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EVALUATION AND COMPARISONS OF RECENT GEOPOTENTIAL SOLUTIONS

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September 1974

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GODDARD SPACE FLIGHT CENTER
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CONTENTS

	Page
INTRODUCTION	1
ANALYSIS TECHNIQUES	1
COMPARISONS	2
INTERCOMPARISON OF GODDARD EARTH MODELS	5
GODDARD EARTH MODELS VERSUS SMITHSONIAN STANDARD EARTH MODELS	11
NAVAL WEAPONS LABORATORY'S (NWL) GRAVITY MODELS VERSUS GEM AND SE MODELS	17
ANALYSIS AND INTERPRETATION	18
CONCLUSIONS	30
ACKNOWLEDGMENTS	31
REFERENCES	31

ILLUSTRATIONS

Figure		Page
1	Degree Correlation Function	12
2	Degree Correlation Function	12
3	Degree Correlation Function	13
4	Spectral Ratio Function	13
5	Degree Variances: GEM 6 and SE III	14
6	Degree Variances: SE III and NWL WGSN 44	14
7	Degree Variances: SE III and NWL 10E	15
8	Degree Variances: GEM 6 and WGSN 44	15
9	Degree Variances: GEM 6 and 10E	16
10	Free Air Gravity Anomalies based on differences between SE III and GEM 6 (2, 0 - 4, 4)	19
11	Free Air Gravity Anomalies based on differences between SE III and GEM 6 (2, 0 - 7, 7)	20
12	Free Air Gravity Anomalies based on differences between SE III and GEM 6 (8, 0 higher)	22
13	Free Air Gravity Anomaly contribution of GEM 6 geopotential coefficients (8, 0) through (16, 16) plus some higher coefficients	23
14	Differences in geoids determined from SE III and GEM 6	24
15	Spectral conversion factor from gravity anomalies to geoidal heights	25
16	Free Air Gravity Anomalies based on differences between GEM 6 and GEM 5	28

Figure		Page
17	$1° \times 1°$ surface gravity coverage. The number of dots in each $5° \times 5°$ square indicates the number of $1° \times 1°$ mean anomalies used in computing the $5° \times 5°$ mean gravity anomaly in that square	29

TABLES

Table		Page
1	Intercorrelation Function of Goddard Earth Models (GEMs) A: Purely Satellite Determined Geopotential Solutions . . .	3
2	B: Combination Solutions	4
	C: Purely Satellite Determined vs. Combination Solutions	4
2	Correlation of Goddard Earth Models (GEMs) with Standard Earth Models	5
3	Correlations of Representative Goddard Earth Model and Standard Earth Model with Naval Weapons Laboratory's Recent Geopotential Solutions	6
4	Spectral Ratio Function: Goddard Earth Models	7
5	Spectral Ratio Function: Goddard Earth Models and Smithsonian Standard Earth Models	7
6	Spectral Function: GEM 6, SE III and NWL Models	8
7	Spectral Ratio Function: Goddard Earth Models Differences	9
8	Spectral Ratio Function: Representative GEM and SE Differences	10

TABLES (continued)

Table		Page
9	Spectral Ratio Function: GEM 6, SE III and NWL Models	11
10	Typical Spectral Contents of Various Frequency Ranges of Some Representative Gravity Models	26
11	Summary of Gravity Model Comparisons with Satellites and Gravimetric Data	27

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ABSTRACT

A statistical evaluation of some of the recent satellite determined gravity models, including some with distinct data base, indicates that the geopotential coefficients of these models are individually meaningful for frequencies with wavenumbers $n = 2$ through 7 certainly and wavenumbers $n = 8$ through 10 probably. Geopotential coefficients in higher frequency ranges while apparently important for computing accurate satellite orbits seem to have little geophysical significance in an individual sense. Differences between various gravity models and those between purely satellite determined geopotential models and their associated combination models show no consistent relationship to surface gravimetric coverage. Additional classical tracking data are important in improving the existing description of the Earth's gravity field but their contribution in extending its frequency range beyond what is now available is uncertain. New tracking data types such as laser, satellite-to-satellite and altimetry data seem to have the potential of improving gravity field description but a quantitative assessment of their contribution is difficult at this stage.

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EVALUATION AND COMPARISONS OF
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INTRODUCTION

Last decade and a half has seen applications of Artificial Earth satellite orbital perturbation techniques in their analytical and numerical form, to determination of the gravity field of the Earth. More recently existing surface gravity data, after some interpolation and extrapolation, have been combined with various forms of satellite tracking data to improve upon these gravity field representations. Since there are so many solutions that differ from each other significantly, it is important to evaluate and compare them and perhaps attempt to select the best representation. Geopotential solutions considered in this paper are Goddard Space Flight Center's GEM solutions (Lerch, et al, 74), Smithsonian Astrophysical Observatory's recent Standard Earth (SE) models (Gaposchkin, 73; Gaposchkin and Lambeck, 70) and Naval Weapons Laboratory's most recent solutions (these solutions are classified but we need only their unclassified statistical parameters for this analysis which were kindly made available by Dr. Richard Anderle of Naval Weapons Laboratory). To test these solutions, the surest standard of comparison, of course, is gravimetric representation of the Earth's gravity field but if we had such a representation, we would not need to obtain these solutions in the first place. Any other test is essentially inferential in nature.

ANALYSIS TECHNIQUES

Let

$$f_1(x) = \text{Est}\, f(x)$$

$$f_2(x) = \text{Est}\, f(x)$$

where Est indicates the estimated value, then

$$R(f_1,f_2) = \frac{\int_x f_1(x)\, f_2(x)\, dx}{\left[\int_x f_1^2(x)\, dx \int_x f_2^2(x)\, dx\right]^{1/2}} = 1 \tag{1}$$

$$\sigma(f_1) = (f_2) \tag{2}$$

$$\sigma(f_1 - f_2) = 0 \tag{3}$$

$$S(f_1, f_2) = 1 \tag{4}$$

$$S(f_1 - f_2, f_1) = 0 \tag{5}$$

$$S(f_1 - f_2, f_2) = 0 \tag{6}$$

where $\sigma(f)$ indicates the degree variance of f(x) and $S(f_1, f_2)$ the spectral ratio function of f_1 (x) and f (x). Inversely if f_1(x) and f_2(x) are independent estimates of the function f(x) and, if the conditions stated in Equations (1) through (6) are satisfied,

$$f_1(x) = f(x) = f_2(x)$$

Note that Equations (1) through (6) will not be satisfied if either f_1(x) or f_2(x) is an incorrect estimate of the function f(x).

On the other hand, the equations will be satisfied if f_1(x) and f_2(x) are not independent and neither is essentially a correct estimate of f(x). In either of the above cases, this test should be supplemented by other analysis.

COMPARISONS

Geopotential models used in the comparisons reported here are Goddard Space Flight Center, GEM solutions GEM 1 through 6, the Smithsonian Standard Earth solutions SE II and SE III, and the Naval Weapons Laboratory's geopotential solutions 10E and WGSN 44. The Naval Weapons Laboratory's solutions are classified; therefore, only some unclassified statistical parameters could be used in this study.

The statistical parameters defined in Equations (1) through (6) are given in Tables 1 through 9. Table 1 gives the correlation functions for intercomparisons of various GEM Solutions. Table 2 lists correlation functions for GEM and SE models. Correlations between selected GEM and SE solutions and the NWL's 10E and WGSN 44 solutions are reported in Table 3. Tables 4 through 6 list spectral ratio functions for various geopotential solutions in the same order. Tables 7 through 9 report spectral ratio functions for some selected difference fields. A few representative correlation curves are shown in Figures 1 through 3 for a quick visual examination. A typical spectral ratio function of two geopotential fields and their differences is plotted in Figure 4. Figures 5 through 9 show degree variances for GEM 6, SE III, 10E, WGSN 44 and their differences.

Of the Goddard Space Flight Center geopotential solutions GEM 1, 3 and 5 are purely satellite derived solutions. These solutions are complete to (12, 12) but have some selected higher degree coefficients up to (22, 14). GEM 2, 4, and 6 are combination solutions, i. e., they are based on satellite tracking data as well as surface gravimetry information; these solutions are complete to (16, 16) with a few selected higher degree coefficients up to (22, 14). Generally various combination solutions are based on their satellite determined predecessors. Smithsonian SE II and SE III models are both combination solutions. The surface gravity data base in these solutions is similar to that in GEM solutions. SE II is complete to (16, 16) with a few higher coefficients. SE III is complete to (18, 18) with a few higher coefficients.

Table 1

Intercorrelation Function of Goddard Earth Models (GEMs)
A: Purely Satellite Determined Geopotential Solutions

n	GEM 5 VS. GEM 1	GEM 5 VS. GEM 3	GEM 3 VS. GEM 1
2	1.0000	1.0000	1.0000
3	1.0000	1.0000	0.9999
4	0.9998	0.9999	0.9998
5	0.9994	0.9995	0.9986
6	0.9983	0.9990	0.9967
7	0.9971	0.9966	0.9922
8	0.9941	0.9943	0.9917
9	0.9918	0.9909	0.9792
10	0.9860	0.9796	0.9616
11	0.9741	0.9653	0.9503
12	0.9463	0.9046	0.8740
13	0.9624	0.9498	0.8938
14	0.9869	0.9424	0.9029
15	0.8594	0.9419	0.7834
16	0.9231	0.9561	0.8869
17	-0.0741	0.9105	0.1780
18	0.9487	0.9509	0.8960
19	0.8671	0.7493	0.6775
20	0.1208	0.8231	0.0181
21	0.8369	0.6281	0.4016
22	0.8433	0.8001	0.5031

B: Combination Solutions

n	GEM 6 VS. GEM 2	GEM 6 VS. GEM 4	GEM 4 VS. GEM 2
2	1.0000	1.0000	0.9999
3	0.9998	1.0000	0.9999
4	0.9996	0.9998	0.9999
5	0.9956	0.9969	0.9961
6	0.9968	0.9978	0.9958
7	0.9752	0.9918	0.9917
8	0.9803	0.9802	0.9883
9	0.8781	0.9502	0.9677
10	0.9039	0.9695	0.9723
11	0.7386	0.9158	0.9662
12	0.7420	0.8586	0.9276
13	0.4961	0.8099	0.9707
14	0.4678	0.7703	0.9637
15	0.2036	0.4998	0.9756
16	0.2970	0.3907	0.9606
17	-0.0366	0.9457	0.8695
18	0.9745	0.9617	0.8569
19	0.8739	0.7885	0.7434
20	0.3398	0.8160	-0.1730
21	0.7212	0.4879	0.3666
22	0.8814	0.7521	0.3880

C: Purely Satellite Determined vs. Combination Solutions

n	GEM 6 VS. GEM 5	GEM 6 VS. GEM 3	GEM 6 VS. GEM 1	GEM 5 VS. GEM 4	GEM 4 VS. GEM 1	GEM 2 VS. GEM 1
2	1.0000	1.0000	1.0000	1.0000	0.9998	1.0000
3	0.9999	0.9999	0.9998	0.9999	0.9999	0.9997
4	0.9999	0.9997	0.9996	0.9999	0.9997	0.9998
5	0.9963	0.9948	0.9956	0.9994	0.9985	0.9950
6	0.9978	0.9951	0.9968	0.8607	0.9964	0.9970
7	0.9858	0.9879	0.9752	0.7044	0.9877	0.9853
8	0.9827	0.9724	0.9803	0.9906	0.9846	0.9840
9	0.9025	0.9067	0.8781	0.9701	0.9552	0.9091
10	0.9289	0.9304	0.9039	0.9646	0.9412	0.9426
11	0.7776	0.8308	0.7386	0.8919	0.8641	0.7995
12	0.8157	0.7751	0.7420	0.8191	0.7480	0.7278
13	0.5327	0.5509	0.4961	0.4702	0.4237	0.5162
14	0.4710	0.4531	0.4678	0.2637	0.2540	0.3171
15	0.2462	0.2538	0.2036	0.2263	0.1761	0.2627
16	0.3069	0.2944	0.2970	0.2835	0.2656	0.3457
17	0.9855	0.9543	-0.0366	0.8972	0.1978	0.2018
18	0.9905	0.9511	0.9745	0.9569	0.9061	0.9867
19	0.9963	0.7791	0.8739	0.7593	0.7003	0.9358
20	0.7958	0.7546	0.3398	0.8273	0.0819	0.7811
21	0.8065	0.4229	0.7212	0.6843	0.4625	0.9337
22	0.9454	0.7665	0.8844	0.7802	0.4842	0.9803

