.508220-020
-.217052-020	-.867362-018	.100000+001	.326637-019	-.490353-019	-.338271-016	.390313-017	.508220-020
-.542101-018	-.627356-015	.272005-014	.100000+001	.108454-016	.310862-014	.222045-015	-.954098-017
.193432-017	.888178-015	.121257-014	.367782-017	.100000+001	-.580092-014	.233147-014	.460786-017
.444692-020	.151788-017	.130104-017	-.296462-020	.121232-019	.100000+001	.346945-017	-.677626-020
.783505-020	.108420-017	.435681-018	.211758-021	-.121232-019	-.173472-017	.100000+001	.677626-020
-.401155-017	-.177636-014	.000000	.216840-017	-.165341-017	-.177636-014	-.532907-014	.100000+001

I, A(J) J=1 , 8 RJ(2), AND RHO ARE LISTED IN BELOW AS FUNCTION OF ITERATION NO.

1	-.1049-01	-.4298-02	.1606-03	-.9708-01	-12.39	-.8569-03	-.3652-03	-12.02	11.80	1.000
2	.1593	-.1271-02	.6898-04	-.3265-01	-.1236-01	-.3432-01	.1538-02	.5970-01	4.921	1.000
3	-.2291-02	-.9756-04	.5199-03	-.2045	.4519	-.5363-01	.4583-01	.2018	3.139	1.000
4	-.2892	.7511-03	.6440-04	.9790-02	.9686	.7121-02	.1479	-.5853	2.728	1.000
5	.4900-01	-.1299-03	.8298-04	-.1783-01	.3777-01	-.3716-01	-.2539-01	-.1103	1.628	1.000
6	.3899-01	-.9781-04	-.3179-04	.8438-02	-.2518	.3460-01	-.2841-01	.9461-01	1.319	1.000
7	.2425-01	-.5840-04	.1016-04	-.5962-03	-.7622-01	.1008-01	-.6926-02	.6005-01	1.277	1.000
8	.7379-02	-.1683-04	-.1656-05	.1581-02	-.3418-01	.3214-02	-.2889-02	.1858-01	1.271	1.000
9	.2663-02	-.4107-05	-.3331-05	.1595-02	-.1536-01	.1505-02	-.8514-03	.4709-02	1.270	1.000
10	.1098-02	-.2527-05	-.4174-06	.2565-03	-.5350-02	.4893-03	-.4471-03	.2906-02	1.270	1.000
11	.4052-03	-.9357-06	-.5475-06	.2597-03	-.2425-02	.2311-03	-.1302-03	.7020-03	1.270	1.000
12	.1722-03	-.3998-06	-.8201-07	.4447-04	-.8472-03	.7658-04	-.6872-04	.4450-03	1.270	1.000

THERE IS CONVERGENCE IN THE LOCAL SENSE FOR I= 12

INPUT SIGNATURE LENGTH = 50

NO. OF ITNS = 12

NORMALIZED STD. DEVIATION = .217463-01

RESULTING PARAMETERS

(FREQUENCY IN RAD/SEC)

(DAMPING IS RATIO TO CRITICAL)

A(1)= .1234+02 E(1)= .5082-05 A(2)= -.7229+02 E(2)= .1991-04 A(3)= .8967+00 E(3)= -.9296-07

A(4)= -.6563+00 E(4)= -.6019-04 A(5)= .1205+01 E(5)= -.3115-03 A(6)= -.6865-01 E(6)= -.5176-03

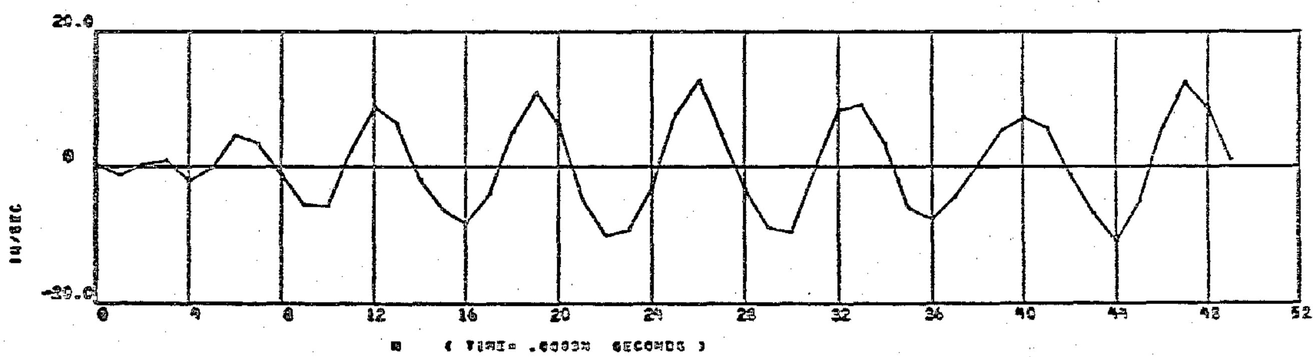
A(7)= .1266+01 E(7)= -.1623-04 A(8)= .2661+00 E(8)= .4194-03 A(

MODE	NATURAL FREQ.	DAMPING
1	73.4830	-.806085-02
2	103.912	-.541334-01

SC4020

GRAPHICAL OUTPUT

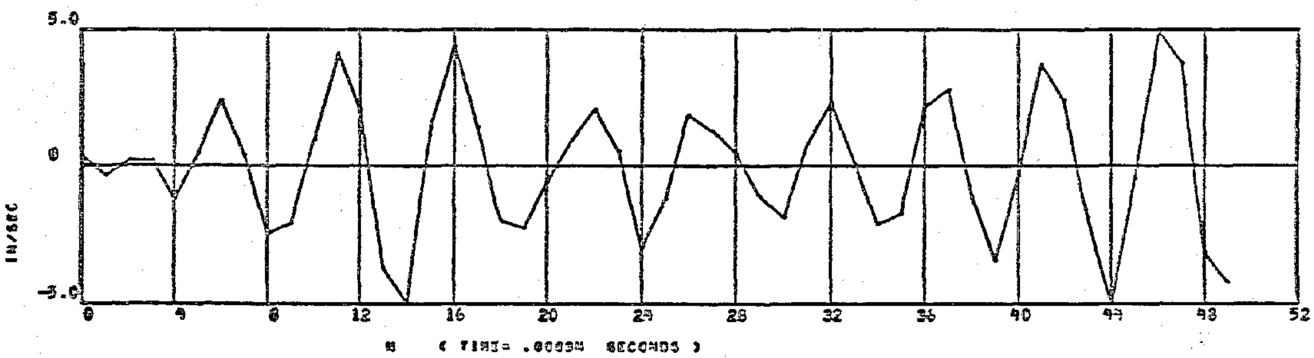
SIMULATED RESPONSE SIGNAL (MODE 1)



