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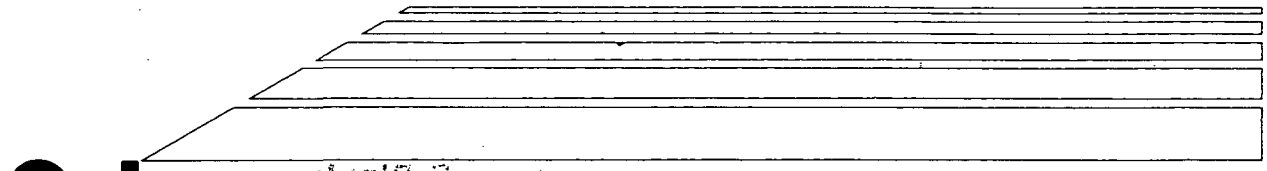

Calspan

# *STUDY OF THE DETAIL CONTENT OF APOLLO ORBITAL PHOTOGRAPHY*

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## ABSTRACT

This final report documents the results achieved during a study of the Detail Content of Apollo Orbital Photography under contract NASW-2434 with NASA Headquarters. This study spanned an 18 month period and was composed of three tasks whose objectives were to assess the effect of anomalous V/H (Velocity to Height ratio) sensor operation or photoreproduction processes upon the detail content of lunar surface photography obtained from the orbiting Command Module. This report includes data and conclusions obtaiend from the Apollo 15, 16 and 17 missions.

The specific tasks undertaken included (1) an evaluation of the residual motion smear in Apollo 15 panoramic photography due to intermittent V/H sensor operation and (2) intercomparison of the detail content of multiple generations of Apollo 16 and 17 photography.

During the Apollo 15 mission the panoramic camera V/H sensor did not operate properly and resulted in anomalously high or pre-set nominal command V/H rates. In some cases a transition between the anomalous rate and the nominal rate occurred during the exposure of a frame. Two such frames (316 and 360) were examined to determine the subsequent effect on the detail content. The evaluation showed that a loss in resolution due to image motion blur and exposure differences within a frame could occur due to the intermittent V/H sensor operation. The lack of correct compensation reduced the resolution by 1/3 (to about 6 meters on the lunar surface). Only when the commanded V/H is equal to the nominal value can the full resolution potential of the panoramic camera be realized; but this may not always occur. Consequently it is concluded that a significant portion of the Apollo 15 panoramic imagery may have lower resolution than the nominal 2 meter value. The user of this imagery should refer to the telemetry data to determine the V/H command situation for the frames he is exploiting and account for the effects of poor resolution if necessary.

An assessment of the detail content of multiple generations of Apollo 16 and 17 photography was made to determine the degradation, if any, in the quality of the photographic products furnished to various users. The original flight film is copied to produce a Master Positive (2P) or Direct Negative (2N) copy. These are the earliest generations which the user may exploit and only a limited number of such copies are produced. Most investigators are likely to use 3rd Generation or 4th Generation Positive copies. Unfortunately 3P and 4P copies of the same type of photography were not available thus preventing a direct comparison of the most likely user products. The evaluation showed that the tone quality of the Master Positive is superior to that of the Direct Negative for the panoramic photography. This superiority results from a greater dynamic range which means that the 2P photography contains higher contrast images over a wider range of average exposure levels than the 2N photography. The data indicates that this difference is preserved in the higher generation products which the user is likely to employ. Tonal quality differences were also present in 2nd Generation copies of metric photography, however, it is not clear that these differences are preserved in the user products. MTF measurements were made in order to compare the fine detail content of the various generations of photography evaluated. Approximately half of these frames showed some loss in fine detail content which is attributed to the reproduction process. An increase in the ground resolution element by a factor of 1.5 was observed; 3 meters for the panoramic photography and 30 meters for the metric photography.

This study successfully demonstrated the development and application of image evaluation methods for orbital photography of the lunar surface. Descriptions of the various elements of software used in this study are included in this report. It is recommended that the development of these techniques be continued for application to future manned and unmanned spacecraft involved in planetary exploration.

# FOREWORD

The research reported herein is the result of a series of tasks performed over an 18 month period under Contract Number NASW-2434 with NASA Headquarters and monitored by Mr. Leon Kosofsky (Code: MAL).

The author is grateful to Mr. Kenneth Hancock of the NASA Johnson Space Center and Mr. Daniel Kinsler of the Lunar Science Institute for furnishing the Apollo orbital photography needed to perform the study. The telemetry data used in the evaluation of the Apollo 15 panoramic photography was obtained from Mr. Bernard Moleburg of the NASA Johnson Space Center while other auxiliary data were obtained from the microfilm files at the Lunar Science Institute. Contributions by others at Calspan Corporation include Mr. Timothy Gallagher who performed the microdensitometry required for the evaluation of the Apollo 16 and 17 photography and Mr. Larry Perletz for assistance in the microdensitometry and analysis associated with the evaluation of the Apollo 15 photography.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

# 1. INTRODUCTION

This final report documents the results achieved during a study of the Detail Content of Apollo Orbital Photography under Contract NASW-2434 with NASA Headquarters. This study was composed of three tasks whose objectives were to assess the effect of a V/H anomaly or photo reproduction process upon the detail content of lunar surface photography obtained from the orbiting Command Module. The report includes results from the photo evaluation of the Apollo 15, 16 and 17 missions. An earlier report[1] presented the results of a similar study of photography from the Apollo 8, 12, 14 and 15 missions. This report also contains a detailed description of the Edge Gradient Spectral Analyses (EGSA) computer program that was employed extensively during these studies.

The specific tasks completed include (1) an evaluation of the residual motion smear in several Apollo 15 panoramic photographs due to intermittent V/H sensor operation and (2) intercomparison of the detail content of multiple generations of Apollo 16 and 17 photography.

Before proceeding we will briefly review the photography obtained during these Apollo missions and its reproduction sequence. In all these missions the Scientific Instrument Module (SIM) Bay contained two cameras; an Optical Bar Panoramic Camera and a Mapping (or Metric) Camera. The Panoramic Camera had a 610 mm (24 inch) focal length lens and employed EK Type 3414 film yielding an approximate resolution capability of 130 cycles/mm. The Mapping Camera had a 76 mm (3 inch) focal length lens and employed EK Type 3400 film. Its resolution capability was about 90 cycles/mm. Both cameras contained internal image motion compensation (IMC) mechanisms. The effect of erratic operation of the Apollo 15 panoramic IMC was assessed in the study.

The original flight film is reproduced for distribution to potential users. Figure 1 shows the reproduction sequence and terminology used. The

flight film is copied to produce a Second Generation or Master Positive (2P) copy. Only a limited number of such copies are generated. They are subsequently copied twice to produce a Fourth Generation (4P) copy which many investigators are likely to use. The flight film is also copied on to reversal film to obtain a Second Generation Direct (2N) Negative. This is copied once to produce a Third Generation (3P) Positive which is also available to investigators. This sequence eliminates one copy step in the hope of improving product quality and reducing production time. An assessment was made of the effect of both reproduction sequences on the detail content of Apollo 16 and 17 photography.

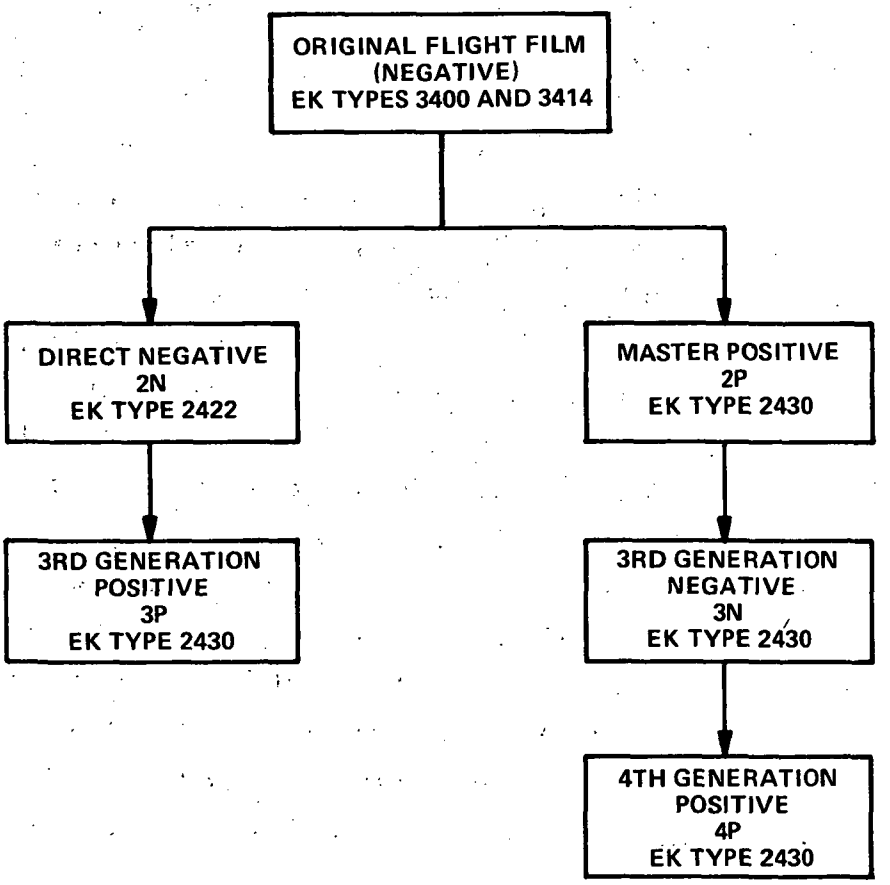



**Figure 1 APOLLO ORBITAL PHOTOGRAPHY REPRODUCTION SEQUENCE**

## 2. EVALUATION TECHNIQUES

All of the methods used in this study to evaluate the Apollo orbital photography were developed during previous contracted studies for NASA and others. These methods are briefly described in this section of the report. In some cases complex computer programs are required. Their operation is discussed and the software detailed in the appendix.

The evaluation of the quality of Apollo Orbital photography can require the measurement of various properties of the imagery. The properties which define image quality can be divided into four general categories: (1) "resolution" or fine detail content, (2) contrast or tone quality, (3) noise level and (4) metric quality. Overall performance can be based upon a composite measure which includes several (or all) of the categories but must be determined with the intended use of the imagery in mind.

The modulation transfer function (MTF) is often used as a measure of image resolution or fine detail content. In this study we used this image quality measure to determine the image blur resulting from erratic operation of the Apollo 15 panoramic camera IMC and to evaluate the nominal resolution in multiple generations of Apollo 16 and 17 metric and panoramic photography.

Contrast or tonal quality can be described as the fluctuation in density due to large area intensity differences in the scene (i.e. low frequency brightness variations). Such fluctuations are frequently used to classify areas or delineate regional boundaries. The most common measure of tonal quality is the Hurter-Driffield or D-Log E response curve. It describes how the apparent scene brightness is reproduced as density in the photograph. The D-Log E response curve was measured for each generation of Apollo 16 and 17 imagery evaluated to assess the effect of the reproduction sequence on tonal quality or large scale detail content.

2.1 Frame Selection - In selecting the frames to be analyzed the primary consideration is the availability of test targets for estimating the MTF. A shadow-to-sunlight edge inside craters will occur in the low sun angle photography. If the sun angle is too high this edge either does not exist or is too close to the rim of the crater to be useful. Similarly at extremely low sun angles the edge will be sufficiently close to the far rim of the crater to make edge analyses impractical. Sun elevation angles between 10° to 15° yield the most satisfactory shadow-to-sunlight edge. Table 1 indicates the frames analyzed during the study.

**Table 1**
**APOLLO FRAMES SELECTED AND ANALYZED**

| MISSION NO. | CAMERA | FRAME NO. | GENERATIONS ANALYZED | OBJECTIVE |
|---|---|---|---|---|
| 15 | PANORAMIC | 316 | 4P | V/H ANOMALY |
| | | 360 | 4P | |
| 16 | PANORAMIC | 4127 | 2P, 2N, 3P | REPRODUCTION EVAULATION |
| | | 5460 | 2P, 2N, 3P | |
| 17 | PANORAMIC | 1633 | 2P, 2N, 3P | |
| | | 3111 | 2P, 2N, 3P | |
| | MAPPING | 180 | 2P, 2N, 4P | |
| | | 2923 | 2P, 2N, 4P | |

2.2 Calculating the MTF from Edges. It is common practice to use edges to calculate the MTF of an optical imaging system both in the laboratory and operationally. The MTF can be calculated using any target by dividing the measured image spectrum by the known spectrum of the object. In the case of an edge gradient analysis no correction for the target is necessary. The derivative of a sharp edge (i.e. gradient) is a narrow pulse (Dirac delta function) whose Fourier transform or spectrum is a constant proportional to the peak-to-peak amplitude of the edge. In selecting the edge for analysis we only require that it be much sharper than the degradation introduced by the optical system being evaluated. In the present case the shadow-to-sunlight edge sharpness is determined by the penumbra effect produced by the finite angular size of the sun. This effect is proportional to the diameter of the

crater. To insure sufficient sharpness we selected craters about 10 resolution elements wide (about 80 microns on panoramic imagery).

The edges are scanned using a microdensitometer and recorded on a strip chart or magnetic tape. The edge sharpness requirement produces targets that are short and curved. Consequently the microdensitometer scanning aperture must be small in both dimensions. If the edges were long, a long but narrow slit could be used to minimize the noise in the edge trace. Since this is not possible the density traces on the strip chart were hand-smoothed and digitized using a Calma digitizer. The data flow is summarized in Figure 2. A special software package, not shown in the figure, supports the Calma to generate a card deck containing the edge traces. The Edge Gradient Spectral Analyses (EGSA) software package shown in the figure consists of two major programs. The initial program reads the digitized density trace, converts it to exposure (illuminance trace) and writes an output tape for subsequent processing by the EGSA program itself. The latter is composed of a main program and several subprograms whose linkage is shown in the macro-flow diagram in Figure 3. In the discussion below we identify the purpose of each of these programs while detailed information is presented in the appendix. This was included in this report for those readers who may be interested in evaluating the detail content of other NASA imagery.

The first major step in the EGSA main program is the input of user data on punched cards in format 2F10.0, 7A4, F10.0. The following variables or arrays are specified for each data set to be processed:

FINV - spatial frequency interval desired for output OTF data in cycles/mm

FMAX - maximum spatial frequency of output OTF data in cycles/mm

OPTION (7) - alpha-numeric array identifying the types of output data desired by user

FCO - cutoff spatial frequency in cycles/mm used in smoothing the line spread

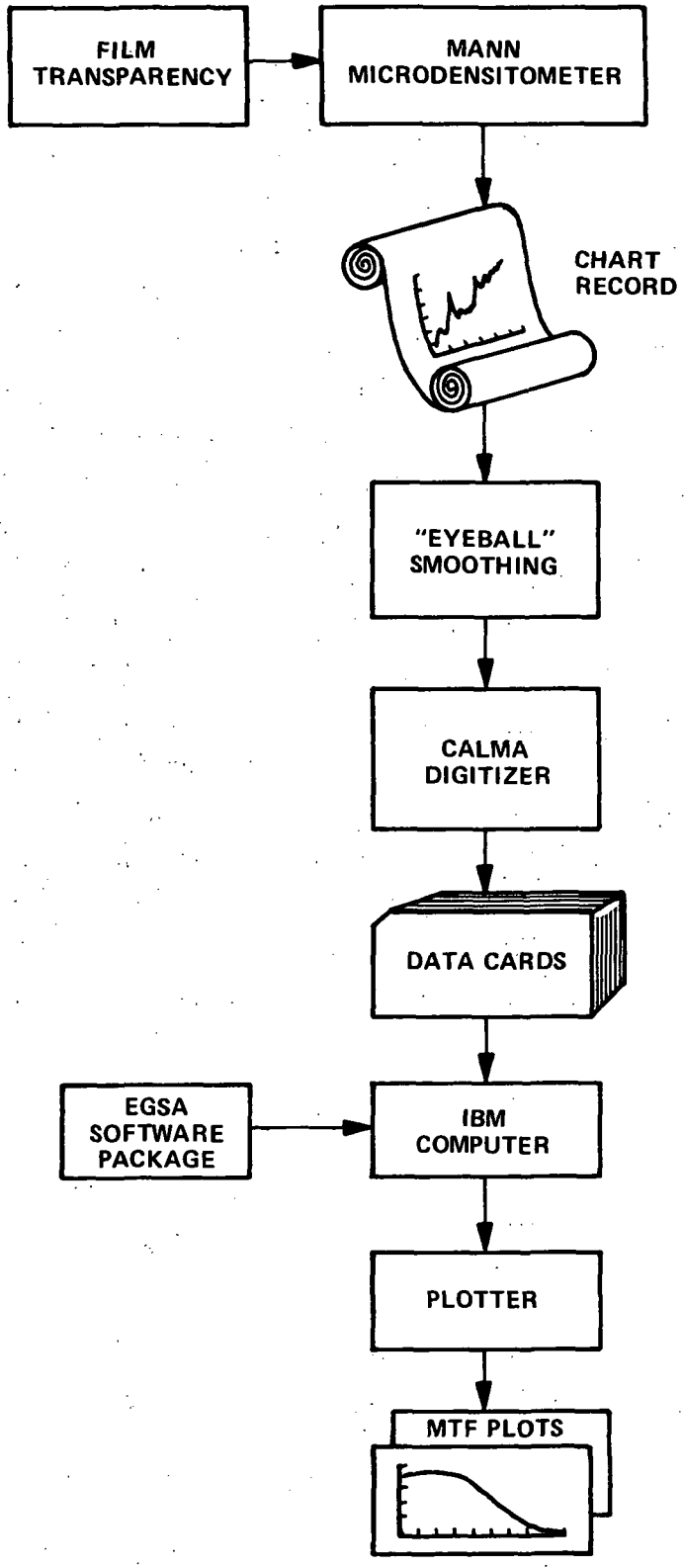



Figure 2 DATA FLOW FOR MTF DETERMINATION FROM EDGE TRACES

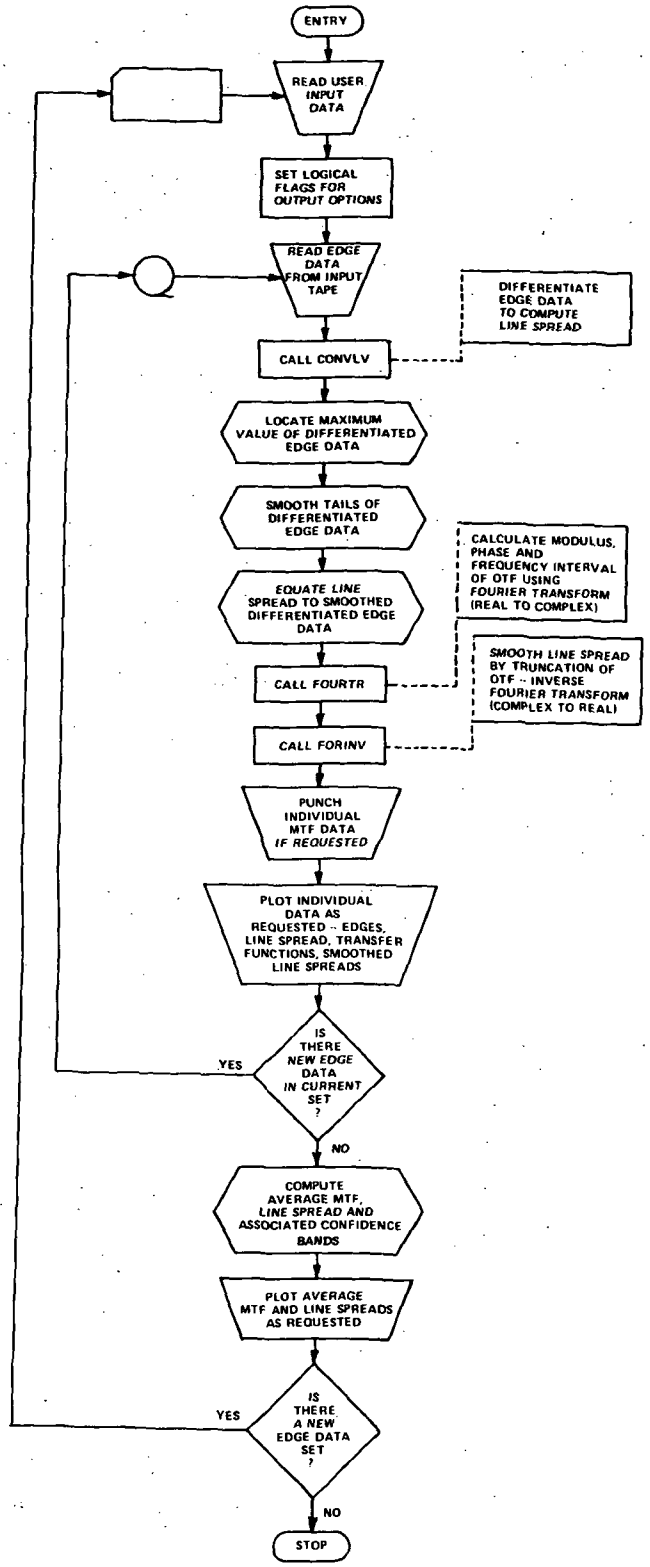



**Figure 3** MACRO FLOW DIAGRAM FOR EGSA MAIN PROGRAM

The edge data is read from an unformatted input data tape and differentiated by Subroutine CONVLV. A special Fourier transform algorithm is employed in Subroutine FOURTR to compute the modulus (MTF) and phase of the Optical Transfer Function (OTF). If requested by the user as an option a smoothed Line Spread is computed using truncation of the OTF at frequency "FCO." The remainder of the main program provides plotted, printed or punched card output data depending upon the options identified by the user. The options available include: plots of the individual edge data, line spreads, smoothed line spreads and transfer functions (MTF only); plots of the average line spreads and MTF; and punched values of the individual and average MTFs.

Provision is made for processing multiple edge data sets under multiple options. The input data tape is read until a null record is encountered indicating the end of the current data set. The output requirements are then completed for that edge data set. The input data tape is read a second time for the next edge data set. If one exists it is processed under the same user specified data and options as the previous set. If a second null record is encountered, however, the program recycles to read new user input data. If no input is provided execution is terminated. For a single edge data set the null records may be deleted and execution will be terminated by the same sequence of operations through recognition of the end of identified files on the data tape and the main stream input.

In order to exercise the EGSA main program sixteen subprograms are required in addition to the standard FORTRAN I/O subprogram. Four of these programs are user supplied and their listings are included in the Appendix. CONVLV performs the convolution between two input functions or an input function and an internally specified function. These include an 11-point smoothing function and an 11-point differentiating function. DXSCAL computes the increment or scale factor (inches/point) for the x-axes of the individual function plots (edges, line spreads and smoothed line spreads). FORINV performs an inverse Fourier transform based upon a special algorithms for

transforming a complex function, inputed as modulus and phase, to a real output functions (i.e. OTF to Line Spread). FOURTR performs a Fourier transform of a real inputed function providing a complex output function as modulus and phase (i.e. Line Spread to OTF). The restrictions on the type of functions permits the use time saving algorithms in both FORINV and FOURTR without restricting the number of points in the input function as the more popular Fast Fourier Transform (FFT) algorithms does.

Ten of the required subprograms are part of the Calspan Corporation Program Library; all but one are used to generate graphic output. AXIS draws an axis with tick marks at one inch intervals, annotates the value of the variable at each of these marks, and provides an identification label. CLEAR fills each byte of a specified block of storage with zeros. This operation could be accomplished, less efficiently, by a sequence of do loops in the EGSA program. EFPLOT is used to close the plot output file after all plot output is completed. LINE actually causes a plot to be drawn, consisting of line segments connecting specified data point coordinates expressed in inches through two data arrays. MGRID provides a grid of variable dimensions and interval size in the X and Y directions independently. NUMBER converts a floating point number and draws the resulting alphanumeric characters on a plot. In the EGSA program it is used to place identification data and scale factors on the output plots. PLOT is a basic plotting routine used to control pen position. In the EGSA program it is used to reset the plotter origin. PLOTER is used to specify the plotting instrument, lined or unlined paper and the type and color of pen desired. SCALE scan the elements in a data array to find maximum and minimum values, computes a optimum scale value based upon plot dimensions (inches) and then changes the array values to corresponding "plotter inches." SYMBOL draws selected symbols or special special characters on a plot. In the EGSA program it is used to title some of the plots.

The remaining two subprograms are IBM supplied FORTRAN IV LIBRARY Subprograms. AMAX1 locates the largest value in an array of real variables. SQRT is a mathematical function that computes the square root of a real argument.

Listings of the first group of subprograms are included in the appendix. The subprograms in the remaining two groups are not included since the user is likely to have access to the same or equivalent programs within his computing facility. The appendix also includes examples of the user supplied input data cards, description of the record sequence for the unformatted input data tape and examples of the output data generated.

2.3 Tone Quality Evaluation Methods - Tone quality is related to density differences of large objects in an image. As the object area decreases in size the OTF reduces the density differences achieved between the object and background. Tone quality is expressed by the net D-Log E response curve which determines the density difference between a large object and the background as illustrated by the sketch in Figure 4. If we assume that apparent scene

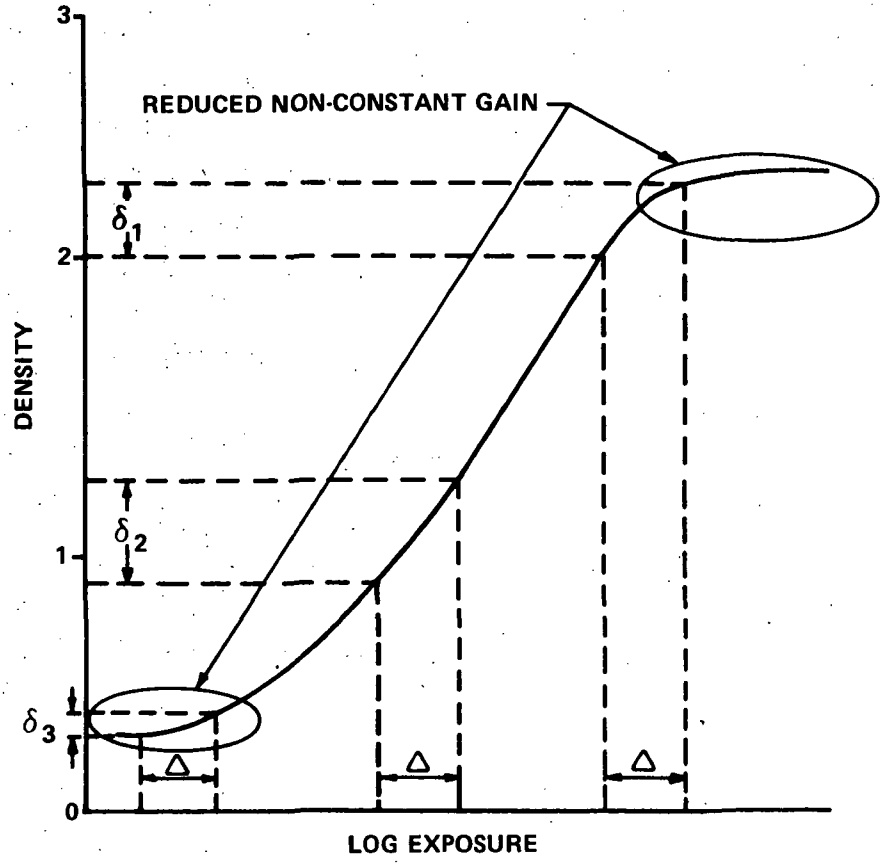



**Figure 4** EFFECT OF D-LOG E RESPONSE CURVE UPON OBJECT CONTRAST AND TONE QUALITY

brightness is such that the difference in log exposure between the object and background is

$$\Delta \equiv \log \frac{E_o}{E_B} \tag{1}$$

then the average exposure level ($= \frac{E_o + E_B}{2}$) influences the density difference. This average exposure level includes the camera parameters (f-number, shutter speed, lens transmission, etc.) as well as the exposure settings used to copy the original film. If the level is too high or too low the density difference (i.e. $\delta_1$ or $\delta_3$ indicated in the Figure) is less than $\delta_2$ achieved by exploiting the middle portion of the D-Log E response curve. This portion is nearly linear and is customarily represented by a straight line equation, namely

$$D = \gamma \log E + k \tag{2}$$

where $\gamma$ is the net film gamma and represents a constant gain between log exposure and density. At the higher and lower exposure levels the response curve departs from linearity; the gain is reduced and no longer is constant.

Differences in log exposure, $\Delta$, can be related to object contrast,

$$C = \frac{E_o - E_B}{E_B} \tag{3}$$

namely,

$$\Delta = \log(C+1) \tag{4}$$

and the resulting density difference, $\delta$, in the constant gain portion of the curve determined by using Eqs. (2) and (4);

$$\delta = \gamma \Delta = \gamma \log(C+1) \tag{5}$$

The ability of the eye to detect an object is approximately proportional to its density difference. In this case the density difference depends only on

the object contrast and the net film gain. In the non-constant gain regions the density difference is reduced and also depends upon the exposure levels. Therefore an appropriate measure of tone quality is the gain achieved as a function of object contrast and average exposure level, namely

$$\gamma(C, E_A) = \delta(C, E_A)/\Delta \tag{6}$$

This quality parameter was calculated in this study using a spline interpolation between the measured step wedge densities (sensitometric calibration). The reader should be made aware that the step wedge densities and resulting response curve can depend upon the type of instrument used in measurement. In general macro-densities measured by a densitometer (such as a MacBeth Model TD102) are not always equal to micro-densities measured by a microdensitometer. The first instrument measures "diffuse" density while the latter generally measures "specular" density. The data in this study were obtained using a microdensitometer and our tone quality evaluation is more appropriate to the use of magnifications (i.e. microscope) when viewing the image rather than unaided viewing.

Calculations of gain factors, Eq. (6), were made at three levels of object/background contrast indicated in Table 2. The high contrast object would represent shadow/sunlight boundaries in Apollo imagery; the mid-contrast, albedo variations associated with bright rays; while the low contrast represents albedo changes caused by 1% reflectance variation about a 10% background reflectance.

**Table 2**
**OBJECT CONTRASTS USED IN TONE QUALITY EVALUATION**

| CONTRAST LEVEL | $E_o/E_B$ | C | $\Delta$ | TYPICAL LUNAR OBJECT |
|---|---|---|---|---|
| HIGH | 6:1 | 5.0 | 0.778 | SUNLIGHT/SHADOW BOUNDARY |
| MID | 1.3:1 | 0.30 | 0.114 | BRIGHT RAY |
| LOW | 1.1:1 | 0.10 | 0.041 | 1% ALBEDO CHANGE |

## 3. EVALUATION OF APOLLO 15 PANORAMIC CAMERA V/H ANOMALY

The Panoramic Camera flown on several Apollo missions was a modified Itek KA-80A camera. During operation its lens system is rotated about the velocity axis of the Command Module at a rate related to the apparent ground speed. When a photograph was taken a capping shutter opened while the lens scanned a 108° arc below the spacecraft producing a frame about 45 inches long. A variable width slit located next to the film acted as a scanning shutter. Its velocity (scanning rate) and width determined the instantaneous exposure time. Image motion compensation (IMC) is provided by a complex electro-mechanical subsystem. Forward motion is compensated by a gimbal that rocks a lens group to stabilize the image. A V/H (velocity/height) sensor measures the apparent ground motion to determine the rate of rocking needed. This sensor also controls the speed of lens system rotation and consequently the rate of film advance which is perpendicular to the flight direction. A second sensor, an exposure meter, measures the brightness of the lunar scene which is combined with the V/H data to control the width of the scanning slit.