Table 2

Correlation of Goddard Earth Models (GEMs) with Standard Earth Models

n	GEM 6 VS. SE III	GEM 6 VS. SE II	GEM 5 VS. SE III	GEM 5 VS. SE II	GEM 4 VS. SE III	GEM 4 VS. SE II	GEM 1 VS. SE III	GEM 1 VS. SE II
2	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000
3	0.9947	0.9999	0.9934	0.9997	0.9947	0.9999	0.9930	0.9996
4	0.9960	0.9971	0.9954	0.9964	0.9954	0.9961	0.9957	0.9964
5	0.9786	0.9907	0.9665	0.9879	0.9673	0.9867	0.9645	0.9861
6	0.9666	0.9675	0.9571	0.9679	0.9559	0.9678	0.9636	0.9656
7	0.9124	0.9561	0.9152	0.9424	0.9107	0.9582	0.9108	0.9293
8	0.6917	0.9430	0.6606	0.9483	0.6343	0.9415	0.6753	0.9413
9	0.8053	0.8189	0.7742	0.8672	0.7919	0.8811	0.7604	0.8828
10	0.5840	0.8310	0.4823	0.7542	0.5060	0.7681	0.5042	0.7355
11	0.7075	0.5204	0.5812	0.6651	0.6426	0.5531	0.5303	0.6587
12	0.5041	0.4473	0.2233	0.3520	0.4322	0.3902	0.1211	0.3050
13	0.7856	0.3828	0.5302	0.4611	0.7269	0.3323	0.4580	0.4168
14	0.8043	0.6070	0.4030	0.5956	0.6808	0.3998	0.4115	0.5836
15	0.6147	0.3628	0.1706	0.1625	0.6884	0.4569	0.0804	0.2119
16	0.5052	0.3238	0.0958	0.1341	0.6800	0.5516	0.1337	0.2061
17	0.2416	0.4276	0.2443	0.4422	0.2193	0.5539	-0.0820	0.4327
18	0.1891	0.2440	0.2172	0.2614	0.2077	0.4068	0.1784	0.2616
19	0.1309	-0.0903	0.1576	-0.0884	-0.2249	-0.2912	0.1346	0.1771
20	0.0787	-0.0142	-0.0668	-0.3519	-0.0892	-0.0205	0.8473	0.4276
21	0.2406	0.0046	0.0025	-0.1759	-0.1488	-0.0324	0.1605	0.3326
22	0.1475	0.0263	0.1260	-0.0075	-0.0629	-0.0065	0.0705	0.1016

Naval Weapons Laboratory's solution 10E is purely satellite derived solution based primarily on doppler data; hence it should provide an independent standard of comparison for GEM and SE models. The NWL solution WGSN 44 is a combination solution.

The statistical parameters reported in Tables 1 through 9 in which any of the GEM 1, 3, or 5 are involved are meaningful to n = 12 only. All other comparisons are valid to n = 16. Comparisons in the higher frequency range (n > 16) are meaningless as these are based only on a limited number of harmonic coefficients.

INTERCOMPARISON OF GODDARD EARTH MODELS

GEM solutions based purely on the satellite data (Table 1A; Figure 1) show a high degree of correlation at all frequencies up to n = 12 (the correlation function for n > 12 should be neglected as it is based only on a few selected coefficients). The spectral ratio function for these fields (Table 4), which is a more sensitive

Table 3

Correlations of Representative Goddard Earth Model and Standard Earth Model with Naval Weapons Laboratory's Recent Geopotential Solutions

n	SSE III VS. NWL WGSN 44	SSE III VS. NWL 10E	GEM 6 VS. NWL WGSN 44	GEM 6 VS. NWL 10E
2	1.0000	1.0000	1.0000	1.0000
3	0.9946	0.9948	0.9998	0.9998
4	0.9961	0.9961	0.9995	0.9995
5	0.9664	0.9681	0.9972	0.9982
6	0.9660	0.9612	0.9977	0.9963
7	0.9192	0.9076	0.9826	0.9816
8	0.6713	0.6954	0.9693	0.9562
9	0.8129	0.8315	0.8796	0.8703
10	0.5695	0.5360	0.9159	0.8900
11	0.5367	0.5107	0.7694	0.7358
12	0.2582	0.3140	0.5556	0.5727
13	0.5782	0.4778	0.5825	0.5398
14	0.4296	0.4223	0.3642	0.3702
15	0.3981	0.2953	0.3021	0.2406
16	0.1296	0.3008	0.3467	0.7191
17	0.2542	0.4262	0.1382	0.3093
18	0.4063	0.1933	0.1854	0.4371
19	0.1728	0.2872	−0.0554	0.0905
20	0.5495	0.4512	−0.1788	−0.0974
21	0.1122	0.4478	−0.1875	0.1950
22	0.3300	−0.0339	−0.0661	0.3092

parameter, is largely in the neighborhood of 100 for frequencies up to n = 12. The spectral ratio function for their differences (Table 7) is close to zero for frequencies up to n = 8 and reasonably close to zero for frequencies up to n = 12. As indicated earlier, the correlation statistics beyond n = 12 are not significant in this case as these solutions are complete to (12, 12) only.

Of the combination GEM solutions which are complete to (16, 16), GEM 6 shows a high degree of correlation with GEM 4 and GEM 2 up to n = 10 and reasonably high correlation ($>$0.7) to n = 12 (Table 1B, Figure 1) beyond which (n $>$ 12) the correlation function becomes irregular. On the other hand, GEM 4 is very highly correlated with GEM 2 up to n = 16 (correlation coefficient $>$0.93). These findings are supported by the spectral ratio function (Table 4, Figure 1) which

Table 4

Spectral Ratio Function: Goddard Earth Models

n	GEM 5 VS. GEM 1	GEM 3 VS. GEM 5	GEM 3 VS. GEM 1	GEM 4 VS. GEM 6	GEM 2 VS. GEM 6	GEM 4 VS. GEM 2	GEM 6 VS. GEM 5	GEM 3 VS. GEM 6	GEM 6 VS. GEM 1	GEM 5 VS. GEM 4	GEM 4 VS. GEM 1	GEM 3 VS. GEM 1	GEM 2 VS. GEM 1
2	100	99	100	100	100	100	99	100	99	100	100	100	100
3	100	99	99	100	100	100	99	100	99	99	99	99	99
4	100	101	101	99	98	101	101	100	100	101	100	101	99
5	97	93	90	99	106	93	94	99	91	93	90	90	97
6	101	98	99	104	102	102	92	105	93	65	97	99	95
7	98	101	99	90	97	93	115	87	113	149	102	99	111
8	98	96	94	83	93	89	125	77	122	104	102	94	114
9	115	88	102	77	83	93	113	78	130	88	101	102	109
10	103	91	94	109	114	95	95	96	98	104	107	94	112
11	115	90	103	92	104	88	92	97	106	85	98	103	111
12	121	77	94	77	94	82	96	80	117	75	91	94	111
13	96	129	118	60	79	76	49	252	47	30	28	118	37
14	41	99	139	40	52	77	29	338	41	12	16	139	21
15	123	171	210	50	76	65	21	779	26	11	13	210	21
16	94	112	105	98	111	88	10	1131	9	10	9	105	10
17	123	31	39	32	282	11	105	30	130	34	42	39	369
18	72	69	49	75	143	52	99	69	71	75	54	49	103
19	226	265	599	239	55	433	101	262	228	241	546	599	126
20	13	23	33	28	223	13	93	24	11	26	3	33	26
21	178	82	146	91	64	143	88	92	158	81	144	146	101
22	100	61	61	59	109	54	110	56	109	64	64	61	119

Table 5

Spectral Ratio Function: Goddard Earth Models and Smithsonian Standard Earth Models

n	GEM 6 VS. SE III	GEM 6 VS. SE II	GEM 5 VS. SE III	GEM 5 VS. SE II	GEM 4 VS. SE II	GEM 3 VS. SE III	GEM 3 VS. SE II	GEM 2 VS. SE III	GEM 2 VS. SE II	GEM 1 VS. SE III	GEM 1 VS. SE II
2	96	98	104	102	98	96	98	97	99	97	99
3	99	98	100	102	97	99	97	99	98	100	98
4	105	103	96	98	103	106	104	104	102	105	103
5	119	82	79	115	81	118	81	127	87	131	90
6	90	78	102	119	81	95	82	92	80	97	83
7	110	89	105	130	80	97	78	108	87	97	78
8	232	78	54	158	66	178	61	216	74	189	64
9	132	143	85	79	111	104	112	111	120	102	110
10	95	112	99	84	123	92	108	110	129	98	115
11	130	159	71	58	146	127	155	136	166	122	150
12	219	124	44	78	96	175	99	207	117	187	106
13	87	104	57	47	63	219	263	69	83	185	222
14	138	143	21	20	57	467	485	72	74	335	348
15	96	124	23	18	62	753	967	74	95	359	461
16	148	204	7	5	200	1678	2309	165	227	1597	2197
17	3081	702	3	15	226	915	208	8693	1980	2357	537
18	367	70	27	142	53	254	49	527	101	513	98
19	217	363	46	28	867	570	951	120	200	95	159
20	476	321	20	29	91	115	77	1061	715	4050	2731
21	80	150	111	59	137	74	139	51	96	51	95
22	59	22	186	499	13	33	12	64	24	54	20

Table 6

Spectral Function: GEM 6, SE III and NWL Models

n	NWL WGSN 44 / SAO SE III	NWL 10E / SAO SE III	NWL WGSN 44 / GEM 6	NWL 10E / GEM 6
2	105	105	102	102
3	103	102	102	101
4	95	96	101	101
5	80	81	96	97
6	108	108	97	98
7	110	101	121	110
8	57	55	132	128
9	87	105	119	143
10	84	99	80	94
11	72	62	118	101
12	51	51	112	112
13	96	101	84	88
14	77	42	106	58
15	104	23	101	22
16	93	57	137	85
17	121	25	3744	781
18	192	36	704	130
19	1216	136	2644	296
20	29	32	136	151
21	668	85	539	68
22	138		82	

shows considerable departures from the reference value of 100 beyond n = 12 in case of comparisons involving GEM 6 and beyond n = 16 in case of GEM 4 versus GEM 2. The spectral ratio function of the differences shows variations of close to 100 percent for frequencies n > 12 except for GEM 4 versus GEM 2 in which case this function shows less than 15% variation for frequencies up to n = 16. This fact is interesting as these solutions are derived from highly correlated GEM 1, GEM 3 and GEM 5 but the set of mean surface gravity anomalies used in GEM 6 is somewhat different from that used in earlier GEM combination solutions.

Comparison of combination solutions with those based purely on satellite orbital data (Table 1C) shows a high degree of correlation to n = 10 and a fairly high correlation to n = 12 (>0.7). Spectral ratio function statistics in Tables 4 and 7 support this high internal consistency (to n = 12) with the notable exception of GEM 4 versus GEM 5 at n = 7 (Table 7). However, this discordance is not noticeable in other comparisons.