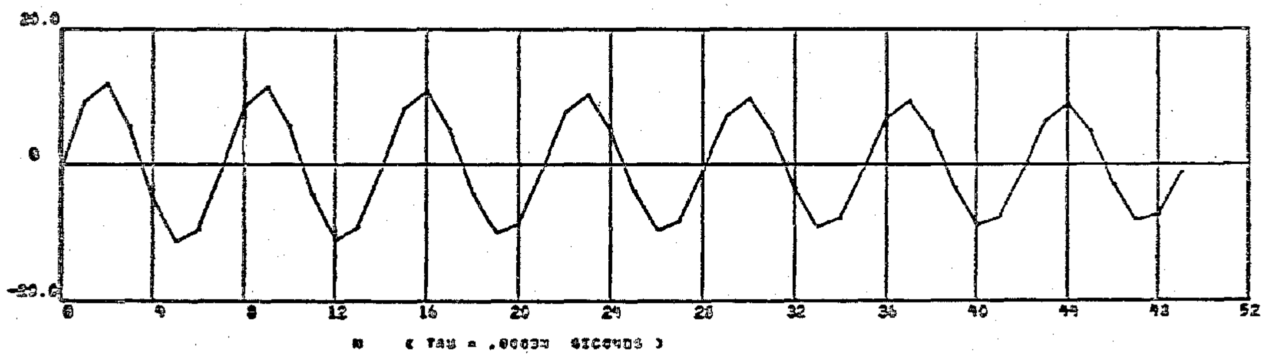
(Set NSF to 1 to Expand Graph to Occupy Full Page)

JOB NO 60232 PAGE 2

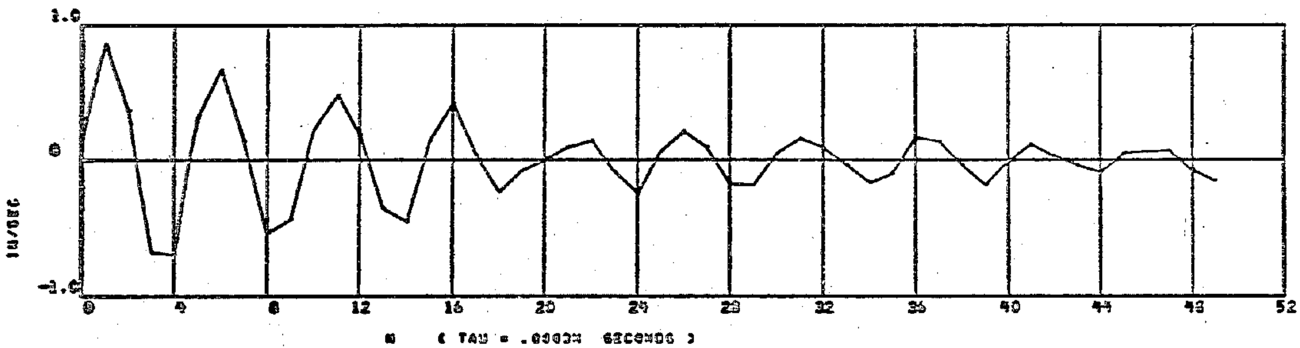
SIMULATED RESPONSE SIGNAL (MODE 2)