During the Apollo 15 mission intermittent operation of the V/H sensor produced an anomaly in a number of the panoramic photographs. One of the tasks undertaken was to assess the effect of these anomalies on the detail content. In the photographs examined they resulted in (1) a loss of resolution due to image blur from uncompensated motion and (2) exposure differences within the frame. The latter effect is clearly visible in Figure 5 which shows Frames 316 and 360 evaluated in this task. These frames were taken at low sun elevation angle and selected to provide shadow-to-sunlight edges for evaluation. The exposure differences are thought to be a direct consequence of the low sun angle. Telemetry data shows that the camera exposure meter estimated the average lunar surface brightness to be about 200 foot-Lamberts for Frame 316 and less than 100 foot-Lamberts for Frame 360. In both cases the telemetry data reproduced in Figure 6(c) shows that the focal plane or scanning slit remained at maximum width (0.3 inches) during the complete exposure of both frames due to the low surface brightness. Thus the abrupt change in the V/H rate

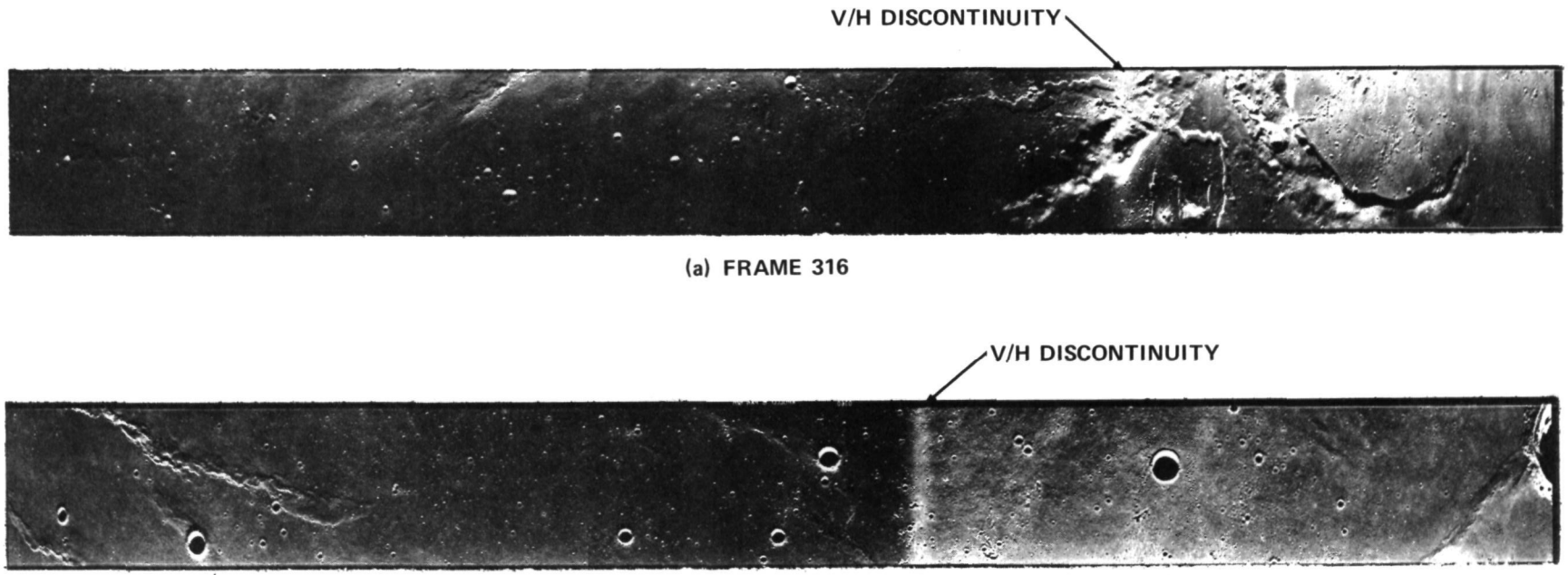



Figure 5 TYPICAL APPOLLO 15 PANORAMIC PHOTOGRAPHS WITH V/H ANOMALIES

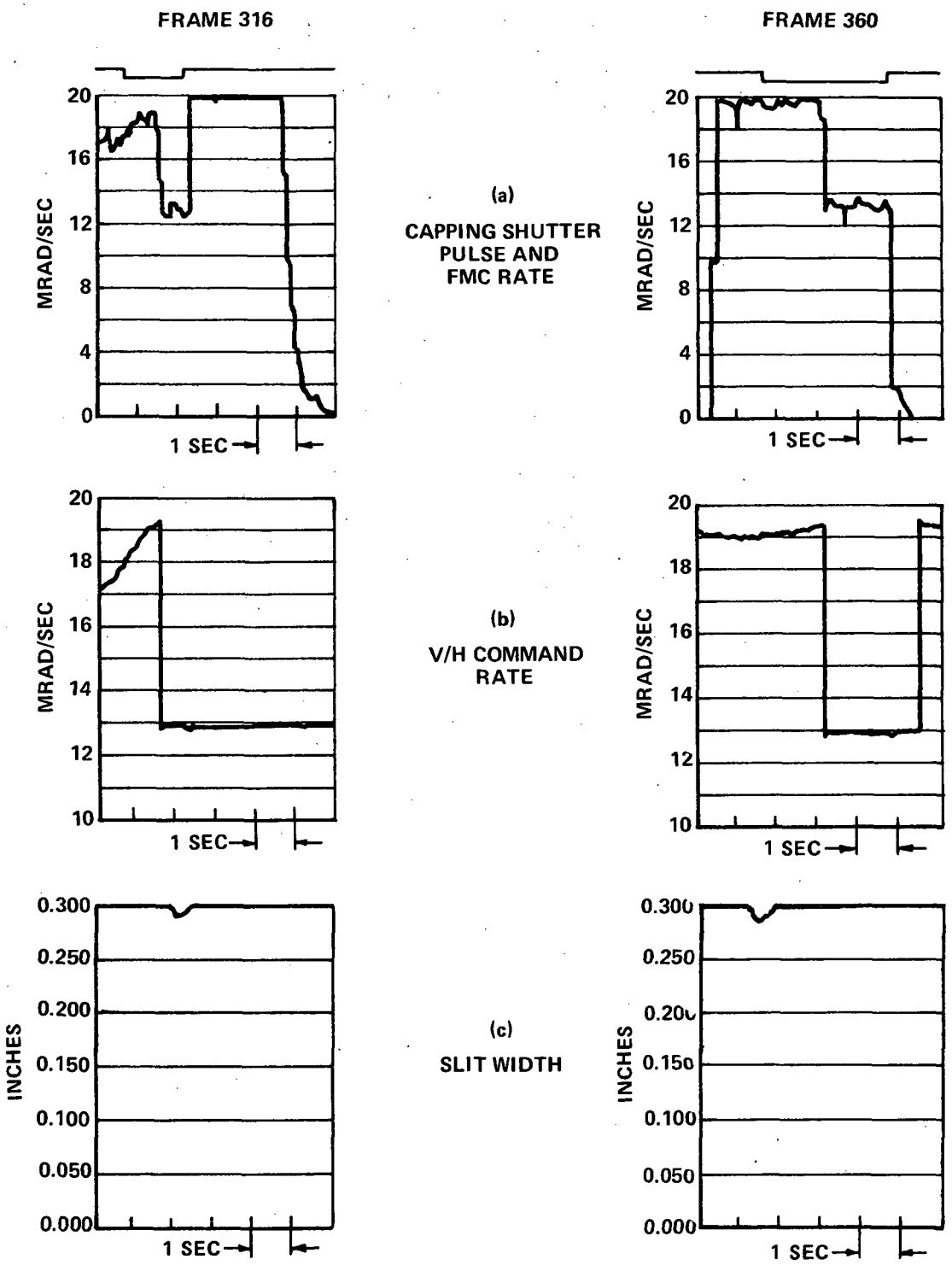



Figure 6 STRIP CHART RECORDS OF TELEMETRY DATA

indicated in 6(b) produced a corresponding change in lens rotation and the velocity of the slit relative to the film without a compensating change in slit width. This resulted in the exposure differences. In frames taken at higher sun angles or brighter lunar surfaces the slit width could be adjusted permitting reduction or elimination of these exposure changes.

The abrupt change in the commanded V/H rate was produced by the V/H sensor signal "going into" or "coming out of" saturation. When the signal exceeded 19.5 mrad/sec (established as the maximum rate that could be encountered in the Apollo missions) the commanded V/H rate was automatically reset to 13.0 mrad/sec (a nominal value) producing the abrupt changes shown in Figure 6(b). When these changes occurred while the capping shutter was open, Figure 6(a), the detail content of the photography was reduced.

To estimate the amount of residual motion blur produced by this V/H rate anomaly, microdensitometer traces were made of the shadow-to-sunlight edges on both sides of the abrupt change in commanded V/H rate. From the telemetry data presented in Figure 6 we see that the V/H sensor signal reached the saturation value and the V/H command was set to the nominal value for both of the frames evaluated. The first portion of Frame 316 is dark and has a commanded V/H rate varying from 17.6 to 19.3 mrad/sec and the latter portion is light with a V/H rate of 13.0 mrad/sec. Similar conditions exist for Frame 360 except that the commanded V/H rate is nearly constant equal to 19.2 mrad/sec in the dark portion of the frame. None of the commanded V/H rates resulted in full compensation of the image motion. The residual blur was measured using the EGSA program and also calculated directly from telemetry and auxiliary data. The results of both of these estimates are compared in this section.

3.1 Residual Blur Estimation - Telemetry and auxillary data was used to estimate the residual blur in both the dark and light portions of each of the frames evaluated. This was achieved by calculating the difference between the uncompensated angular blur and the amount of compensation provided. In performing these calculations, estimates of the angular blur rates and

exposure times were made using the available data sources and measurements of the photographs themselves. We discuss the calculations of the uncompensated angular blur rate, the compensation rate, the exposure time and the residual blur separately in the subsections below.

3.1.1 Uncompensated Angular Blur Rate - In order to compute the angular blur rate due to the velocity of the Command Module provision must be made for the curvature of the lunar surface. In a previous study[1] expressions for the angular blur near the principal point in vertical and forward or aft oblique photography were developed. The expression for the ground blur at the lunar surface is

$$B_G = \frac{Vt_e}{1 + h/r_m} \tag{7}$$

where $V$ is the velocity of the Command Module, $t_e$ is the exposure time, $h$ is the altitude of the Command Module and $r_m$ is the nominal lunar radius (= 1738.1 kilometers). As the altitude decreases, this expression is asymptotic to the more familiar expression in which the blur is proportional to the product of the platform velocity and the exposure time. For Apollo orbital photography, the reduction in the blur due to the interaction of the Command Module altitude and the lunar surface curvature is approximately 6%.

In our earlier study, all assessments were made near the principal point in each photograph and the theoretical expressions developed are only valid under these conditions. In order to evaluate the Apollo 15 V/H nominally, our assessments had to be made away from the principal point in order to evaluate the MTF and residual blur on both sides of the V/H anomaly. Figure 7 shows a sketch where a point in a forward oblique pan photograph is under evaluation. The point is specified by a pitch angle, $\phi$ , and a roll angle, $w$ . In general the pitch angle would be the sum of the camera pitch angle plus an additional term due to the location of the point across the width of the photograph. In the present case however, all assessments were made near the optical axis ground track where the additional term is negligible. The pitch

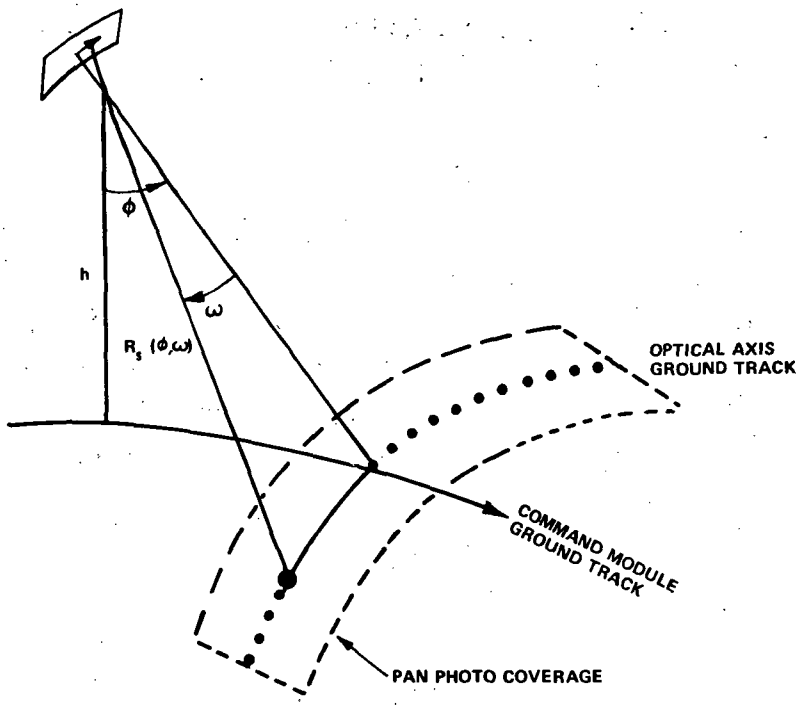



**Figure 7 PAN PHOTO GEOMETRY**

angle will result in a decrease in the apparent ground blur due to foreshortening as discussed below. The roll angle, on the other hand, being perpendicular to the Command Module ground track does not produce any foreshortening effects, however, the ground blur distance will be decreased from that expressed by Eq. (7) due to the fact that the point under assessment is away from the principal point by an amount expressed through the roll angle. For a vertical photograph an additional reduction factor, $\cos\theta$ , must be included in Eq. (7) so that

$$B_G = \frac{V t_e \cos\theta}{1 + h r_m} \tag{8a}$$

where

$$\theta = \sin^{-1}\left[\frac{R_s \sin w}{r_m}\right] \tag{8b}$$

and $R_s$ is the slant range from the Command Module to the point under evaluation. The altitude of the Command Module is sufficiently small

compared to the lunar radius in this case so that this reduction factor is negligible in vertical panoramic photography. For oblique photography, the pitch and roll angles interact producing a more complicated expression for the reduction factor, however, its effect remains negligible.

The next step is the computation of the apparent ground blur by including the effects of foreshortening on the actual ground blur given by Eq. (7). Both the roll and the pitch angle can contribute to the foreshortening effect through the tilt angle given by

$$\cos t = \cos\phi \cos w \tag{9}$$

When the pitch angle is zero, however, the roll angle produces no foreshortening effect since all of the ground blur is perpendicular to the line of sight. The foreshortening factor, $\cos\phi$ , was included in the expressions presented in our previous study. In the presence of a roll angle the foreshortening factor is $\cos t$ as presented by Eq. (9). However, this factor should only be applied to the portion of the blur in the viewing direction. This requires knowledge of the azimuth angle between the direction of flight and the direction of tilt. Kawachi[3] presents the required transformation equations permitting us to write the apparent blur as

$$B_A = \frac{V t_e \left(\sin^2\alpha + \cos^2\alpha \cos^2 t\right)^{1/2}}{1 + h/r_m} \tag{10a}$$

where the azimuth angle, $\alpha$ , is given by,

$$\alpha = \tan^{-1}\left[\frac{\sin w}{\tan\phi}\right] \tag{10b}$$

For the values of roll angles which we are considering, however, this expression can be approximated by the original on-axis equation, namely

$$B_A \approx \frac{V t_e \cos\phi}{1 + h r_m} \tag{10c}$$

This expression is a valid approximation because of two opposing effects; although the foreshortening factor is smaller, i.e. $\cos t$ compared to $\cos\phi$, it is only applied to a smaller portion of the actual ground blur. These two facts compensate for each other, allowing the foreshortening effect to be approximated by $\cos\phi$ with an error less than 0.1%.

The foreshortening effects, including the geometrical equations developed by Kawachi, assume a flat planetary surface. As a result of this assumption, the apparent ground blur will be slightly overestimated. Figure 8a shows the percentage error by which the blur is overestimated for two representative Command Module altitudes. For the present case, the error is less than 0.5% since the pitch angle will be either 0° or 12.5° determined by the mode of camera operation.

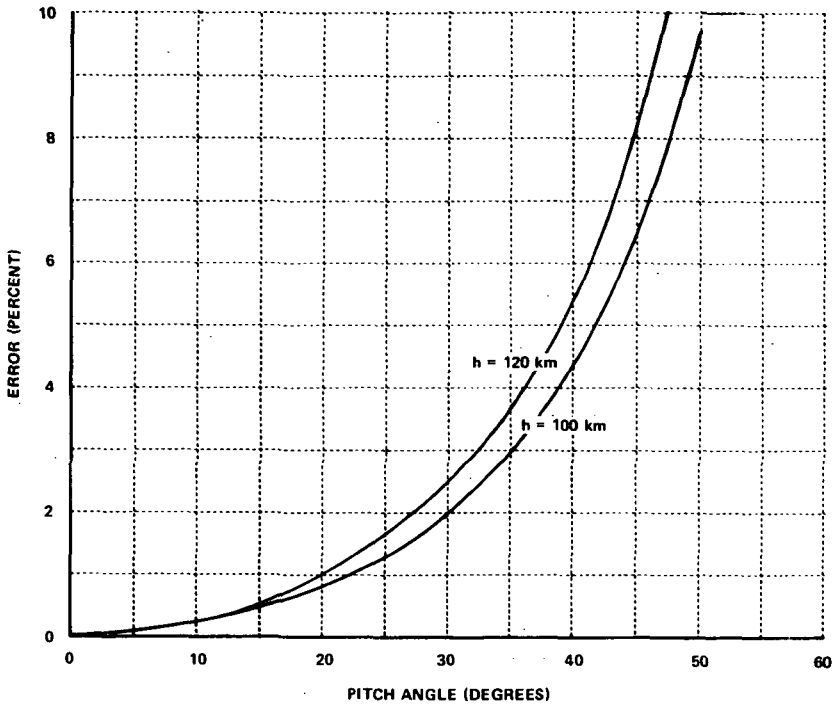
The next step in the calculation of the uncompensated angular blur rate is the conversion of the apparent ground blur to an apparent angular blur by the use of the slant range from the Command Module to the ground point under evaluation. In our earlier study, the expression for slant range employed was

$$R_s \simeq \frac{h}{\cos t} \tag{11}$$

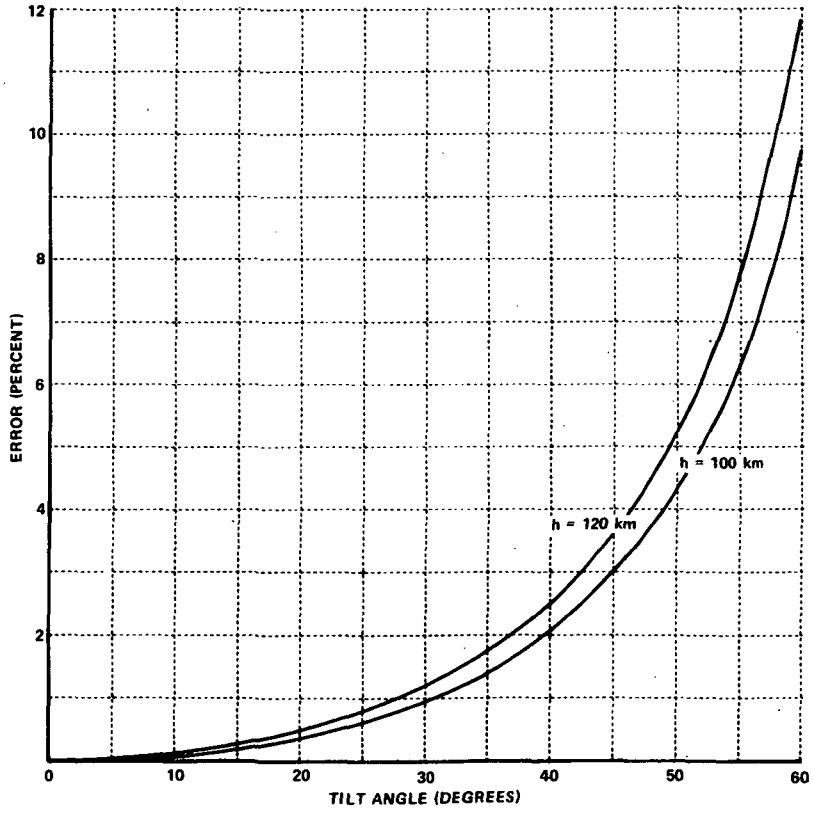
which is based upon the assumption of a flat planetary surface. The actual slant range is larger due to the curvature of the lunar surface and is given by

$$R_s = (h + r_m)\cos t - \left[r_m^2 - (h + r_m)^2 \sin^2 t\right]^{1/2} \tag{12}$$

Figure 8b shows the error produced in the estimation of slant range by assuming a flat lunar surface. The net effect of the assumption is to overestimate the angular blur similar to the effect on foreshortening. In this case, however, errors on the order of 1% are expected since tilt angles can become as high as 30°. Finally, the uncompensated angular blur rate is determined by taking the apparent ground blur of Eq. (10c) and dividing by the slant




(a) FORESHORTENING EFFECT ERROR




(b) SLANT RANGE ERROR

Figure 8 ERRORS RESULTING FROM THE ASSUMPTION OF A FLAT LUNAR SURFACE

range expressed by Eq. (12) and the exposure time, $t_e$ . The uncompensated angular blur rate, $\dot{\beta}$ , is

$$\dot{\beta} = \frac{V \cos \phi}{(1+h/r_m)\left\{(h+r_m)\cos t - \left[r_m^2 - (h+r_m)^2 \sin^2 t\right]^{1/2}\right\}} \tag{13}$$

This expression was employed with the numerical values shown in Table 3 to calculate the blur rate for both the dark and light portions of the frames under evaluation. The values of the Command Module velocity and altitude and the pitch angle were obtained from auxillary data furnished by Mr. K. Hancock at the NASA Johnson Space Center. The roll angles were estimated by measuring the distance of the dark and light evaluation regions from the center of the frame which is directly proportional to the roll angle. The tilt angles are subsequently calculated from Eq. (9).

**Table 3**

**NUMERICAL VALUES USED TO COMPUTE UNCOMPENSATED ANGULAR BLUR RATE**

| VARIABLE | SYMBOL | VALUES | |
|---|---|---|---|
| | | FRAME #316 | FRAME #360 |
| CM VELOCITY | V | 1.6279 km/sec | 1.6305 km/sec |
| CM ALTITUDE | h | 109.22 km | 106.14 km |
| LUNAR RADIUS | $r_m$ | 1738.1 km | 1738.1 km |
| PITCH ANGLE | $\phi$ | $12.45^o$ | $0$ |
| ROLL ANGLES | $W_D$ | $20.4^o$ | $6.0^o$ |
| | $W_L$ | $25.2^o$ | $10.8^o$ |
| TILT ANGLES | $t_D$ | $23.8^o$ | $6.0^o$ |
| | $t_L$ | $27.9^o$ | $10.8^o$ |

The following values resulted; Frame 316

$$\dot{\beta}_D = 12.45 \text{ mrad/sec}$$
$$\dot{\beta}_L = 11.99 \text{ mrad/sec} \tag{14a}$$

and for Frame 360

$$\dot{\beta}_p = 14.39 \quad mrad/sec$$
$$\dot{\beta}_L = 14.20 \quad mrad/sec \tag{14b}$$

An expression equivalent to Eq. (13) could also have been developed by employing the work of Kawachi[3,4]. He derives expressions for the image velocity for an oblique frame camera and notes[4] that "the image velocity for the panoramic photograph is the same as for a frame photograph tangent to it at the point in question." If we are interested in evaluating the velocity near the center of the width of the panoramic frame we can set the transformed film coordinates equal to zero, i.e. $x' = y' = 0$ , in Eqs. (7) and (8) of Ref. 4 and obtain the net image velocity as

$$V_I = \frac{VF}{h} \cos t \left[\sin^2\alpha + \cos^2\alpha \cos^2 t\right]^{1/2} \tag{15}$$

where $F$ is the focal length of the camera and $\alpha$ is defined by Eq. (10b). The uncompensated angular blur rate is determined by dividing by the product of the focal length and the factor $1 + h/r_m$ included previously in Eq. (7). This factor is the principal correction for the curvature of the lunar surface. The resulting expression can also be obtained from Eqs. (10a) and (11) and is therefore equivalent to Eq. (13) except for the neglection of the effect of surface curvature upon slant range.

3.1.2 Blur Compensation Rate - The next step in computing the residual blur is the determination of the compensation rate applied by the camera. In the case of a panoramic camera[2] the blur compensation rate is proportional to the instantaneous rotation angle of the lens, namely,

$$\dot{\beta}_c = \left(\frac{V}{H}\right)_c \cos w \tag{16}$$

This reflects the decrease in angular blur due to the slant range assuming a flat planetary surface. It is not known whether the panoramic camera IMC computer was reprogrammed for the more complex effect of the slant range due

to the curvature of the lunar surface (c.f. Eq. (12)). If so, the compensation rate would be

$$\dot{\beta}_c = \frac{\left(\frac{V}{H}\right)_C}{\left\{\left(1+r_m/h\right)\cos w - \left[\left(\frac{r_m}{h}\right)^2\left(1+\frac{r_m}{h}\right)^2 \sin^2 w\right]^{1/2}\right\}} \qquad (17)$$

where $r_m/h \sim 16$ can be assumed to be constant. Referring back to Figure 8b, we see that the effect is less than 1% for the rotation or roll angles we are considering. However, near the edge of the frame where the rotation angle approaches 54° the error would be as large as 7%. Conversely, the uncompensated blur rate is considerably less in this portion of the panoramic frame and the need for an accurate compensation rate diminished. Consequently, it is assumed that such a modification of the IMC computer was not undertaken since it would offer little advantage. Using Eq. (16), the roll angles presented in Table 3 and the V/H command values from the telemetry data presented in Figure 6, the compensation estimates were estimated for Frame 316 to be

$$\dot{\beta}_{c_D} = 18.00 \ \text{mrad/sec}$$
$$\dot{\beta}_{c_L} = 11.76 \ \text{mrad/sec} \qquad (18a)$$

and for Frame 360

$$\dot{\beta}_{c_D} = 1909 \ \text{mrad/sec}$$
$$\dot{\beta}_{c_L} = 12.77 \ \text{mrad/sec} \qquad (18b)$$

We note that these results are only an approximation since the compensation rate is proportional to the instantaneous rotation angle and a given point in the panoramic frame will receive a varying amount of image motion compensation due to the finite width of the scanning slit. Consequently, Eq. (16) should be integrated over the exposure time of the point under evaluation to determine an average compensation rate, namely

$$\langle \dot{\beta}_c \rangle = \frac{1}{t_e}\int_{t_e} \dot{\beta}_c(t)\,dt \qquad (19)$$

This procedure was executed and the resulting average compensation rates were less than those presented in Eqs. (18a and 18b) by approximately 0.6%. In this case, therefore, the dynamic effect of the motion compensation can be neglected.

3.1.3 Exposure Time Estimation - Three different methods were used to estimate the exposure time of the dark or light portions of each frame under evaluation. As noted previously, the abrupt change in the average density across the V/H anomaly was attributed to a change in exposure time between these two portions of the frames. One method for determining the exposure times is based upon this fact, that is, the density difference is related to the ratio of the exposure times by the expression

$$\mathcal{R} \equiv \frac{t_{eD}}{t_{eL}} = 10^{\delta/\gamma} \tag{20}$$

where $\delta$ is the density difference and $\gamma$ is the gain of the D-Log E response curve. The length of time that the capping shutter is open, $t_c$ is equal to the sum of the times required to expose each portion of the frame, that is,

$$\begin{aligned} t_c &= \frac{L_D}{V_D} + \frac{L_L}{V_L} \\ &= \frac{L_D V_L + L_L V_D}{V_L V_D} \end{aligned} \tag{21}$$

where $L_D$ is the length of the dark portion of the exposed frame, $L_L$ is the length of the light portion of the exposed frame, and $V_D$ , $V_L$ are the velocities of the scanning slit in the respective portions of the frame. Since the width of the scanning slit, $W_S$ , is constant these velocities are inversely proportional to the exposure time and consequently we can equate the variable $\mathcal{R}$ of Eq. (20) to the ratio of the velocities, namely

$$\mathcal{R} = V_L / V_D \tag{22}$$

Combining Eqs. (21) and (22) and eliminating the velocity of the scanning slit

in favor of the ratio of the slit width to the exposure time, we can derive an equation for the exposure in the light portion of the frame

$$t_{e_L} = \frac{W_s}{\mathcal{R} L_D + L_L} t_c \tag{23a}$$

as well as an expression for exposure in the dark portion

$$t_{e_D} \mathcal{R} t_{e_L} \tag{23b}$$

These equations were employed with the numerical values shown in Table 4 to calculate the exposure times for both the dark and light portions of the frames under evaluation. The density difference was measured directly from the frames under evaluation while the gain of the response curve was obtained from Figure 31 of Reference 1 using the curve labeled "4th GEN POS (4P)". The capping shutter time and the scanning slit width were obtained from telemetry data shown previously in Figure 6. The values of the exposure time obtained using these expression are presented in Table 5 along with error estimates derived from first order analyses.

**Table 4**
**NUMERICAL VALUES USED TO ESTIMATE EXPOSURE TIMES**

| VARIABLE (UNITS) | SYMBOL | VALUE | |
|---|---|---|---|
| | | FRAME # 316 | FRAME # 360 |
| DARK-LIGHT DENSITY DIFFERENCE | $\delta$ | 0.42 | 0.36 |
| RESPONSE CURVE GAIN | $\gamma$ | -2.0 | -2.0 |
| EXPOSURE TIME RATIO | $\mathcal{R}$ | 0.61660 | 0.66069 |
| SECTION LENGTHS (INCHES) | $L_D$ | 32 | 26 |
| | $L_L$ | 13 | 19 |
| CAPPING SHUTTER TIME (SECONDS) | $t_c$ | 1.5 | 3.1 |
| SCANNING SLIT WIDTH (INCHES) | $W_S$ | 0.3 | 0.3 |

Table 5
EXPOSURE TIME ESTIMATES

| SOURCE | FRAME #316 | | FRAME #360 | |
|---|---|---|---|---|
| | DARK | LIGHT | DARK | LIGHT |
| EQS (23a) AND (23b) | 8.5 msec ± 9.3% | 13.7 msec ± 7.7% | 17.0 msec ± 5.3% | 25.7 msec ± 4.2% |
| MOLEBURG EQ. (25) | 9.2 msec ± 2.9% | 13.6 msec ± 2.6% | 17.6 msec ± 2.3% | 26.0 msec ± 2.6% |
| ADJACENT FRAME | N/A | 14.0 msec ± 5.0% | N/A | 25.3 msec ± 3.2% |

The second method used to estimate the exposure times is based upon an approximate formula which was furnished by Mr. B. Moleburg at the NASA Johnson Space Center, namely

$$t_e = \frac{K W s}{(V/H)_c} \tag{24}$$

where $K$ =0.59 for the stereo mode and 1.127 for the mono mode. The V/H factor in the denominator of this expression is the command V/H rate obtained from telemetry data; 19.2 milliradians/second for the dark portion and 13.0 milliradians/second for the light portion. The exposure times obtained using this expression are also presented in Table 5 with their associated error estimates.

The final method used to estimate the exposure time is only applicable to the light portion of each frame under evaluation. In both cases, the adjacent frame had an exposure level equivalent to the light portion of the frames under evaluation and consequently the exposure time in the light portion is equal to the exposure time of either adjacent frame. The equation for calculating the exposure time for a frame with a constant V/H command rate is obtained by setting $\mathcal{R}$ =1, $L_D$ =0 and $L_L$ = $L$ =45 inches in Eq. (23a), namely

$$t_e = \frac{Ws}{45}\, t_c \tag{25}$$

The results obtained using this method are also included in Table 5, and we note that the results from all three methods are in excellent agreement. In subsequent evaluations, results obtained from Eqs. (23a) and (23b) are employed.

3.1.4 Residual Blur - It is a straightforward procedure to combine the results derived above in order to determine the residual motion blur at the film plane. The equation for the residual image blur is

$$\delta_r = \left| \dot{\beta} - \dot{\beta}_c \right| t_e F \tag{26}$$

where $F$ is the focal length of the camera (=610 millimeters). This blur produces a Modulation Transfer Function described by the equation

$$\tau(\nu) = \frac{\sin(\pi \nu / \nu_o)}{\pi \nu / \nu_o} \tag{27}$$

which degrades the nominal fine detail content of the photograph. The parameter $\nu_o$ is simply the reciprocal of the residual image blur in cycles/mm. Values obtained for both the residual image blur and the "cutoff" spatial frequency are presented in Table 6.

**Table 6**
**RESIDUAL IMAGE BLUR**

| VARIABLE (UNITS) | FRAME #316 | | FRAME #360 | |
|---|---|---|---|---|
| | DARK | LIGHT | DARK | LIGHT |
| RESIDUAL IMAGE BLUR (mm) | .0288 ± 12% | .0020 ± 44% | .0224 ± 7% | .0488 ± 17% |
| CUTOFF FREQUENCY (CYCLES/mm) | 34.7 ± 4.2 | 512 ± 230 | 44.5 ± 3.1 | 20.5 ± 3.5 |

The expected value of the net MTF can be determined by multiplying the values obtained from Eq. (27) by the product of the MTF for the flight and subsequent copy films and the camera lens. Unfortunately, an MTF for the panoramic camera lens was not available, however, such lenses are noted for their high quality performance and it is very likely that the net MTF is dominated by the performance of the flight and copy films and any residual blur. Consequently, in determining the expected MTF the lens MTF was assumed to be 1.0 at all spatial frequencies evaluated.

A final comment is required concerning the precision of the approximations used in the expressions for the uncompensated angular blur rate and the compensation rate. In both cases only those approximations whose errors were less than 1% were retained. However, since we are employing difference between these two quantities in Eq. (26) a small error can be magnified when a small difference occurs. Eq. (13) for the uncompensated angular blur rate employs two approximations for the inclusion of foreshortening effects; the assumption of a flat planetary surface and equivalence between the off-axis and on-axis reduction factors. For vertical photography, these approximations introduce no errors. However, for oblique photography, the net effect is to over estimate the blur rate by about 0.5%. Eq. (16) for the blur compensation rate neglects the dynamic effects of motion compensation and over estimates by 0.6% compared to the average compensation rate. This occurs for both the oblique and vertical photography. In the case of oblique photography, since both the blur rate and compensation rate are over estimated by about the same percentage, the effect upon the residual image blur is negligible (i.e., about 0.6%) even when there are small differences between the rates. In the case of the vertical photography, only the compensation rate is in error and this is subsequently magnified by the differencing operation resulting in an under estimate of the residual image blur by 5% in the light portion of Frame 360 and an over estimate of the residual image blur by 3% in the dark portion of Frame 360. These errors are still less than the net accuracy of the residual blur estimates shown in Table 6.

3.2 Modulation Transfer Function Estimation - As noted previously in Section 2, a net MTF was estimated for each photo under evaluation. This was accomplished by tracing shadow-to-sunlit edges interior to craters using a microdensitometer and subsequently processing this edge data using the Edge Gradient Spectral Analysis (EGSA) software package described in Section 2.2.

A Mann microdensitometer was used to obtain the required edge data. For this purpose the microdensitometer was used in an improved but unconventional mode of operation. A 41 $\mu$m by 41 $\mu$m source aperture was employed with a 0.4 N.A. influx objective. This offered a magnification of approximately 40x producing a 1.0 $\mu$m square geometric image at the film plane. This image is subsequently broadened by the diffraction effects of the objective whose airy disc has a diameter of 1.5 $\mu$m. Consequently, the image of the source aperture will be a gaussian-like light distribution with a diameter of several micrometers. A 0.25 N.A. objective was used for the efflux optics and the subsequent sensor aperture chosen to minimize stray light and not further define the scanning aperture. In this mode of operation the distorted image of the source aperture is the scanning aperture and the spatial frequency response of the microdensitometer determined accordingly. The over filling of the efflux optics by the influx optics offers an improvement in instrument performance.

Edge data was collected in both the dark and light portions of each of the frames about 1 inch on either side of the suspected V/H discontinuity. This edge data was then hand smoothed and digitized for processing by the EGSA software. The average MTFs resulting from these measurements are presented in Figures 9 and 10 along with the expected MTF determined by the procedures described in Section 3.1.4. In estimating the average MTF through modulus averaging, 10 edges were measured in the dark portion of Frame 316 while 7 edges were measured in the light portion. Similarly, 11 edges were measured in the dark portion of Frame 360 and 10 edges in the light portion.