Table 7

Spectral Ratio Function: Goddard Earth Models Differences

n	GEM 6– GEM 5 VS. GEM 6	GEM 6– GEM 4 VS. GEM 6	GEM 4– GEM 2 VS. GEM 4	GEM 6– GEM 3 VS. GEM 6	GEM 6– GEM 2 VS. GEM 6	GEM 6– GEM 1 VS. GEM 6	GEM 5– GEM 4 VS. GEM 5	GEM 5– GEM 3 VS. GEM 5	GEM 5– GEM 1 VS. GEM 5	GEM 4– GEM 1 VS. GEM 4	GEM 3– GEM 1 VS. GEM 3	GEM 2– GEM 1 VS. GEM 2
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	0	1	0	0	0	1	1	1
6	1	0	1	1	0	1	26	0	0	1	1	1
7	3	2	2	3	2	6	77	1	1	2	2	3
8	5	5	3	8	3	6	2	1	1	3	2	4
9	21	13	6	23	14	29	6	2	3	9	4	19
10	13	6	6	14	4	19	7	5	3	12	8	13
11	42	18	7	34	17	54	20	8	6	27	10	43
12	36	34	14	52	25	56	33	23	12	48	25	57
13	74	57	7	70	45	78	78	10	7	83	24	74
14	78	108	8	80	71	81	93	12	6	96	26	92
15	98	159	8	95	104	105	95	14	32	100	83	97
16	90	123	8	91	100	91	92	9	14	93	23	88
17	3	77	53	87	27	239	29	93	239	116	117	391
18	2	11	28	16	14	6	9	16	11	21	23	3
19	1	40	223	42	17	64	105	46	65	319	367	16
20	39	147	124	208	137	88	41	196	103	100	102	46
21	36	108	155	121	35	76	57	83	54	133	149	13
22	11	74	97	74	36	24	39	59	31	87	82	5

Table 8

Spectral Ratio Function: Representative GEM and SE Differences

n	GEM 6–SE III VS. GEM 6	GEM 6–SE II VS. GEM 6	GEM 5–SE III VS. GEM 5	GEM 5–SE II VS. GEM 5	GEM 1–SE III VS. GEM 1	GEM 1–SE II VS. GEM 1	GEM 2–SE III VS. GEM 2	GEM 2–SE II VS. GEM 2	GEM 4–SE II VS. GEM 4
2	0	0	0	0	0	0	0	0	0
3	1	0	1	0	1	0	1	0	0
4	1	1	1	1	1	1	1	1	1
5	6	3	7	3	10	3	8	2	3
6	7	7	8	7	7	7	6	8	7
7	18	8	17	12	18	14	19	12	9
8	121	11	57	12	104	13	128	12	13
9	47	47	42	32	48	25	34	19	25
10	81	36	103	54	98	57	84	35	52
11	68	127	73	98	105	88	75	77	112
12	170	124	114	149	254	143	163	109	120
13	40	126	77	178	160	198	39	142	110
14	49	98	84	326	284	230	45	132	97
15	75	143	106	588	429	470	49	90	90
16	125	211	102	2038	1590	2104	83	60	144
17	2913	575	94	535	2536	436	8458	88	160
18	395	129	104	126	532	146	562	149	94
19	278	497	125	492	169	214	170	141	1139
20	542	426	126	572	3072	2384	626	54	194
21	137	249	210	315	128	130	118	162	245
22	136	119	251	120	144	111	148	493	112

Table 9

Spectral Ratio Function: GEM 6, SE III and NWL Models

n	SAO SE III - NWL WGSN 44 / SAO SE III	SAO III - NWL 10E / SAO III	GEM 6 - WGSN 44 / GEM 6	GEM 6 - MWL 10E / GEM 6
2	0	0	0	0
3	1	1	0	0
4	1	1	0	0
5	7	7	1	0
6	7	8	1	1
7	17	19	5	4
8	56	52	9	12
9	35	34	27	34
10	80	91	15	21
11	80	82	57	60
12	114	105	93	91
13	83	106	77	87
14	102	87	131	102
15	123	95	140	99
16	167	112	156	53
17	165	83	3687	719
18	180	112	706	130
19	1197	169	2806	367
20	71	83	2773	264
21	710	99	727	136
22	161	172	193	102

These intracomparisons, however, show merely internal consistency of the GEM solutions. Since these solutions are not independent of each other, the high correlations cannot be interpreted in terms of the degree of accuracy of these solutions. It is interesting to note, however, that the earliest (GEM 1) and the latest (GEM 6) solutions (from amongst the solutions analyzed here) show such a high degree of correlation in spite of vastly improved data fed into the later solutions.

GODDARD EARTH MODELS VERSUS SMITHSONIAN STANDARD EARTH MODELS

The GEM solutions and the SE models are derived principally from the same type of satellite and surface gravity data, though the amount of data used in the more recent GEM solutions is larger than that used in the SE models and the methods of analysis used in the two sets of solutions are somewhat different.

