

## **General Disclaimer**

### **One or more of the Following Statements may affect this Document**

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

(NASA-CR-142458) APPLICATION OF MODERN  
CONTROL THEORY TO SCHEDULING AND  
PATH-STRETCHING MANEUVERS OF AIRCRAFT IN THE  
NEAR TERMINAL AREA (Massachusetts Inst. of  
Tech.) 28 p HC \$3.75

N75-15626

Unclass  
88454

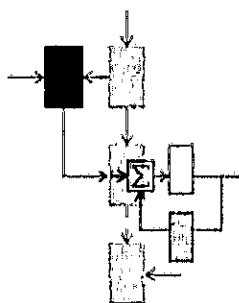
CSCL 17B 63/74

October, 1974

Report ESL-R-574

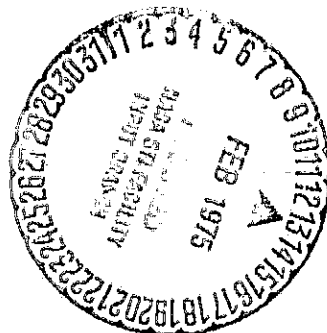
MIT OSP Projects 76265  
and 73935

Nasa Grant NGL-22-009-124  
AFOSR Grant 72-2273



## APPLICATION OF MODERN CONTROL THEORY TO SCHEDULING AND PATH-STRETCHING MANEUVERS OF AIRCRAFT IN THE NEAR TERMINAL AREA

*Michael Athans*



*Electronic Systems Laboratory*

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASSACHUSETTS 02139**

*Department of Electrical Engineering*

*Decision and Control Sciences Group*

October, 1974

Report ESL-R-574

APPLICATION OF MODERN CONTROL THEORY  
TO SCHEDULING AND PATH-STRETCHING MANEUVERS OF  
AIRCRAFT IN THE NEAR TERMINAL AREA

by

Michael Athans

This research was conducted in the Decision and Control Sciences Group of the M.I.T. Electronic Systems Laboratory with support by the NASA Ames Research Center under grant NGL-22-009-124 and by the Air Force Office of Scientific Research under grants AFOSR 69-1724 (A) and 72-2273.

Electronic Systems Laboratory  
Department of Electrical Engineering  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139

APPLICATION OF MODERN CONTROL THEORY  
TO SCHEDULING AND PATH-STRETCHING MANEUVERS OF  
AIRCRAFT IN THE NEAR TERMINAL AREA

by

Michael Athans

Director, Electronic Systems Laboratory  
Professor of Electrical Engineering  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139 USA

ABSTRACT

The purpose of this paper is to present a design concept for the dynamic control of aircraft in the near terminal area. An arbitrary set of nominal air routes, with possible multiple merging points, all leading to a single runway is considered.

The system allows for the automated determination of acceleration/deceleration of aircraft along the nominal air routes, as well as for the automated determination of path-stretching delay maneuvers.

In addition to normal operating conditions the system accommodates

- (a) variable commanded separations over the outer marker (to allow for takeoffs between successive landings)
- (b) emergency conditions (in the sense that an aircraft is given partial or complete priority for landing).

The system design is based upon the combination of three distinct optimal control problems: (a) a standard linear-quadratic problem, (b) a parameter optimization problem, and (c) a minimum-time rendezvous problem.

Simulation results involving twelve aircraft under both normal and emergency conditions will also be presented.

I. INTRODUCTION

This paper considers a class of problems encountered in air traffic control. Specifically, we examine the problem of coordinating the flight trajectories of jet aircraft in the vicinity of an airport. It is assumed that the aircraft enter the NTA (near terminal area), properly separated by the en-route ATC centers, and they all desire to land in a single runway. Furthermore, it is assumed that the same runway is to be used for both takeoffs and landings. Special emphasis is focused upon heavy traffic conditions, which imply that some aircraft must undergo path-stretching and holding maneuvers while awaiting landing.

The motivation for this research was provided by system delays encountered in the current operation of the ATC systems during heavy traffic conditions. At present, there are small time delays which are associated with "unstacking" the aircraft, since the aircraft at the bottom of the stack may have the wrong heading at the time of the command to "leave the stack". These small time delays can, however, become significant when many aircraft are holding; hence, this contributes to an effective reduction in the landing rate. These accumulated delays cannot be blamed upon the air traffic controllers since, in heavy traffic conditions, they cannot monitor the detailed trajectory of each aircraft that is holding. Rather, it is the result of overall system uncoordination due to the many decisions that must be carried out continuously by the human controllers.

This lack of system coordination is also exhibited when a particular aircraft in a queue develops an emergency. In such cases, it is often desirable to have this aircraft land first and this necessitates the reordering of the aircraft which were supposed to land before the troubled one. The determination of the proper trajectories for all aircraft concerned is a complex task which, in heavy traffic conditions, can tax the ability of the most capable human.

Similar situations arise when a sudden wind shift requires a change in the landing runway. Once more, the rescheduling of the aircraft is a complex decision task.

The ever increasing demand for air travel has motivated the definition of required changes in the current and contemplated ATC system. The so-called "Alexander Report" (see reference 1) contains a description of the short-term and long-term problems in ATC. It also contains a discussion of the gradual changes that must be carried out to improve

- (a) airport runway design
- (b) improved ground and airborne instrumentation and displays
- (c) improved landing aids (e.g., scanning microwave landing systems)
- (d) system automation

There is very little doubt, that digital computers will find an ever increasing role in air traffic control. In fact, the Alexander Report (ref. 1, p. 84) concluded that the total computational demands can be met by currently available digital computers. However, it points out that research is needed to define in what precise form the computer can be used to alleviate the load on the human ATC controller.

From a conceptual viewpoint, there is no reason why the ATC system cannot be completely automated. One can envision an automated set of command and control systems in which digital computers obtain all the information from the ground and airborne sensors, smooth the data, compute trajectories for all aircraft, and transmit the appropriate commands to the autopilot of each and every aircraft.

However, such a completely automated system neglects the economic and political constraints that exist. It takes many years to develop and test any new equipment and even more time to negotiate and change all international agreements that exist. Hence, it is safe to assume that such a completely automated system even if developed now will not take effect for at least 20-30 years. Thus, any proposed innovations must take into account the need for gradual evolution and the fact that man-machine interaction will be an essential part of the ATC system for many years to come.

## II. THE ROLE OF THE COMPUTER AS A DECISION MAKING TOOL

It is the contention of this paper that the digital computer can be used as an effective tool that carries out routine decisions and advises the human of suggested courses of action via a display. It is most important, however, that the basis for the automated suggested decisions are such that as technological advances in displays, instrumentation, and autopilots are gradually incorporated into the ATC system, that these changes do not necessitate drastic changes in the way that the decisions are being made. For example, if improved accuracies in locating the aircraft become possible by advanced sensors, and this can be used to reduce the safe separation distance, this should not require a completely new conceptual and algorithmic package for system coordination.

There are many system studies for improved scheduling and trajectory control (see reference 2 for typical proposals). However, most of the suggested approaches basically attempt to have the digital computer execute decisions in essentially the same manner that a human does. This introduces an artificial element of complexity in the algorithms because the human basic decision rules cannot be defined precisely and are modulated to a large extent by past rules, regulations, constraints,\* and "we always did it this way" doctrines. Even if such computational algorithms could be developed, then they would not necessarily be in tune with the future automatic control capabilities that can be realized by hardware advances.

The approach taken in this paper is to attack the problem of automated decision making by the digital computer essentially as though all decisions could be implemented completely automatically. This would define the "best possible" way that the aircraft could be controlled. However, these automated decisions are only presented as suggestions, via displays, to the human controller, and eventually the pilot. The human essentially can follow the suggested actions or he can ignore them. The decision making algorithm should be a general one; yet have free parameters that can be adjusted by having realistic man-machine simulation experiments. In this manner, at any phase of implementation, one would be able to "tune" the algorithm parameters to be in phase with the human constraints. On the other hand, as more and more true automation is introduced in the system, this would only necessitate changes in certain algorithm parameters rather than a complete overhaul of the decision making algorithm.

## III. STRUCTURE OF THIS PAPER

The general ideas presented in the previous section will be examined in detail for a specific sub-problem in air traffic control. In Section IV we define the problem considered in the NTA as well as the assumptions. In Section V we deduce the index of the aircraft as a function of their desired order of landing (under normal and emergency conditions). In Section VI we formulate the basic decisions that must be executed. Section VII summarizes the method of approach which is based upon the solution of three deterministic optimal control problems, and develops the quantitative formulation. Sections VIII, IX, and X contain a discussion of the algorithm and its possible gradual implementation into the current and contemplated ATC system. Section XI discusses the simulation results.

## IV. PROBLEM DEFINITION

The system to be considered is concerned with the trajectory control of aircraft in a region of airspace (e.g., 50-75 miles radius) in the vicinity of an airport called the NTA region. In this paper we assume that a single runway has been selected to handle takeoffs and landings. We also assume that there is a point in the airspace, the outer marker, which defines the point at which all aircraft that must land have to arrive. We shall not be concerned with the aircraft motion between the outer marker and the runway. Thus, once an aircraft passes over the outer marker, it is removed from the NTA control center.

It is assumed that aircraft enter the NTA region at specific points along its boundary; prior to that, they are being controlled by the appropriate en-route center.

### 4.1 Nominal Air Route Structure

Figure 1 illustrates the NTA region. As shown in Figure 1, we assume that there is a well defined set of nominal air-routes which connect each entry point at the boundary of the NTA region to the outer marker. It is possible to have two or more air-routes merge at a node point in this tree like structure.

The air routes are trajectories in three dimensional space. Hence, a particular air route may involve altitude changes. Also, the air routes may be made out of curved trajectories rather than straight lines. This is important because the potential scanning-beam microwave system can be used to define curved trajectories.

\*The state of the art of "artificial intelligence" in computer science is far away from being able to have a digital computer duplicate complex human decision elements.

The nominal air route tree structure should be interpreted as the flight path that would be followed by an aircraft if there was no other traffic. It is then tacitly assumed that the aircraft should stay on their nominal air route unless it is imperative to leave it for path-stretching and/or holding maneuvers.

## 4.2 Aircraft Characteristics

In this paper we shall only consider aircraft with similar flight characteristics. For example, only commercial jets (DC-8's, 707's, 727's, DC-9's, etc.). The problem of mixed aircraft population (e.g., jets, 4 propeller, 2 propeller, 1 propeller) will not be covered here (see references (10) and (12)).

This single population assumption implies that all aircraft under consideration have similar speed and maneuverability characteristics.

## 4.3 Minimum-Separation

We shall assume that all aircraft must be mutually separated by at least  $D_{\min}$  distance units for safety reasons. We assume that the enroute centers deliver aircraft to the NTA region properly separated. It is the task of the NTA center to keep them properly separated subsequently.

## 4.4 Desired Speed

For reasons that will become apparent later on, and for the sake of overall system coordination, we shall establish a single desired constant speed  $\bar{v}$ , (e.g., 160 knots). The speed  $\bar{v}$  should be substantially higher than the stall velocity and landing velocity. It is assumed that speed reduction from  $\bar{v}$  is possible between the outer marker and the runway. We remark that all aircraft will be controlled such that all speed deviations from  $\bar{v}$  are nulled out.

The extension of these ideas to the multiple speed problem, using altitude separation, has been briefly considered in references 10 and 12.

## 4.5 Maneuverability Constraints

In heavy traffic conditions, one would expect that some of the aircraft will have to undergo path stretching and/or holding trajectories which will take them temporarily away from its nominal air route. Hence, we must take into account constraints on the aircraft maneuverability. For this reason, we assume that a minimum turn radius,  $R_{\min}$ , can be defined for each aircraft which establishes the tightest circular trajectory that the aircraft can execute at or near the speed  $\bar{v}$ . The minimum turn radius is not necessarily the one that can be physically attained by the aircraft; it should be construed as the one that conforms with FAA regulations as well as passenger comfort.

## 4.6 Further Assumptions

It is assumed that, at any instant of time, the NTA control center knows

- a) the identity of each aircraft
- b) its true position and velocity
- c) its nominal future air route

## V. IDENTITY AND LANDING INDICES

We commence our quantitative development by considering a simple scheme that determines the landing order of the aircraft.

### 5.1 Identity Index

Let  $t$  denote the current instant of time. Let  $N$  denote the number of aircraft in the NTA region (the number  $N$  changes with time; it is increased by one every time a new aircraft enters the NTA region).

Let  $k = 1, 2, \dots, N$  be an arbitrary identity index. For example,  $k = 4$  means TWA 43.

### 5.2 Positions and Velocities

Let  $z_k(t)$  denote the distance-to-go, along the nominal air route, to the outer marker. Since we assumed that we knew the identity of each aircraft, its nominal air route and its location,  $z_k(t)$  can be readily computed (see Figure 2).

Let  $v_k(t)$  denote the actual speed of the  $k$ -th aircraft at time  $t$  along the air route.