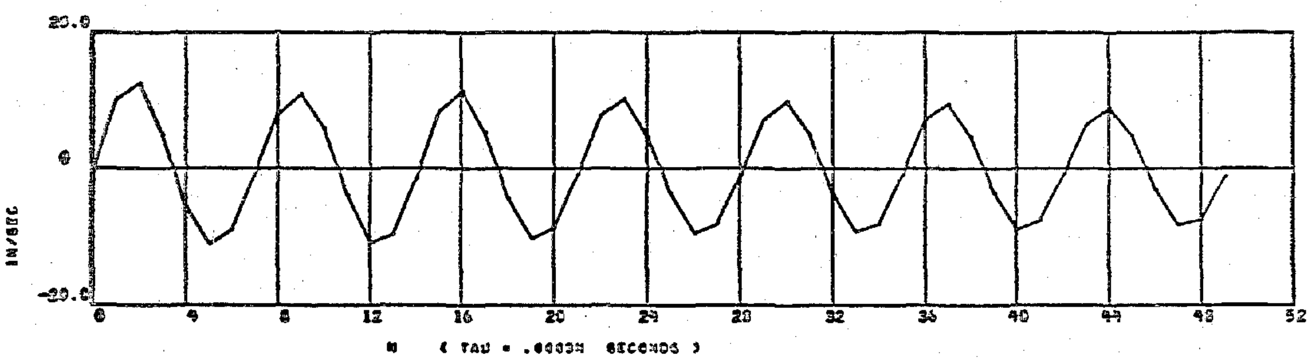
MODE 1 SIGNATURE



MODE 2 SIGNATURE



SYSTEM SIGNATURE



PEAK PLOT OF MODE 1 SIGNATURE FOR SEGMENT 1

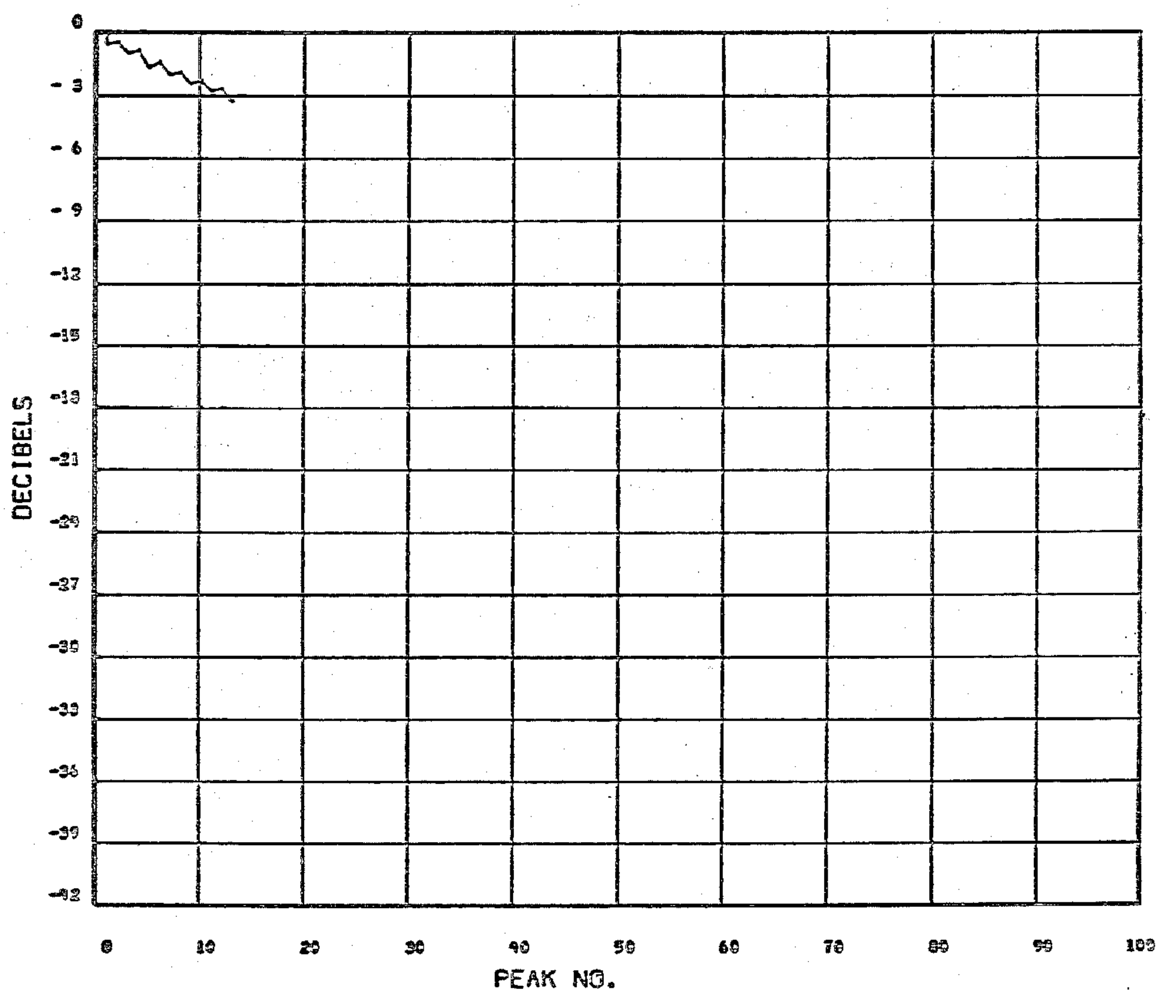
SIMULATION SIGNAL PARAMETERS

FCN FORM	FREQ	RANDOM	BOXCAR FORCES
NA(1)= 2140	NU(1) = 535.0	R(1.1)=0.050	LD(1) = $9.99999 \times 10^{+05}$
NA(2)= 2140	NU(2) = 861.0	R(1.2)=0.010	LW(1) =
NB = 0	DAMPING	R(2.1)=0.050	LD(2) = $9.99999 \times 10^{+05}$
NC = 0	Z(1) = 0.0317	R(2.2)=0.000	LW(2) =
INIT COND	Z(2) = 0.0058	AV(1) =0.000	CYCLIC FORCES
P(1) = 0.000	OTHER	SD(1) =1.000	C(1) =0.000
Q(1) =48.000	D OMGA= 0.1257	AV(2) =0.000	N1P =0.100
P(2) = 0.000	A(1) = 24.6	SD(2) =1.000	TH(1) =0.000
Q(2) = 0.000	A(2) = 93.3		C(2) =0.000
	MU(1) = 0.0260		N2P =0.100
	MU(2) = 0.0520		TH(2) =0.000