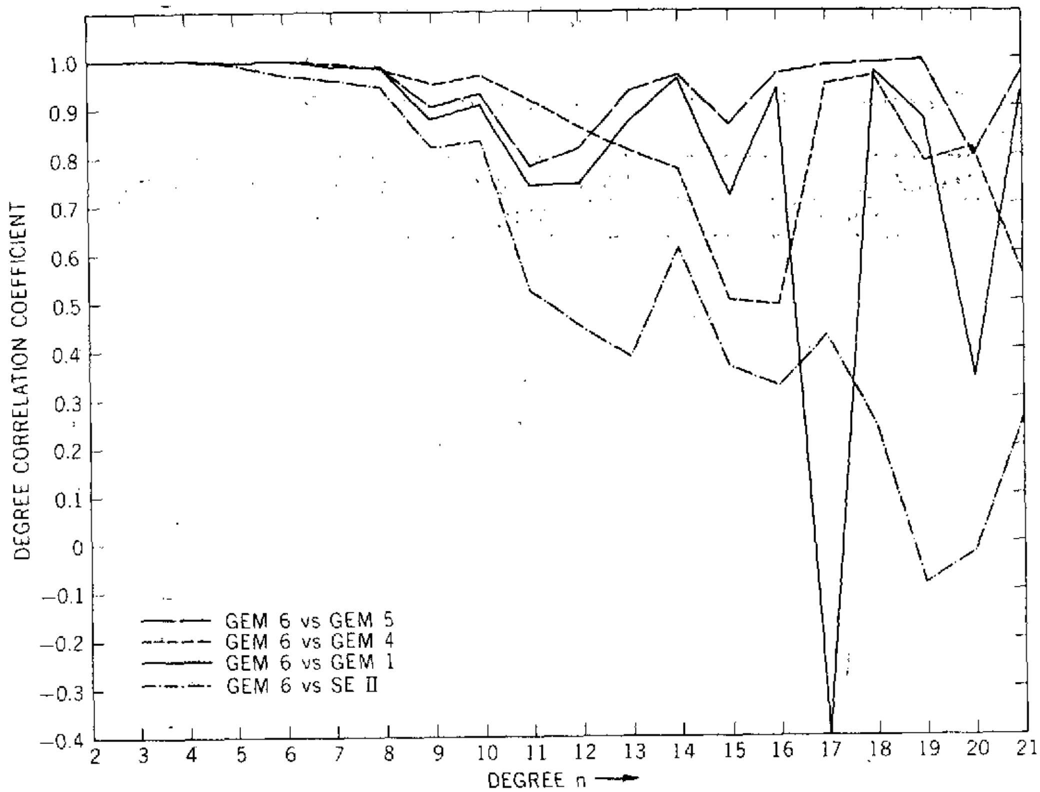

Figure 1. Degree Correlation Function

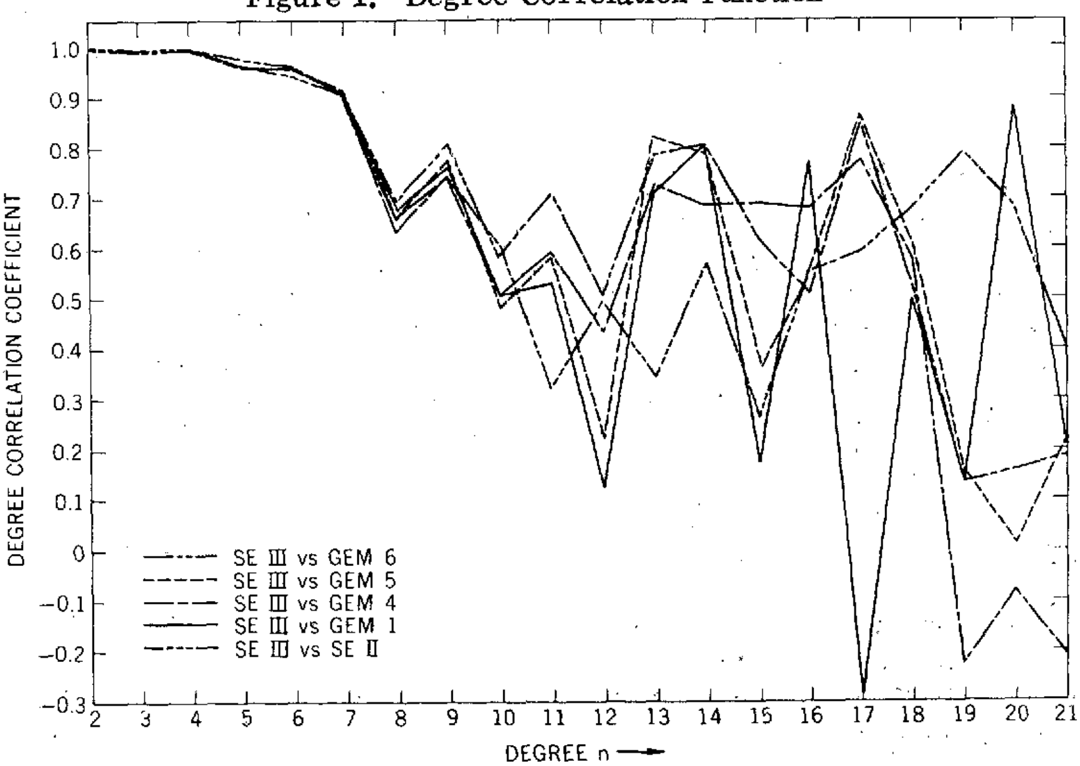

Figure 2. Degree Correlation Function

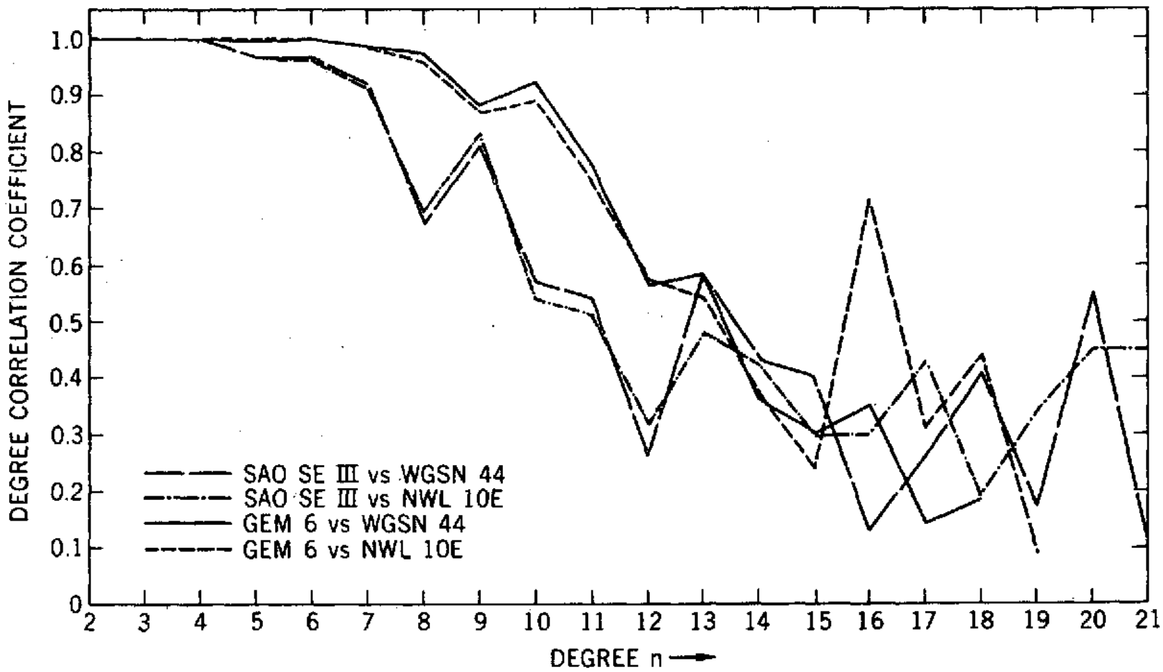

Figure 3. Degree Correlation Function

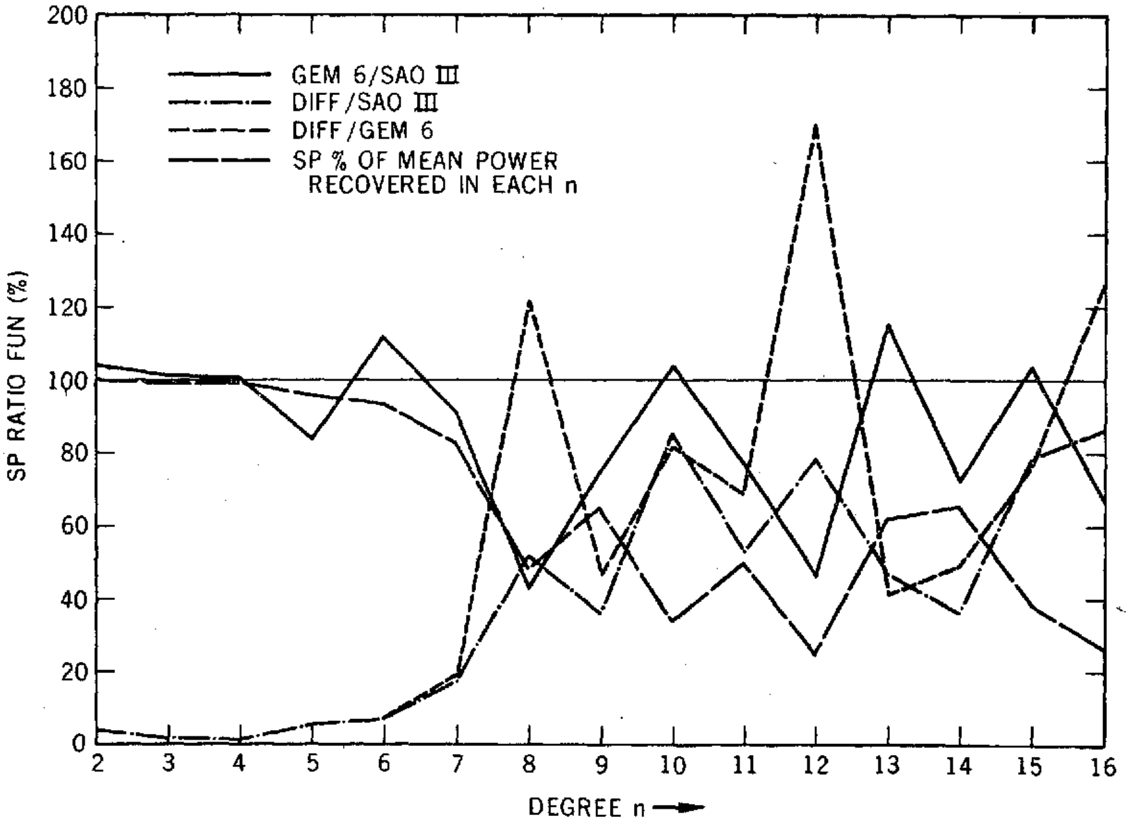

Figure 4. Spectral Ratio Function

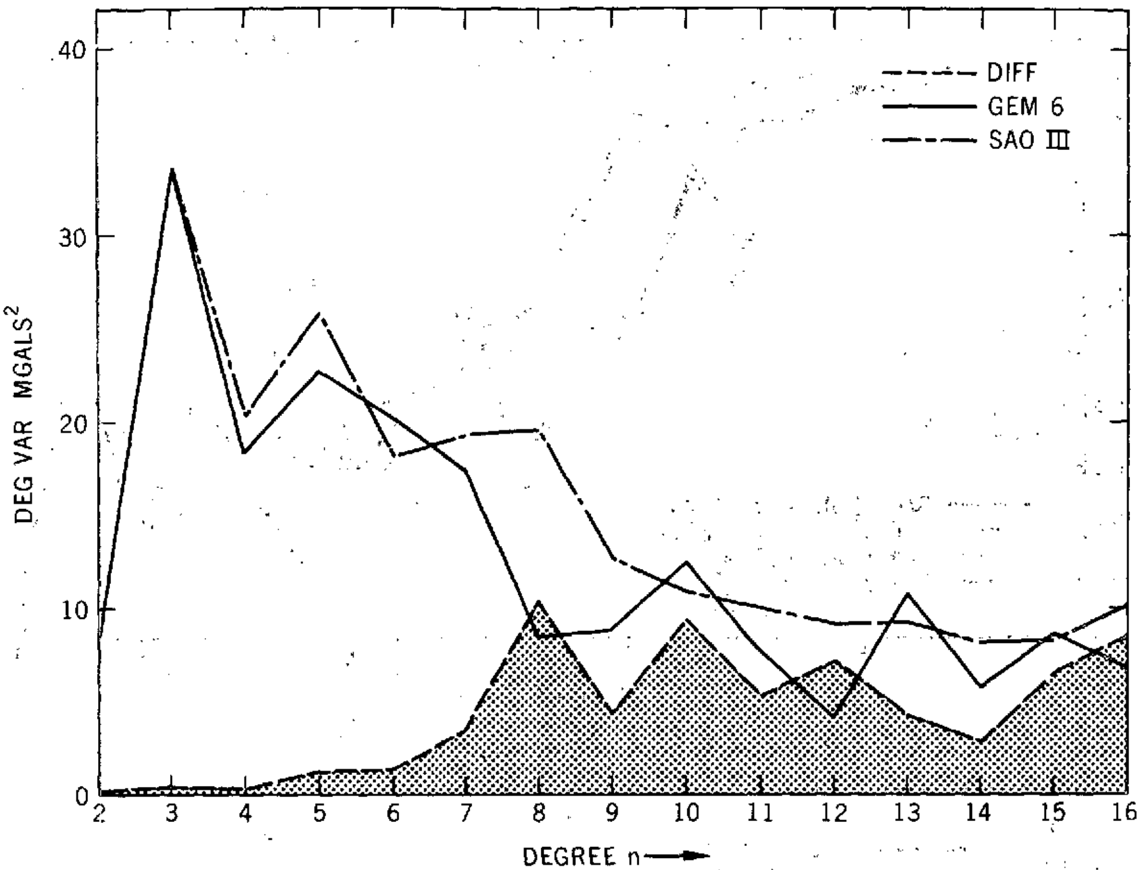

Figure 5. Degree Variances: GEM 6 and SE III

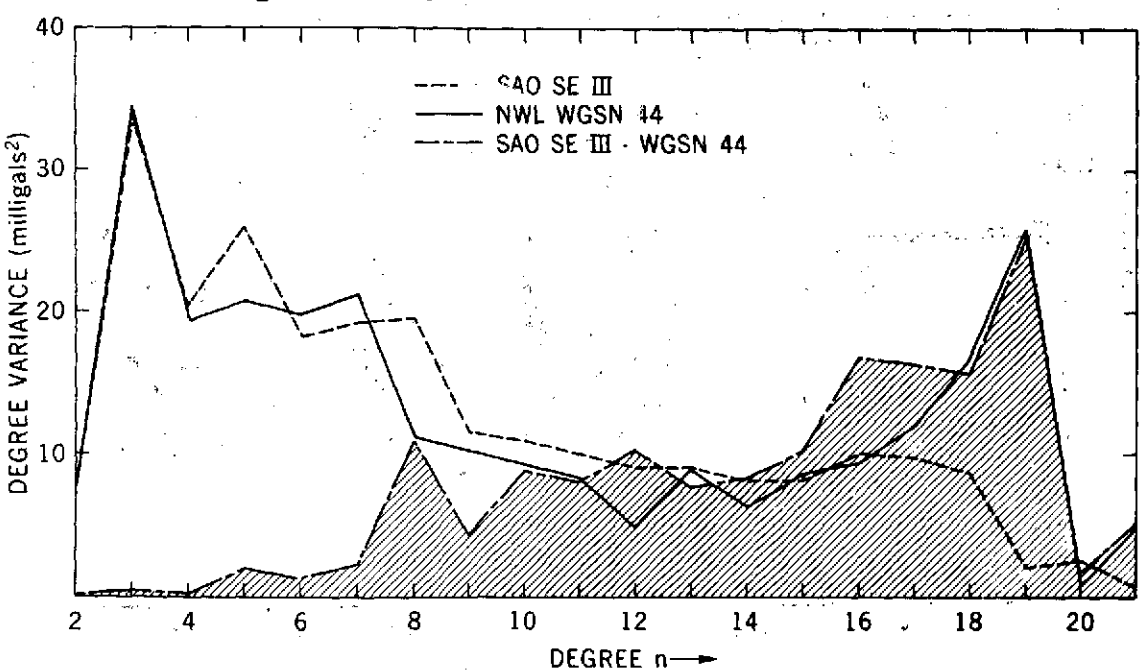

Figure 6. Degree Variances: SE III and NWL WGSN 44

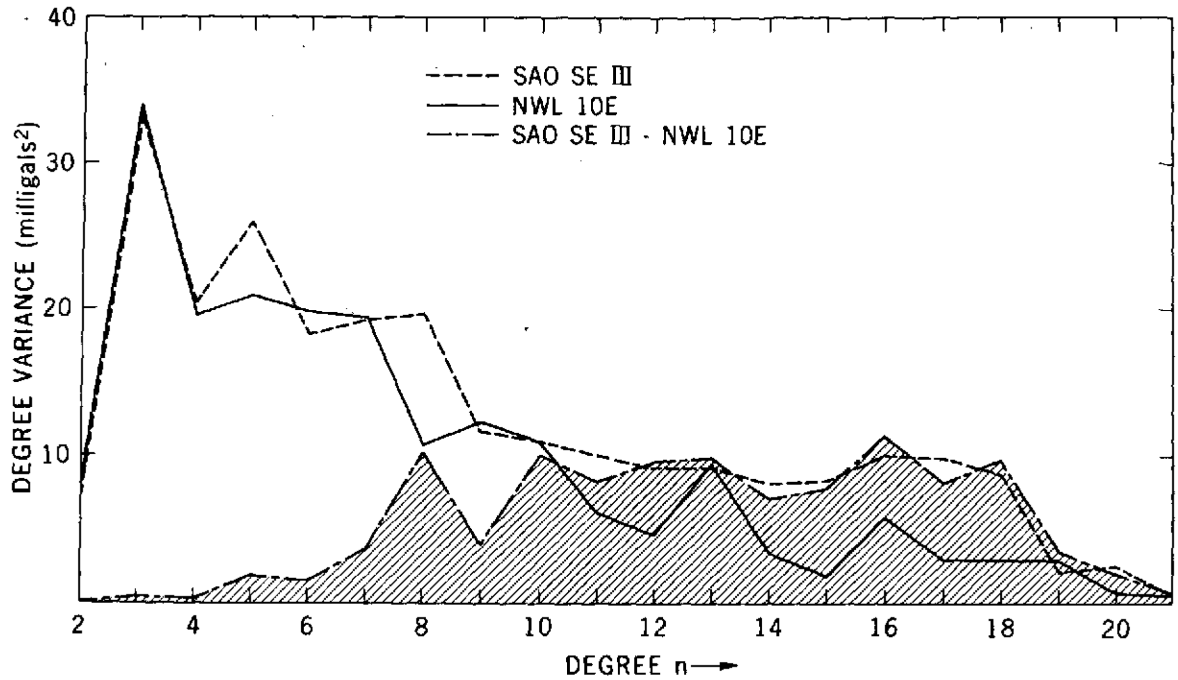

Figure 7. Degree Variances: SE III and NWL 10E

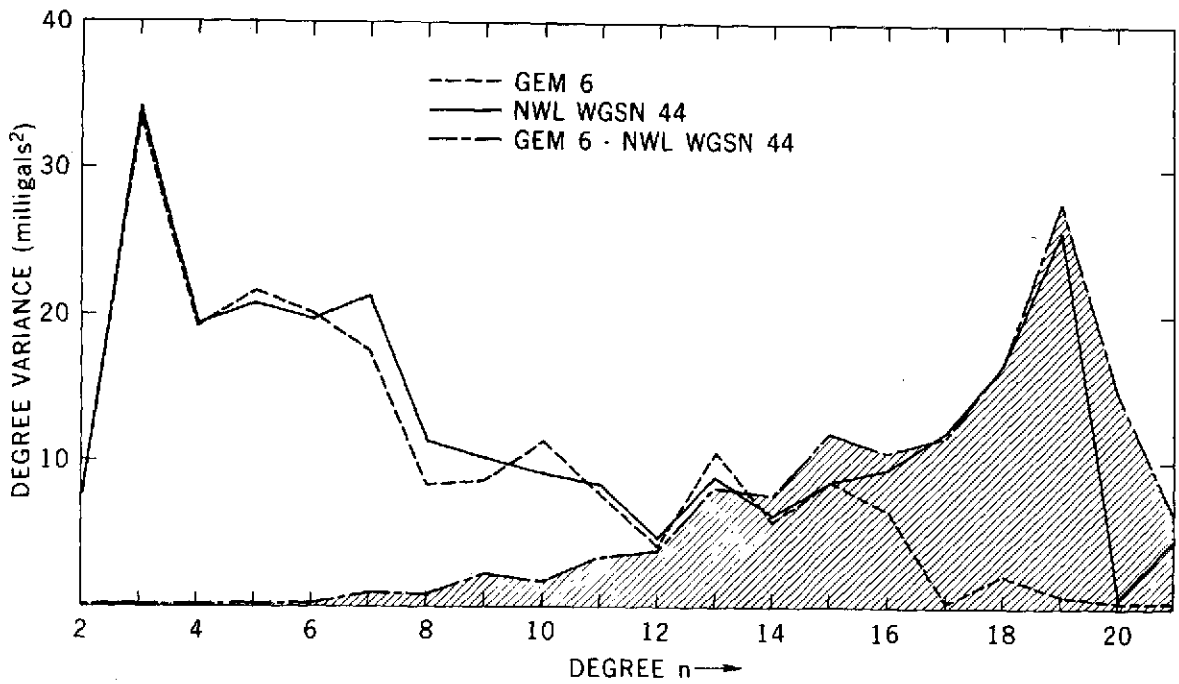

Figure 8. Degree Variances: GEM 6 and WGSN 44

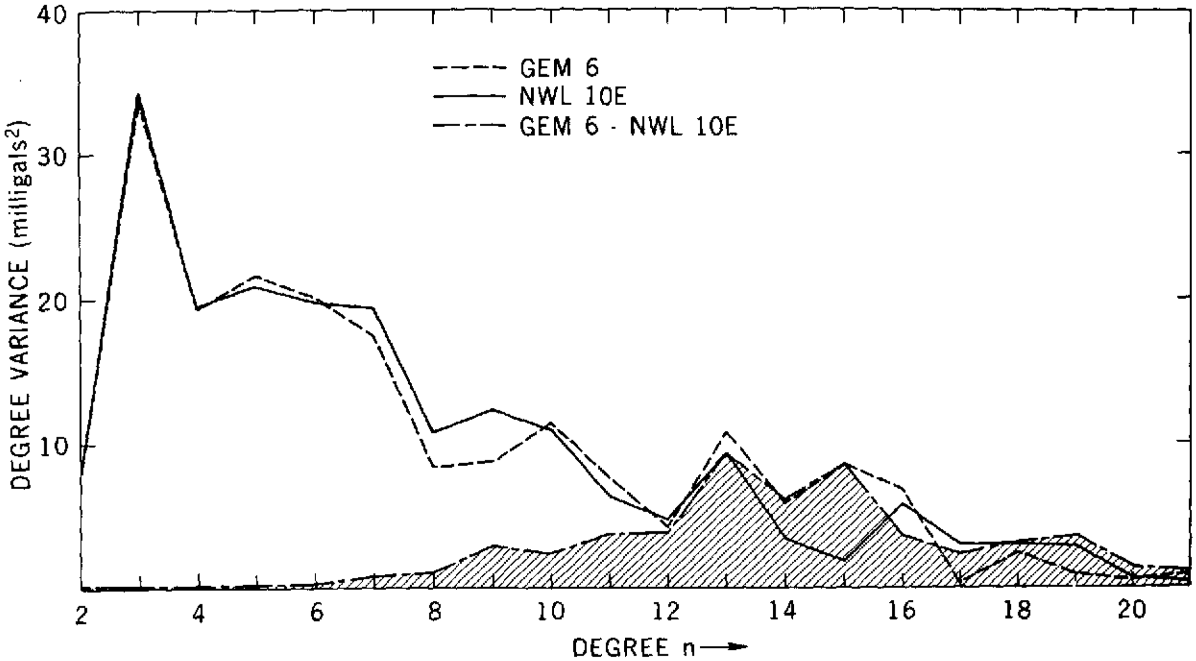

Figure 9. Degree Variances: GEM 6 and 10E

Thus, although the two sets of solutions do not constitute independent standards of comparison against each other, their comparisons will yield useful insights.

The comparison statistics for GEM and SE models are listed in Tables 2, 5, and 8. Some representative curves are illustrated in Figures 1, 2, and 4. The correlation function between GEM 6 and SE III is high (>0.9) to wavenumber n = 7 beyond which it falls off showing inconsistent peaks at n = 9, 11, 13, and 14 (Table 2). The spectral ratio function (Table 5) shows less than 20 percent variation to n = 7 but shows significant departures for higher frequencies. The spectral ratio function of the differences (Table 8) corroborates this pattern in that the differences are practically zero to n = 4, less than 20 percent to n = 7 and significantly higher for n > 8. The degree variances of the differences are close to zero to n = 7 but reach about the same amplitude as those of the total fields for n = 8 and higher as illustrated in Figure 5. The comparisons of SE III with GEM 5, GEM 4, GEM 3, GEM 2, and GEM 1 follow exactly analogous patterns in all the three correlations parameters (Tables 2, 5 and 8; Figures 2 and 4).

Comparison of GEM 6 and SE II shows correlation coefficients of >0.9 to n = 8 and those >0.8 to n = 10 (Table 2). The spectral ratio function (Table 5) departs from its reference value of 100 by about 20 percent up to n = 8. The spectral ratio function of the differences departs less than 10 percent to n = 7 and 11 percent for n = 8 (Table 8). This function is again practically at its reference values up to n = 4 (Table 8). This is corroborated by the correlation function plot in Figure 1 and the spectral ratio function values given in Table 5. Comparisons of SE II with GEM 5, GEM 4, GEM 3, GEM 2, and GEM 1 show even higher consistency for frequencies up to n = 10. Note that SE II and SE III as well as GEM 2, 4 and 6 are all combination solutions while GEM 1, 3 and 5 are purely satellites derived gravity models.

NAVAL WEAPONS LABORATORY'S (NWL) GRAVITY MODELS VERSUS GEM AND SE MODELS

The Naval Weapons Laboratory's gravity models are derived principally from doppler tracking data and are independent enough from the Goddard and Smithsonian efforts to institute a fairly reasonable standard of comparison in relation to GEM and SE solutions. Unfortunately, however, NWL's gravity fields are classified. Therefore, only those statistical parameters of the NWL gravity fields which cannot be used to derive their information content are available for this study.

The NWL gravity model 10E is a purely satellite derived gravity model based principally on doppler tracking data. The NWL gravity model WGSN 44 is a combination solution. These two models are compared against GEM 6 and SE III. The correlation coefficient (Table 3) between SE III and WGSN 44 is high to wavenumber n = 7; the spectral ratio function is close to 100 to n = 4 and shows less than 20 percent variation to n = 7; the spectral ratio function of the differences is practically zero to n = 4, varies less than 10 percent to n = 6 and less than 20 percent at n = 7.