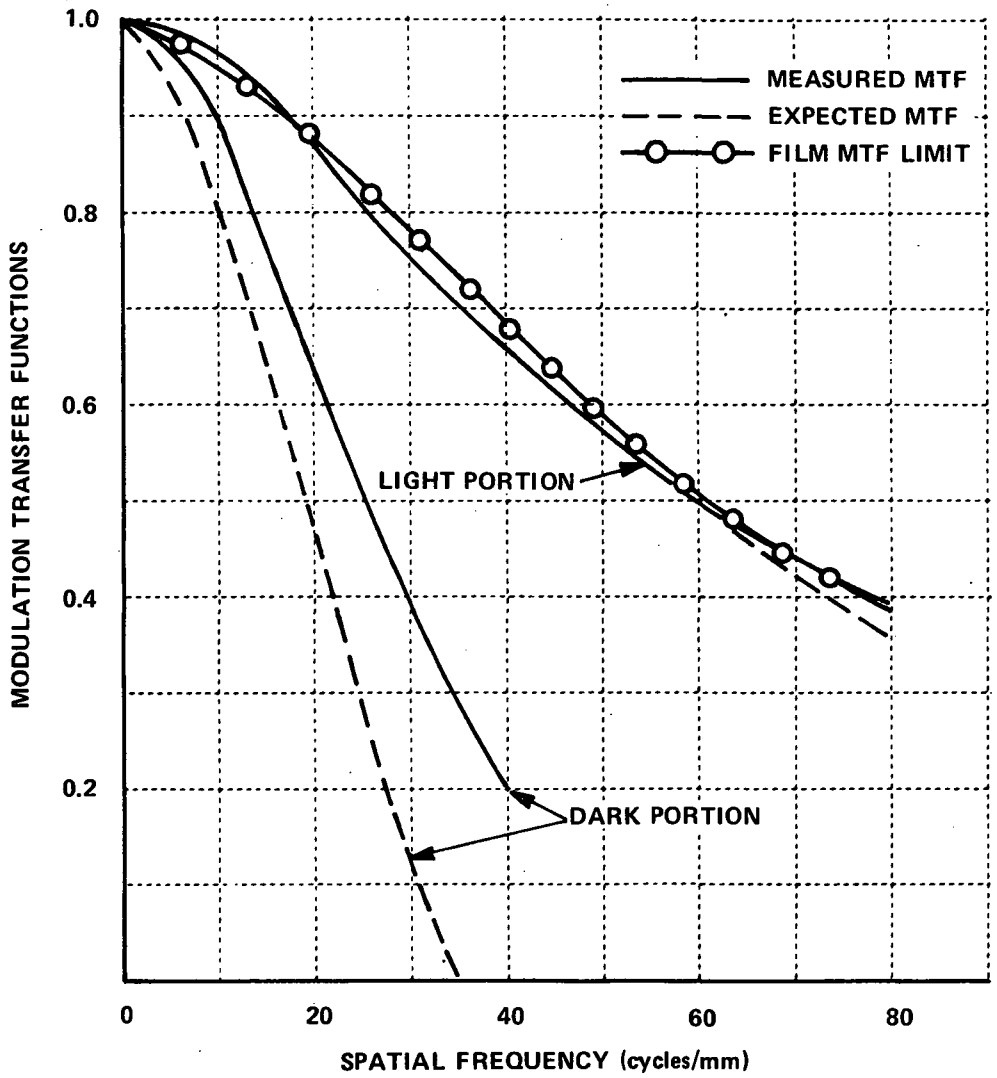



Figure 9 MEASURED AND EXPECTED MTF's FOR APOLLO 15 FRAME 316 WITH V/H ANOMOLY

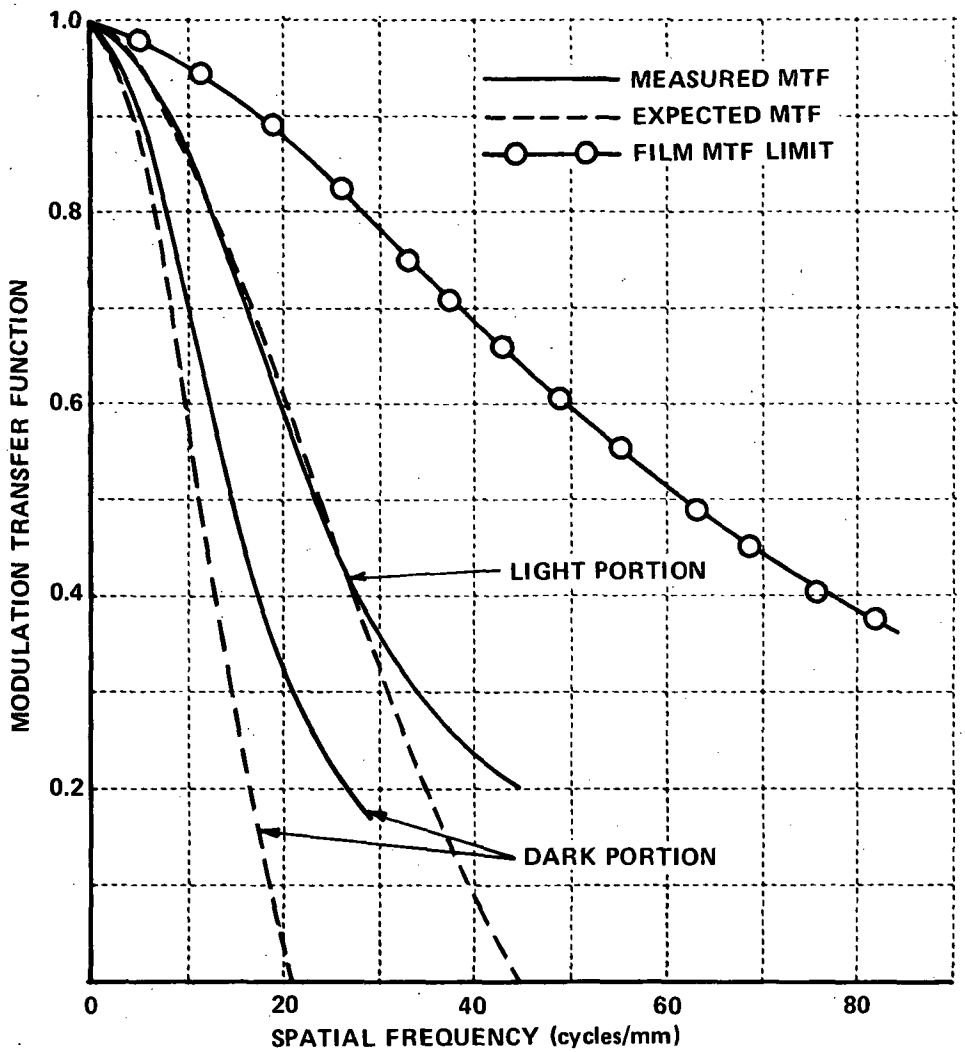



Figure 10 MEASURED AND EXPECTED MTF's FOR APOLLO 15 FRAME 360 WITH V/H ANOMOLY

Also shown in the Figures is the net film MTF limit for the 4th Generation Positive copies employed in the evaluation. We note that there is excellent agreement between the expected and measured MTF values in all four instances. For the light portion of Frame 316 the measured and expected MTF values are essentially identical and equal to the net film MTF limit over the range of spatial frequencies shown. This is a result of the nearly perfect compensation provided in this case. In all the other cases the compensation was poor and the resolution reduced by 1/3 (to about 6 meters on the lunar surface).

Other low sun angle photography known to have V/H discontinuities include Frames 8852 (Rev. 4) and 344 (Rev. 72). When such a discontinuity did not occur the V/H command signal was either at the nominal value (13.0 mrad/sec) similar to the light portions of the frames evaluated or varying about an anomalously high value similar to the dark portions. Telemetry data indicates that the latter situation occurred at least for Frames 345, 346 and 347 (Rev. 72). Only when it is equal to the nominal value can the full "resolution" potential of the panoramic camera be realized; but this may not always occur (refer to the MTF for the light portion of Frame 360). The user of Apollo 15 panoramic data should refer to the telemetry data to determine the V/H command situation for the frames he is exploiting and account for the effects of poorer resolution if necessary.

## 4. COMPARISON AMONG MULTIPLE GENERATIONS OF APOLLO 16 AND 17 PHOTOGRAPHY

One of the specific tasks accomplished during this study was an assessment of the detailed content of multiple generations of Apollo 16 and 17 photography. The purpose of this assessment was to determine the degradation, if any, in the quality of the photographic products furnished to various users. The original flight film is copied to produce a Master Positive (2P) or Direct Negative (2N) copy. These are the earliest generations which a user may exploit and only a limited number of such copies are generated. The Master Positive is subsequently copied twice to produce a 4th Generation Positive (4P) which most investigators are likely to use. The Direct Negative is also copied once to produce a Third Generation Positive (3P) which is also available to investigators. This latter sequence eliminates one copy step in the hope of improving product quality or reducing production time. Unfortunately the characteristics of the reversal film required to produce the 2N copy may limit the detail content and no actual improvement in product quality obtained. The purpose of this task was to assess the effect of both reproduction sequences on the detail content of Apollo 16 and 17 photography.

An inter-comparison of the total quality and the fine detail content was made for Apollo 16 panoramic photography, Apollo 17 panoramic photography and Apollo 17 metric photography. The frames selected and subsequently evaluated were shown previously in Table 1. Unfortunately 3P and 4P copies of the same type of photography were not available at the Lunar Science Institute thus preventing a direct comparison of the most likely user products. Instead a comparison of preceeding generations (2N and 2P) was made. The results of the evaluations are discussed in detail in the subsections below.

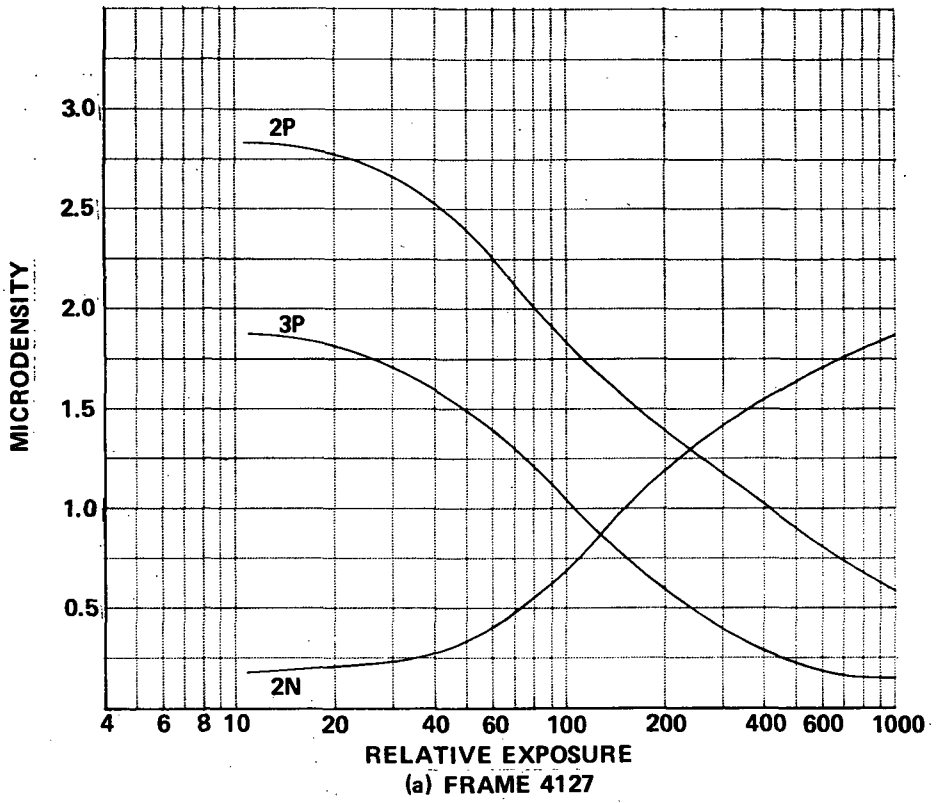
4.1 Product Tonal Quality - The methodology used to assess the tonal quality of the Apollo 16 and 17 photography was discussed previously in Section 2. The most common measure of tonal quality is the Hurter-Driffield or D-Log E response curve. It specifies how the apparent scene brightness is reproduced as density in the photograph. The D-Log E response curve was measured for each frame evaluated. The results obtained are presented in

Figures 11 through 13. These curves were generated from microdensitometer scans of step wedges placed on the original flight film and subsequently copied during each duplication step. Consequently, the relative exposure refers to the apparent brightness of scene elements at the original flight film. The exposure of the first step was arbitrarily assigned a value of 1.0. Since there is an increment of 0.15 in the log exposure between the 21 steps, the last step has a relative exposure of 1000.

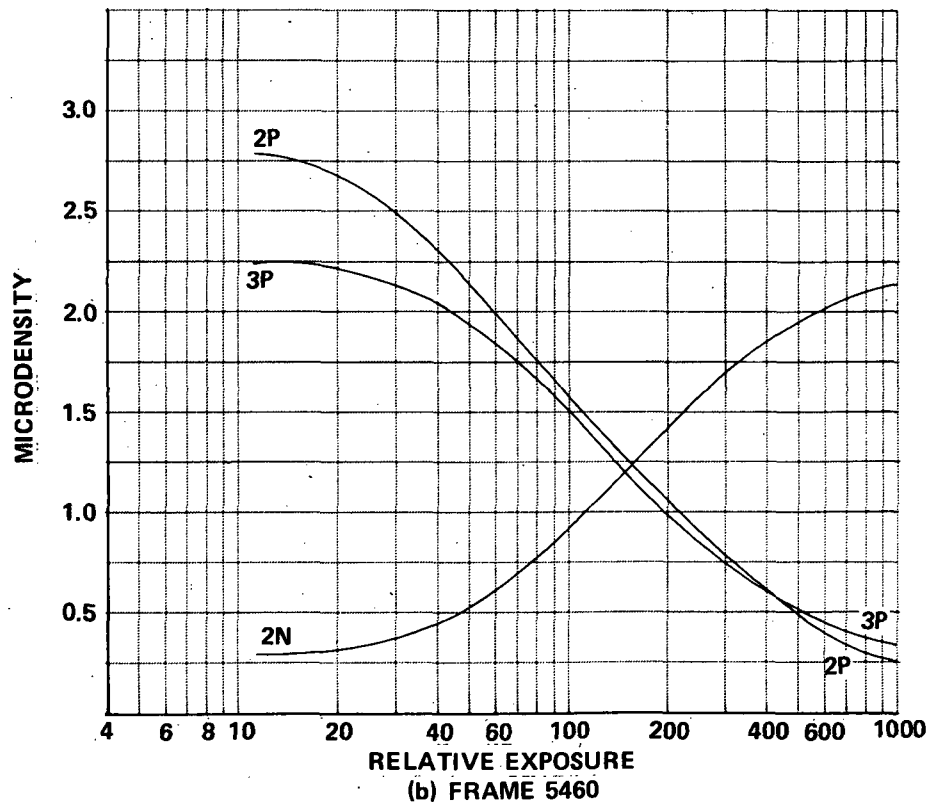
A software program was written that calculates the gain factors expressed by Eq. (6) using step wedge densities (sensitometric calibration) supplied as input. The input values were obtained from the curves shown in Figures 10 through 13 created by visually passing a smooth curve through the actual measured step wedge densities. Spline interpolation functions are used to compute density values for exposure levels between the individual steps. These interpolation functions yield results equivalent to forcing a flexible straight line (spline) through the data points and tend to avoid the "waviness" often encountered by other curve fitting techniques.

The tone quality evaluation program uses the resulting coefficients of the spline regression to estimate the density difference between the image and the background for various contrast levels identified previously in Table 2. The average exposure levels, i.e. the average of the object and background exposure values, were set equal to the relative exposure levels of each of the steps in the step tablet. The density difference at each average exposure level is divided by $\Delta$ (c.f. Table 2) to determine the gain.

The gain as a function of the average exposure level was identical for the low and mid-contrast objects for all the cases examined. The gain functions for the second generation photography are presented in Figures 14 through 16. In describing tonal quality the term "dynamic range" is frequently employed. This is loosely defined as the exposure region corresponding to the linear portion of the D-Log E curve. In this study we defined the dynamic

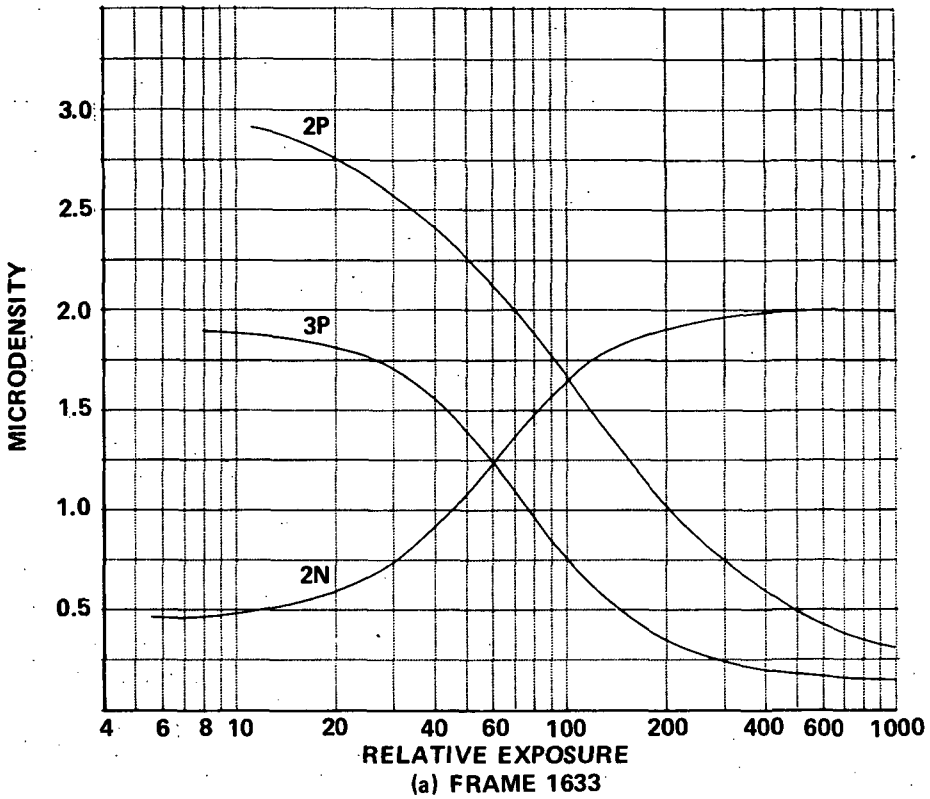



(a) FRAME 4127

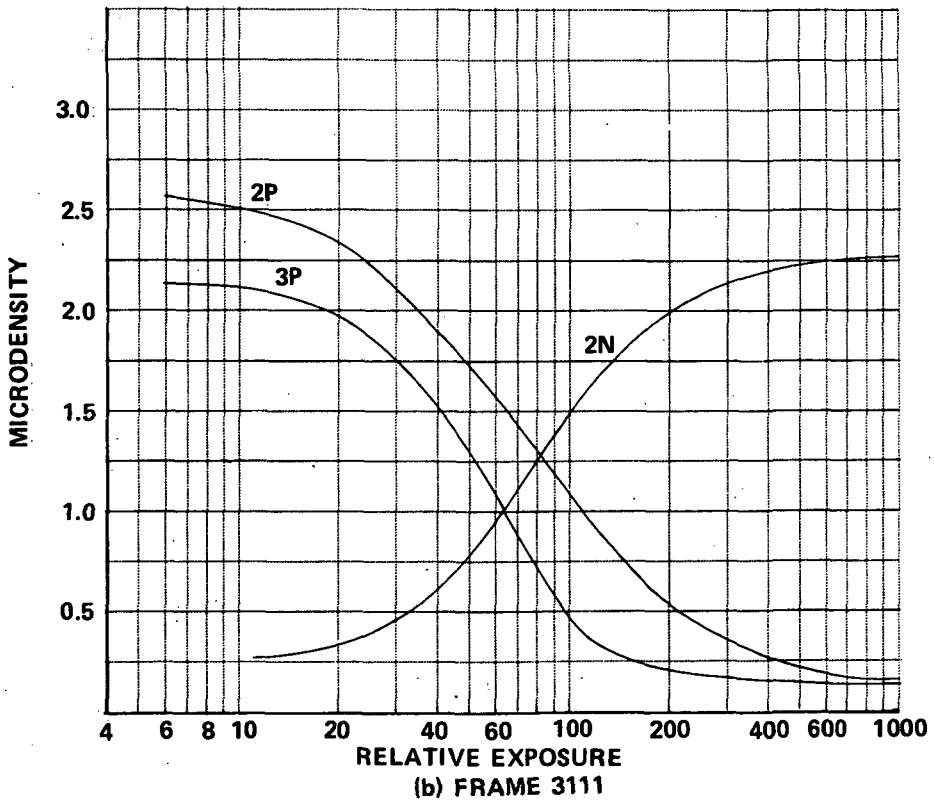



(b) FRAME 5460

**Figure 11 SENSITOMETRIC CALIBRATION - APOLLO 16 PANORAMIC PHOTOGRAPHY**

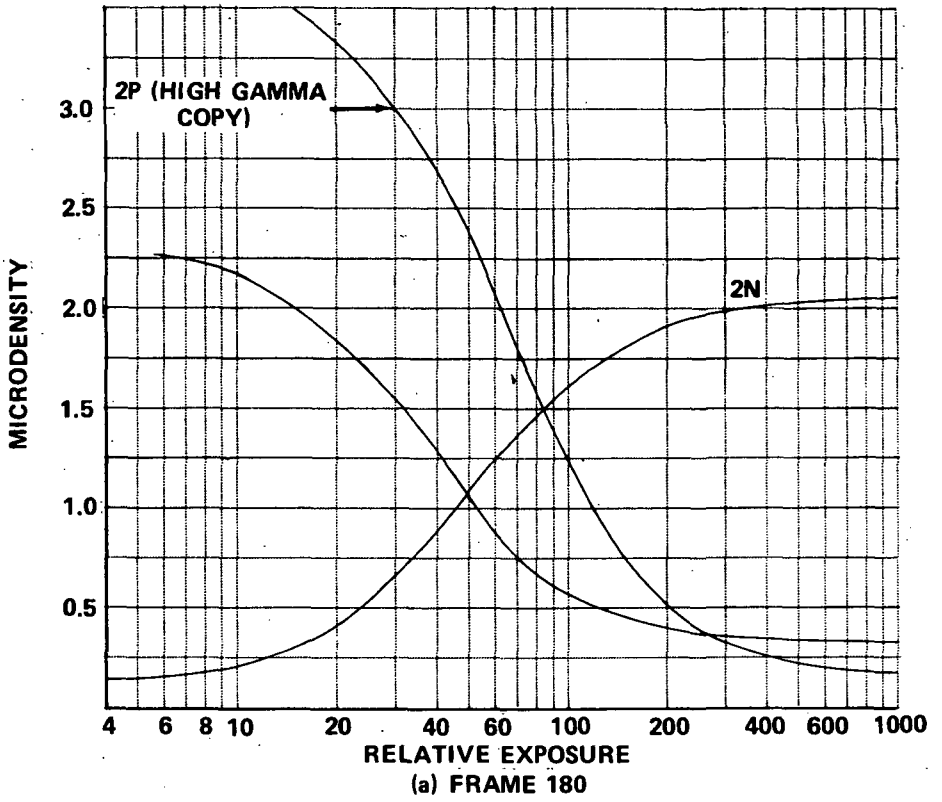



(a) FRAME 1633

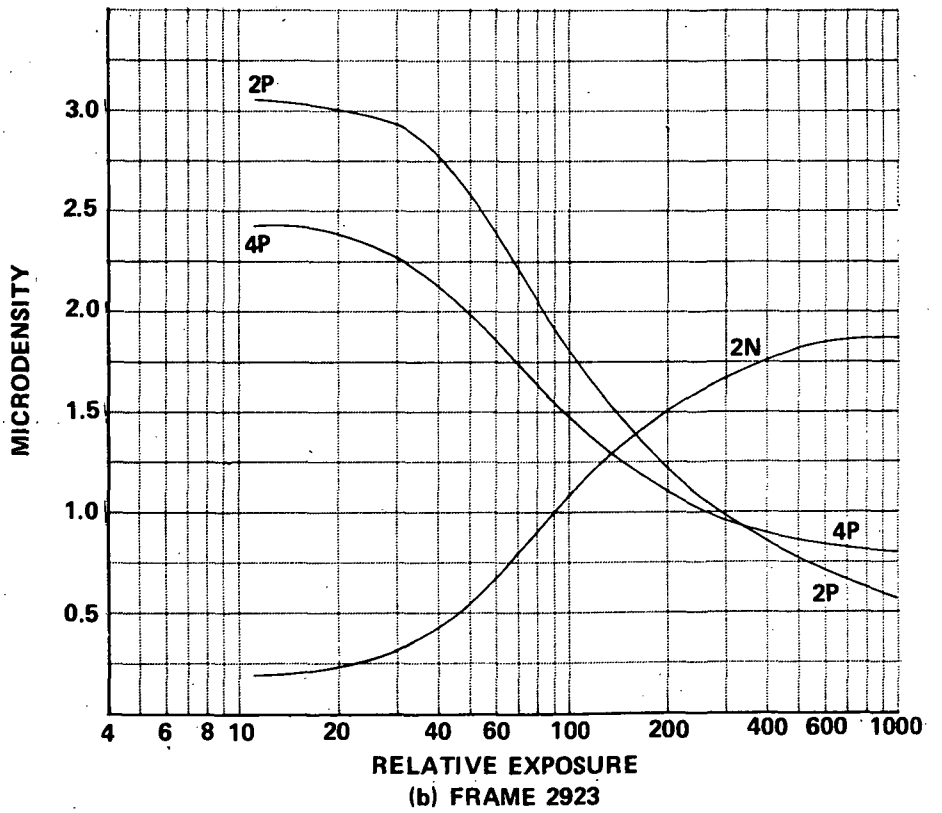



(b) FRAME 3111

**Figure 12 SENSITOMETRIC CALIBRATION - APOLLO 17 PANORAMIC PHOTOGRAPHY**




(a) FRAME 180




(b) FRAME 2923

Figure 13 SENSITOMETRIC CALIBRATION - APOLLO 17 METRIC PHOTOGRAPHY

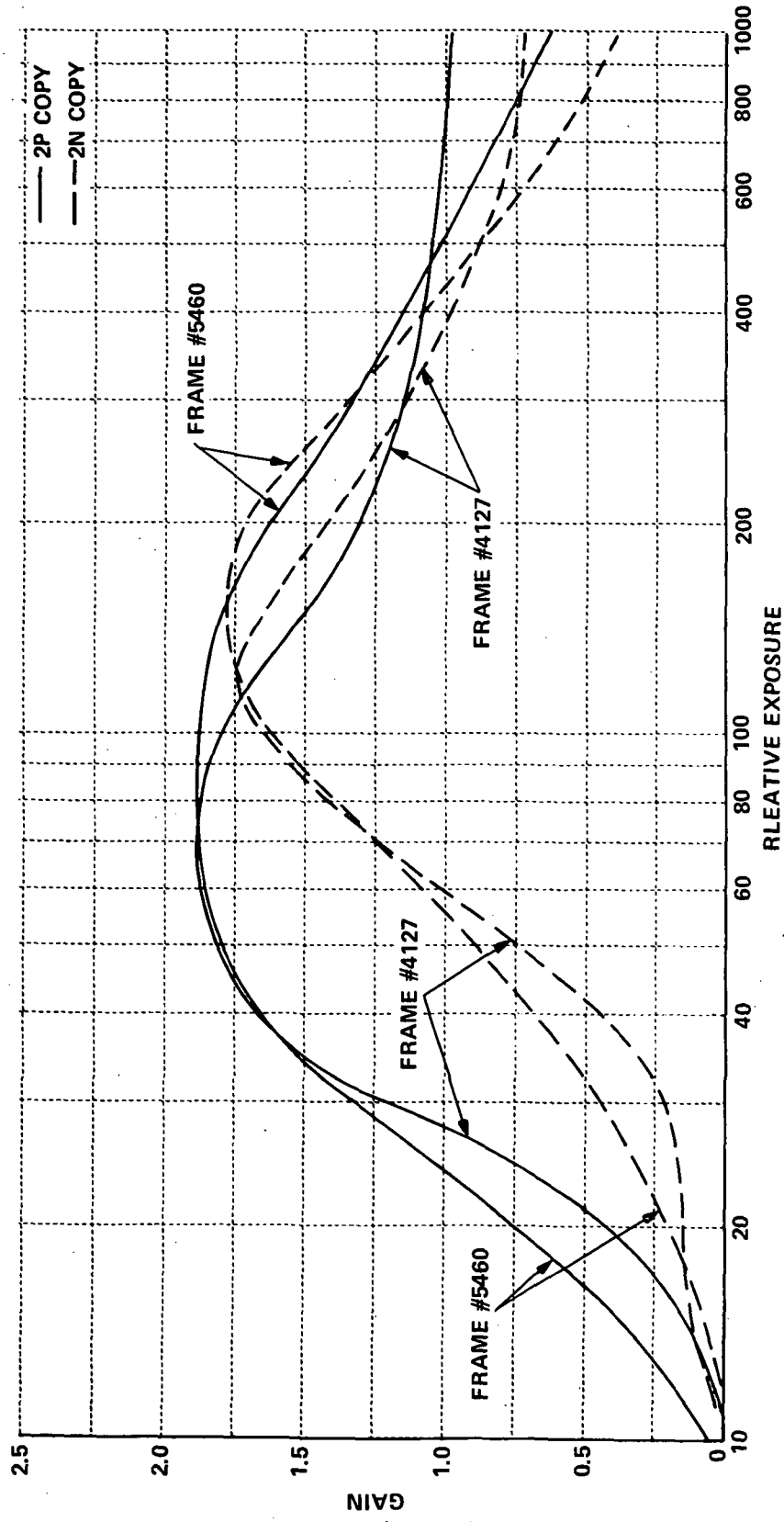



Figure 14 GAIN FOR LOW AND MID-CONTRAST OBJECTS – APOLLO 16 2nd GENERATION PANORAMIC PHOTOGRAPHY

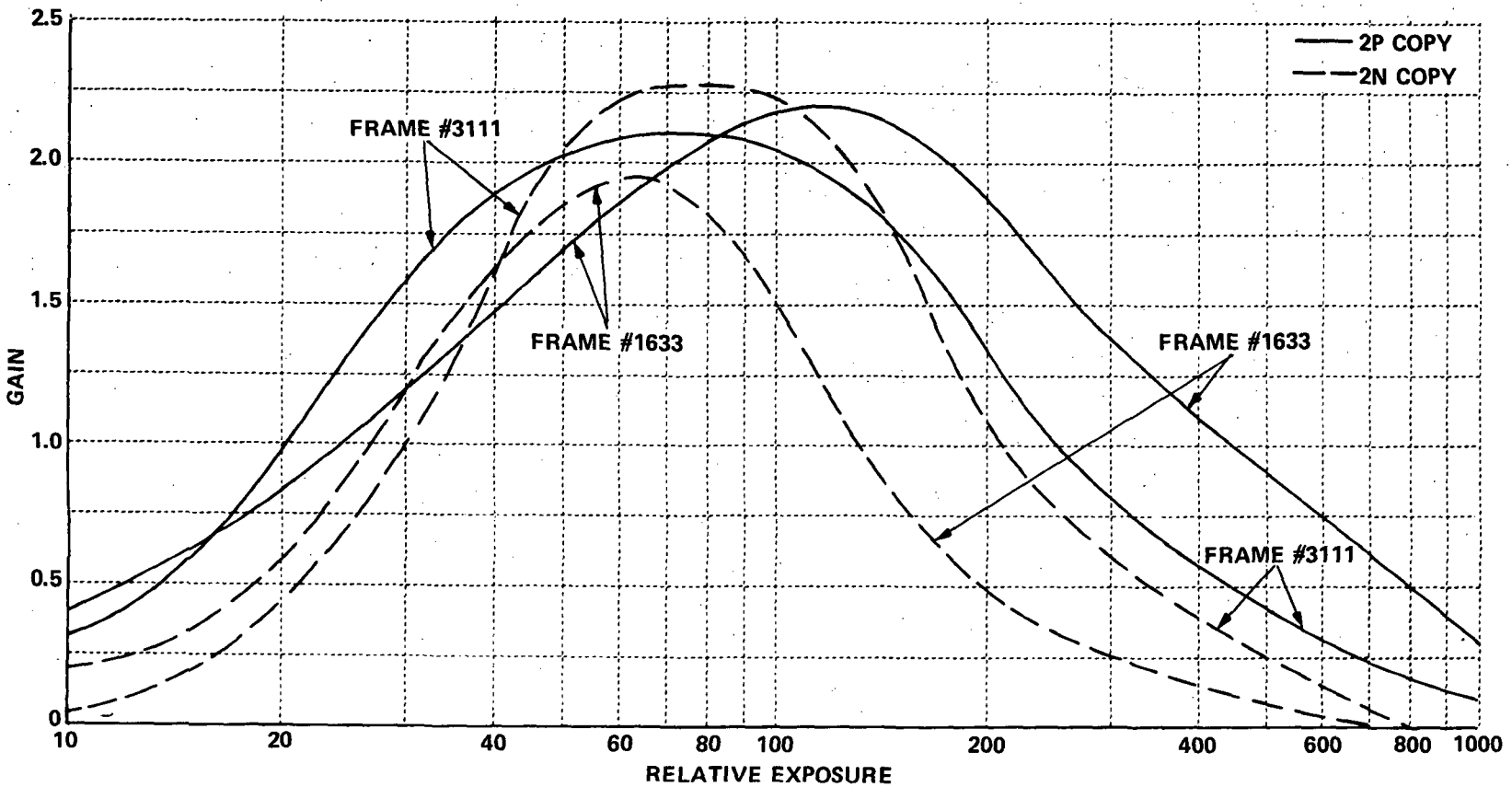



Figure 15 GAIN FOR LOW AND MID-CONTRAST OBJECTS – APOLLO 17 2nd GENERATION PANORAMIC PHOTOGRAPHY

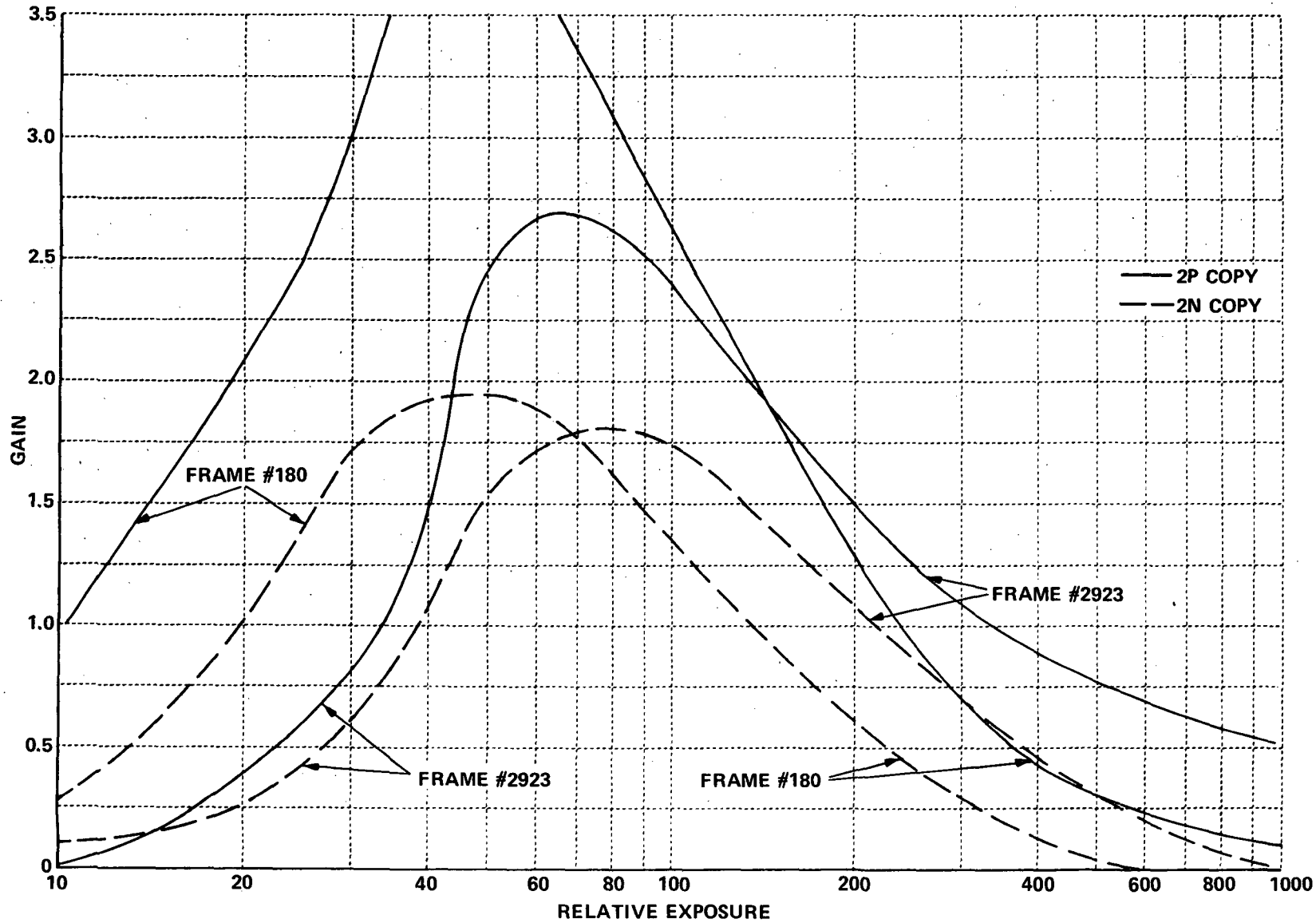



Figure 16 GAIN FOR LOW AND MID-CONTRAST OBJECTS – APOLLO 17 2nd GENERATION METRIC PHOTOGRAPHY

range more quantitatively as the exposure interval between the points where the gain functions dropped to 50% of their peak value. The average value for the peak gain and the dynamic range for each type of photograph evaluated is presented in Table 7. It is clear from examination of Figures 14 and 15 that both the 2P and 2N photography have about the same peak gain but the 2P photography has a much greater dynamic range. This is also evident from the numerical values presented in Table 7. We can conclude, therefore, that the tone quality of the Master Positive is superior to the Direct Negative for the panoramic photography. In the case of the metric photography, it is not clear from Figure 16 that a difference in dynamic range exists and the values in Table 7 show an equivalence of the dynamic range. However, the peak gain of the Master Positive photography is clearly greater. The roll containing Frame 180 was, in fact, processed to yield a "high gamma" copy. In this case we conclude that the Master Positive is superior due to the peak gain rather than any difference in dynamic range.