To be notationally consistent, we agree to measure distances-to-go as being positive and speeds as being positive. In this case, a higher speed would decrease the distance-to-go faster. Hence, the precise relationship between  $z_k(t)$  and  $v_k(t)$  is

$$\frac{d}{dt} z_k(t) = -v_k(t) \quad (1)$$

### 5.3 Landing Sequence Index

We next define another index  $i = 1, 2, \dots, N$  that indexes the aircraft according to their landing order.

(First to land, second to land, third to land, etc.)

We shall distinguish two landing sequences

- a) The natural landing sequence
- b) The emergency landing sequence.

In the natural landing sequence, the planes that can land first, do land first. In an emergency landing sequence, we assumed that an aircraft (say the  $k$  one) develops an emergency condition. In this case, we may wish to clear its air route, and have it land first; the remaining  $N-1$  aircraft land after  $k$  in their natural landing sequence. Under different circumstances the plane under emergency conditions may be given partial priority. For example, under a natural landing sequence, the plane in question might have been 6th in line, and we may advance it to be 3rd in line.

The determination of the natural landing sequence is simple, once we have assumed that all aircraft will be controlled so that their speed is reduced to  $\bar{v}$ . Since  $z_k(t)$  is the actual distance to go for the  $k$ -th aircraft, and since  $\bar{v}$  is its desired speed along the air route, one can compute the estimated time of arrival to the outer marker  $T_k$  by

$$\hat{T}_k = \frac{z_k(t)}{\bar{v}}; k = 1, 2, \dots, N \quad (2)$$

The plane with the smallest  $\hat{T}_k$  is indexed by  $i = 1$ , the plane with the next smallest  $\hat{T}_k$  is indexed by  $i = 2$ , etc. This can be used to define the natural landing sequence; in case of emergency, the plane  $k$  is indexed by  $i = j$  and the rest by the natural landing sequence.

So we shall let  $i = 1, 2, \dots, N$  index the desired landing sequence (natural or not). Thus, the  $i$ -th and  $i + 1$ -st planes pass successively over the outer marker and land (first  $i$ , then  $i + 1$ ).

From now on, the aircraft will be indexed by their desired landing index  $i$  rather than by their identity index  $k$ .

## VI. THE BASIC DECISION AND CONTROL PROBLEM

Up to this point, we have established a desired landing order. We have not specified what should be the desired separation (in time or distance) between two planes that land successively.

### 6.1 Desired Future Separations

The desired separation between successive aircraft pairs would depend on the demands for takeoffs on the same runway. If the runway is used exclusively for landings, then the desired separation over the outer marker of two successive aircraft should be  $D_{\min}$  (the minimum desired separation). On the other hand, if one wishes to interject one or more takeoffs, after the  $i$ -th plane has landed, and before the  $i+1$ -st plane lands, then the desired separation of the  $i$ -th and  $i+1$ -st planes in the vicinity of the outer marker must be much greater than  $D_{\min}$ .

For this reason, we assume a cooperative system at the airport that will coordinate takeoffs and landings. As far as the NTA control center is concerned, we shall assume that it is given\* a set of desired separations.

$$d_1, d_2, \dots, d_{N-1} \quad (3)$$

with

$$d_i \geq D_{\min}; i = 1, 2, \dots, N-1 \quad (4)$$

which specify the desired distance separations between successive aircraft, near the vicinity of the outer marker. Thus, the number  $d_i$  specifies the desired separation of the  $i$ -th and  $i+1$ -st aircraft at the outer marker. Of course, if no takeoffs are contemplated then

$$d_i = D_{\min} \text{ for all } i \quad (5)$$

### 6.2 The Basic Decision and Control Problem

We now summarize the variables and parameters assumed known to the NTA ATC center and the basic questions that arise.

Given:

- a) The number  $N$  of aircraft in the NTA region that must land

\*of course, this can be accomplished by having the NTA center transmit to the airport center the estimated times of arrival  $T_k$  of the  $N$  aircraft.

- b) The landing order index  $i = 1, 2, \dots, N$
- c) The distances-to-go  $z_1(t), z_2(t), \dots, z_N(t)$
- d) The velocities  $v_1(t), v_2(t), \dots, v_N(t)$
- e) The desired constant speed  $\bar{v}$  (a parameter)
- f) The future desired separations (parameters)  
 $d_1, d_2, \dots, d_{N-1} (d_i \geq D_{\min})$
- g) The minimum separation distance  $D_{\min}$  (a parameter)

Find:

The commands to be transmitted to each aircraft such that

- a) velocity deviations from  $\bar{v}$  are nulled out
- b) The desired separations  $d_i$  are eventually attained.

### 6.3 Constraints

Some position control can be exercised by changing the acceleration and, hence, the velocity of the aircraft along the air route. However, large position changes cannot be accomplished by having the aircraft stay on its assigned air route; in particular, excessive slow down is impossible by severe decelerations since stall velocity constraints must be observed. Hence, only minor position control can be executed by aircraft that stay on the air route. This implies, that in a heavy traffic environment, certain aircraft must execute path-stretching or holding-type maneuvers. These aircraft must then temporarily leave their nominal air route "to waste time".

### 6.4 Basic Questions

In view of the above discussion, it follows that the NTA-ATC center must determine the answers to the following fundamental questions:

1. Which aircraft must stay on their nominal air route and which must leave it?
2. If a particular aircraft stays on its air route, what acceleration/deceleration should it undergo to null out velocity errors and to start correcting now for future separation errors?
3. If a particular aircraft must temporarily leave its air route, what should it do? How are path stretching trajectories obtained so that they do not violate the minimum turn radius constraint? How long does the aircraft have to stay away from its nominal air route? When and where does it return?
4. How can we guarantee that the aircraft are always separated by at least  $D_{\min}$  units of distance?

The approach to the development of a decision and control algorithm is outlined in the following section. We use the theory of optimal control to develop this algorithm. Then we comment on its possible implementation in the current and evolutionary ATC systems.

## VII. THE OPTIMIZATION ALGORITHMS

In this section we develop the basic algorithms that provide in a relatively simple and automated way the answers to the basic questions that would define the strategies to be employed by the NTA-ATC center.

The particular formulation employed has been the outgrowth of more of five years of investigations. Technically, the algorithm is the combination of the solution of three distinct optimization problems. The formulation of the optimization problems has been subjectively selected so as to lead to as simple an implementation as possible. Some of the technical details and derivations can be found in references 3, 4, 5, 6 and 8. In this paper, only the problem formulation, solution statement, and possible implementation is stressed. Needless to say, the validity of both the assumptions and of the formulation can only be determined by simulations and man-machine studies.

The algorithm essentially proceeds in two steps.

### Step 1.

The determination of which aircraft should stay on their nominal air routes and which must leave them is obtained from the solution of a standard "linear-quadratic" (reference 13) optimal control problem coupled together with an optimization of part of the initial state vector. The solution to this problem establishes the ideal motion and positions of all aircraft along their nominal air routes, and provides us with the means of obtaining the desired acceleration/deceleration commands for the aircraft that do stay on their air routes.

## Step 2.

By comparing the actual positions of the aircraft with their desired ones, one can readily deduce which aircraft must undergo severe position corrections and hence must leave the air route. The ideal aircraft motion, determined in Step 1 is used to formulate a minimum-time rendezvous problem. The solution to this time-optimal problem, yields simple (turn and straight line) trajectories to be followed by the aircraft in question.

### 7.1 Linearized Dynamics Along Air Routes

We shall use the landing order index  $i = 1, 2, \dots, N$  to keep track of the aircraft. For each aircraft, we define the following variables:

- $z_i(t)$ : distance-to-go of the  $i$ -th aircraft at time  $t$  to the outer marker
- $v_i(t)$ : instantaneous velocity of the  $i$ -th aircraft at time  $t$ , along the air-route
- $m_i$ : mass of  $i$ -th aircraft (assumed constant)
- $g_i(v_i(t))$ : drag force acting on  $i$ -th aircraft, a nonlinear function of the velocity (effect of altitude is neglected)
- $f_i(t)$ : thrust force applied to  $i$ -th aircraft along air route only
- $u_i(t)$ : net acceleration or deceleration commanded to  $i$ -th aircraft along air route

From the assumed polarity convention on the position and velocity variables (see eq. (11)) we have

$$\frac{d}{dt} z_i(t) = -v_i(t) \quad (6)$$

and from Newton's Law

$$m_i \frac{d}{dt} v_i(t) = -g_i(v_i(t)) + f_i(t) \quad (7)$$

It has been stressed that all aircraft will be controlled such that any speed deviations from the desired constant speed  $\bar{v}$  will be nulled out. So let

$$\delta v_i(t) = v_i(t) - \bar{v} \quad (8)$$

denote the velocity error of the  $i$ -th aircraft.

The drag force  $g_i(v_i(t))$  is the net effect of the natural drag and of any wind that produces a force. The drag at the desired velocity  $\bar{v}$  is simply  $g_i(\bar{v})$ . Hence, a nonzero thrust  $\bar{f}_i \triangleq g_i(\bar{v})$  is required to keep the  $i$ -th aircraft flying at the constant speed  $\bar{v}$ . In order to accelerate,  $f_i(t)$  must be larger than  $\bar{f}_i$ . To decelerate  $f_i(t)$  must be smaller than  $\bar{f}_i$ . We remark that the necessary adjustments to  $\bar{f}_i$  can be made to account for mean wind variations.

By linearizing the drag force at the desired velocity  $\bar{v}$  we can approximate equation (7) by the linear equation

$$m_i \frac{d}{dt} (\bar{v} + \delta v_i(t)) = -g_i(\bar{v}) - \left. \frac{\partial g_i}{\partial v_i} \right|_{\bar{v}} \delta v_i(t) + f_i + m_i u_i(t) \quad (9)$$

where we have introduced the acceleration  $u_i(t)$  by defining

$$f_i(t) \triangleq \bar{f}_i + m_i u_i(t) \quad (10)$$

The quantity  $\left. \frac{\partial g_i}{\partial v_i} \right|_{\bar{v}}$  is simply the slope of the drag force at the velocity  $\bar{v}$ . Figure 3 shows this curve for the KC-135 (military version of the Boeing 707). Note that the drag force is essentially linear in the region  $\bar{v} = 160$  knots over a relatively wide range of speeds. This indicates that the linear equation (9) is an excellent approximation to the nonlinear equation (7) in the vicinity of  $\bar{v}$ .

Since  $\bar{f}_i \triangleq g_i(\bar{v})$ , then equation (9) can be written as

$$\frac{d}{dt} \delta v_i(t) = -\beta_i \delta v_i(t) + u_i(t) \quad (11)$$

where the constant  $\beta_i$  is defined by

$$\beta_i \triangleq \frac{1}{m_i} \left. \frac{\partial g_i}{\partial v_i} \right|_{\bar{v}} \quad (12)$$

Remark: Different jet aircraft (e.g., stretched DC-8 vs. DC-9) are characterized by different values of  $\beta_i$ . However, it appears that for all currently available jets, their corresponding values of  $\beta_i$  are roughly the same. The reason is that larger aircraft have a bigger drag reference area and, hence, larger drag coefficient

$$\left. \frac{\partial g_i}{\partial v_i} \right|_{\bar{v}} \quad (13)$$

However, larger aircraft have also a larger mass  $m_i$ . This is the reason that one would expect

$$\beta_i \approx \bar{\beta} \text{ for all } i = 1, 2, \dots, N \quad (14)$$

If (14) is indeed true, this has a marked influence on the amount of on-line computation that must be carried out. We shall discuss this point in more detail in Section X.

In summary, the motion of each aircraft, indexed by  $i$ , along its nominal air route, is characterized by the pair of linear constant coefficient differential equations

$$\begin{cases} \frac{d}{dt} z_i(t) = -\bar{v} - \delta v_i(t) \\ \frac{d}{dt} \delta v_i(t) = -\beta_i \delta v_i(t) + u_i(t) \end{cases} \quad (15)$$

Consider the  $i$ -th and  $i+1$ -st aircraft. As before we denote their current distance-to-go by  $z_i(t)$  and  $z_{i+1}(t)$ . Recall that these two aircraft should be separated by a specified distance  $d_i$  in the vicinity of the outer marker.

We define the current separation distance error  $\delta z_i(t)$  as follows:

$$\delta z_i(t) \triangleq z_{i+1}(t) - z_i(t) - d_i \quad (16)$$

The value  $\delta z_i(t)$  represents, at the present time  $t$ , the deviation of the  $i$ -th and  $i+1$ -st aircraft from their eventual desired separation  $d_i$ . If  $\delta z_i(t) > 0$ , this means that the aircraft are "too far apart" while if  $\delta z_i(t) < 0$ , then the aircraft are "too close together". We remark that these errors do not pertain to the actual current separation of the aircraft in the NTA region. Rather, it is the current error, from a future desired value, measured along the nominal air routes (at the present time, the two aircraft may be in completely different air routes as illustrated in Figure 4).

The advantage of having a current value for the separation distance error,  $\delta z_i(t)$ , is that appropriate corrective action can be taken early, so that by the time the aircraft are indeed near the outer marker, then they will be separated by the desired distance  $d_i$ .