NWL 10E shows a high correlation with SE III to n = 7; the spectral ratio function for the two fields is very close to 100 up to n = 4, varies less than 20 percent between n = 5 to 7; the spectral ratio function of the differences is practically zero to n = 4, varies less than 20 percent between n = 5 to 7. The degree variances of the difference fields as illustrated in Figures 6 and 7, are close to zero up to n = 7 and reach about the same amplitude as the total fields for higher frequencies. For frequencies $n > 7$, the correlations decay rapidly, the spectral ratio function departs significantly from its reference value and the spectral ratio function of the differences rises to large values. The correlation coefficient between GEM 6 and WGSN 44 is high ($\gtrsim 0.9$) to n = 10 (Table 3). The spectral ratio function is close to 100 up to n = 6 (Table 6); the spectral ratio function of the difference fields is practically zero to n = 6, less than 20 percent to n = 10 except for n = 9 for which it is 27 percent (Table 9). The degree variances of the differences are close to zero to n = 8, reasonably small for n = 9 and 10 and equal or exceed the amplitude of those of the parent fields for higher frequencies (Figure 8). For frequencies higher than n = 10 or 11, the correlations decay rapidly and the spectral ratio functions show large variations.

GEM 6 shows a high degree of correlation ($\gtrsim 0.9$) with 10E up to n = 10 (Table 3); the spectral ratio function between the two fields (Table 6) is practically at its reference value to n = 6; the spectral ratio function of the difference fields (Table 9) is zero to n = 6 and is in the neighborhood of 20 percent to n = 10 except for n = 9. The degree variances of differences (Figure 9) are close to zero to n = 8, reasonably small to n = 10 and acquire or exceed the amplitudes of those of the parent fields for higher frequencies. Also, for higher frequencies the correlations are small and the variations of the spectral ratio function from their reference values are large.

ANALYSIS AND INTERPRETATION

For all geopotential models compared here, the correlations are practically equal to one up to wavenumber n = 4. For the same frequency range, the spectral ratio function of the total fields is practically equal to its reference value of 100 and the spectral ratio function of the differences is approximately equal to zero. The degree variances of the differences are also zero. All this indicates that the various geopotential solutions are completely consistent in this frequency range. This is most convincingly shown in Figure 10 which shows the gravity differences in milligals between GEM 6 and SE III solutions for this frequency range. Notice that the maximum amplitudes of these differences are ±2 milligals while most of the differences are in the neighborhood of 0 milligal as indicated by the rms value of close to zero for these differences.

Between the frequency range n = 5 to n = 7 the correlation function, though slightly less than 1, is still very close to one. The spectral ratio function of the total fields is in the neighborhood of its reference value and variations shown by the spectral ratio function of the differences are less than 20 percent. This is supported by the difference degree variances. Thus the various geopotential solutions are nearly identical in this frequency range. The extent of their variations is shown in Figure 11, which illustrates differences between GEM 6 and SE III in the frequency range of n = 2 to n = 7. Notice that while the maximum amplitudes of the differences reach 8 milligals, the differences are generally far less in amplitude and the rms of the differences is only 2.3 milligals.

The comparison studies indicate that in the frequency range n = 8 to 12 the various GEM solutions are internally consistent as would be expected. GEM solutions also show good agreement with the NWL solutions to n = 10. But the

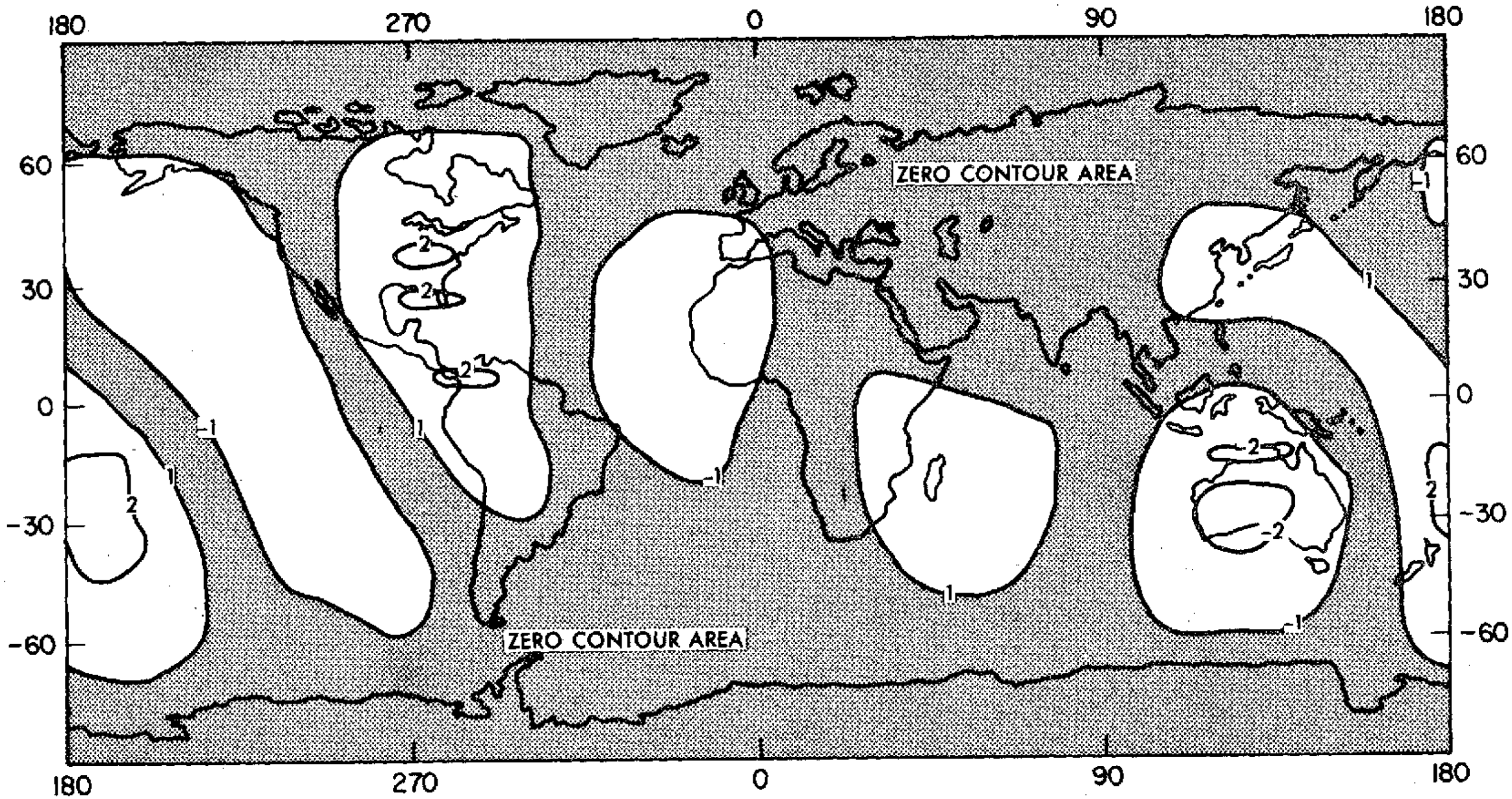

Figure 10. Free Air Gravity Anomalies based on differences between SE III and GEM 6 (2, 0 - 4, 4)