**Table 7**

**TONE QUALITY COMPARISON**

**(LOW AND MID CONTRAST OBJECTS)**

| PHOTO TYPE | | PEAK GAIN | DYNAMIC RANGE (dB) |
|---|---|---|---|
| APOLLO 16 PAN | 2N | 1.8 | 9.9 |
| | 2P | 1.9 | 15.5 |
| | 3P | 1.8 | 9.6 |
| APOLLO 17 PAN | 2N | 2.1 | 7.5 |
| | 2P | 2.2 | 11.1 |
| | 3P | 2.4 | 6.5 |
| APOLLO 17 METRIC | 2N | 1.9 | 8.3 |
| | 2P | 2.7 | 8.4 |
| | 4P | 2.1 | 7.5 |

Similar measurements were made on the 3P and 4P copies where they were available. These data were not included in Figures 14 through 16 for the sake of clarity but are presented in Table 7. The quality of these products generally correlates with the corresponding Second Generation photography. There is little change in the peak gain between these generations, but a definite decrease in the dynamic range for the higher generation products. An exception is the 2P to 4P reproduction of the Apollo 17 metric photograph where an apparent drop in the peak gain is noted. Consequently, we cannot infer that the tonal quality differences present in the Second Generations are preserved in the 3P and 4P user products for the metric photography.

The gain functions for a high contrast object (e.g. sunlight-shadow boundary) have lower peak gains and greater dynamic range (c.f. Table 8) but support the conclusions reached above.

**Table 8**

**TONE QUALITY COMPARISON**

**(HIGH CONTRAST OBJECTS)**

| PHOTO TYPE | | PEAK GAIN | DYNAMIC RANGE (dB) |
|---|---|---|---|
| APOLLO 16 PAN | 2N | 1.5 | 13.0 |
| | 2P | 1.7 | > 15.7 |
| | 3P | 1.5 | 12.0 |
| APOLLO 17 PAN | 2N | 1.7 | 9.7 |
| | 2P | 2.0 | 12.3 |
| | 3P | 1.9 | 9.2 |
| APOLLO 17 METRIC | 2N | 1.6 | 10.7 |
| | 2P | 2.6 | 10.7 |
| | 4P | 1.6 | 10.7 |

4.2 Fine Detail Content - MTF measurements were made in order to compare the detail content of the various generations of photography evaluated. These measurements were made by scanning shadow-to-sunlit edges near the center of each frame using a microdensitometer. The mode in which the instrument was operated was described previously in Section 3.2 in connection with similar measurements on Apollo 15 panoramic photography. The edge data

obtained were processed using the EGSA software as discussed previously in Section 2.2. Table 9 shows the amount of edge data collected and processed for each of the photographs evaluated. The MTFs measured for the Apollo 17 metric photography are presented in Figure 17. Included in the Figure is the MTF limit for the film alone. The significance of the departure of the measured MTFs from the film limit was assessed using the 95% confidence bands calculated by the EGSA main program (see the Appendix). The systematic affect at the low spatial frequencies (0-30 cycles/mm) where the measured functions exceed the limit is a result of the manual smoothing and the noise level. The top and bottom of the edge are masked by the noise and the observer tends to employ small radii of curvature thereby truncating the tails of the spread functions and producing this effect. Consequently the confidence bands are examined only above 30 cycles/mm to assess significance. Both the Direct Negative (a) and Master Positive (b) MTF's are not significantly lower than the film limit except for the 2P copy of Frame 180. The 4th Generation Positive MTF's (c) are both significantly less than the film imposed limit. In these three cases the estimated ground resolution is about 30 meters (versus a nominal 20 meter resolution). Since half of the copies have nominal resolution the loss in fine detail must have occurred during the reproduction processes.

**Table 9**

**EDGE DATA COLLECTED AND PROPOSED FOR APOLLO 16 AND 17 PHOTOGRAPHY**

| TYPE / FRAME | DIRECT NEGATIVE | MASTER POSITIVE | 3rd GEN. POSITIVE | 4th GEN. POSITIVE |
|---|---|---|---|---|
| 180 | 8 | 7 | – | 10 |
| 2293 | 8 | 8 | – | 8 |
| 1633 | 10 | 9 | 6 | – |
| 3111 | 6 | 6 | 3 | – |
| 4127 | 5 | 10 | 9 | – |
| 5460 | 10 | 8 | 7 | – |

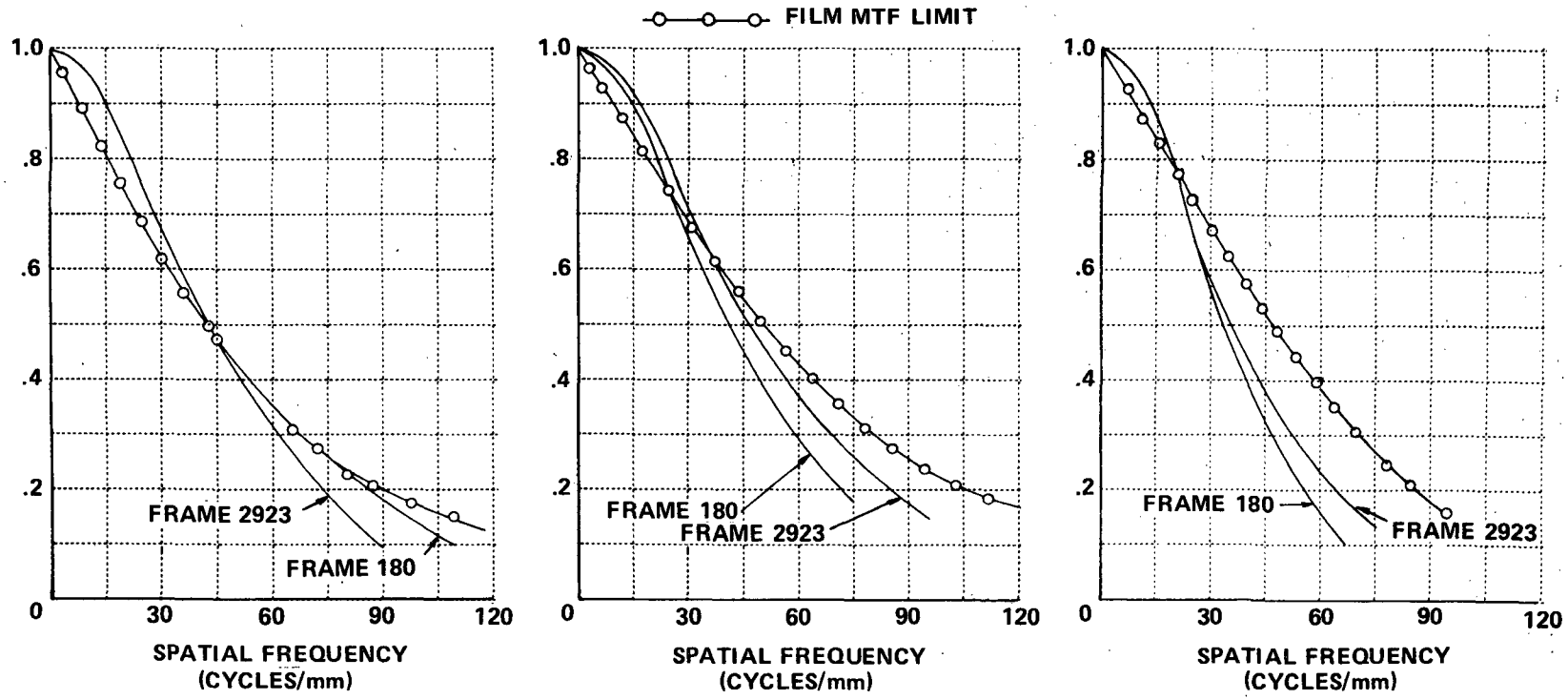



Figure 17 MODULATION TRANSFER FUNCTIONS - APOLLO 17 METRIC PHOTOGRAPHY

Figure 18 presents the MTF results for the Apollo 17 panoramic photography. For both frames the Direct Negative, Master Positive and 3rd Generation Positive MTF's are not significantly different from the respective film limits. This photography has the maximum possible fine detail content or about 2 meter "resolution."

Figure 19 shows the MTFs obtained for the Apollo 16 panoramic photography evaluated. In this case all of the copies examined showed a significantly lower response than that imposed by the photographic films involved. For the Direct Negative copies the ground resolution element is estimated to be 4 meters while the Master Positive and 3rd Generation Positive copies have a 3 meter resolution. Whether this reduced fine detail content is present in the original flight film it is unknown since that photography was not available for evaluation. We note that the Direct Negative copies evaluated are not necessarily those used to produce the 3rd Generation Positive copies (which apparently have better resolution).

Based upon the results presented above it is concluded the reproduction processes may have produced a loss in fine detail. About half the frames evaluated showed some loss. In these cases the ground resolution element increased by a factor of 1.5 to 2.

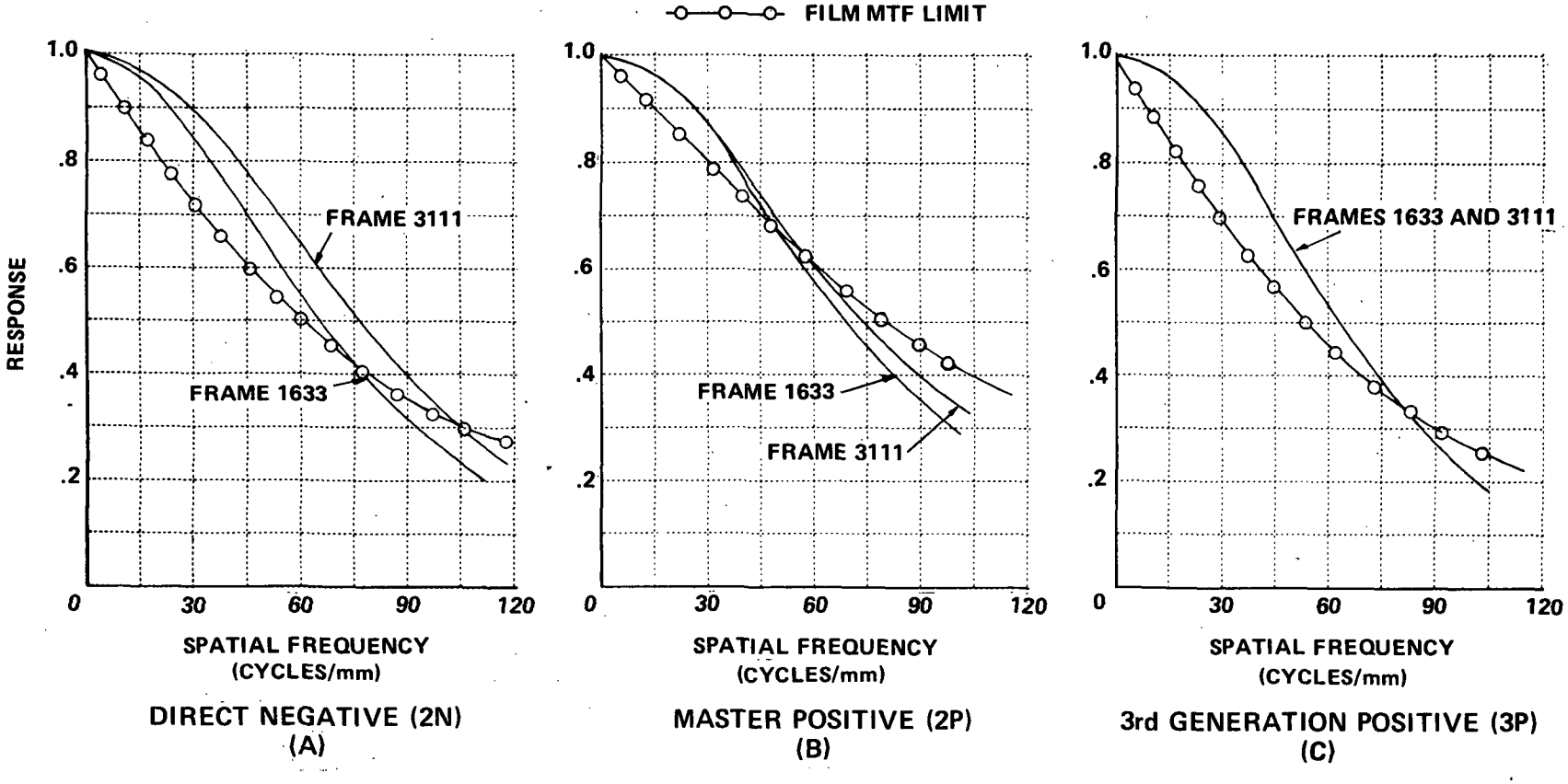



Figure 18 MODULATION TRANSFER FUNCTIONS - APOLLO 17 PANORAMIC PHOTOGRAPHY

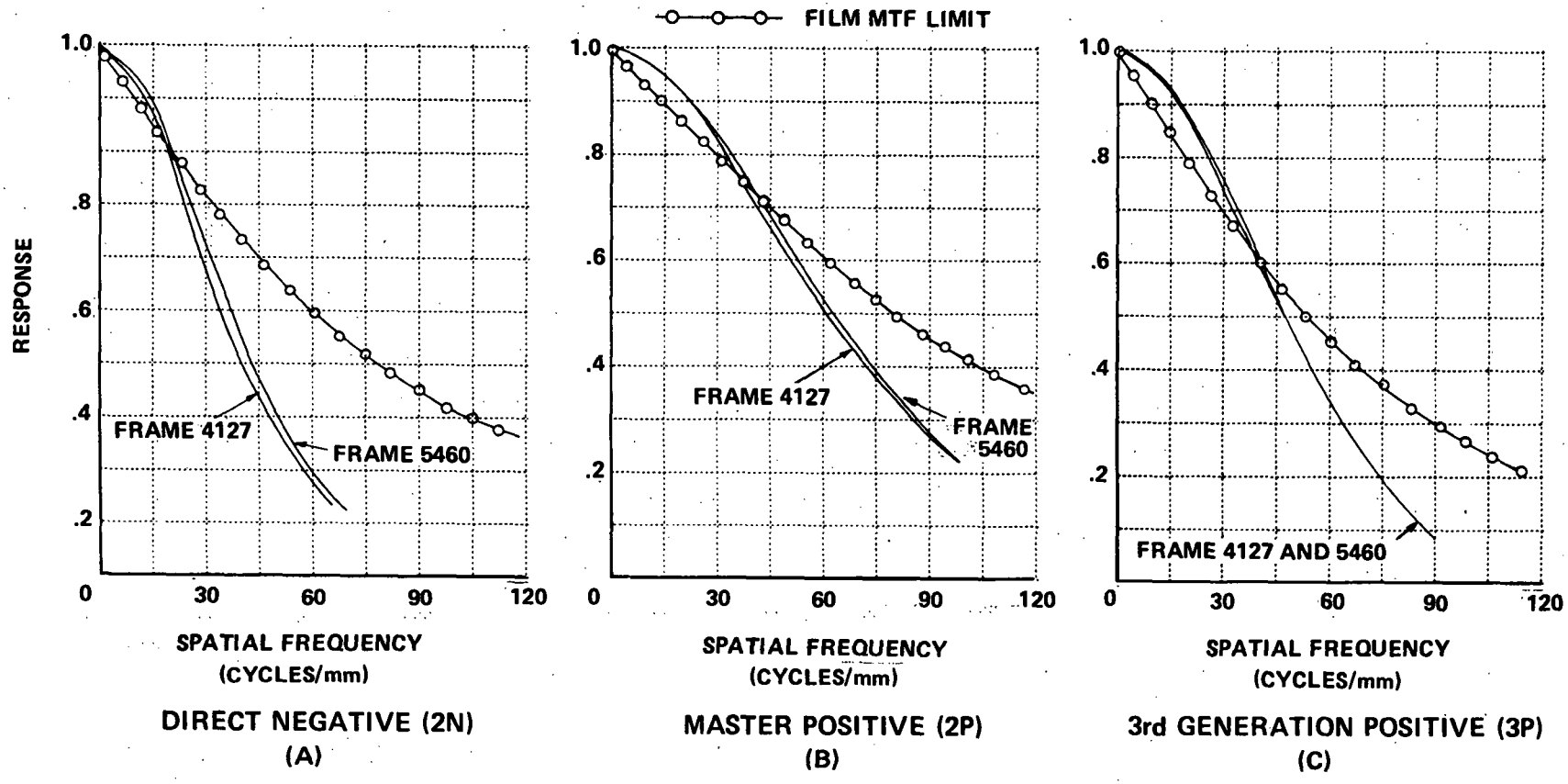



Figure 19 MODULATION TRANSFER FUNCTIONS - APOLLO 16 PANORAMIC PHOTOGRAPHY

# 5. CONCLUSIONS

The development and application of image evaluation methods for assessing the detail content of Apollo Orbital Photography has been demonstrated. Edge analyses using shadow-to-sunlight edges interior to craters was successfully used to evaluate the fine detail content for Apollo 15, 16 and 17 imagery. A method for evaluating tone quality was developed using a gain factor as a function of object contrast and average exposure level that can be related to object detectability.

The anomalous operation of the V/H sensor during the Apollo 15 mission introduced several degrading effects into the panoramic imagery; a loss in resolution due to image motion blur and exposure differences within a frame. The latter effect is present in those frames where an abrupt change in the V/H rate occurs. Two such frames were evaluated and exhibited a 0.4 density difference. It is estimated that this difference would be less in high sun angle photography although no such frames were examined.

The fine detail content was seriously degraded by the anomalous V/H sensor operation. The commanded V/H rate was either anomalously high or equal to the pre-set nominal rate. This resulted in either over or under compensation and a reduction in the resolution by 1/3 (to about 6 meters on the lunar surface). In one instance the nominal V/H rate was sufficiently close to the actual rate to permit nearly perfect compensation. Only when the commanded V/H rate is equal to the nominal value can the full resolution potential of the panoramic camera be realized; but this may not always occur. When the command rate varies about an anomalously high value a loss in resolution can be expected for the full frame. The user of Apollo 15 panoramic imagery should refer to the telemetry data to determine the V/H command situation for the frames he is exploiting and account for the effects of poorer resolution if necessary.

As a result of the comparison among multiple generations of Apollo 16 and 17 photography it was concluded that the tone quality of the Master Positive is superior to that of the Direct Negative for the panoramic photography. This difference is attributed to a greater dynamic range of the 2P photography. This means that lunar objects have a greater contrast over a wider range of average exposure levels in the 2P photography compared to the 2N photography. Unfortunately 3P and 4P copies of the same type of photography were not available thus preventing a direct comparison of the most likely user products. However, the quality of these products generally correlates with the corresponding 2nd Generation Photography, and we can infer that the tonal quality differences are preserved in higher generation user products. Such differences were not evident in the metric photography.

MTF measurements made to compare the detail content of the various generations revealed that half of the imagery examined had a significant loss in fine detail content. These losses were found in both the metric and panoramic photography and are probably introduced during the reproduction process. In the case of the Apollo 16 panoramic imagery all copies showed loss of fine detail and it was not possible to assess whether this loss was present in the flight imagery since it was not available for evaluation. The net effect was an increase in the ground resolution element by an approximate factor of 1.5; 3 meters for the panoramic photography and 30 meters for the metric photography.

The success of the image evaluation methods in assessing the detail content of Apollo orbital photography and the importance of this detail content to potential users suggests that NASA should continue to develop the methods for applications for the future manned and unmanned spacecraft involved in planetary or earth exploration.

## REFERENCES


1. Kinzly, R. E., Study of the Detail Content of Apollo Orbital Photography. Report No. VT-2912-0-1, Calspan Corporation (formerly Cornell Aeronautical Laboratory, Inc.), 29 September 1972.

2. Jensen, N., Optical and Photographic Reconnaissance Systems. J. Wiley and Sons, Inc., 1968.

3. Kawachi, D. A., Image Motion and Its Compensation for the Oblique Frame Camera. Photogrammetric Engineering, Vol. 31, No. 1, January 1965, pp. 154.

4. Kawachi, D. A., Image Motion Due to Camera Rotation. Photogrammetric Engineering, Vol. 31, No. 5, September 1965, pp. 861.

# APPENDIX
# SOFTWARE DESCRIPTIONS

This Appendix contains descriptions of eight programs used in the evaluation of the Detail Content of the Apollo Orbital Photography. The purpose, description of inputs, logic and processing, and a sample of the input and output data are included, if appropriate, for each of the programs. All of the programs were written using FORTRAN IV Language. Source and cross reference listings are included for seven of these programs.

Seven of the programs are part of the Edge Gradient Spectral Analysis (EGSA) software package. This package consists of two main programs and five subprograms and is described in Sections 1.0 through 2.0 of this Appendix. Section 8.0 describes the main program used to evaluate the tone quality of Apollo Orbital Photography.

1.0 Main Program DTAPE - This program is part of the Edge Gradient Spectral Analysis software package. It is one of two main programs in the package and is used to create an input data tape for subsequent processing by the main EGSA program. The input to this program is assumed to be edge trace data (i.e., density values at uniform spatial increments) supplied on card input. Sensitometric calibration (step wedge data) is also required for program execution. These data are used by function subprogram EXPOS to convert the density values to relative exposure values which are subsequently written onto an output data tape. No printed output data is furnished by this program.

Fiugre 20 is a listing of program DTAPE. The function of the major blocks are identified by the comments included in the coding. The program initially reads the sensitometric calibration data supplied by the user in card form. An example of a typical input data set is shown in Figure 21. The initial card of this data deck specifies the number of points, NSW, and the sense of the photographic image (i.e., positive or negative). The variable SENSE is actually a dual purpose variable. It is used to indicate the sense of the image; a positive value corresponds to a positive image and a negative

```
C ********************************************************************************
C ********************************************************************************
C **                                                                            **
C **  MAIN PROGRAM - DTAPE (INPUT DATA TAPE CREATION)                           **
C **                                                                            **
C **       THIS PROGRAM IS PART OF THE EDGE GRADIENT SPECTRAL ANALYSIS          **
C **  SOFTWARE PACKAGE.  ITS PURPOSE IS TO CREATE AN INPUT DATA TAPE FOR        **
C **  THE MAIN EGSA PROGRAM.  THIS DATA TAPE CONTAINS THE EDGE DATA TO BE       **
C **  PROCESSED.  VALUES OF DENSITY VERSUS POSITION OBTAINED FROM               **
C **  MICRODENSITOMETER TRACES ARE READ AND CONVERTED TO RELATIVE               **
C **  EXPOSURE USING SENSITOMETRIC CALIBRATION DATA ALSO PROVIDED AS            **
C **  INPUT.  FUNCTION 'EXPOS' IS REQUIRED FOR EXECUTION.  THE OUTPUT          **
C **  TAPE WRITTEN CONTAINS TWO TYPES OF RECORDS.  THE FIRST SPECIFIES          **
C **  EDGE IDENTIFICATION PARAMETERS, THE NUMBER OF POINTS AND THE              **
C **  INCREMENT BETWEEN POINTS.  THE SECOND TYPE CONTAINS THE CONVERTED         **
C **  EDGE DATA.                                                                **
C **                                                                            **
C **                                                                            **
C **                      ** VARIABLE LIST **                                   **
C **                                                                            **
C **  INPUT VARIABLES                                                           **
C **                                                                            **
C **       DX      =INCREMENT BETWEEN POINTS IN THE EDGE TRACE IN MM.           **
C **       E       =EDGE TRACE DATA INPUT AS DENSITY VALUES.                    **
C **       NED     =OPTIONAL EDGE IDENTIFICATION PARAMETER.                     **
C **       NID     =OPTIONAL EDGE IDENTIFICATION PARAMETER.                     **
C **       NP      =NUMBER OF POINTS IN THE EDGE TRACE.  MUST BE < OR = 500     **
C **                LIMITED BY DIMENSION OF "E" ARRAY.                          **
C **       NSW     =NUMBER OF POINTS IN SENSITOMETRIC CALIBRATION DATA.         **
C **                MUST BE < OR = 21.                                          **
C **       SENSE   =A DUAL PURPOSE VARIABLE INDICATING THE SENSE OF THE         **
C **                OF THE IMAGE; SENSE >0 INDICATES A POSITIVE AND             **
C **                <0 A NEGATIVE.  IF THE DIFFERENCE BETWEEN STEPS IN THE      **
C **                WEDGE IS 0.15 IN LOG EXPOSURE THE MAGNITUDE OF SENSE        **
C **                SHOULD BE SET TO 1.0; OTHERWISE IT CAN BE USED AS A         **
C **                MULTIPLYING FACTOR TO ADJUST THE DIFFERENCE IN LOG          **
C **                EXPOSURE BETWEEN STEPS.                                     **
C **       SWD     =SENSITOMETRIC CALIBRATION DENSITY VALUES.                   **
C **                                                                            **
C **                                                                            **
C **       AUTHOR: R.E. KINZLY                                                  **
C **       DATE: DECEMBER 30, 1971                                              **
C **       REVISED: MARCH 13, 1974                                              **
C **                APRIL 1, 1974                                               **
C **                                                                            **
C ********************************************************************************
C ********************************************************************************
C
C
ISN 0002          DIMENSION E(500),SWD(21)
ISN 0003          DATA DUM/0.0/,NDUM/0/
ISN 0004        1 FORMAT(3I5,F9.0)
ISN 0005        2 FORMAT(4(10X,F10.0))
ISN 0006        3 FORMAT(I2,F6.0)
ISN 0007        4 FORMAT(14F5.0,10X)
```

**Figure 20** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM DTAPE (PAGE 1 OF 3)

```
C  ********************************************************************************
C  *                                                                              *
C  *  READ SENSITOMETRIC CALIBRATION DATA                                         *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0008          5 READ(5,3,END=30) NSW,SENSE
ISN 0009            CALL CLEAR(E(1),SWD(21))
ISN 0010            READ(5,4) (SWD(I),I=1,NSW)
C
C  ********************************************************************************
C  *                                                                              *
C  *  READ EDGE IDENTIFICATION PARAMETERS,NUMBER OF POINTS AND INCREMENT          *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0011          6 READ(5,1) NED,NID,NP,DX
ISN 0012            IF(NP.EQ.0) GO TO 20
C
C  ********************************************************************************
C  *                                                                              *
C  *  READ EDGE DATA (DENSITY VERSUS POSITION)                                    *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0014            READ(5,2) (E(I),I=1,NP)
C
C  ********************************************************************************
C  *                                                                              *
C  *  CONVERT DENSITY TO RELATIVE EXPOSURE USING SENSITOMETRIC CALIBRATION         *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0015            DO 10 I=1,NP
ISN 0016         10 E(I)=EXPOS(E(I),SENSE,SWD,NSW)
C
C  ********************************************************************************
C  *                                                                              *
C  *  WRITE OUTPUT DATA TAPE RECORDS                                              *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0017            WRITE(1) NED,NID,NP,DX
ISN 0018            WRITE(1) (E(I),I=1,NP)
ISN 0019            GO TO 6
ISN 0020         20 END FILE 1
ISN 0021            GO TO 5
C
C  ********************************************************************************
C  *                                                                              *
C  *  WRITE DUMMY OUTPUT RECORD (LAST FILE) AND TERMINATE EXECUTION               *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0022         30 WRITE(1) NDUM,NDUM,NDUM,DUM
ISN 0023            END FILE 1
ISN 0024            STOP
ISN 0025            END
```

**Figure 20. SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM DTAPE (PAGE 2 OF 3)**

```
                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
E       0002  0009  0014  0016  0016  0018
I       0010  0010  0010  0014  0014  0014  0015  0016  0016  0018  0018  0018
DX      0011  0017
NP      0011  0012  0014  0015  0017  0018
DUM     0003  0022
NED     0011  0017
NID     0011  0017
NSW     0008  0010  0016
SWD     0002  0009  0010  0016
NDUM    0003  0022  0022  0022
CLEAR   0009
EXPOS   0016
SENSE   0008  0016


 LABEL   DEFINED   REFERENCES
     1   0004      0011
     2   0005      0014
     3   0006      0008
     4   0007      0010
     5   0008      0021
     6   0011      0019
    10   0016      0015
    20   0020      0012
    30   0022      0008
```

**Figure 20** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM DTAPE (PAGE 3 OF 3)

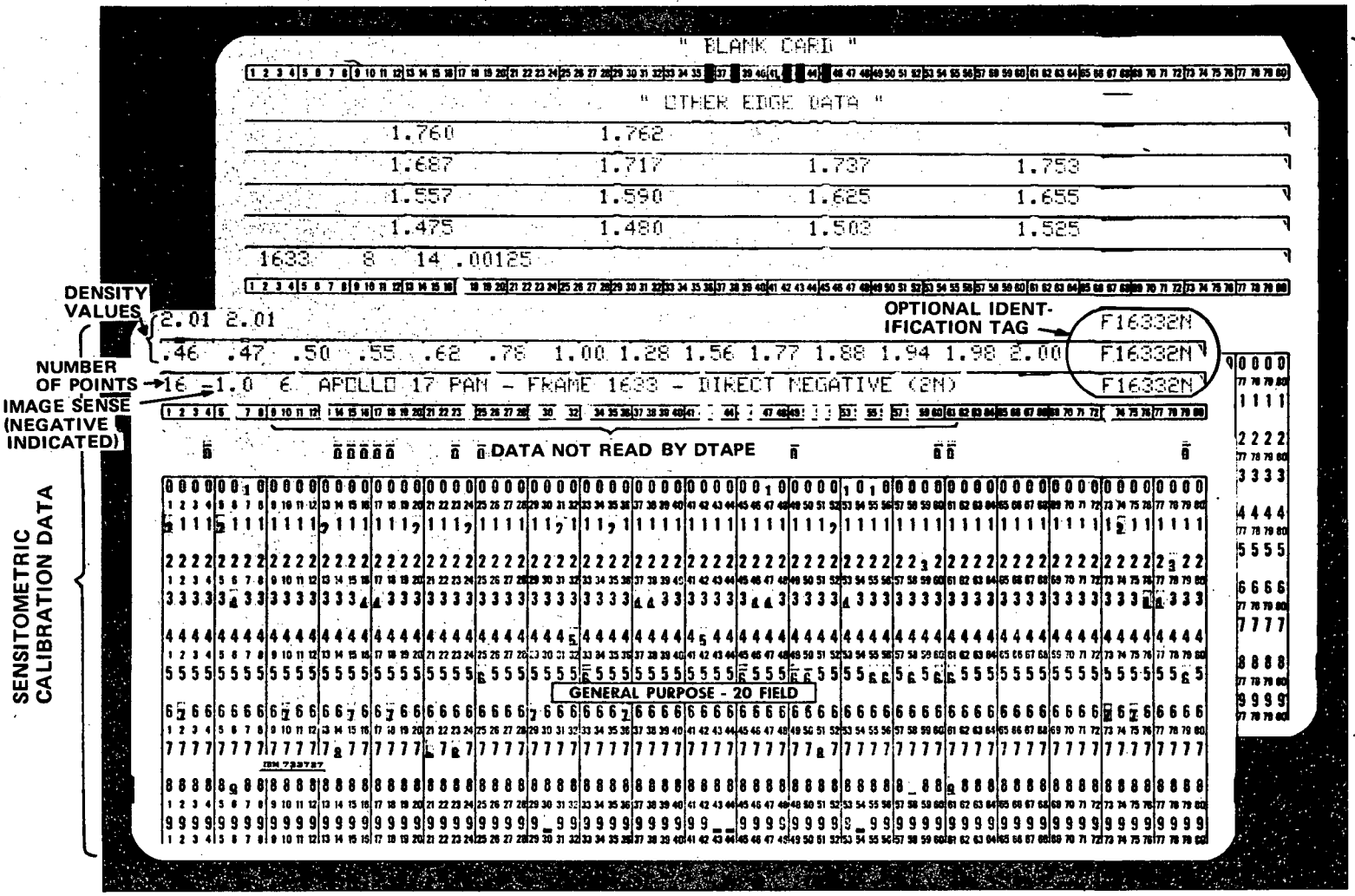



Figure 21 TYPICAL DTAPE PROGRAM INPUT DATA SET

value to a negative and to control the difference in log exposure between the points in the sensitometric calibration data. The difference in log exposure between points is computed as 0.15*SENSE and consequently if this difference is 0.15, the most likely value, the magnitude of SENSE should be set equal to 1.0. The subsequent cards in ths data deck contain density values for each point which are read into the SWD(I) array.