From eqs. (16) and (15) we can compute the time rate of change of the position errors

$$\frac{d}{dt} \delta z_i(t) = \delta v_i(t) - \delta v_{i+1}(t) \quad (17)$$

since  $\bar{v}$  and  $d_i$  are constants. Eq. (17) shows the way that velocity errors contribute to the time rate of change of position errors for the  $i$ -th and  $i+1$ -st aircraft.

### 7.3 The System Error Dynamics

We have now defined a position error  $\delta z_i(t)$  for each pair of aircraft, and a velocity error  $\delta v_i(t)$  for each and every aircraft. For any given set of accelerations  $u_i(t)$ ,  $i = 1, 2, \dots, N$ , we can compute the dynamic propagation of the position and velocity errors by means of the following set of  $2N-1$  simultaneous differential equations

$$\left. \begin{aligned} \frac{d}{dt} \delta z_1(t) &= \delta v_1(t) - \delta v_2(t) \\ \frac{d}{dt} \delta z_2(t) &= \delta v_2(t) - \delta v_3(t) \\ &\dots\dots\dots \\ \frac{d}{dt} \delta z_{N-1}(t) &= \delta v_{N-1}(t) - \delta v_N(t) \end{aligned} \right\} \text{(position error equations)}$$

$$\left. \begin{aligned} \frac{d}{dt} \delta v_1(t) &= -\beta_1 \delta v_1(t) + u_1(t) \\ \frac{d}{dt} \delta v_2(t) &= -\beta_2 \delta v_2(t) + u_2(t) \\ &\dots\dots\dots \\ \frac{d}{dt} \delta v_N(t) &= -\beta_N \delta v_N(t) + u_N(t) \end{aligned} \right\} \text{(velocity error equations)} \quad (18)$$

This set of equations can be written in vector form as follows. Define the position error vector  $\delta \underline{z}(t)$ , the velocity error vector  $\delta \underline{v}(t)$  and the acceleration vector  $\underline{u}(t)$  by:

$$\delta \underline{z}(t) \triangleq \begin{bmatrix} \delta z_1(t) \\ \delta z_2(t) \\ \dots \\ \delta z_{N-1}(t) \end{bmatrix}; \quad \delta \underline{v}(t) \triangleq \begin{bmatrix} \delta v_1(t) \\ \delta v_2(t) \\ \dots \\ \delta v_N(t) \end{bmatrix}; \quad \underline{u}(t) \triangleq \begin{bmatrix} u_1(t) \\ u_2(t) \\ \dots \\ u_N(t) \end{bmatrix} \quad (19)$$

Also define the constant matrices

$$\underline{A}_{12} \triangleq \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix} \quad (N-1) \times N \text{ matrix} \quad (20)$$

$$\underline{A}_{22} \triangleq \begin{bmatrix} -\beta_1 & 0 & \dots & 0 \\ 0 & -\beta_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -\beta_N \end{bmatrix} \quad N \times N \text{ matrix} \quad (21)$$

Using this notation the set of equations (18) can be written in vector form as follows:

$$\frac{d}{dt} \begin{bmatrix} \delta \underline{z}(t) \\ \delta \underline{v}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{0} & \underline{A}_{12} \\ \underline{0} & \underline{A}_{22} \end{bmatrix}}_{\underline{A}} \begin{bmatrix} \delta \underline{z}(t) \\ \delta \underline{v}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \underline{0} \\ \underline{I} \end{bmatrix}}_{\underline{B}} \underline{u}(t) \quad (22)$$

which is of the standard state variable form

$$\dot{\underline{x}}(t) = \underline{A} \underline{x}(t) + \underline{B} \underline{u}(t) \quad (23)$$

The above development indicates that the propagation of the position and velocity errors as a function of the accelerations, is governed by linear time-invariant differential equations.

#### 7.4 The System Quadratic Index of Performance

Let us assume for the time being that all aircraft are constrained to move along their nominal air routes. Thus, we assume that all position and velocity errors are small. This means that all aircraft at the present time  $t$  are flying with speeds near  $\bar{V}$  and that they are almost properly separated. Then by adjusting the components of the acceleration vector  $\underline{u}(t)$ , we can expect to be able to make the required minor corrections in speed and position so that the position error vector  $\delta \underline{z}(t)$  and velocity error vector  $\delta \underline{v}(t)$ ,  $t > t$ , are almost zero; this can be accomplished without leaving the air routes.

An extremely simple and coordinated way of deducing the required acceleration vector  $\underline{u}(t)$  as a function of the actual errors  $\delta \underline{z}(t)$  and  $\delta \underline{v}(t)$ , at each and every instant of time, is by solving for  $\underline{u}(t)$  as the solution of a standard linear-quadratic optimal control problem (reference [7], Chapter 9 or reference [3]) over an infinite time interval. The use of linear quadratic problems for small error regulation has been proven successful in many applications. Its main advantage from an implementation viewpoint is that the acceleration vector  $\underline{u}(t)$  can be determined using constant gain feedback from the measured error vectors  $\delta \underline{z}(t)$  and  $\delta \underline{v}(t)$ .

The standard quadratic cost functional,  $J$ , penalizes the system for all position and velocity errors, and for excessive use of accelerations by the following scalar index of performance

$$J = \int_t^{\infty} \left[ \sum_{i=1}^{N-1} q_i \delta z_i^2(\tau) + \sum_{i=1}^N p_i \delta v_i^2(\tau) + \sum_{i=1}^N u_i^2(\tau) \right] d\tau \quad (24)$$

where  $t$  is the present time and the constant weightings  $q_i$  and  $p_i$  are all positive.

If we define the weighting matrices

$$\underline{Q} \triangleq \begin{bmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & q_{N-1} \end{bmatrix}, \quad \underline{P} \triangleq \begin{bmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & p_N \end{bmatrix}, \quad \underline{R} \triangleq \begin{bmatrix} \underline{\tilde{Q}} & \underline{\tilde{Q}} & \underline{\tilde{Q}} \\ \underline{\tilde{Q}} & \underline{\tilde{P}} & \underline{\tilde{P}} \\ \underline{\tilde{Q}} & \underline{\tilde{P}} & \underline{\tilde{P}} \end{bmatrix} \quad (25)$$

then the quadratic cost functional  $J$  can be written as

$$J = \int_t^{\infty} \{ \delta \underline{z}'(\tau) \underline{Q} \delta \underline{z}(\tau) + \delta \underline{v}'(\tau) \underline{P} \delta \underline{v}(\tau) + \underline{u}'(\tau) \underline{R} \underline{u}(\tau) \} d\tau \quad (26)$$

## 7.5 The Optimal Control Problem and its Solution

The precise statement of the optimal control problem is:

Given the system (22) and the cost functional (26). Find

$\underline{u}(\tau)$ ,  $t \leq \tau < \infty$ , such that  $J$  is minimized.

It is well known (see, for example, reference [7], Chapter 9) that the solution to this linear-quadratic optimal control problem is obtained as follows:

Step 1: Let  $\underline{K}$  be a  $(2N-1) \times (2N-1)$  constant matrix. Form the algebraic matrix Riccati equation

$$\underline{0} = \underline{K} \underline{A} + \underline{A}' \underline{K} + \underline{Q} - \underline{K} \underline{B} \underline{B}' \underline{K} \quad (27)$$

where  $\underline{A}$  and  $\underline{B}$  are defined in eq. (22) and  $\underline{Q}$  in eq. (25).

Step 2: Determine the (unique) symmetric positive definite solution matrix of (27) using one of the standard numerical techniques (see, for example, reference [11]).

Next, decompose the  $(2N-1) \times (2N-1)$  solution matrix  $\underline{K}$  into

$$\underline{K} = \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} \\ \underline{K}_{12}' & \underline{K}_{22} \end{bmatrix}; \quad \underline{K}_{11} = (N-1) \times (N-1); \quad \underline{K}_{22} = N \times N \quad (28)$$

Then, the optimal acceleration vector at any time  $\tau \geq t$  is given by

$$\underline{u}(\tau) = - \underline{K}_{12}' \delta \underline{z}(\tau) - \underline{K}_{22} \delta \underline{v}(\tau) \quad (29)$$

where  $\delta \underline{z}(\tau)$  and  $\delta \underline{v}(\tau)$  are the actual (measured) position and velocity error vectors at time  $\tau$ .

This scheme provides us with the means for computing acceleration commands for the aircraft provided the system errors are "small" so that it is not necessary for any aircraft to leave its nominal air route. We shall see Section 8 what happens if the errors are large so that path stretching maneuvers are required.

## 7.6 The Minimum Value of the Quadratic Cost

Another key property of the linear-quadratic optimal control problem is that one can readily compute the value of  $J$  in eq. (26), provided that the optimal acceleration (29) is utilized. It is well known that the minimum value of  $J$  at time  $t$  (present) is given by

$$J = \delta \underline{z}'(t) \underline{K}_{11} \delta \underline{z}(t) + 2 \delta \underline{z}'(t) \underline{K}_{12} \delta \underline{v}(t) + \delta \underline{v}'(t) \underline{K}_{22} \delta \underline{v}(t) \quad (30)$$

where  $\delta \underline{z}(t)$  and  $\delta \underline{v}(t)$  are the present position and velocity error vectors, and the matrices  $\underline{K}_{11}$ ,  $\underline{K}_{12}$ ,  $\underline{K}_{22}$  are defined by eq. (28).

The quadratic nature of the minimum cost (30) is essential for the simple development of the coordination strategy of the proposed ATC algorithms.

## VIII. IDEAL AIRCRAFT LOCATIONS AND MOTIONS ALONG AIR ROUTES

In this section we shall use the results of the solution of the linear-quadratic optimization problem and start the development of the algorithm that will culminate in the overall means of scheduling and coordinating all the aircraft.

The basic question that will be answered in this section is:

Under heavy traffic conditions, which aircraft must leave their nominal air route for path-deviating maneuvers?

### 8.1 The Importance of the Minimum Value of the Cost

Let us examine carefully the equation of the minimum cost  $J$  given by eq. (30). Since the  $K$  matrix is positive definite, the value of  $J$  will always be non negative; in fact,

$$J = 0 \text{ if and only if } \underline{\delta z}(t) = \underline{0}, \underline{\delta v}(t) = \underline{0} \quad (31)$$

which can only happen if at the present time all aircraft travel at exactly  $\bar{v}$  ( $\underline{\delta v}(t) = \underline{0}$ ) and are properly separated ( $\underline{\delta z}(t) = \underline{0}$ ). Under these conditions the optimal acceleration is (see eq. (29))

$$\underline{u}(t) = \underline{0} \quad (32)$$

and this results in

$$\underline{u}(\tau) = \underline{0}, \underline{\delta z}(\tau) = \underline{0}, \underline{\delta v}(\tau) = \underline{0} \text{ for all } \tau \geq t \quad (33)$$

Thus, all aircraft should continue to travel along their air routes at  $\bar{v}$  and, as they pass over the outer marker, they will be separated exactly by the desired separations  $d_1, d_2, \dots, d_{N-1}$ .

In general, the value of the positive number  $J$  in eq. (22) is a measure of the degree of system uncoordination. For example, suppose that all aircraft at time  $t$  travel at  $\bar{v}$  (i.e.,  $\underline{\delta v}(t) = \underline{0}$ ) but suppose that due to heavy takeoff demands all the position errors  $\delta z_i(t)$  are negative, which means that all aircraft are too close to each other (see Section 7.2). The effect of this is that  $J$  is a large number, since when  $\underline{\delta v}(t) = \underline{0}$  then  $J = \underline{\delta z}'(t) K_{11} \underline{\delta z}(t)$ . The larger in magnitude the position errors, the larger the value of the quadratic cost  $J$ . In this situation, what should have happened is that the 2nd, 3rd, ...,  $N$ th aircraft should have been further back along their respective nominal air routes.

### 8.2 Ideal Aircraft Position Error Vector

This train of thought leads to the following question:

Suppose that one could arbitrarily place each aircraft at some point on their nominal air route at time  $t$ ; where should each aircraft be placed?

For each aircraft indexed by  $i$ , we have its actual distance-to-go  $z_i(t)$  at the present time  $t$ . We define  $z_i^*(t)$  to be the ideal distance-to-go along the air route. We can then define (compare with eq. (16)), the ideal position error  $\delta z_i^*(t)$  between the  $i$ -th and  $i+1$ -st aircraft by

$$\delta z_i^*(t) \triangleq z_{i+1}^*(t) - z_i^*(t) - d_i$$

and the ideal position error vector  $\underline{\delta z}^*(t)$

$$\underline{\delta z}^*(t) \triangleq \begin{bmatrix} \delta z_1^*(t) \\ \delta z_2^*(t) \\ \dots \\ \delta z_{N-1}^*(t) \end{bmatrix} \quad (35)$$

Once more suppose that  $\underline{\delta v}(t) = \underline{0}$  (no speed whatsoever). Then, the ideal position error vector  $\underline{\delta z}^*(t)$  is clearly  $\underline{\delta z}^*(t) = \underline{0}$ , because from time  $t$  on the aircraft would continue to travel at  $\bar{v}$  and end-up properly separated. Note that, if  $\underline{\delta v}(t) = \underline{0}$ , then  $\underline{\delta z}^*(t) = \underline{0}$  corresponds to the absolute minimum (zero) of the cost functional (22).