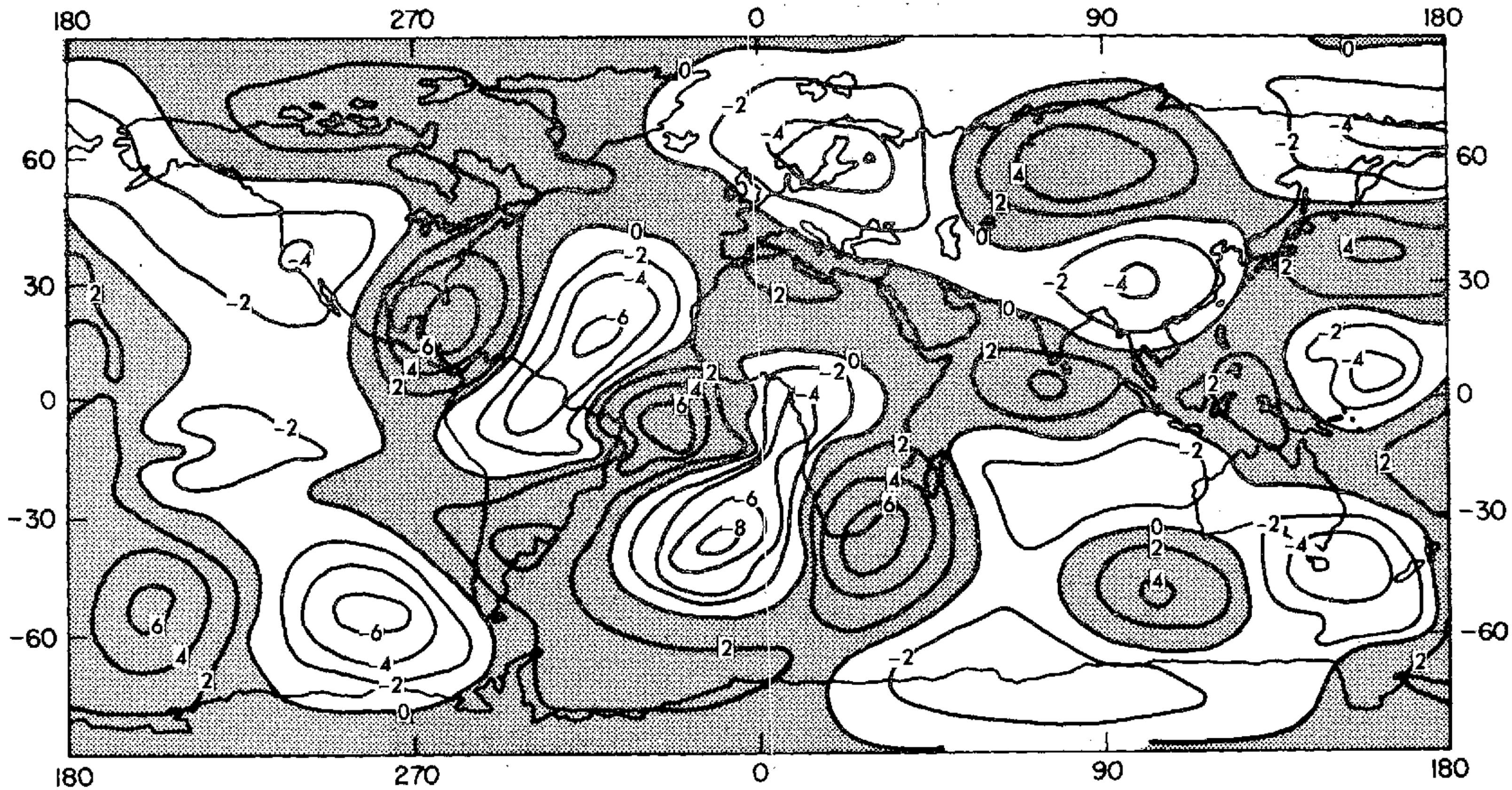

Figure 11. Free Air Gravity Anomalies based on differences between SE III and GEM 6 (2, 0 - 7, 7)

differences between SE III and either GEM or NWL models become significant at n = 8. This is demonstrated in Figure 12 which shows the gravity differences in GEM 6 and SE III for frequencies n = 8 and higher. Notice that the maximum amplitudes reach 36 milligals and whereas these amplitude maxima occur in the southern hemisphere, the gravity differences have comparable amplitudes and frequencies in the northern hemisphere. These gravity anomaly differences have an rms value of 9.3 milligals.

Figure 13 shows the total gravity contribution of GEM 6 in the same frequency range. Compare it with Figure 12 and notice that the amplitudes of these gravity anomalies are comparable with those of gravity anomaly differences in Figure 12. Also that the anomalous patterns, though not identical, are intriguingly similar. The rms value of these gravity anomalies in 9 milligals – very close to the rms value of the gravity anomaly differences shown in Figure 12.

The differences in geoidal heights arising from gravity anomaly differences shown in Figure 12 are given in Figure 14. The reason that geoidal differences are milder in amplitudes as well as gradients than the corresponding gravity anomaly differences is that the process of transforming the gravity anomalies into geoidal heights is equivalent to passing the gravity anomalies through the type of filter (in a spectral form) shown in Figure 15. This process tends to accentuate the effects of long wavelength gravity anomalies and scale down the effects of the shorter wavelengths. It is for this reason that geoidal comparisons are not regarded as an ideal instrument for comparative and evaluative investigations of the various gravity solutions.

The characteristics derived from the comparison of GEM 6 with SE III are shown more or less by all other solutions though the value of n may change somewhat in each case. Table 10 shows in a summary form, the concentration of spectral energy in the different frequency ranges of representative gravity solutions. For GEM 6, the total power in the frequency range of n = 2 through 7 is 119.8 milligals^2 or 876 meters^2. For the same frequency range the total power of the GEM 6 and SE III differences is 6.3 milligals^2 or 14 meters^2. Thus, the differences consitute only 5.3 percent (or 1.6 percent for geoid) of the total spectral energy in this frequency range. For the same frequency range, total spectral power in WGSN 44 and 10E is 123 milligals^2 and 121.3 milligals^2, respectively. Their differences from GEM 6, 1.1 milligals^2 and 1 milligals^2, respectively are less than 1 percent of the total spectral power for these frequencies. Same holds for geoidal comparisons (Table 10). This, together with the comparison studies, is interpreted to mean that the coefficients in this frequency range are well-determined in all recent geopotential solutions, though GEM 6 seems to test better against the independently obtained gravity solutions of the Naval Weapons Laboratory.

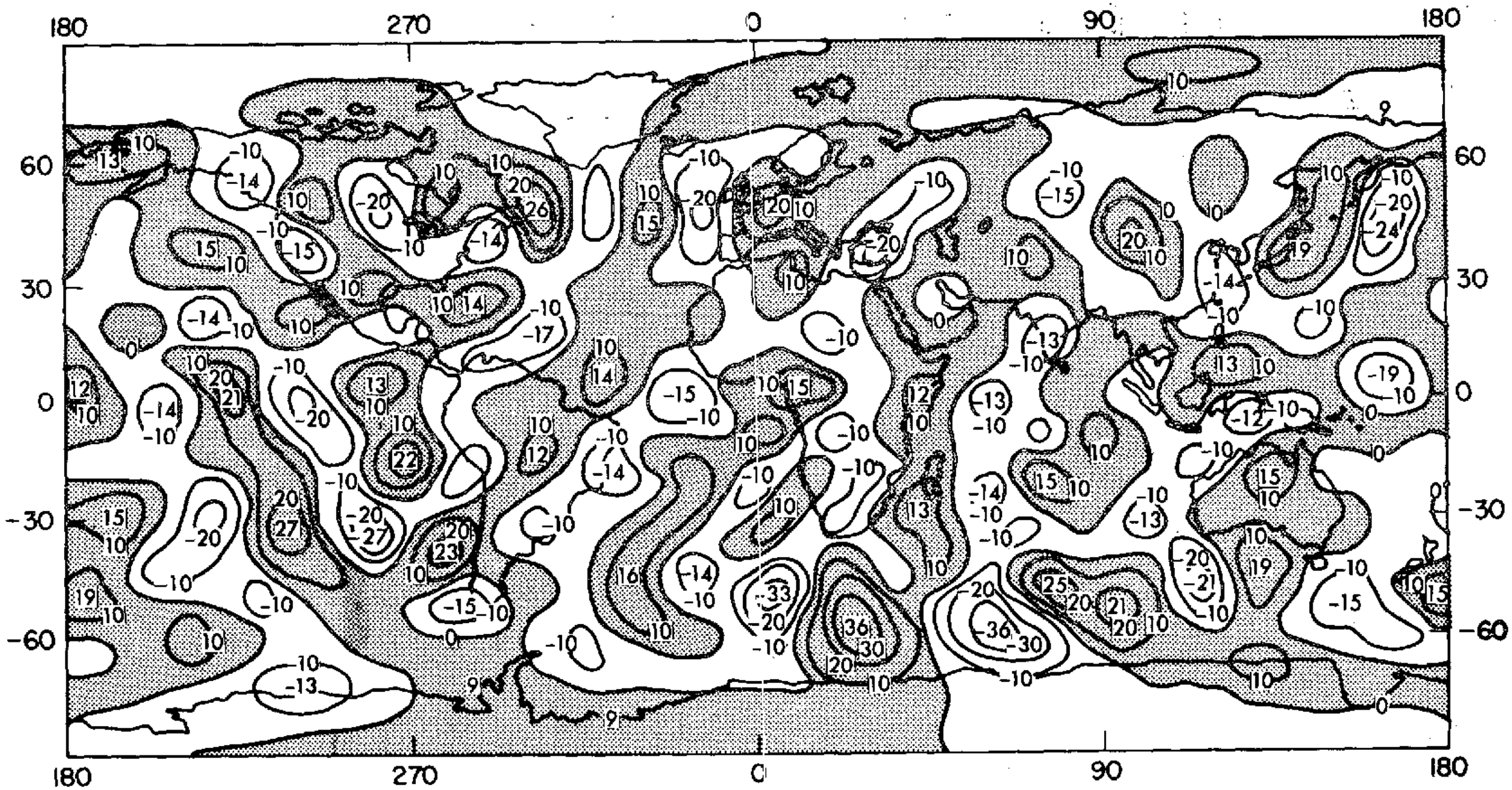

Figure 12. Free Air Gravity Anomalies based on differences between SE III and GEM 6 (8, 0 higher)