The next two major steps in the program involve reading edge identification parameters, the number of points in the edge data and the increment between points as well as the edge data itself. These data must also be supplied by the user in card deck form. The initial card shown in Figure 21 contains two integer identification parameters, the integer NP equal to the number of points in the edge data and DX, the increment between points. In the Apollo photographic evaluation the identification parameters were set equal to the frame number and a sequential number identifying the edge trace made on that frame of photography. This card is followed by a sequence of cards containing the actual edge data. Provision is made for multiple edge traces in any one edge data set. After the last trace a blank card must be inserted to separate edge data sets. The user is free to add other edge data sets to the input by following this blank card with the new sensitometric calibration data and the associated edge data.

After reading the edge data, the program converts the density values to relative exposure by calling function subprogram EXPOS. The converted edge data are written onto an output tape and the program recycled to read the next edge trace identification parameters and data. This process is repeated until a zero is encountered for the number of points in an edge trace. This identifies the end of the current data set and the program places an end of file on the output data tape. It then recycles to read new sensitometric calibration data for the next data set. If no data is encountered, the program writes a dummy output record as the last file on the output tape and terminates execution.

2.0 Function Subprogram EXPOS - This program is part of the Edge Gradient Spectral Analysis software package. It supports Program DTAPE in the creation of an input data tape for subsequent processing by the main EGSA program. The purpose of this function subprogram is to convert a density value supplied as input into a relative exposure value using sensitometric calibration data. No direct input data must be furnished by the user. The input data is supplied through the calling program. No printed output data is furnished by this program.

Figure 22 is a listing of program EXPOS. The function of the major blocks are identified by the comments included in the coding. The computing path branches depending upon the difference between the input value of density to be converted and the density value of the current point in the sensitometric data. That point is identified by the integer variable I. If these two density values are equal, the exposure for that point is computed and returned to the calling program. If the input density value is greater, the value of I is increased by one and a comparison between the input density and the density value of the next point in the step wedge is made. This process is repeated until the density of the point in the sensitometric calibration is greater than the input density or until the last point of the data is reached. If the latter occurs, the exposure for this last point is computed and returned to the calling program. If a point in the sensitometric calibration data of greater density than the input value is found the program performs a linear interpolation between that point and the previous adjacent point which bracket the value of the input density. If the current point in the sensitometric calibration data has a value which exceeds the input density, a check is made to determine if the density value lies between the current point and the adjacent point and if not, the index I is decreased by one and the comparison made again. This process is repeated until the input density value is bracketed by two points in the sensitometric calibration. Should the input density value be less than the first point in the calibration data (lowest density value) the relative exposure is set equal to one and returned to the calling program.

```
C      FUNCTION EXPOS(D,SENSE,SWD,NSW)
C
C ******************************************************************************
C ******************************************************************************
C **                                                                          **
C **  FUNCTION SUBPROGRAM EXPOS                                               **
C **                                                                          **
C **       THIS FUNCTION SUBPROGRAM CONVERTS DENSITY INTO EXPOSURE            **
C **  USING MEASURED DENSITIES OF A STEP WEDGE HAVING 21 STEPS OR            **
C **  LESS.  DIFFERENCES IN LOG EXPOSURE BETWEEN STEPS IS 0.15*SENSE.         **
C **                                                                          **
C **                                                                          **
C **                        ** VARIABLE LIST **                               **
C **                                                                          **
C **  INPUT VARIABLES                                                         **
C **                                                                          **
C **       D       =INPUT DENSITY TO BE CONVERTED. RETURNED TO THE CALLING    **
C **                PROGRAM UNCHANGED.                                        **
C **       SENSE   =A DUAL PURPOSE VARIABLE INDICATING THE SENSE OF THE       **
C **                OF THE IMAGE; SENSE >C INDICATES A POSITIVE AND           **
C **                <0 A NEGATIVE.  IF THE DIFFERENCE BETWEEN STEPS IN THE    **
C **                WEDGE IS 0.15 IN LOG EXPOSURE THE MAGNITUDE OF SENSE      **
C **                SHOULD BE SET TO 1.0; OTHERWISE IT CAN BE USED AS A       **
C **                MULTIPLYING FACTOR TO ADJUST THE DIFFERENCE IN LOG        **
C **                EXPOSURE BETWEEN STEPS.                                   **
C **       SWD     =DENSITY ARRAY OF THE INDIVIDUAL STEPS IN THE WEDGE        **
C **                IN INCREASING DENSITY.  INPUT THROUGH CALLING LIST.       **
C **       NSW     =NUMBER OF STEPS IN WEDGE.  INPUT THROUGH CALLING LIST.    **
C **                MUST BE < OR = 21.                                        **
C **                                                                          **
C **  OUTPUT VARIABLES                                                        **
C **                                                                          **
C **       EXPOS   =OUTPUT EXPOSURE VALUE                                     **
C **                                                                          **
C **  MAJOR INTERNAL VARIABLES                                                **
C **                                                                          **
C **       ENT     =LOGICAL VARIABLE FLAGING THE CALL SEQUENCE TO THE         **
C **                SUBROUTINE.  FOR THE FIRST CALL ENT = FALSE; FOR          **
C **                SUBSQUENT CALLS ENT = TRUE.                               **
C **                                                                          **
C **                                                                          **
C **       AUTHOR: R. E. KINZLY                                               **
C **       DATE: MAY 18, 1973                                                 **
C **       REVISED: MARCH 12, 1974                                            **
C **                                                                          **
C **                                                                          **
C ******************************************************************************
C ******************************************************************************
C
ISN 0002        FUNCTION EXPOS(D,SENSE,SWD,NSW)
ISN 0003        DIMENSION SWD(21)
ISN 0004        LOGICAL ENT/.FALSE./
ISN 0005        DLE=0.15*SENSE
ISN 0006        IF(ENT) GO TO 7
ISN 0008        ENT=.TRUE.
ISN 0009        I=0
```

**Figure 22 SOURCE AND CROSS REFERENCE LISTINGS FOR EXPOS FUNCTION SUBPROGRAM (PAGE 1 OF 3)**

```
ISN 0010          5 I=I+1
ISN 0011            IF(I.GE.NSW) GO TO 20
ISN 0013          7 IF(D-SWD(I))15,10,5
                C
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  COMPUTE EXPOSURE FOR INPUT DENSITY EQUAL TO DENSITY OF SWD(I)          *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0014         10 EXPOS=10.**(-DLE*(I-1))
ISN 0015            IF(I.GT.1) I=I-1
ISN 0017            RETURN
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  COMPUTE EXPOSURE FOR INPUT DENSITY BETWEEN SWD(I-1) AND SWD(I) BY      *
                C  *  USING LINEAR INTERPOLATION                                             *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0018         15 IF(I.EQ.1) GO TO 25
ISN 0020            IF(D.LT.SWD(I-1)) GO TO 30
ISN 0022            EXPOS=10.**(-DLE*(I-1-(SWD(I)-D)/(SWD(I)-SWD(I-1))))
ISN 0023            I=I-1
ISN 0024            RETURN
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  COMPUTE EXPOSURE FOR INPUT DENSITY BEYOND MAXIMUM WEDGE DENSITY        *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0025         20 EXPOS=10.**(-DLE*(NSW-1))
ISN 0026            I=I-1
ISN 0027            RETURN
ISN 0028         25 EXPOS=1.0
ISN 0029            RETURN
ISN 0030         30 I=I-1
ISN 0031            GO TO 7
ISN 0032            END
```

Figure 22 SOURCE AND CROSS REFERENCE LISTINGS FOR EXPOS FUNCTION SUBPROGRAM (PAGE 2 OF 3)

```
                    *****F O R T R A N    C R O S S    R E F E R E N C E    L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
D       0002  0013  0020  0022
I       0009  0010  0010  0011  0013  0014  0015  0015  0015  0018  0020  0022  0022  0022  0022  0023  0023  0026  0026
        0030  0030
DLE     0005  0014  0022  0025
ENT     0004  0004  0006  0008
NSW     0002  0011  0025
SWD     0002  0003  0013  0020  0022  0022  0022
EXPOS   0002  0014  0022  0025  0028
SENSE   0002  0005


 LABEL   DEFINED   REFERENCES
    5    0010      0013
    7    0013      0006  0031
   10    0014      0013
   15    0018      0013
   20    0025      0011
   25    0028      0018
   30    0030      0020
```



**Figure 22 SOURCE AND CROSS REFERENCE LISTINGS FOR EXPOS FUNCTION SUBPROGRAM (PAGE 3 OF 3)**

The relative exposures supplied by this program are 1.0 or greater for negative imagery and between 1.0 and 0.0 for positive imagery. The points in the sensitometric calibration data must be provided in order of increasing density.

3.0 Main Program EGSA - This program is part of the Edge Gradient Spectral Analysis software package. It is one of two main programs in the package and is used to compute an Optical Transfer Function (OTF) from edge trace data. The complex OTF is expressed as modulus and phase functions. The edge data is read from an unformatted data tape created by program DTAPE described previously in Section 1.0 of this Appendix. In addition to these data, the user must supply card input which identifies various data options he elects to receive from the program. Both printed and plotted output data are furnished as output options.

Figure 23 is a listing of program EGSA. The function of the major blocks are identified by the comments included in the coding. A macro flow diagram for this program was presented as Figure 3 on page 7 of the main text.

In order to exercise this program, sixteen subprograms are required. Four of these programs are user supplied and are discussed in detail in the following subsections. These include CONVLV which convolves an input function with an 11-point differentiating filter, DXSCAL which determines the increment or scale factor (inches/point) for the x-axis of several plots, FORINV which performs an inverse Fourier transform and FOURTR which calculates a Fourier transform of a real input function providing the complex output function as modulus and phase.

Of the twelve remaining subprograms, nine are used to generate graphic output. A brief description of the purpose of each of these programs is presented in Section 2.2 of the main text. Since these programs are peculiar to the data processing facility, no further discussion is provided in this Appendix. Two of the subprograms are IBM supplied FORTRAN IV Library subprograms

```
C ***********************************************************************
C **                                                                   **
C **  MAIN PROGRAM - EGSA (EDGE GRADIENT SPECTRAL ANALYSIS)            **
C **                                                                   **
C **       THIS PROGRAM COMPUTES AN OPTICAL TRANSFER FUNCTION (OTF) FROM **
C **  EDGE TRACE DATA.  THE COMPLEX OTF IS EXPRESSED AS MODULUS, 'TF',  **
C **  AND PHASE, 'PH'.   THE EDGE DATA IS READ FROM AN UNFORMATED TAPE, **
C **  DIFFERENTIATED AND FOURIER TRANSFORMED TO COMPUTE THE OTF.  INDIVIDUAL **
C **  MODULI CAN BE AVERAGED IF MULTIPLE EDGES ARE CONTAINED IN A SINGLE **
C **  DATA SET.  THE INDIVIDUAL EDGES, THEIR DERIVATIVES (OR LINE SPREADS) **
C **  AND RESULTING MODULI ARE PLOTTED IF REQUESTED BY THE USER.  OTHER **
C **  OUTPUT OPTIONS INCLUDE PLOTS OF THE AVERAGE MODULUS AND ITS 95%   **
C **  CONFIDENCE BAND AND THE AVERAGE LINE SPREAD AND CONFIDENCE BAND AS **
C **  WELL AS PUNCHED CARDS OF THE AVERAGE AND INDIVIDUAL MODULI.       **
C **                                                                   **
C **                                                                   **
C **                      ** VARIABLE LIST **                          **
C **                                                                   **
C **  INPUT VARIABLES                                                  **
C **                                                                   **
C **       DX      =INCREMENT BETWEEN EDGE DATA POINTS IN MM.  MUST REMAIN **
C **                CONSTANT IN ANY ONE DATA SET.                      **
C **       E       =EDGE TRACE DATA (EXPOSURE VALUES).                **
C **       FCO     =CUTOFF SPATIAL FREQUENCY USED IN SMOOTHING THE LINE **
C **                SPREAD.                                            **
C **       FINV    =SPATIAL FREQUENCY INTERVAL REQUESTED BY THE USER IN **
C **                CYCLES/MM.                                         **
C **       FMAX    =MAXIMUM SPATIAL FREQUENCY OF OTF DESIRED BY THE USER. **
C **                EXPRESSED IN CYCLES/MM.                            **
C **       NED     =OPTIONAL EDGE TRACE IDENTIFICATION VARIABLE.      **
C **                VALUE REPRODUCED ON PLOTTED AND PRINTED OUTPUT.    **
C **       NID     =OPTIONAL EDGE TRACE IDENTIFICATION VARIABLE.      **
C **       NP      =NUMBER OF POINTS IN EDGE TRACE DATA; < OR = 900.  **
C **       OPTION  =ALPHA NUMERIC CHARACTERS IDENTIFYING USER SELECTED **
C **                OUTPUT OPTIONS.                                    **
C **                                                                   **
C **  MAJOR INTERNAL VARIABLES                                         **
C **                                                                   **
C **       ALS     =AVERAGE LINE SPREAD FUNCTION.                     **
C **       ALSM    =AVERAGE LINE SPREAD CONFIDENCE BAND (LOWER BOUND). **
C **       ALSP    =AVERAGE LINE SPREAD CONFIDENCE BAND (UPPER BOUND). **
C **       ATF     =AVERAGE MODULUS OF THE OTF (OR MTF).              **
C **       ATFM    =AVERAGE MTF CONFIDENCE BAND (LOWER BOUND).        **
C **       ATFP    =AVERAGE MTF CONFIDENCE BAND (UPPER BOUND).        **
C **       DNU     =ACTUAL SPATIAL FREQUENCY INTERVAL OF OTF IN CYCLES/MM. **
C **       KNT     =COUNTER EQUAL TO THE NUMBER OF EDGE TRACES IN A DATA SET. **
C **       NCF     =NUMBER OF POINTS IN DIFFERENTIATED EDGE TRACE.    **
C **       NPL     =NUMBER OF INDIVIDUAL FUNCTION PLOTS REQUESTED BY USER. **
C **       NPLS    =NUMBER OF POINTS IN THE STORED OR UNSMOOTHED LINE SPREAD. **
C **       NPTF    =NUMBER OF POINTS IN THE OTF.  (=1+NUMBER OF HARMONICS) **
C **       NSL     =NUMBER OF POINTS IN THE SMOOTHED LINE SPREAD FUNCTION. **
C **       PH      =PHASE OF THE OTF.                                 ON. **
C **       SL      =LINE SPREAD FUNCTION (SMOOTHED VALUE IF OPTION REQUESTED). **
C **       SSL     ="STORED" OR UNSMOOTHED LINE SPREAD FUNCTION.      **
C **       TF      =MODULUS OF THE OTF.                               **
C **                                                                   **
```

**Figure 23** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 1 OF 12)

```
C **                                                                    **
C **       AUTHORS: R. E. KINZLY                                        **
C **                M. J. MAZUROWSKI                                    **
C **       DATE: JULY, 1966                                             **
C **       REVISIONS: AUGUST, 1973                                      **
C **                                                                    **
C ************************************************************************
C
ISN 0002        DIMENSION XP(900),ATF(900),ETF(900),SL(900),E(900),CF(900),TF(900)
               *,ATFM(900),ATFP(900),PH(900),DUM(1),XPI(900),XLI(900),OPTION(7),
               *ALS(900),ELS(900),ALSP(900),ALSM(900),XL(900),SSL(900),
               *XS(900),OPT(7),XLE(900)
ISN 0003        LOGICAL LTV(7)/7*.FALSE./
ISN 0004        DATA OPT/4H   E,4H   L,4H   T,4H  SL,4H  AL,4H  AT,4H  PT/
ISN 0005     2000 FORMAT(2F10.0,7A4,F10.0)
ISN 0006     3000 FORMAT(//10X,2I10 / (10X,10F10.3))
ISN 0007     3010 FORMAT(I10)
ISN 0008     3020 FORMAT(F5.3,11F6.3)
ISN 0009     3030 FORMAT(1H1,7X,'---> THE FREQUENCY INTERVAL OF THE OTF HAS BEEN CHA
               *NGED FROM',F6.2,' CYCLES/MM TO',  F6.2, ' CYCLES/MM,' / 13X, 'THE
               *EDGE DATA SET IDENTIFICATION PARAMETERS ARE: NED= ',I10,', NID= ',
               *I10 )
ISN 0010     3032 FORMAT(8X, '---> USER REQUEST FOR AN AVERAGE LINE SPREAD HAS BEEN
               *DELEATED.' )
ISN 0011     3034 FORMAT( 8X, '---> USER REQUEST FOR AN AVERAGE MTF HAS BEEN DELEATE
               *D.')
ISN 0012        CALL PLOTER(1,1,10)
C
C  ************************************************************************
C  *                                                                      *
C  *  READ USER INPUT DATA AND SPECIFIED OPTIONS                          *
C  *                                                                      *
C  ************************************************************************
C
ISN 0013     10 READ(5,2000,END=550)FINV,FMAX,(OPTION(I),I=1,7),FCO
ISN 0014        NPL=0
C
C  ************************************************************************
C  *                                                                      *
C  *  INITIALIZE LOGICAL ARRAY 'LTV' DESCRIBING OUTPUT OPTIONS            *
C  *                                                                      *
C  ************************************************************************
C
ISN 0015        DO 16 I=1,7
ISN 0016     16 LTV(I)=.FALSE.
C
C  ************************************************************************
C  *                                                                      *
C  *  SET 'LTV' ARRAY ELEMENTS TO 'TRUE' FOR OPTIONS REQUESTED            *
C  *                                                                      *
C  ************************************************************************
C
ISN 0017        DO 18 I=1,7
ISN 0018        DO 18 J=1,7
ISN 0019     18 IF(OPTION(I).EQ.OPT(J))LTV(J)=.TRUE.
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 2 OF 12)

```
C  ********************************************************************************
C  *                                                                              *
C  *  BEGIN PROCESSING CURRENT EDGE DATA SET                                      *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0021        NPTF=FMAX/FINV+1.5
ISN 0022        DF=FMAX/5.
ISN 0023     20 KNT=0
ISN 0024        CALL CLEAR(ATF(1),ATF(900))
ISN 0025        CALL CLEAR(ETF(1),ETF(900))
ISN 0026        CALL CLEAR(ALS(1),ALS(900))
ISN 0027        CALL CLEAR(ELS(1),ELS(900))
C
C  ********************************************************************************
C  *                                                                              *
C  *  READ EDGE DATA PARAMETERS                                                   *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0028     50 READ(1,ERR=550,END=400)NED,NID,NP,DX
ISN 0029        IF(NP.EQ.0) GO TO 400
C
C  ********************************************************************************
C  *                                                                              *
C  *  READ EDGE DATA FROM TAPE                                                    *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0031        NP=NP+12
ISN 0032        READ(1,ERR=550,END=550)(E(I),I=13,NP)
ISN 0033        KNT=KNT+1
C
C  ********************************************************************************
C  *                                                                              *
C  *  ADJUST EDGE DATA FOR NUMBER OF POINTS TO BE LOST IN DIFFERENTIATION         *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0034        DO 55 I=1,12
ISN 0035        E(I)=E(13)
ISN 0036     55 E(NP+I)=E(NP)
ISN 0037        NP=NP+12
ISN 0038        IF(LTV(1)) CALL SCALE(E,NP,2.,DUMMY,DUMMY,1)
C
C  ********************************************************************************
C  *                                                                              *
C  *  DIFFERENTIATE EDGE DATA                                                     *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0040        CALL CONVLV(E,NP,DUM,1,CF,NCF,3)
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 3 OF 12)

```
C  ************************************************************************
C  *                                                                      *
C  *  COMPUTE THE NUMBER OF POINTS NEEDED IN LINE SPREAD TO OBTAIN DESIRED *
C  *  FREQUENCY INTERVAL, 'FINV', FOR OTF                                 *
C  *                                                                      *
C  ************************************************************************
C
ISN 0041          NPLS=1./(FINV*DX)+0.5
C
C  ************************************************************************
C  *                                                                      *
C  *  SET UP X-AXIS INTERVALS AND ARRAYS FOR INDIVIDUAL EDGES AND LINE    *
C  *  SPREADS AND AVERAGE LINE SPREAD IF REQUESTED BY USER                *
C  *                                                                      *
C  ************************************************************************
C
ISN 0042          IF(.NOT.(LTV(5).OR.LTV(2).OR.LTV(1))) GO TO 70
ISN 0044          IF(KNT.EQ.1) CALL DXSCAL(2.5,NPLS,DX,DXTI,DXI)
ISN 0046          CALL DXSCAL(2.5,NP,DX,DXTE,DXE)
ISN 0047          DO 60 I=1,NPLS
ISN 0048          XL(I)=(1-I-NPLS/2)*DX
ISN 0049       60 XLE(I)=(I-1)*DXE
ISN 0050          IF(KNT.NE.1) GO TO 62
ISN 0052          DO 61 I=1,NPLS
ISN 0053       61 XLI(I)=(I-1)*DXI
ISN 0054       62 CALL SCALE(XL,NPLS,6.,XM,DI,1)
ISN 0055       70 CALL CLEAR(SL(1),SL(NPLS))
ISN 0056          CALL CLEAR(SSL(1),SSL(NPLS))
C
C  ************************************************************************
C  *                                                                      *
C  *  TEST FOR DECREASING EDGE DATA AND ADJUST IF NECESSARY               *
C  *                                                                      *
C  ************************************************************************
C
ISN 0057          IF(E(NP)-E(1).GT.0.) GO TO 72
ISN 0059          DO 71 I=1,NCF
ISN 0060       71 CF(I)=-CF(I)
C
C  ************************************************************************
C  *                                                                      *
C  *  LOCATE THE MAXIMUN VALUE OF THE DERIVATIVE OF THE EDGE DATA         *
C  *                                                                      *
C  ************************************************************************
C
ISN 0061       72 BGL=0.0
ISN 0062          DO 80 I=1,NCF
ISN 0063          IF(CF(I).LT.BGL) GO TO 80
ISN 0065          BGL=CF(I)
ISN 0066          LBIG=I
ISN 0067       80 CONTINUE
C
C  ************************************************************************
C  *                                                                      *
C  *  LOCATE AND SMOOTH THE NEGATIVE VALUES IN THE TAILS OF THE           *
C  *  DERIVATIVE, IF ANY                                                  *
C  *                                                                      *
C  ************************************************************************
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 4 OF 12)

```
ISN 0068          ITEST=LBIG
ISN 0069       90 ITEST=ITEST+1
ISN 0070          IF(ITEST.GT.NCF-1) GO TO 101
ISN 0072          IF(CF(ITEST).GE.0.) GO TO 90
ISN 0074          CF(ITEST)=(CF(ITEST-1)+CF(ITEST)+CF(ITEST+1))/3.
ISN 0075          IF(CF(ITEST).GE.0.) GO TO 90
ISN 0077      101 IHI=ITEST-1
ISN 0078          ITEST=LBIG
ISN 0079      105 ITEST=ITEST-1
ISN 0080          IF(ITEST.LT.2) GO TO 110
ISN 0082          IF(CF(ITEST).GE.0.) GO TO 105
ISN 0084          CF(ITEST)=(CF(ITEST-1)+CF(ITEST)+CF(ITEST+1))/3.
ISN 0085          IF(CF(ITEST).GE.0.) GO TO 105
ISN 0087      110 ILO=ITEST+1
C
C  ***************************************************************************
C  *                                                                         *
C  *  ADJUST THE NUMBER OF POINTS IN THE DERIVATIVE TO INCLUDE THE PORTION   *
C  *  BETWEEN 'ILO' AND' IHI' AND EQUATE THE LINE SPREADS TO THIS PORTION    *
C  *                                                                         *
C  ***************************************************************************
C
ISN 0088          NCF=IHI-ILO+1
ISN 0089      112 KP=(NPLS+1)/2
ISN 0090          KLO=LBIG-ILO+1
ISN 0091          KHI=IHI-LBIG
ISN 0092          IB=KP-KLO
ISN 0093          IE=ILO-1
ISN 0094          IF((KLO.GT.KP).OR.(KHI.GT.NPLS-KP)) GO TO 114
ISN 0096          DO 113 I=1,NCF
ISN 0097      113 SL(IB+I)=CF(IE+I)
ISN 0098          GO TO 116
ISN 0099      114 IF(KLO.GT.KHI) GO TO 115
ISN 0101          NPLS=2*KHI
ISN 0102          FI=1.0/(FLOAT(NPLS)*DX)
ISN 0103          WRITE(6,3030) FINV,FI,NED,NID
ISN 0104          IF(LTV(5)) WRITE (6,3032)
ISN 0106          LTV(5)=.FALSE.
ISN 0107          IF(LTV(6)) WRITE (6,3034)
ISN 0109          LTV(6)=.FALSE.
ISN 0110          FINV=FI
ISN 0111          GO TO 112
ISN 0112      115 NPLS=2*KLO
ISN 0113          FI=1.0/(FLOAT(NPLS)*DX)
ISN 0114          WRITE(6,3030) FINV,FI,NED,NID
ISN 0115          IF(LTV(5)) WRITE (6,3032)
ISN 0117          LTV(5)=.FALSE.
ISN 0118          IF(LTV(6)) WRITE (6,3034)
ISN 0120          LTV(6)=.FALSE.
ISN 0121          FINV=FI
ISN 0122          GO TO 112
C
C  ***************************************************************************
C  *                                                                         *
C  *  NORMALIZE LINE SPREAD AND EQUATE TO "STORED" LINE SPREAD               *
C  *                                                                         *
C  ***************************************************************************
```

**Figure 23** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 5 OF 12)

```
ISN 0123        116 SUM=0.0
ISN 0124            DO 117 I=1,NPLS
ISN 0125        117 SUM=SUM+SL(I)
ISN 0126            DO 118 I=1,NPLS
ISN 0127            SL(I)=SL(I)/SUM
ISN 0128        118 SSL(I)=SL(I)
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  COMPUTE STATISTICS FOR THE AVERAGE LINE SPREAD                         *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0129            IF(.NOT.LTV(5)) GO TO 130
ISN 0131            DO 120 I=1,NPLS
ISN 0132            ALS(I)=ALS(I)+SL(I)
ISN 0133        120 ELS(I)=ELS(I)+SL(I)*SL(I)
ISN 0134        130 IF(LTV(2))CALL SCALE(SSL,NPLS,2.,DUMMY,DUMMY,1)
ISN 0136            IF(.NOT.(LTV(3).OR.LTV(4).OR.LTV(6))) GO TO 210
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  FOURIER TRANSFORM LINE SPREAD TO OBTAIN THE COMPLEX OPTICAL TRANSFER    *
                C  *  FUNCTION                                                               *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0138            CALL FOURTR(SL,NPLS,DX,TF,PH,DNU,NH)
ISN 0139            IF(.NOT.LTV(4)) GO TO 160
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  COMPUTE  THE NUMBER OF HARMONICS FOR THE SMOOTHED LINE SPREAD          *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0141            NHM=FCO/FINV
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  APPLY INVERSE TRANSFORM TO OBTAIN THE SMOOTHED LINE SPREAD             *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0142            CALL FORINV(TF,PH,NHM,DNU,SL,NSL,DXSLS,1)
ISN 0143            DO 140 I=1,NSL
ISN 0144        140 SL(I)=ABS(SL(I))
ISN 0145            CALL SCALE(SL,NSL,2.,DUMMY,DUMMY,1)
ISN 0146            IF(KNT.NE.1) GO TO 180
ISN 0148            CALL DXSCAL(2.5,NSL,DXSLS,DXSI,DXS)
ISN 0149            DO 150 I=1,NSL
ISN 0150        150 XS(I)=(I-1)*DXS
ISN 0151        160 IF(KNT.NE.1) GO TO 180
ISN 0153            DO 170 I=1,NPTF
ISN 0154            XP(I)=(1-I)*DNU/DF
ISN 0155        170 XPI(I)=2.5+XP(I)*0.5
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 6 OF 12)

```
C  ********************************************************************************
C  *                                                                              *
C  *  COMPUTE SUMS RELATED TO THE AVERAGE MTF AND CONFIDENCE BAND                 *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0156      180 TF1=TF(1)
ISN 0157          DO 200 I=1,NPTF
ISN 0158          TF(I)=TF(I)/TF1
ISN 0159          ATF(I)=ATF(I)+TF(I)
ISN 0160      200 ETF(I)=ETF(I)+TF(I)**2
C
C  ********************************************************************************
C  *                                                                              *
C  *  PUNCH INDIVIDUAL MTF IF REQUESTED BY USER                                   *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0161          IF(LTV(7)) WRITE(6,3000)NED,NID,(TF(I),I=1,NPTF)
C
C  ********************************************************************************
C  *                                                                              *
C  *  SCALE INDIVIDUAL MTF FOR PLOTTING                                           *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0163          DO 205 I=1,NPTF
ISN 0164      205 TF(I)=TF(I)*2.0
C
C  ********************************************************************************
C  *                                                                              *
C  *  PLOT INDIVIDUAL FUNCTIONS AS SPECIFIED BY USER                              *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0165      210 IF(MOD(KNT,15).NE.1) GO TO 216
ISN 0167          FNED=NED
ISN 0168          NDS=NED
C
C  ********************************************************************************
C  *                                                                              *
C  *  ESTABLISH THE NUMBER OF INDIVIDUAL FUNCTION PLOTS REQESTED                  *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0169          NPL=0
ISN 0170          DO 212 I=1,4
ISN 0171      212 IF(LTV(I)) NPL=NPL+1
ISN 0173          IF(NPL.EQ.0) GO TO 50
C
C  ********************************************************************************
C  *                                                                              *
C  *  DRAW AXIS FOR EACH TYPE OF INDIVIDUAL FUNCTION PLOT REQUESTED               *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0175          CALL PLOT(0.,0.,0)
ISN 0176          XPL=-8.5
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 7 OF 12)