However, the determination of the ideal position error vector  $\underline{\delta z}^*(t)$  is not as easy when  $\underline{\delta v}(t) \neq \underline{0}$  (some aircraft go too fast, others too slow). In this case, the ideal position of each aircraft depends on the velocity errors since it would take some time to correct these velocity errors.

What we seek is a systematic and simple way to determine the ideal position error vector  $\underline{\delta z}^*(t)$ . In the proposed algorithm the following philosophy is adopted:

Suppose we are given  $\underline{\delta v}(t)$ . Since the magnitude of  $J$  in Eq. (22) is a measure of the degree of system uncoordination,

define  $\underline{\delta z}^*(t)$  to be the vector that leads to the smallest possible value of  $J$ .

In other words,  $\underline{\delta z}^*(t)$  is defined by

$$\begin{aligned} & \underline{\delta z}^{*'}(t) K_{11} \underline{\delta z}^*(t) + 2 \underline{\delta z}^{*'}(t) K_{12} \underline{\delta v}(t) + \underline{\delta v}^{*'}(t) K_{22} \underline{\delta v}(t) \\ & \leq \underline{\delta z}'(t) K_{11} \underline{\delta z}(t) + 2 \underline{\delta z}'(t) K_{12} \underline{\delta v}(t) + \underline{\delta v}'(t) K_{22} \underline{\delta v}(t) \end{aligned} \quad (36)$$

Clearly, we can find  $\underline{\delta z}^*(t)$  as the root of the vector equation

$$\underline{0} = \frac{\partial J}{\partial \underline{\delta z}(t)} = K_{11} \underline{\delta z}(t) + K_{12}' \underline{\delta v}(t) \quad (37)$$

or

$$\underline{\delta z}^*(t) = -K_{11}^{-1} K_{12}' \underline{\delta v}(t) \quad (38)$$

#### Remarks

- (a)  $K_{11}^{-1}$  exists, because  $K$  is positive definite
- (b) when  $\underline{\delta v}(t) = \underline{0}$ , then  $\underline{\delta z}^*(t) = \underline{0}$  as commented above, and as is intuitively obvious.
- (c) The ideal position error vector  $\underline{\delta z}^*(t)$  depends on all velocity error vectors.
- (d) The on-line computational requirements of (38) are small once  $K_{11}^{-1}$  and  $K_{12}'$  have been computed.

### 8.3 Computation of Ideal Plane Locations

Equation (38) defines the ideal position error vector  $\underline{\delta z}^*(t)$  given the actual velocity error vector  $\underline{\delta v}(t)$ . This is not enough to specify the desired aircraft locations  $\underline{z}_i^*(t)$  on their air route.

In case of relatively large velocity mismatch one would expect that some aircraft should have been "ahead of themselves" while others "in back of themselves". For the planes that should have been "ahead of themselves" very little can be done if one wishes to avoid severe accelerations, decelerations, and speed changes or cutting across air routes. On the other hand, if a plane should be "in back of itself", then such a plane can temporarily waste time by leaving the air route for an appropriate time and their returning to it (how this is done exactly will be discussed in Section 9).

This philosophy can be used to develop the following algorithm which defines which planes can stay on their nominal air route and which must leave it to waste time.

#### Planes 1 and 2 (i=1)

Let  $i=1$ . Consider the lead plane and its true distance-to-go  $z_1(t)$ . We assume that the first plane to land should not undergo any path-stretching maneuvers so we set

$$\hat{z}_1^*(t) = z_1(t) \quad (39)$$

(i.e., the desired location of the lead plane coincides with its actual current location).

Now consider the second plane (indexed by  $i=2$ ). Given  $d_1$ ,  $\hat{z}_1^*(t)$  from eq. (39), and  $\underline{\delta z}_1^*(t)$  from eq. (38), we can see that eq. (34) yields

$$z_2^*(t) = \underline{\delta z}_1^*(t) + \hat{z}_1^*(t) + d_1 \quad (40)$$

Note that  $z_2^*(t)$  is the ideal distance-to-go for the second-in-line plane. Comparing it with its actual distance-to-go,  $z_2(t)$  we compute

$$x_2(t) \triangleq z_2^*(t) - z_2(t) \quad (41)$$

Case 1: If  $x_2(t) \leq 0$ , this means that the second aircraft should have been ahead of itself; in this case, there is no reason for it to leave its nominal air-route because we cannot possibly place it at  $z_2^*(t)$ . So we set  $\hat{z}_2^*(t) = z_2(t)$  and we proceed with investigating planes 2 and 3.

Case 2: If  $x_2(t) > 0$ , this means that the second plane should have been behind itself. In this case, plane 2 is commanded to leave its nominal air route, and its desired location  $\hat{z}_2^*(t)$  is that found in (40), i.e.,

$$\hat{z}_2^*(t) = z_2^*(t) \quad (42)$$

In either case, we have a value for  $\hat{z}_2^*(t)$  and we proceed to examine plane 3.

#### Planes 2 and 3 ( $i=2$ )

We have  $\hat{z}_2^*(t)$  found above;  $d_2$ , the desired separation between planes 2 and 3; and,  $\delta z_2^*(t)$  from eq. (38). From Eq. (34) we find the ideal location  $z_3^*(t)$  of the 3rd plane.

$$z_3^*(t) = \delta z_2^*(t) + \hat{z}_2^*(t) + d_2 \quad (43)$$

From  $z_3^*(t)$  and the actual distance-to-go  $z_3(t)$  we compute

$$x_3(t) \triangleq z_3^*(t) - z_3(t) \quad (44)$$

Case 1: If  $x_3(t) \leq 0$ , then plane 3 stays in its air route and we set  $\hat{z}_3^*(t) = z_3(t)$ .

Case 2: If  $x_3(t)$ , plane 3 should be behind itself; it is commanded to leave its air route and its ideal position  $\hat{z}_3^*(t)$  is that of eq. (42), i.e.,

$$\hat{z}_3^*(t) = z_3^*(t) \quad (45)$$

This algorithm can be used until all  $N$  aircraft have been tested. Figure 5 shows this part of the algorithm. This simple algorithm finds

- (a) which planes stay on their air route  
(when  $x_i(t) = z_i^*(t) - z_i(t) \leq 0$ )
- (b) which planes must leave the air route  
(when  $x_i(t) = z_i^*(t) - z_i(t) > 0$ )

#### 8.4 Computation of Ideal Motion along Air Routes

The preceding algorithm has established the desired location of each and every aircraft, as defined by the desired distance-to-go  $\hat{z}_i^*(t)$ , as the present time  $t$ . The subsequent desired motion of the aircraft for  $t \geq t$  can also be found by using the optimal acceleration command  $\underline{u}(t)$ , determined in Section 7.5 from the solution of the linear-quadratic optimal control problem.

If we substitute eq. (29) into eq. (22) we find that the position and velocity error vectors,  $\delta z(t)$  and  $\delta v(t)$ , are defined by means of the closed-loop differential equation.

$$\frac{d}{dt} \begin{bmatrix} \delta z(t) \\ \delta v(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I & A_{12} \\ -K'_{12} & I & A_{22} - K_{22} \end{bmatrix}}_{A_{c1}} \begin{bmatrix} \delta z(t) \\ \delta v(t) \end{bmatrix} \quad (46)$$

Let  $A_{c1}$  denote the  $(2N-1) \times (2N-1)$  closed loop system matrix defined by (46); note that it is a constant matrix.

The algorithm of Section 8.3 has defined an ideal distance-to-go for each aircraft,  $\hat{z}_i^*(t)$ . This can be used to compute a desired position error between aircraft  $i$  and  $i+1$  by

$$\delta \hat{z}_i^*(t) = \hat{z}_{i+1}^*(t) - z_i^*(t) - d_i \quad (47)$$

and hence, a desired position error vector  $\delta z^*(t)$ .

The desired motion of the aircraft along their air routes can now be found from the solution of eq. (46) for any time  $t \geq t$

$$\begin{bmatrix} \delta \hat{z}^*(\tau) \\ \delta \hat{v}^*(\tau) \end{bmatrix} = e^{A_{c1}(\tau-t)} \begin{bmatrix} \delta z^*(t) \\ \delta v(t) \end{bmatrix} ; \tau \geq t \quad (48)$$

Knowledge of  $\delta \hat{z}^*(\tau)$  can be once more translated into distance-to-go information. The overall computation is indicated in the flow chart of Figure 6.

#### 8.5 Possible Utilization of the Results up to Now

These results can be utilized in the current ATC system in the following way. (At the present time, the ground controller observes the actual motion of the aircraft on his radarscope.)

A digital computer program can be devised using the information contained in the flow charts of Figures 5 and 6. The outcome of the digital computer program can be used to superimpose on the current

radarscope display (perhaps using a different color)

- a) The desired location of each aircraft at the present time  $t$ , from knowledge of the desired distances to go  $\hat{z}_i^*(t)$
- b) The desired future motion of each aircraft at any time  $\tau \geq t$  in the future, from knowledge of the  $\hat{z}_i^*(\tau)$  distances.

The human ground controller can then monitor the actual and desired motion of the aircraft along the air routes simultaneously. He can transmit appropriate voice commands to the pilots so that their actual motion eventually coincides with the desired motion. We shall see in Section 9, how this task can be aided for the aircraft that should have been behind themselves and which, therefore, must undergo path stretching or holding type maneuvers.

This computer-aided decision system is also compatible with a possible evolutionary ATC system currently under investigation at M.I.T. (see for example ref. 9). In this envisioned system, the current weather radar display will be modified so that one can transmit from the ground, and display to the pilot, the location of his aircraft and of the neighboring ones. In this manner, it is hoped that each pilot can do his "own air traffic control". In such a system, one can transmit to the pilot not only the actual situation but also the desired situation. Each pilot could be then given the responsibility of matching the motion of his own aircraft to the desired one and executing these maneuvers in a safe manner. Figure 7 illustrates the type of display information that can be provided. In addition, fast prediction capability can be used to indicate where each aircraft should be in a fixed time in the future (e.g., 30 second prediction).

Additional remarks about the computational requirements will be given at the end of the paper.

#### IX. DETERMINATION OF PATH STRETCHING/HOLDING TRAJECTORIES

The algorithm presented up to now simply determines the desired locations of the aircraft along the nominal air routes. As discussed before, especially in heavy traffic situations, it is reasonable to expect that many aircraft should have been further back along their nominal air routes. In view of velocity limitations, these aircraft then must temporarily leave their air route, waste time by some sort of maneuver, and then reenter their nominal air route at some future time when it is possible that their actual motion coincides with the desired one. (see Figure 8)

This problem can be viewed as requiring each aircraft to "rendezvous" (more or less) with its desired moving location along the nominal air route. Although one can rely on the ground controller and/or pilot to figure out the appropriate trajectories to accomplish this rendezvous, it is clear that any computer aided decision rules and suggestions will be welcomed for the execution of these tasks.

This section presents such an algorithm that automatically generates nominal time-wasting trajectories. Once more these trajectories can be displayed on the ground ATC display and, perhaps, on future airborne displays.

##### 9.1 Philosophy

Consider an aircraft whose actual distance-to-go,  $z(t)$ , is smaller than its desired distance-to-go,  $\hat{z}^*(t)$ , so that (see Section 8.3)

$$\begin{aligned} x(t) &\triangleq \hat{z}^*(t) - z(t) > 0 \\ \hat{z}^*(t) &= z^*(t) \end{aligned} \quad (49)$$

which means that this aircraft must temporarily leave its nominal air route.

As discussed before, this aircraft must return to the nominal air route at some future time  $\tau \geq t$ . At that time, it is necessary to have

$$\hat{z}^*(\tau) = z(\tau) \quad (50)$$

which implies that the actual location of the aircraft coincides with its desired location. The key question is: how this can be accomplished in a manner that the required trajectories are easy to implement in real time, and consistent with the constraints on the motion of the aircraft.

The first assumption that one can make is that:

The rendezvous should be accomplished in minimum time

One can defend this assumption on the grounds that:

- (a) Nominal air routes are often established because the pilot has the most accurate navigation information; hence, minimizing the time spent away from a nominal air route is satisfactory from the navigation accuracy viewpoint.
- (b) In the vicinity of an airport, pilots dislike the execution of too many maneuvers unless absolutely necessary; for any given traffic conditions, knowledge that the time required to execute path stretching trajectories is minimum can be comforting to the pilot and passengers, especially in bad weather conditions.

The second point to be kept in mind is that this problem should only reduce nominal flight paths to aid the human; hence, exact rendezvous is not absolutely necessary. As long as the aircraft returns to the air route in the vicinity of its desired location, one can rely on the ability of the feedback system to make a certain amount of corrections along the air route. This philosophy, and the desire to obtain a simple algorithm lead us to the second assumption:

Both the aircraft and its desired moving image are moving at the constant speed  $\bar{v}$  (recall that all aircraft will be controlled to move at  $\bar{v}$ .)

In practice, velocities will be somewhat different than  $\bar{v}$ . This will cause some mismatch in the rendezvous (some aircraft will return slightly ahead and some slightly behind their desired location on the nominal air route.)