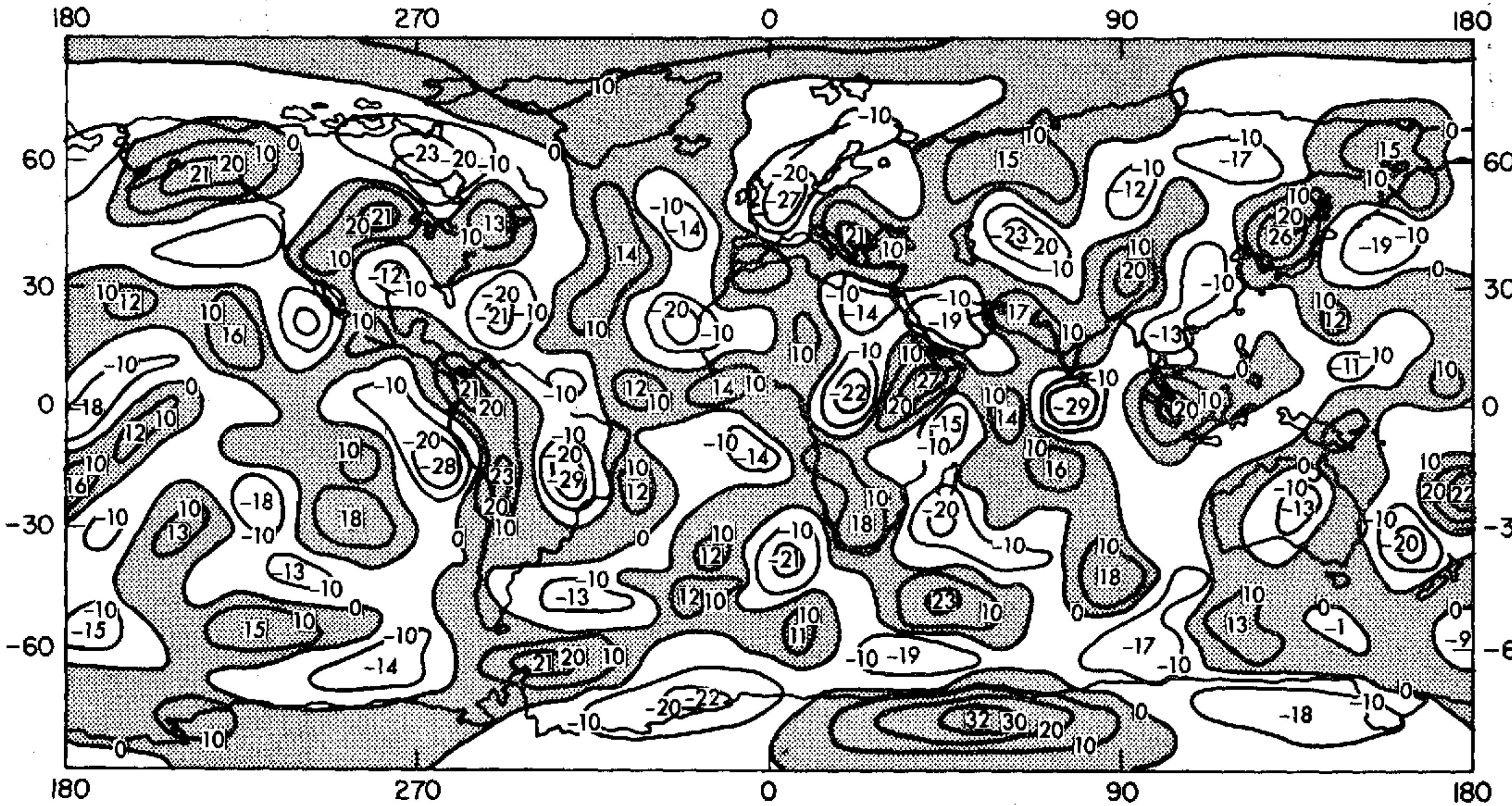

Figure 13. Free Air Gravity Anomaly contribution of GEM 6 geopotential coefficients (8, 0) through (16, 16) plus some higher coefficients

DIFFERENCES IN GEOIDS DETERMINED FROM SAO III AND GEM 6
CONTOUR INTERVAL = 5 METERS

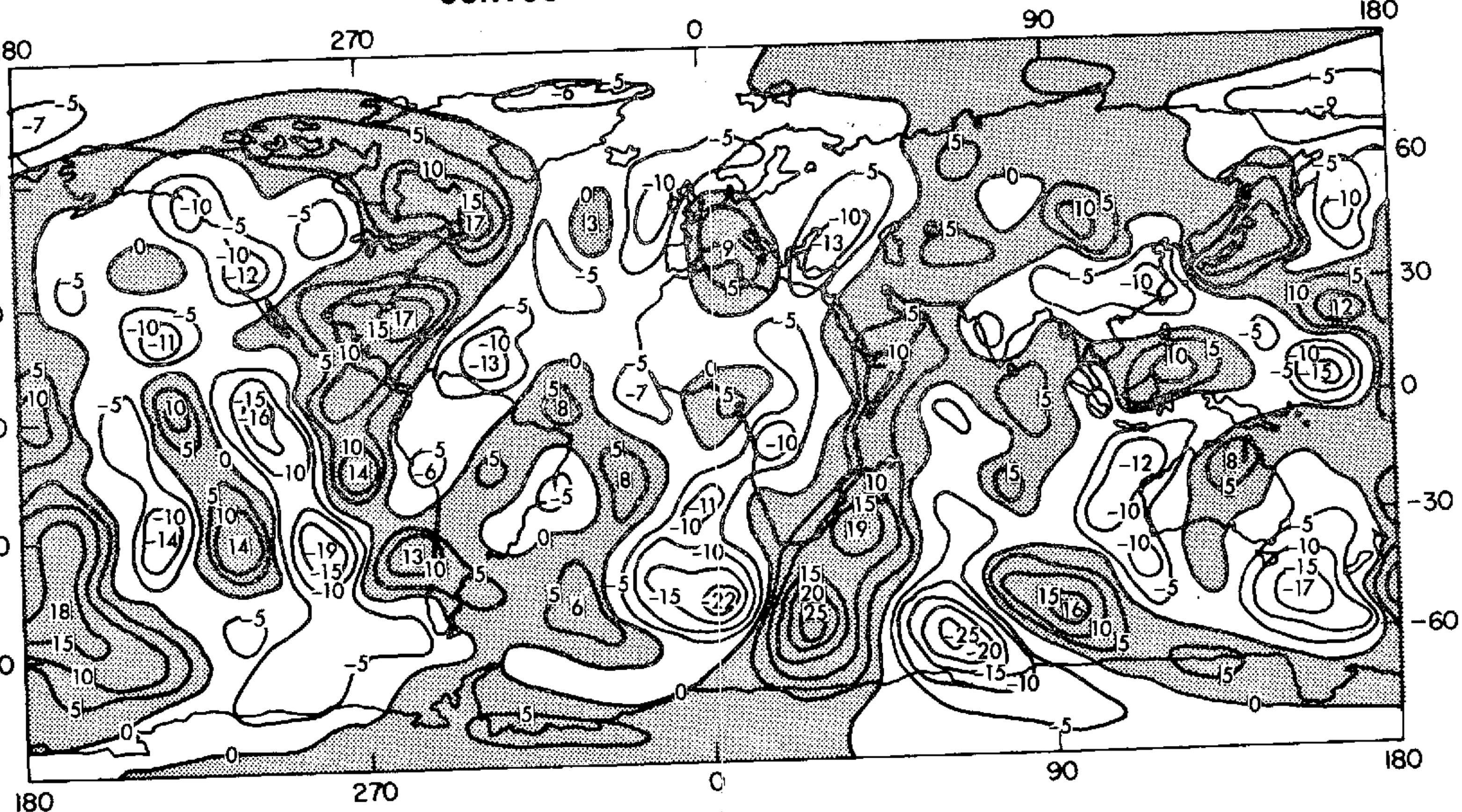

Figure 14. Differences in geoids determined from SE III and GEM 6

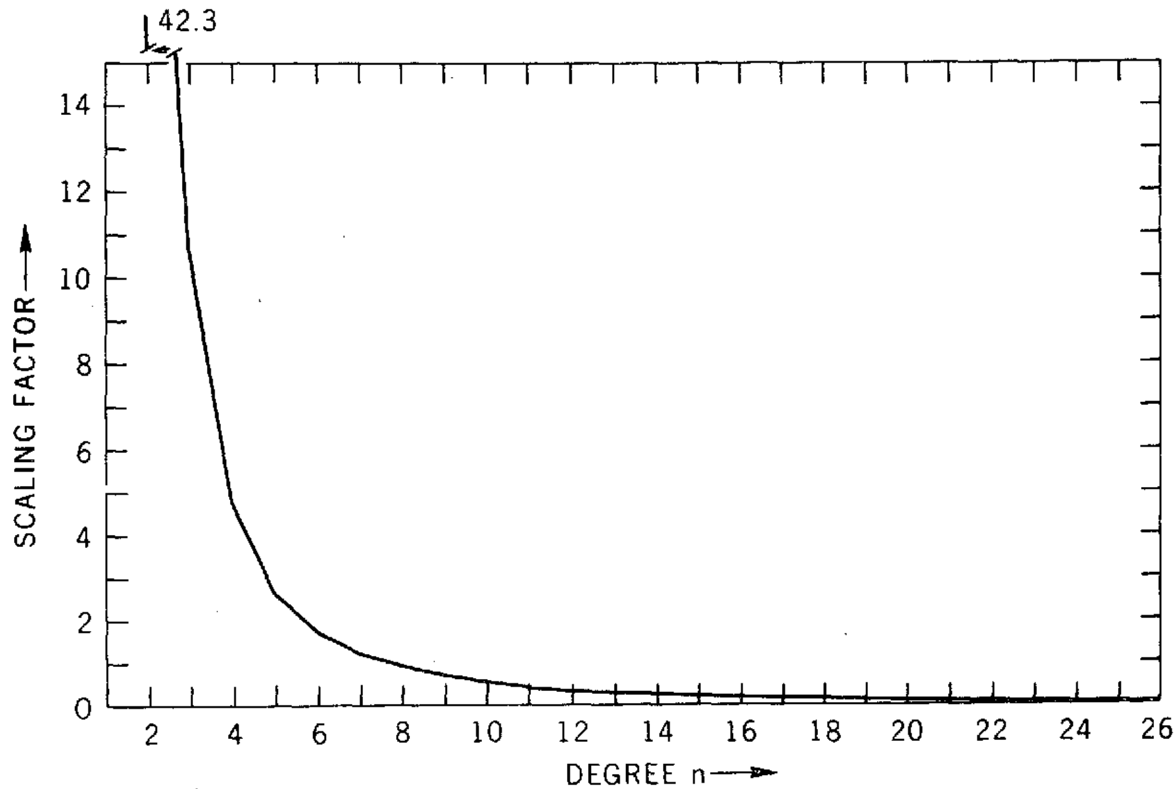

Figure 15. Spectral conversion factor from gravity anomalies to geoidal heights

However, the picture is different for higher frequencies. For $n \geqslant 9$ the total spectral energy in GEM 6 is 79 milligals^2, that in the differences between GEM 6 and SE III is 86 milligals^2 so that the power in differences is 109 percent of the total power of the solution itself. Since GEM 6 is complete only to (16, 16) and SE III to (18, 18) it could perhaps be argued that the different cutoff frequency ranges magnify the differences spuriously. However, truncation of the two solutions at (16, 16) does not change the above results in any significant manner. GEM 6 seems to compare somewhat better with WGSN 44 and 10E in which case the spectral power of the differences constitutes about 96 percent and 70 percent of the total spectral powers, respectively. But the important point here is that in the higher frequency range ($n \geqslant 8$ or 10 depending upon the models being considered), the difference spectra are of the same order of magnitude as the spectra of the total fields. This has, of course, also been demonstrated earlier in Figures 12 and 13 which show that amplitudes of the gravity anomaly differences between GEM 6 and SE III for $n \geqslant 8$ are of the same order of magnitude as the gravity contribution of higher frequency coefficients ($n \geqslant 8$)

Table 10

Typical Spectral Contents of Various Frequency Ranges of Some Representative Gravity Models

	GRAVITY Mgals^2			GEOID METERS^2		
	n = 0 – 7	n = 8 →	TOTAL	n = 0 – 7	n = 8 →	TOTAL
GEM 6	119.8	79.0	198.8	876.0	32.5	908.5
GEM 6 - SAO III	6.3	86.0	92.3	14.0	30.1	44.1
(GEM 6 - SAO III / GEM 6) %	5.3	109.0	46.0	1.6	93.0	4.9
NWL WGSN 44	123.0	137.0	260.0	889.4	42.3	931.7
GEM 6 - WGSN 44	1.1	130.8	131.9	1.8	25.6	27.4
(GEM 6 - WGSN 44 / GEM 6) %	0.9	95.5	50.7	0.2	60.5	2.9
NWL 10E	121.3	75.3	196.6	885.8	34.0	919.8
GEM 6 - NWL 10E	1.0	52.8	53.8	1.7	15.3	17.0
(GEM 6 - NWL 10E / GEM 6) %	0.8	69.5	27.4	0.2	45.0	1.9

of GEM 6 or SE III. Since NWL 10E and WGSN 44 are classified it is not possible to study their individual gravity differences with respect to GEM 6 or SE III. But the comparison statistics (Tables 3, 6 and 9), indicate that GEM 6 is consistent with the NWL gravity fields up to n = 10 beyond which the difference coefficients acquire the same amplitude as the coefficients of the parent fields. It is thus clear that the individual values of these higher frequency harmonic coefficients should be treated with caution.