RANDOMDEC PARAMETERS

METHODS	0
NO. OF SEGMENTS	16
TOTAL TIMES TRIGGERED	234
NO. OF ITRNS ON AVG SIG	1

MAXIMUM PEAK VALUE = $2.85063 \times 10^{+03}$



PEAK PLOT OF MODE 2 SIGNATURE FOR SEGMENT 1

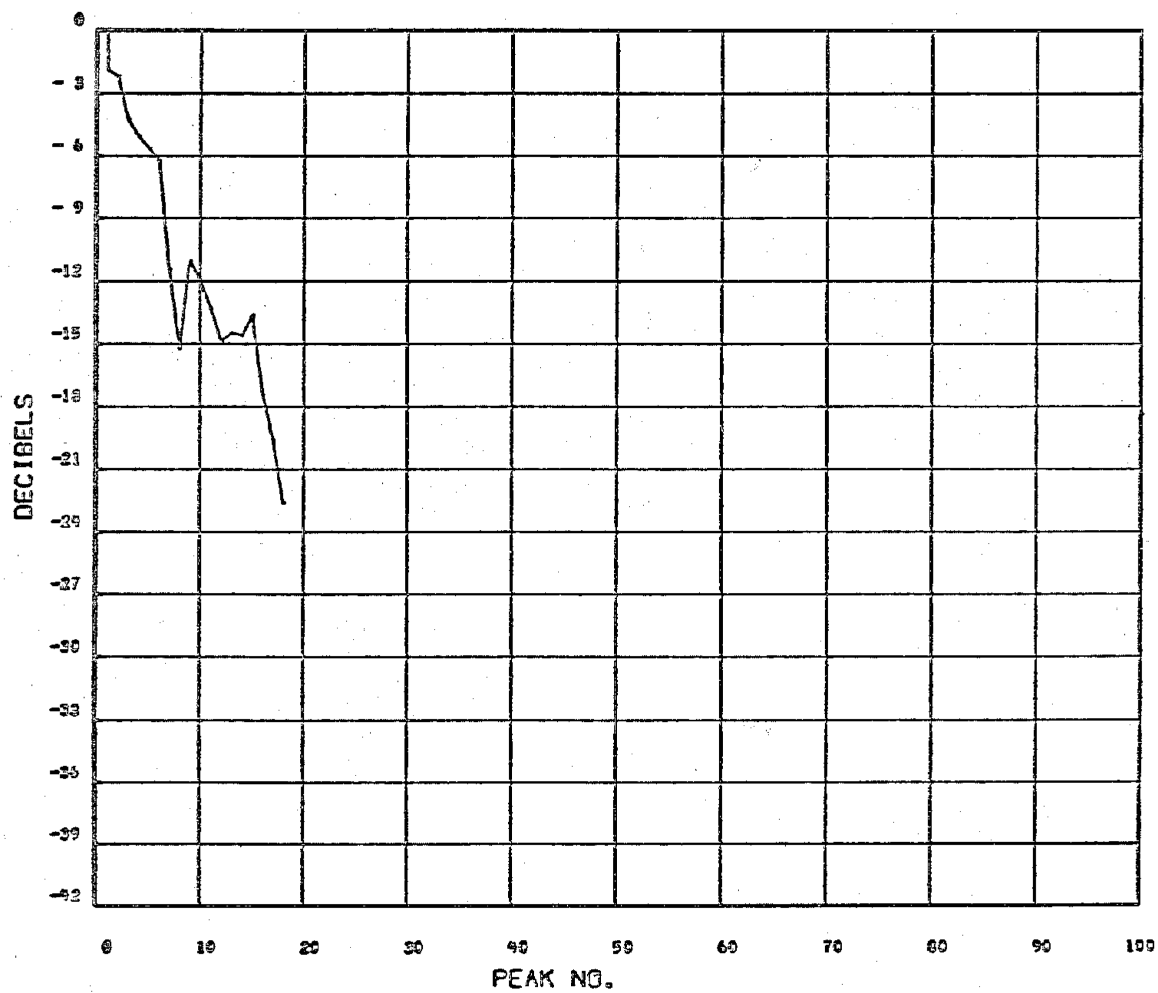
SIMULATION SIGNAL PARAMETERS

FCN FORM	FREQ	RANDOM	BOXCAR FORCES
NA(1)= 2140	NU(1) = 535.0	R(1,1)=0.050	LD(1) = $9.99999 \times 10^{+05}$
NA(2)= 2140	NU(2) = 861.0	R(1,2)=0.010	LW(1) =
NB = 0	DAMPING	R(2,1)=0.050	LD(2) = $9.99999 \times 10^{+05}$
NC = 0	Z(1) = 0.0317	R(2,2)=0.000	LW(2) =
INIT COND	Z(2) = 0.0058	AV(1) =0.000	CYCLIC FORCES
P(1) = 0.000	OTHER	SD(1) =1.000	C(1) =0.000
Q(1) =48.000	D OMGA= 0.1257	AV(2) =0.000	N1P =0.100
P(2) = 0.000	A(1) = 24.6	SD(2) =1.000	TH(1) =0.000
Q(2) = 0.000	A(2) = 93.3		C(2) =0.000
	MU(1) = 0.0260		N2P =0.100
	MU(2) = 0.0520		TH(2) =0.000

RANDOMDEC PARAMETERS

METHODS	0
NO. OF SEGMENTS	16
TOTAL TIMES TRIGGERED	234
NO. OF ITRNS ON AVG SIG	1

MAXIMUM PEAK VALUE = $2.01806 \times 10^{+02}$



PEAK PLOT OF SYSTEM SIGNATURE FOR SEGMENT 1

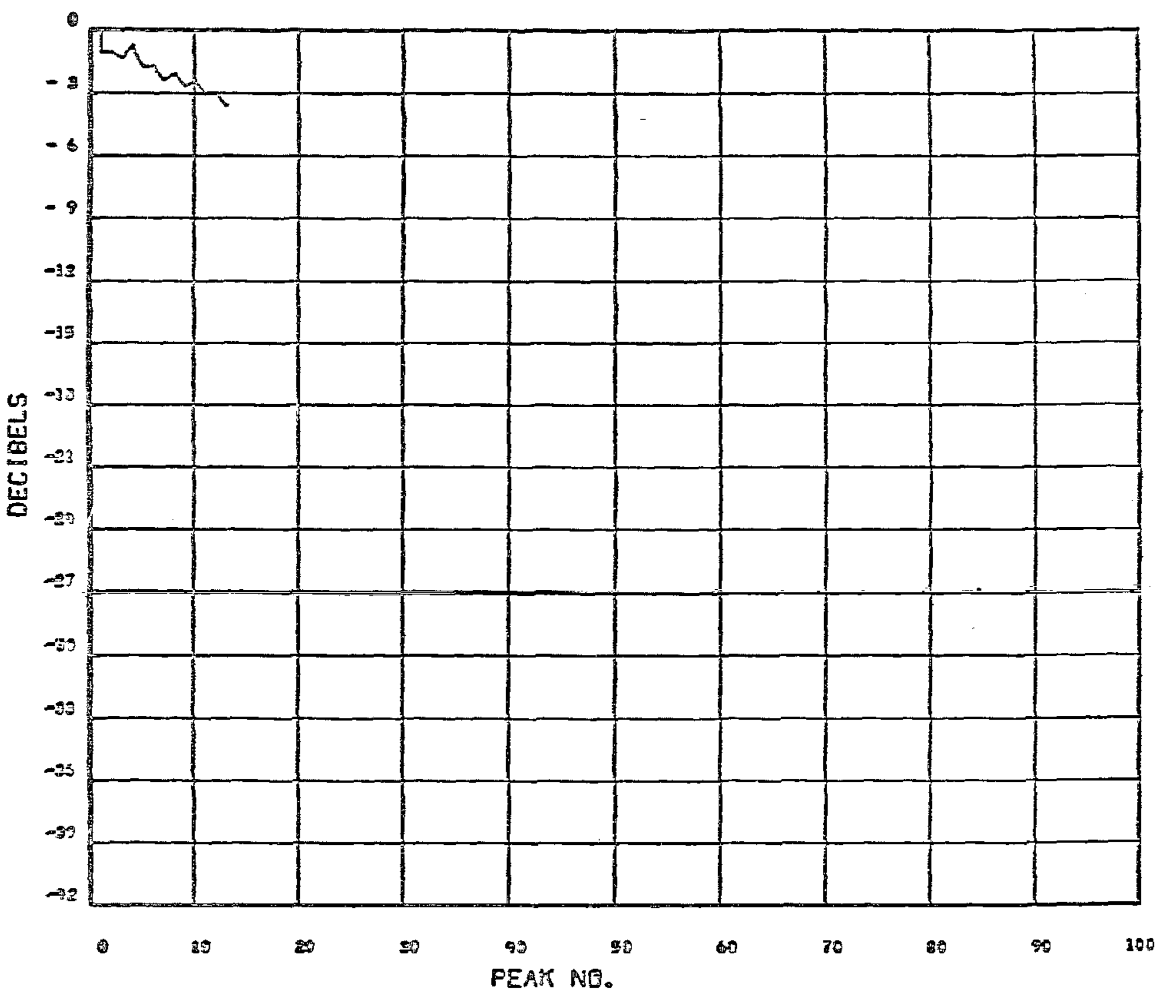
SIMULATION SIGNAL PARAMETERS

FCN FORM	FREQ	RANDOM	BOXCAR FORCES
NA(1)= 2190	NU(1) = 535.0	R(1.1)=0.050	LO(1) = $9.99999 \times 10^{+05}$
NA(2)= 2190	NU(2) = 361.0	R(1.2)=0.010	LW(1) =
NS = 0	DAMPING	R(2.1)=0.050	LO(2) = $9.99999 \times 10^{+05}$
NC = 0	Z(1) = 0.0317	R(2.2)=0.000	LW(2) =
INIT COND	Z(2) = 0.0058	AV(1) =0.000	CYCLIC FORCES
P(1) = 0.000	OTHER	SD(1) =1.000	C(1) =0.000
Q(1) =93.000	D OMGA= 0.1257	AV(2) =0.000	N1P =0.100
P(2) = 0.000	A(1) = 24.6	SD(2) =1.000	TH(1) =0.000
Q(2) =43.000	A(2) = 93.3		C(2) =0.000
	MU(1) = 0.0260		N2P =0.100
	MU(2) = 0.0520		TH(2) =0.000