Even under these assumptions, there are many questions that remain:

1. How long should the aircraft stay away from its nominal air route (What is the minimum value of  $t-t^*$ ?)
2. At what location does the aircraft return to its nominal air route?
3. How are radius-of-turn (maneuverability) limitations taken into account?
4. Are the path stretching or holding trajectories easy to fly?
5. Are the turning commands easy to understand and implement?

All these questions will be answered by solving a minimum time nonlinear rendezvous problem.

## 9.2 Flight Dynamics

We shall assume that path stretching maneuvers will be executed at a plane defined by the nominal air route. Figure 9 shows the suggested coordinate system; the negative x-axis coincides with the air route and the y-axis is along the local "horizontal" plane defined by the air route.

The motion of the aircraft in the x-y plane is modelled as the motion of a constant speed ( $\bar{v}$ ) particle. It changes direction by controlling the rate of change of the heading angle  $\phi(t)$ . In problems of this type one can ignore the short term aircraft dynamics and use the following set of equations to describe the aircraft motion; (these equations have been used many times in a variety of aerospace applications to obtain nominal trajectories):

$$\begin{aligned}\dot{x}(t) &= -\bar{v} \cos\phi(t) \\ \dot{y}(t) &= \bar{v} \sin\phi(t) \\ \dot{\phi}(t) &= w(t)\end{aligned}\tag{51}$$

Thus,  $z(t)$ ,  $y(t)$ , and  $\phi(t)$  are the state variables,  $w(t)$  the control variable, and  $\bar{v}$  a constant parameter.

We assume that (near  $\bar{v}$ ) the aircraft maneuverability is constrained by a minimum turn radius  $R_{\min}$ . This can then be expressed as the following constraint on the control variable  $w(t)$  in eq. (51):

$$-\frac{\bar{v}}{R_{\min}} \leq w(t) \leq \frac{\bar{v}}{R_{\min}}\tag{52}$$

## 9.3 Formulation of the Minimum Time Rendezvous Problem

At the initial time,  $t$  (= present time), the aircraft flies along its nominal air route hence, the initial conditions are

$$\begin{aligned}y(t) &= 0 \\ \phi(t) &= 0\end{aligned}\tag{53}$$

and  $z(t)$  is the actual distance-to-go.

Also at the initial time, we know the desired distance-to-go  $z^*(t) > z(t)$ . The desired motion of the aircraft is along the air route. Our assumption that the desired motion is with a speed  $\bar{v}$  allows us to write

$$\hat{z}^*(\tau) = \hat{z}^*(t) - \bar{v}(\tau-t); \tau > t\tag{54}$$

Hence the precise formulation of the minimum time problem is:

Find the control  $w(\sigma)$ ,  $t \leq \sigma \leq \tau^*$ , that satisfies the constraints (52), such that

$$\left. \begin{aligned} z(\tau^*) &= \hat{z}^*(\tau^*) \\ y(\tau^*) &= 0 \\ \phi(\tau^*) &= 0 \\ \text{and} \\ \tau^* &= \text{minimum} \end{aligned} \right\} \quad (55)$$

#### 9.4 Solution of the Minimum Time Rendezvous Problem

The above optimization problem can be readily solved using the maximum principle. The details of the derivation can be found in the thesis by Porter, reference [8]. Only the solution to the problem is included here.

The optimal trajectories are made up of

- (a) straight lines and/or
- (b) circles whose radius is the minimum turning radius,  $R_{\min}$

Hence, the time-optimal rendezvous trajectories are extremely simple and easy to execute as far as a pilot is concerned.

There are three types of trajectories that arise in this problem depending on the difference  $x(t)$  between ideal distance-to-go of the aircraft  $\hat{z}^*(t)$  (found in Section 8.3) and its actual distance-to-go  $z(t)$  at the present time  $t$ , i.e.,

$$x(t) = \hat{z}^*(t) - z(t) \quad (56)$$

##### Case I: Oscillation Type Maneuver

This case occurs when

$$0 < x(t) < 2\pi R_{\min} \quad (57)$$

which occurs when the ideal location of the aircraft is not too far behind itself. In this case, the optimal strategy is determined as follows. From the actual value of the distance  $x(t)$  find the angle  $\alpha$ ,  $0 \leq \alpha \leq 180^\circ$ , which is the root of the equation

$$\alpha - \sin \alpha = \frac{x(t)}{4R_{\min}} \quad (58)$$

Then, the required path stretching strategy is

- (a) turn hard left for  $\alpha$  degrees (or  $\alpha R_{\min}/\sqrt{v}$  seconds), then
- (b) turn hard right for  $2\alpha$  degrees (or  $2\alpha R_{\min}/\sqrt{v}$  seconds), then
- (c) turn hard left for  $\alpha$  degrees (or  $\alpha R_{\min}/\sqrt{v}$  seconds).

The type of the resultant trajectory is shown in Figure 10; it consists of three circular arcs executed at the minimum turn radius (equally well one could execute a right-left-right turn policy).

In this type of maneuver the (minimum) time  $\tau^*$  at which the aircraft returns to its air route is given by

$$\tau^* = t + \frac{4\alpha R_{\min}}{\sqrt{v}} \text{ seconds} \quad (59)$$

where  $\alpha$  is the solution of eq. (58).

##### Case II: Circular Maneuver

This case occurs when

$$x(t) = 2\pi R_{\min} \quad (60)$$

The optimal strategy in this case for the aircraft to undergo a  $360^\circ$  degree left (or right) hard turn at the minimum turn radius. The (minimum) time  $\tau^*$  at which the maneuver is completed is given by

$$\tau^* = \frac{2\pi R_{\min}}{\sqrt{v}} \text{ seconds} \quad (61)$$

Figure 11 shows this type of trajectory.

### Case III: Fly-Around Maneuver

This case occurs when

$$x(t) > 2\pi R_{\min} \quad (62)$$

i.e., when the aircraft should be significantly behind itself. In this case, the optimal strategy is

- turn hard left for  $180^\circ$  degrees (or  $\pi R_{\min}/\bar{v}$  seconds), then
- fly straight for a distance of  $x(t) - 2\pi R_{\min}$  (antiparallel to the air route) or for a time  $(x(t) - 2\pi R_{\min})/2\bar{v}$  seconds, then
- turn hard left for  $180^\circ$  degrees (or  $\pi R_{\min}/\bar{v}$  seconds)

This holding-type trajectory is illustrated in Figure 12. The maneuver is finished at the (minimum) time of

$$t^* = t + \frac{2\pi R_{\min}}{\bar{v}} + \frac{x(t) - 2\pi R_{\min}}{2\bar{v}} \quad (63)$$

Of course, if the distance that should be flown antiparallel to the air route is very large one can execute one of more circular maneuvers, provided that no aircraft under emergency status utilizes the same portion of the air route during the maneuver.

Figure 13 summarizes this information for a KC-135 at 160 knots. The minimum turn radius is

$$R_{\min} = \frac{5.333}{2\pi} \text{ N.M.} \quad (64)$$

which corresponds to a 2 minute  $360^\circ$  turn. It is interesting to note the discontinuity of the minimum required maneuver time at the critical distance  $2\pi R_{\min}$  (it takes less time if the error in desired location is 6 N.M. than say 5 N.M.)

### 9.5 Discussion

These exceedingly simple maneuvers can of course be computed extremely rapidly by a digital computer algorithm. This is illustrated by the flow diagram of Figure 14.

The commands and suggested trajectories can be introduced to the display of the ground controller, once more to aid his decision making. He can then communicate to the pilot the simple turn-turn-turn or turn straight-turn commands. In the possible evolutionary ATC system, these commands and trajectories can also be transmitted to a display in the cockpit. Eventually, all of these commands can be introduced directly to the aircraft autopilot.

### X. FURTHER DISCUSSION

The essential computational algorithm has been presented in the preceding two sections. In this section we elaborate upon its usage, updating requirements, as well as the type of research that is still needed, before these ideas can be fully implemented.

#### 10.1 The Choice of the Weighting Constants

In Equation (24), the quadratic cost functional  $J$  contains the positive constants  $q_1, q_2, \dots, q_{N-1}$ , which are used to penalize position errors, and  $p_1, p_2, \dots, p_{N-1}$ , which are used to penalize for velocity errors. In general, velocity errors should be penalized much more heavily than position errors so that the essential assumption that all planes fly at  $\bar{v}$  is justified, and since severe position errors can be alleviated by path stretching maneuvers.

It is suggested that the same weight should be assigned to all velocity errors, i.e., one should use

$$p_1 = p_2 = \dots = p_{N-1} = p \quad (65)$$

By the same token, one could use the same weight to all position errors. However, this is not advisable. In general position errors are more critical for the planes that are to land earlier than others, which means that they are indexed by small values of the landing index  $i$ . Thus, the following relation should be used in the selection of the  $q_i$ 's:

$$q_1 \geq q_2 \geq \dots \geq q_{N-1} \quad (66)$$

Since velocity errors are the most important ones we should also have

$$p > q_1 \quad (67)$$

(A ratio of 20 to 1 is suggested).

Of course all these suggestions are tentative. Research is underway to determine the proper value of these constants by conducting man-machine simulation experiments; in essence, these numbers should be determined by the ability of the human pilot to be able to follow the suggested motion.

## 10.2 Computation of the Riccati Solution Matrix

Once the weights  $p_i$  and  $q_i$  have been selected, the matrix  $Q$  in Eq. (27) is determined. The  $B$  matrix, see eq. (22), depends only on the number  $N$  of the aircraft. The  $A$  matrix, see eqs. (22), (20), and (21), depends on the number  $N$  of the aircraft and on the physical identity of the jets. The reason is that the  $A_{22}$  submatrix in Eq. (21) depends on the drag coefficient-to-mass ratio  $\hat{B}_i$ . If indeed the  $\hat{B}_i$ 's are drastically different in value, then the Riccati equation must be solved again

- (a) every time a new aircraft enters the NTA region, or
- (b) every time an aircraft crosses the outer marker

On the other hand, if assumption (14) is true, which means that all the  $\hat{B}_i$  are essentially the same, then the  $A$  and  $B$  matrices depend only on the number  $N$  of aircraft and not on their physical identity.

In this case, one can precompute and store the solution of the Riccati equation (27) for all reasonable values of  $N$  that one may anticipate. Hence, at each instant of time by counting the aircraft one can pull out of the computer memory or tape the appropriate  $K$  matrix.

In fact, one can precompute and store:

- (a) The key matrix  $-K_{11}^{-1} K_{12}'$  that according to eq. (38) defines the ideal position error vector  $\hat{z}^*(t)$  in terms of the actual velocity error  $\delta v(t)$
- (b) The gain matrices  $-K_{12}'$  and  $K_{22}$  that generate the optimal acceleration vector  $u(t)$  according to eq. (29).
- (c) The closed loop matrix exponential  $e^{A_{cl}(\tau-t)}$  for several values of the prediction time  $\tau-t$  (e.g.,  $\tau-t = 15$  secs.,  $\tau-t = 30$  secs., etc.)

Under these assumptions then, the most time consuming digital computer calculations do not have to be done on-line. The remainder of the logic and calculations are extremely trivial and can be done by simple and cheap digital computers.

## 10.3 Updating the Algorithm

As mentioned before, the number  $N$  of aircraft changes with time. Every time a new aircraft enters the region  $N$  is increased by one; every time an aircraft flies over the outer marker,  $N$  is decreased by one.

Let us suppose that the algorithm has been working for a while and let us suppose that at the present time  $t$  a new aircraft enters or leaves so that  $N$  must be updated. At this instant of time, some of the existing aircraft are on their nominal air routes while others are undergoing path stretching maneuvers. Ideally (and this is a subject for future research) the desired positions of all aircraft should be updated. However, this would require the piling up of extra maneuvers for the aircraft that are currently "holding".

For this reason, whenever a new aircraft enters the region, (indexed by  $N+1$ ) it is reasonable to compute its desired location only with respect to the previously last plane (indexed by  $N$ ). This scheme would then utilize the previously computed desired position  $\hat{z}_N^*(t)$  of the last aircraft to determine the action required on the  $N+1$ -st aircraft.

Similarly, when an aircraft passes the outer marker, it is suggested that only a simple re-indexing takes place rather than a complete reevaluation of all desired positions.

This course of action, although suboptimal, has the advantage that each aircraft would undergo at most a single path stretching maneuver under normal conditions. It is only during an emergency or change in the landing runway that a complete new reevaluation of the system is required.

This then suggests another possibility for future research. Namely, to modify the proposed algorithm so as to handle the  $N$  aircraft in interlaced substring control for ground vehicles discussed in reference [4].

It should be stressed that additional research and extensive simulation studies are necessary to evaluate the potential usefulness of the suggested algorithm. The advantage of the algorithm essentially lies in its simplicity from the viewpoint of on-line computation and its flexibility in being adapted to an increasingly automated ATC system.

## XI. Simulation Results

In this section we present some simulation results for twelve aircraft using the following assumptions.

All aircraft are identical (Boeing 707-320B Pax version) with empty operating weight of 138,323 lbs.