```
ISN 0177            DO 214 I=1,NPL
ISN 0178            XPL=XPL+8.5
ISN 0179            CALL MGRID(0.5,XPL+5.6,10.,2.5,2.,2.5,1)
ISN 0180            CALL MGRID(0.5,XPL+3.0,10.,2.5,2.,2.5,1)
ISN 0181            CALL MGRID(0.5,XPL+0.4,10.,2.5,2.,2.5,1)
ISN 0182          214 CALL NUMBER(0.15,XPL+4.5,0.14,FNED,-90.,-1)
ISN 0183            XR=0.0
ISN 0184            CALL PLOT(8.5,5.6,-3)
ISN 0185            XOR=0.0
ISN 0186            YOR=0.0
ISN 0187            GO TO 218
ISN 0188          216 IF(NPL.EQ.0) GO TO 50
ISN 0190            YOR=0.0
ISN 0191            XOR=-2.0
ISN 0192            IF(MOD(KNT,5).NE.1) GO TO 218
ISN 0194            XOR=8.0
ISN 0195            YOR=-2.6
ISN 0196          218 CALL PLOT(XOR,YOR+XR,-3)
ISN 0197            XR=0.0
ISN 0198            NPLT=NPL
ISN 0199            IF(LTV(1)) GO TO 230
ISN 0201          221 IF(LTV(2)) GO TO 231
ISN 0203          222 IF(LTV(3)) GO TO 232
ISN 0205          223 IF(LTV(4)) GO TO 233
ISN 0207          230 NPLT=NPLT-1
ISN 0208            CALL LINE(E,XLE,NP,1)
ISN 0209            CALL NUMBER(0.8,2.4,0.07,DXTE,-90.,3)
ISN 0210            KC=1
ISN 0211            GO TO 240
ISN 0212          231 NPLT=NPLT-1
ISN 0213            CALL LINE(SSL,XLI,NPLS,1)
ISN 0214            IF(KNT.EQ.1) CALL NUMBER(0.8,2.4,0.07,DXTI,-90.,3)
ISN 0216            KC=2
ISN 0217            GO TO 240
ISN 0218          232 NPLT=NPLT-1
ISN 0219            CALL LINE(TF,XPI,NPTF,1)
ISN 0220            IF(KNT.EQ.1) CALL NUMBER(1.8,0.75,.07,FMAX,-90.,1)
ISN 0222            KC=3
ISN 0223            GO TO 240
ISN 0224          233 NPLT=NPLT-1
ISN 0225            CALL LINE(SL,XS,NSL,1)
ISN 0226            IF(KNT.EQ.1) CALL NUMBER(0.8,2.4,.07,DXSI,-90.,3)
ISN 0228            IF(KNT.EQ.1) CALL NUMBER(1.0,2.4,0.07,FCO,-90.,1)
ISN 0230          240 IF(NPLT.EQ.0) GO TO 50
ISN 0232            CALL PLOT(0.,8.5,-3)
ISN 0233            XR=XR-8.5
ISN 0234            GO TO (221,222,223),KC
ISN 0235          400 IF(KNT.EQ.0) GO TO 10
ISN 0237            IF(.NOT.(LTV(5).OR.LTV(6))) GO TO 20
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  CALCULATE AVERAGE MTF AND 95% CONFIDENCE BAND                          *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0239            IF(.NOT.LTV(6)) GO TO 460
ISN 0241            DO 450 I=1,NPTF
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 8 OF 12)

```
ISN 0242          ATF(I)=ATF(I)/KNT
ISN 0243          DEV=AMAX1((ETF(I)-KNT*ATF(I)**2),0.0)
ISN 0244          DEV=2*SQRT(DEV/(KNT-1))
ISN 0245          DEV=DEV/SQRT(FLOAT(KNT))
ISN 0246          ATFM(I)=ATF(I)-DEV
ISN 0247        450 ATFP(I)=ATF(I)+DEV
ISN 0248        460 IF(.NOT.LTV(5)) GO TO 490
                C  ***************************************************************************
                C  *                                                                         *
                C  *  CALCULATE AVERAGE LINE SPREAD AND 95% CONFIDENCE INTERVAL              *
                C  *                                                                         *
                C  ***************************************************************************
                C
                C
ISN 0250          DO 480 I=1,NPLS
ISN 0251          ALS(I)=ALS(I)/KNT
ISN 0252          DEV=AMAX1(ELS(I)-KNT*ALS(I)**2,0.0)
ISN 0253          DEV=2*SQRT(DEV/(KNT-1))
ISN 0254          DEV=DEV/SQRT(FLOAT(KNT))
ISN 0255          ALSP(I)=ALS(I)+DEV
ISN 0256        480 ALSM(I)=ALS(I)-DEV
ISN 0257          BGL=0.0
ISN 0258          DO 482 I=1,NPLS
ISN 0259          IF(ALSP(I).LT.BGL) GO TO 482
ISN 0261          BGL=ALSP(I)
ISN 0262          LBIG=I
ISN 0263        482 CONTINUE
ISN 0264          CALL SCALE(ALSP,NPLS,5.,SLM,DL,1)
ISN 0265          SF=ALSP(LBIG)/BGL
ISN 0266          DO 484 I=1,NPLS
ISN 0267          ALS(I)=ALS(I)*SF
ISN 0268        484 ALSM(I)=ALSM(I)*SF
ISN 0269          IF(.NOT.LTV(6)) GO TO 500
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  PUNCH AND PRINT AVERAGE MTF IF REQUESTED                               *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0271        490 IF(LTV(7)) WRITE(6,3000)NDS,KNT,(ATF(I),I=1,NPTF)
ISN 0273          IF(LTV(7)) WRITE(7,3010)NDS
ISN 0275          IF(LTV(7)) WRITE(7,3020)(ATF(I),I=1,NPTF)
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  PLOT AVERAGE MTF AND CONFIDENCE INTERVAL IF REQUESTED                  *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0277          DO 495 I=1,NPTF
ISN 0278          ATF(I)=5.0*ATF(I)
ISN 0279          ATFP(I)=5.0*ATFP(I)
ISN 0280        495 ATFM(I)=5.0*ATFM(I)
ISN 0281          CALL PLOT(0.,0.,0)
ISN 0282          CALL PLOT(2.5,6.5,-3)
ISN 0283          CALL AXIS(0.,0.,2HNU,-2,5.,-90.,0.,DF)
ISN 0284          CALL AXIS(0.,0.,3HTAU,3,5., 0.,0.,0.2)
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 9 OF 12)

```
ISN 0285          CALL MGRID(0.,-5.,5.,5.,1.,1.,1)
ISN 0286          CALL LINE(ATFP,XP,NPTF,1)
ISN 0287          CALL LINE(ATF,XP,NPTF,1)
ISN 0288          CALL LINE(ATFM,XP,NPTF,1)
ISN 0289          CALL SYMBOL(-1.5,-1.2,0.14,22HAVERAGE OF     SAMPLES,-90.,22)
ISN 0290          CALL NUMBER(-1.5,-2.52,0.14,FLOAT(KNT),-90.,-1)
ISN 0291          CALL NUMBER(5.5,-2.16,0.14,FNED,-90.,-1)
ISN 0292          IF(.NOT.LTV(5)) GO TO 20
                  C
                  C  ***************************************************************************
                  C  *                                                                         *
                  C  *  PLOT AVERAGE LINE SPREAD AND CONFIDENCE INTERVAL IF REQUESTED          *
                  C  *                                                                         *
                  C  ***************************************************************************
                  C
ISN 0294        500 CALL PLOT(0.,0.,0)
ISN 0295          CALL PLOT(2.5,7.,-3)
ISN 0296          CALL AXIS(0.,0.,1HX,-1,6.,-90.,XM,DI)
ISN 0297          CALL AXIS(0.,0.,4HL(X),4,5., 0.,SLM,DL)
ISN 0298          CALL PLOT(0.,-6.,-3)
ISN 0299          CALL LINE(ALS,XL,NPLS,1)
ISN 0300          CALL LINE(ALSP,XL,NPLS,1)
ISN 0301          CALL LINE(ALSM,XL,NPLS,1)
ISN 0302          CALL PLOT(0.,6.,-3)
ISN 0303          CALL SYMBOL(-1.5,-1.7,0.14,22HAVERAGE OF     SAMPLES,-90.,22)
ISN 0304          CALL NUMBER(-1.5,-3.02,0.14,FLOAT(KNT),-90.,-1)
ISN 0305          CALL NUMBER(5.5,-2.66,0.14,FNED,-90.,-1)
ISN 0306          GO TO 20
                  C
                  C  ***************************************************************************
                  C  *                                                                         *
                  C  *  TERMINATE EXECUTION                                                    *
                  C  *                                                                         *
                  C  ***************************************************************************
                  C
ISN 0307        550 CALL EFPLOT
ISN 0308          STOP
ISN 0309          END
```

Figure 23 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 10 OF 12)

```
                *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
E       0002  0032  0035  0035  0036  0036  0038  0040  0057  0057  0208
I       0013  0013  0013  0015  0016  0017  0019  0032  0032  0032  0034  0035  0036  0047  0048  0048  0049  0049  0052
        0053  0053  0059  0060  0060  0062  0063  0065  0066  0096  0097  0097  0124  0125  0126  0127  0127  0128  0128
        0131  0132  0132  0132  0133  0133  0133  0133  0143  0144  0144  0149  0150  0150  0153  0154  0154  0155  0155
        0157  0158  0158  0159  0159  0159  0160  0160  0160  0161  0161  0161  0163  0164  0164  0170  0171  0177  0241
        0242  0242  0243  0243  0246  0246  0247  0247  0250  0251  0251  0252  0252  0255  0255  0256  0256  0258  0259
        0261  0262  0266  0267  0267  0268  0268  0271  0271  0271  0275  0275  0275  0277  0278  0278  0279  0279  0280
        0280
J       0018  0019  0019
CF      0002  0040  0060  0060  0063  0065  0072  0074  0074  0074  0074  0075  0082  0084  0084  0084  0084  0085  0097
DF      0022  0154  0283
DI      0054  0296
DL      0264  0297
DX      0028  0041  0044  0046  0048  0102  0113  0138
FI      0102  0103  0110  0113  0114  0121
IB      0092  0097
IE      0093  0097
KC      0210  0216  0222  0234
KP      0089  0092  0094  0094
NH      0138
NP      0028  0029  0031  0031  0032  0036  0036  0037  0037  0038  0040  0046  0057  0208
PH      0002  0138  0142
SF      0265  0267  0268
SL      0002  0055  0055  0097  0125  0127  0127  0128  0132  0133  0133  0138  0142  0144  0144  0145  0225
TF      0002  0138  0142  0156  0158  0158  0159  0160  0161  0164  0164  0219
XL      0002  0048  0054  0299  0300  0301
XM      0054  0296
XP      0002  0154  0155  0286  0287  0288
XR      0183  0196  0197  0233  0233
XS      0002  0150  0225
ABS     0144
ALS     0002  0026  0026  0132  0132  0251  0251  0252  0255  0256  0267  0267  0299
ATF     0002  0024  0024  0159  0159  0242  0242  0243  0246  0247  0271  0275  0278  0278  0287
BGL     0061  0063  0065  0257  0259  0261  0265
DEV     0243  0244  0244  0245  0245  0246  0247  0252  0253  0253  0254  0254  0255  0256
DNU     0138  0142  0154
DUM     0002  0040  0040
DXE     0046  0049
DXI     0044  0053
DXS     0148  0150
ELS     0002  0027  0027  0133  0133  0252
ETF     0002  0025  0025  0160  0160  0243
FCO     0013  0141  0228
IHI     0077  0088  0091
ILO     0087  0088  0090  0093
KHI     0091  0094  0099  0101
KLO     0090  0092  0094  0099  0112
KNT     0023  0033  0033  0044  0050  0146  0151  0165  0192  0214  0220  0226  0228  0235  0242  0243  0244  0245  0251
        0252  0253  0254  0271  0290  0304
LTV     0003  0003  0016  0019  0038  0042  0042  0042  0104  0106  0107  0109  0115  0117  0118  0120  0129  0134  0136
        0136  0136  0139  0161  0171  0199  0201  0203  0205  0237  0237  0239  0248  0269  0271  0273  0275  0292
MOD     0165  0192
NCF     0040  0059  0062  0070  0088  0096
NDS     0168  0271  0273
NED     0028  0103  0114  0161  0167  0168
NHM     0141  0142
NID     0028  0103  0114  0161
```

**Figure 23** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 11 OF 12)

```
                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
NPL     0014  0169  0171  0171  0173  0177  0188  0198
NSL     0142  0143  0145  0148  0149  0225
OPT     0002  0004  0019
SLM     0264  0297
SSL     0002  0056  0056  0128  0134  0213
SUM     0123  0125  0125  0127
TF1     0156  0158
XLE     0002  0049  0208
XLI     0002  0053  0213
XOR     0185  0191  0194  0196
XPI     0002  0155  0219
XPL     0176  0178  0178  0179  0180  0181  0182
YOR     0186  0190  0195  0196
ALSM    0002  0256  0268  0268  0301
ALSP    0002  0255  0259  0261  0264  0265  0300
ATFM    0002  0246  0280  0280  0288
ATFP    0002  0247  0279  0279  0286
AXIS    0283  0284  0296  0297
DXSI    0148  0226
DXTE    0046  0209
DXTI    0044  0214
FINV    0013  0021  0041  0103  0110  0114  0121  0141
FMAX    0013  0021  0022  0220
FNED    0167  0182  0291  0305
LBIG    0066  0068  0078  0090  0091  0262  0265
LINE    0208  0213  0219  0225  0286  0287  0288  0299  0300  0301
NPLS    0041  0044  0047  0048  0052  0054  0055  0056  0089  0094  0101  0102  0112  0113  0124  0126  0131  0134  C138
        0213  0250  0258  0264  0266  0299  0300  0301
NPLT    0198  0207  0207  0212  0212  0218  0218  0224  0224  0230
NPTF    0021  0153  0157  0161  0163  0219  0241  0271  0275  0277  0286  0287  0288
PLOT    0175  0184  0196  0232  0281  0282  0294  0295  0298  0302
SQRT    0244  0245  0253  0254
AMAX1   0243  0252
CLEAR   0024  0025  0026  0027  0055  0056
DUMMY   0038  0038  0134  0134  0145  0145
DXSLS   0142  0148
FLOAT   0102  0113  0245  0254  0290  0304
ITEST   0068  0069  0069  0070  0072  0074  0074  0074  0074  0075  0077  0078  0079  0079  0080  0082  0084  0084  0084
        0084  0085  0087
MGRID   0179  0180  0181  0285
SCALE   0038  0054  0134  0145  0264
CONVLV  0040
DXSCAL  0044  0046  0148
EFPLOT  0307
FORINV  0142
FOURTR  0138
NUMBER  0182  0209  0214  0220  0226  0228  0290  0291  0304  0305
OPTION  0002  0013  0019
PLOTER  0012
SYMBOL  0289  0303
```

**Figure 23** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM EGSA (PAGE 12 OF 12)

and should be available to most users. The last subprogram, CLEAR, fills a specified block of storage with zeros. If necessary, an equivalent operation can be accomplished by a sequence of do loops in Program EGSA.

The program initially reads user supplied input data on punched cards. The following variables are specified for each data set to be processed:

FINV - spatial frequency interval desired for output OTF data in cycles/mm

FMAX - maximum spatial frequency of output OTF data in cycles/mm

OPTION (7) - alpha-numeric array identifying the types of output data desired by user

FCO - cutoff spatial frequency in cycles/mm used in smoothing the line spread

Seven output options are available to the user. These are specified by employing one or more of a set of alpha-numeric characters on the punched card input. These characters must be right hand justified in the appropriate field and may be listed in any order. The following characters are allowed:

E - specifies plots of the individual input edge traces of the data set

L - specifies plots of the individual line spread data resulting from differentiation of the edge data

T - specifies plots of the individual MTFs* resulting from Fourier transformation of the line spread functions

SL - specifies plots of smooth line spread functions

AL - specifies a plot of the average line spread and its associated confidence bands

AT - requests a plot of the average MTF and its confidence bands

PT - requests the values of the individual and average MTFs

* The Modulation Transfer Function (MTF) is simply the modulus of the complex OTF.

on punched cards and printed values of the average MTF. The values of the average MTF are not punched or printed unless the option for a plot is also specified.

The program sets elements in a logical array, LTV, to true for the options requested if they are properly specified on the input data card. Figure 24 contains an example of this data card where the user has selected plots of the individual MTFs and the average MTF. This input card also specifies the spatial frequency interval as 5 cycles/mm and a maximum spatial frequency of 150 cycles/mm. The cutoff spatial frequency variable is unspecified or equal to zero, indicating that the user has not elected to smooth the line spreads consistent with the deletion of the SL option on the card. The value for the frequency cutoff is, in fact, ignored if this option is not specified.

The next major step in the program is the initialization of parameters and clearing of arrays prior to beginning the actual processing of an edge data set. Identification and data records are then read from an input data tape created using main program DTAPE described previously in Section 1.0. The edge data are then adjusted for the number of points that will be lost due to differentiation. The differentiation is accomplished by subroutine CONVLV described in the next section. Although the current version contains an 11-point, internally specified differentiating filter the dimensions allow a maximum of a 25-point filter. The adjustment in the edge data is commensurate with the maximum size filter.

After differentiation the next major step in the execution involves the set-up of x-axis arrays for the output plots of several of the individual functions. This is followed by manipulation of the differentiated edge data including a non-stationary 3-point smoothing of negative values in its tails. Only the non-negative portion of the edge data is retained and equated to the smoothed and unsmoothed line spread functions. The line spread function is then Fourier transformed by calling FOURTR to obtain the modulus and phase functions of the OTF. If the user has requested a smooth line spread, this is computed by truncation of the OTF and subsequent inverse Fourier transformation. The

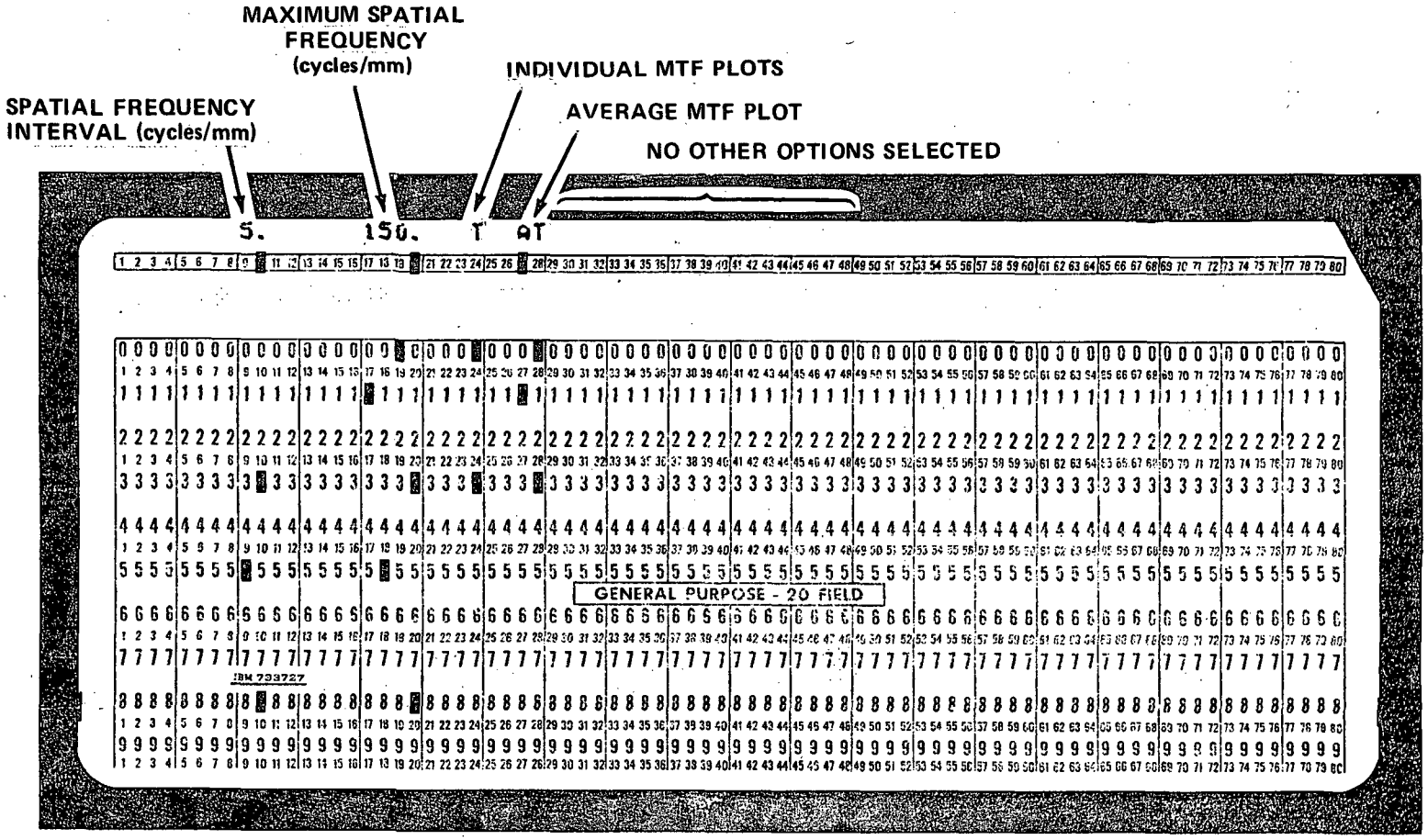
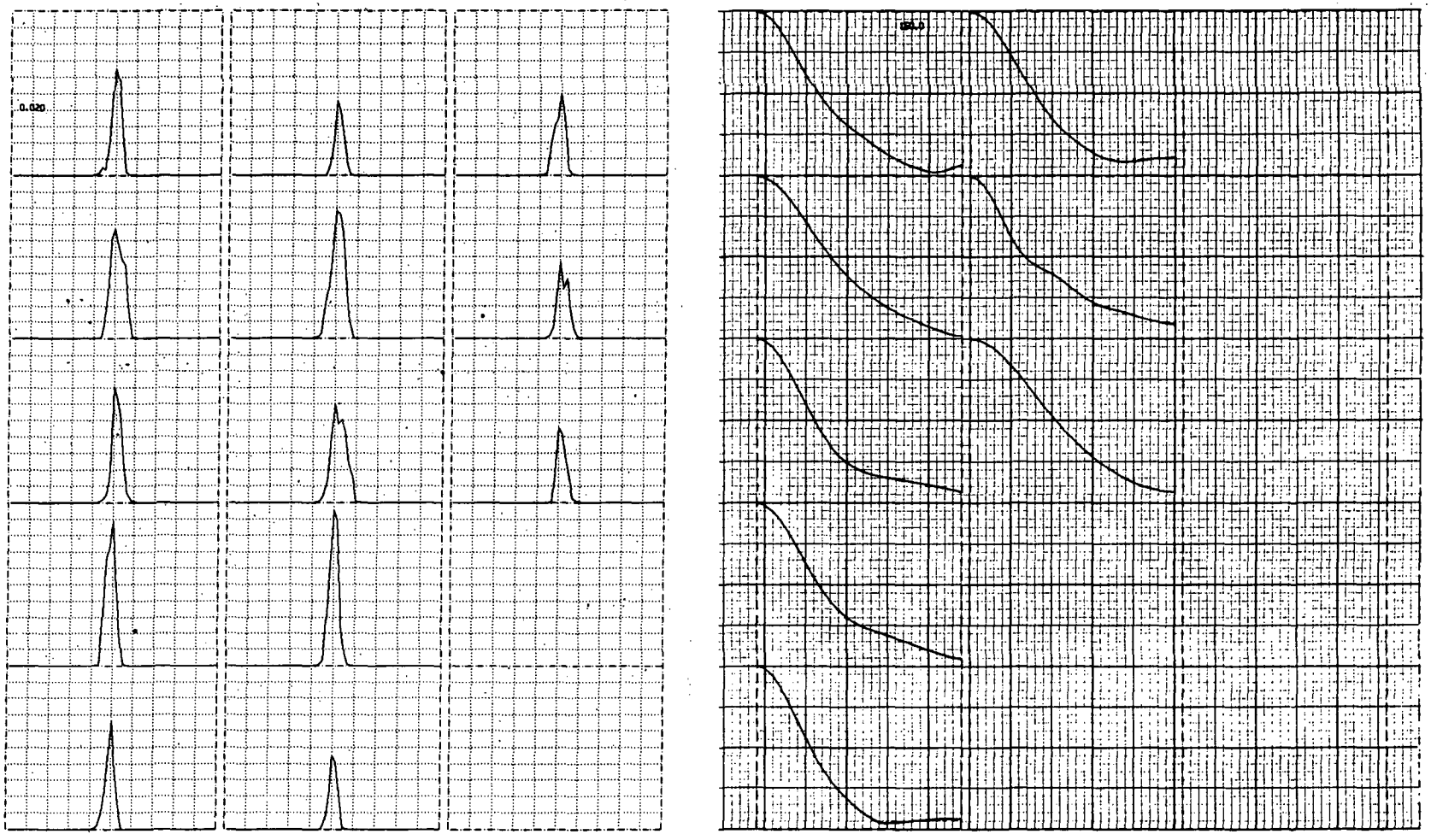



**Figure 24 TYPICAL EGSA PROGRAM INPUT DATA CARD**

inverse transformation is accomplished by subroutine FORINV. This calculation is primarily a cosmetic operation and produces no new information about the optical system performance that is not contained in the unsmoothed line spread or the complex OTF.

The next steps in the program involve the production of output data as requested by the user. These include punched cards of the individual MTF and plots of the individual edges, line spreads, smoothed line spreads or MTF. The program then recycles to read the next edge trace in the current edge data set and repeats the processing cycle. If no additional edge data exists, the calculation and plotting of the average line spread and a 95% confidence band and the average MTF and its confidence band are accomplished if these options were requested by the user. The program recycles to process the next edge data set, if any, in the same manner. If no additional data sets are identified the program terminates.

Examples of the plots of individual function data generated by the program are presented in Figure 25. Part (a) shows individual line spread functions which resulted during an analysis of the reproduction of Apollo 15 photography and Part (b) shows individual MTF data resulting during a similar analysis of Apollo 17 photography. The original plots fill an 8 x 11 1/2 inch page and were reduced for reproduction here. Similar plots are generated for the individual edges and smoothed line spreads if requested. A number is placed at the bottom of each page which corresponds to the identification number NED specified by the user. In the examples shown this is the number of the frame from which the edge data was acquired. The top left hand plot in both cases also contains a number specifying the total length of the x-axis. In the case of edges, line spreads or smoothed line spreads, this length is expressed in millimeters and for MTFs as spatial frequency in cycles/millimeters. Figure 26 contains an example of an average MTF plot generated from the individual MTFs in Figure 25(b) as part of the evaluation of Apollo 17 photography. The upper and lower curves define the 95% confidence band. The number placed at the top of a plot corresponds to the user specified variable NED and




(a) LINE SPREAD FUNCTIONS

(b) MTFS

**Figure 25 EXAMPLE OF INDIVIDUAL FUNCTION PLOTS FROM THE EGSA PROGRAM**

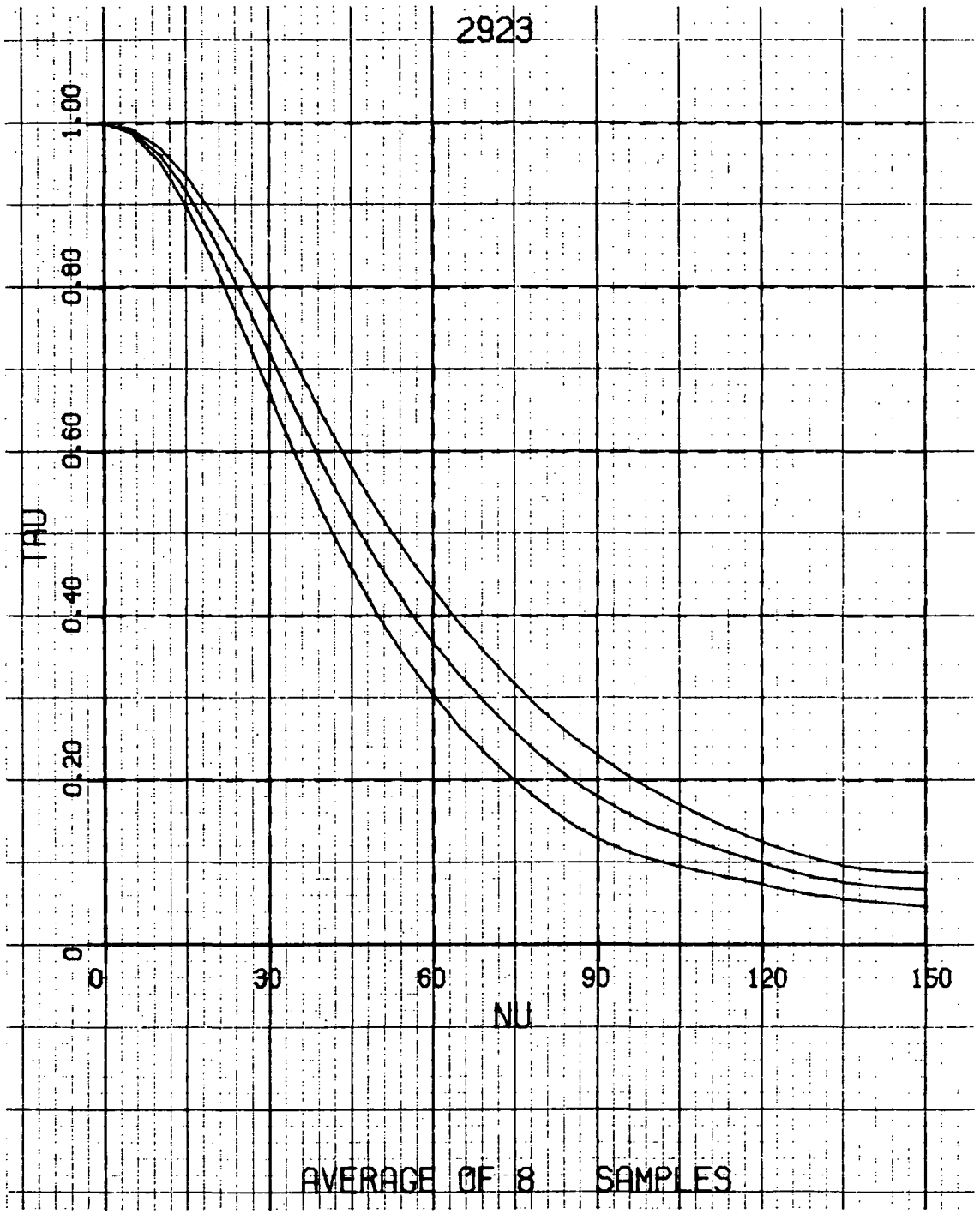



**Figure 26 EXAMPLE OF AN AVERAGE MTF PLOT FROM THE EGSA PROGRAM**

in this case is equal to the corresponding frame number. The number of individual edge traces which were used to compute the average MTF is indicated on the bottom of the plot. A plot with a similar format is generated for the average line spread function if that output option is requested.

4.0 Subroutine CONVLV - This program is part of the Edge Gradient Spectral Analysis software package. It supports Program EGSA in the calculation of an OTF from edge trace data. The purpose of this subroutine is to differentiate the edge trace data using an internally specified differentiation filter. No direct input data must be furnished by the user. The input data is supplied through the calling program. No printed output data is furnished by this program.

Figure 27 is a listing of subroutine CONVLV. The function of the major blocks are identified by the comments included in the coding. This subroutine has capabilities that are not exploited by the EGSA main program. It can perform the convolution between two input functions or an input function and an internally stored filter. The internal filter options include an 11-point smoothing filter and an 11-point differentiating filter. A maximum of 25-point filters are permitted by the dimension statement and can be inserted by changing the appropriate data statement. The options available in the convolving operation are controlled by the variable KCNTRL as follows:

1 - specifies the convolution between two input functions
2 - causes the primary input function to be smoothed by the internal filter
3 - specifies differentiation of the primary input function by the appropriate internal filter
4 - indicates smoothing of the primary input function followed by differentiation.

The computing task branches depending upon the value of the variable KCNTRL. When the specified option involves the convolution between two input functions

```
C ********************************************************************************
C ********************************************************************************
C **                                                                            **
C **  SUBROUTINE CONVLV                                                         **
C **                                                                            **
C **       THIS SUBROUTINE IS PART OF THE EDGE GRADIENT SPECTRAL ANALYSIS       **
C **  (EGSA) SOFTWARE PACKAGE.  THIS PACKAGE ESTIMATES THE OPTICAL              **
C **  TRANSFER FUNCTION (OTF) FOR ANY OPTICAL SYSTEM USING A SET OF EDGE        **
C **  DATA MEASURED AT THE OUTPUT OF THE SYSTEM.  THIS SUBROUTINE               **
C **  PERFORMS THE CONVOLUTION BETWEEN TWO INPUT FUNCTIONS OR AN INPUT          **
C **  FUNCTION AND AN INTERNALLY STORED FUNCTION.  THE INTERNAL FUNCTION        **
C **  OPTIONS INCLUDE AN 11-POINT SMOOTHING FILTER AND AN 11-POINT              **
C **  DIFFERENTIATING FILTER.  UP TO 25-POINT FITLERS CAN BE INSERTED BY        **
C **  CHANGING THE APPROPIATE DATA STATEMENTS.  THE CONVOLVING OPERATION IS     **
C **  SPECIFIED BY THE VARIABLE "KCNTRL" AS FOLLOWS:                            **
C **       =1 CONVOLUTION OF TWO INPUT FUNCTIONS                                **
C **       =2 SMOOTHING OF INPUT FUNCTION BY INTERNAL FILTER                    **
C **       =3 DIFFERENTIATION OF INPUT FUNCTION BY INTERNAL FILTER               **
C **       =4 SMOOTHING FOLLOWED BY DIFFERENTIATION                             **
C **                                                                            **
C **                                                                            **
C **                      ** VARIABLE LIST **                                   **
C **                                                                            **
C **  INPUT VARIABLES                                                           **
C **                                                                            **
C **       FUN1    =PRIMARY INPUT FUNCTION.  RETURNED TO THE CALLING            **
C **                PROGRAM UNCHANGED.  MUST BE DIMENSIONED IN THE CALLING      **
C **                PROGRAM.                                                    **
C **       FUN2    =SECONDARY INPUT FUNCTION.  MUST HAVE FEWER POINTS THAN      **
C **                THE PRIMARY INPUT FUNCTION.  RETURNED TO THE CALLING        **
C **                PROGRAM UNCHANGED.  MUST BE DIMENSIONED IN THE CALLING      **
C **                PROGRAM.                                                    **
C **       KCNTRL =INTEGER FLAG IDENTIFYING THE EXECUTION OPTION SELECTED       **
C **                BY THE USER.                                                **
C **       N1      =NUMBER OF POINTS IN FUN1.                                   **
C **       N2      =NUMBER OF POINTS IN FUN2.                                   **
C **                                                                            **
C **  OUTPUT VARIABLES                                                          **
C **                                                                            **
C **       CFUN    =CONVOLVED FUNCTION.  MUST BE DINENSIONED IN THE CALLING     **
C **                PROGRAM.                                                    **
C **       NC      =NUMBER OF POINTS IN CFUN.                                   **
C **                                                                            **
C **                                                                            **
C **       AUTHORS: R. E. KINZLY                                                **
C **                M. J. MAZUROWSKI                                            **
C **       DATE: APRIL, 1966                                                    **
C **                                                                            **
C ********************************************************************************
C ********************************************************************************
C
ISN 0002        SUBROUTINE CONVLV(FUN1,N1,FUN2,N2,CFUN,NC,KCNTRL)
ISN 0003        DIMENSION FUN1(1), FUN2(1), CFUN(1), FIL(25), SMF(25), DIF(25)
```

Figure 27 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE CONVLV (PAGE 1 OF 4)