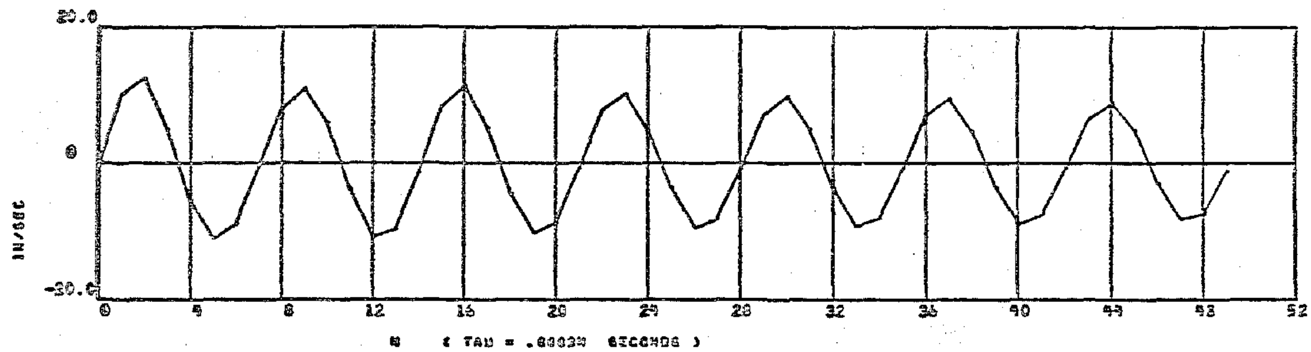
RANDOMDEC PARAMETERS

METHODS	0
NO. OF SEGMENTS	16
TOTAL TIMES TRIGGERED	234
NO. OF ITRNS ON AVG SIG	1

MAXIMUM PEAK VALUE = $2.93328 \times 10^{+03}$

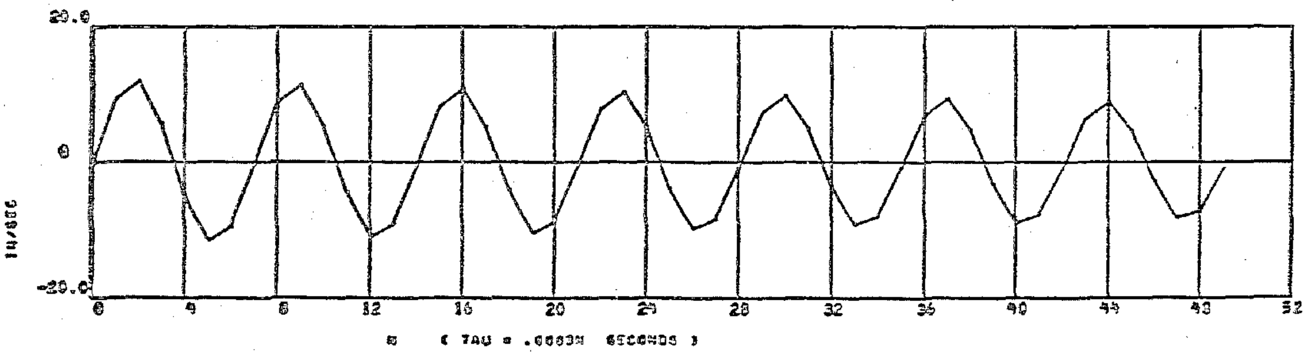


INPUT SIGNATURE TO BE CURVE-FITTED



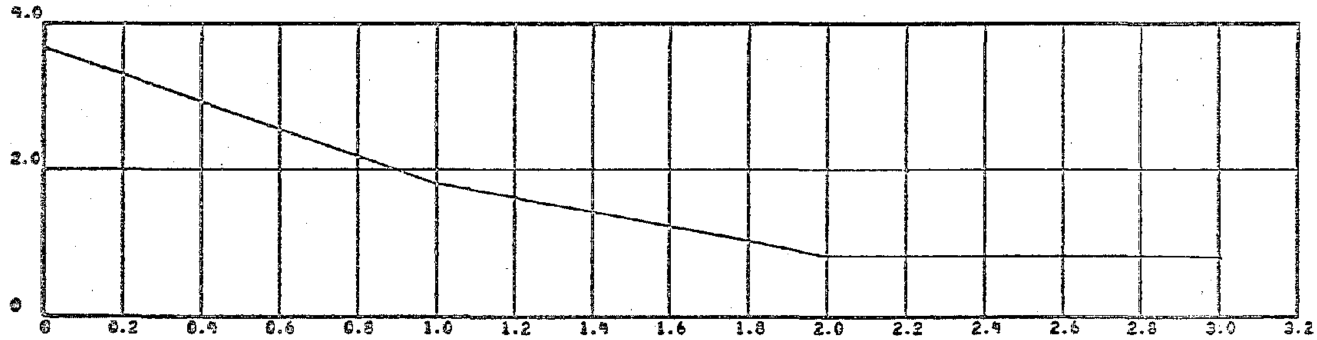
JOB NO 60232 PAGE 42

LEAST-SQUARE APPROXIMATION ON ITERATION 5

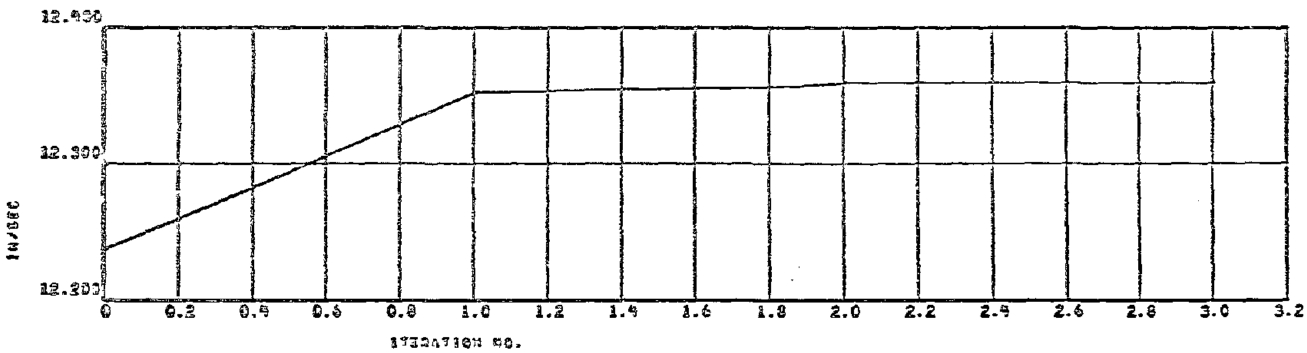