The desired speed was  $\bar{v} = 160$  knots. For this case,  $\beta_1 = 3$  and

$$\beta = 0.05167 \text{ sec}^{-1}$$

All position and velocity weights were selected equal. Thus, in the quadratic cost functional listed

$$q_1 = q = 0.018^4$$

$$p_1 = p = 0.058^2$$

which was found to give satisfactory control.

Also, we set

$$d_1 = D_{\min} = 3 \text{ n.m.}$$

i.e., no takeoffs were allowed between landings.

### 11.1 Normal Landing Case

The initial configuration of the aircraft is shown in Fig. 15, only ten out of the twelve aircraft are shown. At time  $t=0$  the locations and speeds of the aircraft are summarized in Table I.

| Aircraft number            | 1   | 2   | 3    | 4    | 5     | 6   | 7    | 8   | 9     | 10  | 11  | 12  |
|----------------------------|-----|-----|------|------|-------|-----|------|-----|-------|-----|-----|-----|
| Initial Velocities (knots) | 163 | 162 | 163  | 166  | 166   | 169 | 172  | 172 | 163   | 175 | 178 | 190 |
| Initial positions (n.m.)   | 3   | 6.1 | 8.8  | 10.5 | 12    | 20  | 23.1 | 29  | 31    | 36  | 43  | 50  |
| Ideal positions (n.m.)     | 3   | 6.1 | 9.15 | 12.5 | 15.15 | 20  | 23.1 | 29  | 32.05 | 36  | 43  | 50  |

Table I Actual and ideal situation for a normal landing procedure

Figures 16 to 18 show the response of the aircraft every twenty seconds. Both the actual locations of the aircraft (denoted by crosses) and their desired images on the air routes (denoted by circles).

In Fig. 15 we summarize the situation at time  $t=0$ . The numbers indicate the natural landing sequence. The primed numbers denote the desired location of the aircraft along the air route, next to the circles. Note that aircraft 3 and 4 were closer than 3 n.m. so that they have to be separated. This requires aircraft 5 to waste time so that no conflict at merging will occur. The slight delay required by aircraft 9 is to avoid conflict with aircraft 8 at the merging node.

Figure 16 illustrates what happens during the first minute. Once more, we display both the desired motion (circles) and the actual motion of the aircraft (crosses) every twenty seconds. Note that aircraft 3, 4, 5, and 9 have to execute oscillation maneuvers. Only aircraft 3 completes its maneuver at the end of the first minute and it is now correctly separated from aircraft 2.

Figure 17 illustrates the motion during the second minute. Note that aircraft 4 and 9 complete their oscillation maneuvers and they are correctly separated (4 with respect to 3, and 9 with respect to 8). Aircraft 5 still undergoes its oscillation maneuver to avoid conflict with aircraft 4 at the merging node.

Figure 18 illustrates the motion during the third minute. Aircraft 5 has completed its oscillation maneuver, and is correctly separated from aircraft 4. In the absence of any subsequent changes, the aircraft will proceed along their nominal air routes, accomplishing all subsequent merging safely, and will pass over the outer marker correctly separated by 3 n.m.

### 11.2 Emergency Case

In this case we assumed that the aircraft which would normally would have been fifth in natural landing order (see Table I) develops a minor emergency and is reassigned a new landing priority,  $i=3$ . The initial conditions are now summarized in Table II.

| Aircraft number            | 1   | 2   | 3   | 4     | 5     | 6     | 7     | 8   | 9     | 10  | 11  | 12  |
|----------------------------|-----|-----|-----|-------|-------|-------|-------|-----|-------|-----|-----|-----|
| Initial velocities (knots) | 163 | 162 | 166 | 163   | 166   | 169   | 172   | 172 | 163   | 175 | 178 | 190 |
| Initial positions (n.m.)   | 3   | 6.1 | 12  | 8.8   | 10.5  | 20    | 23.1  | 29  | 31    | 36  | 43  | 50  |
| Ideal positions (n.m.)     | 3   | 6.1 | 12  | 15.01 | 18.08 | 21.01 | 24.01 | 29  | 32.05 | 36  | 43  | 50  |

Table II Actual and ideal positions when No. 3 is in emergency

Note that this creates a "squeeze" on aircraft #4 and #5. The responses are summarized in Figures 19 to 22.

The simulation results indicate that the total propagation of the adverse situation created by the emergency is localized easily, since the trajectories of aircraft 8 to 12 are not significantly changed from their normal operation. This, we feel, illustrates the efficiency of this method.

Consultation with some experts in ATC has produced an opinion that if this emergency situation occurred in real life the human controller would tend to completely direct aircraft 4 and 5. This then would have created a reduction in the capacity, because there is quite a bit of space unused between aircraft 3 and 6. This method of solution illustrates that although aircraft 4 and 5 have to undergo flyaround maneuvers, only minor oscillation maneuvers have to be undertaken by aircraft 6 and 7 (as compared to the normal case). Due to the existing capacity aircraft 8 to 12 are not affected. Once more note that in spite of the major perturbation created, all mergings are accomplished safely.

## XII. CONCLUSIONS

A simple computer-aided decision algorithm has been proposed for the ATC problem in the near terminal area. The algorithm appears to be practical from a computational standpoint and adaptable to evolutionary ATC changes. Additional research and simulations is required to prove its potential usefulness, through conduct of pilot acceptance experiments, and incorporating major stochastic effects.

## XIII. REFERENCES

1. Report of Department of Transportation Air Traffic Control Advisory Committee, Volumes 1 and 2, December 1969.
2. Special Issue on Air Traffic Control, Proc. IEEE, Vol. 58, No. 3, March 1970
3. Athans, M., "System Aspects of Air Traffic Control" Proc. 4th Hawaii International Conference on Systems Sciences, Honolulu, Hawaii, January 1971
4. Athans, M., Levine, W.S., and Levis, A.H., "A System for the Optimal and Suboptimal Position and Velocity Control for a String of High-Speed Vehicles" Proc. 5th International Analog Computation Meeting, Lausanne, Switzerland, pp. 746-760, August, 1967
5. Levine, W.S. and Athans, M., "On the Optimal Error Regulation of a String of Moving Vehicles", IEEE Trans. on Automatic Control, Vol. AC-11, No. 3, pp. 355-361, July, 1966
6. Athans, M., "A Unified Approach to the Vehicle Merging Problem" Transportation Research, Vol. 3, pp. 123-133, 1969
7. Athans, M., and Falb, P.L., Optimal Control, McGraw-Hill Book Company, New York, 1966
8. Porter, L.W., "On Optimal Scheduling and Holding Strategies for the Air Traffic Control Problem" S.M. Thesis, Dept. of Electrical Engineering, M.I.T., Cambridge, Mass. September, 1969
9. Imrich, T., "Air Traffic Control Project" (Working Paper) Electronics Systems Lab., M.I.T., Cambridge, Mass. July 2, 1970
10. Sarris, A.H., "On Optimal Scheduling and Air Traffic Control in the Near Terminal Area", September 1971, ESL-R-459, S.M. Thesis, M.I.T.
11. Kleinman, D.L., "An Iterative Technique for Riccati Equation Computations" Tech. Memo. DLK-1, Bolt, Beranek, and Newman, Cambridge, Mass., June 30, 1970
12. A.H. Sarris and M. Athans, "An Algorithm for Terminal Air Traffic Control" Report ESL-P-466, MIT Electronic Systems Laboratory, Cambridge, Mass, December 1971
13. M. Athans, "The Role and Use of the Linear-Quadratic-Gaussian Problem in Control System Design" IEEE Trans. on Auto. Control Vol. AC-16, December 1971, pp. 529-552

14. M. Athans and L.W. Porter, "An Approach to Semiautomated Optimal Scheduling and Holding Strategies for Air Traffic Control" Proc. 1971 Joint Automatic Control Conference, St. Louis, Missouri, pp. 536-545 (also available DEC-AD 721474)

#### XIV. ACKNOWLEDGEMENTS

The contribution of L.W. Porter, A.H. Sarris, and A.A. Lopez-Toledo to this project are greatly acknowledged.

This research was conducted in the Decision and Control Sciences group of the M.I.T. Electronic Systems Laboratory with support provided by the NASA Ames Research Center under grant NGL-22-009-124 and by the Air Force Office of Scientific Research under grants AFCSR 69-1724 (A) and 72-2273.

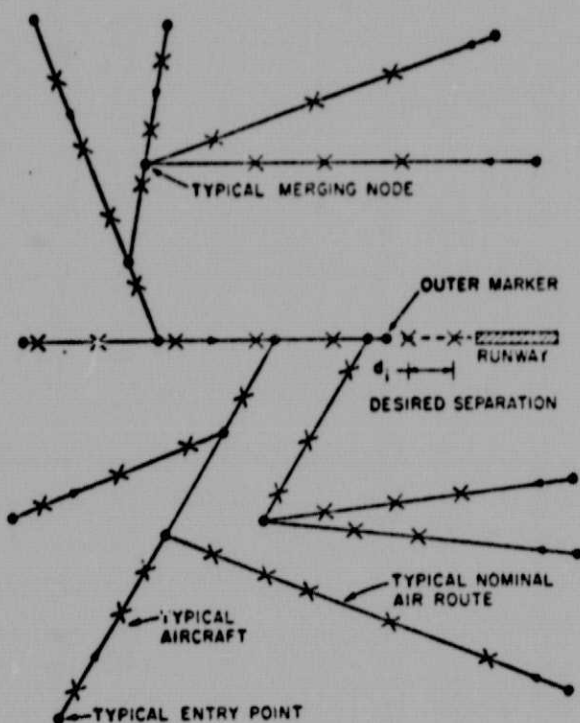


Fig. 1 Air Route Structure in Near Terminal Region

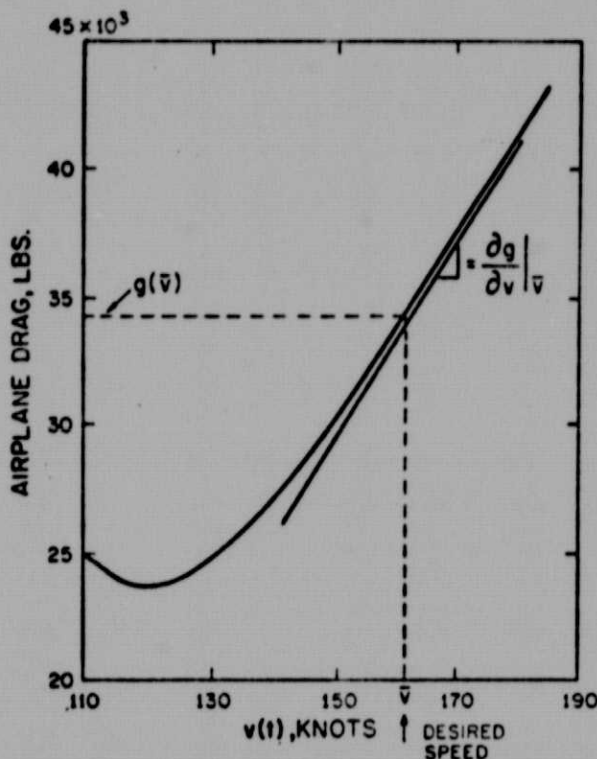
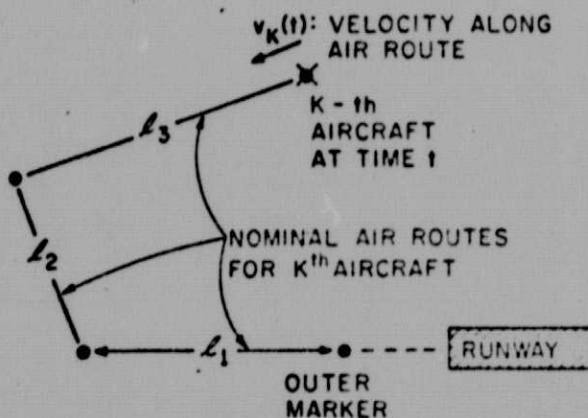