```
C  ********************************************************************************
C  *                                                                              *
C  *  SMOOTHING FILTER DEFINITION (DATA STATMENT)                                 *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0004          DATA SMF(1),SMF(2),SMF(3),SMF(4),SMF(5),SMF(6),SMF(7),SMF(8),
                 *SMF(9),SMF(10),SMF(11)/-0.0005616,-0.0005366,0.02395,-0.1049,
                 *0.2266,0.72108,0.2266,-0.1049,0.02395,-0.0005366,-0.0005616/,
                 *NSM/11/
C
C  ********************************************************************************
C  *                                                                              *
C  *  DIFFERENTIATING FILTER DEFINITION (DATA STATMENT)                           *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0005          DATA DIF(1),DIF(2),DIF(3),DIF(4),DIF(5),DIF(6),DIF(7),DIF(8),
                 *DIF(9),DIF(10),DIF(11)/0.00079365,-0.0099206,0.059524,-0.2381,
                 *0.833333,0.0,-0.833333,0.2381,-0.059524,0.0099206,-0.00079365/,
                 *NDF/11/
C
C  ********************************************************************************
C  *                                                                              *
C  *  TRANSFER CONTROL TO THE APPROPIATE PROGRAM SECTION BASED UPON THE            *
C  *  VALUE OF "KCNTRL"                                                           *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0006          GO TO (1,20,30,20),KCNTRL
C
C  ********************************************************************************
C  *                                                                              *
C  *  CONVOLVE TWO INPUT FUNCTIONS "FUN1" AND "FUN2"                              *
C  *                                                                              *
C  ********************************************************************************
C
ISN 0007           1 NC=N1+N2+1
ISN 0008             DO 3 I=1,NC
ISN 0009           3 CFUN(I)=0.0
ISN 0010             IF(N1.LT.N2) RETURN
ISN 0012             NCF1 = NC-1
ISN 0013             DO 8 I=2,NCF1
ISN 0014             CFUN(I) =0.0
ISN 0015             KK = 1
ISN 0016             IF(I.GT.N2+1)  KK=I-N2
ISN 0018             KKK = I-1
ISN 0019             IF(I.GT.N1+1)  KKK=N1
ISN 0021             DO 7 J = KK,KKK
ISN 0022             L=I-J
ISN 0023           7 CFUN(I) = CFUN(I) + FUN1(J)* FUN2(L)
ISN 0024           8 CONTINUE
ISN 0025             RETURN
```

**Figure 27 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE CONVLV (PAGE 2 OF 4)**

```
C  ************************************************************************
C  *                                                                      *
C  *  SMOOTH INPUT FUNCTION "FUN1" USING THE INTERNAL FILTER "SMF"        *
C  *                                                                      *
C  ************************************************************************
C
ISN 0026        20 NFP = NSM
ISN 0027           DO 21  I=1,NFP
ISN 0028        21 FIL(I) = SMF(I)
ISN 0029        22 LOSS = (NFP-1)/2
ISN 0030           NFM1 = N1-LOSS
ISN 0031           NC = N1
ISN 0032           LP1=LOSS+1
ISN 0033           GO TO 40
C
C  ************************************************************************
C  *                                                                      *
C  *  DIFFERENTIATE THE INPUT FUNCTION "FUN1" USING THE INTERNAL FILTER "DIF"  *
C  *                                                                      *
C  ************************************************************************
C
ISN 0034        30 NFP=NDF
ISN 0035           DO 31 I=1,NFP
ISN 0036        31 FIL(I) = -DIF(I)
ISN 0037           IF(KCNTRL.EQ.3) GO TO 22
ISN 0039           DO 33 I=1,N1
ISN 0040        33 FUN1(I)=CFUN(I)
ISN 0041           KCNTRL=3
ISN 0042           GO TO 22
C
C  ************************************************************************
C  *                                                                      *
C  *  PERFORM THE CONVOLUTION OF THE INPUT FUNCTION AND EITHER INTERNAL    *
C  *  FILTER                                                              *
C  *                                                                      *
C  ************************************************************************
C
ISN 0043        40 DO 48 I=LP1,NFM1
ISN 0044           CFUN(I) = 0.0
ISN 0045           KK = I-LOSS
ISN 0046           KL = I +LOSS
ISN 0047           JI=0
ISN 0048           DO 47  J=KK,KL
ISN 0049           JI=JI+1
ISN 0050        47 CFUN(I) = CFUN(I)+FUN1(J)*FIL(JI)
ISN 0051        48 CONTINUE
ISN 0052           DO 49 I=1,LOSS
ISN 0053           II=NC-LOSS+I
ISN 0054           CFUN(I)=CFUN(LOSS+1)
ISN 0055        49 CFUN(II) = CFUN(II-1)
ISN 0056           IF(KCNTRL.EQ.4) GO TO 30
ISN 0058           RETURN
ISN 0059           END
```

**Figure 27** SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE CONVLV (PAGE 3 OF 4)

```
                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
I       0008  0009  0013  0014  0016  0016  0018  0019  0022  0023  0023  0027  0028  0028  0035  0036  0036  0039  0040
        0040  0043  0044  0045  0046  0050  0050  0052  0053  0054
J       0021  0022  0023  0046  0050
L       0022  0023
II      0053  0055  0055
JI      0047  0049  0049  0050
KK      0015  0016  0021  0045  0048
KL      0046  0048
NC      0002  0007  0008  0012  0031  0053
N1      0002  0007  0010  0019  0019  0030  0031  0039
N2      0002  0007  0010  0016  0016
DIF     0003  0005  0005  0005  0005  0005  0005  0005  0005  0005  0005  0005  0036
FIL     0003  0028  0036  0050
KKK     0018  0019  0021
LP1     0032  0043
NDF     0005  0034
NFP     0026  0027  0029  0034  0035
NSM     0004  0026
SMF     0003  0004  0004  0004  0004  0004  0004  0004  0004  0004  0004  0004  0028
CFUN    0002  0003  0009  0014  0023  0023  0040  0044  0050  0050  0054  0054  0055  0055
FUN1    0002  0003  0023  0040  0050
FUN2    0002  0003  0023
LOSS    0029  0030  0032  0045  0046  0052  0053  0054
NCF1    0012  0013
NFM1    0030  0043
CONVLV  0002
KCNTRL  0002  0006  0037  0041  0056


 LABEL   DEFINED   REFERENCES
    1     0007      0006
    3     0009      0008
    7     0023      0021
    8     0024      0013
   20     0026      0006  0006
   21     0028      0027
   22     0029      0037  0042
   30     0034      0006  0056
   31     0036      0035
   33     0040      0039
   40     0043      0033
   47     0050      0048
   48     0051      0043
   49     0055      0052
```

**Figure 27 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE CONVLV (PAGE 4 OF 4)**

expressed mathematically as

$$f_c(x) = \int_{D_g} f(y)\, g(x-y)\, dy \tag{28}$$

the first or primary function in the calling list must contain more points than the second function. If this is not the case the program returns zeros for all points in the convolved function. Equation 28 is implemented as

$$f_{c,i} = \sum_{n=1}^{n_g} f_{i-n_g+n-1} \cdot g_{n_g-n+1} \; ; \; i = 1, \cdots, n_f + n_g + 1 \tag{29}$$

where $f_n = 0.0$ for $n \leq 0$ or $n \geq n_f + 1$ and $f_{c,\, n_f + n_g + 1}$ $f_{c,1} = 0.0$ . The assumption that the primary function is zero outside of the range of its input should be compensated for in the calling program if it is not valid. This is accomplished by deleting points at the beginning and/or the end of the convolved function array returned to the calling program. Note that the total number of points in the returned function is one more than the sum of the number of points in each individual input function.

When either the differentiation or smoothing operations have been specified Equation 28 is implemented as

$$f_{c,i} = \sum_{n=1}^{n_g} f_{i-n_g+n-1} \cdot g_n \; ; \; i = \frac{n_g+1}{2}, \cdots, n_f - \frac{n_g-1}{2}$$

and (30)

$$f_{c,1} = f_{c,2} = \cdots = f_{c,\, \frac{n_g+1}{2}}$$

$$f_{c,n_f} = f_{c,n_f-1} = \cdots = f_{c,\, n_f - \frac{n_g-1}{2}}$$

where $g_n$ is set equal to the appropriate internally stored filter and $n_g$ is the number of points in the filter array. Note that $n_g - 1$ points, divided equally between the beginning and the end of the convolved function

are not computed by Equation 30 but arbitrarily set to the nearest computable point. Therefore, although $n_f$ points are returned in the convolved function array, some of these points contain artificial information and may require special treatment in the calling program.

The major difference between Equations 29 and 30, beside the method of treatment of the points at the beginning and end of the convolved function array, is the folding of the secondary function $g$ . In Equation 29 this folding is accomplished through manipulation of the index while this is not required in Equation 30 because of symmetry properties of the internally stored smoothing and differentiating filters. The secondary function is actually set equal to the negative of the stored differentiating filter in order to accomplish the required folding.

Note that both Equation 29 and Equation 30 do not account for the effect of the differential, $d_y$ , present in Equation 28. This must be accomplished in the calling program if required.

5.0 Subroutine DXSCAL - This program is part of the Edge Gradient Spectral Analysis software package. It supports Program EGSA in the calculation of the OTF from edge trace data. The purpose of this subroutine is to determine the best scale factor for the x-axis for plots of individual functions requested by the user as an output option. No direct input data must be furnished by the user. The input data is supplied through the calling program. No printed output data is furnished by this program.

No explicit listing of this program is provided since its requirements may depend upon the user's computing facility similar to the other plotting subroutines. Its operation, however, is straightforward and will permit a user to code an equivalent program.

Three calls to DXSCAL are made by Program EGSA. The calling sequence is DXSCAL (XL, NP, DX, DXT, DXI) where

XL = the length of the x-axis in inches (input variable)

NP = the number of points in the array to be plotted (input variable)

DX = the increment between points (input variable)

DXT = the total length of the x-axis in units of DX (output variable)

DXI = the scale for plotting in inches/point (output variable)

The purpose of the subrouting is to compute "convenient" values for DXT and DXI. DXI is chosen so that DX/DXI is the smallest number in the set ($10^m$, $1.5*10^m$, $2*10^m$, $2.5*10^m$, $4*10^m$, $5*10^m$, $8*10^m$, m an integer) that makes the data fit an axis of length XL, i.e. subject to the restriction that (NP-1) * DXI $\leq$ XL. This scaling ensures that the plotted function will fill at least half of the XL inches.

DXI is used directly by Program EGSA to compute an x-axis array for the plots of individual edges, line spreads or smoothed line spreads. The DXT variable is placed on the plots so that the axis can be labeled by user if desired. The units of DX and hence DXT in these cases are mm.

6.0 Subroutine FORINV - This program is part of the Edge Gradient Spectral Analysis software package. It supports Program EGSA in the calculation of an OTF from edge trace data. The purpose of this subroutine is to Fourier transform complex spectra (modulus and phase) to obtain a real output function. Specifically it is used by Program EGSA to compute the smooth line spread function from the truncated OTF. No direct input data must be furnished by the user. The input data is supplied through the calling program. No printed output data is furnished by this program.

Figure 28 is a listing of subroutine FORINV. The function of the major blocks are identified by the comments included in the coding. The algorithm employed in this program is specifically designed for transforming a complex to a real function. The function to be transformed is expressed as modulus and phase functions. Consequently the inverse Fourier transform can be written as

$$I(x) = \int \tau(\nu)\, e^{-i(2\pi\nu x - \phi(\nu))}\, d\nu \qquad (31a)$$

```
C ***************************************************************************
C **                                                                       **
C **  SUBROUTINE FORINV                                                    **
C **                                                                       **
C **       THIS SUBROUTINE IS PART OF THE EDGE GRADIENT SPECTRAL ANALYSIS  **
C **  (EGSA) SOFTWARE PACKAGE.  THIS PACKAGE ESTIMATES THE OPTICAL         **
C **  TRANSFER FUNCTION (OTF) FOR ANY OPTICAL SYSTEM USING A SET OF EDGE   **
C **  DATA MEASURED AT THE OUTPUT OF THE SYSTEM.  THIS SUBROUTINE          **
C **  COMPUTES THE INVERSE FOURIER TRANSFORM USING AN ALGORITHM FOR        **
C **  TRANSFORMING A COMPLEX FUNCTION TO A REAL FUNCTION.  AN OPTION FOR   **
C **  SMOOTHING THE MODULUS OF THE FUNCTION TO BE TRANSFORMED PRIOR TO     **
C **  TRANSFORMATION CAN BE SELECTED BY SETTING "KLIP" TO NON-ZERO INTEGER. **
C **  THE FUNCTION TO BE TRANSFORMED IS ASSUMED TO BE COMPLEX AND INPUTED  **
C **  AS MODULUS, "TAU", AND PHASE, "PHI", ARRAYS.  IF A REAL FUNCTION IS  **
C **  SUPPLIED IT IS ASSUMED TO BE SYMMETRIC ABOUT THE FIRST ELEMENT IN THE **
C **  ARRAY.  A MAXIMUM OF 500 POINTS IN EACH OF THE INPUT ARRAYS IS       **
C **  PERMITTED DUE TO THE DIMENSIONS OF THE INTERNAL WORKING ARRAYS "A,    **
C **  B, VCOS AND VSIN".                                                   **
C **                                                                       **
C **                                                                       **
C **                    ** VARIABLE LIST **                               **
C **                                                                       **
C **  INPUT VARIABLES                                                      **
C **                                                                       **
C **       TAU    =MODULUS OF FUNCTION TO BE TRANSFORMED.  RETURNED TO THE **
C **               CALLING PROGRAM UNCHANGED.  MUST BE DIMENSIONED IN THE  **
C **               CALLING PROGRAM.                                        **
C **       PHI    =PHASE OF THE FUNCTION TO BE TRANSFORMED.  RETURNED TO   **
C **               THE CALLING PROGRAM UNCHANGED.  MUST BE DIMENSIONED IN  **
C **               THE CALLING PROGRAM.                                    **
C **       NHARM  =NUMBER OF "HARMONICS" IN THE FUNCTION TO BE TRANSFORMED. **
C **               MUST BE < OR = 500.  THE ACTUAL NUMBER OF POINTS IN     **
C **               THE "TAU" AND "PHI" ARRAYS IS ASSUMED TO BE NHARM+1.    **
C **               RETURNED TO THE CALLING PROGRAM UNCHANGED.              **
C **       DELNU  =INTERVAL BETWEEN THE POINTS OF THE FUNCTION ARRAYS.     **
C **               USUALLY EXPRESSED IN SPATIAL FREQUENCY AS CYCLES/MM.    **
C **               RETURNED TO THE CALLING PROGRAM UNCHANGED.              **
C **       KLIP   =CONTROL INTEGER FOR MODULUS SMOOTHING FACTOR.  IF KLIP=0 **
C **               NO SMOOTHING IS EMPLOYED.  RETURNED TO THE CALLING      **
C **               PROGRAM UNCHANGED.                                      **
C **                                                                       **
C **  OUTPUT VARIABLES                                                     **
C **                                                                       **
C **       FUNC   =REAL TRANSFORMED FUNCTION.  MUST BE DIMENSIONED IN THE  **
C **               CALLING PROGRAM.                                        **
C **       NFUN   =NUMBER OF POINTS IN THE TRANSFORMED FUNCTION; EQUAL TO  **
C **               2*NHARM.  ( < OR = 1000 )                               **
C **       DELF   =THE INCREMENT BETWEEN POINTS IN THE TRANSFORMED FUNCTION. **
C **               USUALLY EXPRESSED IN MM.  CALCULATED AS 1/(2*NHARM*DELNU). **
C **                                                                       **
C **                                                                       **
C **       AUTHORS: R. E. KINZLY                                           **
C **                M. J. MAZUROWSKI                                       **
C **       DATE: JULY, 1966                                                **
C **                                                                       **
C ***************************************************************************
```

Figure 28. SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FORINV (PAGE 1 OF 4)

```
ISN 0002          SUBROUTINE FORINV(TAU,PHI,NHARM,DELNU,FUNC,NFUN,DELF,KLIP)
ISN 0003          DIMENSION TAU(1),PHI(1),FUNC(1),A(501),B(501),VCOS(251),VSIN(251),
                 *SIGF(501)
ISN 0004          DATA PI/3.14159265/
ISN 0005          DELF=0.0
ISN 0006          NFUN=0
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  DELETE EXECUTION IF "NHARM" > 500                                      *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0007          IF(NHARM.GT.500) RETURN
ISN 0009          NHP1 = NHARM +1
ISN 0010          HNO = NHARM
ISN 0011          IF(KLIP.EQ.0) GO TO 41
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  DEFINE SINC FUNCTION "SMOOTHING" FACTORS IF REQUESTED BY USER          *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0013          SIGF(1)=1.0
ISN 0014          DO 10 I=2,NHP1
ISN 0015          FI = I-1
ISN 0016       10 SIGF(I)=SIN(PI*FI/HNO)/(PI*FI/HNO)
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  CALCULATE COEFFICIENTS, A(I) AND B(I), FOR THE SINE AND COSINE TERMS   *
                C  *  OF THE INVERSE TRANSFORM.                                              *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0017          DO 40 I=1,NHP1
ISN 0018          A(I) = 2.*TAU(I)*COS(PHI(I))*SIGF(I)
ISN 0019       40 B(I) = 2.*TAU(I)*SIN(PHI(I))*SIGF(I)
ISN 0020          GO TO 43
ISN 0021       41 DO 42 I=1,NHP1
ISN 0022          A(I) = 2.*TAU(I)*COS(PHI(I))
ISN 0023       42 B(I) = 2.*TAU(I)*SIN(PHI(I))
ISN 0024       43 A(1) = A(1)/2.0
                C
                C  ***************************************************************************
                C  *                                                                         *
                C  *  SET UP COSINE AND SINE ARRAYS OVER FIRST QUADRANT TO BE USED TO        *
                C  *  COMPUTE THE INVERSE TRANSFORM.                                         *
                C  *                                                                         *
                C  ***************************************************************************
                C
ISN 0025          NHCS=NHARM/2 +1
ISN 0026          DO 50 M = 1,NHCS
ISN 0027          VCOS(M) = COS(PI*FLOAT(M-1)/HNO)
ISN 0028       50 VSIN(M) = SIN(PI*FLOAT(M-1)/HNO)
```

Figure 28 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FORINV (PAGE 2 OF 4)

```
C  ***************************************************************************
C  *                                                                         *
C  *  CALCULATE THE INVERSE TRANSFORM --- REAL FUNCTION.                     *
C  *                                                                         *
C  ***************************************************************************
C
ISN 0029          NFUN = 2*NHARM
ISN 0030          DO 100 J = 1,NFUN
ISN 0031          SUM = 0.0
ISN 0032          NN = -NHARM +J
ISN 0033          NNA = IABS(NN)
ISN 0034          DO 80 I=1,NHP1
ISN 0035          NNH=NNA*(I-1)
ISN 0036          MD = NNH/(2*NHARM)
ISN 0037          L = IABS(NNH-2*MD*NHARM)
ISN 0038          KSIGN = 2*L/NHARM+1
ISN 0039          SN = ISIGN(1,NN)
C
C  ***************************************************************************
C  *                                                                         *
C  *  BRANCH ACCORDING TO THE QUADRANT OF THE ARGUMENT                       *
C  *                                                                         *
C  ***************************************************************************
C
ISN 0040          GO TO (51,52,53,54),KSIGN
ISN 0041       51 SUM = SUM+A(I)*VCOS(L+1)+B(I)*SN*VSIN(L+1)
ISN 0042          GO TO 80
ISN 0043       52 L = NHARM-L
ISN 0044          SUM = SUM-A(I)*VCOS(L+1)+B(I)*SN*VSIN(L+1)
ISN 0045          GO TO 80
ISN 0046       53 L = L - NHARM
ISN 0047          SUM = SUM-A(I)*VCOS(L+1)-B(I)*SN*VSIN(L+1)
ISN 0048          GO TO 80
ISN 0049       54 L = 2*NHARM -L
ISN 0050          SUM = SUM+A(I)*VCOS(L+1)-B(I)*SN*VSIN(L+1)
ISN 0051       80 CONTINUE
ISN 0052      100 FUNC(J) = SUM
ISN 0053          DELF=1.0/(2.0*HNO*DELNU)
ISN 0054          RETURN
ISN 0055          END
```



**Figure 28 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FORINV (PAGE 3 OF 4)**

```
                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
A       0003  0018  0022  0024  0024  0041  0044  0047  0050
B       0003  0019  0023  0041  0044  0047  0050
I       0014  0015  0016  0017  0018  0018  0018  0019  0019  0019  0019  0021  0022  0022  0022  0023  0023  0023
        0034  0035  0041  0041  0044  0044  0047  0047  0050  0050
J       0030  0032  0052
L       0037  0038  0041  0041  0043  0043  0044  0044  0046  0046  0047  0047  0049  0049  0050  0050
M       0026  0027  0027  0028  0028
FI      0015  0016  0016
MD      0036  0037
NN      0032  0033  0039
PI      0004  0016  0016  0027  0028
SN      0039  0041  0044  0047  0050
COS     0018  0022  0027
HND     0010  0016  0016  0027  0028  0053
NNA     0033  0035
NNH     0035  0036  0037
PHI     0002  0003  0018  0019  0022  0023
SIN     0016  0019  0023  0028
SUM     0031  0041  0041  0044  0044  0047  0047  0050  0050  0052
TAU     0002  0003  0018  0019  0022  0023
DELF    0002  0005  0053
FUNC    0002  0003  0052
IABS    0033  0037
KLIP    0002  0011
NFUN    0002  0006  0029  0030
NHCS    0025  0026
NHPI    0009  0014  0017  0021  0034
SIGF    0003  0013  0016  0018  0019
VCOS    0003  0027  0041  0044  0047  0050
VSIN    0003  0028  0041  0044  0047  0050
DELNU   0002  0053
FLOAT   0027  0028
ISIGN   0039
KSIGN   0038  0040
NHARM   0002  0007  0009  0010  0025  0029  0032  0036  0037  0038  0043  0046  0049
FORINV  0002


 LABEL   DEFINED   REFERENCES
   10    0016      0014
   40    0019      0017
   41    0021      0011
   42    0023      0021
   43    0024      0020
   50    0028      0026
   51    0041      0040
   52    0043      0040
   53    0046      0040
   54    0049      0040
   80    0051      0034  0042  0045  0048
  100    0052      0030
```

Figure 28 SOURCE AND CORSS REFERENCE LISTINGS FOR SUBROUTINE FORINV (PAGE 4 OF 4)

and the discrete representation as

$$I(x) = \sum_{n=-n^*}^{n^*} \tau_n \; e^{-i(2\pi \nu_n x - \phi_n)} \tag{31b}$$

where $\tau_n = \tau(\nu_n)$, $\phi_n = \phi(\nu_n)$ , and $\nu_{-n} = -\nu_n$ . Note that the effect of the differential, $d\nu$ , in Equation 31a has been suppressed in Equation 31b. Since $I(x)$ is real the following conditions must be met

$$\begin{aligned} \tau_n &= \tau_{-n} \\ \phi_n &= -\phi_{-n} \\ \phi_o &= 0.0 \end{aligned} \tag{32}$$

Expanding Equation 31b and using these conditions we can write the discrete form of the inverse transform as

$$I(x) = \sum_{n=0}^{n^*} A_n \cos(2\pi \nu_n x) + \sum_{n=0}^{n^*} B_n \sin(2\pi \nu_n x) \tag{33}$$

where $A_0 = \tau_0$ , $A_n = 2\tau_n \cos\phi_n$ , $B_0 = 0.0$ , $B_n = 2\tau_n \sin\phi_n$ and the factor of 2 accounts for the summation over the negative values of $n$ . For coding purposes this equation is rewritten with summation over positive indices by letting $i = n+1$ , namely

$$I(x) = \sum_{i=1}^{n^*+1} A_i \cos(2\pi \nu_i x) + \sum_{i=1}^{n^*+1} B_i \sin(2\pi \nu_i x) \tag{34}$$

and the coefficients $A_i$ and $B_i$ are appropriately redefined. Letting

$$x_j = (-n^* + j)\Delta_x \; ; \; j = 1 \, , \cdots , \, 2n^* \tag{35a}$$

$$\nu_i = (i-1)\Delta_\nu \; ; \; i = 1 \, , \cdots , \, n^*+1 \tag{35b}$$

and requiring that

$$\Delta_x = \frac{1}{2n^*\Delta_\nu} \tag{35c}$$

we can write

$$2\pi\, v_i\, x_j = \frac{\pi (i-1)(-n^*+j)}{n^*} \tag{35d}$$

Setting $m_{ij} = (i-1)(-n^*+j)$ in Equation 35d, the expression for the inverse transform can be written

$$I(x_j) = \sum_{i=1}^{n^*+1} A_i \cos\left(\frac{\pi m_{ij}}{n^*}\right) + \sum_{i=1}^{n^*+1} B_i \sin\left(\frac{\pi m_{ij}}{n^*}\right) \tag{36}$$

The sine and cosine terms need only be computed over the first quadrant, i.e. $0 \le m_{ij} \le \frac{n^*}{2}$ and an appropriate algorithm developed for selecting the appropriate element from this array when the argument lies in another quadrant. Note that $m_{ij}$ can be less than zero and is accounted for in the coding by changing the sign of the second summation in Equation 36. In order to select the appropriate sine or cosine term and identify the quadrant the initial step involves a transformation of $m_{ij}$ from the range $[0, n^{*2}]$ to $[0, 2n^*)$ through the expression

$$L = m_{ij} - 2n^* \left[\!\left[\frac{m_{ij}}{2n^*}\right]\!\right] \tag{37}$$

where $[\![\ ]\!]$ represents the greatest integer which is less than the enclosed argument. The quadrant can then be subsequently specified by the expression

$$K = \left[\!\left[\frac{2L}{n^*}\right]\!\right] + 1 \;;\quad 0 \le K \le 4 \tag{38}$$

and the value of L given by Equation 37 appropriately modified depending upon the quadrant to select the appropriate sine and cosine terms to perform the summations indicated by Equation 36.

The restriction of performing an inverse transform from a complex to a real function, i.e. conditions of Equation 32 and the required relationship between the spatial and frequency increments, Equation 35c, permit the development of this algorithm which results in a considerable savings in computation

time. As the listing in Figure 28 indicates the user has an option of smoothing the modulus prior to inverse transformation. If this option is elected the initial step in the program is the setup of the smoothing factors SIGF(I). This is followed by the calculation of the coefficients A(I) and B(I) identified at Equation 33. The sine and cosine factors appearing in Equation 36 are then computed for argument values in the first quadrant and the calculation of both sum terms of the inverse transform of Equation 36 completed. In calculating the sums the program branches depending upon the value of KSIGN which is equivalent to Equation 38. After all the sum terms are computed, the spatial increment is defined using Equation 35c and execution is returned to the calling program.

7.0 Subroutine FOURTR - This program is part of the Edge Gradient Spectral Analysis software package. It supports Program EGSA in the calculation of an OTF from edge trace data. The purpose of this subroutine is to Fourier transform a real input function to obtain a complex spectrum expressed by modulus and phase functions. It is specifically employed by Program EGSA to calculate individual OTF from differentiated edge trace data. No direct input data must be furnished by the user. The input data is supplied through the calling program. No printed output data is furnished by this program.

Figure 29 is a listing of subroutine FOURTR. The function of the major blocks are identified by the comments included in the coding. The algorithm employed in this program is specifically designed for transforming a real to a complex function and results in considerable savings in execution time compared to a more direct implementation. If we assume that the transformed function will be written in terms of a modulus function $\tau(\nu)$ and a phase function $\phi(\nu)$ the Fourier transform can be expressed by the equation

$$\tau(\nu)\, e^{i\phi(\nu)} = \int_{-\infty}^{\infty} I(x)\, e^{2\pi i \nu x}\, dx \tag{39a}$$

The corresponding discrete representation is

$$\tau(\nu)\, e^{i\phi(\nu)} = \Delta_x \sum_{n=-n^*}^{n^*} I(x_n)\, e^{2\pi i \nu x_n} \tag{39b}$$

```
C ********************************************************************************
C **                                                                            **
C **  SUBROUTINE FOURTR                                                         **
C **                                                                            **
C **       THIS SUBROUTINE IS PART OF THE EDGE GRADIENT SPECTRAL ANALYSIS       **
C **  (EGSA) SOFTWARE PACKAGE.  THIS PACKAGE ESTIMATES THE OPTICAL              **
C **  TRANSFER FUNCTION (OTF) FOR ANY OPTICAL SYSTEM USING A SET OF EDGE        **
C **  DATA MEASURED AT THE OUTPUT OF THE SYSTEM.  THIS SUBROUTINE               **
C **  COMPUTES THE FOURIER TRANSFORM OF A REAL FUNCTION USING A SPECIAL         **
C **  ALGORITHM.  THE FUNCTION IS INPUTED THROUGH THE ARRAY "DATA".  A          **
C **  MAXIMUM OF 1000 POINTS IS PERMITTED DUE TO THE DIMENSION OF THE           **
C **  INTERNAL WORKING ARRAYS "F, G, A, B, VCOS, AND VSIN".                     **
C **                                                                            **
C **                                                                            **
C **                      ** VARIABLE LIST **                                   **
C **                                                                            **
C **  INPUT VARIABLES                                                           **
C **                                                                            **
C **       DATA   =FUNCTION TO BE TRANSFORMED.  RETURNED TO THE CALLING         **
C **               PROGRAM WITH EXTRA POINT ADDED IF FURNISHED WITH AN          **
C **               ODD NUMBER OF POINTS.  MUST BE DIMENSIONED IN THE            **
C **               CALLING PROGRAM.                                             **
C **       DX     =INCREMENT BETWEEN POINTS IN THE FUNCTION ARRAY.  RETURNED    **
C **               TO THE CALLING PROGRAM UNCHANGED.                            **
C **       NDATA  =NUMBER OF POINTS IN DATA ARRAY.  RETURNED TO THE CALLING     **
C **               PROGRAM INCREASED BY ONE IF ORIGINALLY ODD.                  **
C **                                                                            **
C **  OUTPUT VARIABLES                                                          **
C **                                                                            **
C **       DELNU  =INCREMENT BETWEEN POINTS OF THE TRANSFORMED FUNCTION.        **
C **               USUALLY EXPRESSED AS SPATIAL FREQUENCY IN CYCLES/MM.         **
C **               CALCULATED AS 1/(2*NHARM*DX) = 1/(NDATA*DX).                 **
C **       NHARM  =NUMBER OF "HARMONICS" IN TRANSFORM.  CALCULATED AS           **
C **               NDATA/2.  THE NUMBER OF POINTS IN THE TRANSFORM IS           **
C **               ACTUALLY NHARM+1.                                            **
C **       PHI    =PHASE OF THE TRANSFORM.  MUST BE DIMENSIONED IN THE          **
C **               CALLING PROGRAM.                                             **
C **       TAU    =MODULUS OF THE TRANSFORM.  MUST BE DIMENSIONED IN THE        **
C **               CALLING PROGRAM.                                             **
C **                                                                            **
C **                                                                            **
C **       AUTHORS: R. E. KINZLY                                                **
C **                M. J. MAZUROWSKI                                            **
C **       DATE: JULY, 1966                                                     **
C **                                                                            **
C **                                                                            **
C ********************************************************************************
C
ISN 0002        SUBROUTINE FOURTR(DATA,NDATA,DX,TAU,PHI,DELNU,NHARM)
ISN 0003        DIMENSION DATA(1),TAU(1),PHI(1),F(501),G(501),A(501),B(501),
               *VCOS(251),VSIN(251)
ISN 0004        DATA PI/3.14159265/
ISN 0005        DELNU=0.0
ISN 0006        NHARM=0
```

Figure 29 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FOURTR (PAGE 1 OF 4)

```
C  ************************************************************************
C  *                                                                      *
C  *  DELETE EXECUTION IF "NDATA" > 1000                                  *
C  *                                                                      *
C  ************************************************************************
C
ISN 0007          IF(NDATA.GT.1000) RETURN
C
C  ************************************************************************
C  *                                                                      *
C  *  ADJUST THE NUMBER OF POINTS IN THE "DATA" ARRAY TO BE EVEN.         *
C  *                                                                      *
C  ************************************************************************
C
ISN 0009          IF(NDATA/2.EQ.(NDATA+1)/2)  GO TO 8
ISN 0011          NDATA=NDATA+1
ISN 0012          DATA(NDATA)=DATA(NDATA-1)
ISN 0013        8 NHARM=NDATA/2
C
C  ************************************************************************
C  *                                                                      *
C  *  SET UP THE "F" AND "G" SUM AND DIFFERENCE ARRAYS THAT ARE THE       *
C  *  COEFFICIENTS OF THE REAL AND IMAGINARY PARTS OF THE FOURIER TRANSFORM.*
C  *                                                                      *
C  ************************************************************************
C
ISN 0014          NHP1 = NHARM+1
ISN 0015          F(1) = DATA(NHARM)
ISN 0016          G(1) = 0.0
ISN 0017          DO 10 J=2,NHARM
ISN 0018          NHP = NHARM+J-1
ISN 0019          NHM = NHARM-J+1
ISN 0020          F(J) = DATA(NHP)+ DATA(NHM)
ISN 0021       10 G(J) = DATA(NHP)- DATA(NHM)
ISN 0022          F(NHP1) = DATA(NDATA)
ISN 0023          G(NHP1) = 0.0
C
C  ************************************************************************
C  *                                                                      *
C  *  SET UP COSINE AND SINE ARRAYS OVER FIRST QUADRANT TO COMPUTE THE    *
C  *  REAL AND IMAGINARY TERMS.                                           *
C  *                                                                      *
C  ************************************************************************
C
ISN 0024          NHCS=NHARM/2 +1
ISN 0025          HNO = NHARM
ISN 0026          DO 20 M=1,NHCS
ISN 0027          VCOS(M) = COS(PI*FLOAT(M-1)/HNO)
ISN 0028       20 VSIN(M) = SIN(PI*FLOAT(M-1)/HNO)
C
C  ************************************************************************
C  *                                                                      *
C  *  COMPUTE THE REAL AND IMAGINARY TERMS OF THE TRANSFORM AT NHARM+1    *
C  *  POINTS CORRESPONDING TO FREQUENCIES FROM 0.0 TO NHARM*DELNU.        *
C  *                                                                      *
C  ************************************************************************
```

**Figure 29 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FOURTR (PAGE 2 OF 4)**