JOB NO 60232 PAGE 43

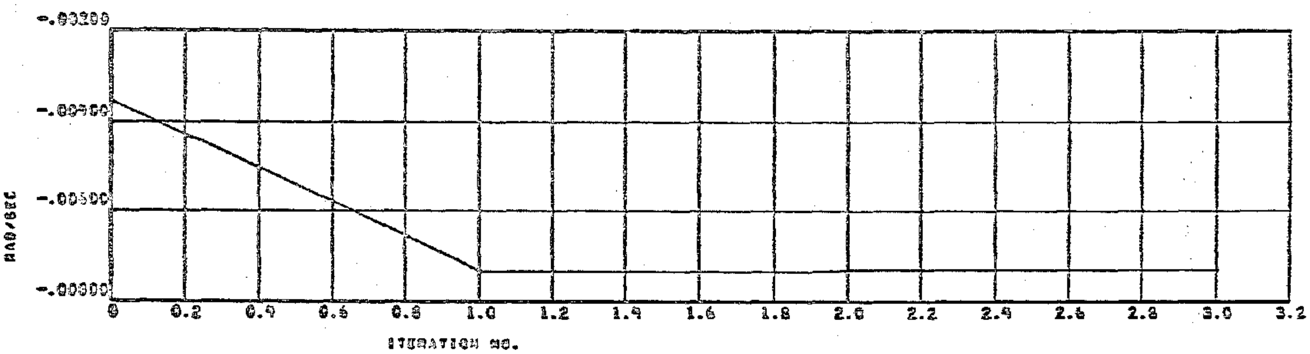
COMPUTED VALUE OF LOG(MEAN-SQUARED ERROR) FOR EACH ITERATION



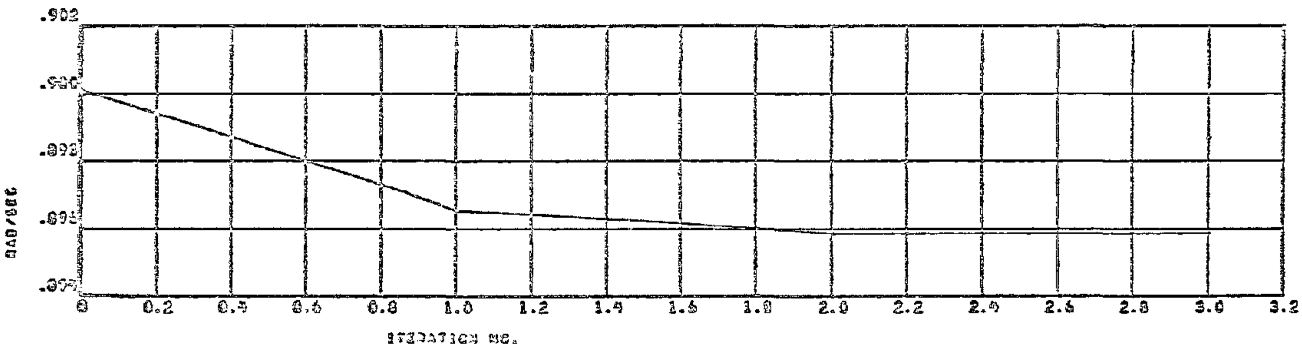
CONVERGENCE PATH OF A(1)



CONVERGENCE PATH OF A(2)

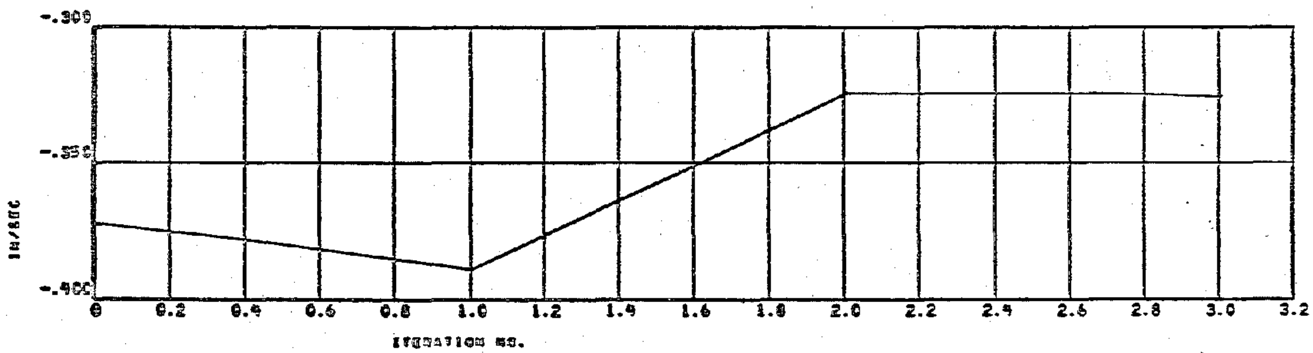


CONVERGENCE PATH OF A(3)

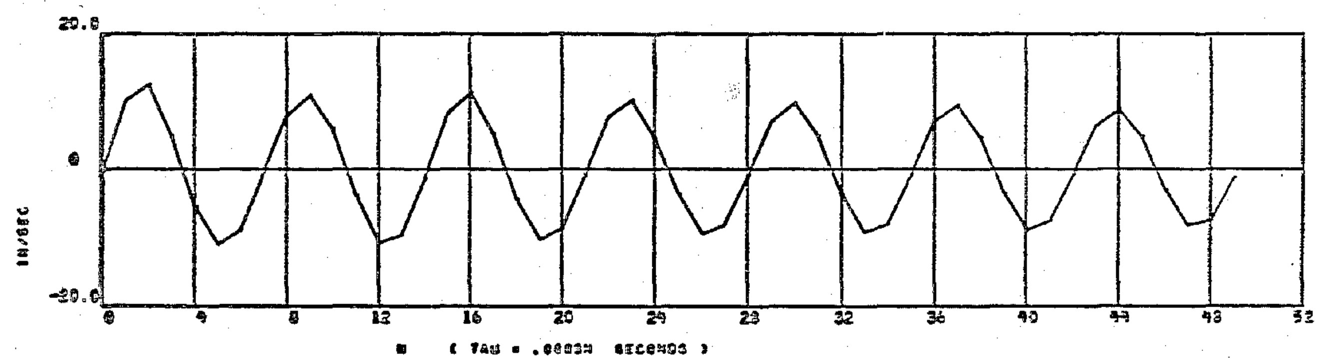


JOB NO 60232 PAGE 47

CONVERGENCE PATH OF A(4)