Fig. 3 Drag vs. Velocity for KC-135 (Boeing 707)



$$z_k(t) = l_1 + l_2 + l_3; \quad v_k(t) = -\frac{d}{dt} z_k(t)$$

Fig. 2 Definition of Distance-to-go,  $z_k(t)$  for k-th Aircraft

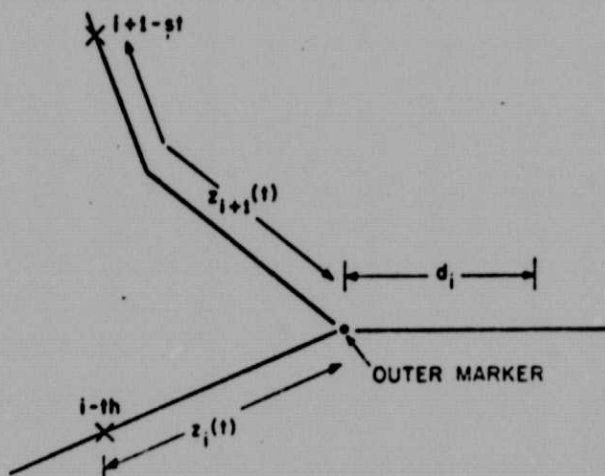


Fig. 4 Definition of Position Error

$$\delta z_i(t) = z_{i+1}(t) - z_i(t) - d_i$$

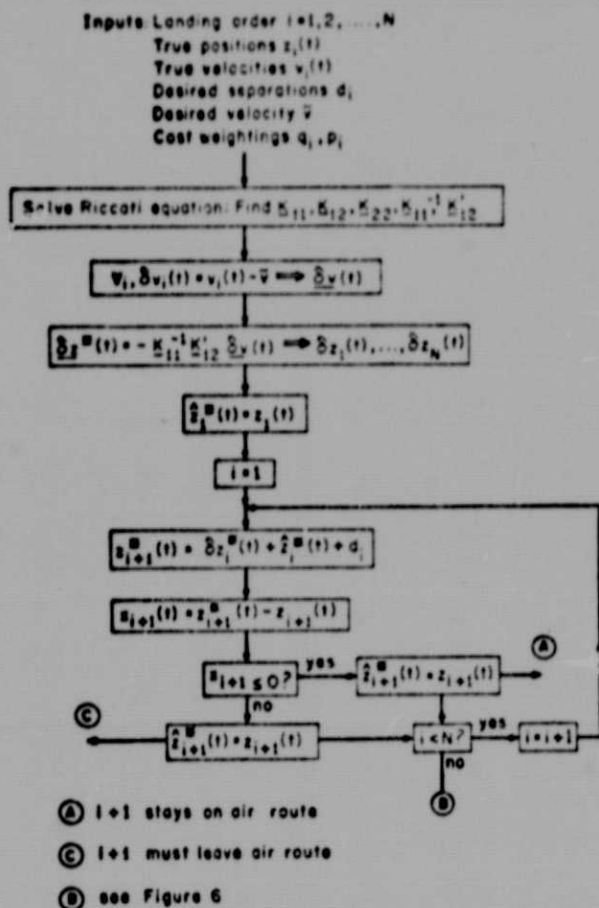


Fig. 5 Calculation of Desired Positions  $\hat{z}_i^*(t)$

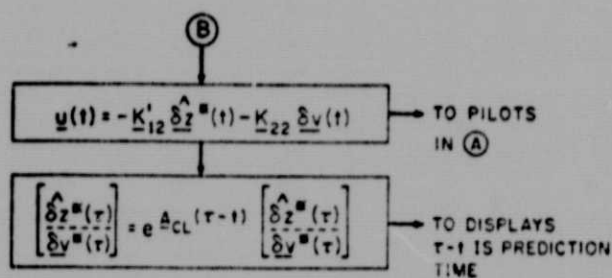


Fig. 6 Calculation of accelerations  $u(t)$ ; transmitted to pilots that stay on nominal air routes.

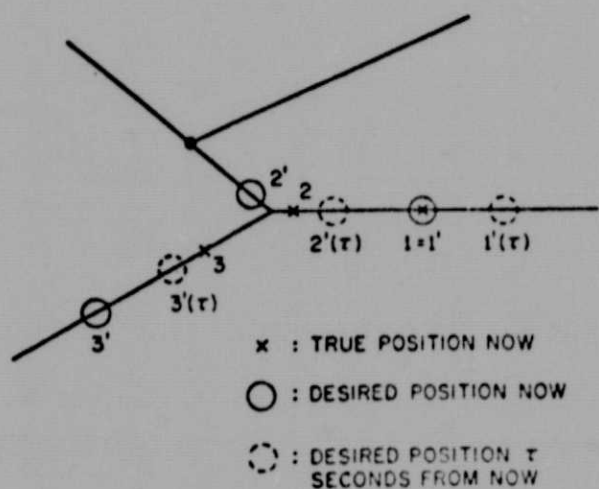
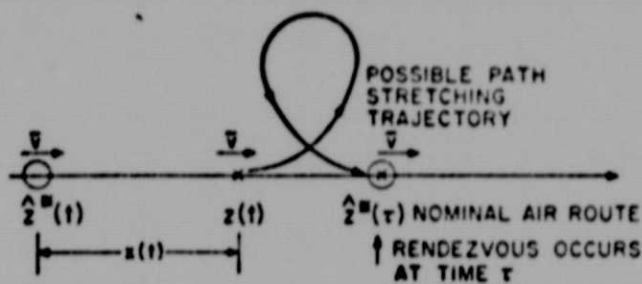


Fig. 7 Possible Display Information



t: PRESENT TIME

$\tau$ : FUTURE TIME

$z(t)$ : PRESENT ACTUAL POSITION  
 $\hat{z}^*(t)$  PRESENT IDEAL POSITION } ALONG AIR ROUTE  
 $\hat{z}^*(\tau)$  FUTURE IDEAL POSITION

Fig. 8 Illustration of Path Stretching Rendezvous Maneuver

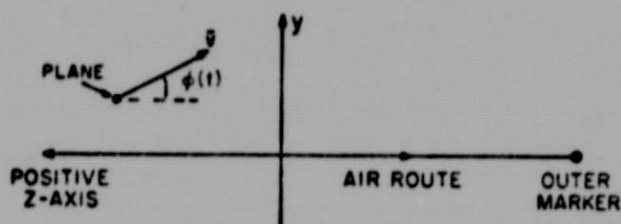


Fig. 9 Coordinate System for Path Stretching Maneuvers

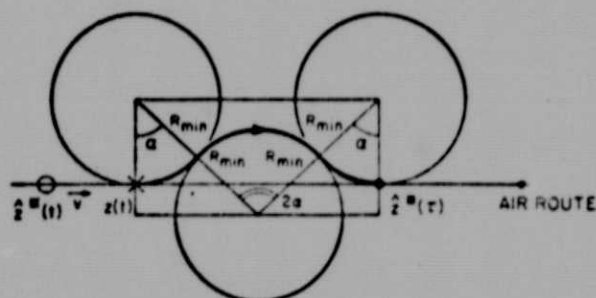


Fig. 10 Oscillation Maneuver

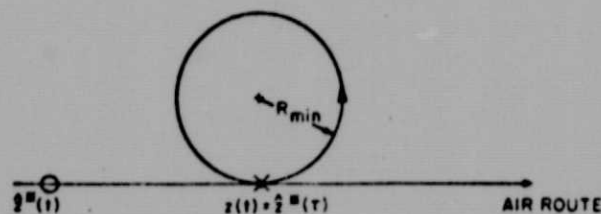


Fig. 11 Circular Maneuver

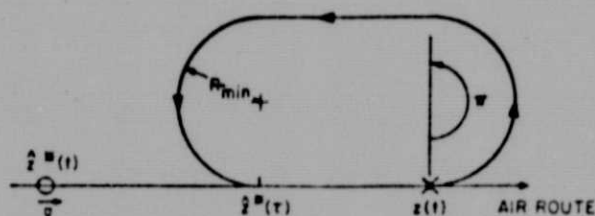


Fig. 12 Fly-around Maneuver

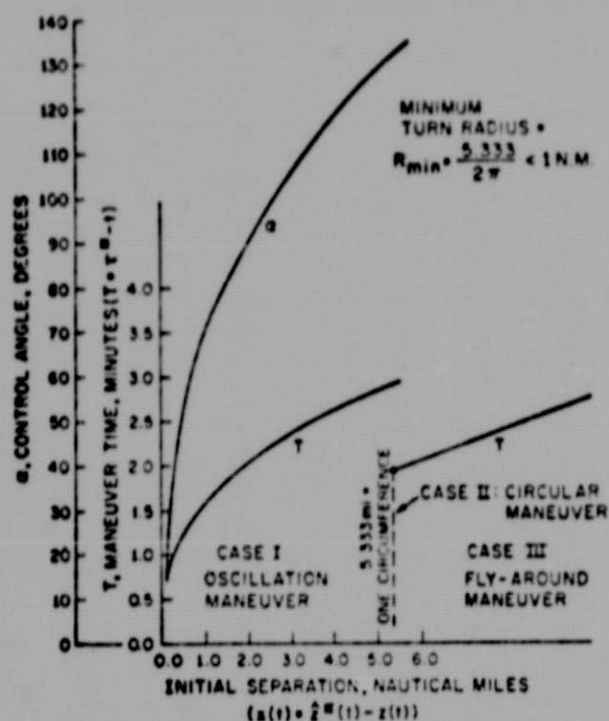


Fig. 13 Maneuver time and control angle vs. initial separation at two minute turn rate and 160 knots for both target and aircraft