It is interesting to see the relationship between the surface gravity data used in a typical combination solution and the gravity anomaly differences between this typical combination solution and the associated purely satellite determined solution. Figure 16 shows the gravity anomaly differences between GEM 6 and GEM 5 solutions. The basic distribution of the surface gravimetric data used in the GEM 6 combination solution is shown in Figure 17. There seems to be no consistent relationship between surface gravimetric data coverage and the gravity anomaly differences.

Let us now examine Table 11 which gives rms values of observation residuals for weekly orbital arcs based on optical data for 23 satellites, 11 daily arcs based on USB Doppler data for ERTS-1 satellite, 22 BE-C short arcs based on laser data, long term zonal perturbations on 21 satellites and rms of residuals with respect to 5° × 5° surface gravity anomalies (Lerch et al, personal communication). Note that ERTS-1 Doppler arcs and BE-C laser arcs were not used in the computation of gravity models considered here and thus deserve special weight. With the exception of GEM 4, the orbital residuals from all other GEM solutions are nearly equal and while those based on GEM 5 and GEM 6 show some improvement relative to GEM 3 and GEM4, they seem to show no recognizable improvement over GEM 1.

Table 11

Summary of Gravity Model Comparisons with Satellites and Gravimetric Data

MODELS		OPTICAL DATA ON WEEKLY ARCS FOR 23 SATELLITES (SECONDS OF ARC)	U S B DOPPLER* DATA ON 11 DAILY ERTS-1 ARCS (CM./SEC.)	LASER DATA* ON 22 BEC SHORT ARCS (METERS)	LONG TERM GOAL PERTURBATIONS ON 21 SATELLITES (RELATIVE MEASURE)	5° TERRESTIAL GRAVITY ANOMALIES (MGAL)
GEM	1	2.54	5.9	1.33	3.62	12.5
GEM	3	2.71	5.9	2.00	2.92	12.3
GEM	4	3.10	7.2	4.05	2.89	12.2
GEM	5	2.37	5.9	1.54	3.13	12.3
GEM	6	2.74	5.5	1.65	2.97	11.6
SAO S.E.	II	3.44	10.3	2.51	5.49	12.8
SAO S.E.	III	–	11.2	–	–	12.5

*DATA FOR THESE TWO CATEGORIES WERE INDEPENDENT OF THE SOLUTIONS FOR ALL MODELS.

The rms with respect to gravimetric data in computed from $\langle (g_T - g_S)^2 \rangle$ where g_T denotes the 5° × 5° mean gravity anomaly based on surface gravity data and g_S is the corresponding gravity value computed from the specific geopotential model. GEM 6 seems to agree with the surface gravity data somewhat better than the other models. But it is probably due to the fact that the set of mean surface gravity anomalies used in the computation of GEM 6 is the same as that used in the computation of rms residuals, whereas it is somewhat different from that used in obtaining earlier GEM and SE solutions.

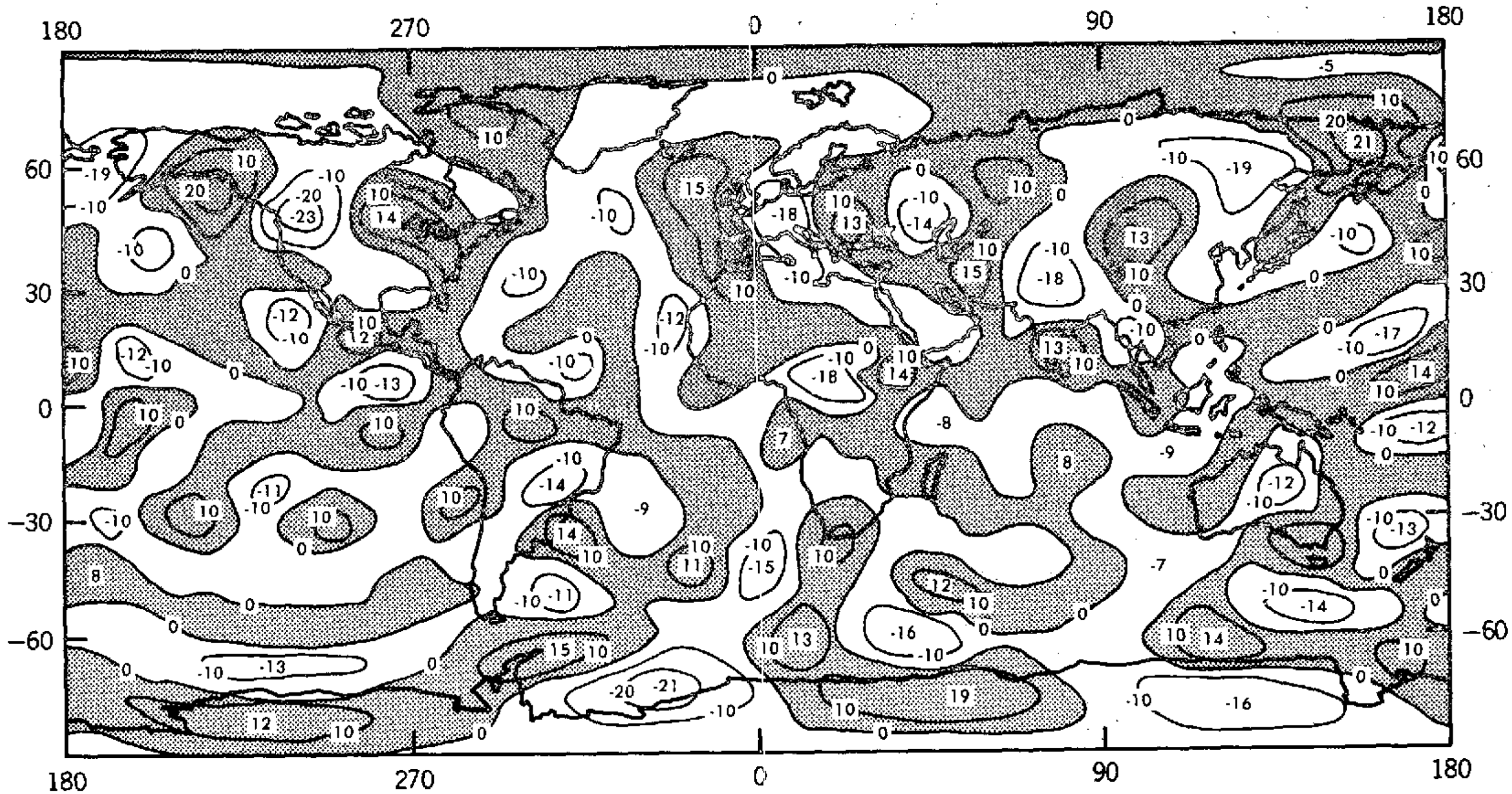

Figure 16. Free Air Gravity Anomalies based on differences between GEM 6 and GEM 5

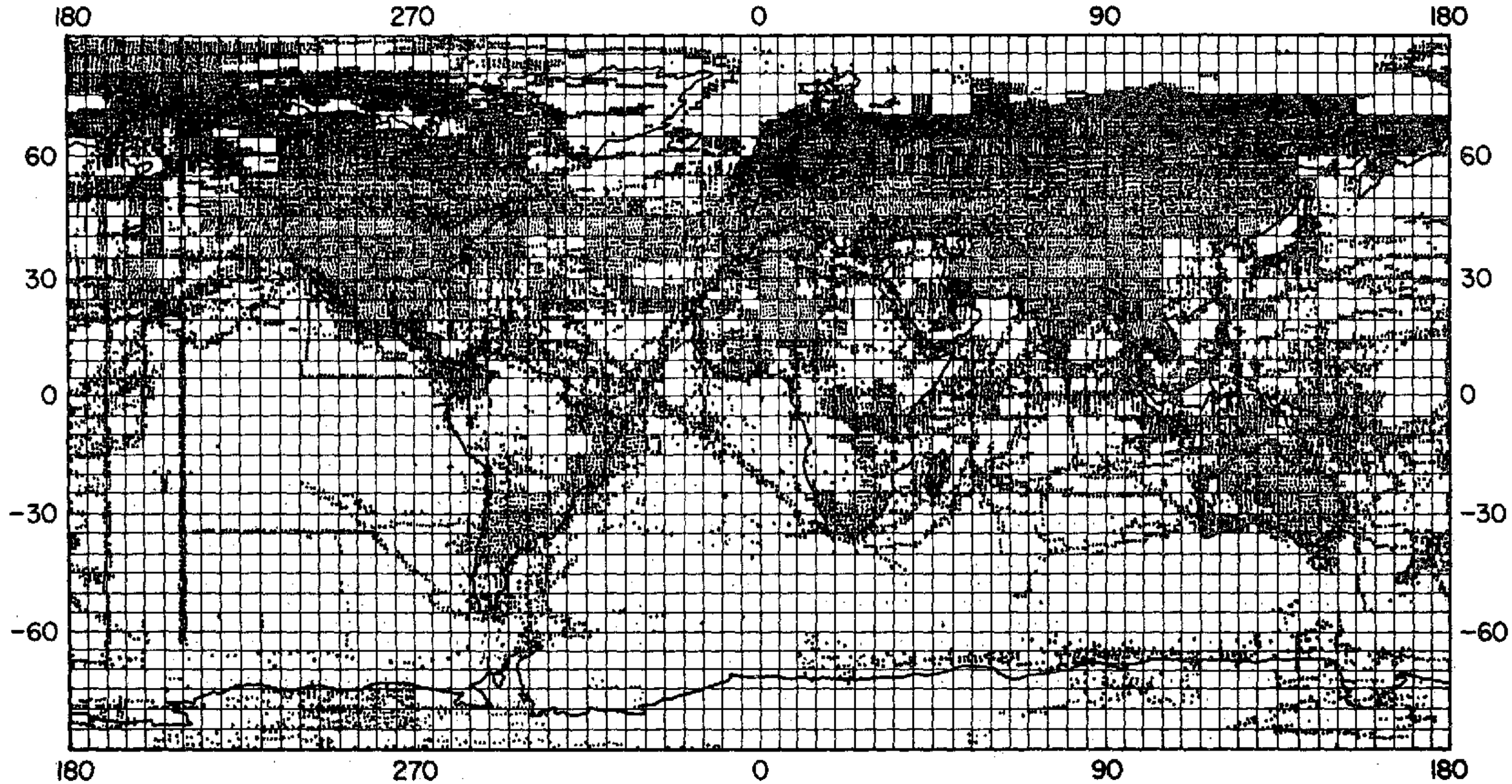

Figure 17. 1° × 1° surface gravity coverage. The number of dots in each 5° × 5° square indicates the number of 1° × 1° mean anomalies used in computing the 5° × 5° mean gravity anomaly in that square.

CONCLUSIONS

1. The spherical harmonic coefficients of Earth's gravity field up to $n = 4$ seem to have been determined accurately.

2. The geopotential coefficients corresponding to wavenumbers $n = 5$ through 7 seem to have been determined quite accurately though they show minor differences from one solution to the other.

3. The geopotential coefficients for wavenumbers $n = 8$ through 10 seem to be determined fairly accurately in GEM 6 and NWL solutions.

4. For frequencies higher than $n \geqslant 11$, the various geopotential solutions seem to be very divergent and although a cumulative contribution of these frequencies seems to make a marked improvement in the satellite orbital residuals, the significance of their individual values and their geophysical contribution are not clear.

5. While the satellite orbital data make the predominant contribution to the geopotential coefficients up to wavenumber $n = 10$, the higher frequencies (particularly $n > 12$) seem to be primarily controlled by the surface gravimetric contribution. The full potential of this contribution, however, does not appear to be realized in combination solutions.

6. While the value of the additional classical tracking data in improving the description of the Earth's gravity field in long wavelength components cannot be overstated, their contribution in extending the range of frequency of Earth's gravity field description is uncertain. New tracking data types such as laser, satellite-to-satellite and altimetry data seem to have the potential of improving the frequency range of Earth's gravity field description but a quantitative assessment of their impact is difficult at this stage.

7. On the basis of analysis reported here, it is difficult to select a particular GEM solution over other GEM solutions because of their almost identical test results (Table 11). For geophysical studies, however, GEM 6 is recommended because of its more updated and extensive data base. For studies based on orbital dynamics, either GEM 1 or GEM 5 seem suitable.

   NWL 10E or WGSN 44 are not available for such studies.

I must end this paper with a note of caution. The analysis reported here is not against an absolute standard but on a relative basis so that each field which is being tested itself forms a standard of comparison for other fields in the evaluation process.

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