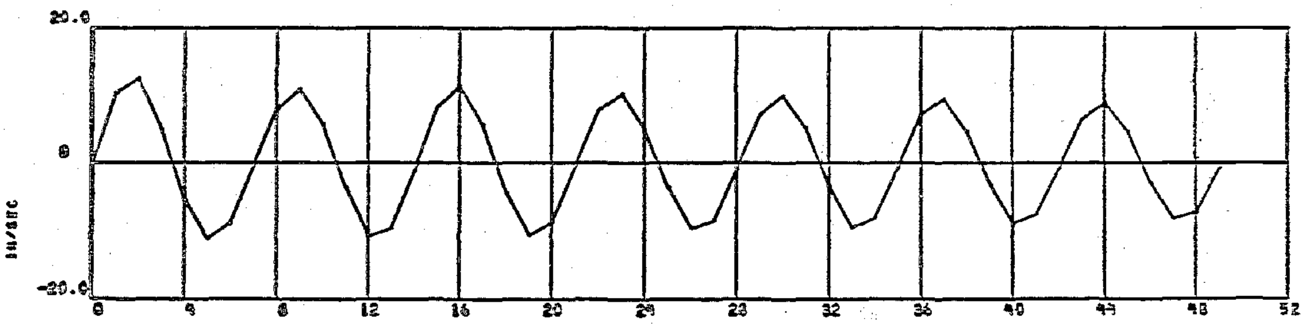


INPUT SIGNATURE TO BE CURVE-FITTED

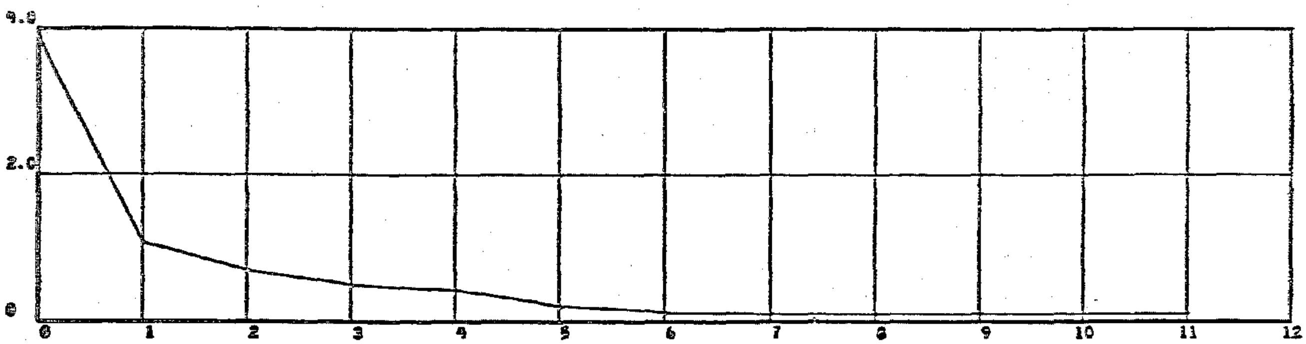


JOB NO 60232 PAGE 49

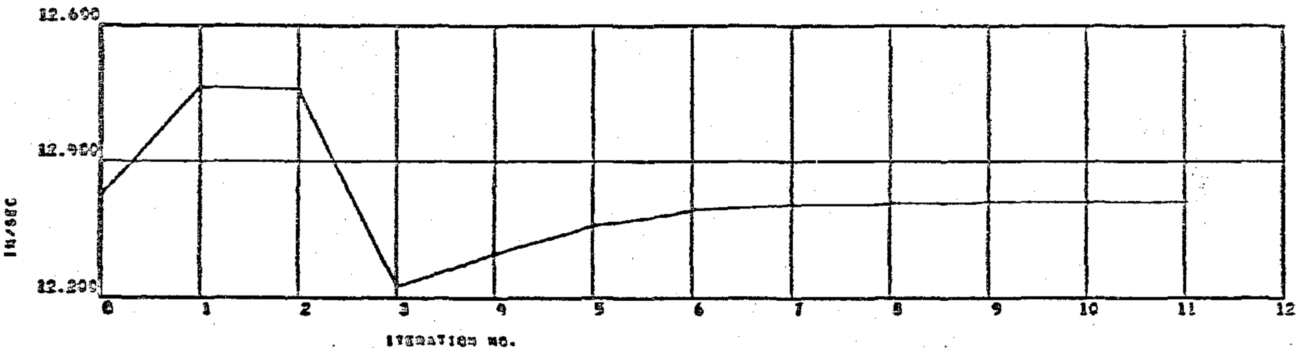
LEAST-SQUARE APPROXIMATION ON ITERATION 13



COMPUTED VALUE OF LOG(MEAN-SQUARED ERROR) FOR EACH ITERATION

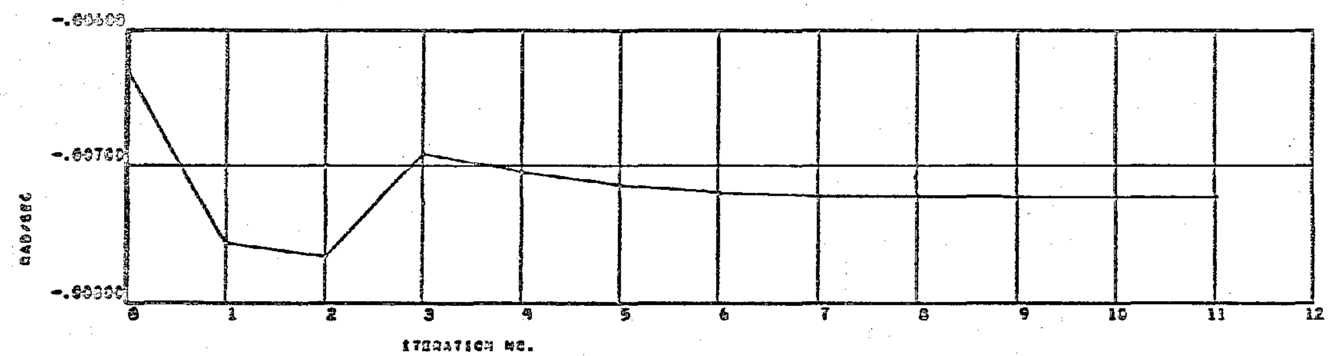


CONVERGENCE PATH OF A(1)