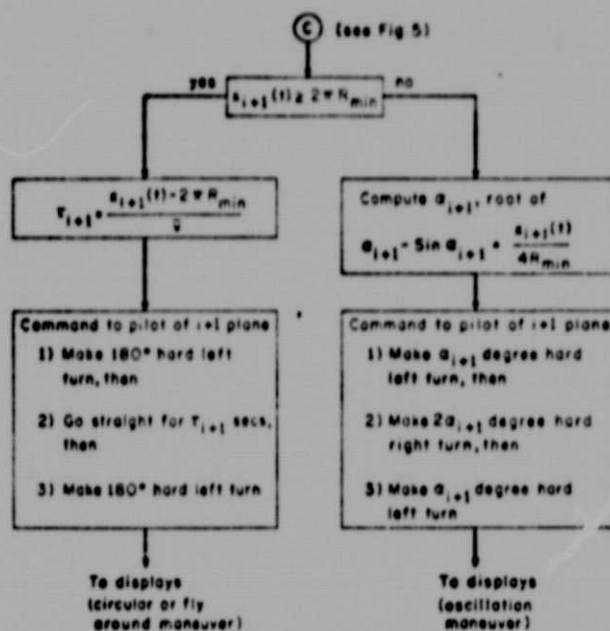


Fig. 14 Computation of Path Stretching Commands and Maneuvers

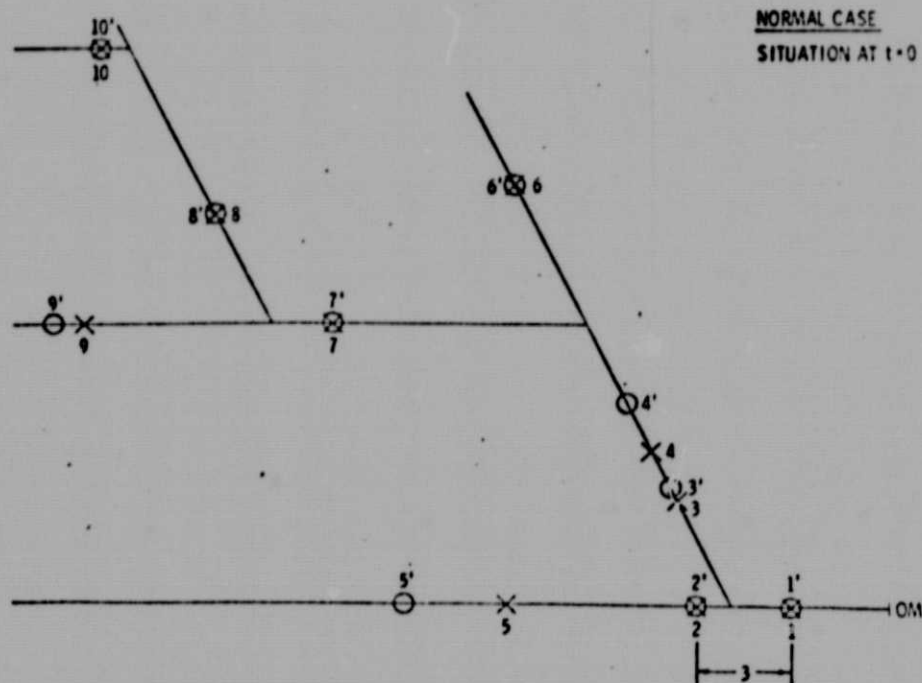


Figure 15

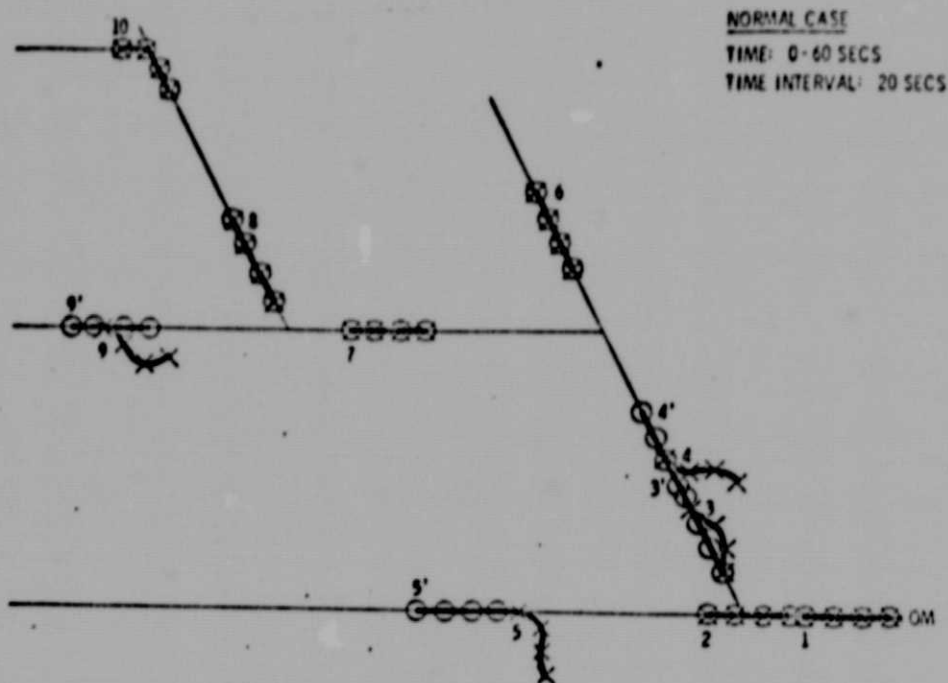


Figure 16

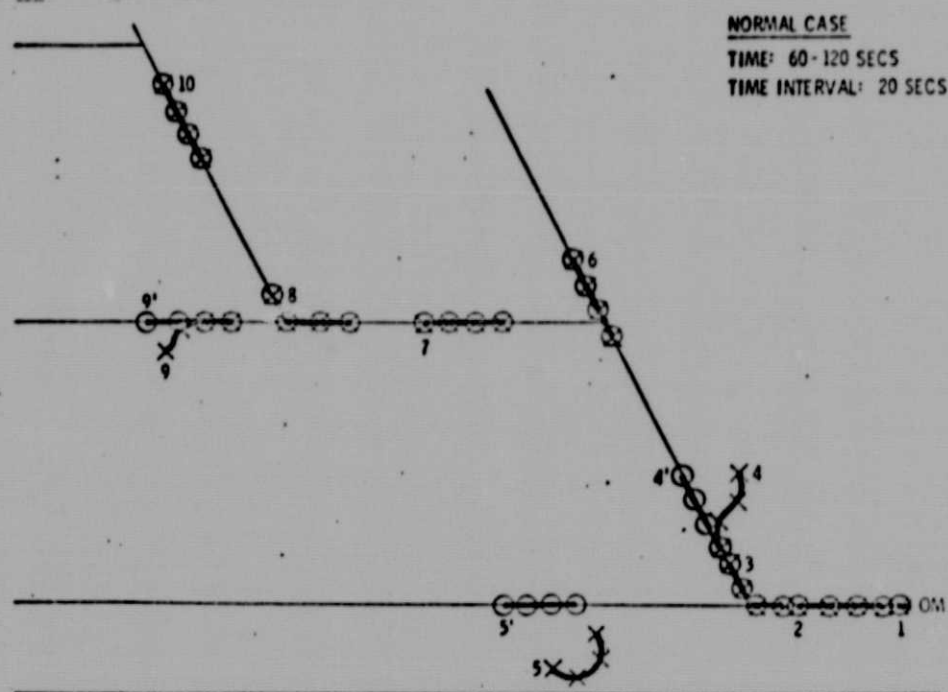


Figure 17

NORMAL CASE  
 TIME: 120-180 SECS  
 TIME INTERVAL: 20 SECS

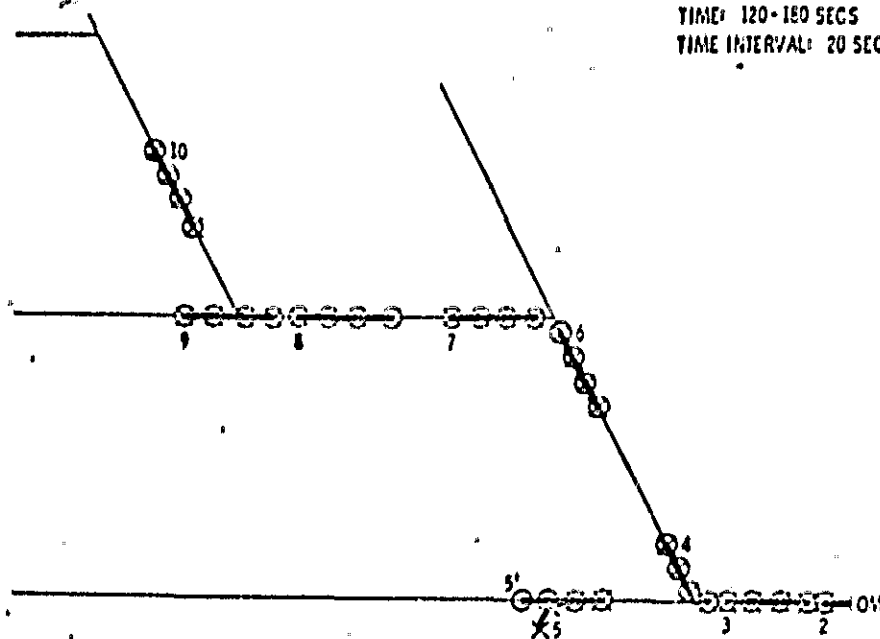
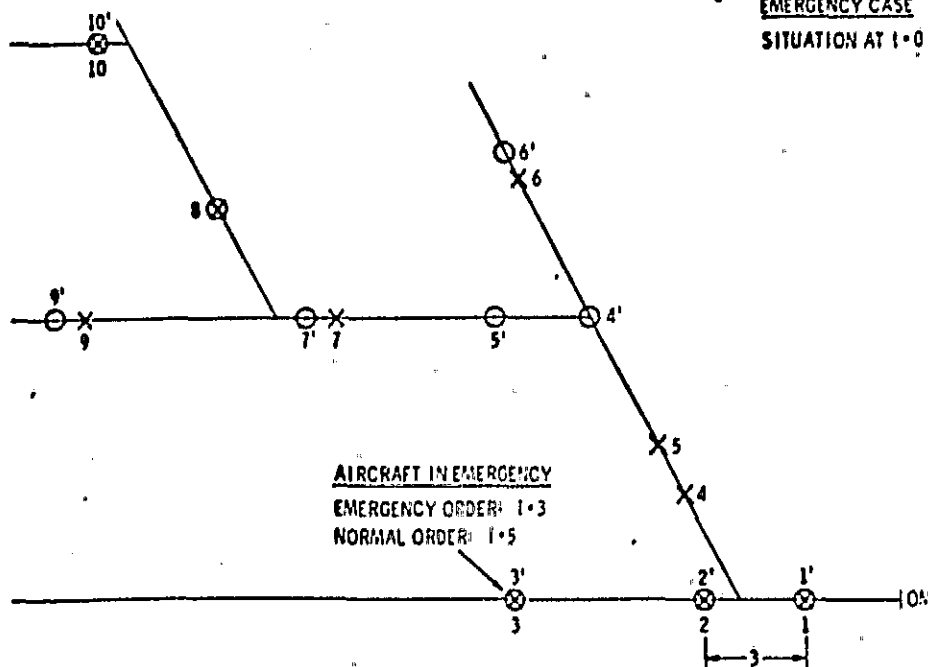


Figure 18

EMERGENCY CASE  
 SITUATION AT 1-0



AIRCRAFT IN EMERGENCY  
 EMERGENCY ORDER: 1-3  
 NORMAL ORDER: 1-5

Figure 19

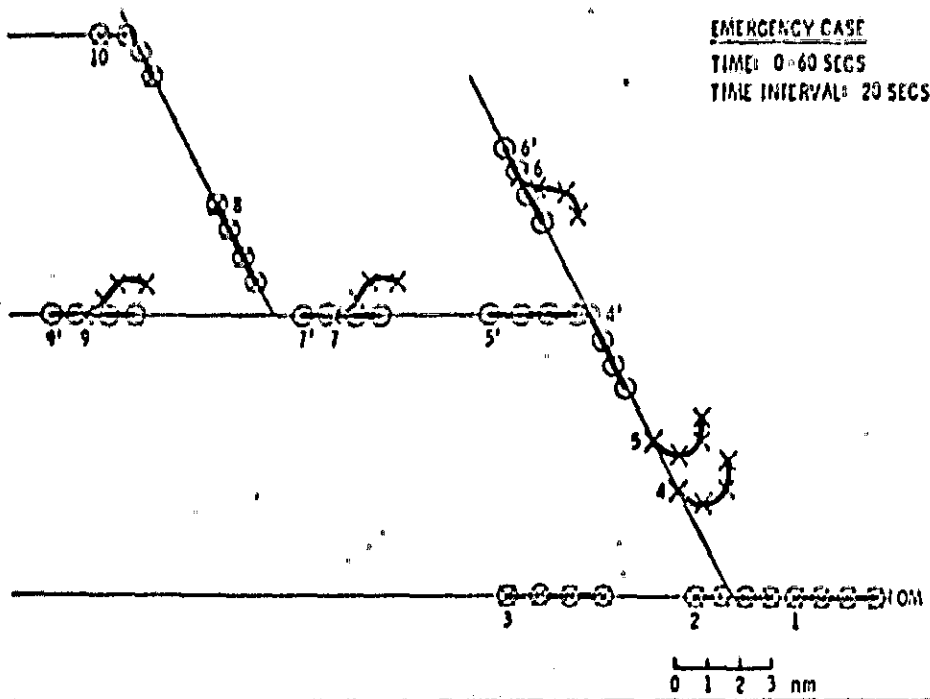


Figure 20

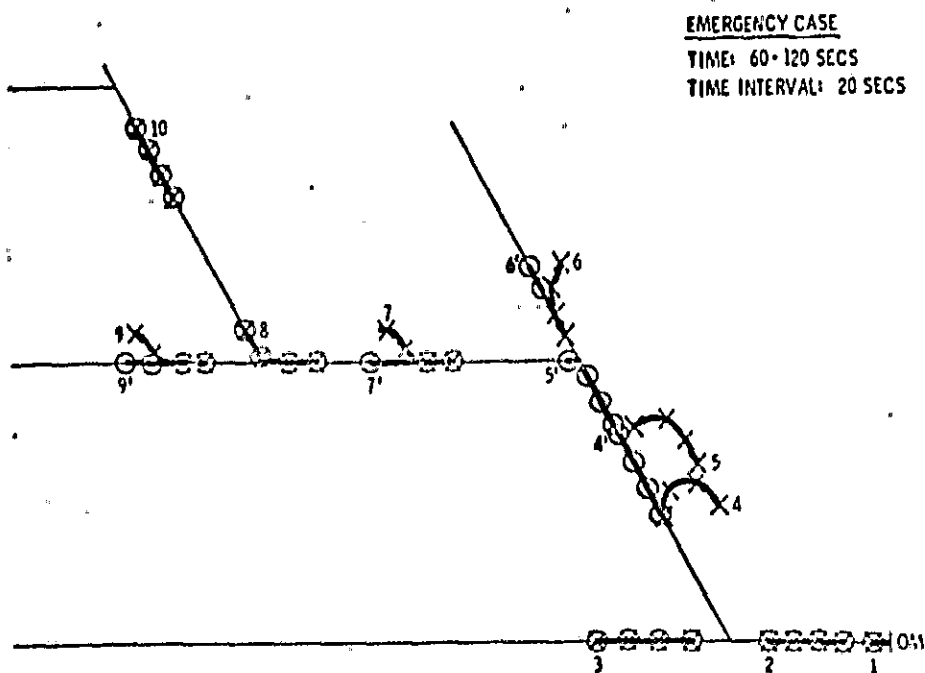


Figure 21

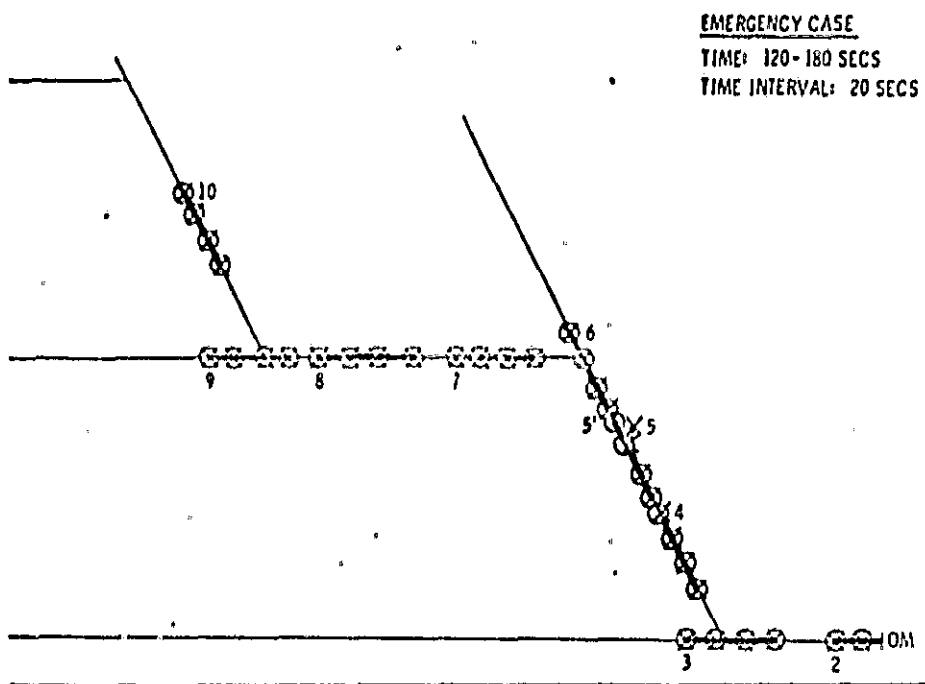


Figure 22