```
ISN 0029          HNO = 2*NHARM
ISN 0030          SUM = 0.0
ISN 0031          DO 30 J=1,NHP1
ISN 0032       30 SUM = SUM + F(J)
ISN 0033          A(1) = SUM/HNO
ISN 0034          B(1) = 0.0
ISN 0035          DO 50 I=2,NHARM
ISN 0036          SUMA = 0.0
ISN 0037          SUMB = 0.0
ISN 0038          DO 40 J=1,NHP1
ISN 0039          MD  =(I-1)*(J-1)/(2*NHARM)
ISN 0040          L = (I-1)*(J-1)-2* MD*NHARM
ISN 0041          KSIGN = 2*L/NHARM +1
                  C
                  C  ***************************************************************************
                  C  *                                                                         *
                  C  *  BRANCH ACCORDING TO THE QUADRANT OF THE ARGUMENT                       *
                  C  *                                                                         *
                  C  ***************************************************************************
                  C
ISN 0042          GO TO (31,32,33,34),KSIGN
ISN 0043       31 SUMA = SUMA+F(J)*VCOS(L+1)
ISN 0044          SUMB = SUMB+G(J)*VSIN(L+1)
ISN 0045          GO TO 40
ISN 0046       32 L = NHARM-L
ISN 0047          SUMA = SUMA-F(J)*VCOS(L+1)
ISN 0048          SUMB = SUMB+G(J)*VSIN(L+1)
ISN 0049          GO TO 40
ISN 0050       33 L = L-NHARM
ISN 0051          SUMA = SUMA-F(J)*VCOS(L+1)
ISN 0052          SUMB = SUMB-G(J)*VSIN(L+1)
ISN 0053          GO TO 40
ISN 0054       34 L = 2*NHARM-L
ISN 0055          SUMA = SUMA+F(J)*VCOS(L+1)
ISN 0056          SUMB = SUMB-G(J)*VSIN(L+1)
ISN 0057       40 CONTINUE
ISN 0058          A(I) = SUMA/HNO
ISN 0059          B(I) = SUMB/HNO
ISN 0060       50 CONTINUE
ISN 0061          SUM = 0.0
ISN 0062          DO 60 J=1,NHP1
ISN 0063       60 SUM = SUM+F(J)*(-1.)**(J-1)
ISN 0064          A(NHP1) = SUM/(2.*HNO)
ISN 0065          B(NHP1) = 0.0
                  C
                  C  ***************************************************************************
                  C  *                                                                         *
                  C  *  CALCULATE THE MODULUS AND PHASE FUNCTIONS FROM THE REAL AND IMAGINARY  *
                  C  *  TERMS.                                                                 *
                  C  *                                                                         *
                  C  ***************************************************************************
                  C
ISN 0066          DO 70 I=1,NHP1
ISN 0067          TAU(I) = SQRT(A(I)**2+B(I)**2)
ISN 0068          IF(B(I).EQ.0.) GO TO 61
ISN 0070          PHI(I) = ATAN2(B(I),A(I))+PI*(1.-SIGN(1.,B(I)))
ISN 0071          GO TO 70
ISN 0072       61 PHI(I)=(1.-SIGN(1.,A(I)))*PI/2.
```

Figure 29 SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FOURTR (PAGE 3 OF 4)

```
ISN 0073        70 CONTINUE
ISN 0074           DELNU = 1./(FLOAT(NDATA)*DX)
ISN 0075           RETURN
ISN 0076           END
```

*****FORTRAN CROSS REFERENCE LISTING*****

| SYMBOL | INTERNAL STATEMENT NUMBERS |
|---|---|
| A | 0003 0033 0058 0064 0067 0070 0072 |
| B | 0003 0034 0059 0065 0067 0068 0070 0070 |
| F | 0003 0015 0020 0022 0032 0043 0047 0051 0055 0063 |
| G | 0003 0016 0021 0023 0044 0048 0052 0056 |
| I | 0035 0039 0040 0058 0059 0066 0067 0067 0067 0068 0070 0070 0070 0070 0072 0072 |
| J | 0017 0018 0019 0020 0021 0031 0032 0038 0039 0040 0043 0044 0047 0048 0051 0052 0055 0056 0062 0063 0063 |
| L | 0040 0041 0043 0044 0046 0046 0047 0048 0050 0050 0051 0052 0054 0054 0055 0056 |
| M | 0026 0027 0027 0028 0028 |
| DX | 0002 0074 |
| MD | 0039 0040 |
| PI | 0004 0027 0028 0070 0072 |
| COS | 0027 |
| HNO | 0025 0027 0028 0029 0033 0058 0059 0064 |
| NHM | 0019 0020 0021 |
| NHP | 0018 0020 0021 |
| PHI | 0002 0003 0070 0072 |
| SIN | 0028 |
| SUM | 0030 0032 0032 0033 0061 0063 0063 0064 |
| TAU | 0002 0003 0067 |
| DATA | 0002 0003 0012 0012 0015 0020 0020 0021 0021 0022 |
| NHCS | 0024 0026 |
| NHP1 | 0014 0022 0023 0031 0038 0062 0064 0065 0066 |
| SIGN | 0070 0072 |
| SQRT | 0067 |
| SUMA | 0036 0043 0043 0047 0047 0051 0051 0055 0055 0058 |
| SUMB | 0037 0044 0044 0048 0048 0052 0052 0056 0056 0059 |
| VCOS | 0003 0027 0043 0047 0051 0055 |
| VSIN | 0003 0028 0044 0048 0052 0056 |
| ATAN2 | 0070 |
| DELNU | 0002 0005 0074 |
| FLOAT | 0027 0028 0074 |
| KSIGN | 0041 0042 |
| NDATA | 0002 0007 0009 0009 0011 0011 0012 0012 0013 0022 0074 |
| NHARM | 0002 0006 0013 0014 0015 0017 0018 0019 0024 0025 0029 0035 0039 0040 0041 0046 0050 0054 |
| FOURTR | 0002 |

| LABEL | DEFINED | REFERENCES |
|---|---|---|
| 8 | 0013 | 0009 |
| 10 | 0021 | 0017 |
| 20 | 0028 | 0026 |
| 30 | 0032 | 0031 |
| 31 | 0043 | 0042 |
| 32 | 0046 | 0042 |
| 33 | 0050 | 0042 |
| 34 | 0054 | 0042 |
| 40 | 0057 | 0038 0045 0049 0053 |
| 50 | 0060 | 0035 |
| 60 | 0063 | 0062 |
| 61 | 0072 | 0068 |
| 70 | 0073 | 0066 0071 |

**Figure 29. SOURCE AND CROSS REFERENCE LISTINGS FOR SUBROUTINE FOURTR (PAGE 4 OF 4)**

where $x_o = 0.0$, $x_{-n} = -x_n$, and $\Delta_x$ is the increment between points. Note that the effect of the differential, $d_x$, is included in the discrete representation. Since $I(x)$ is real, the summation can be divided into real and imaginery terms, namely

$$\tau(\nu)\, e^{i\phi(\nu)} = A(\nu) + i\, B(\nu) \tag{40a}$$

where

$$A(\nu) = \Delta_x \sum_{n=-n^*}^{n^*} I(x_n) \cos(2\pi\nu x_n) \tag{40b}$$

and

$$B(\nu) = \Delta_x \sum_{n=-n^*}^{n^*} I(x_n) \sin(2\pi\nu x_n) \tag{40c}$$

Through algebraic manipulation the sum over the negative and positive indices can be combined and for coding purposes the summation rewritten using positive indices by letting $j = n+1$ ; $j = 1, \cdots, n^*+1$. Equations 40b and 40c become

$$A(\nu) = \Delta_x \sum_{j=1}^{n^*+1} F_j \cos(2\pi\nu x_j) \tag{41}$$

$$B(\nu) = \Delta_x \sum_{j=1}^{n^*+1} G_j \sin(2\pi\nu x_j) \tag{42}$$

where

$$F_j = \begin{cases} I(0) ; & j = 1 \\ I(x_j) + I(-x_j) ; & j \geq 2 \end{cases} \tag{43}$$

$$G_j = \begin{cases} 0.0 ; & j = 1 \\ I(x_j) - I(-x_j) ; & j \geq 2 \end{cases} \tag{44}$$

The modulus and phase of the Fourier transform can be written in terms of the real and imaginery terms by referring to Equation 40a, namely

$$\tau(\nu) = \left(A(\nu)^2 + B(\nu)^2\right)^{1/2} \tag{45}$$

$$\phi(\nu) = \tan^{-1} \frac{B(\nu)}{A(\nu)} \tag{46}$$

The arguments of the sine and cosine factors appearing in Equations 41 and 42 are treated in the same fashion described in the previous section, namely setting

$$x_j = (j-1)\,\Delta_x \;;\quad j = 1, \cdots, n^*+1 \tag{47a}$$

$$\nu_i = (i-1)\,\Delta_\nu \;;\quad i = 1, \cdots, n^*+1 \tag{47b}$$

and requiring that

$$\Delta_x = \frac{1}{2n^*\Delta_\nu} \tag{47c}$$

we can write the argument of the sine and cosine terms as

$$2\pi\nu_i x_j = \frac{\pi(i-1)(j-1)}{n^*} = \frac{\pi m_{ij}}{n^*} \tag{47d}$$

In this case, however, $m_{ij}$ is not negative and has values in the range $[0, n^{*2}]$. The equations for the real and imaginery terms become

$$A_i' = A_i\,\Delta_\nu = \frac{1}{2n^*} \sum_{j=1}^{n^*+1} F_j \cos\left(\frac{\pi m_{ij}}{n^*}\right) \tag{48}$$

$$B_i' = B_i\,\Delta_\nu = \frac{1}{2n^*} \sum_{j=1}^{n^*+1} G_j \sin\left(\frac{\pi m_{ij}}{n^*}\right) \tag{49}$$

The sine and cosine terms need only be computed over the first quadrant, i.e. $0 \le m_{ij} \le \frac{n^*}{2}$, and an appropriate algorithm developed for selecting the correct element from this array when the argument lies in another quadrant.

We define

$$L = m_{ij} - 2n^* \left[\!\left[ \frac{m_{ij}}{2n^*} \right]\!\right] \tag{50}$$

where $[\![\ ]\!]$ represents the greatest integer which is less than the enclosed argument. The quadrant is then specified by the expression

$$K = \left[\!\left[ \frac{2L}{n^*} \right]\!\right] + 1 \quad ; \quad 0 \leq K \leq 4 \tag{51}$$

and the value of $L$ from Equation 50 appropriately modified depending upon the quadrant to select the correct sine and cosine terms from the stored arrays.

The restriction of Fourier transforming a real function to obtain a complex spectrum and the required relationship between spatial and frequency increments, Equation 47c, are required to develop the algorithm described and permit a considerable savings in computation time. The elements of the modulus and phase arrays are calculated using a modified form of Equations 45 and 46, namely

$$\tau_i = \left(A_i'^2 + B_i'^2\right)^{1/2} = \Delta_\nu \left(A_i^2 + B_i^2\right)^{1/2} \tag{52}$$

$$\phi_i = \tan^{-1} B_i'/A_i' = \tan^{-1} B_i/A_i \tag{53}$$

Note that the phase array is equivalent to that described by Equation 46. However, the modulus has been multiplied by the spatial frequency increment $\Delta_\nu$ and appropriate action should be taken in the calling program if required.

The first step in the program is to adjust the number of points in the input function array to be even. This is not a requirement of the mathematics but executed only as an interface convenience with subroutine FOUINV described in the previous section. The origin of the spatial axis,

i.e. $x=0$, is assumed to be located at the midpoint of the array. The next step in the program is the establishment of the sum and difference arrays expressed by Equations 43 and 44 and subsequently used to calculate the real and imaginery parts of the Fourier transform. This is followed by the computation of the sine and cosine terms within the first quadrant. The real and imaginery terms can then be computed from Equations 48 and 49 and subsequent branching depending upon the quadrant number specified by the variable KSIGN calculated using Equation 51. The sign of the sine and cosine terms are adjusted in accordance with the quadrant number and selected from the corresponding array using the transformed value of $L$ (c.f. Equation 50). The final step in the program is the calculation of the modulus and phase arrays using Equations 52 and 53 and the definition of the spatial frequency increment based upon Equation 47c. Execution is then returned to the calling program.

8.0 Main Program TONEQ - This program provides data which can be used to evaluate the tone quality of a photograph. It computes the gain achieved as a function of object contrast and average exposure level. Input data on punched cards must be supplied by the user. Printed values of the gain factors are furnished in tabular form by the program.

Figure 30 is a listing of TONEQ. The function of the major blocks are identified by the comments included in the coding. In order to exercise this program two subprograms that are part of the Calspan Corporation Program Library are required. Detailed descriptions of these programs will not be furnished in this Appendix, however, the user is likely to have equivalent programs available in the library at his computing facility. CLEAR fills each byte of a specified block of storage with zeros. This operation can be accomplished by substituting a sequence of do loops in the TONEQ program. SPLN46 calculates a set of interpolation functions for a set of input points. These interpolation functions produce a continuous function that has a continuous first and second derivative over the interval spanned by the input data. The function tends to avoid waviness often encountered by other curve fitting techniques. The input data points must be reasonably accurate or

```
C ********************************************************************************
C ********************************************************************************
C **                                                                            **
C **  MAIN PROGRAM - TONEQ (TONE QUALITY EVALUATION)                            **
C **                                                                            **
C **       THIS PROGRAM GENERATES MEASURES OF TONE QUALITY FOR A PHOTOGRAPH     **
C **  UNDER EVALUATION.  THE MEASURE IS THE GAIN FACTOR FOR VARIOUS             **
C **  CONTRAST OBJECTS AS A FUNCTION OF THE AVERAGE EXPOSURE LEVEL.  THE        **
C **  GAIN IS THE RATIO OF THE OBJECT-TO-BACKGROUND DENSITY DIFFERENCE TO       **
C **  THE DIFFERENCE IN RELATIVE LOG EXPOSURE.  SENSITOMETRIC CALIBRATION       **
C **  DATA ARE REQUIRED AS INPUT.  THE OBJECT CONTRAST VALUES (EXPRESSED        **
C **  AS LOG EXPOSURE DIFFERENCES) ARE INITIALIZED BY A DATA STATEMENT;         **
C **  THE CURRENT VALUES WERE USED IN THE EVALUATION OF APOLLO 16 AND 17        **
C **  ORBITAL PHOTOGRAPHY.  A MAXIMUM OF THREE VALUES ARE PERMITTED.            **
C **                                                                            **
C **                                                                            **
C **                      ** VARIABLE LIST **                                   **
C **                                                                            **
C **  INPUT VARIABLES                                                           **
C **                                                                            **
C **       D       =DENSITY VALUES OF THE SENSITOMETRIC CALIBRATION DATA.       **
C **       N1      =THE NUMBER OF THE STEP CORRESPONDING TO THE INITIAL         **
C **                 POINT IN THE INPUT CALIBRATION DATA.                       **
C **       NSW     =NUMBER OF POINTS IN SENSITOMETRIC CALIBRATION DATA.         **
C **       SENSE   =A DUAL PURPOSE VARIABLE INDICATING THE SENSE OF THE         **
C **                 OF THE IMAGE; SENSE >0 INDICATES A POSITIVE AND            **
C **                 <0 A NEGATIVE.  IF THE DIFFERENCE BETWEEN STEPS IN THE     **
C **                 WEDGE IS 0.15 IN LOG EXPOSURE THE MAGNITUDE OF SENSE       **
C **                 SHOULD BE SET TO 1.0; OTHERWISE IT CAN BE USED AS A        **
C **                 MULTIPLYING FACTOR TO ADJUST THE DIFFERENCE IN LOG         **
C **                 EXPOSURE BETWEEN STEPS.                                    **
C **       TITLE   =AN OPTIONAL ARRAY FOR AN IDENTIFICATION LABEL               **
C **                                                                            **
C **  MAJOR INTERNAL VARIABLES                                                  **
C **                                                                            **
C **       C1,C2,C3  =COEFFICIENTS OF THE CUBIC SPLINE INTERPOLATION            **
C **                  POLYNOMINALS.                                             **
C **       DD      =COMPUTED GAIN FACTORS.                                      **
C **       DEL     =OBJECT CONTRAST VALUES EXPRESSED AS LOG EXPOSURE            **
C **                 DIFFERENCES.                                               **
C **                                                                            **
C **                                                                            **
C **       AUTHOR: R. E. KINZLY                                                 **
C **       DATE: SEPTEMBER, 1973                                                **
C **                                                                            **
C **                                                                            **
C ********************************************************************************
C ********************************************************************************
```

**Figure 30** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM TONEQ (PAGE 1 OF 4)

```
ISN 0002          COMMON X,D,C1,C2,C3,DD
ISN 0003          DIMENSION X(21),D(21),C1(21),C2(21),C3(21),DD(21,3),TITLE(17),
                 *DEL(3)
                C
                C  ************************************************************************
                C  *                                                                      *
                C  *  OBJECT CONTRAST DEFINITION (DATA STATEMENT)                         *
                C  *                                                                      *
                C  ************************************************************************
                C
ISN 0004          DATA DEL/0.0413927,0.1139434,0.7781512/
ISN 0005         1 FORMAT(1H1,7X,'D-LOGE RESPONSE CURVE SPLINE REGRESSION' // 10X,
                 *17A4 // 10X, 'LOGE', 5X,'DENSITY', 9X, '**********   COEFFICIENTS
                 *  **********' / )
ISN 0006         3 FORMAT(I2,F6.2,I2,2X,17A4)
ISN 0007         4 FORMAT(14F5.0,10X)
ISN 0008         5 FORMAT(10X, F4.2, 7X, F4.2, 3F16.4)
ISN 0009         6 FORMAT(/// 8X,'TONE QUALITY EVALUATION PROGRAM' // 12X, 17A4 //
                 *12X, 'RELATIVE', 11X, 'LOW', 12X, 'MID', 11X, 'HIGH' / 12X, 'EXPOS
                 *URE', 9X,       'CONTRAST', 7X, 'CONTRAST', 7X, 'CONTRAST' / 9X, '(
                 *DELTA LOGE =)     (', F6.4, ')       (', F6.4, ')       (', F6.4,
                 * ')' /)
ISN 0010         7 FORMAT(13X,F7.3,3F15.2)
                C
                C  ************************************************************************
                C  *                                                                      *
                C  *  READ USER INPUT DATA                                                *
                C  *                                                                      *
                C  ************************************************************************
                C
ISN 0011        10 READ(5,3,END=50) NSW,SENSE,N1,(TITLE(N),N=1,17)
ISN 0012          CALL CLEAR(X(1),DD(21,3))
ISN 0013          IF(SENSE.GT.0.0) N1=22-N1
                C
                C  ************************************************************************
                C  *                                                                      *
                C  *  READ SENSITOMETRIC CALIBRATION DATA (DENSITY VALUES)                *
                C  *                                                                      *
                C  ************************************************************************
                C
ISN 0015          READ(5,4) (D(N),N=1,NSW)
                C
                C  ************************************************************************
                C  *                                                                      *
                C  *  CALCULATE CORESPONDING RELATIVE LOG EXPOSURE VALUES                 *
                C  *                                                                      *
                C  ************************************************************************
                C
ISN 0016          X1=0.0
ISN 0017          IF(SENSE.GT.0.0) X1=3.0*SENSE
ISN 0019          DX=-0.15*SENSE
ISN 0020          X1=X1+(N1-1)*DX
ISN 0021          DO 14 N=1,NSW
ISN 0022        14 X(N)=X1+(N-1)*DX
```

**Figure 30** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM TONEQ (PAGE 2 OF 4)

```
C  ************************************************************************
C  *                                                                      *
C  *  OBTAIN SPLINE INTERPOLATION FUNCTIONS FOR SENSITOMETRIC CALIBRATION  *
C  *  DATA AND PRINT OUT COEFFICIENTS IN TABULAR FORM                     *
C  *                                                                      *
C  ************************************************************************
C
ISN 0023          CALL SPLN46(0,X1,DUM,DUM,DUM,X,D,NSW,C1,C2,C3)
ISN 0024          WRITE(6,1) (TITLE(N),N=1,17)
ISN 0025          WRITE(6,5) (X(J),D(J),C1(J),C2(J),C3(J),J=1,NSW)
C
C  ************************************************************************
C  *                                                                      *
C  *  COMPUTE GAIN FACTORS AT AVERAGE EXPOSURE LEVELS EQUAL TO EACH POINT  *
C  *  IN THE CALIBRATION DATA AND THREE OBJECT CONTRAST LEVELS             *
C  *                                                                      *
C  ************************************************************************
C
ISN 0026          DX=ABS(DX)
ISN 0027          DO 20 J=1,3
ISN 0028          XD=DEL(J)/2.0
ISN 0029          NI=XD/DX
ISN 0030          IF(SENSE.GT.0.0) XD=-XD
ISN 0032          DO 20 N=1,NSW
ISN 0033          NH=N+NI
ISN 0034          IF(NH.GT.NSW) NH=NSW
ISN 0036          XH=X(N)+XD-X(NH)
ISN 0037          DH=D(NH)+C1(NH)*XH+C2(NH)*XH**2+C3(NH)*XH**3
ISN 0038          NL=N-NI-1
ISN 0039          IF(NL.LT.1) NL=1
ISN 0041          XL=X(N)-XD-X(NL)
ISN 0042          DL=D(NL)+C1(NL)*XL+C2(NL)*XL**2+C3(NL)*XL**3
ISN 0043       20 DD(N,J)=(DH-DL)/DEL(J)
C
C  ************************************************************************
C  *                                                                      *
C  *  PRINT OUT GAIN FUNCTIONS IN TABULAR FORM                            *
C  *                                                                      *
C  ************************************************************************
C
ISN 0044          WRITE(6,6) (TITLE(I),I=1,17),(DEL(J),J=1,3)
ISN 0045          DO 40 I=1,NSW
ISN 0046          XX=10.**X(I)
ISN 0047       40 WRITE(6,7) XX,(DD(I,J),J=1,3)
ISN 0048          GO TO 10
C
C  ************************************************************************
C  *                                                                      *
C  *  TERMINATE EXECUTION                                                 *
C  *                                                                      *
C  ************************************************************************
C
ISN 0049       50 STOP
ISN 0050          END
```

**Figure 30** SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM TONEQ (PAGE 3 OF 4)

```
                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

SYMBOL  INTERNAL STATEMENT NUMBERS
D       0002  0003  0015  0023  0025  0037  0042
I       0044  0044  0044  0045  0046  0047
J       0025  0025  0025  0025  0025  0025  0025  0027  0028  0043  0043  0044  0044  0044  0047  0047  0047
N       0011  0011  0011  0015  0015  0015  0021  0022  0022  0024  0024  0024  0032  0033  0036  0038  0041  0043
X       0002  0003  0012  0022  0023  0025  0036  0036  0041  0041  0046
C1      0002  0003  0023  0025  0037  0042
C2      0002  0003  0023  0025  0037  0042
C3      0002  0003  0023  0025  0037  0042
DD      0002  0003  0012  0043  0047
DH      0037  0043
DL      0042  0043
DX      0019  0020  0022  0026  0026  0029
NH      0033  0034  0034  0036  0037  0037  0037  0037
NI      0029  0033  0038
NL      0038  0039  0039  0041  0042  0042  0042  0042
N1      0011  0013  0013  0020
XD      0028  0029  0030  0030  0036  0041
XH      0036  0037  0037  0037
XL      0041  0042  0042  0042
XX      0046  0047
X1      0016  0017  0020  0020  0022  0023
ABS     0026
DEL     0003  0004  0028  0043  0044
DUM     0023  0023  0023
NSW     0011  0015  0021  0023  0025  0032  0034  0034  0045
CLEAR   0012
SENSE   0011  0013  0017  0017  0019  0030
TITLE   0003  0011  0024  0044
SPLN46  0023


 LABEL   DEFINED   REFERENCES
    1      0005      0024
    3      0006      0011
    4      0007      0015
    5      0008      0025
    6      0009      0044
    7      0010      0047
   10      0011      0048
   14      0022      0021
   20      0043      0027  0032
   40      0047      0045
   50      0049      0011
```

**Figure 30 SOURCE AND CROSS REFERENCE LISTINGS FOR MAIN PROGRAM TONEQ (PAGE 4 OF 4)**

smooth since the subroutine forces the function to pass through these points and consequently cannot perform any smoothing of the input data. The interpolation functions are piecewise cubic and yield about the same curve one would obtain by forcing a flexible straight edge (a spline) through the set of input points. For additional information about spline interpolation procedures the reader is referred to DeBoor, Carl; Bicubic Spline Interpolation Journal of Mathematics and Physics, Vol. 41, No. 3, March 1962, pp. 212. Subroutine SPLN46 is used by this program to obtain interpolation functions for the sensitometric calibration data.

The initial step in the program is to read user input data which specifies the number of points in the sensitometric calibration data, the sense of the imagery (i.e. positive or negative) and the number from 1 to 21 of the first step in the calibration data. The user may also include a title to identify the photograph under evaluation if he so desires. Figure 31 shows a typical input data set. This example is taken from that used in the evaluation of the Apollo 17 panoramic photography. The first two cards compose the initial data block and the first card indicates that there are 14 points in the sensitometric calibration data measured on a negative image. The number of the step for the first data point is 8. The fact that the magnitude of the variable SENSE is 1.0 indicates that the difference in relative log exposure between points in the sensitometric calibration is equal to 0.15. The optional identification label indicates that the data were obtained from a Direct Negative roll of Apollo 17 panoramic photography that contains Frame 3111. An optional identification tag can be placed in columns 73 through 80 to aid in data set identification. The figure shows a second data block for another photograph; in this case a Third Generation Positive copy of the same frame. This can be followed by additional data blocks for each photograph under evaluation.

The next step in the program is the input of the sensitometric calibration value (density values). This data is contained on the second card of the first data block and the second and third cards in the second data

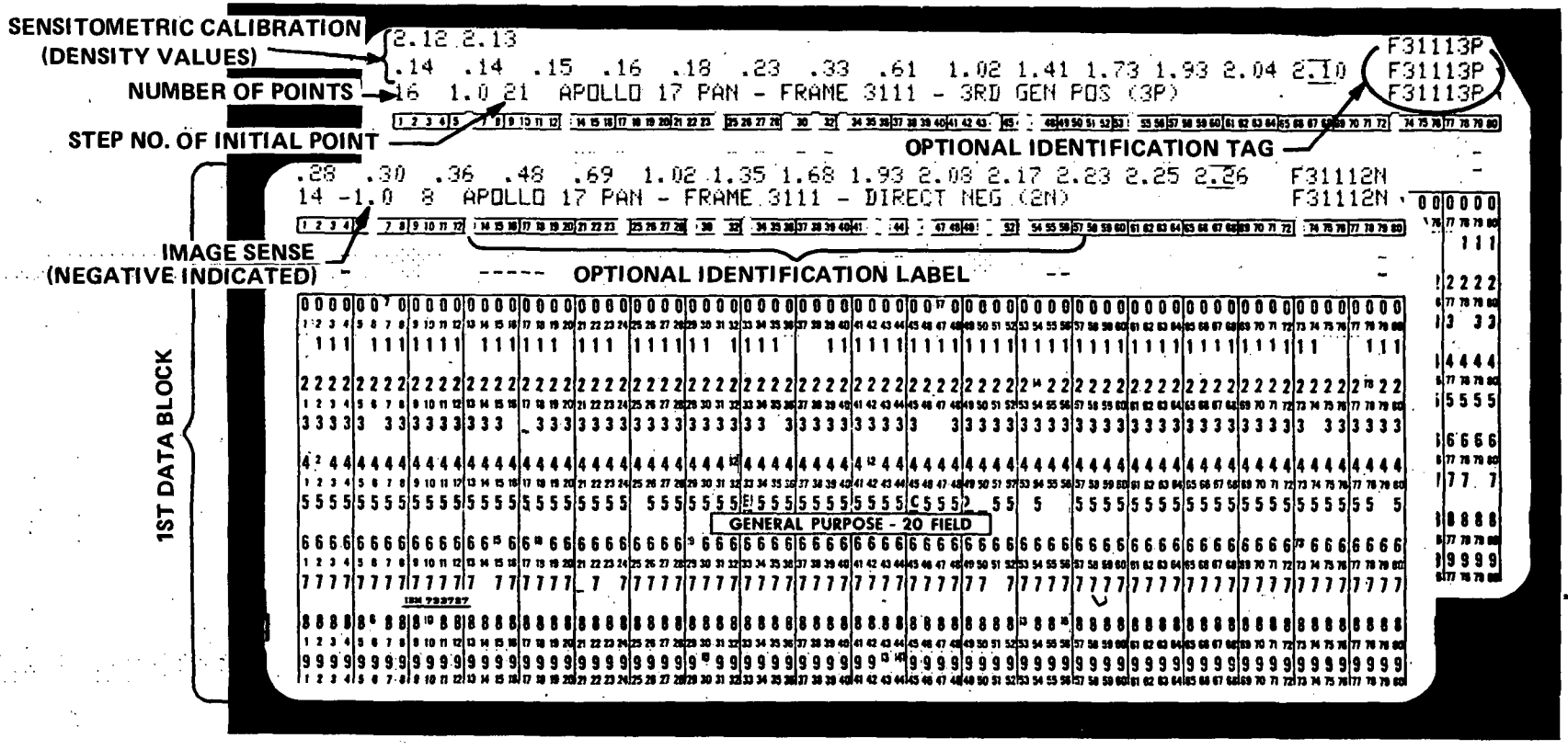



Figure 31 TYPICAL TONEQ PROGRAM INPUT DATA SET

block. The program then calculates an array of relative log exposure values X(N) in the range 0.0 to 3.0 that correspond to the input density values. By calling subroutine SPLN46 the coefficients of the spline interpolation functions for the sensitometric calibration data are obtained and subsequently printed out in tabular form. Figure 32 shows the printout generated for the first data block of Figure 31. The next step in the program involves the computation of the gain factors at an average exposure level equal to that of each point in the calibration data and three different object contrast levels. The levels of object contrast are specified by a data statement at the beginning of the program. Once the calculations have been completed the gain factors are printed out in tabular form. An example of this printout is included in Figure 32 for the first data block identified in Figure 31. The program then recycles to process the next data block if one is supplied by the user. If not, the program terminates execution.

D-LOGE RESPONSE CURVE SPLINE REGRESSION

APOLLO 17 PAN - FRAME 3111 - DIRECT NEG (2N) F31112N

| LOGE | DENSITY | ********** | COEFFICIENTS | ********** |
|---|---|---|---|---|
| 1.05 | 0.28 | 0.0832 | 0.0000 | 2.2274 |
| 1.20 | 0.30 | 0.2336 | 1.0023 | 0.7149 |
| 1.35 | 0.36 | 0.5825 | 1.3241 | 0.8384 |
| 1.50 | 0.48 | 1.0363 | 1.7014 | 4.8206 |
| 1.65 | 0.69 | 1.8721 | 3.8707 | -11.2321 |
| 1.80 | 1.02 | 2.2752 | -1.1837 | 4.5509 |
| 1.95 | 1.35 | 2.2272 | 0.8642 | -6.9708 |
| 2.10 | 1.68 | 2.0160 | -2.2726 | -0.3734 |
| 2.25 | 1.93 | 1.3090 | -2.4406 | 2.5386 |
| 2.40 | 2.08 | 0.7482 | -1.2982 | 2.0702 |
| 2.55 | 2.17 | 0.4984 | -0.3666 | -1.9309 |
| 2.70 | 2.23 | 0.2581 | -1.2355 | 2.6912 |
| 2.85 | 2.25 | 0.0691 | -0.0245 | 0.0544 |
| 3.00 | 2.26 | 0.0654 | 0.0 | 0.0 |

TONE QUALITY EVALUATION PROGRAM

APOLLO 17 PAN - FRAME 3111 - DIRECT NEG (2N) F31112N

| RELATIVE EXPOSURE (DELTA LOGE =) | LOW CONTRAST (0.0414) | MID CONTRAST (0.1139) | HIGH CONTRAST (0.7782) |
|---|---|---|---|
| 11.220 | 0.08 | 0.09 | 0.39 |
| 15.849 | 0.23 | 0.24 | 0.46 |
| 22.387 | 0.58 | 0.59 | 0.78 |
| 31.623 | 1.04 | 1.05 | 1.20 |
| 44.668 | 1.87 | 1.86 | 1.58 |
| 63.095 | 2.27 | 2.26 | 1.85 |
| 89.125 | 2.23 | 2.22 | 1.90 |
| 125.892 | 2.01 | 2.00 | 1.70 |
| 177.827 | 1.31 | 1.31 | 1.36 |
| 251.187 | 0.75 | 0.76 | 0.97 |
| 354.812 | 0.50 | 0.50 | 0.59 |
| 501.185 | 0.26 | 0.26 | 0.34 |
| 707.942 | 0.07 | 0.07 | 0.20 |
| 999.994 | 0.07 | 0.07 | 0.11 |

**Figure 32 EXAMPLE OF TONEQ PROGRAM PRINTED OUTPUT**