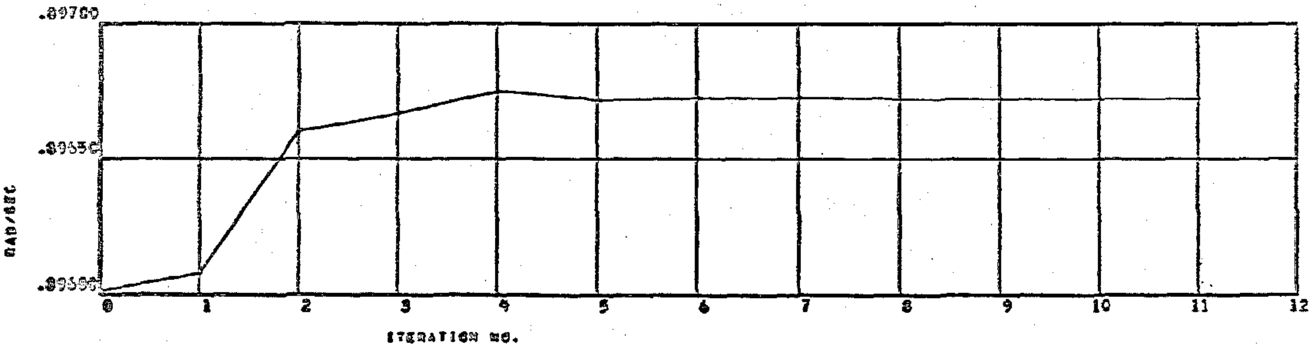


JOB NO 60232 PAGE 52

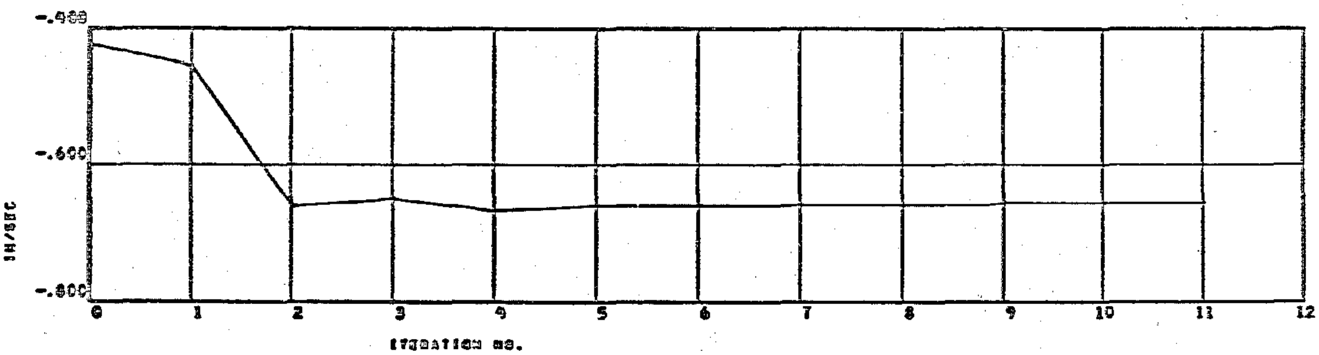
CONVERGENCE PATH OF A(2)



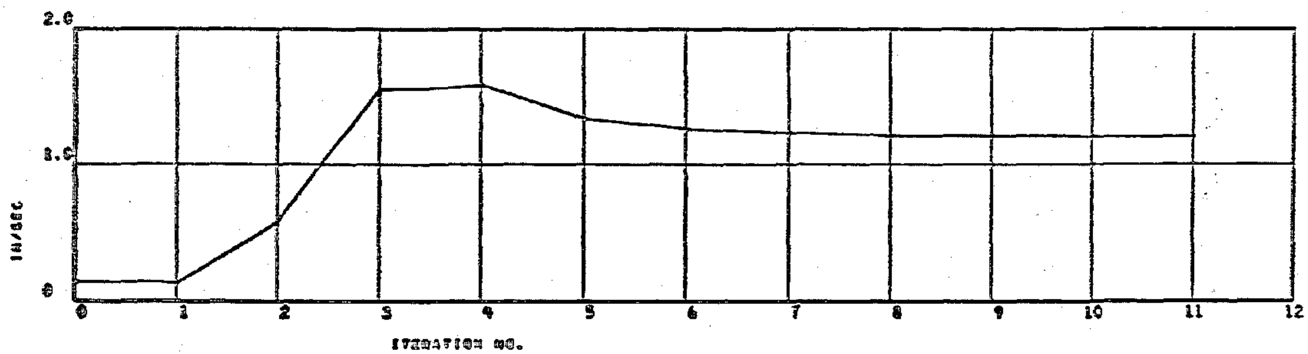
CONVERGENCE PATH OF A(3)



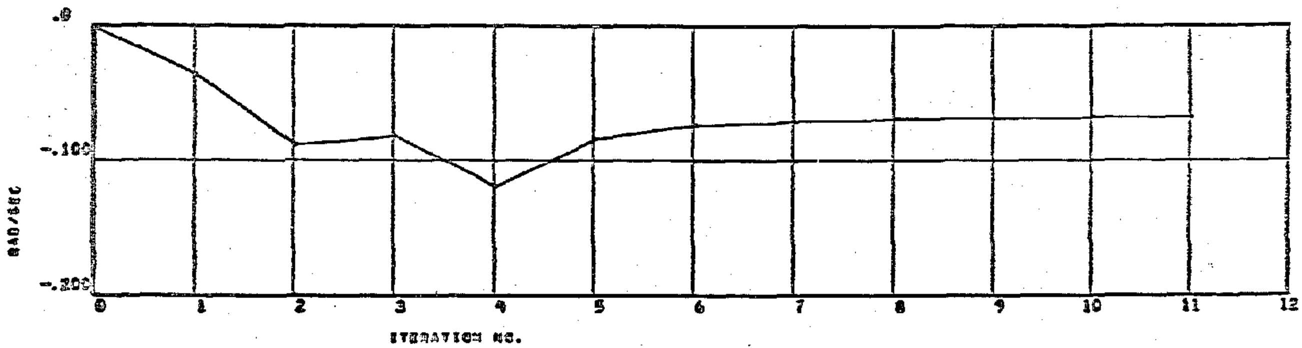
CONVERGENCE PATH OF A(4)



CONVERGENCE PATH OF A(5)



CONVERGENCE PATH OF A(6)



CONVERGENCE PATH OF A(7)